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A Column Generation Approach for Operational Flight Scheduling and Aircraft Maintenance Routing

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Abstract: Aircraft is one of the most expensive resources owned by an airline which should be properly planned. The aircraft maintenance routing problem (AMRP) generates aircraft routes to serve scheduled flights, while satisfying the strict maintenance requirements. However, in operations, the pre-determined aircraft routes are usually disrupted due to unplanned maintenance requirements or insufficient remaining legal flying time to maintenance stations. Thus, airlines often have to re-route aircraft in real time. This study proposes a new aircraft re-routing approach to fulfil the maintenance requirements arising in the operational stage. Specifically, maintenance stations are capacity-constrained, while airlines could allocate maintenance resources (like staff and equipment) to other airports with additional costs. Besides, flights could be re-scheduled (i.e., cancelled with a high penalty), while the model endeavors to minimize the impact of recovery actions on the original plan. To achieve this, specialized flight networks are constructed, and a column generation-based algorithm is developed to obtain high-quality solutions within short computational times. Computational

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experiments show that the solutions obtained by the proposed algorithm are optimal or near-optimal with an optimality gap of 0.3% on average. In addition, some managerial insights on allocating maintenance resources to other airports to fulfil aircraft maintenance demands in operations are discussed.

Keywords: airline recovery; flight scheduling; aircraft maintenance routing; column generation.

1. Introduction

The airline scheduling problem is often decomposed into four sequential problems (Jamili, 2017; Munari & Alvarez, 2019; Wen et al., 2020; Wen et al., 2021): flight scheduling, fleet assignment, aircraft maintenance routing, and crew scheduling (Sun et al., 2020). Aircraft is the core resource of an airline which should be properly planned to ensure that the strict maintenance requirements would not be violated when serving flights. The aircraft maintenance checks vary across countries and airlines (Jamali et al., 2021; Shaukat et al., 2020). For instance, Liang et al. (2011) state that many U.S. airlines require a daily check for every 24-60 flying hours, including a walkaround inspection and checks on lights & emergency equipment, etc., while Federal Aviation Administration (FAA) imposes an A check for every 60 flying hours (Barnhart, Boland, et al., 1998). Typically, from a tactical planning perspective, the AMRP prescribes that aircraft can follow a cyclic path and visit a *maintenance station* (i.e., an airport with qualified manpower and sufficient equipment to conduct the corresponding maintenance check) on a regular basis to fulfil the maintenance requirement.

However, the aviation industry is full of uncertainties. Many unplanned events force airlines to modify their pre-determined tactical aircraft routes in real operations (Kammoun & Rezg, 2018; Yetimoğlu & Aktürk, 2021). For instance, bad weather or accidents during previous flights, unplanned mechanical failures, or emergency maintenance checks required by airlines, aircraft manufacturers, and authorities all impose unexpected maintenance requirements which are not planned in the tactical planning stage. Besides, disruptions like flight delays and traffic congestions lengthen flight flying times (Chung et al., 2017; Khan et al., 2019a, 2019b; Khan et al., 2021). Thus, the pre-determined aircraft routes may become infeasible as there is no proper maintenance opportunity for the unexpected requirements or an aircraft could not arrive at the scheduled maintenance station in time. Thus, airlines often have to re-route aircraft in daily operations to fulfill maintenance demands while covering the scheduled flight as much as possible. To be specific, airlines usually apply automatic recording systems to keep track of the real-time status of each individual aircraft. When the remaining legal flying time reaches a certain threshold, the aircraft will be labeled as a "*high-time*" aircraft. Then, in the recovery procedure, the high-time aircraft will be re-routed to proper maintenance stations to carry out maintenance checks (Sarac et al., 2006). However, maintenance stations are often capacity-constrained in various aspects, like available man-hour, parking slots, available equipment. Eltoukhy et al. (2018) point out that the maintenance resource availability constraint is a major factor which affects the feasibility of an aircraft routing plan. Conflicts between the workload assigned to a maintenance station with its capacity are commonly seen in the industry, which is even more severe in the operational stage as many unforeseen maintenance requirements arise day by day. To ensure the travel safety and avoid the high penalties of violating maintenance requirements, many airlines would allocate maintenance resources (like staff and equipment) to other airports with high costs (named as the *SMR strategy* in this study). That is, if a high-time aircraft is expected to finish its duty at a maintenance-infeasible airport (either an unqualified airport or a qualified airport with insufficient maintenance resources), the airline will send a team of qualified personnel with sufficient equipment there[2]. The maintenance team can be sent (allocated) to the prescribed airport by any means, like moving along with the flights operated by that aircraft. However, adopting the SMR strategy is expensive. First, some seats on the aircraft that could generate profits by transporting passengers are occupied by maintenance staff if they move along with the aircraft. Second, travel and accommodation allowances should be paid to the maintenance staff. Third, the maintenance personnel schedules of the airline are disrupted. Thus, airlines have to balance between the additional expenditures generated by adopting the SMR strategy and the costs of cancelling flights. Moreover, when conducting recovery, airlines usually endeavor to keep the original flight schedule and aircraft routing unchanged. That is, a re-scheduling/routing plan is preferred if it induces the minimum impact on the original plan. This leaves great challenges for airlines to adjust the tactical plan to meet the urgent needs arising in operations.

Although the AMRP has been extensively studied, relatively less attention has been paid to the operational aircraft re-routing problem. This study aims to propose a new aircraft re-routing approach to fulfil the maintenance requirements arising in the operational stage by both capacity-constrained maintenance stations and the SMR strategy. Flights could be cancelled with a high penalty, while the impact on the original plan is minimized. To achieve this,

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[2] Note that airlines could also seek for third-party maintenance service providers' assistance to conduct maintenance checks at other airports with high costs.

specialized flight networks are constructed for high-time aircraft to guarantee maintenance opportunities, and a column generation-based algorithm is developed to obtain high-quality solutions within short computational times. Note that some existing AMRP studies have considered the usage of deadhead flights to reposition an aircraft to a different station where it is needed to cover a flight leg or where it can take maintenance checks, which is totally different from the SMR strategy proposed in this study. The flight network construction logic (e.g., flight connection arcs, maintenance arcs) is also different for these two strategies.

As we investigate an operational AMRP in this study, the planning horizon is set as one day. That is, given a list of high-time aircraft, a list of non-high-time aircraft, and initial airports, the goal of the recovery problem is to re-route the aircraft to cover the scheduled flights (note that flights are allowed to be cancelled with high penalties), while ensuring that all the high-time aircraft can conduct maintenance checks at the end of the day. Thus, both flights and aircraft need to be re-planned. The maintenance check could be conducted by either a resource-feasible maintenance station or the maintenance resources sent to the airport by the airline. As pointed out by Sarac et al. (2006), the time horizon of the aircraft recovery problem could be even shorter, like several hours, as disruptions usually occur during the middle of a day, while a longer horizon, like a week, is more like a tactical planning problem that is too optimistic for the disruptions frequently occurring in daily operations.

This paper is organized as follows. First of all, the previous related literature is reviewed from three perspectives in Section 2. Then, Section 3 builds the mathematical model for the operational flight scheduling and aircraft routing problem. Then, a column generation-based solution algorithm is developed in Section 4, while computational experiments are demonstrated in Section 5. Finally, Section 6 draws conclusions for this work.

2. Literature review

In this section, we review the related literature from three aspects. First, as our study relates to AMRP, we first review the aircraft maintenance routing studies and discuss the major factors that affect the AMRP formulations. Besides, we survey the operational aircraft maintenance recovery studies because we also explore a re-routing problem. Last, as our study allows the cancellation of scheduled flights, which relates to the integrated flight scheduling and aircraft routing problems, we review the recent integrated airline scheduling studies.

