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Log-Average-SNR Ratio and Cooperative Spectrum Sensing

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Abstract: **In this paper, we analyze the spectrum-sensing performance of a cooperative cognitive radio (CR) network consisting of a number of CR nodes and a fusion center (FC). We introduce the "log-average-SNR ratio" that relates the average SNR of the CR-node-FC link and that of the primary-user-CR-node link. Assuming that the FC utilizes the K-out-of-N rule as its decision rule, we derive exact expressions for the sensing gain and the coding gain — parameters used to characterize the CR network performance at the high SNR region. Based on these results, we determine ways to optimize the performance of the CR network.**

Index Terms: **Coding gain, cognitive radio, log-average-SNR ratio, missing-detection probability, sensing gain, spectrum sensing.**

I. INTRODUCTION

COgnitive radio (CR) has been recently proposed as a smart and agile technology which allows non-legitimate users to utilize the licensed bands [1]. To ensure that the operations of the primary users are not affected, the CR user must possess the spectrum-sensing capability, i.e., the ability of detecting the presence of primary signals in the bands of interest. Energy detection is a common method for spectrum sensing because of its low infrastructure cost [2]. The detector measures the energy of the incoming signal and compares it with a threshold, which is associated with an acceptable probability of false alarm. However, when there exists fading and/or shadowing effect in the channel, the sensing performance of a single detector will degrade significantly and the detection task will become very difficult [3]. For this reason, cooperative spectrum-sensing techniques with multiple CR nodes have been proposed to enhance the sensing performance. Some classical algorithms that fuse the local decisions used in the distributed detection and make a global decision have been considered and developed in cooperative spectrum sensing [3]-[16].

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In particular, the sensing-gain (or diversity-gain) concept, which is a fundamental performance indicator widely adopted in the field of wireless communications, has been extended and applied to the analysis of spectrum-sensing performance [9], [11]. In [9], [11], the gains of some spectrum sensing schemes with and without cooperation have been determined quantitatively. However, the analysis is only limited to the case where (i) the sensing channels between the primary-user (PU) and the CR nodes follow Rayleigh fading, and (ii) the reporting channels between the CR nodes and the fusion center (FC) are error-free. In practice, the reporting channels also suffer from noise and possibly interference. Therefore, in this paper and [10], we relax the aforementioned restrictions. In addition to the sensing gain, we extend and develop another important parameter called the "coding gain".

In cooperative spectrum sensing, the FC often employs the so-called "K-out-of-N counting rule" as its decision fusion rule, where K is the number of CR nodes that claims the PC is present and N is the total number of cooperative CR nodes. The K-out-of-N counting rule, with the "Or rule", "And rule" and "Majority rule" as its special cases, is the optimal fusion rule at the FC when identical tests and identically distributed observations are available at the detectors [17]. The probability of false alarm and the probability of missing detection under the K-out-of-N rule have been widely studied. Moreover, the detection performance of the cooperative system is reflected in the form of receiver-operating-characteristic (ROC) curves. However, quantifying the improvement due to cooperation in a detection system has not been simple.

Supposing a PU signal exists in a spectrum and the cooperative CR network fails to detect it, a secondary user may reuse this spectrum. If the secondary user sends a signal in the spectrum which is being used by the PU, the signal of the primary user will be disrupted, causing great inconvenience to the PU. We aim at minimizing the chance that this scenario occurs and therefore, in our study, we treat the overall missing-detection probability as a more important parameter. Our primary target is thus to evaluate the rate that the overall missing-detection probability decreases when the SNR is increased. We further define such a rate at the high SNR region as the sensing gain. By analyzing the sensing gain of the cooperative CR network when different decision rules are implemented, researchers can easily evaluate and compare the overall missing-detection probabilities when the SNR is large.

In this paper, we analyze the spectrum-sensing performance of a cooperative CR network consisting of a number of CR nodes and a fusion center (FC). We focus on the performance of the cooperative CR network at the medium and high SNR regions, which is different from [13]. For the convenience of

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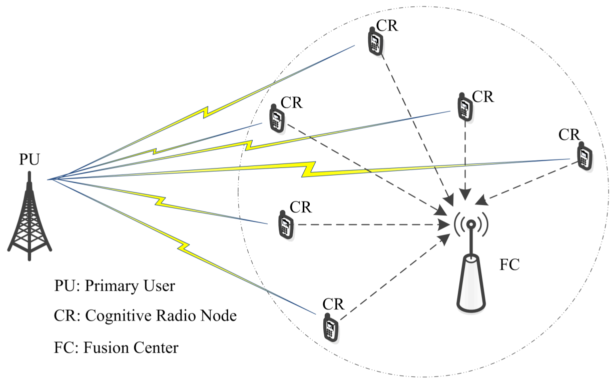

