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Auction-based Parking Mechanisms Considering Withdrawal Behaviors

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Abstract

Auction-based parking mechanisms (ABPM) have been recognized as promising solutions to the parking problem. As the operator of the ABPM, the parking platform should not only be well-operated to provide parking services for demanders in need but also maintain commercial viability for sustainable operations. Motivated by this, we propose three ABPMs, namely, the ABPM without withdrawal right, the ABPM with costless withdrawal right, and the ABPM with non-free withdrawal right. Particularly, we characterize the parking demanders' withdrawal behaviors in the ABPMs. This behavior is motivated by the demanders' negative utilities derived from the auction sequence, where the demanders will first submit bid prices and are then allowed to withdraw from the parking market after the announcement of auction results when they are aware of their true valuations. We derive the equilibrium bidding strategies and the platform's expected revenues under the above three mechanisms and compare them in terms of the analytical results. It has been found that the introduction of withdrawal right will stimulate demanders to raise their bids for winning and the ABPM with non-free withdrawal right generates the highest revenue under certain conditions. A case study of parking in Beijing Financial Street has been conducted to illustrate the findings and explore managerial implications, including adapting the ABPM with non-free withdrawal right, narrowing the parking supply-demand gap, and enhancing travel experiences on the way to parking.

Keywords: parking mechanism; platform operation; withdrawal behavior; revenue boost

1. Introduction

Parking management is crucial for the future of urban mobility because almost every vehicle trip begins and ends in a parking area. A well-designed parking mechanism not only helps to match parking supplies with demands by offering parking services but also generates sufficient revenue for platform operations (Xiao and Xu, 2023). Over the past decades, various parking mechanisms have been proposed for different parking scenarios (Simićević et al., 2013; Antolín et al., 2018; Tian et al., 2018; Xu et al., 2021). Among these mechanisms, the auction-based parking mechanism (ABPM), which leverages the auction market to facilitate transactions by enacting a set of parking rules, has been found efficient for parking management (Kokolaki et al., 2014; Xiao et al., 2018; Tan et al., 2021). Although the auction approach is well-organized with straightforward rules and helps to speed up the transaction process (Pueboobpaphan et al., 2019; Ivaldi et al., 2022), the wide adoption of ABPM in real-world applications is hindered by economic inviability and unfavorable restrictions. Current ABPMs, marketed in some cities like Batesville and Shipshewana, often face economic uncertainties (Hashimoto et al., 2013; Yang et al., 2019; Rizvi et al., 2021). In addition, the parking demanders are not allowed to cancel their bids and have to pay full parking fees even though they quit parking. This may turn down potential demanders who prefer flexible parking services. Therefore, how to design profitable and favorable parking mechanisms considering demanders' withdrawal behaviors is one of the most pressing challenges faced by the auction-based parking platform.

Despite fruitful research outputs on parking management, auction-based parking studies, however, are still lacking. Current literature about auction-based parking focused on the trading process (Hashimoto et al., 2012), revenue management (Mansoori et al., 2014), pricing strategies (Xiao et al., 2020), and demand disturbance (Shao et al., 2020). These prior studies concerned parking management at the operational level and provided rigid services for parking demanders without allowing withdrawals. This degrades the effectiveness of ABPM in addressing parking problems, resulting in a waste of resources and discouraging demanders from using the platform. In reality, withdrawal behaviors (or order cancellations) are quite common in the real world (Chiew et al., 2017), but only a few studies have ever considered withdrawal behaviors in the parking market. Very recently, Lai et al. (2021) developed an operating model for parking service providers and examined the impact of random withdrawal on parking operations. The designed parking mechanism, however, was not auction-based. In addition, the withdrawal behaviors in their work were exogenously given to focus on the impact of random events. In real auctions with economic considerations, participants' withdrawal behaviors are normally derived from their endogenous motivations with negative utilities (Crowley and Sade, 2004). One stream of research was focusing on this endogenous behavior in general auctions. For example, Harstad and Rothkopf (1995) presumed that allowing withdrawals would benefit auction service providers. Motivated by this, Asker (2000) designed costless withdrawal rules for the general auction market based on analytical models and laboratory experiments. He found that the auction market with costless withdrawal right would raise the expected revenue for auction service providers. Schwarz (2021) further designed non-free withdrawal rules for a specified Talmudic auction market. A simple pre-designed rule was proposed to cover the auction service provider's loss by transferring it to withdrawers in the form of a withdrawal fee. However, this setting of the withdrawal fee is not obtained under the market equilibrium framework.

In this study, we make the first attempt to design ABPMs for flexible and optional parking services allowing demanders' withdrawal behaviors based on market equilibrium analysis. The demanders are considered to confirm their true valuations for the parking slots after the announcement of the auction results. Those who have negative utilities will withdraw from the market. Incorporating the parking features, we mathematically model three ABPMs, namely, the ABPM without withdrawal right, the ABPM with costless withdrawal right, and the ABPM with non-free withdrawal right, and derive the equilibrium bidding strategies and the platform's expected revenues. The comparisons of the three ABPMs regarding the equilibrium bidding strategies and the platform's revenues are made and managerial implications for generating sufficient revenue are explored. The contributions of this study are threefold.

- First, parking demanders' endogenous withdrawal behaviors are characterized and modeled in the ABPM. Apart from the conventional private value in general auctions, we further incorporate parking features to consider the value affected by external circumstances on the way to parking. The parking demanders submit their bids based on the above two values before the auction and realize their true valuations after the auction results announcement, of those with negative utilities will withdraw from the market.

- Second, three ABPMs are designed under the market equilibrium framework. The ABPM without withdrawal right, the ABPM with costless withdrawal right, and the ABPM with non-free withdrawal right are mathematically modeled. Comparisons are made in terms of the equilibrium bidding strategies and the platform's expected revenues to figure out the most profitable one.
- Third, pathways to commercial viability for parking platform operations are proposed. Managerial implications are derived to boost the platform's revenue, including the adaptation of ABPM with non-free withdrawal right, the reduction of the parking supply-demand gap, and the enhancement of travel experiences on the way to parking.

The remainder of the paper is organized as follows. Section 2 presents the assumptions and problem description of the ABPM. Three parking mechanisms, i.e., the ABPM without withdrawal right, the ABPM with costless withdrawal right, and the ABPM with non-free withdrawal right, are modeled and analyzed in Section 3. Section 4 provides analytical discussions concerning the impact of withdrawal behaviors and the impact of external circumstances. Section 5 elaborates on a case study as well as managerial implications. Section 6 concludes this study with further research directions.

2. Assumptions and Problem Description

2.1 Auction-based parking mechanism

The parking scenario considered in this study is specified for parking demanders with the daily working pattern requested for one-day parking, where the parking duration is assumed the same for all parking demanders, e.g., eight hours from 9:00 am to 5:00 pm in the CBD areas. A parking platform in the parking scenario operates m parking slots and provides parking services to n parking demanders. Following the ParkingAuction application (http://www.parkingauction.com) and current parking auction studies (e.g., Shao et al. (2020)), we approach the parking model as the first-price sealed auction, where winning demanders are determined by their bid prices from high to low and pay their bid prices. Specifically, the ABPM proposed in this paper composes of the following steps:

Step 1: Bid price submission

The parking demanders submit their bid prices before the deadline set by the parking platform, e.g., 10:00 pm of the day before the parking day.

Step 2: Winner determination and auction results announcement

The parking platform follows the first-price sealed auction approach to determine the winning demanders and announce the auction results once the auction terminates. If $n \le m$, all demanders can park at the slot and pay their own bid prices; otherwise, only the first-ranked m demanders can park at the slot and pay their bid prices.

Step 3: Parking implementation

The parking demanders are allowed to cancel their parking orders before the parking implementation, e.g., from 10:00 pm to 9:00 am on the next day. The winning demanders who decide to withdraw from the parking market are requested to pay the withdrawal fee c once they quit parking, while other winners are regulated to pay their bid prices before they park at the allocated slots.

2.2 Bid price basis

In Step 1, the parking demanders submit bid prices based on their expected valuations for the parking slots. These expected valuations are often specified as private values in conventional parking auction studies, e.g., Xiao and Xu (2022). Although the private value can characterize demanders' 'willingness to pay' based on their income and preferences, etc., it fails to capture the parking feature. In the auction-based parking scenario, parking demanders will have two reference values for the parking slots, including the mentioned private values and the values affected by external circumstances. The consideration of external circumstance-related values is derived from the fact that parking slots will be accessed by demanders through a driving period, which will affect demanders' evaluations for slots, e.g., the demanders who face severe traffic congestion will be likely to withdraw the parking booking. For simplicity, we consider external circumstance-related values can be predicted with the assumption that the value is approximately stable in a certain region.

The parking demanders in the proposed ABPM have not realized their true valuations for the parking slots before the auction facing their private values and the values affected by external circumstances. Two states of State 1 and State 2 are involved in the ABPM, where State 1 refers to the situation that parking demanders evaluate parking slots based on their private values and State 2 refers to the situation that parking demanders evaluate slots based on the values affected by external circumstances. Their submitted bids in Step 1 will naturally be the expected valuations with both a value component in State 1 and a value component in State 2. In particular, the private value in State 1 is heterogenous and expressed as v_i for demander i, while the value affected by external circumstances E is assumed the same for all demanders in this paper considering that the demanders driving on the same day will have the same travel experiences. Inspired by Asker (2000), we mathematically formulate their expected valuations as the summation of each state's weighted value, where a known probability is associated in each state with α for State 2 and $1-\alpha$ for State 1 with $\alpha \in [0,1]$.