2.1 Aircraft maintenance routing

As aircraft is one of the most expensive resources owned by airlines, the academia has conducted abundant research on the AMRP with the fast development of operations research (OR) in the past several decades. There are several major factors that influence the problem definition and model formulation adopted in the existing AMRP studies.

First, aircraft maintenance checks vary across airlines with different time lengths and frequency requirements (e.g., the time interval between two checks, which could be measured by D days, maximum flying hours, maximum number of take-offs, etc.). These different maintenance considerations naturally impose significant impacts on the AMRP studies. For example, in Haouari et al. (2011), both routine checks (every 3-4 calendar days) and short-term A checks are considered. However, the modelling approach for these two checks are totally different. For routine checks, the authors state that routine checks are only conducted during night, while the flight schedule of the considered airline can ensure that every aircraft would stay overnight at a routine-check-feasible airport. Thus, it is no need to model routine checks explicitly. However, short-term A checks shall be carried out every 60 hours, and not all airports are A-check-feasible station. Therefore, special resource vectors should be created for labels during the path extension procedure, in order to satisfy the A check requirements (Haouari et al., 2011).

The time horizon of the scheduling problem is another major factor. Generally, the existing literature considers daily-repeated flight schedules, weekly-repeated flight schedules, one-day horizon, four-day horizon, or one-week horizon. The time horizon greatly affects how the AMRP can be depicted. For example, if a daily-repeated flight schedule is considered, the AMRP is usually formulated as a network flow problem based on the time-space network developed by Hane et al. (1995), in which the flow balance constraints ensures that aircraft can circulate in the network. Usually, the maintenance check requirement is fulfilled by assuming that maintenance is conducted during night (i.e., by traversing an overnight/wraparound arc). Liang et al. (2011) propose that if an aircraft needs to take a maintenance check every D days, then the daily-repeated flight schedule could be copied for D days, while overnight maintenance arcs are created to link the end of each day with the starting of the first day. Based on the daily AMRP network, Liang and Chaovalitwongse (2013) revise the network structure so that weekly schedules could be considered. However, the maintenance checks that can be modeled by the time-space based networks are overnight checks, while only the number of calendar days between two maintenance checks could be considered. Thus, the string-based approach developed by Barnhart, Boland, et al. (1998) is recommended if airlines allow daytime maintenance or other maintenance interval requirements are imposed (like the accumulated flying hours and number of take-offs). In the string-based approach, a string is a sequence of flights that starts from and ends at a maintenance station, while an augmented string is a string with the maintenance check time attached to the end of the last flight in the string. Flight events are modelled as nodes at each maintenance station, while ground arcs are created to link the flight event nodes. Besides, by counting the number of aircraft in the air and on the ground at a time point, the fleet size restriction could be satisfied (Barnhart, Boland, et al., 1998). The flight schedule considered in Barnhart, Boland, et al. (1998) could be T-day repeated where T could be any positive integer. By imposing flow balance constraints at each flight event node, the model ensures that aircraft could circulate in the network while satisfying various maintenance requirements. Focusing on an one-day horizon, Desaulniers et al. (1997) consider that an aircraft should start from and end at the same airport based on a connection network, while flights could depart within pre-determined time windows. A set-partitioning type model and a time-constrained multi-commodity network flow model are proposed, which are solved by column generation and Dantzig-Wolfe decomposition, respectively (Desaulniers et al., 1997). Considering a one-week period, Haouari et al. (2011) formulate an assignment-based model, as well as set-partitioning-based model, which are solved by Benders decomposition and Branch-and-price, respectively. In the problem studied by Haouari et al. (2011), only the long-term maintenance checks are explicitly considered. That is, the model will identify aircraft routes that cover the pre-scheduled long-term maintenance checks for each aircraft within the considered one-week period. Moreover, deadhead flights are utilized in Haouari et al. (2011), in which two flight nodes could be connected even if the arrival airport of the first flight is not the departure airport of the second flight, to serve the maintenance check purposes and flight coverage purposes, or to satisfy the aircraft number requirements to be used for the next day at each airport. On the other hand, some studies consider the four-day horizon as it is a commonly required that a maintenance check shall be conducted every four calendar days (Eltoukhy et al., 2018; Talluri, 1998).

2.2 Operational aircraft maintenance recovery and robust routing

In the tactical planning stage, aircraft are arranged to serve specific flight while satisfying the various maintenance requirements, usually with the aim of minimizing operations costs or maximizing profits (like through revenues). However, in real operations, disruptions are common which make the planned aircraft itinerary infeasible. Thus, aircraft recovery is necessary to maintain daily operations.

Haouari et al. (2011) state that the AMRP in the considered airline usually plans for long-term maintenance checks, while during daily operations, short-term maintenance (also named as A check) should be considered which requires the revision of the long-term AMRP solutions. Thus, they explore an operational aircraft maintenance re-routing problem for one day. To be specific, Haouari et al. (2011) consider that A checks should be conducted every 60 flying hours, while airlines generally keep track of the accumulated flying hours for each individual aircraft, and re-route the aircraft that are labeled with low legal remaining flying hours to ensure that they could land at a maintenance-feasible airport at the end of the day. The number of aircraft that can be used for the next day's tasks is also formulated as a constraint in Haouari et al. (2011). Similarly, Sarac et al. (2006) study the aircraft re-routing problem encountered in daily operations, in which both available man-hours and number of available slots for different maintenance types at each overnight station are considered. Besides, those “non-high-time” aircraft whose remaining legal flying hours are sufficient (e.g., two days or more) are not restricted to be used in the given day in their re-routing model. A set-partitioning based formulation is developed, while a problem structured based branching scheme is explored to relieve the computational burden of the pricing problem in Sarac et al. (2006).

Liang et al. (2018) point out that the recovery time horizon is usually one to four days. Using a connection network, Liang et al. (2018) aim to utilize strategic flight delays, flight cancellations, flight swaps, and planned maintenance swaps to deal with the disruption sources like airport flow control, airport-on-grounds, and airport/time mismatch. A column generation approach is developed to solve the linear-relaxation of the aircraft recovery model developed in Liang et al. (2018), while the integer programming technique is used to obtain integer solutions based on the solutions of the last restricted master problem. Moreover, in order to consider the airport capacity and maintenance swaps, special shortest path algorithms are developed in Liang et al. (2018).

In fact, aircraft recovery is critical when disruption happens in real operations as sequential recovery actions such as crew and passenger recovery are badly dependent on it. Some aircraft recovery actions such as reassigning fleet type for disrupted aircraft will complicate crew and passenger recovery. Rosenberger et al. (2003) proposed a revised aircraft routing model which aims to minimize the re-routing and cancellation cost during the aircraft recovery stage without resigning fleet types to help avoid disruptions to crew pairing and passenger itineraries in real operations. A heuristic called aircraft selection heuristic is developed to determine the subset of aircraft for optimization before new routes are constructed for the aircraft, by which large recovery instances can be solved quickly.