Fig. 1. Cooperative spectrum sensing in a cognitive radio network

explanation, we first assume that the sensing channels as well as the report channels are independent identically distributed (i.i.d.), and all CR nodes have identical false-alarm probabilities and missing-detection probabilities. Then, we investigate the overall missing-detection probability at the FC when the parameters used in the K-out-of-N decision rule changes. We introduce the "log-average-SNR ratio" (LASR) that relates the average SNR of the CR-node-FC link and that of the primary-user-CR-node link. Assuming that the FC utilizes the K-out-of-N rule to infer the absence or the presence of a primary signal, we derive an expression for the asymptotic probability of missing detection under a general fading environment. Based on the result, we can determine the sensing gain of the network. We further quantify the sensing performance by evaluating the coding gain of the network at the high SNR region. Based on the analytical results, the improvements accomplished by the introduction of extra CR nodes are quantified. We also derive the minimum transmitted power for each CR node that can achieve the highest sensing gain. We further observe that cooperative CR networks with higher "sensing gains" perform better in terms of overall missing-detection probability in both medium and high SNR regions. In addition, we also extend our mentioned-above analysis to the non-homogeneous scenario where both the sensing channels and the report channels may be independent but not identically distributed (i.n.i.d.).

The organization of this paper is as follows. In Sect. II, we introduce the model of the cooperative CR network and the expressions for the missing-detection probabilities and false-alarm probabilities. In Sect. III, we define the sensing gain and the coding gain, and derive analytical expressions of these two parameters for the system under study. In Sect. IV, we generalize the analysis of Sect. III. Sect. V presents and discusses the numerical results under different scenarios. Finally, Sect. VI provides the conclusions.

II. SYSTEM MODEL

With reference to Fig. 1, we consider a cooperative CR network with N CR nodes and a fusion center (FC). Each CR node performs energy detection independently and sends its local binary decision to the FC in the wireless network. The FC makes a binary decision on the received signal from each CR node, and then makes a final decision based on all of these binary decisions. We assume that the FC applies the K-out-of-N rule to infer the absence or presence of a primary user (PU). As the name implies, the K-out-of-N rule will make a positive decision when K or more of the N decoded decisions are positive. We also assume that the channels between the PU and the CR nodes are subject to i.i.d. fading. Similarly, the channels between the CR nodes and the FC are i.i.d. but can be different from those between the PU and the CR nodes.

Spectrum-sensing processing over the cooperative CR network is accomplished in two successive stages. In the first stage, the i-th CR node ($i = 1, 2, \cdots, N$) performs local spectrum sensing with energy detection to get a local binary decision. We denote the energy threshold by λ and the number of samples in a sensing duration by $2u$. We also denote the received SNR of the primary signal at the i-th CR node by $\gamma_1^{(i)}$. We further define $f_{\gamma_1^{(i)}}(\gamma_1^{(i)})$ and $\bar{\gamma}_1^{(i)}$, respectively, as the probability density function (PDF) and the mean of the received SNR. Note that because of the i.i.d. channel property between the PU and each CR node, all CR nodes have the same PDF and the same mean, i.e.,

$$\bar{\gamma}_1^{(1)} = \bar{\gamma}_1^{(2)} = \cdots = \bar{\gamma}_1^{(N)} = \bar{\gamma}_1. \quad (1)$$

We make use of $P_f^{(i)}$ and $P_m^{(i)}$, respectively, to denote the false-alarm probability and missing-detection probability of the i-th CR node. Thus, we have [18]

$$P_f^{(1)} = P_f^{(2)} = \cdots = P_f^{(N)} = \frac{\Gamma\left(u, \frac{\lambda}{2}\right)}{\Gamma(u)} = P_f \quad (2)$$

$$\begin{aligned} P_m^{(1)} &= P_m^{(2)} = \cdots = P_m^{(N)} = P_m \\ &= \int_0^\infty \left[1 - Q_u\left(\sqrt{2\gamma_1^{(1)}}, \sqrt{\lambda}\right)\right] f_{\gamma_1^{(1)}}(\gamma_1^{(1)}) d\gamma_1^{(1)} \end{aligned} \quad (3)$$

where $\Gamma(\cdot)$ and $\Gamma(\cdot,\cdot)$ denote the gamma function [19, eq.(8.310.1)] and the incomplete gamma function [19, eq.(8.350.2)], respectively; $Q_u(\cdot,\cdot)$ is the generalized Marcum Q-function of order u which is given by [20]

$$Q_u(w, v) = \frac{1}{w^{u-1}} \int_v^\infty x^u e^{-\frac{x^2+w^2}{2}} I_{u-1}(wx)\, dx \quad (4)$$

with $I_{u-1}(\cdot)$ being the modified Bessel function of order $u-1$ [19, eq.(8.445)].