2.3 Withdrawal behavior consideration

After being notified of the auction results in Step 2, the winning demanders have enough cooling-off time to re-evaluate the parking slots and confirm their true valuations in Step 3. For ease of model formulation, we follow Asker (2000), focusing on the examination of auctions with withdrawal rights, to characterize withdrawal behaviors in the ABPMs. The determination of the true valuation after the auction result announcement is set as either the private value or the value affected by external circumstances. This setting is assumed for ease of model formulations, which is inspired by the reference price effects in general trading markets that most people will determine their valuations for the commodities according to available reference values (Kalyanaram and Winer, 1995). In this way, the winning demanders will know their real utilities by calculating the difference between the true valuations and their bid prices. If the real utilities are nonnegative, the demanders will remain in the parking market because they can benefit from the mechanism; otherwise, they will withdraw from the parking market to avoid utility losses. Based on the above descriptions, we illustrate the timeline and the flow chart of the ABPM considering withdrawal behaviors in Figures 1 and 2, respectively.

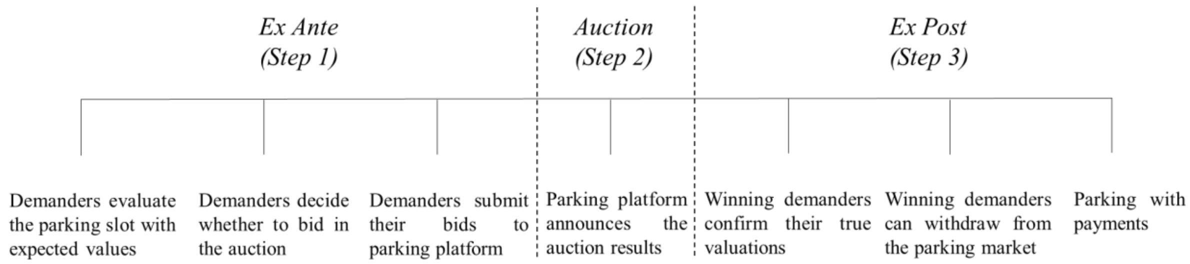


Figure 1. The timeline of the ABPM considering withdrawal behavior

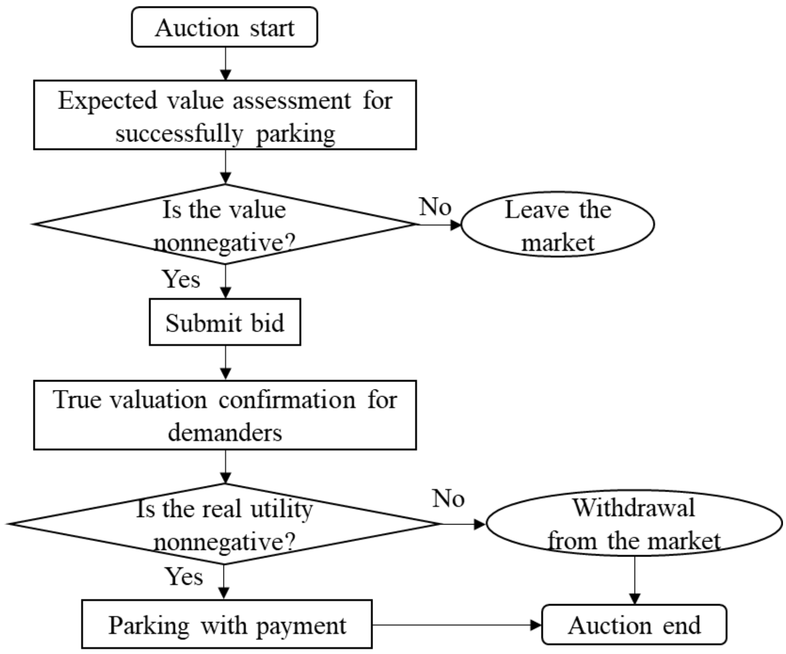


Figure 2. The flow chart of the ABPM considering withdrawal behavior

In this paper, we are concerned about how the demanders will bid and whether the parking platform can benefit from the ABPM. Therefore, we will investigate the equilibrium bidding strategy and the platform's expected revenue under the ABPM. Bidding strategies represent a series of bidding behaviors that a demander adopts with a certain purpose in an auction. Among these strategies, the equilibrium bidding strategy is defined as the behaviors resulting in a situation where a demander cannot increase his or her utility by unilaterally altering the behavior under the condition that all other demanders keep their behaviors unchanged. Considering that the withdrawal right is allowed in the ABPM, the equilibrium bidding strategy in this paper not only includes the equilibrium bid price when the demander remains in the market but also the usage of the withdrawal right. The parking platform receives revenue from the demanders and its expected revenue is defined as the sum of the payment in each state multiplied by that state's probability. To investigate the impact of withdrawal behaviors in the auction-based parking market, we mathematically model three ABPMs and compare them in terms of the equilibrium bidding strategies and the platform's expected revenues.

For easy reference, notations throughout the manuscript are listed in Table 1.

Table 1. Notations

Notations	Interpretations
m	The number of parking slots
n	The number of parking demanders
v_i	The demander $i's$ private value for the parking slots
E	The value affected by external circumstances on the way to parking
α	The probability associated in the state with external circumstances
w_i	The expected value of demander i
v^*	The reserve price
c	The withdrawal fee
$\pi_i\|_{win}$	The real utility of the winning demander i
d_i	The bid price of demander i
R	The parking platform's expected revenue
$D(v_i)$	The function of bid price with respect to the private value

$V(d_i)$	The inverse bid price function

3. Model Formulation

We formulate the model of the ABPM without withdrawal right, the ABPM with costless withdrawal right, and the ABPM with non-free withdrawal right in this section. The equilibrium bidding strategies and the platform's expected revenues are derived accordingly.

3.1. Mechanism I: the ABPM without withdrawal right

The demanders in this mechanism cannot cancel their bids and have to pay the full parking fees if they choose to quit parking later.

3.1.1. Equilibrium bidding strategy under Mechanism I

Recall that the private value of demander i in State 1 and the value affected by external circumstances in State 2 are v_i and E, respectively. Each state is associated with a known probability, i.e., α for State 2 and $1-\alpha$ for State 1. For simplicity, we normalize the private value to $[0,1]$ and derive the expected value of demander i as:

$$w_i = (1-\alpha)v_i + \alpha E \tag{1}$$

Each demander needs to decide whether to bid in the auction based on his or her expected value. If the expected value is negative, the demander will face a negative utility and will not bid. Therefore, the threshold of participation can be obtained as follows by setting $w_i \geq 0$:

$$v_i \geq -\frac{\alpha E}{1-\alpha} \tag{2}$$

It can be seen that when $E < 0$, i.e., the external circumstance is unfavorable such as rainstorms and traffic congestion, some demanders whose private value is less than the reserve price $v^* = -\frac{\alpha E}{1-\alpha}$ will not bid because their private values cannot ensure the nonnegative utility constraints. When $E \geq 0$, i.e., the external circumstance is favorable such as low-density traffic and smooth travel experience, all demanders will bid in the auction because the private values are nonnegative and the reserve price in this case equals 0.

After being notified of the auction results, demanders confirm their true valuations. Take demander i as an example, his or her true valuation for the parking slot is either v_i or E. Therefore, the real utility of the winning demander i under Mechanism I is given by:

$$\pi_{iI}\big|_{win} = \begin{cases} v_i - d_i, & \text{in State 1} \\ E - d_i, & \text{in State 2} \end{cases} \tag{3}$$

where d_i represents the bid price of demander i and subscript 'I' stands for Mechanism I.

Following Vickrey (1961), we assume that the equilibrium bidding strategy in the auction-based parking market is strictly monotonic and symmetric. Let $D(v_i)$ represent the function of bid price with respect to the private value to be followed by all demanders and $V(d_i)$ be the inverse bid price function. It can be easily seen that the probability of demander i winning in the auction is 1 when the demands are no more than supplies and otherwise $\left[V(d_i)\right]^{n-m}$ according to Vickrey (1961), Asker (2000), and Krishna (2010). We can then define the expected utility of the demander i as the expectation of his utility from the outcome given the probability of winning in the auction, which is formulated by:

$$\pi_i(v_i, d_i) = \begin{cases} (1-\alpha)v_i + \alpha E - d_i, & \text{if } n \le m \\ \left[(1-\alpha)v_i + \alpha E - d_i\right] \cdot \left[V(d_i)\right]^{n-m}, & \text{otherwise} \end{cases} \tag{4}$$

Note that the demander's equilibrium bidding strategy is derived based on the expected utility of Eq. (4) in Step 1 instead of the real utility of Eq. (3) in Step 3. Differentiating Eq. (4) with respect to d_i yields the first-order condition as follows:

$$\frac{\partial \pi_i(v_i, d_i)}{\partial d_i} = \begin{cases} -1, & \text{if } n \le m \\ (n-m)v_i'(d_i)\left[(1-\alpha)v_i + \alpha E - d_i\right](v_i)^{n-m-1} - (v_i)^{n-m}, & \text{otherwise} \end{cases} \tag{5}$$

Here, the assumption of symmetry allows $V(d_i)$ to be directly replaced by v_i. In this paper, we will only consider the case of $n > m$, considering that parking demands are normally more than supplies in downtowns and the demander's bidding strategy and platform's revenue in the case of $n \le m$ can be easily deduced as zero[1]. In theory, Eq. (5) in the case of $n > m$ should be set as zero to maximize the demander's utility, which is given by

$$(n-m)v_i'(d_i)\left[(1-\alpha)v_i+\alpha E-d_i\right](v_i)^{n-m-1}-(v_i)^{n-m}=0 \tag{6}$$