In addition to recovery problem, some studies also try to insert robustness into the tactical plans of aircraft. For example, Liang et al. (2015) propose a robust aircraft routing model for weekly-repeated flight schedules where maintenance checks shall be conducted for every three days. Robustness is achieved by minimizing the propagated delay of flights within each day based on data analytics of historical data, while a new method to compute propagated delay is developed. The maintenance requirements are satisfied by the network construction strategies that ensure an aircraft will traverse an overnight maintenance arc no more than three days. A column generation approach is developed to generate promising line-of-flights for each day in Liang et al. (2015). Yan and Kung (2016) propose a robust optimization model for the aircraft routing problem, in which the objective is to minimize the possible maximum total propagated delay. To incorporate the correlated and non-linear propagated delay into the model and make the solution approach tractable, a column and row generation framework is developed based on the decomposition methods. Each time a new robustness constraint (with the consideration of the optimal primary delay) is added to the relaxed master problem, while the column generation procedure is applied to obtain new feasible routes until no more violated constraint is found.

2.3 Integrated airline scheduling problems

Although the sequential scheduling approach can significantly reduce the problem scale and solution complexity for airlines, the high correlations among the sub-scheduling problems may lead to poor solution quality (or even infeasibility). Thus, in recent years, increasing research is devoted to exploring integrated airline scheduling problems. Specifically, AMRP is usually integrated with the fleet assignment problem and the crew scheduling problem. For instance, focusing on a regional airline with three operators operating at CanaryIslands, Salazar-González (2014) propose an integrated decision framework for the fleet assignment, aircraft routing, and crew pairing problem. Note that the fleet assignment problem considered in Salazar-González (2014) is to assign flights to a specific operator, instead of assigning fleet type to flights. The flight schedule considered in Salazar-González (2014) is a daily schedule, while the author solve the integrated routing problem day by day, and impose that the number of aircraft and crew members available could be obtained according to the previous day's solution. A mixed integer linear programming model is developed for the integrated problem, while a heuristic is developed for solutions in Salazar-González (2014). Based on the routing solutions, Salazar-González (2014) further studies the crew rostering problem. As for the merit of integrating the aircraft routing and crew pairing problem, Salazar-González (2014) point out that the aircraft/crew changes along an itinerary could be minimized. Besides, Mercier et al. (2005) build aircraft routes and crew pairings simultaneously for daily flight schedules. The authors build time-space networks for both aircraft and crew, while flights are copied by D times where D equals the maximum number of calendar days between two maintenance checks for aircraft, and the maximum number of calendar days that a crew pairing could last. For the AMRP, a source node and a sink node are used to facilitate the generation of aircraft paths. As pointed out by Mercier et al. (2005), for a daily AMRP, the number of calendar days involved in an aircraft route is equal to the number of aircraft required by this route. Moreover, the aircraft-crew integrated decision framework developed by Mercier et al. (2005) enables crew members to serve two consecutive flights with a connection time that is shorter than the minimum legal transit time if the two flights are operated by the same aircraft (named as short connections). On the other hand, if the flight connection time is larger than the minimum legal transit time but shorter than the ideal transit time for crew members, penalties will not be imposed only when the two flights are served by the same aircraft (Mercier et al., 2005). A similar study could be found in Cordeau et al. (2001), in which the Benders decomposition method is used to solve the integrated problem where the AMRP is the master problem while the crew pairing problem is the sub-problem. Kenan et al. (2018) study a flight scheduling (flights are allowed to be rejected), fleet assignment (consider the revenue produced by serving a flight by an aircraft type), and aircraft routing (require that each aircraft begins and ends at the same airport, which is a maintenance station) integrated problem. A one day connection network is built for this integrated problem, while flight propagated delay is minimized through a column generation approach (Kenan et al., 2018). It is worth mentioning that each aircraft type is given an initial airport, while the number of aircraft for each type is limited, to facilitate the modelling of maintenance checks implicitly in Kenan et al. (2018).

The traditional integrated scheduling regarding aircraft routing is based on generic aircraft, i.e., same type of fleet, not yet specific to the tail number of each aircraft. However, such kind of integrated scheduling is usually based on a short planning horizon where only the most frequent maintenance requirements are involved. For those less frequent maintenance, tail-dependent scheduling is supposed to be considered (Ruther et al. 2017). On the other hand, for those integrated scheduling of aircraft and crew, adjustments of the crew connections to the aircraft routes in the tail assignment process may happen due to less consideration of the less frequent maintenance. Differently, Ruther et al. (2017) propose an integrated aircraft routing, crew pairing, and tail assignment problem at the operational level, in which each aircraft and group of crews are separately modeled with a specified pricing problem (PP). To tackle with the unpredictable disruption from the aircraft, Ahmed et al. (2018) explore an integrated robust operational solution for the airline by a novel objective function, which aims to minimize the total penalties of short connections of crews and aircraft. A polynomial-size mixed-integer nonlinear programming model is built to solve the proposed integrated aircraft routing and crew paring problem. A reformulation-linearization technique is applied so that the model can be resolved by the commercial solver directly. Simulation-based evaluation is carried out to show the superiority of the robust model compared to the non-robust one.

Based on the previous study of Salazar-González 2014, Cacchiani and Salazar-González

(2017) further explore the exact methods to solve an integrated scheduling of fleet assignment, aircraft routing and crew pairing per day for a regional carrier, in which the objective is to minimize the aircraft and crew operations cost while maximizing the robustness of the integrated schedules. A novel arc-path model is built on both arc-based variables (aircraft routes) and path-based variables (crew pairing). For the arc-path model, column generation is used to obtain the lower bound while some heuristic is applied to obtain the upper bound. The authors further solve a reduced MILP model in which the dynamic programming procedure is used to select out those path-based variables, whose reduced cost is less than the gap of upper and lower bounds, to solve the arc-path model to optimality.

2.4 Research gaps and contribution

Based on the existing literature, we found that AMRP has been extensively studied from different perspectives like the requirements of distinct aircraft maintenance checks, recovery actions for daily disruptions, and considerations of integrated mechanism among different decision stages. However, existent literature usually treats the maintenance requirements as the passive restrictions in the operational stage. Differently, this study innovatively proposes the SMR strategy under the real situation of capacity-constrained maintenance stations, which aims to explore a novel aircraft re-routing approach to fulfill the maintenance requirements in the operational stage via a flexible and movable maintenance mechanism. Besides, we propose a column generation based algorithm to solve the new problem with good solution quality. This study may further bring about useful managerial insights on the cooperation of different parties like airlines and maintenance service providers for maintenance resource sharing at the operational level.