In the second stage, the i-th CR node sends its local decision to the FC and the FC decodes the binary decision based on the received signal. We make use of $\bar{\gamma}_2^{(i)}$ and $P_b^{(i)}$ to denote the average received SNR and the probability of a decoding error at the FC. Assume that all CR nodes transmit with same power. Due to the i.i.d. property of the CR-node-FC (CR-FC) links, we have

$$\bar{\gamma}_2^{(1)} = \bar{\gamma}_2^{(2)} = \cdots = \bar{\gamma}_2^{(N)} = \bar{\gamma}_2. \quad (5)$$

and

$$P_b^{(1)} = P_b^{(2)} = \cdots = P_b^{(N)} = P_b. \tag{6}$$

Suppose that the local decision is sent with a one-bit information using BPSK modulation. Then, P_b is given by [21]

$$P_b = \int_0^\infty Q\left(\sqrt{2\gamma_2^{(1)}}\right) f_{\gamma_2^{(1)}}(\gamma_2^{(1)}) d\gamma_2^{(1)} \tag{7}$$

where the function $f_{\gamma_2^{(1)}}(\gamma_2^{(1)})$ represents the PDF of the received SNR $\gamma_2^{(1)}$ at the FC from the first CR node.

For the i-th PU-CR-FC link, the probability of false alarm $\tilde{P}_f^{(i)}$ and the probability of missing detection $\tilde{P}_m^{(i)}$ are given, respectively, by

$$\tilde{P}_f^{(i)} = (1 - P_f)P_b + P_f(1 - P_b) = \tilde{P}_f \tag{8}$$

and

$$\tilde{P}_m^{(i)} = P_m(1 - P_b) + (1 - P_m)P_b = \tilde{P}_m. \tag{9}$$

Finally, based on the N decoded decisions, the FC makes its final binary decision with the K-out-of-N rule. The overall probability of false alarm P_F and the overall probability of missing detection P_M at the FC under the K-out-of-N rule can then be expressed, respectively, as

$$P_F = \sum_{j=K}^{N} \binom{N}{j} \left(\tilde{P}_f\right)^j \left(1 - \tilde{P}_f\right)^{N-j} \tag{10}$$

and

$$P_M = \sum_{j=0}^{K-1} \binom{N}{j} \left(1 - \tilde{P}_m\right)^j \left(\tilde{P}_m\right)^{N-j}. \tag{11}$$

When $\bar{\gamma}_2$ is large, $P_b \to 0$ and $\tilde{P}_f \approx P_f$. Also, P_f is usually very small and we can approximate $1 - P_f$ by unity. Hence, when $\bar{\gamma}_2$ is large, we can re-write (10) as

$$P_F \approx \sum_{j=K}^{N} \binom{N}{j} (P_f)^j \approx \binom{N}{K} (P_f)^K. \tag{12}$$

To further shed light on the effect of the parameters N and K in the detection performance, we will derive an asymptotic expression for the overall probability of missing detection P_M in Sect. III.

Note that we can also evaluate the overall missing-detection probability P_M for a given overall false alarm probability P_F [13]. First, with a given P_F, we calculate the false alarm probability $\tilde{P}_f$ for each single PU-CR-FC link by making use of (10). Then, we apply (8) to find the corresponding false alarm probability P_f of each CR node (assuming that P_b is known). Further, we compute the threshold λ if u is known (or compute u if λ if is given) based on (2). Finally, we determine the overall missing-detection probability P_M with the help of (3), (9) and (11). Moreover, by varying other parameters such as K, an optimal overall missing-detection probability can be found.

III. ANALYSIS OF ASYMPTOTIC SPECTRUM SENSING PERFORMANCE

It is shown from [21] that the average symbol error probability(SEP) of an uncoded or coded wireless transmission system at the high SNR region may be approximately expressed as

$$P_e \approx (C \cdot \bar{\gamma})^{-D} \tag{13}$$

where D is referred to as the diversity gain, C is termed the coding gain, and $\bar{\gamma}$ is the average SNR. The diversity gain D determines the slope of the SEP versus average SNR curve at high SNR in a log-log scale. On the other hand, the coding gain D in decibels determines the shift of the curve in SNR relative to a benchmark SEP curve of $\bar{\gamma}^{-D}$. The two gain concepts can be naturally extended to a spectrum sensing system. It should be noticed that we refer to the corresponding D as the sensing gain in this paper and [9].

A. *Definitions of sensing gain and coding gain*

In order to conveniently quantify the influence of cooperation detection on the spectrum sensing performance, we define a "log-average-SNR ratio" (LASR) as the ratio between the log value of the average SNR of each CR node signal at the FC ($\log \bar{\gamma}_2$) and the log value of the average SNR of the PU signal at each CR node ($\log \bar{\gamma}_1$). Denoting the LASR by h, we can write

$$h = \frac{\log \bar{\gamma}_2}{\log \bar{\gamma}_1}. \tag{14}$$

We only consider reasonable scenarios in which both $\bar{\gamma}_1$ and $\bar{\gamma}_2$ are larger than 0 dB, i.e., $\log \bar{\gamma}_1, \log \bar{\gamma}_2 > 0$. Thus the LASR h is always a positive constant when $\bar{\gamma}_1$ and $\bar{\gamma}_2$ are fixed. Moreover, by increasing the transmission powers of the CR nodes, the average SNR received at the FC (i.e., $\bar{\gamma}_2$) improves and hence the value of h increases.

