Size effect on the statistical distribution of stress and strain in microforming

Z.Y. Feng [1], H. Li [2], D. Zhang [2], M.W. Fu [1 #]

[1] Department of Mechanical Engineering, Research Institute for Advanced Manufacturing, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, China
[2] State Key Laboratory of Solidification Processing, School of Materials Science and Engineering, Northwestern Polytechnical University, Xi'an 710072, China
[#]Email: mmmwfu@polyu.edu.hk

Abstract. The frequency distribution of local stress or strain across the micromechanical field in plastic deformation tends to universally follow a normal or lognormal distribution, regardless of the variety of microstructural inhomogeneity. However, it has not been reported how size effect (SE) influences the grain-scale statistical distribution of stress and strain in microforming, and thus an in-depth investigation is further needed. Taking the polycrystalline Cu sheets with thickness t = 0.1–1.5 mm and t/d = 1–29 as the case materials, this study implements the full-field CPFE simulation incorporated with a size-dependent dislocation-based constitutive model and statistical analyses to explore the influence of SE on the frequency distribution of grain-scale stress and strain in micro-scaled plastic deformation. With increasing t/d, the stress frequency consistently follows a normal distribution. In contrast, the frequency distributions of strain and dislocation density undergo a transformation from a lognormal distribution to an approximately normal distribution. The results indicate that the distribution law of stress is dominantly influenced by the dislocation density, while that of strain is determined by a multiplicative process of slip activities. The established knowledge will help to elucidate the nature of the distribution law of stress or strain, and to seek for effective approaches to alleviate the scatter and uncertainty of deformed part geometry during a microforming process.

Keywords: Size effect, Microforming, Crystal plasticity, Stress/strain frequency, Statistical distribution.

1 Introduction

Product miniaturization in many industrial sectors such as micro-electromechanics, electronics, and medical devices puts forward a growing need and a higher requirement for micro metal parts, and thus promotes the prosperity of the microforming technology [1, 2]. However, when the part or its feature size is scaled down from macro to micro-scale, size effect (SE) renders deformation behaviors or mechanical properties of micro-scaled structures deviate from those of their macroscale analogues [1, 3-5]. Particularly, SEs induced grain-scale plastic heterogeneities result in scattering characteristics and uncertain product geometries, and thus impede the mass production of high-performance microparts. However, despite grain-scale plastic heterogeneities due to the variety of microstructures or the anisotropy of individual grains, the statistical distributions of some micromechanical quantities in plastic deformation such as local stress and strain were reported to be normal or lognormal [6, 7]. Therefore, to elucidate the nature of the distribution law of local stress and strain and alleviate the negative influence of SEs, it is necessary to investigate the SEs on the statistical distributions of grain-scale stress and strain in microforming.

It has been reported that some grain-scale mechanical quantities in deformed materials follow a normal or a lognormal distribution. The DIC experiment on the dual-phase steel manifested that the frequency of local plastic strain of both the ferrite phase and the martensite phase presents a bell shape resembling a normal distribution [8]. Crystal plasticity finite element (CPFE) simulations [9] and HR-EBSD experiments [10] on polycrystalline copper showed that the statistical distributions of stress and strain components, as well as hydrostatic and effective stress differences, are Gaussian-like. On the other hand, Tang et al. [7] illustrated that the local plastic strain in five representative steels follows a lognormal distribution regardless of the diversity of the phase contents and deformation mechanisms. The crystal plasticity-fast Fourier transform (CP-FFT) and CPFE simulation results also displayed that the distributions of plastic strain are generally closer to a lognormal law regardless of the crystal structures [11, 12]. Thus, the statistical law of stress or strain across the microstructural field in plastic deformation remains ambiguous, and an in-depth study is needed.