3. The model

Consider a set of flights ($f \in F$) to be scheduled for an operation day. The set of aircraft is denoted by $k \in K$, while all the possible routes for aircraft k is R_k, indexed by r. An aircraft is in the high-time list if its remaining legal flying time is below a threshold. The maintenance checks to be conducted could be either pre-planned ones or unexpected requirements that arise in the operational stage. The set of high-time aircraft is denoted by K_h, while the set of non-high-time aircraft is denoted by K_n. For a high-time aircraft $k \in K_h$, c_r^k is the cost of assigning route r to aircraft k, which is divided into three parts: the operating costs of aircraft k flying the flights contained in the route ($\sum_{f \in F_r} c_f^k$), aircraft change penalties ($\sum_{f \in F_r} o_f^k w_f$), and maintenance costs (m_r^k). That is, $c_r^k = \sum_{f \in F_r} c_f^k + \sum_{f \in F_r} o_f^k w_f + m_r^k$ $(\forall k \in K_h)$. Note that F_r is the set of flights contained in route r. It is noted that to reduce the impact of recovery operations on the original plan, we assign a penalty cost w_f to flight f if it is covered by an aircraft which is different from the originally assigned one. Accordingly, o_f^k is used to represent whether the flight is scheduled to be operated by the aircraft assigned in the original plan. If $o_f^k = 1$, it means the aircraft assigned to flight f is changed. If $o_f^k = 0$, it means the aircraft assigned to flight f remains unchanged. Besides, for maintenance costs, we consider that when aircraft k is scheduled to conduct the maintenance at a feasible maintenance station ($n \in N$), the maintenance costs is the basic cost $m_r^k = b_k$. Each feasible maintenance station is restricted by the available man-hour P_n, while aircraft k requires h_n^k man-hour to complete the maintenance check at station n. On the other hand, if the aircraft carries out the maintenance check at a maintenance-infeasible airport by the maintenance resources (personnel and equipment) sent by the airline (i.e., the SMR strategy), additional cost (d_k) should be attached to this route, i.e., $m_r^k = b_k + d_k$. Note that the "maintenance-infeasible airport" could be either an airport which is not a maintenance station or a maintenance airport without enough resources/capacities. For a non-high-time aircraft ($k \in K_n$), its route cost is the aircraft change penalties plus the flight operating costs. Following a large body of the AMRP literature, we consider that the maintenance checks are carried out at the end of the day (Haouari et al., 2011; Kenan et al., 2018; Liang & Chaovalitwongse, 2013). Besides, q_f represents the penalty of cancelling flight f, which can represent the compensation for passengers, the damage to airline reputation, etc. Similarly, we apply q_k to aircraft k if it is not used in the solution. For high-time aircraft, on one hand, a large idling cost for a high-time aircraft can help promote this aircraft to move for a maintenance check. On the other hand, this cost can represent the maintenance check cost if this aircraft stays at the initial airport without any duty (implying that the airline has to allocate maintenance resources to this initial airport to conduct maintenance checks for this high-time aircraft). For non-high-time aircraft, imposing a penalty cost for idling can help improve the resource utilization of the airline. Actually, this penalty can also be set as zero according to airlines' preference. Moreover, binary coefficient a_{rf}^{k} stands for whether route r of aircraft k covers flight f, while e_{rn}^{k} represents whether route r of aircraft k ends at maintenance station n and conducts maintenance check at that station. Note that a non-high-time aircraft may end at a maintenance station without any maintenance activities. In such situations, the non-high-time aircraft would not occupy any resources of the maintenance station. Binary decision variables x_r^k stand for whether route r of aircraft k is selected or not, while y_f represent whether flight f will be cancelled by the algorithm or not. The notation used in this study is summarized in Appendix.

With the notation introduced above, the operational flight scheduling and aircraft routing problem can be formulated as in Eq. (3-1) to Eq. (3-5). First, the objective function Eq. (3-1) minimizes the overall costs of the routes selected for high-time aircraft (i.e., $\sum_{k\in K_h}\sum_{r\in R_k}(\sum_{f\in F_r} c_f^k + \sum_{f\in F_r} o_f^k w_f + m_r^k)x_r^k$), the overall costs of the routes selected for non-high-time aircraft (i.e., $\sum_{k\in K_n}\sum_{r\in R_k}(\sum_{f\in F_r} c_f^k + \sum_{f\in F_r} o_f^k w_f)x_r^k$), the total flight cancellation penalties (i.e., $\sum_{f\in F} q_f y_f$), and the total penalties for unused aircraft (i.e., $\sum_{k\in K} q_k y_k$). Second, Eqs. (3-2) are the flight coverage constraint, requiring that each flight ($f \in F$) is either serviced by an aircraft (i.e., $\sum_{k\in K}\sum_{r\in R_k} a_{rf}^k x_r^k = 1$) or cancelled (i.e., $y_f = 1$). Third, Eqs. (3-3) ensures that each aircraft is assigned with no more one route. Then, the man-hour availability at each maintenance station is respected through applying Eqs. (3-4), in which the left-hand-side is the total man-hour required for the high-time aircraft that is assigned to maintenance station n to carry out the maintenance check. Last, the solution space for the decision variables is regulated in Eqs. (3-5).

	$\sum_{k\in K_h}\sum_{r\in R_k}(\sum_{f\in F_r} c_f^k + \sum_{f\in F_r} o_f^k w_f + m_r^k)x_r^k$ $+\sum_{k\in K_n}\sum_{r\in R_k}(\sum_{f\in F_r} c_f^k + \sum_{f\in F_r} o_f^k w_f)x_r^k$ $+\sum_{f\in F} q_f y_f + \sum_{k\in K} q_k y_k$		(3-1)
s.t.	$\sum_{k\in K}\sum_{r\in R_k} a_{rf}^k x_r^k + y_f = 1$	$\forall f \in F$	(3-2)
	$\sum_{r\in R_k} x_r^k + y_k = 1$	$\forall k \in K$	(3-3)
	$\sum_{k\in K}\sum_{r\in R_k} h_n^k e_{rn}^k x_r^k \le P_n$	$\forall n \in N$	(3-4)
	$x_r^k, y_f, y_k \in \{0,1\}$	$\forall k \in K, \forall r \in R_k, \forall f \in F$	(3-5)

It is seen that the operational flight scheduling and aircraft routing model formulated above is a set-partitioning type formulation, which is well-known to be NP-hard due to the exponential number of possible decision variables (i.e., vast number of feasible aircraft routes). It is inefficient or even infeasible to enumerate all routes and find a subset of cost-minimized routes. Therefore, we develop a column generation-based solution algorithm to efficiently solve the operational flight scheduling and aircraft routing problem in the next section.

4. Solution approach

Column generation is a powerful tool to solve large-scale linear programming problems without encountering the difficulty of enumerating the vast number of potential variables (Barnhart, Johnson, et al., 1998; Sarac et al., 2006). The mechanism of column generation is briefly explained as follows. First of all, the large-scale linear programming problem is divided into a master problem and a sub-problem (or many sub-problems) (Desaulniers et al., 2006; Liang et al., 2018). The master problem is initiated with a small number of variables, so that the mater problem is also named as the restricted master problem. Solving the restricted master problem (e.g., by using the simplex method), the dual prices for each constraint are obtained and passed to the sub-problem which is usually formulated as a resource constrained shortest path problem in airline scheduling problems. Considering the resource window for each node, the aim of the sub-problem is to identify the most negative path in the network with the dual prices generated by the master problem. The *resources* could be flying time, the number of take-offs, the number of calendar days, etc., in AMRP, and could be duty working time, duty flight number, pairing elapse time, etc., in the crew pairing problem (Anbil et al., 1998; Lavoie et al., 1988). In this study, the remaining legal flying time of an aircraft is the resource we consider during path generation. That is, for high-time aircraft, before the remaining legal flying time becomes zero, the aircraft would be assigned with a maintenance opportunity. The identified negative paths (routes) are then added into the solution pool of the restricted master problem, while the next iteration is triggered until no more negative path could be found (implying that the master problem has reached optimality). If the solutions obtained by column generation is fractional, then the branch-and-bound technique is invoked to get integer solutions. If the column generation is called at every node of the branch-and-bound tree, then the overall algorithm is named as branch-and-price. Exact branching is exactly time-consuming. Thus, heuristic branching strategies, like diving column fixing and inter-task fixing, are commonly used. Many studies also apply the mixed integer programming technique on the last restricted master problem, without invoking column generation at each branch-and-bound tree, in order to obtain integer solutions quickly (Liang et al., 2015; Liang et al., 2018). In the following, we present the details of the restricted master problem and the sub-problem for each aircraft.

