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Stochastic Modeling and Analysis of Unicast Performance in 802.11p VANETs

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***Abstract*—802.11p inter-vehicle communications is attracting growing research attentions. Most of previous works focused on studying the broadcasting performance, whereas the unicast performance is seldom addressed. Furthermore, most of conventional studies fail to consider a realistic vehicular traffic distribution. In view of this inadequacy, this paper proposes an 802.11p unicast modeling methodology under a practical stochastic traffic model and demonstrates its feasibility. Our analytical results show that the vehicular distribution on a road with signal control has strong influence on unicast performance.**

I. Introduction

Vehicular ad-hoc network (VANET) has been becoming a hot research field, as it has the potential to greatly improve road safety and traffic/transportation efficiency [1]. 802.11p [2], an amendment to the IEEE 802.11 standard used in Wireless LAN, defines the physical and MAC layers in the Dedicated Short Range Communications (DSRC) protocol stack for inter-vehicle communications in VANETs. In DSRC, the 5.9 GHz band is divided into seven channels, with one used as the control channel (CCH) for safety message transmission and the others used as service channels (SCH) for non-safety information transmission. In DSRC, the CCH and the SCH switch with each other every 50 ms according to the IEEE 1609.4 standard [3].

In VANET, broadcast is widely used in road safety and traffic efficiency applications [4]. Thus far, a body of prior works are devoted to the study of broadcasting performance in VANETs (e.g., [5, 6]). However, in some applications (e.g., infotainment applications [7]), unicast is more preferable. For example, some Internet data may be exchanged via unicast between a road side unit (RSU) and the cars within its communication range. The main reason for using unicast instead of broadcast is that unicast with a retransmission mechanism is capable of ensuring successful transmission.

The two-dimensional (2-D) Markov chain was commonly used in many previous studies to delineate the process of transmission contention in CSMA/CA protocols. Bianchi [8] proposed a 2-D Markov Chain model for analyzing the performance of 802.11. Han et al. [9] also established a similar 2-D Markov model to analyze the throughput of 802.11p VANET. Wang et al. [10] designed a different 2-D Markov modeling method by considering the probability that the channel is busy in the sensing period.

However, all of the above works did not consider VANET with practical vehicle traffic distribution. The distribution of vehicles on the road actually affects the performance of VANET. Hence, modeling traffic to reflect the actuality is critical for the analysis of VANET performance. Ma [4] put forward a one-dimensional (1-D) highway traffic model in which vehicles are randomly distributed according to the Poisson process. Nonetheless, this model only depicted a homogeneous scenario which is not realistic enough. Our prior work [11] proposed a more reasonable stochastic traffic model considering the effect of traffic lights on the road and interactions of vehicles. With this stochastic traffic model, the vehicular traffic distribution can be analyzed, and a vehicular density profile (a function of space and time that describes the stochastic fluctuations of the car density over a road) can be obtained. In [5, 6], we coupled 802.11p VANET with the stochastic traffic model in [11] to evaluate the broadcasting efficiency for safety messages.

In this paper, we model 802.11p unicast and analyze its performance under the aforementioned stochastic traffic model. The major contributions of this paper are three-fold:

(i) We combine the 2-D Markov chain, which is similar to that in [8, 9], with the stochastic traffic model for our analysis. We analyze the unicast transmission probability for a vehicle at a given location. The relationship between the transmission probability and the collision probability (caused by other concurrent transmissions or hidden terminals) are derived with respect to the vehicular density knowledge.

(ii) We consider a common interference model on top of the stochastic traffic model. With this interference model, another relationship between the collision probability and the transmission probability is obtained.

(iii) The combination of the two relationships above allows the computation of transmission and collision probabilities at different locations with respect to the mean vehicular density profile, and important performance metrics such as delay and throughput can be further obtained.