Although SE-affected deformation behaviors in microforming have been investigated extensively, SEs on the statistical distribution of grain-scale stress or strain have not well been revealed. CP-FFT simulations conducted by Zhang et al. [13] revealed that the increase of grain size renders the well bell-shaped distribution of stress and strain rather irregular. However, in the CP-FFT simulations of Tang et al. [11], it seems that an increased *t/d* only shifts the distribution peak but cannot alter the general lognormal profile. It is reported that there exists a lower limit size for the presence of a lognormal distribution law of plastic strain in bulk materials [7, 12]. But the effect of grain size on the stress or strain distribution was not revealed and the applied strain was relatively small (only 1.55%). Therefore, the question remains how SE influences the statistical distribution of grain-scale stress and strain, and further the scattering characteristics of mechanical responses in microforming.

To obtain a statistical view of SEs on the grain-scale stress and strain distribution in microforming, the current study implements the full-field CPFE simulation incorporated with a size-dependent dislocation-based model to investigate the effect of sheet thickness and grain size on multiple micromechanical variables. The frequencies of grain-scale stress and strain of Cu sheets with $t/d = 1–29$ obtained from CP simulations were analyzed statistically via the data fitting using a normal or lognormal function. The established knowledge will help to elucidate the nature of the distribution law of local stress and strain, and to seek for the effective approaches to alleviate the SEs influenced scatter and uncertainty of part geometry during a microforming process.

2 Crystal plasticity model and simulation schemes

2.1 Dislocation-based crystal plasticity model

In the dislocation density-based model (DDBM), the Orowan equation typically is served as the kinetic equation designated by [14, 15]:

$$\dot{\gamma} = \rho b_s v_0 \exp\left\{-\frac{Q_s}{k_B T}\left[1-\left(\frac{|\tau|-\tau_{pass}}{\tau_{sol}}\right)^p\right]^q\right\}\operatorname{sgn}(\tau) \tag{1}$$

where ρ : the density of mobile dislocations, b_s : the magnitude of the Burgers vector, v_0 : the reference velocity, Q_s : the activation energy for slip, k_B : the BOLTZMANN constant, T: the temperature, τ_{pass} : the passing stress, τ_{sol} : the solid solution strength, τ : the resolved shear stress. The fitting parameters p and q are set as 1.

The passing stress τ_{pass} is given by [11]:

$$\tau_{pass}^{\alpha} = \tau_0 + \frac{K_{hp}}{\sqrt{d}} + c_1 G b_s \sqrt{\sum_{\beta}^{N_s} \chi^{\alpha\beta} \rho^{\beta}} \tag{2}$$

where τ_0 : the initial resistance on slip system as grain size d is extremely large, K_{hp} : the Hall-Petch slope, G: the shear modulus, c_1 is a fitting parameter. $\chi^{\alpha\beta}$: the interaction coefficient, 1.0 for coplanar slip systems and 1.4 for non-coplanar slip systems.

The evolution law of dislocation density ρ is expressed by [2, 3]:

$$\dot{\rho}^{\alpha} = \frac{|\dot{\gamma}^{\alpha}|}{b_s \Lambda_s^{\alpha}} - \Omega \rho^{\alpha} |\dot{\gamma}^{\alpha}| \tag{3}$$

$$\frac{1}{\Lambda_s^{\alpha}} = \frac{1}{d} + \frac{1}{L_s}\sqrt{\sum_{\beta}^{N_s} \chi^{\alpha\beta} \rho^{\beta}} \tag{4}$$

where Λ_s^{α} : the mean free space, Ω and L_s are fitting parameters.

Model calibration and CP simulations were conducted by using the open-source materials simulation kit DAMASK [15]. The details were elaborated in the previous work [3]. Model parameters for pure copper are: C_{11}=186,000 MPa, C_{12}=93,000 MPa, C_{44}=46,500 MPa, b_s=2.54×10^{-10} m, c_1=0.03, Q_s=3.7×10^{-19} J, K_{hp} =4.5×10^{4} MPa·m$^{1/2}$, ρ_0 =1.0×10^{10} m^{-2}, τ_0 =1.0 MPa, τ_{sol} =2.0 MPa, Ω =7.0, L_s =3.0.