4.1 Restricted master problem

The restricted master problem is the linear relaxed version of the operational flight scheduling and aircraft routing model formulated in Section 3, as shown in Eq. (4-1) to Eqs. (4-5). The restricted master problem is initialized with a sub-set of aircraft routes. In this study, we build an artificial aircraft route covering all flights with a very large cost to initiate the model. This initial large-cost route will be priced out when new aircraft routes are generated and inserted into the solution pool. Note that this initial bad route would always be priced out as flights are allowed to be cancelled, while the cost of this initial route is far larger than the flight cancellation cost. The dual prices associated with Eq. (4-2) to Eqs. (4-4) are denoted by β_f (for each flight), δ_k (for each aircraft), and γ_n (for each maintenance station), respectively. The dual prices are passed to the sub-problem for each aircraft to seek for new routes that have the potential to lower down the objective value.

	$\sum_{k\in K_h}\sum_{r\in R_k}(\sum_{f\in F_r} c_f^k + \sum_{f\in F_r} o_f^k w_f + m_r^k)x_r^k + \sum_{k\in K_n}\sum_{r\in R_k}(\sum_{f\in F_r} c_f^k + \sum_{f\in F_r} o_f^k w_f)x_r^k + \sum_{f\in F} q_f y_f + \sum_{k\in K} q_k y_k$	(4-1)
s.t.	$\sum_{k\in K}\sum_{r\in R_k} a_{rf}^k x_r^k + y_f = 1 \quad \forall f \in F$	(4-2)
	$\sum_{r\in R_k} x_r^k + y_k = 1 \quad \forall k \in K$	(4-3)
	$\sum_{k\in K}\sum_{r\in R_k} h_n^k e_{rn}^k x_r^k \leq P_n \quad \forall n \in N$	(4-4)
	$x_r^k, y_f, y_k \geq 0 \quad \forall k \in K, \forall r \in R_k, \forall f \in F$	(4-5)

4.2 Sub-problem

As mentioned, the objective of the sub-problem is to identify a better column (that is, an aircraft route in the AMRP) that can help reduce the objective value using the dual prices obtained from solving the restricted master problem. In this study, the sub-problem is equivalent to identifying a route for an (or several) aircraft with a negative reduced cost. The reduced cost (θ_r^k) for route r of aircraft k is formulated as in Eq. (4-6). More specifically, the reduced cost for a route of a high-time aircraft is detailed in Eq. (4-7), while that for a route of a non-high-time aircraft is presented in Eq. (4-8). Specifically, it is seen that compared with non-high-time aircraft, the reduced cost for a high-time aircraft involves the maintenance cost as well as the dual prices related to the maintenance station if used.

$\theta_r^k = c_r^k - \sum_{f\in F} a_{rf}^k \beta_f - \delta_k - \sum_{n\in N} h_n^k e_{rn}^k \gamma_n$	$\forall k \in K, \forall r \in R_k$	(4-6)
$\theta_r^k = \sum_{f\in F_r} c_f^k + \sum_{f\in F_r} o_f^k w_f + m_r^k - \sum_{f\in F} a_{rf}^k \beta_f - \delta_k - \sum_{n\in N} h_n^k e_{rn}^k \gamma_n$	$\forall k \in K_h, \forall r \in R_k$	(4-7)
$\theta_r^k = \sum_{f\in F_r} c_f^k + \sum_{f\in F_r} o_f^k w_f - \sum_{f\in F} a_{rf}^k \beta_f - \delta_k$	$\forall k \in K_n, \forall r \in R_k$	(4-8)

The sub-problem is then transformed to solving a resource constrained shortest path problem (RCSTP), where the cost of each flight arc is set as $(-\beta_f)$, the cost of each aircraft is set as $(-\delta_k)$, the cost of each maintenance station for each aircraft is set as $(-h_n^k\gamma_n)$, while the path cost is defined as θ_r^k. At each column generation iteration, the path with the most negative cost is identified (actually, all non-dominated paths with negative costs could be found). Note that for a non-high-time aircraft, the path extension procedure is not restricted by the legal flying time constraint. However, for a high-time aircraft, it should be guaranteed that the aircraft could either stay at a maintenance station with sufficient resources or stay at a maintenance-infeasible airport with the SMR strategy before it uses up the remaining legal flying time. The flight networks most commonly used in AMRPs are the time-space networks and connection networks (Barnhart, Boland, et al., 1998; Hane et al., 1995). In a time-space network, nodes represent flight departures and arrivals, while arcs represent flights or overnights. On the other hand, connection networks apply nodes as flights, while the arcs connecting flights represent the turn-around time between two consecutive flights. In this study, we apply the connection network for path generation. Generally, the connection network contains a dummy source node (*s*), a dummy sink node (*t*), flight nodes, and flight arcs. Next, we will demonstrate how to construct the connection networks for each type of aircraft in order to achieve the above-mentioned purposes.

Connection network for non-high-time aircraft $k \in K_n$

The network for non-high-time aircraft is relatively simple. Given an initial location of the aircraft, the dummy source node is connected with all flights that depart from the initial airport through route starting arcs to represent the beginning of a route. Flight nodes are connected through connection arcs if the arrival airport of the previous flight is the departure airport of the following flight, while the connection time between these two flights is at least the minimum required turn time. Besides, each flight is connected with the sink node through a route ending arc, as there is no restriction on non-high-time aircraft regarding the ending airport in the day. A simple example is shown in **Figure 1**, where solid lines represent the route starting arcs and route ending arcs, while dash lines are flight connection arcs. In this example, the initial location of the aircraft is the departure airport of flight 1 and flight 2 (i.e., station A). Flights 1&4, 2&3, and 3&5 could be flown consecutively by an aircraft as the turn times between those flight pairs are legal. Besides, the aircraft could end its duty in the day after flying any flight. Thus, all flights could be connected with the sink node no matter whether the ending station is a maintenance station or not. Along the generation of routes, once the originally assigned aircraft for a flight is not the currently considered one, an aircraft change penalty w_f is added to the path cost.

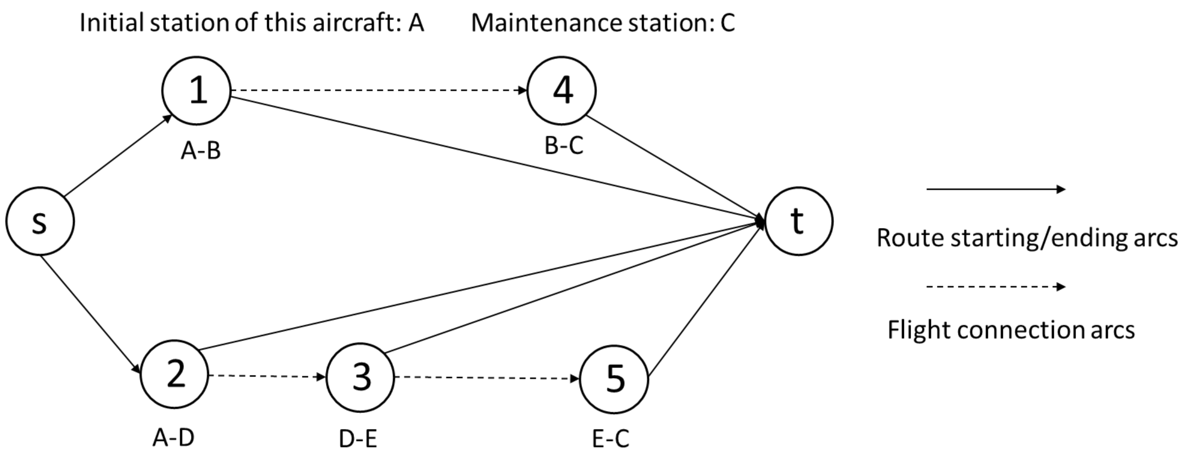

Figure 1. A connection network for a non-high-time aircraft.