However, we cannot directly derive the equilibrium bid price from this equation because of the differential notation $v_i'(d_i)$. Luckily, the envelope theorem (Michael, 2001) provides an approach to derive the equilibrium bid price d_i^* indirectly. It is an efficient theorem to address the optimization problem with the indirect objective function for a given set of parameters, where the indirect objective function is an "envelope" of the set of optimized objective functions generated by varying the parameters of the model. Briefly, it is an alternative approach to tackle the optimization problem when the direct objective function cannot be formulated, e.g., the functions presented in Eq. (6). The envelope theorem demonstrates that the changes in the parameter rather than the variable will contribute to the change of the continuously differentiable function (Michael, 2001). In this study, $\pi_i(v_i, d_i)$ given by Eq. (4) can be viewed as a continuously differentiable function, where d_i is the variable and v_i is the parameter. According to the modeling procedure proposed by Milgrom and Segal (2002) and the analogy principle regarding the relationship between the differentiable function, the variable, and the parameter, we can obtain the following equation:

$$\frac{\partial \pi_i(v_i,d_i)}{\partial v_i}=(1-\alpha)\left[V(d_i)\right]^{n-m} \tag{7}$$

Again, we cannot derive the bid price from Eq. (7), but Milgrom and Segal (2002) further provided a theorem based on the identified relationship between the differentiable function, the variable, and the parameter (Eq. (7) in this study) to reformulate the utility function of demander i given a specific value d_i^*, which satisfies the following integral condition:

[1] The negative value of Eq. (5) in the case of $n \le m$ implies that the utility of the demander is a decreasing function with respect to the bid price. This indicates that the demanders will bid zero to maximize their utilities and thus lead to zero platform's expected revenue. The reason for this is that demanders in the market are in the dominant position and can choose any strategy to maximize their utilities.

$$\pi_i\left(v_i, d_i^*\right) - \pi_i\left(v^*, d_i^*\right) = \int_{v^*}^{v_i} (1-\alpha) x^{n-m} dx \tag{8}$$

where the integrand is derived from Eq. (7). Recall that v^* denotes the reserve price and follows $\pi_i\left(v^*, d_i^*\right) = 0$ because the demander with a reserve price is indifferent to winning and losing. By combining Eq. (4) (the case of $n > m$), we have

$$d_i^*\left(v_i\right) = (1-\alpha) v_i + \alpha E - \frac{1}{\left(v_i\right)^{n-m}} \int_{v^*}^{v_i} (1-\alpha) x^{n-m} dx \tag{9}$$

To derive the final equilibrium bid price, we must substitute the value of v^* into Eq. (9). Recall that in Mechanism I, $v^* = -\frac{\alpha E}{1-\alpha}$ when $E < 0$ and $v^*=0$ when $E \geq 0$. Therefore, the equilibrium bid price of demander i under Mechanism I can be calculated as follows:

$$d_{iI}^* = \begin{cases} (1-\alpha)\dfrac{n-m}{n-m+1} v_i + \alpha E, & \text{if } E \geq 0 \\ (1-\alpha)\left[\dfrac{n-m}{n-m+1} v_i + \dfrac{1}{(n-m+1)\left(v_i\right)^{n-m}}\left(\dfrac{-\alpha E}{1-\alpha}\right)^{n-m+1}\right] & \\ +\alpha E, & \text{if } E < 0 \text{ and } v_i \geq -\dfrac{\alpha E}{1-\alpha} \end{cases} \tag{10}$$

Based on Eq. (10), we have noticed that the equilibrium bid price of the demander in the ABPM is a weighted sum of the optimal bid price in each state. For $E \geq 0$, $\frac{n-m}{n-m+1} v_i$ is the optimal bid price of State 1, which is verified by multi-unit first-price sealed auction studies (Krishna, 2010). E can be regarded as the optimal bid price of State 2. The probabilities of State 1 and State 2 are $1-\alpha$ and α respectively, which can be regarded as the weight in each state. For $E < 0$, the bid price of State 1 is that of the standard optimal bid price with a reserve price of $\frac{-\alpha E}{1-\alpha}$. E is again the optimal bid price of State 2. The two probabilities of $1-\alpha$ and α are also the weights. This observation provides a simplified approach to determining the equilibrium bid price, that is, to identify the probability and bid price in each state and then integrate all states together.

3.1.2. Platform's expected revenue under Mechanism I

The expected revenue of the parking platform in this paper is defined as the sum of the payment in each state multiplied by that state's probability. According to Eq. (10), the expected revenue when $E \geq 0$ is formulated as $\int_0^1 d_{iI}^* n(v_i)^{n-m} dv_i$, where $(v_i)^{n-m}$ represents the probability of demander i winning in the auction. Similarly, the expected revenue in the case of $E<0$ and $v_i \geq -\frac{\alpha E}{1-\alpha}$ is formulated as $\int_{-\frac{\alpha E}{1-\alpha}}^1 d_{iI}^* n(v_i)^{n-m} dv_i$. The parking platform's expected revenue is then given by

$$R_I = \begin{cases} (1-\alpha)\dfrac{n(n-m)}{(n-m+1)(n-m+2)} + \alpha\dfrac{nE}{n-m+1}, & \text{if } E \geq 0 \\ (1-\alpha)\dfrac{n(n-m)\left[1-\left(\dfrac{-\alpha E}{1-\alpha}\right)^{n-m+2}\right] + n(n-m+2)\left(1+\dfrac{\alpha E}{1-\alpha}\right)\left(\dfrac{-\alpha E}{1-\alpha}\right)^{n-m+1}}{(n-m+1)(n-m+2)} & \\ +\alpha\dfrac{nE}{n-m+1}\left[1-\left(\dfrac{-\alpha E}{1-\alpha}\right)^{n-m+1}\right], & \text{if } E<0 \text{ and } v_i \geq -\dfrac{\alpha E}{1-\alpha} \end{cases} \tag{11}$$

3.2. Mechanism II: the ABPM with costless withdrawal right

3.2.1. Equilibrium bidding strategy under Mechanism II

The demanders in this mechanism can freely cancel their bids without any cost. Given the two states to confirm the true valuations, the real utility of the winning demander i changes from Eq. (3) to Eq. (12) as follows:

$$\pi_{iII}\big|_{win} = \begin{cases} v_i - d_i, & v_i - d_i \geq 0 \text{ in State 1} \\ 0, & v_i - d_i < 0 \text{ in State 1} \\ E - d_i, & E - d_i \geq 0 \text{ in State 2} \\ 0, & E - d_i < 0 \text{ in State 2} \end{cases} \tag{12}$$

Eq. (12) means that whenever the demander realizes a loss, this loss can be reduced to zero. We then divide the demanders into two groups, i.e., $v_i \leq E$ and $v_i > E$, as follows:

Case 1: $v_i \leq E$

In this case, a rational demander prefers to submit a bid less than E for nonnegative utility, no matter which state is evaluated. The best response from any competing demander is to bid marginal higher as $d_i + \varepsilon$. By doing so, the competing demanders face the following two states: If State 2 is evaluated, all demanders have the incentive to bid higher in response to a bid less than E, which forms of Bertrand competition (Meunier and Carre, 2013) to drive all bids up to at least the level of E. If State 1 is evaluated, the competing demander will fall into a situation of raising bids to E and then withdrawing from the market to earn zero utility. To sum up, the equilibrium bid price of demanders with $v_i \le E$ is E.

Case 2: $v_i > E$

We further consider the following two situations regarding the value of E. For $E < 0$, the demanders will bid according to their standard strategies for the multi-unit first-price sealed auction if State 1 is evaluated, i.e., $\frac{n-m}{n-m+1}v_i$, while will choose to withdraw from the parking market because of negative utilities if State 2 is eventuated; For $E \ge 0$, the bid price E induces the lower bound of any bidding strategy with zero utility according to the analysis with $v_i \le E$. This makes the equilibrium bid price of the demander with $v_i > E$ is isomorphic to the equilibrium bid price of the demander with a reserve price of E. By setting α=0 and $v^* = E$ in Eq. (9), the bid price of demander i when $v_i > E \ge 0$ is formulated, i.e., $\frac{n-m}{n-m+1}v_i + \frac{E^{n-m+1}}{(n-m+1)(v_i)^{n-m}}$. If State 1 is evaluated, the demanders will remain in the market because $v_i - \left[\frac{n-m}{n-m+1}v_i + \frac{E^{n-m+1}}{(n-m+1)(v_i)^{n-m}}\right] > 0$. If State 2 is evaluated, the demanders will withdraw from the market because $E - \left[\frac{n-m}{n-m+1}v_i + \frac{E^{n-m+1}}{(n-m+1)(v_i)^{n-m}}\right] < 0$. Therefore, the equilibrium bid price of demander i under Mechanism II is given by

$$d_{iII}^{*}=\begin{cases}E, & \text{if } v_i \le E\\ \dfrac{n-m}{n-m+1}v_i+\dfrac{E^{n-m+1}}{(n-m+1)(v_i)^{n-m}}, & \text{if } v_i > E \text{ and } E \ge 0\\ \dfrac{n-m}{n-m+1}v_i, & \text{if } v_i > E \text{ and } E < 0\end{cases} \tag{13}$$

Note that Eq. (13) excludes the strategies of demanders who choose to withdraw from the market because they contribute nothing to the revenue. This is also the reason why the probability of each state is not involved in Eq. (13).

3.2.2. Platform's expected revenue under Mechanism II

Three cases, i.e., $E<0$, $0\le E<1$, and $E\ge 1$, are discussed here based on the value range of v_i and the relationship between v_i and E of Eq. (13).

Case 1: $E\ge 1$

According to Eq. (13), the demanders who remain in the market will bid E. The expected revenue of the platform is then given by

$$R_{II}=\int_0^1 \alpha E n(v_i)^{n-m}\,dv_i=\alpha\frac{nE}{n-m+1} \tag{14}$$

Note that only the demanders whose true valuations are the value in State 2 will remain in the market and pay the fees to the platform.