2.2 Simulation schemes

Table 1 details the 2D simulation scheme of Cu sheets with different t and t/d. The length of all sheets was set to 5 mm. Only the sheet thickness t is changed, as follows: 0.1, 0.4, 0.8, 1.5 mm. Grain size is changed such that t/d is increased from 1 to 29. The displacement boundary condition with a global strain of 0.2 was adopted. The polycrystalline geometric models were produced by a standard Poisson-Voronoi tessellation using open-source software Neper [16]. The FE element type of CPE4R was adopted.

Table 1. Simulation scheme of copper sheets with different t and t/d.

Uniaxial tension model (DDBM, length = 5 mm, element type: CPE4R, δ: mesh size)				
t (mm)	0.1	0.4	0.8	1.5
d (μm)	5.56–100	3.33–400	3.33–800	1.67–1,500
t/d	1–9	1–13	1–19	1–29
δ	0.005	0.01	0.02	0.03

3 Results and discussion

Statistical analyses on the frequency distributions of multiple micromechanical variables were conducted by using the general normal distribution, whose probability density function (PDF) is expressed by

$$f(x) = y_0 + \frac{1}{\sqrt{2\pi}w} e^{-\frac{(x-x_c)^2}{2w^2}} \tag{5}$$

or the general lognormal distribution, whose PDF is

$$f(x) = y_0 + \frac{1}{\sqrt{2\pi}wx} e^{-\frac{(\ln(x/x_c))^2}{2w^2}} \tag{6}$$

where x_c is the mean, and w is the standard deviation.

3.1 Grain-scale stress statistics

Fig. 1 displays the effect of t/d on the frequency distribution of von Mises stress and corresponding fitting curves in different sheets of t = 0.1, 0.4, 0.8, and 1.5 mm at the global strain of 0.15. Generally, the stress frequencies in all sheets with different t/d consistently agree with the normal distribution. The larger t/d leads to a better agreement of stress frequency with the normal distribution. Stress distributions in sheets with t/d = 1 have the most irregularity while that in sheets with t/d greater than 13 have a relatively better bell shape. As the grain-scale stress in metallic materials is mainly influenced by the elastic deformation of crystal lattice according to the generalised Hooke's law, the normal distribution law of stress frequency is reasonably attributed to the normal distribution of elastic strain [7, 17]. On the other hand, from a statistical point of view, the increased t/d leads to the larger mean x_c and standard deviation w of the corresponding Gaussian fitting function, i.e., higher strength of materials and the more scattered local stress due to the increasing difference of dislocation accumulation. An explanation from the perspective of dislocation density will be given in Section 3.3.

3.2 Grain-scale strain statistics

Fig. 2 demonstrates the effect of t/d on the frequency distribution of von Mises strain in different sheets of t = 0.1, 0.4, 0.8, 1.5 mm at the global strain of 0.15. The strain frequency distributions in all sheets are bell-shaped and have a good agreement with the corresponding fitting curves except that of t/d = 1, which is extraordinarily similar to the distribution law of stress. Intuitively, increased t/d leads to higher peak values, the slight shifting of fitting curves from left to right, and the gradually disappearing long-tail phenomenon at large strain, which is associated with the obvious shear band and severe strain localization. It means that more material points experience a strain as large as the externally applied strain, and more homogenous deformation is achieved on the whole due to the less severe strain localization. This observation is also in agreement with the grain size effect on global material responses, i.e., finer grains and larger *t/d* values effectively enhance the homogenous deformation, which delays the initiation of plastic instability, resulting in better ductility [3].

Statistically, as *t/d* is increased from 3 to 19 or 29, the strain frequency distributions transform from the lognormal (*t/d* $\leq$ 9) to the normal distribution (*t/d* $\geq$ 13). As shown in t = 0.8 mm, when *t/d* is larger than 13, the strain frequency has a better fitting with normal distribution than the lognormal distribution. This phenomenon is different from the findings in the literature [6, 7], which merely reported the lognormal distribution law of local strain in plastic deformation and did not consider the size effect. It also indicates that the local plastic strain undergoes a transformation from a multiplicative accumulation process to an approximately additive accumulation process.