II. The Stochastic Traffic Model

This paper considers a one-directional, single-lane and semi-infinite road scenario, as depicted in Fig. 1. The road is divided into many segments, with a traffic light located at the junction of any two adjacent road segments. Vehicles arrive at location 0, move toward the right direction, and can join or leave the road at junctions. Let $C^{+}(x, t)$ and $C^{-}(x, t)$ be the number of cars joining and leaving the region (0, x] for time from $-\infty$ to t respectively; also let $N(x, t)$ and $Q(x, t)$ be the number of vehicles moving into (0, x] at time t and the number of vehicles moving past location x until time t. Then, we have

$$N(x,t) = C^{+}(x,t) - C^{-}(x,t) - Q(x,t), \quad (1)$$

$$\frac{dn(x,t)}{dt} = c^{+}(x,t) - c^{-}(x,t) - \frac{\partial v(x,t)}{\partial x} n(x,t). \quad (2)$$

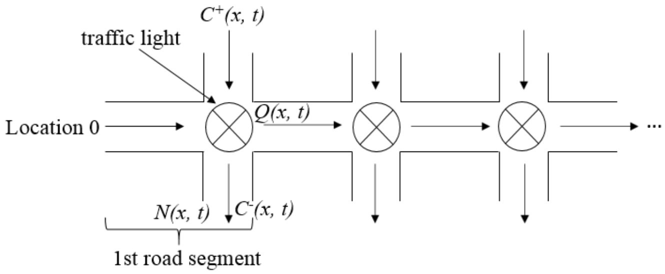

Figure 1. The 1-D Road condition considered.

In (2), $n(x,t) = \frac{\partial N(x,t)}{\partial x}$ is the vehicular density at location x; $c^+(x,t) = \frac{\partial^2 C^+(x,t)}{\partial x \partial t}$ and $c^-(x,t) = \frac{\partial^2 C^-(x,t)}{\partial x \partial t}$ denote the joining rate density and the leaving rate density respectively; $q(x,t) = \frac{\partial Q(x,t)}{\partial t}$ is the flow rate at x and is given by $q(x,t) = n(x,t)v(x,t)$ [11], where $v(x, t)$ is the vehicular velocity at location x and time t. All variables in (2) are random variables, and we can apply expectation to (2). From now on, we use $n(x,t)$ to denote $\mathbb{E}[n(x,t)]$ for simplicity. The ordinary differential equation (ODE) in (2) can be solved to find the vehicular density profile $n(x,t)$.

If the arrival process at location 0 is a Poisson process, and $v(x,t)$ is independent of $n(x,t)$, then, the mean number of vehicles in the range $(x_1, x_2]$ is given by

$$E[N(x_1,x_2,t)] = \overline{N}(x_1,x_2,t) = \int_{x_1}^{x_2} n(x,t)dx. \quad (3)$$

We refer the readers to [11] for more details regarding the stochastic traffic model.

III. The Markov Model of 802.11p Unicast

Here we consider non-safety message transmission (e.g., Internet multimedia) in 802.11p VANET by unicast. The SCH is used for this purpose. In 802.11p, the CSMA/CA medium access mechanism is employed (see details in [9]), and there are four channel access priorities used for four data classes. Different priorities are assigned with different sizes of the Contention Window (CW) and the Arbitration Inter-Frame Space (AIFS). Moreover, time is divided into slots of 13 μs in 802.11p. In this paper, we only focus on the packet transmission in the highest-priority access class. Scenarios of multiple priorities and/or multiple channels can be analyzed by extending our work. In the following analysis, we consider the transmission on the channel under saturated condition and analyze the corresponding transmission or collision rate through a Markov model.

Similar to [9, 10], we employ a 2-D Markov chain to model the 802.11p unicast in this paper as shown in Fig. 2. From the figure, we can obtain a relationship between the transmission probability τ for a car at location a to transmit a packet at the beginning of a time slot and the collision probability q for the packet to be collided. We use $<i, k>$ to denote each state in Fig. 2, of which i means the back-off stage (the number of collisions that the packet has encountered), and k is the back-off counter value. Let p be the probability that the channel sensed busy by the car at location a, m be the maximum exponential back-off stages, and f be the maximum number of attempts after stage m and before the packet is dropped. By denoting the maximum contention window size at stage i as w_i, we have $w_i = 2^i w_0$ for $0 \leqslant i \leqslant m$, and $w_i = 2^m w_0$ for $m < i \leqslant m+f$, where $w_0 = CW_{min}+1$ and $w_m = CW_{max}+1$.