Case 2: $0\le E<1$

In this case, the demanders with $v_i\le E$ will bid E and those with $v_i>E\ge 0$ will bid $\dfrac{n-m}{n-m+1}v_i+\dfrac{E^{n-m+1}}{(n-m+1)(v_i)^{n-m}}$. The platform's expected revenue is then given by

$$\begin{aligned}R_{II}&=\int_0^E \alpha E n(v_i)^{n-m}\,dv_i+\int_E^1(1-\alpha)\left[\frac{n-m}{n-m+1}v_i+\frac{E^{n-m+1}}{(n-m+1)(v_i)^{n-m}}\right]n(v_i)^{n-m}\,dv_i\\&=\alpha\frac{nE^{n-m+2}}{n-m+1}+(1-\alpha)\frac{n(n-m)\left(1-E^{n-m+2}\right)+n(n-m+2)(1-E)E^{n-m+1}}{(n-m+1)(n-m+2)}\end{aligned} \tag{15}$$

If $v_i\le E$, only the demanders whose true valuations are the value in State 2 will remain in the market, otherwise, those whose true valuations are the value in State 1 will remain in the market.

Case 3: $E < 0$

The demanders whose true valuations are private values in State 1 will remain in the market and bid $\frac{n-m}{n-m+1}v_i$ in this case. The expected revenue of the platform is given by

$$R_{II} = \int_0^1 (1-\alpha)\frac{n-m}{n-m+1}v_i n(v_i)^{n-m}\, dv_i = (1-\alpha)\frac{n(n-m)}{(n-m+1)(n-m+2)} \tag{16}$$

To sum up, the expected revenue of the parking platform under Mechanism II is given by

$$R_{II} = \begin{cases} \alpha \frac{nE}{n-m+1}, & \text{if } E \geq 1 \\ \alpha \frac{nE^{n-m+2}}{n-m+1} + (1-\alpha)\frac{n(n-m)(1-E^{n-m+2}) + n(n-m+2)(1-E)E^{n-m+1}}{(n-m+1)(n-m+2)}, & \text{if } 0 \leq E < 1 \\ (1-\alpha)\frac{n(n-m)}{(n-m+1)(n-m+2)}, & \text{if } E < 0 \end{cases}$$

(17)

3.3. Mechanism III: the ABPM with non-free withdrawal right

3.3.1. Equilibrium bidding strategy under Mechanism III

The demanders in this mechanism will pay the withdrawal fee c when canceling the bids. For ease of modeling, we also normalize the withdrawal fee, i.e., $c \in [0,1]$. Given the two states, the real utility of the winning demander i changes from Eq. (3) to Eq. (18) as follows:

$$\pi_{iIII}\big|_{win} = \begin{cases} v_i - d_i, & v_i - d_i \geq -c \text{ in State 1} \\ -c, & v_i - d_i < -c \text{ in State 1} \\ E - d_i, & E - d_i \geq -c \text{ in State 2} \\ -c, & E - d_i < -c \text{ in State 2} \end{cases} \tag{18}$$

Rational demanders will consider these potential results presented in Eq. (18) before submitting the bids to earn at nonnegative utilities. Following the classification of Mechanism II, we also divide the demanders into two groups, i.e., $v_i \leq E$ and $v_i > E$.

Case 1: $v_i \leq E$

According to the Bertrand competition analysis, the demanders have incentives to bid higher up to E if State 2 is evaluated. This analysis is valid for the case of $c \le E$ because demanders will bid c when $c > E$ and suffer negative utilities. The demanders are further divided into the following two groups if State 1 is evaluated: If $c > v_i$, the demanders will choose to remain in the market because the high cost of canceling parking orders hinders demanders' withdrawal behaviors. They will then bid according to their standard strategies in auctions, i.e., $\frac{n-m}{n-m+1}v_i$. If $c \le v_i$, the analysis is similar to the case of $v_i > E \ge 0$ under Mechanism II, in which the equilibrium bid price of the demander with $c \le v_i$ is isomorphic to the equilibrium bid price of the demander with a reserve price of c. By setting α=0 and $v^* = c$ in Eq. (9), the bid price of demander i is $\frac{n-m}{n-m+1}v_i + \frac{c^{n-m+1}}{(n-m+1)(v_i)^{n-m}}$.

Case 2: $v_i > E$

We consider the following two cases, i.e., $E < 0$ and $E \ge 0$. For $E < 0$, demanders will bid the same price as that of Case 1 if State 1 is evaluated, i.e., bid $\frac{n-m}{n-m+1}v_i$ if $c > v_i$ and bid $\frac{n-m}{n-m+1}v_i + \frac{c^{n-m+1}}{(n-m+1)(v_i)^{n-m}}$ otherwise, while will bid without certain analytical solutions if State 2 is evaluated due to negative utilities suffered. We thus cannot derive the complete bidding strategy when $E < 0$ because the equilibrium bid price of the parking demander is a weighted sum of the optimal bid price in each state. For $E \ge 0$, we have the following two discussions: If $c > E$, no equilibrium bidding strategies can be found. If $c \le E$, the demanders will bid E when State 2 is evaluated and bid $\frac{n-m}{n-m+1}v_i + \frac{c^{n-m+1}}{(n-m+1)(v_i)^{n-m}}$ when State 1 is evaluated like Case 1.

The equilibrium bid prices of demander i under Mechanism III are then given by

$$d_{iIII}^{*}=\begin{cases}\alpha E+(1-\alpha)\dfrac{n-m}{n-m+1}v_i, & \text{if } c>v_i \text{ and } c\le E\\ \alpha E+(1-\alpha)\left[\dfrac{n-m}{n-m+1}v_i+\dfrac{c^{n-m+1}}{(n-m+1)(v_i)^{n-m}}\right], & \text{if } c\le v_i \text{ and } c\le E\end{cases} \tag{19}$$

3.3.2. Platform's expected revenue under Mechanism III

Given the constraint $c \le E$, we formulate the platform's expected revenue in two parts, i.e., the revenue from demanders with $v_i < c$ and with $v_i \ge c$, which is given by

$$\begin{aligned} R_{III} &= \int_0^c \left[\alpha E+(1-\alpha)\frac{n-m}{n-m+1}v_i\right] n(v_i)^{n-m}\,dv_i \\ &\quad + \int_c^1 \left\{\alpha E+(1-\alpha)\left[\frac{n-m}{n-m+1}v_i+\frac{c^{n-m+1}}{(n-m+1)(v_i)^{n-m}}\right]\right\} n(v_i)^{n-m}\,dv_i \\ &= \alpha\frac{nE}{n-m+1}+(1-\alpha)\frac{n(n-m)+n(n-m+2)(1-c)c^{n-m+1}}{(n-m+1)(n-m+2)} \end{aligned} \tag{20}$$

It can be found that the platform's expected revenue under Mechanism III is influenced by the value of c. The optimal withdrawal fee is discussed in the following proposition.

Proposition 1. *The optimal withdrawal fee of Mechanism III is* E *when* $E<\dfrac{n-m+1}{n-m+2}$ *and* $\dfrac{n-m+1}{n-m+2}$ *otherwise.*

This proposition provides a significant implication for the parking platform regarding how to enforce the withdrawal fee if Mechanism III is chosen. Let R_{III}^{*} denote the maximal revenue that can be achieved by the parking platform when Mechanism III is employed and c^{*} be the optimal withdrawal fee that induces the maximal revenue. Then, we have the following conclusions: (i) if $E<\dfrac{n-m+1}{n-m+2}$, it follows that $c^{*}=E$ and $R_{III}^{*}=\dfrac{\alpha nE}{n-m+1}+\dfrac{(1-\alpha)n\left[(n-m)+(n-m+2)(1-E)E^{n-m+1}\right]}{(n-m+1)(n-m+2)}$; (ii) if $E\ge\dfrac{n-m+1}{n-m+2}$, it follows that $c^{*}=\dfrac{n-m+1}{n-m+2}$ and $R_{III}^{*}=\dfrac{\alpha nE}{n-m+1}+\dfrac{(1-\alpha)n\left[(n-m)+\left(\dfrac{n-m+1}{n-m+2}\right)^{n-m+1}\right]}{(n-m+1)(n-m+2)}$.

Detailed proof can be found in Appendix A.

4. Analytical Discussions

The proposed three mechanisms are closely interrelated. Mechanism III will reduce to Mechanism I if the withdrawal fee c is enforced as an infinitely large value and Mechanism III will reduce to Mechanism II if c is set to zero. We first compare them in terms of the equilibrium bidding strategies and the platform's expected revenues to identify the impact of withdrawal behaviors in Subsection 4.1. We then discuss the impact of the external circumstance concerning how it affects parking demanders' withdrawal behaviors and the parking platform's revenue in Subsection 4.2.

4.1. The impact of withdrawal behaviors

(i) Comparing the equilibrium bidding strategies

According to Eqs. (10), (13), and (19), we have the following proposition:

Proposition 2. *The ABPM with costless withdrawal right always results in the highest bids among the three mechanisms, followed by the ABPM with non-free withdrawal right and the ABPM without withdrawal right in a descending order.*

This proposition identifies the differences in bidding strategies under three mechanisms: the demanders under Mechanism II and Mechanism III will bid more aggressively than the demanders under Mechanism I and the demanders under Mechanism III will bid conservatively compared with the demanders under Mechanism II. In brief, the introduction of the withdrawal right encourages demanders to raise their bids for winning, whereas the charge for withdrawal from the market will stimulate them to lower their bids to assure nonnegative utilities. Detailed proof can be found in Appendix B.