Given a constant h, we denote the link formed by (i) a sensing link from the primary user to a CR node and (ii) the corresponding communication link from the CR node to the FC, i.e., a PU-CR-FC link, as a single composite link. We also define the sensing gain $d(h)$ of the composite link as the slope of the probability-of-missing-detection $\tilde{P}_m$ curve at the high SNR region when plotted versus the average received SNR of the PU signal at each CR node $\bar{\gamma}_1$ in a log-log scale [22], i.e.,

$$d(h) = -\lim_{\bar{\gamma}_1 \to \infty} \frac{\log \tilde{P}_m}{\log \bar{\gamma}_1}. \tag{15}$$

Also, we define the corresponding coding gain $c(h)$ as

$$c(h) = \lim_{\bar{\gamma}_1 \to \infty} \left(\tilde{P}_m \cdot \bar{\gamma}_1^{d(h)}\right)^{-1/d(h)}. \tag{16}$$

Similarly, for the cooperative CR network, we define the sensing gain $D(h)$ and the coding gain $C(h)$, respectively, as

$$D(h) = -\lim_{\bar{\gamma}_1 \to \infty} \frac{\log P_M}{\log \bar{\gamma}_1}. \tag{17}$$

and

$$C(h) = \lim_{\bar{\gamma}_1 \to \infty} \left(P_M \cdot \bar{\gamma}_1^{D(h)}\right)^{-1/D(h)}. \tag{18}$$

We further introduce two common notations as follows [23]. For two positive functions $a(x)$ and $b(x)$, $a(x) = O(b(x))$ means that $\limsup_{x\to\infty} a(x)/b(x) < \infty$, whereas $a(x) \sim b(x)$ means that $\lim_{x\to\infty} a(x)/b(x) = 1$. Using these notations together with the definitions of the sensing gain and the coding gain, we have

$$\tilde{P}_m = O(\bar{\gamma}_1^{-d(h)}) \tag{19}$$

$$\tilde{P}_m \sim (c(h)\bar{\gamma}_1)^{-d(h)} \tag{20}$$

$$P_M = O(\bar{\gamma}_1^{-D(h)}) \tag{21}$$

$$P_M \sim (C(h)\bar{\gamma}_1)^{-D(h)}\,. \tag{22}$$

Hence, by evaluating both the sensing gain and the coding gain, the exact performance of a CR network at the high SNR region can be found. If two CR networks have the same sensing gain, the one with a larger coding gain will give a lower overall missing-detection probability for the same SNR.

B. Analysis of the sensing gain

Under a general fading environment, the diversity gain and the coding gain for a wireless transmission system can be analyzed in a unified form [21]. Suppose that the diversity gain of the fading channel between each CR node and the FC equals d_2, i.e.,

$$P_b = O(\bar{\gamma}_2^{-d_2}). \tag{23}$$

Combining (14) and (23), we have

$$P_b = O(\bar{\gamma}_1^{-hd_2}). \tag{24}$$

Furthermore, we let d_1 be the sensing gain of each CR detector [9]. Thus,

$$P_m = O(\bar{\gamma}_1^{-d_1}). \tag{25}$$

Combining (9), (24) and (25), the probability of missing detection for the PU-CR-FC composite link $\tilde{P}_m$ can be written as

$$\tilde{P}_m = O(\bar{\gamma}_1^{-d(h)}) \tag{26}$$

where $d(h)$, the sensing gain of the composite link, is given by

$$d(h) = \min\{d_1, hd_2\} = \begin{cases} hd_2, & \text{for } h < d_1/d_2; \\ d_1, & \text{for } h = d_1/d_2; \\ d_1, & \text{for } h > d_1/d_2. \end{cases} \tag{27}$$

We then consider the cooperative CR network. From (11), it can be readily shown that the overall probability of missing detection P_M has the following upper and lower bounds:

$$P_M \le \left(\sum_{j=0}^{K-1}\binom{N}{j}\right)\left(\tilde{P}_m\right)^{N-K+1} \tag{28}$$

$$\begin{aligned} P_M &\ge \binom{N}{K-1}\left(1-\tilde{P}_m\right)^{K-1}\left(\tilde{P}_m\right)^{N-K+1} \\ &\approx \binom{N}{K-1}\left(\tilde{P}_m\right)^{N-K+1}. \end{aligned} \tag{29}$$

Using these two bounds, P_M can be expressed as

$$P_M = O\left(\tilde{P}_m^{N-K+1}\right). \tag{30}$$

Combining (26) and (30), we have

$$P_M = O\left(\bar{\gamma}_1^{-D(h)}\right) \tag{31}$$

where $D(h)$, the sensing gain of the cooperative CR network, is given by

$$D(h) = d(h) \times (N-K+1). \tag{32}$$

When $h > d_1/d_2$, it is obvious from (27) that

$$D(h) = d_1 \times (N-K+1)\,. \tag{33}$$

In practice, we can increase the transmission power of the CR nodes, hence increasing the average received SNR at the FC $\bar{\gamma}_2$ and the LASR h. If we raise the transmission power such that there is almost no decision error occurring at the FC, i.e., $\bar{\gamma}_2$ is high enough, then $d(h) = d_1$ and thus (33) holds. Alternatively, to achieve a sensing gain of $d_1 \times (N-K+1)$, according to (15), we can simply adjust the transmission power of the CR nodes such that $h = d_1/d_2$ in the high SNR region. Obviously, such a setting is much more power-efficient.