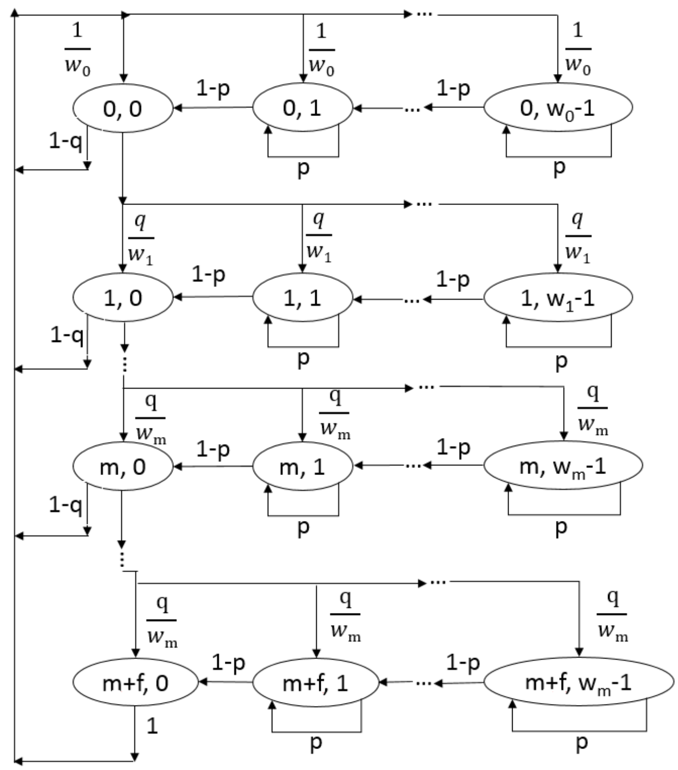

Figure 2. The 2-D Markov chain for 802.11p unicast.

Through solving the stationary distribution of the Markov chain in Fig. 2, we obtain a relationship between τ and q as follows.

$$\tau = \frac{1-q^{m+f+1}}{1-q}\left[\frac{1-q^{m+f+1}}{1-q} + \frac{1}{2(1-p)}\left(w_0\frac{1-(2q)^{m+1}}{1-2q} - \frac{1-q^{m+1}}{1-q}\right) + \right.$$

wm−12(1−p)qm+1(1−qf)1−q]−1, m≠0. (4)

For the highest-priority access class, we have CW_{min}=3 and CW_{max}=7 [12]. Then w_0=4, w_m=8 and m=1. For simplicity, we set f=∞, i.e., packets are permitted to be re-transmitted until successful. Substituting the four parameters above into (4), the relation between τ and q at location a is given by

$$\tau = \frac{2-2p}{5-2p+4q}. \quad (5)$$

The channel is sensed busy by the car at location a (or simply car a) if there is at least one car transmitting in the time slot within car a's sensing range R_I. According to the empirical result in [11], we assume that the number of cars in a road segment is Poisson distributed. Then, we have

$$p = 1 - \sum_{n=0}^{\infty} P(n \text{ cars in the sensing range})\left(1-\tau_{R_I}\right)^n$$
$$= 1 - \sum_{n=0}^{\infty} \frac{\overline{N}(R_I)^n e^{-\overline{N}(R_I)}}{n!}\left(1-\tau_{R_I}\right)^n$$
$$= 1 - e^{-\overline{N}(R_I)\tau_{R_I}},$$

(6) where τ_{R_I} is the average transmission probability for all cars within car a's sensing range, and $\overline{N}(R_I)$ is the mean number of cars in the sensing range of car a. Since a time slot of 13 μs is extremely small, $n(x,t)$ barely varies within a slot.