(ii) Comparing the platform's expected revenues

According to Eqs. (11), (17), and Eq. (20), we have the following proposition:

Proposition 3. *The ABPM with costless withdrawal right results in higher revenue than the ABPM without withdrawal right when* $E < 0$ *and lower revenue otherwise; the ABPM with non-free withdrawal right results in higher revenue than the ABPM without withdrawal right* *and the ABPM with costless withdrawal right when* $c \le E$.

This proposition figures out the most beneficial mechanism with the largest revenue: among the three ABPMs, Mechanism III is found to generate the largest revenue under certain conditions. Detailed proof of this proposition is provided in Appendix C.

4.2. The impact of external circumstances

Considering that Mechanism III generates the largest revenue among the three mechanisms in certain cases, we only discuss the impact of external circumstances on parking demanders' withdrawal behaviors and the parking platform's revenue in this mechanism. In particular, the results without consideration of external circumstances can be calculated by setting $\alpha = 0$ in Eqs. (19)-(20), which is given by

$$d'^{*}_{iIII} = \begin{cases} \dfrac{n-m}{n-m+1} v_i, & \text{if } c > v_i \text{ and } c \le E \\ \left[\dfrac{n-m}{n-m+1} v_i + \dfrac{c^{n-m+1}}{(n-m+1)(v_i)^{n-m}} \right], & \text{if } c \le v_i \text{ and } c \le E \end{cases} \tag{21}$$

$$R'_{III} = \frac{n(n-m) + n(n-m+2)(1-c)c^{n-m+1}}{(n-m+1)(n-m+2)} \tag{22}$$

The differences between the results with and without the consideration of external circumstances of Mechanism III can be obtained as follows:

$$\Delta d_{iIII} = d^{*}_{iIII} - d'^{*}_{iIII} = \begin{cases} \alpha E - \alpha \dfrac{n-m \;\; v_i}{n-m+1}, & \text{if } c > v_i \text{ and } c \le E \\ \dfrac{\alpha E \;\; n-m+1 \;\; - \alpha \left[\;\; n-m \;\; v_i + c^{1-m+n} v_i^{m-n} \right]}{n-m+1}, & \text{if } c \le v_i \text{ and } c \le E \end{cases} \tag{23}$$

$$\Delta R_{III} = R_{III} - R'_{III} = \frac{\alpha N c^{n-m+1} \;\; n-m+2 \;\;\; c-1 \;\; + \alpha N \left[\;\; n-m \;\;\; E-1 \;\; + 2E \right]}{n-m+2 \;\;\; n-m+1} \tag{24}$$

According to Eqs. (23)-(24), we have the following proposition:

Proposition 4. *External circumstances on the way to parking may positively or negatively affect the parking demanders' equilibrium bid prices and the platform's expected revenue, which are dependent on the detailed values of the State 2 and the withdrawal fee.*

For the comparison result concerning the parking demanders' equilibrium bid prices (see Eq. (23)), we can find that $\Delta d_{iIII} > 0$ when $c > v_i$ and $c \le E$, indicating that the consideration of external circumstances will encourage demanders to raise their bid prices. The result on the other condition (i.e., $c \le v_i$ and $c \le E$), however, is dependent: $\Delta d_{iIII} > 0$ can be found when $E - v_i \quad n - m \ + E > c^{n-m+1} v_i^{m-n}$ while $\Delta d_{iIII} \le 0$ can be found otherwise. This demonstrates that the consideration of external circumstances may or may not raise demanders' bid prices, which is dependent on the situation with specific E and c. For the comparison result concerning the parking platform's revenue (see Eq. (24)), we can also find the result is dependent: $\Delta R_{III} > 0$ can be found when $c^{n-m+1} \quad n - m + 2 \quad c - 1 > n - m \quad 1 - E \ - 2E$ while $\Delta R_{III} \le 0$ can be found otherwise. This indicates that the consideration of external circumstances may or may not boost the platform's expected revenue.

4.3. Further discussions about the comparisons of three mechanisms

In this subsection, we further consider two concerns based on the mechanisms proposed in Section 3, including the discussions concerning travel costs and fairness issues. Comparisons of the three mechanisms for these two concerns are intuitively discussed.

(i) Discussions concerning travel costs

This study focuses on parking demanders' perceived values for slots, where both the private values and the values affected by external circumstances will potentially affect the expected valuations. Further consideration can be extended to cover the travel costs of demanders. Generally speaking, the large travel costs will hinder demanders from canceling parking orders because these travel costs can be regarded as sunk costs once the withdrawal right is used. Intuitively, more parking demanders, of whom have already suffered large travel costs, are expected to choose Mechanism I, followed by Mechanisms III and II. Policymakers who designed ABPMs with withdrawal right are encouraged to measure the average travel costs of demanders to estimate their market shares, which can be explored in future studies.

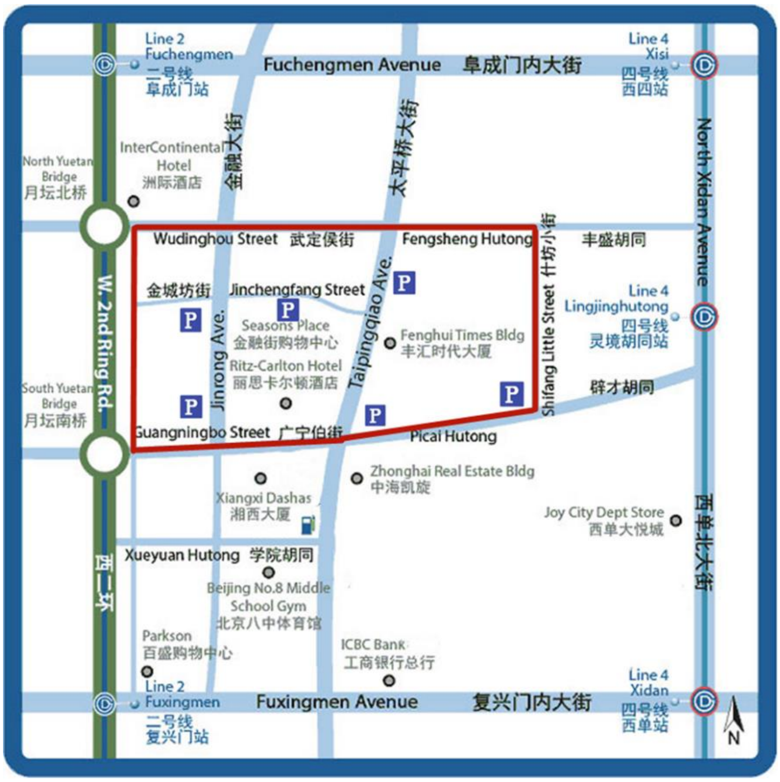
(ii) Discussions concerning fairness issues

This study focuses on the parking demanders' equilibrium bidding strategies and the platform's expected revenues in a static environment. Further consideration can be extended to a dynamic environment. Generally speaking, deploying recurrent parking auctions, especially allowing withdrawal behaviors, will stimulate parking demanders to bid in the auctions. This might lead to a low probability of winning for demanders as the auction-based parking market evolves with market expansion, where the related fairness issues can be explored. Intuitively, Mechanism I will induce the highest winning rate in auctions for demanders, followed by Mechanisms III and II in a descending order. This can be explained by the fact that the mechanism without withdrawal right will hinder some potential demanders from choosing the auction market due to the rigid rule, while the mechanism with costless and non-free withdrawal right will encourage demanders to bid in the auction due to no or fewer penalties. In other words, policymakers of Mechanisms II and III should concern with fairness issues regarding demanders' winning rate in auctions, which is a novel topic in future studies.

5. Case Study and Managerial Implications

We perform a case study focusing on the parking in the core area of Beijing Financial Street (see Figure 3). Six main public parking lots in this area provide a total of 1022 available slots, i.e., $m = 1,022$ (http://service.jtw.beijing.gov.cn/uservice/app/service/parkingResult), which cannot satisfy the estimated parking demands of 1150 vehicles, i.e., $n = 1,150$. The parking demanders in this area will first face external circumstances on the way to parking, e.g., traffic congestion, extreme weather with snow and rain, temporary traffic control, optimization of traffic lights, etc. Based on the real conditions of Xicheng District (the location of Beijing Financial Street), we estimate the value affected by these circumstances as 0.2, i.e., $E = 0.2$, which means the comprehensive influence of these external circumstances based in Beijing is favorable for parking demanders. We also estimate the same proportion of the private value and the value affected by external circumstances, i.e., $\alpha = 50\%$. For comparison, we normalize the private value and withdrawal fee as $v \in [0,1]$ and $c \in [0,1]$. Specifically, the withdrawal fee is assumed to be set at a minor value of 0.1, i.e., $c = 0.1$, while the private value in the basic case study is regarded as a variable ranging from 0 to 1 to cover all demanders.

In particular, this case study is based on available real data, e.g., the number of parking slots, and reasonable estimations, e.g., the parking demands. Kindly note that we have normalized the private value of parking demanders for ease of model building, making some related parameters also normalized values. Therefore, we did not present the units in the following tables and figures.



Source: Beijing Municipal Commission of Transport

Figure 3. Illustration of the core area of Beijing Financial Street

The results of equilibrium bid prices and the platform's expected revenues of the three mechanisms are first compared in Subsection 5.1. Sensitivity analysis with respect to significant parameters is then presented in Subsection 5.2. Managerial implications are finally provided in Subsection 5.3 accordingly.