We also observe from (32) that the detection performance is improved with the cooperation among the N CR nodes. The sensing gain increases from $d(h)$ in the case of a single composite link to $d(h) \times (N-K+1)$ when cooperation is introduced. When $K = 1$ ("Or" rule), the full sensing gain, i.e., $N \times d(h)$, can be accomplished and the overall false-alarm probability becomes the largest among $1 \le K \le N$. As K increases, a smaller sensing gain is attained and the overall false-alarm probability decreases.

C. Analysis of coding gain

For the fading channel between each CR node and the FC, the coding gain is related to the diversity gain d_2 and can be written as [21]

$$c_2 = b_2\left(\frac{a_2 2^{d_2-1}\Gamma(d_2+1/2)}{\sqrt{\pi}d_2}\right)^{-1/d_2} \tag{34}$$

where a_2 is a positive constant and can be determined by the PDF of the channel gain; b_2 is a positive constant related to the modulation scheme and equals two when BPSK modulation is used. Moreover, for the sensing link from a PU to a CR node, the corresponding coding gain is related to the sensing gain d_1 and is given by [9]

$$c_1 = (A-B)^{-\frac{1}{d_1}} \tag{35}$$

where

$$A = a_1\left(\frac{1}{2}\right)^{d_1-1}\int_0^\infty\left[1-Q\left(t,\sqrt{\lambda}\right)\right]t^{2d_1-1}dt; \tag{36}$$

$$B = a_1\Gamma(d_1)e^{-\frac{\lambda}{2}}\sum_{i=1}^{u-1}\frac{\lambda^i\Phi\left(d_1;i-1,\frac{\lambda}{2}\right)}{2^i\Gamma(i+1)}; \tag{37}$$

with a_1 being a positive constant determined by the PDF of the channel gain; $Q(\cdot,\cdot)$ being the Marcum Q-function of order 1; and $\Phi(\cdot,\cdot,\cdot)$ being the confluent hypergeometric function [19, eq.(9.210.1)].

Based on (9), (15), (34) and (35), the coding gain $c(h)$ of a single composite link in the CR network can therefore be shown equal to

$$c(h) = \begin{cases} c_2^{\frac{1}{h}}, & \text{for } h < d_1/d_2, \\ \left(c_2^{-d_2} + c_1^{-d_1}\right)^{-\frac{1}{d_1}}, & \text{for } h = d_1/d_2, \\ c_1, & \text{for } h > d_1/d_2. \end{cases} \tag{38}$$

When $\bar{\gamma}_1$ and $\bar{\gamma}_2$ are large, the probability of missing detection P_m at the CR node, i.e., (3), will be small and the overall probability of missing detection P_M at the FC expressed by (11) will be dominated by the term $\binom{N}{K-1}\left(\tilde{P}_m\right)^{N-K+1}$. Hence, we have

$$P_M \sim \binom{N}{K-1}\left(\tilde{P}_m\right)^{N-K+1}. \tag{39}$$

Combining (20) and (39), we have

$$\begin{aligned} P_M &\sim \binom{N}{K-1}\left(c(h)\bar{\gamma}_1\right)^{-d(h)\times(N-K+1)} \\ &= \left(c(h)\cdot\binom{N}{K-1}^{-1/D(h)}\bar{\gamma}_1\right)^{-D(h)}. \end{aligned} \tag{40}$$

Comparing (22) and (40), the coding gain $C(h)$ for the whole CR network is readily shown equal to

$$C(h) = c(h)\cdot\binom{N}{K-1}^{-1/D(h)}. \tag{41}$$

In particular, when the "Or rule" is adopted, i.e., $K = 1$, we have

$$C(h) = c(h) \tag{42}$$

which implies that the coding gain has not been affected by the cooperation of the CR nodes.

IV. PERFORMANCE ANALYSIS FOR THE NON-HOMOGENEOUS SCENARIO

A. Performance expressions

In this section, we generalize the performance analysis of Sect. III to the non-homogeneous scenario in which there are i.n.i.d. fading sensing and reporting channels. Under this situation, the PU-CR-FC links are allowed to have different missing-detection probabilities or false-alarm probabilities. For this reason, we need to rewrite out expressions of performance metrics.