Connection network for high-time aircraft $k \in K_h$

The network structure for a high-time aircraft is more complicated as we need to model the maintenance activity at a maintenance station, as well as the SMR strategy to ensure that each high-time aircraft could be maintained before the remaining legal flying time is used up in the day. Thus, we introduce two types of maintenance arcs, normal maintenance arcs and SMR maintenance arcs, as shown in **Figure 2**. In this figure, the arrival airport of flight 4 and flight 5 are maintenance stations. The normal maintenance arc links the flights arriving at airports that can serve as a maintenance station with the sink node, while the SMR maintenance arc links all flights with the sink node. These two types of maintenance arcs ensure that every high-time aircraft could end with a maintenance opportunity at the end of the operation day. Accordingly, the cost of a normal maintenance arc is set as the basic maintenance costs of aircraft k (i.e., b_k), while that of a SMR maintenance arc is set as the basic maintenance cost plus the additional cost when aircraft k adopts the SMR maintenance strategy (i.e., $b_k + d_k$). Note that the flights arriving at a maintenance station are also connected with the sink node through a SMR maintenance arc (see flight 4 and flight 5 in **Figure 2**), implying that when there is insufficient man-hour at the corresponding maintenance station, a high-time aircraft could end at that airport by applying the maintenance personnel/equipment sent by the airline. If the available resources of a maintenance station are lower than the minimum requirements of all aircraft, then the normal maintenance arcs of this maintenance station could be eliminated. In the example of **Figure 2**, the aircraft could operate flights 1 and 4, ending at the maintenance station C (either adopting the SMR strategy or using the maintenance services provided by station C). It can also only operate flight 1, and end at the arriving airport of flight 1 (that is, station B). The maintenance personnel sent by the airline will conduct the maintenance check for this aircraft at the arriving airport of flight 1 (that is, station B). Similar to non-high-time aircraft, along the generation of routes, an aircraft change penalty w_f is derived if the aircraft is not the one originally assigned to a flight. It should be pointed out that through the aircraft change penalty cost, one could reduce the impact of the recovery plan on the original schedule.

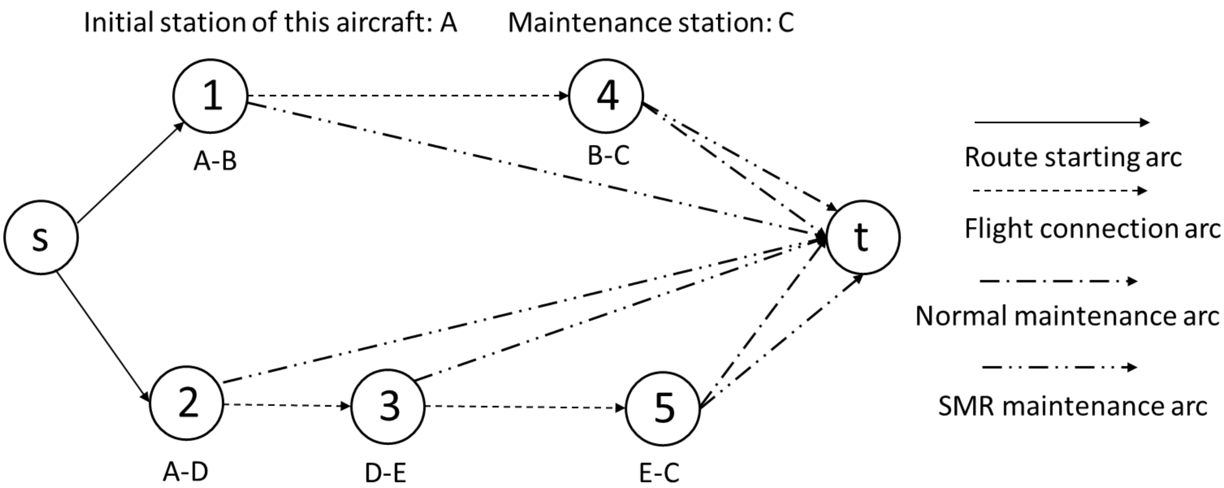

Figure 2. A connection network for a high-time aircraft.

The RCSTP is usually solved by the labelling algorithm (Irnich & Desaulniers, 2005). The labelling algorithm is further clarified as the label setting algorithm and the label correcting algorithm according to the path extension method. In this study, we apply the label correcting algorithm. A label L_i represents a partial path arriving at node i, denoted by $L_i = [T(L_i), \theta_r^k(L_i)]$, where $\theta_r^k(L_i)$ is the reduced cost for the partial path at node i. $T(L_i)$ stands for remaining legal flying time, and it should satisfy the resource window ($[0, T]$) regulated at node i (T is the maximum allowed flying time between two maintenance checks). Starting from the trivial initial label only containing the source node, new labels are created during the path extension procedures if they are resource-feasible. As there are usually more than one label existing for each node (i.e., a node can be reached from the source node through different paths), dominance tests are conducted to remove unpromising labels to reduce searching efforts and accelerate the labeling algorithm. Note that if two labels are identical (reduced costs and resource consumptions are the same), one of them will be kept. To accelerate the overall solution procedure, the partial pricing strategy could be adopted. That is, at each iteration, only a subset of sub-problems for some aircraft are invoked as long as a number of negative paths could be identified. At the end of the column generation procedure, the sub-problems for all aircraft will be examined to guarantee that no more negative paths could be identified to ensure optimality. Note that in this study, we add all the negative paths identified to the restricted master problem.

5. Computational experiments

This section presents analyses that demonstrate the performances of the proposed operational flight scheduling and aircraft routing model through computational experiments based on real-world collected flight schedules. Experiments were conducted on a PC with Windows 10 operation system and Intel (R) Core (TM) i7-10510U CPU @ 1.80GHz @ 2.30 GHz (16 GB RAM). The implementations are coded in Java programming language. The restricted master problem is solved using CPLEX Concert Technology in IBM ILOG CPLEX Optimization Studio (Version 20.1.0).

5.1 Instances characteristics

The test instances are summarized in Table 1. Note that our objective is to reschedule aircraft in daily operations. Thus, all the instances used in our study span one day. There are four major instances, I1, I2, I3, and I4, with 53 flights, 103 flights, 190 flights, and 238 flights in total. All these flight schedules involve two airports (both are maintenance stations with limited available man-hour, as shown in the fifth column of Table 1). Note that although the number of flights does not increase too many from 53 to 238, the scale and complexity of the flight network increases significantly. This is because with only two airports serving as the departure and arrival stations of all flights, the number of feasible connections between flights grows exponentially. Thus, it is seen later that the computational time required by larger instances also increases exponentially. Besides, each instance is characterized by the number of non-high-time aircraft, the number of high-time aircraft, as well as the average maintenance required time by the high-time aircraft. As mentioned, due to the resource restriction at the maintenance stations, as well as the aircraft route constraints, some high-time aircraft may not be able to stay at a resource-feasible maintenance station at the end of the day (or when its legal remaining flying time is used up). Thus, for the test reason, we further construct four sub-instances (A, B, C, and D) for each major instance by adding additional 50% available man-hour for the maintenance stations. For example, I1-A is featured by 600-minute average available man-hour, implying that the total available man-hour for the two maintenance stations is 1200 minutes. In I1-B, I1-C, and I1-D, the average available man-hours for the two maintenance stations are then 900, 1350, and 2025 minutes, respectively. Besides, we observe that the high-time aircraft in I1 requires 5*320=1600 minutes maintenance time in total. Therefore, it is concluded that the SMR strategy is essential in this case. On the other hand, I1-D operates with 2025-minute average available man-hour, which is larger than 1600. Thus, in I1-D, the maintenance stations have the potential to fulfil all the maintenance demand of the high-time aircraft (but not guaranteed due to aircraft route constraints).