Thus, we omit t in $n(x,t)$for simplicity, and from (3), we have $\overline{N}(R_I) = \int_{a-R_I}^{a} n(x)dx + \int_{a}^{a+R_I} n(x)dx$.

IV. The Interference Model for Transmission

For an ongoing unicast transmission from car a to car x, if there is a third car, b, transmits within the sensing range of car x, then it interferes with the signal reception at car x, or collision happens at car x. Here, car b is called the interference source or interferer. We model the interference by analyzing the possible locations of interferers and their effect on the transmission of car a. This analysis will provide us another relationship between the collision probability q and the transmission probability τ. For simplicity, we here consider the data transmission in the backward direction, as the forward transmission can be analyzed similarly.

Fig. 3 illustrates an interference scenario, where car a is transmitting to car x located within the transmission range R_S, and an interferer, either car b or c, is located within the sensing range R_I of car x. Obviously, the possible locations of interferers can be divided into three non-overlapping regions: $[a–R_S, a)$, $(a, x+R_I]$, and $[x–R_I, a–R_S)$. Note that as x varies within $[a–R_S, a)$, the regions $(a, x+R_I]$ and $[x–R_I, a–R_S)$ will vary accordingly.

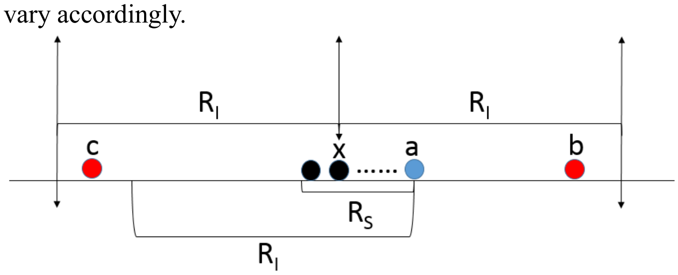

Figure 3. An interference scenario.

As we assume cars arrive according to a Poisson process, the probability that there are i cars in the transmission range of car a is

$$P\{i \text{ cars in } [a - R_S, a)\} = \frac{\overline{N}(R_S)^i e^{-\overline{N}(R_S)}}{i!}. \quad (7)$$

where $\overline{N}(R_S) = \int_{a-R_S}^{a} n(x)dx$. We assume that the i cars in $[a - R_s, a)$ have equal probability to be chosen as the receiver of the unicast from car a, i.e., the k-th car within the i cars be chosen as the receiver has a probability

$$P(k) = \frac{1}{i}. \quad (8)$$

In the following, we go through the analysis for the three non-overlapping interference regions one by one.

A. Interferers in Region 1: $[a–R_S, a)$

If there is at least one interferer in this region, the packet reception at car x will be ruined wherever the interferers are located. When car a transmits with probability τ, the probability that the receiver (the k-th car) is interfered is then given by

$$P(k_{Intf1}) = \tau P(k)[1 - (1 - \tau_{R_S})^i], \quad (9)$$

where τ_{R_S} is the average transmission probability of cars in Region 1. Note that (9) is a function of the number of cars i in $[a - R_s, a)$. Hence, combining (7) and (9), we have the overall interference probability in region 1 as follows.

$$P_1 = \sum_{i=1}^{\infty} P\{i \text{ cars in } [a - R_S, a)\} \sum_{k=1}^{i} P(k_{Intf1}). \quad (10)$$