5.1. Basic results

We first present the results of the equilibrium bid prices of demanders under the three ABPMs in Figure 4. It can be found from this figure that the bid prices of demanders under Mechanism II are always the highest, followed by the bid prices under Mechanism III and Mechanism I in a descending order. This is consistent with the finding of Proposition 2, which illustrates that the allowance of the withdrawal right encourages demanders to raise their bids for winning. In particular, the non-free withdrawal right (Mechanism III) makes the demanders bid conservatively compared with the costless withdrawal right (Mechanism II) because demanders under Mechanism III must count their costs to withdraw from the parking market to ensure nonnegative utilities. We also emphasize that the values in Figure 3 are not real values because some parameters, e.g., withdrawal fee and private value, are normalized, but it doesn't hinder the effectiveness of comparisons of three mechanisms regarding bid prices for the case of parking in Beijing Financial Street.

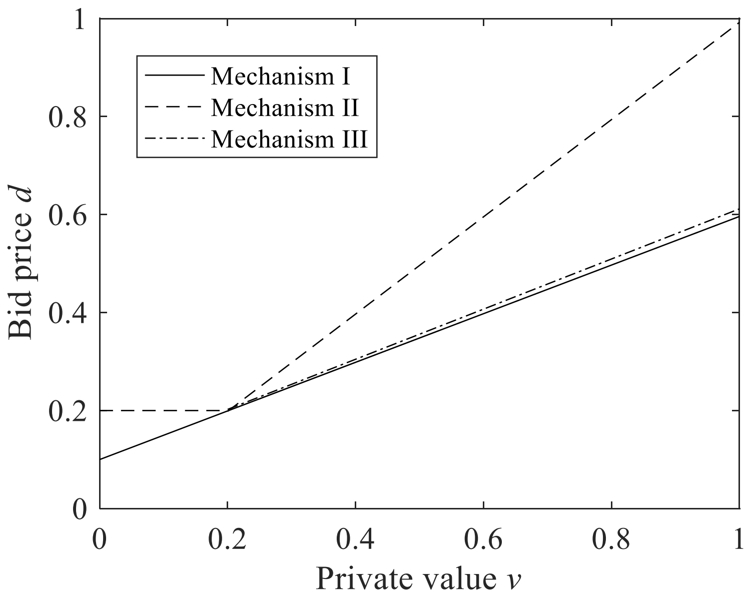


Figure 4. Equilibrium bid prices of demanders under the three ABPMs

The results of the platform's revenues of the three ABPMs are presented in Table 2. Considering that the platform's revenues are not influenced by private values, we can directly obtain the unique revenue of each mechanism. It can be found from this table that Mechanism III is the most beneficial one among the three ABPMs, followed by Mechanism I and Mechanism II in a descending order. This result is consistent with the finding of Proposition 3 under the condition of $c < E$, which indicates that the parking platform operated in Beijing Financial Street is encouraged to adopt the ABPM with non-free withdrawal right.

Table 2. The revenue of the parking platform under the three ABPMs

Mechanism	I	II	III
Platform's revenue	5.28	4.39	5.29

5.2. Sensitivity analysis of the parking platform's revenue

Generating more revenues is the main task of the parking platform. Based on the comparisons of the three ABPMs, Mechanism III should be implemented. According to Eq. (20), the supply-demand gap $(n-m)$, the value affected by external circumstances E, the withdrawal fee c, and the probability of State 2 α play significant roles in determining the revenue. Therefore, we present the sensitivity analysis with respect to the four parameters for Mechanism III.

(i) Sensitivity analysis with respect to the supply-demand gap

We set the supply-demand gap $(n-m)$ as the variable ranges from 0 to 256. Other parameters are the same as the basic setting, i.e., α=50%, $n=1{,}150$, $E=0.2$, and $c=0.1$. A sensitivity analysis of the platform's revenue with respect to the supply-demand gap is presented in Figure 5. It can be found from the figure that the decrease of $(n-m)$ will help to generate more revenue for the parking platform under Mechanism III. The value of R^* is gradually increased with the decrease of $(n-m)$ and the increment rate becomes larger when the gap decreases. This indicates that the parking platform is encouraged to mitigate the gap between parking demands and supplies by providing more slots.

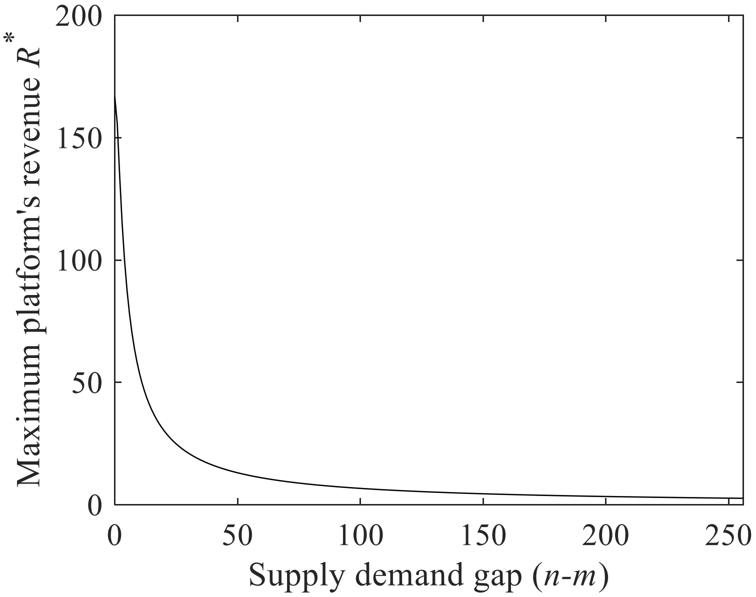


Figure 5. Sensitivity analysis of the platform's revenue with respect to the supply-demand gap

(ii) Sensitivity analysis with respect to the value affected by external circumstances

We set the value affected by external circumstances E as the variable ranges from 0.1 to 0.3 because the constraint $0.1 = c \le E$ is held. Other parameters are also the same as the basic setting. A sensitivity analysis of the platform's revenue with respect to E is presented in Figure 6. A linear increase of R^* with the increase of E can be found from this figure, which indicates that the positive travel experiences on the way to the final parking slots with large value affected by external circumstances are always favored. Comparing the results of Figures 5 and 6, we find that the decrease of $(n-m)$ results in higher revenue than the increase of E in a reasonable region, e.g., the decrease of $(n-m)$ from 50 to 10 results in large revenue compared with the revenue due to the increase of E from 0.2 to 0.3.

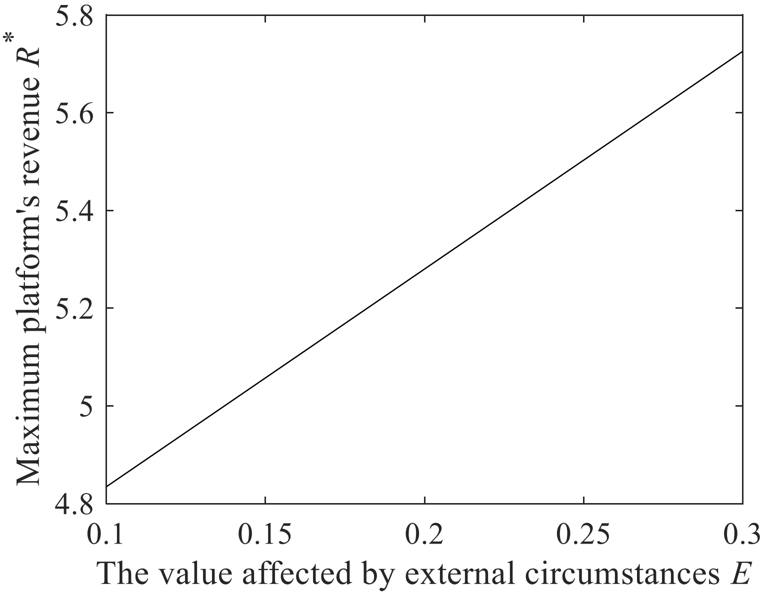


Figure 6. Sensitivity analysis of the platform's revenue with respect to the value affected by external circumstances

(iii) Sensitivity analysis with respect to the withdrawal fee

We set the withdrawal fee c as the variable ranges from 0 to 1. A sensitivity analysis of the platform's revenue with respect to c is presented in Figure 7. It can be found from this figure that R^* will first increase and then decrease with the increase of c from 0 to 1, the largest revenue can be found by setting $c = 0.992$. This implies that the largest withdrawal fee, i.e., $c = 1$, does not necessarily lead to the largest platform's revenue. This should be attributed to the fact that a larger withdrawal fee will drive some demanders to bid conservatively. In particular, in this specific case of parking in Beijing Financial Street, the parking platform should set the withdrawal fee as $c^* = E = 0.2$ because the value affected by external circumstances in this case is at a low level, i.e., $E = 0.2 < \frac{n-m+1}{n-m+2} = 0.992$. To further boost the platform's revenue, the parking platforms in Beijing Financial Street are encouraged to increase E together with the increase of c in a proper range.