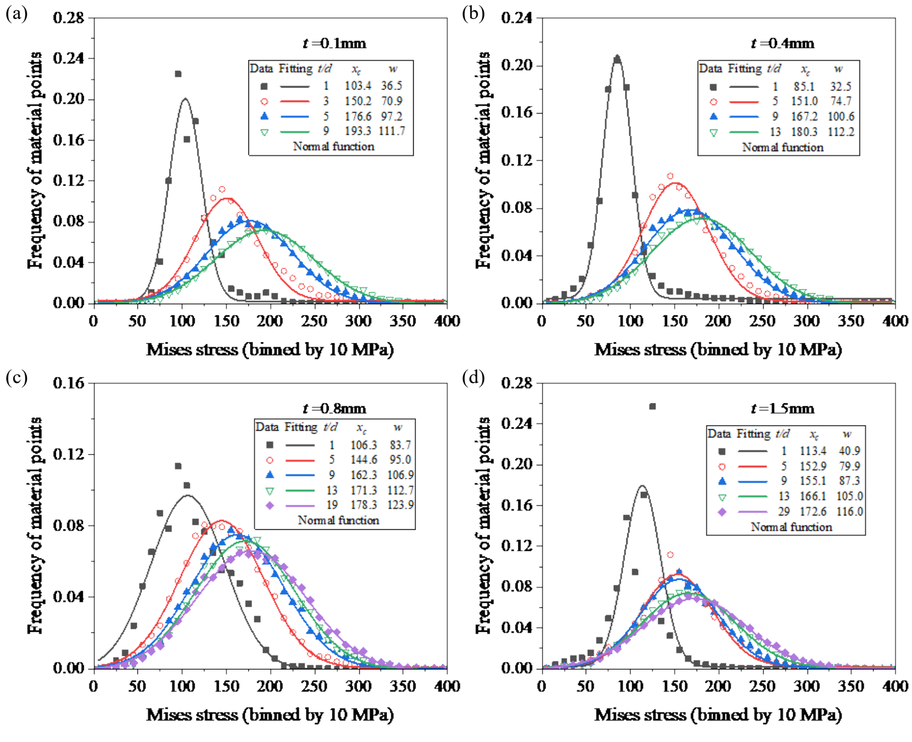

Fig. 1. Effect of *t/d* on the frequency distribution of von Mises stress at the global strain of 0.15, (a) t = 0.1 mm, *t/d* = 1, 3, 5, 9; (b) t = 0.4 mm, *t/d* = 1, 5, 9, 13; (c) t = 0.8 mm, *t/d* = 1, 5, 9, 13, 19; (d) t = 1.5 mm, *t/d* = 1, 5, 9, 13, 29. The bin size is 10 MPa.