B. Interferers in Region 2: $(a, x+R_I]$

In this case, whether the packet reception at car x is ruined depends on whether there is any car transmits in Region 2. Let $S(x)$ be the probability that no car transmits in Region 2. Specifically, we have

$$\begin{aligned} S(x) &= \sum_{n=0}^{\infty} P(n \text{ cars in Region 2}) P(n \text{ cars do not transmit}) \\ &= \sum_{n=0}^{\infty} \frac{\overline{N}_S(x)^n e^{-\overline{N}_S(x)}}{n!} (1 - \tau_S)^n \\ &= e^{-\tau_S \overline{N}_S(x)}. \end{aligned} \quad (11)$$

where $\overline{N}_S(x) = \int_{a}^{x+R_I} n(s)ds$ and τ_S are the average number of cars in Region 2 and the average transmission probability of cars in the region, respectively. According to [11], given the vehicular density profile $n(x)$, we can identify the probability density function (PDF) for a car located at x within the transmission range $[a - R_s, a)$ of car a as follows.

$$f(x) = \begin{cases} \frac{n(x)}{\overline{N}(R_S)}, & a - R_S \le x < a \\ 0, & \text{otherwise} \end{cases}. \quad (12)$$

Thus, combining (11) and (12), the probability that the k-th car located at x in $[a - R_s, a)$ is interfered by cars in Region 2 is given by

$$P(k_{Intf2}) = \tau P(k) \int_{a-R_S}^{a} f(x)[1 - S(x)]dx. \quad (13)$$

Also note that (13) is a function of the number of cars i in Region 1. Hence, the overall interference probability from Region 2 is as follows.

$$P_2 = \sum_{i=1}^{\infty} P\{i \text{ cars in } [a - R_S, a)\} \sum_{k=1}^{i} P(k_{Intf2}). \quad (14)$$

C. Interferers in Region 3: $[x–R_I, a–R_S)$

The packet reception at car x will be ruined if there is at least one interferer in this region. What should be noted is that, from Fig. 3, it is possible that some interferers may be located beyond the sensing range of car a, and they are actually the hidden nodes for the transmission from car a to car x. The hidden nodes can also cause interference to car x. Let $T(x)$ be the probability that no car transmits in Region 3, we have

$$\begin{aligned} T(x) &= \sum_{n=0}^{\infty} P(n \text{ cars in Region 3}) P(n \text{ cars do not transmit}) \\ &= \sum_{n=0}^{\infty} \frac{\overline{N}_T(x)^n e^{-\overline{N}_T(x)}}{n!} (1 - \tau_T)^n \\ &= e^{-\tau_T \overline{N}_T(x)}. \end{aligned} \quad (15)$$

where $\overline{N}_T(x) = \int_{x-R_I}^{a-R_S} n(s)ds$ and τ_T are the average number of cars in Region 3 and the average transmission probability of cars in the region, respectively. Similar to the case in Region 2 above, we can derive the conditional interference probability in Region 3 (16) and the overall interference probability in the region (17) as follows.

$$P(k_{Intf3}) = \tau P(k) \int_{a-R_S}^{a} f(x)[1 - T(x)]dx, \quad (16)$$

$$P_3 = \sum_{i=1}^{\infty} P\{i \text{ cars in } [a - R_S, a)\} \sum_{k=1}^{i} P(k_{Intf3}). \quad (17)$$

Eventually, the overall probability of collision for the transmission from car a to car x is given by

$$q = 1 - (1 - P_1)(1 - P_2)(1 - P_3). \quad (18)$$

We can see that (18) provides another relationship of the collision probability q as a function of the transmission probability τ. This is being solved numerically in our model together with (5) for the values of q and τ in the system.

V. Delay and Throughput Analysis

The unicast delay is defined as the duration from the time a packet is generated at the transmitter to the time the Acknowledgement (ACK) for this packet sent by the receiver is received by the transmitter. The signal propagation delay assumed negligible in our study, as the transmission range is about 200 m.