Now we define $\mathbf{S}$ as the peripheral set of the indices of the PU-CR-FC links, i.e., $\mathbf{S} = \{1, 2, \ldots, N\}$. We further use the sets $\mathbf{S}^1$ and $\mathbf{S}^0$, respectively, to store the indices of the PU-CR-FC links with the decoded outputs indicating the presence and absence of the PU. So we have that $\mathbf{S}^1 \bigcup \mathbf{S}^0 = \mathbf{S}$. Furthermore, we denote by $\mathbf{S}(j)$ the set of all possible sets of $\mathbf{S}^1$ with cardinality j. Thus the cardinality of $\mathbf{S}(j)$ is equal to $\binom{N}{j}$. Consequently, under the K-out-of-N rule, the overall probability of false alarm P_F and the overall probability of missing detection P_M at the FC can be expressed, respectively, as

$$P_F = \sum_{j=K}^{N} \sum_{\mathbf{S}^1\in\mathbf{S}(j)} \prod_{i\in\mathbf{S}^1}\left(\tilde{P}_f^{(i)}\right)\prod_{i\notin\mathbf{S}^1}\left(1-\tilde{P}_f^{(i)}\right) \tag{43}$$

and

$$P_M = \sum_{j=0}^{K-1} \sum_{\mathbf{S}^1\in\mathbf{S}(j)} \prod_{i\in\mathbf{S}^1}\left(1-\tilde{P}_m^{(i)}\right)\prod_{i\notin\mathbf{S}^1}\left(\tilde{P}_m^{(i)}\right) \tag{44}$$

where $\tilde{P}_f^{(i)}$ and $\tilde{P}_m^{(i)}$ still denote the false alarm probability and the mission detection probability for the ith PU-CR-FC link, respectively. At this time they are expressed as

$$\tilde{P}_f^{(i)} = \left(1-P_f^{(i)}\right)P_b^{(i)} + P_f^{(i)}\left(1-P_b^{(i)}\right) \tag{45}$$

and

$$\tilde{P}_m^{(i)} = \left(1-P_m^{(i)}\right)P_b^{(i)} + P_m^{(i)}\left(1-P_b^{(i)}\right). \tag{46}$$

B. Analysis of sensing gain

Under the i.n.i.d. situation, we redefine the two common average received SNRs for all PU-CR links and all CR-PU links, respectively, as

$$\bar{\gamma}_1 = \frac{1}{N}\sum_{i=1}^{N}\bar{\gamma}_1^{(i)} \tag{47}$$

and

$$\bar{\gamma}_2 = \frac{1}{N}\sum_{i=1}^{N}\bar{\gamma}_2^{(i)}. \tag{48}$$

Furthermore, the average received SNRs of the i-th PU-CR link and the i-th CR-FC link can be written, respectively, as

$$\bar{\gamma}_1^{(i)} = u_1^{(i)}\bar{\gamma}_1 \tag{49}$$

and

$$\bar{\gamma}_2^{(i)} = u_2^{(i)}\bar{\gamma}_2. \tag{50}$$

It should be pointed out that the SNR coefficients $\{u_1^{(i)}, i = 1, 2, \cdots, N\}$ and $\{u_2^{(i)}, i = 1, 2, \cdots, N\}$ should satisfy, respectively

$$\sum_{i=1}^{N} u_1^{(i)} = N \tag{51}$$

and

$$\sum_{i=1}^{N} u_2^{(i)} = N. \tag{52}$$

We still denote the LASR by $h = \log\bar{\gamma}_2/\log\bar{\gamma}_1$. For the i-th PU-CR-FC link, the missing detection probability of the i-CR node and the probability of a decoding error at the FC can be written, respectively, as

$$P_m^{(i)} = O(\bar{\gamma}_1^{-d_1}) \tag{53}$$

and

$$P_b^{(i)} = O(\bar{\gamma}_2^{-d_2}). \tag{54}$$

Then the missing detection probability for the composite link can be given by

$$\tilde{P}_m^{(i)} = O(\bar{\gamma}_1^{-d(h)}) \tag{55}$$

where $d(h) = \min\{d_1, hd_2\}$ is just the sensing gain of the composite link. Finally, with the help of (44), the overall probability of missing detection P_M at the FC can be also expressed as (31), and thus the sensing gain of the cooperative CR network for the non-homogeneous scenario is still $D(h) = d(h) \times (N-K+1)$.

C. Analysis of coding gain

Based on (44), we can obtain by following a similar line of reasoning as in Sect. III

$$P_M \sim \sum_{\mathbf{S}^1 \in \mathbf{S}(K-1)} \prod_{i \notin \mathbf{S}^1} \left(\tilde{P}_m^{(i)}\right)$$

$$= \left(\left(\sum_{\mathbf{S}^1 \in \mathbf{S}(K-1)} \left(\prod_{i \notin \mathbf{S}^1} c^{(i)}(h) \right)^{-d(h)} \right)^{-\frac{1}{D(h)}} \bar{\gamma}_1 \right)^{-D(h)} \tag{56}$$

where $c^{(i)}(h)$ denotes the coding gain of the i-th composite link in the CR network, and it can be equal to

$$c^{(i)}(h) = \begin{cases} (c_2 u_2^{(i)})^{\frac{1}{h}} & \text{for } h < d_1/d_2 \\ \left((c_2 u_2^{(i)})^{-d_2} + (c_1 u_1^{(i)})^{-d_1}\right)^{-\frac{1}{d_1}} & \text{for } h = d_1/d_2 \\ c_1 u_1^{(i)} & \text{for } h > d_1/d_2 \end{cases} \tag{57}$$

Note that c_1 and c_2 in (57) are defined in (35) and (34), respectively.