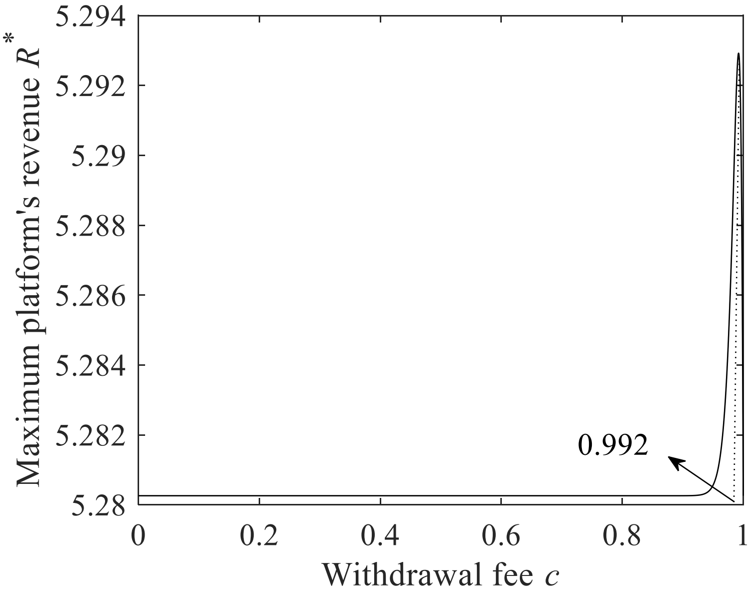


Figure 7. Sensitivity analysis of the platform's revenue with respect to the withdrawal fee

(iv) Sensitivity analysis with respect to the probability of State 2

We set the probability of State 2 α as the variable ranges from 0 to 1. A sensitivity analysis of the platform's revenue with respect to α is presented in Figure 8. It can be found from this figure that the decrease of α contribute to increasing the platform's revenue. This means that the auction-based parking platform prefers the low probability of external circumstances. Specifically, the platform gains the highest revenue with zero α. It indicates that external circumstances in this specific case would negatively affect the platform's revenue. In this case, measures to mitigate the negative impact of external circumstances should be adopted.

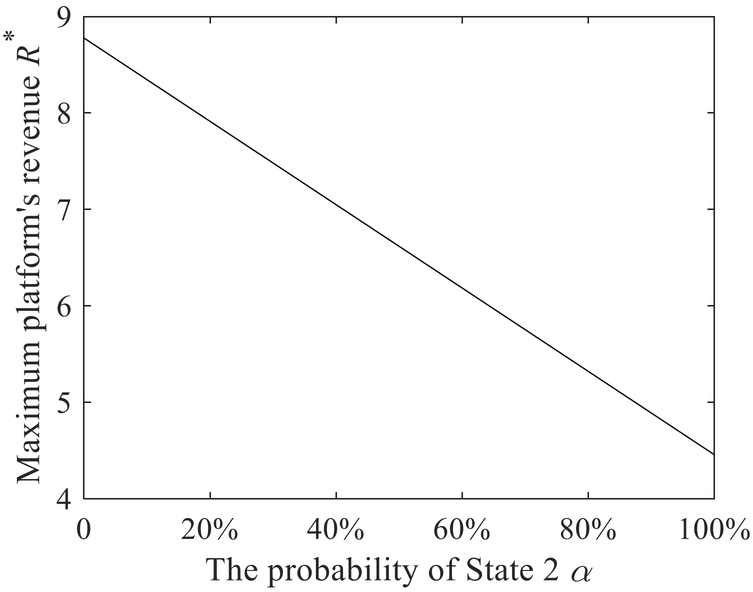


Figure 8. Sensitivity analysis of the platform's revenue with respect to the probability of State 2

5.3. Managerial implications

To boost the parking platform's revenue for sustainable development, some managerial measures are provided accordingly.

(i) Adopt the ABPM with non-free withdrawal right and set the optimal withdrawal fee

This insight provides a direct implication of which mechanism should be adopted. Based on the theoretical comparisons of three ABPMs and the case study specified for the parking in Beijing Financial Street, the parking platforms are encouraged to adopt the ABPM with non-free withdrawal right considering that this mechanism generates the highest revenue among the three ABPMs. The implementation of this mechanism lies in the enforcement of the withdrawal fee, where the fee should be set as E when the value affected by external circumstances is at a low level, i.e., $E < \frac{n-m+1}{n-m+2}$, while should be set as $\frac{n-m+1}{n-m+2}$ when the value affected by external circumstances is at a high level, i.e., $E \geq \frac{n-m+1}{n-m+2}$. The parking platforms can easily determine the optimal withdrawal fee to achieve the highest revenue.

(ii) Narrow the parking supply-demand gap

This insight provides an implication for the parking platforms regarding how many parking slots should be supplied. Considering the relationship between parking demands and parking supplies, i.e., the value of $(n-m)$, will affect the platform's revenue, it is thus of profound significance to provide an appropriate number of parking slots to achieve the maximum revenue. Supported by the sensitivity analysis, the parking platforms are encouraged to mitigate the parking supply-demand gap by providing sufficient parking slots because the small parking demand-supply gap will contribute to boosting the platform's revenue. The parking platform can rent spare private slots from residential areas but will suffer the costs related to acquiring the slots' using right. If the revenue gains obtained by this measure can offset the costs, renting spare private slots would be a viable approach to promote the revenue of the platform.

(iii) Enhance travel experiences on the way to parking

This insight provides an implication for the parking platforms' responses to external circumstances. Considering that a large positive value of E will lead to a large revenue, the parking platforms are suggested to enhance parking demanders' travel experiences on the way to the slot. For example, the parking platform can improve the resilience of the parking system by providing a clear indication for the location of the parking slot along the road and real-time information about traffic conditions for the parking demanders, which can help demanders to avoid congested roads for short travel time. This could be realized by collaborating with the navigation company but the platform will suffer the costs related to acquiring the navigation services. Again, collaborating with the navigation company would be a viable approach for boosting the platform's revenue if the revenue gains can offset the costs.

6. Conclusions

In this study, we made the first attempt to model the withdrawal behaviors in the ABPM. Incorporating parking features, parking demanders' expected valuations for the parking slots before the auction include the conventional private values in general auctions and the values affected by external circumstances. They will confirm true valuations for the parking slots after the announcement of the auction results. Those with negative utilities will withdraw from the parking market. To better characterize demanders' withdrawal behavior and examine its impact on the parking market, we propose three mechanisms, namely, the ABPM without withdrawal right, the ABPM with costless withdrawal right, and the ABPM with non-free withdrawal right. Comparisons of the three mechanisms in terms of the equilibrium bidding strategies and the platform’s expected revenues are then conducted, where the findings can be derived with the following conclusions: The allowance of the withdrawal right encourages bid raising and the ABPM with non-free withdrawal right generates the highest revenue under certain conditions. Further discussions regarding the parking platform’s operations are discussed, including the enforcement of the optimal withdrawal fee under the ABPM with non-free withdrawal right ($c = E$ when $E < \frac{n-m+1}{n-m+2}$ while $c = \frac{n-m+1}{n-m+2}$ when $E \geq \frac{n-m+1}{n-m+2}$) and the quantitative impact of significant parameters on the platform’s profit (the decrease of parking supply-demand gap and the increase of the value in State 2 contribute to boosting the platform's profit). Managerial implications based on the above findings are proposed accordingly, including adapting the ABPM with non-free withdrawal right, narrowing the parking supply-demand gap, and enhancing travel experiences on the way to parking.

To further explore the withdrawal behaviors in the auction-based parking market, several aspects are worthy of investigation for future studies. First, this study focused on the macroscopic economic performances of the parking platform such as the revenue. The microscopic issues about operations management such as how to fill the parking vacancies due to the usage of the withdrawal right can be pursued. Second, we assumed that all demanders were risk-neutral for simplicity, which can be further relaxed by considering risk-averse demanders in the ABPMs. Last, laboratory/field experiments can be adopted to simulate the random withdrawal behaviors of parking demanders and their bid prices in the ABPM with non-free withdrawal right.

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Appendix A. Proof of Proposition 1

According to Eq. (20), the platform's expected revenue under Mechanism III is influenced by c. Let $F(c)=\alpha\frac{nE}{n-m+1}+(1-\alpha)\frac{n(n-m)+n(n-m+2)(1-c)c^{n-m+1}}{(n-m+1)(n-m+2)}$; we find that $F'(c)=(1-\alpha)\frac{nc^{n-m}}{n-m+1}\left[(n-m+1)-c(n-m+2)\right]$. Two stationary points, i.e., $c=0$ and $c=\frac{n-m+1}{n-m+2}$, are found by setting $F'(c)=0$. Moreover, $F(c)$ is a monotonic increasing function in $0\le c\le\frac{n-m+1}{n-m+2}$ and is a monotonic decreasing function in $\frac{n-m+1}{n-m+2}<c\le 1$. Given the constraint $c\le E$, we discuss the following two cases regarding the relationship of E and $\frac{n-m+1}{n-m+2}$.

Case 1: $E<\frac{n-m+1}{n-m+2}$

The optimal withdrawal fee is set as $c^*=E$ and the highest revenue of the parking platform is calculated by $R_{III}^*=\frac{\alpha nE}{n-m+1}+\frac{(1-\alpha)n\left[(n-m)+(n-m+2)(1-E)E^{n-m+1}\right]}{(n-m+1)(n-m+2)}$.

Case 2: $E\ge\frac{n-m+1}{n-m+2}$

The optimal withdrawal fee is set as $c^*=\frac{n-m+1}{n-m+2}$ and the highest revenue of the parking platform is calculated by $R_{III}^*=\frac{\alpha nE}{n-m+1}+\frac{(1-\alpha)n\left[(n-m)+\left(\frac{n-m+1}{n-m+2}\right)^{n-m+1}\right]}{(n-m+1)(n-m+2)}$.

Appendix B. Proof of Proposition 2

Three lemmas are provided to support Proposition 2.

Lemma B.1. *The demanders in the ABPM with costless withdrawal right will bid more* *aggressively than the demanders in the ABPM without withdrawal right.*

Proof. We compare the bid prices under Mechanisms I and II. Three cases, i.e., $0 \le v_i \le E$, $0 \le E < v_i$, and $E < 0 \le v_i$, are discussed.

Case 1: $0 \le v_i \le E$

$d_{iII}^* - d_{iI}^* = (1-\alpha)\left(E - \frac{n-m}{n-m+1} v_i\right) \ge (1-\alpha)\frac{E}{n-m+1} \ge 0$, which means $d_{iII}^* \ge d_{iI}^*$.