At the beginning of every time slot, we assume that the channel may be busy for T slots (where T denotes the transmission time of a packet) with probability p, or idle for one slot (13 μs) with probability $1–p$. So, we define the 'expected time slot' here of length $pT + (1 - p)$ similar to [8]. Let j be the number of expected time slots for car a to contend before transmitting in the back-off stage mentioned in Section III, $(1–\tau)^j \tau$ be the probability that car a does not transmit until the $(j+1)$-th expected time slot, we derive the expected contention time in the back-off stage as follows.

$$\overline{CT} = \sum_{j=0}^{\infty} j[pT + (1 - p)]\,(1 - \tau)^j \tau. \quad (19)$$

The delay incurred for a failed transmission caused by collision is expressed as

$$T_C = \overline{CT} + T. \quad (20)$$

The delay for a successful transmission is given by

$$T_S = \overline{CT} + T + SIFS + ACK, \quad (21)$$

where SIFS stands for the Short Inter-Frame Space. The total unicast delay comprises of a number of T_C and one T_S, or is equal to T_S if there is no collision. Then, the delay D can be expressed as

$$D = nT_C + T_S, \text{where } n \geq 0. \quad (22)$$

The probability that there are n occurrences of collisions before a successful transmission is $q^n(1 - q)$, where q is collision probability. Finally the expected unicast delay for a packet transmitted by car a is obtained as follows.

$$E[D] = \sum_{n=0}^{\infty} D * q^n(1 - q). \quad (23)$$

After obtaining the delay, we can easily compute the expected throughput of car a as follows

$$E[\rho] = \frac{L}{E[D]}, \quad (24)$$

where L is the size of a packet.

VI. Numerical Analysis

We assume that the mean density profile at a certain time obtained from the stochastic traffic model as illustrated in Fig. 4 is used for the computation of delay and throughput. At location 0, the vehicular density n = 34 cars/km. A traffic light is set to be located at x = 2 km, thus there is a huge bunch of cars before the traffic light's location. The values for various parameters used in this analysis are listed in Table 1.

Fig. 5 and Fig. 6 plot respectively the approximated analytical delay and throughput with homogeneous vehicle distribution against different car density levels. Both figures reveal the general condition that the network performance is strongly affected by the vehicular density. When the density increases, the average time for a node to contend to access the channel is longer, therefore, the delay will increase and throughput will drop.

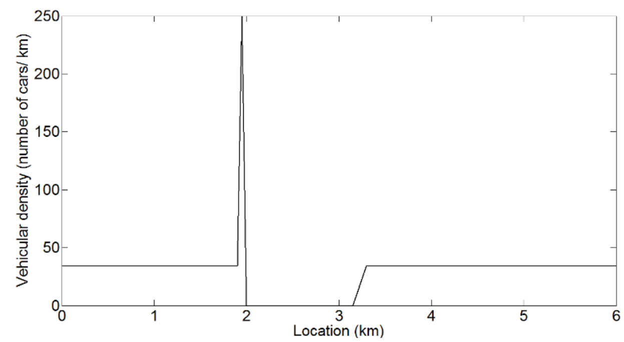

Figure 4. Mean density profile captured at a certain time.

TABLE I. Parameters of Analysis

CW_{min}	3
CW_{max}	7
R_S	0.2 km
R_I	0.5 km
Time slot	13 μs
Packet size	512 bytes
Data rate	6 Mbps
SIFS	32 μs
ACK	30 bytes

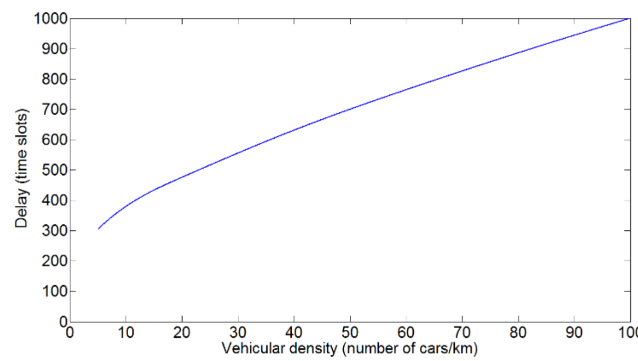

Figure 5. Analytical delay with homogeneous vehicular distribution.