V. NUMERICAL RESULTS

In this section, we present some numerical results. We assume that all channels are suffering from Rayleigh fading. Thus, both (i) the diversity gain of the channel between each CR node and the FC, and (ii) the diversity gain of the channel between the primary user and each CR node are unity, i.e., $d_2 = d_1 = 1$. Moreover, we assume that 10 samples are taken in each sensing duration (i.e., $u = 5$). Also, BPSK modulation is assumed for the link between each CR node and the FC.

A. $\bar{\gamma}_1$ is the same as $\bar{\gamma}_2$

We first consider the homogeneous scenario when the average received SNR at each CR node is the same as that received at the FC from each CR node, i.e., $\bar{\gamma}_1 = \bar{\gamma}_2$, and consequently the LASR $h = \log\bar{\gamma}_2/\log\bar{\gamma}_1 = 1$. We fix the probability of false alarm for each CR node to be 0.01, i.e., $P_f = 0.01$. In Fig. 2 and Fig. 3, we plot, respectively, the overall probability of false alarm P_F versus the average received SNR $\bar{\gamma}_1$ at the CR node when the "Or rule" and the K-out-of-N rule are employed. As expected, the overall false-alarm probability P_F increases as N increases when the "Or rule" is used as the decision rule. P_F also increases when K decreases under the K-out-of-N rule. Furthermore, as the average received SNR $\bar{\gamma}_1$ at each CR node increases, so does $\bar{\gamma}_2$ (since the LASR h is fixed at unity) and hence the probability of a decoding error at the FC will goes to

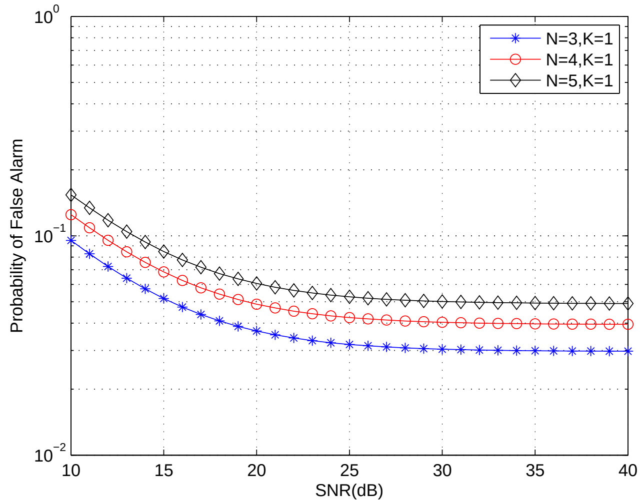

Fig. 2. Overall probability of false alarm P_F versus the average received SNR $\bar{\gamma}_1$ at the CR node under the "Or rule" for different number of CR nodes N. $P_f = 0.01$, $d_2 = d_1 = 1, \bar{\gamma}_1 = \bar{\gamma}_2$ and the log-average-SNR ratio (LASR) $h = \log\bar{\gamma}_2/\log\bar{\gamma}_1 = 1$.

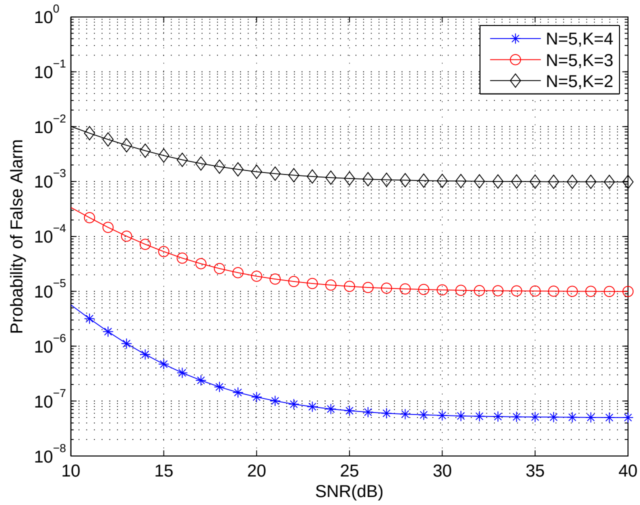

Fig. 3. Overall probability of false alarm P_F versus the average received SNR $\bar{\gamma}_1$ at the CR node under the K-out-of-N rule for different values of K. $P_f = 0.01$, $N = 5, d_2 = d_1 = 1, \bar{\gamma}_1 = \bar{\gamma}_2$ and the log-average-SNR ratio (LASR) $h = \log\bar{\gamma}_2/\log\bar{\gamma}_1 = 1$.

zero, i.e., $P_b \to 0$. Since P_f is very small (0.01 in our case), it can be seen from Fig. 2 and Fig. 3 that the overall false-alarm probability P_F converges to the expression given in (12), i.e., $\binom{N}{K}(P_f)^K$. For example, when $N = 5, K = 2$, P_F should converge to $\binom{5}{2}(0.01)^2 = 10^{-3}$, which matches exactly with the results shown in Fig. 3.