Table 1. Instance characteristics.

Index	Number of flights	Number of airports	Number of maintenance stations	Average available man-hour in min	Number of non-high-time aircraft	Number of high-time aircraft	Average maintenance required time in min
I1-A	53	2	2	600	8	5	320
I1-B				900			
I1-C				1350			
I1-D				2025			
I2-A	103	2	2	1200	14	10	320
I2-B				1800			
I2-C				2700			
I2-D				4050			
I3-A	190	2	2	1800	30	14	323
I3-B				2700			
I3-C				4050			
I3-D				6075			
I4-A	238	2	2	1500	45	12	330

I4-B	2250
I4-C	3375
I4-D	5063

5.2 Solution time analysis

The computational performance of the proposed column generation based algorithm is displayed in detail in Table 2. It is obvious to reveal that, with the increase of the number of the flights from 53 to 238, the computation time increases exponentially with the instance size, namely a 323% growth. The number of columns generated to obtain optimality is increased dramatically, namely, from 6000 to 400000. In particular, for the sub-instances of I1, I2, and I3, the corresponding computation times are relatively close to each other. However, for I4, there is a big difference for the computation time among the subinstances (e.g., 273s is required by I4-A, while 793s is spent by I4-B). The reason behind is that, when there is limited (i.e., small feasible solution space) or sufficient (i.e., less constraints on the feasible solutions) average available man-hour, the time used for searching the optimal solution is much less. When the number of the average available man-hour is at a medium level, on one hand, the searching space of the feasible solutions is increased; on the other hand, more constraints are involved on the maintenance man-hour, which incurs a larger computational burden.

The fourth column of Table 2 presents the time spent on the sub-problems (i.e., the resource-constrained shortest path problem), while the fifth column summarizes the overall time used to insert the identified new potential columns into the restricted master problem. As for I1-I3, the computation times for the corresponding sub-problems are all less than those spent on new column insertion. This is because the computation times for the sub-problems of the small-size instances are relatively short. However, for I4, much time is needed to run the resource-constrained shortest path problem in the corresponding large network.

Moreover, it is verified that, for different problem sizes, the solutions obtained by the proposed column generation based algorithm are optimal or near-optimal. The biggest optimality gap is within 1.8%, and 0.3% on average.

Table 2. Computation times and optimality gap.

Index	Time consumed (s)				CG iterations	Number of variables		Optimality gap
	CG	MIP	Sub-problem	New column insertion		Final master problem	Generated	
I1-A	0.9	0.2	0.2	0.5	15	6070	6003	1.8%
I1-B	0.9	0.4	0.2	0.5	12	6262	6195	1.1%
I1-C	0.8	0.2	0.1	0.4	11	5563	5496	0.0%
I1-D	0.8	0.1	0.1	0.4	14	5754	5687	0.0%
I2-A	12.5	4.5	2.6	6.5	21	54671	54543	0.4%
I2-B	12.2	6.4	2.5	6.7	20	55854	55726	0.1%
I2-C	12.2	2.5	2.8	6.2	19	52432	52304	0.0%
I2-D	12.2	1.7	2.5	6.1	25	51484	51356	0.0%
I3-A	128.2	51.2	41.8	55.4	28	247061	246826	0.1%
I3-B	117.4	51.6	42.8	50.9	28	253716	253481	0.0%
I3-C	129.0	85.0	43.1	54.0	29	244391	244156	0.0%
I3-D	113.3	51.9	37.4	49.3	28	244148	243913	0.0%
I4-A	273.5	111.7	105.4	109.7	34	383473	383177	0.1%
I4-B	793.8	226.0	382.5	283.9	39	397006	396710	0.0%
I4-C	733.6	199.6	352.5	287.7	36	411938	411642	0.0%
I4-D	292.2	121.7	126.0	106.2	36	394052	393756	0.0%

5.3 Model performances

As is shown in Table 3, there is no flight being cancelled for all instances, while all aircraft are used by the algorithm. In particular, for the sub-instance A of all instances (i.e., I1-A, I2-A, I3-A and I4-A), due to the lack of available man-hour for the maintenance stations, more SMR maintenance checks are utilized. When there is sufficient man-hour available at the maintenance stations (i.e., in sub instances of B, C, and D), it is less essential to adopt the SMR strategy. Moreover, except for I1, it is obvious to uncover that the level of utilization of the maintenance stations is high for all sub-instances A. However, for I1-A, only 52% of the available man-hour is used in the maintenance stations. It may be caused by the structure of the flight schedules. Furthermore, it is interesting to find that, for the cases with super-sufficient man-hour available in the maintenance stations (i.e., I1-D, I2-D, I3-D, I4-D), the level of man-hour utilization becomes lower.

Table 3. Solution details.

Index	Number of uncovered flights	Number of unused aircraft	Number of SMR used	Utilization of maintenance station 1	Utilization of maintenance station 2
I1-A	0	0	3	52%	52%
I1-B	0	0	1	74%	69%
I1-C	0	0	0	23%	95%
I1-D	0	0	0	0%	79%
I2-A	0	0	4	81%	81%
I2-B	0	0	0	89%	89%
I2-C	0	0	0	25%	94%
I2-D	0	0	0	16%	62%
I3-A	0	0	4	89%	87%
I3-B	0	0	0	70%	97%
I3-C	0	0	0	16%	95%
I3-D	0	0	0	6%	69%
I4-A	0	0	4	88%	83%
I4-B	0	0	0	87%	89%
I4-C	0	0	0	49%	68%
I4-D	0	0	0	32%	45%

To further analyze the impacts of the average maintenance required time, flight cancellation cost and aircraft idling cost on the optimal solution (especially on the utilization of maintenance stations, the number of SMR used, the number of flights cancelled and the number of aircraft unused), extended numerical experiments are conducted based on instance I2-A (serving as the benchmark). The corresponding parameter settings of the instances are shown in Table 4. In particular, instances I2-A-1 to I2-A-4 specify the setting of the average maintenance required time being in an increasing trend. Instances I2-A-5 to I2-A-8 and I2-A-9 to I2-A-12 specify the settings of the flight cancellation cost and aircraft idling cost being in a decreasing manner, respectively. Note that for each instance, only the specific parameter is substituted with the corresponding value shown in the table. Note that the computational performances of these extended analyses on I2-A are shown in Appendix.

Table 4. Instance characteristics for extended analysis on I2-A.

Index	Average maintenance required time in min	Index	Flight cancellation cost	Index	Aircraft idling cost
benchmark	320	benchmark	100000	benchmark	10000
I2-A-1	360	I2-A-5	10000	I2-A-9	1000
I2-A-2	400	I2-A-6	1000	I2-A-10	100
I2-A-3	440	I2-A-7	100	I2-A-11	10
I2-A-4	480	I2-A-8	0	I2-A-12	0

As we can see from the results of instances I2-A-1 to I2-A-4 in Table 5, when the average maintenance required time is increased, the aircraft rely more on the SMR maintenance strategy, while the utilization of the maintenance stations becomes lower. This is because, on one hand, when the average maintenance required time is increased, it has more impacts on the original flight/aircraft schedules, while adopting the SMR strategy helps reduce the changes of the original flight/aircraft schedule, thus being preferred by the algorithm. On the other hand, the available man-hour at maintenance stations becomes more restricted when the maintenance demand of aircraft increases. From the results of instances I2-A-5 to I2-A-8, it is interesting to uncover that, when the flight cancellation cost is decreased, the number of SMR maintenance used keeps unchanged, but the number of uncovered flights is increased dramatically with a relatively low utilization of the maintenance stations. The results are reasonable from the cost-oriented perspective. As the flight cancellation cost is decreased, the optimal decision for the airline is to cancel some flights. However, in the case of extreme low penalty cost, the service level of the passengers and the overall profit of the airline maybe damaged to a large extent. Furthermore, compared to the flight cancellation cost, the aircraft idling cost imposes little impact on the optimal solution.