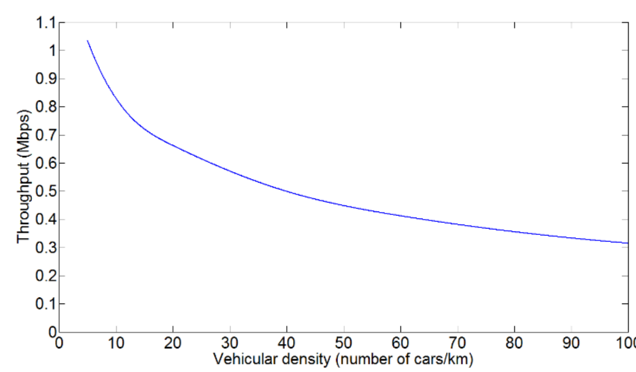

Figure 6. Analytical throughput with homogeneous vehicular distribution.

Fig. 7 shows the approximated analytical delay with the vehicular density profile in Fig. 4 at different locations. From this figure, we can see that the non-homogeneous vehicular distribution significantly affects the delay. The transmission delay starts to ascend at location near 1.5 km as there are more cars within the sensing range (0.5 km). But sharply beyond 1.5 km, the delay descends as there are less number of cars at the road junction due to the traffic signal. Before location 3.5 km, delay has a small increase as the density rises slightly. Fig. 8 shows how the throughput varies according to the density profile, which is inversely proportional to the delay.

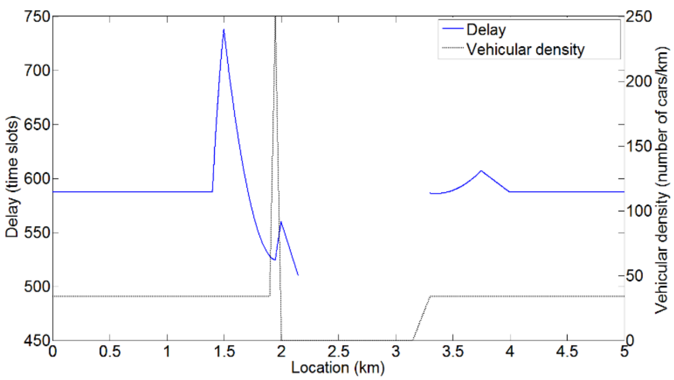

Figure 7. Analytical delay with non-homogeneous vehicular distribution.

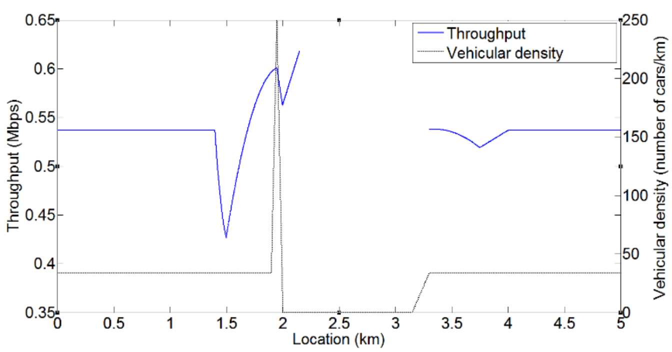

Figure 8. Analytical throughput with non-homogeneous vehicular distribution.

VII. CONCLUSION

In this paper, 802.11p VANET unicast is modeled by a 2D Markov chain coupled with a stochastic traffic model that can reflect practical on-road vehicular distribution. In addition, an interference model is proposed to characterize the effect of packet collisions with respect to the vehicular traffic. By jointly solving the two relationships between the transmission probability and collision probability acquired from the Markov model and interference model, the expected unicast transmission delay and throughput can be readily computed. Therefore, given the urban road configuration and traffic information (e.g., vehicle arrival rate, vehicle speed, traffic signal setting, etc.), the proposed methodology can be utilized to predict unicast performance in 802.11p VANETs for various kinds of system planning and control.

As future work, we endeavor to further enhance the models by considering other practical factors such as probabilistic link connectivity, multi-queue and multi-channel contention, as well as the optimization of transmission power and various protocol parameters in 802.11p for optimal performance.

ACKNOWLEDGMENT

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