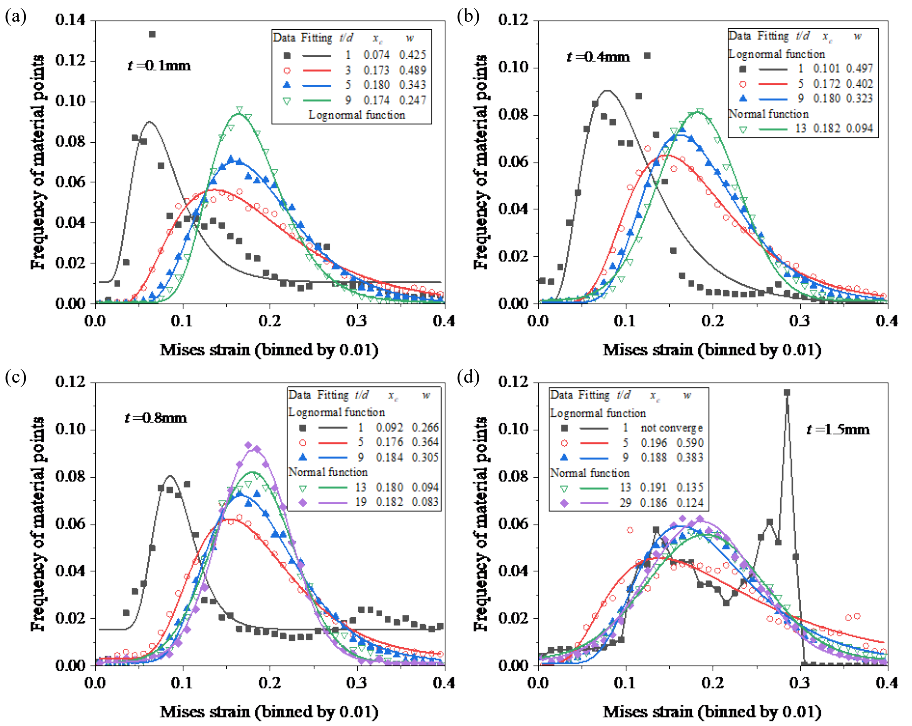

Fig. 2. Effect of *t/d* on the frequency distribution of von Mises strain at the global strain of 0.15, (a) t = 0.1 mm, *t/d* = 1, 3, 5, 9; (b) t = 0.4 mm, *t/d* = 1, 5, 9, 13; (c) t = 0.8 mm, *t/d* = 1, 5, 9, 13, 19; (d) t = 1.5 mm, *t/d* = 1, 5, 9, 13, 29. The bin size is 0.01.

3.3 Statistical dislocation density

The dislocation density is calculated by $\rho=\sum_{\alpha}^{N_s}\sum_{\beta}^{N_s}\chi^{\alpha\beta}\rho^{\beta}$, which means a sum of contributions containing the dislocation density of each slip system multiplied by the interaction coefficient between each system and all other systems, including itself (self-interaction). Fig. 3 present the effect of *t/d* on the frequency distributions of dislocation density ρ in different sheets of t = 0.1, 0.4, 0.8, 1.5 mm at the global strain of 0.15. It is shown that ρ in sheets of *t/d* =1 has a bad fitting with the lognormal distribution, especially for sheets of t = 0.4 and 1.5 mm. With increased *t/d* from 3 to 19 or 29, the distribution law of ρ transforms from a lognormal to a normal distribution ($t/d \geq 9$). Although a mild long-tail phenomenon remains in the sheet of greater *t/d*, the Gaussian function is more suitable for the fitting of frequency distribution of ρ. It can be deduced from a comparison of Fig. 1 and Fig. 3 that von Mises stress during the plastic deformation is dominantly influenced by the dislocation density ρ. After revisiting mathematical expressions of the shear strain rate in Eq. (1) and the passing stress in Eq. (2), it can be found that local stress in plastic deformation stage is mainly governed by ρ while local strain is affected by slip activities, which render the plastic strain as a multiplicative accumulation process. However, the square root of ρ narrows the gap between great and small values, thus the lognormal distribution of ρ could result in the same distribution law of local strain but not directly lead to the same distribution pattern of local stress. The spatial distributions of ρ displayed in Fig. 4 show that a general greater value of ρ and more hot spots dispersed due to the increasing difference of dislocation accumulation are found in sheets of larger *t/d*, indicating the higher material strength and the more scattered stress distribution in Fig. 1.