Table 5. Solution details for further analysis on I2-A.

Index	Number of uncovered flights	Number of unused aircraft	Number of SMR used	Utilization of M1	Utilization of M2
I2-A-1	0	0	4	91%	88%
I2-A-2	0	0	4	98%	98%
I2-A-3	0	0	6	75%	72%
I2-A-4	0	0	6	82%	79%
I2-A-5	14	0	4	78%	78%
I2-A-6	79	0	4	78%	78%
I2-A-7	79	0	4	78%	78%
I2-A-8	79	0	4	78%	78%
I2-A-9	0	0	4	81%	78%
I2-A-10	0	0	4	81%	81%
I2-A-11	0	0	4	81%	78%
I2-A-12	0	0	4	81%	81%

6. Conclusions

Aircraft is the core resource of an airline which should be properly planned to ensure that it complies with the strict maintenance requirements when serving flights. Typically, from a tactical planning perspective, the AMRP prescribes that aircraft can follow a cyclic path and visit a maintenance station on a regular basis to fulfil the maintenance requirement. However, the aviation industry is full of uncertainties. Many unplanned events force airlines to modify their pre-determined tactical aircraft routes in real operations. This study considers that unexpected maintenance requirements arise in daily operations, or an aircraft could not arrive at a maintenance station as planned due to disruptions, which triggers the need to re-route aircraft in real time to fulfill maintenance requirements while covering the scheduled flight as much as possible. Moreover, maintenance stations are often capacity-constrained in various aspects, like available man-hour, parking slots, available equipment. To ensure travel safety and avoid high penalties of violating maintenance requirements, many airlines allocate maintenance resources (like manpower and equipment) to the destination airports of aircraft when there is no proper maintenance station could be utilized (named as the SMR strategy in this study). However, allocating maintenance resources to other airports is expensive. Thus, airlines have to balance between the additional expenditures generated by re-allocation of maintenance resources and the costs of cancelling flights. Moreover, when conducting recovery, airlines usually endeavor to keep the original flight schedule and aircraft routing unchanged. This study proposes a new aircraft re-routing approach to fulfil the maintenance requirements arising in the operational stage either through the SMR strategy or the maintenance services provided by maintenance stations. Besides, flights are re-scheduled if necessary (i.e., cancelled with a high penalty), while the impact on the original plan is minimized. To achieve this, specialized flight networks are constructed for high-time aircraft to guarantee maintenance opportunities, while a column generation-based algorithm is developed to obtain high-quality solutions within short computational times. The sub-problems of the column generation approach are modelled as resource-constrained shortest path problems which are solved by the labelling algorithm. To validate the computational efficiency of the proposed solution algorithm as well as the model performances, computational experiments based on real-world collected flight schedules are conducted. Results show that, even for big-size problems, the solutions obtained by the proposed column generation based algorithm are optimal or near-optimal with an optimality gap of 0.3% on average. Finally, note that although we have found that the maintenance workload (i.e., manhours) is often uncertain in real operations[3] (Khan et al., 2021), the related uncertainty is not considered in the current study as the maintenance is performed overnight which is regarded to be enough to absorb the disruptions.

Appendix – Notation

The notation used in this study is summarized in Table A as below.

Table A. Notation.

F	The set of flights to be scheduled, indexed by f.
K	The set of aircraft to be re-routed, indexed by k.
K_h	The set of high-time aircraft.
N	The set of feasible maintenance station, indexed by n.
F_r	The set of flights contained in route r.
R_k	The set of possible routes for aircraft k, indexed by r.

[3] From our data analysis based on the maintenance data provided by our industrial partner, we found that most maintenance workloads follow the normal distribution.

c_r^k	The cost of assigning route r to aircraft k.
c_f^k	The operating costs of aircraft k flying flight f.
w_f	The aircraft change penalties of flight f.
o_f^k	The binary coefficient representing whether the flight is scheduled to be operated by the aircraft assigned in the original plan.
m_r^k	The maintenance costs of aircraft k.
b_k	The basic maintenance costs of aircraft k at a maintenance station.
d_k	The additional cost if aircraft k carries a maintenance personnel.
P_n	The man-hour restriction for maintenance station n.
h_n^k	The number of man-hours required by aircraft k at maintenance station n.
d_k	The additional cost derived by the SMR strategy.
q_f	The penalty of cancelling flight f.
q_k	The idling cost of aircraft k.
a_{rf}^k	The binary coefficient representing whether route r of aircraft k covers flight f.
e_{rn}^k	The binary coefficient representing whether route r of aircraft k ends at maintenance station n and conducts maintenance check at that station.
x_r^k	Binary decision variable, whether route r of aircraft k is selected or not.
y_k	Binary decision variable, whether aircraft k is used or not.
y_f	Binary decision variable, whether flight f is cancelled or not.
β_f	The dual price for each flight.
δ_k	The dual price for each aircraft.
γ_n	The dual price for each maintenance station.

Appendix – Table A1

Table A1 shows the computational performance of the proposed column generation based algorithm for the extended instances regarding the sensitivity analysis of the number of average available maintenance man-hour, the cost of flight cancellation, and the cost of the aircraft idling. We find that there is little impact on the computational complexity when the number of average available maintenance man-hour and the aircraft idling cost are changed. However, when the penalty for the flight cancellation is decreased, the computation time is reduced to a great extent with a rather smaller number of columns needed for attaining the optimality. The reason behind is that when the penalty cost for the flight cancellation is very small, the problem complexity is reduced as many of the originally-scheduled flights are not considered at all during the solution procedure. For most of the instances, the solution quality is good as the optimality gap is within 0.41%.

Table A1. Computation times and optimality gap for further analysis on I2-A.

Index	Time consumed (s)				CG iterations	Number of variables		Optimality gap
	CG	MIP	Sub-problem	New column insertion		Final master problem	Generated	
I2-A-1	15.4	4.4	3.7	7.9	21	56436	56308	0.18%
I2-A-2	24.5	6.2	2.7	18.7	20	53610	53482	0.05%
I2-A-3	14.8	5.5	3.0	7.2	24	58067	57939	0.41%
I2-A-4	12.3	4.2	2.3	6.5	23	55171	55043	0.28%
I2-A-5	8.4	3.3	1.9	4.7	14	36522	36394	0.52%
I2-A-6	0.5	0.2	0.1	0.4	2	2624	2496	2.20%
I2-A-7	0.4	0.1	0.1	0.3	2	2012	1884	4.10%
I2-A-8	0.4	0.1	0.1	0.3	2	1947	1819	4.53%
I2-A-9	12.0	4.4	2.4	6.4	19	55302	55174	0.39%
I2-A-10	14.2	4.9	2.9	7.5	21	58109	57981	0.39%
I2-A-11	13.6	4.4	2.7	7.0	21	57052	56924	0.39%
I2-A-12	12.3	4.1	2.4	6.2	24	53829	53701	0.39%

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