Case 2: $0 \le E < v_i$

Suppose $d_{iII}^* \le d_{iI}^*$, we derive that $-\frac{E^{n-m+1}}{(n-m+1)(v_i)^{n-m}} \ge \alpha\left(E + \frac{n-m}{n-m+1} v_i\right)$, which is a contradiction as it implies that a nonpositive number is greater than a positive number. Therefore, we have $d_{iII}^* > d_{iI}^*$.

Case 3: $E < 0 \le v_i$

Suppose $d_{iII}^* > d_{iI}^*$; we derive that $\frac{1-\alpha}{(n-m+1)(v_i)^{n-m}}\left(\frac{-\alpha E}{1-\alpha}\right)^{n-m+1} + \alpha E < \alpha \frac{n-m}{n-m+1} v_i$.

Let $G(E) = \frac{1-\alpha}{(n-m+1)(v_i)^{n-m}}\left(\frac{-\alpha E}{1-\alpha}\right)^{n-m+1} + \alpha E$, $d_{iII}^* > d_{iI}^*$ is established when $G(E) < 0$ in $-\frac{v_i(1-\alpha)}{\alpha} \le E < 0$. We further derive that $G'(E) = \alpha\left[1 - \frac{1}{(v_i)^{n-m}}\left(\frac{-\alpha E}{1-\alpha}\right)^{n-m}\right]$ and find that $G'(E) = 0$ when $E = -\frac{v_i(1-\alpha)}{\alpha}$ and $G'(E) > 0$ when $E = 0$. $G(E)$ is thus monotonic with a positive slope and $G(E) < G(0) = 0$, which means $d_{iII}^* > d_{iI}^*$.

The proof of Lemma B.1 is completed. □

Lemma B.2. *The demanders under the ABPM with non-free withdrawal right will bid more aggressively than the demanders under the ABPM without withdrawal right.*

Proof. We compare the bid prices under Mechanisms I and III. Given the constraint $0 \le c \le E$, two cases, i.e., $0 \le c \le v_i$ and $0 \le v_i < c$, are discussed.

Case 1: $0 \le c \le v_i$

$$d_{iIII}^* - d_{iI}^* = (1-\alpha)\frac{c^{n-m+1}}{(n-m+1)(v_i)^{n-m}} \ge 0$$, which means $d_{iIII}^* \ge d_{iI}^*$.

Case 2: $0 \le v_i < c$

$d_{iIII}^* - d_{iI}^* = 0$, which means $d_{iIII}^* = d_{iI}^*$.

The proof of Lemma B.2 is completed. □

Lemma B.3. *The demanders under the ABPM with non-free withdrawal right will bid more aggressively than the demanders under the ABPM with costless withdrawal right.*

Proof. We compare the bid prices under Mechanisms II and III. Given the constraint $0 \le c \le E$, three cases, i.e., $0 \le v_i < c \le E$, $0 \le c \le v_i \le E$, and $0 \le c \le E < v_i$, are discussed.

Case 1: $0 \le v_i < c \le E$

$$d_{iIII}^* - d_{iII}^* = (\alpha - 1)E + (1-\alpha)\frac{n-m}{n-m+1}v_i < -\frac{(1-\alpha)E}{n-m+1} < 0$$, which means $d_{iIII}^* < d_{iII}^*$.

Case 2: $0 \le c \le v_i \le E$

$$d_{iIII}^* - d_{iII}^* = (\alpha - 1)E + (1-\alpha)\left[\frac{n-m}{n-m+1}v_i + \frac{c^{n-m+1}}{(n-m+1)(v_i)^{n-m}}\right] \le (1-\alpha)(v_i - E) \le 0$$,

which means $d_{iIII}^* \le d_{iII}^*$.

Case 3: $0 \le c \le E < v_i$

Suppose $d_{iIII}^* > d_{iII}^*$; we derive that $-\alpha\frac{n-m}{n-m+1}v_i > \frac{E^{n-m+1} + (1-\alpha)c^{n-m+1}}{(n-m+1)(v_i)^{n-m}}$, which is a contradiction as it implies that a negative number is greater than a nonnegative number.

Therefore, we have $d_{iIII}^* \le d_{iII}^*$.

The Lemma B.3 is completed. □

Proposition 2 is jointly supported by Lemmas B.1- B.3.

Appendix C. Proof of Proposition 3

Three lemmas are provided to support Proposition 3.

Lemma C.1. *The ABPM with costless withdrawal right results in higher revenue than the ABPM without withdrawal right when* $E < 0$ *and otherwise lower revenue.*

Proof. We compare the platform's expected revenues under Mechanisms I and II. Three cases, i.e., $E \ge 1$, $0 \le E < 1$, and $-\frac{1-\alpha}{\alpha} v_i \le E < 0$, are discussed.

Case 1: $E \ge 1$

$R_{II} - R_I = -(1-\alpha)\frac{n(n-m)}{(n-m+1)(n-m+2)} \le 0$, which means $R_I \ge R_{II}$.

Case 2: $0 \le E < 1$

Let $H(E) = R_{II} - R_I$, we find that $H'(E) = \frac{\alpha n}{n-m+1}\left(E^{n-m+1} - 1\right) + 2(\alpha - 1) n E^{n-m+1}$ $+(\alpha-1) n E^{n-m} < 0$. $H(E)$ must be monotonous in $0 \le E < 1$ with a negative slope. Thus, $H(E) < H(0) = 0$ and $R_I > R_{II}$.

Case 3: $-\frac{1-\alpha}{\alpha} v_i \le E < 0$

Let $I(E) = R_{II} - R_I$, we find that $I'(E) = \alpha n\left(-\frac{\alpha E}{1-\alpha}\right)^{n-m} - \frac{\alpha n(n-m)}{n-m+1}\left(-\frac{\alpha E}{1-\alpha}\right)^{n-m+1} - \frac{\alpha n}{n-m+1}$ and $I''(E) = -\frac{\alpha^2 n(n-m)}{1-\alpha}\left(1 + \frac{\alpha E}{1-\alpha}\right)\left(-\frac{\alpha E}{1-\alpha}\right)^{n-m-1}$. In the region $-\frac{1-\alpha}{\alpha} \le E < 0$, $I''(E) < 0$ is established and $I'(E) < I'\left(-\frac{1-\alpha}{\alpha}\right) = 0$, which means $I(E)$ is monotonous in

$-\frac{1-\alpha}{\alpha} \le E < 0$ with a negative slope. Considering that $v_i \in [0,1]$, we further derive that $I(E)$ must be monotonous in $-\frac{1-\alpha}{\alpha} v_i \le E < 0$ with a negative slope, indicating $I(E) > I(0) = 0$ and $R_I < R_{II}$.

To sum up, Lemma C.1 is supported by the conclusion that $R_I \ge R_{II}$ when $E \ge 0$ and $R_I < R_{II}$ when $-\frac{1-\alpha}{\alpha} v_i \le E < 0$. □

Lemma C.2. *The ABPM with non-free withdrawal right results in higher revenue than the ABPM without withdrawal right when* $c \le E$.

Proof. We compare the platform's expected revenues under Mechanisms I and III. Given the constraint $0 \le c \le E$, only one case is discussed. We have $R_{III} - R_I = (1-\alpha)\frac{n(1-c)c^{n-m+1}}{n-m+1} \ge 0$, which means $R_{III} \ge R_I$ and Lemma C.2 is valid. □

Lemma C.3. *The ABPM with non-free withdrawal right results in higher revenue than the ABPM with costless withdrawal right when* $c \le E$.

Proof. We compare the platform's expected revenues under Mechanisms II and III. Given the constraint $0 \le c \le E$, two cases, i.e., $0 \le E < 1$ and $E \ge 1$, are discussed.

Case 1: $0 \le c \le E < 1$

$$R_{III} - R_{II} = \frac{\alpha n E}{n-m+1} + (1-\alpha)n\frac{(n-m)E^{n-m+2} + (n-m+2)(1-c)c^{n-m+1}}{(n-m+1)(n-m+2)} - \frac{nE^{n-m+1}\left[E(2\alpha-1)+(1-\alpha)\right]}{n-m+1}.$$

We consider two cases with the value of α. When $1/2 \le \alpha < 1$, we have $R_{III} - R_{II} > \alpha\frac{nE\left(1-E^{n-m}\right)}{n-m+1} + (1-\alpha)n\frac{(n-m)E^{n-m+2} + (n-m+2)(1-c)c^{n-m+1}}{(n-m+1)(n-m+2)} \ge 0$, which means $R_{III} > R_{II}$. When $0 \le \alpha < 1/2$, we have $R_{III} - R_{II} > \frac{\alpha n E}{n-m+1}$ $+(1-\alpha)n\frac{(n-m)E^{n-m+2} + (n-m+2)(1-c)c^{n-m+1}}{(n-m+1)(n-m+2)} - \frac{(1-\alpha)nE^{n-m+1}}{n-m+1}$. Let $J(E) = \frac{\alpha n E}{n-m+1}$

$+(1-\alpha)n\frac{(n-m)E^{n-m+2}+(n-m+2)(1-c)c^{n-m+1}}{(n-m+1)(n-m+2)}-\frac{(1-\alpha)nE^{n-m+1}}{n-m+1}$ and it can be found that $J''(E)<0$. Therefore, $J'(E)>J'(1)>0$ and $J(E)\geq J(0)\geq 0$, which means $R_{III}\geq R_{II}$.

Case 2 $0\leq c\leq 1\leq E$

$R_{III}-R_{II}=(1-\alpha)\frac{n(n-m)+n(n-m+2)(1-c)c^{n-m+1}}{(n-m+1)(n-m+2)}\geq 0$, which means $R_{III}\geq R_{II}$.

To sum up, Lemma C.3 is valid. □

Proposition 3 is jointly supported by Lemmas C.1- C.3.

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