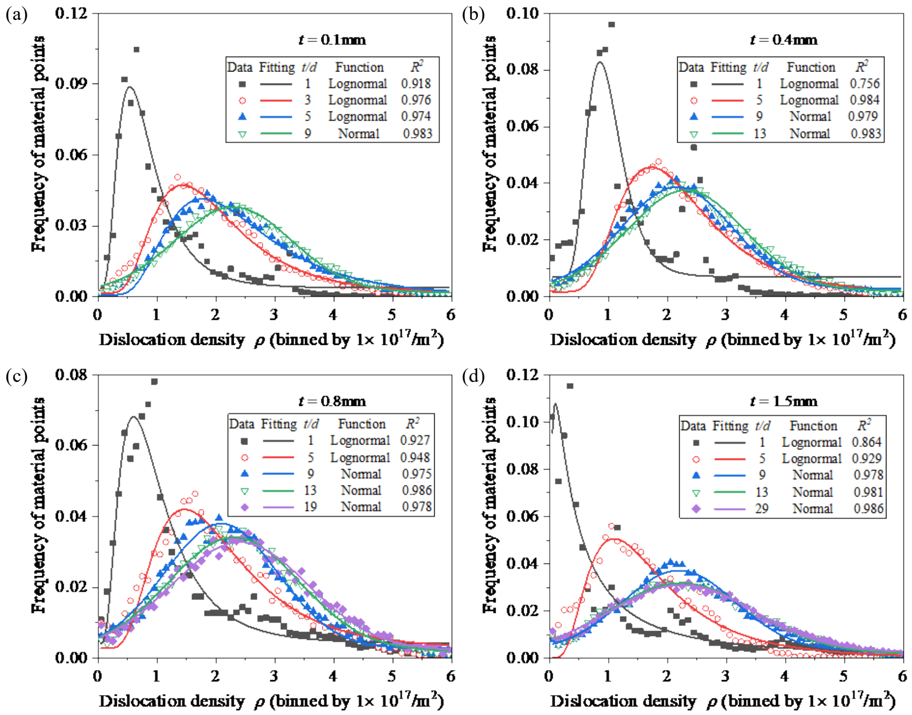

Fig. 3. Effect of *t/d* on the frequency distribution of dislocation density ρ at the global strain of 0.15, (a) $t = 0.1$ mm, $t/d = 1, 3, 5, 9$; (b) $t = 0.4$ mm, $t/d = 1, 5, 9, 13$; (c) $t = 0.8$ mm, $t/d = 1, 5, 9, 13, 19$; (d) $t = 1.5$ mm, $t/d = 1, 5, 9, 13, 29$. The bin size is $1\times10^{17}/\text{m}^2$.

4 Conclusion

Taking the polycrystalline Cu sheets with $t = 0.1$–1.5 mm and $t/d = 1$–29 as the case materials, this study implements the full-field CPFE simulation incorporated with a size-dependent dislocation-based constitutive model and statistical analyses to explore the influence of SE on the frequency distribution of von Mises stress and strain, and dislocation density in micro-scaled plastic deformation. The results reveal that SEs have the potential to modify the probability density function of micromechanical variables, resulting in an altered distribution pattern. At small *t/d* such as 1 at the global strain of 0.15, the distributions of all variables are generally irregular. As *t/d* is increased from 3 to 29, the stress frequency consistently conforms to a normal distribution. In contrast, the frequencies of strain and dislocation density undergo a transformation from a lognormal distribution (multiplicative accumulation process) to an approximately normal distribution (additive accumulation process). The established knowledge will help to elucidate the nature of distribution law of local stress or strain, and to seek for effective approaches to mitigate the scatter and uncertainty of deformed part geometry in a microforming process.

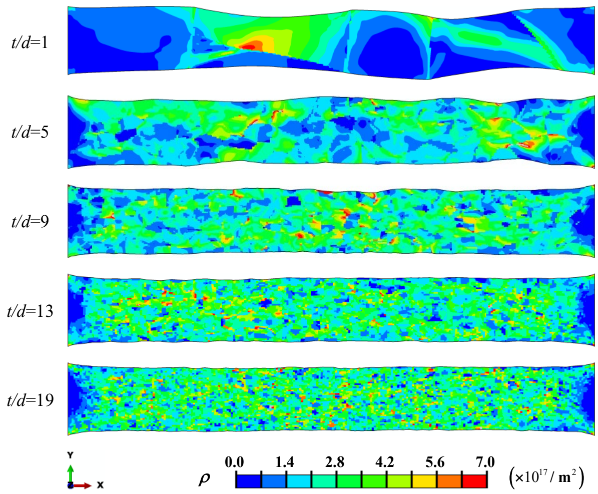

Fig. 4. Effect of *t/d* on the spatial distribution of dislocation density ρ in sheets of $t = 0.8$ mm, *t/d* = 1, 5, 9, 13, 19 at the global strain of 0.15.

Acknowledgements

The authors acknowledge the funded project of ZE1W from The Hong Kong Polytechnic University, the project of 51835011 from National Natural Science Foundation of China, and the project of 15223520 from the General Research Fund of Hong Kong Government.

References

1. M. W. Fu, J. L. Wang (2021). Size effects in multi-scale materials processing and manufacturing. Int. J. Mach. Tools Manuf., 167, 103755.
2. X.F. Tang, S.Q. Shi, M.W. Fu (2020). Interactive effect of grain size and crystal structure on deformation behavior in progressive micro-scaled deformation of metallic materials, Int. J. Mach. Tools Manuf., 148, 103473.
3. Z.Y. Feng, H. Li, D. Zhang, et al. (2022). Multi-aspect size effect transition from micro to macroscale: Modelling and experiment, Int. J. Plast., 156, 103364.

4. X. Shang, H. Zhang, Z. Cui, et al. (2020). A multiscale investigation into the effect of grain size on void evolution and ductile fracture: Experiments and crystal plasticity modeling, Int. J. Plast., 125, 133-149.
5. L. Peng, Z. Xu, Z. Gao, et al. (2018). A constitutive model for metal plastic deformation at micro/meso scale with consideration of grain orientation and its evolution, Int. J. Mech. Sci., 138-139, 74-85.
6. A. Tang, H. Liu, G. Liu, et al. (2020). Lognormal Distribution of Local Strain: A Universal Law of Plastic Deformation in Material, Phys Rev Lett, 124(15), 155501.
7. J. Chen, A.M. Korsunsky (2021). Why is local stress statistics normal, and strain lognormal? Mater. Design, 198, 109319.
8. H. Ghadbeigi, C. Pinna, S. Celotto, et al. (2010). Local plastic strain evolution in a high strength dual-phase steel, Mater. Sci. Eng. A, 527, 5026-5032.
9. K.-S. Zhang, Y.-K. Shi, J.W. Ju (2013). Grain-level statistical plasticity analysis on strain cycle fatigue of a FCC metal, Mech. Mater., 64, 76-90.
10. V.V.C. Wan, M.A. Cuddihy, J. Jiang, et al. (2016). An HR-EBSD and computational crystal plasticity investigation of microstructural stress distributions and fatigue hotspots in polycrystalline copper, Acta Mater., 115 45-57.
11. X.F. Tang, L.F. Peng, S.Q. Shi, et al. (2019). Influence of crystal structure on size dependent deformation behavior and strain heterogeneity in micro-scale deformation, Int. J. Plast., 118 147-172.
12. J. Chen, Z. Wang, A.M. Korsunsky (2022). Multiscale stress and strain statistics in the deformation of polycrystalline alloys, Int. J. Plast., 152, 103260.
13. H. Zhang, J. Liu, D. Sui, et al. (2018). Study of microstructural grain and geometric size effects on plastic heterogeneities at grain-level by using crystal plasticity modeling with high-fidelity representative microstructures, Int. J. Plast., 100, 69-89.
14. E. Orowan (1934). Zur Kristallplastizität I–III. Zeitschrift für Physik, 89, 605–659.
15. F. Roters, M. Diehl, P. Shanthraj, et al. (2019). DAMASK–The Düsseldorf Advanced Material Simulation Kit for modeling multi-physics crystal plasticity, thermal, and damage phenomena from the single crystal up to the component scale, Comp. Mater. Sci., 158, 420-478.
16. R. Quey, P. R. Dawson, F. Barbe (2011). Large-scale 3D random polycrystals for the finite element method: generation, meshing and remeshing. Comput. Method Appl. M., 200 (17-20), 1729–1745.
17. J., Oddershede, S. Schmidt, H. F. Poulsen, et al. (2011). Grain-resolved elastic strains in deformed copper measured by three-dimensional X-ray diffraction. Mater. Charact., 62(7), 651-660.