Next, we fix the overall probability of false alarm at 0.05, i.e., $P_F = 0.05$. Figure 4 and Figure 5, respectively, plot the overall probability of missing detection P_M versus the average received SNR $\bar{\gamma}_1$ at the CR node when the "Or rule" and the K-out-of-N rule are employed. In both figures, the asymptotic results given by (40) and the exact results given by (11) are shown. We observe that the curves match very well at the high SNR re-

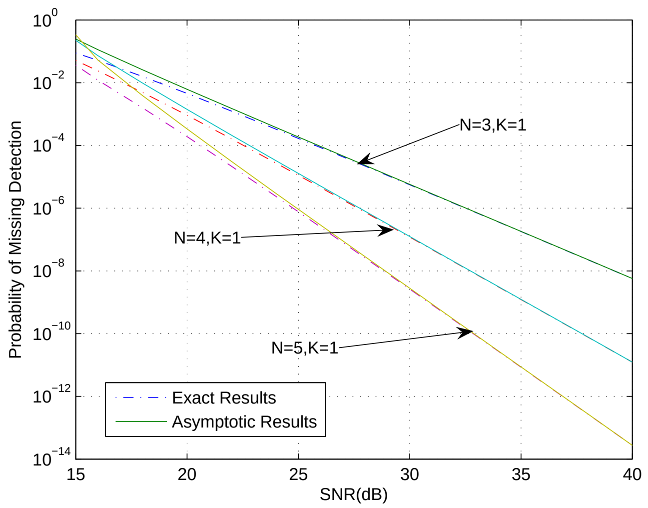

Fig. 4. Overall probability of missing detection P_M versus the average received SNR $\bar{\gamma}_1$ at the CR node under the "Or rule" for different number of CR nodes N. $P_F = 0.05$, $d_2 = d_1 = 1, \bar{\gamma}_1 = \bar{\gamma}_2$ and the log-average-SNR ratio (LASR) $h = \log\bar{\gamma}_2/\log\bar{\gamma}_1 = 1$.

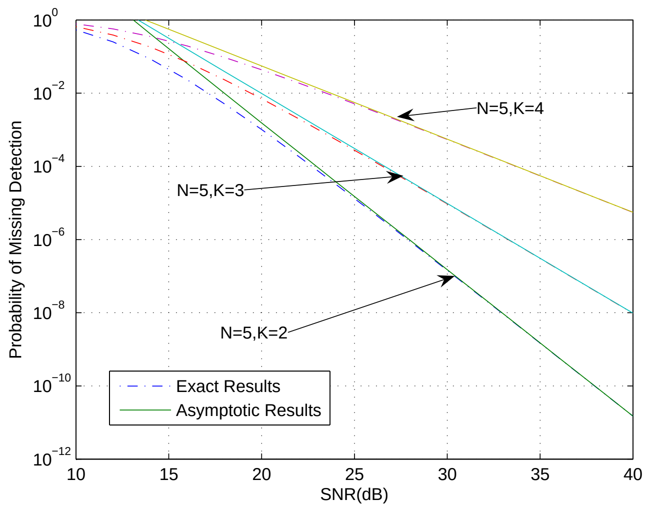

Fig. 5. Overall probability of missing detection P_M versus the average received SNR $\bar{\gamma}_1$ at the CR node under the K-out-of-N rule for different values of K. $P_F = 0.05, N = 5, d_2 = d_1 = 1, \bar{\gamma}_1 = \bar{\gamma}_2$ and the log-average-SNR ratio (LASR) $h = \log\bar{\gamma}_2/\log\bar{\gamma}_1 = 1$.

gion, indicating that the sensing gain and the coding gain have been derived accurately. Figure 4 also shows that the curves attain a larger slope at the high SNR region as the number of CR nodes (i.e., N) increases. Moreover, the slope for each curve at the high SNR region equals N. The results verify that a larger N produces a larger sensing gain and that the potential performance improvement of N times can be accomplished when the "Or rule" is used. In Fig. 5, the results indicate that the slope of the curve at the high SNR region (i.e., the sensing gain) decreases as K increases. Thus, to minimize the probability of missing detection, K should be reduced. In other words, the "Or rule" should be the optimal rule for the FC to make a decision when considering the overall probability of missing detection. The results in Fig. 4 and Fig. 5 further indicate that cooperative CR networks with higher "sensing gains" perform better in terms of overall missing-detection probability in both medium and high SNR regions.

B. $\bar{\gamma}_1$ can be different from $\bar{\gamma}_2$

In what follows, we set $P_F = 0.05$, $K = 3$ and $N = 5$ in the K-out-of-N rule, and consider the homogeneous scenario when the average received SNR at each CR node ($\bar{\gamma}_1$) can be different from that received at the FC from each CR node ($\bar{\gamma}_2$). Consequently, the LASR h can be greater than, equal to or less than unity. Figure 6 plots the exact overall probability of missing detection (11) versus the average SNR $\bar{\gamma}_1$ at the CR node for different values of h. The results show that when $h \geq 1 = d_1/d_2$, the curves are close to one another and the sensing gain remains the same. As can be seen from the figure, the slopes of such curves equal 3 at the high SNR region, which is the same as that predicted in (32), i.e., $D(h) = d_1 \times (N-K+1) = 3$. When $h < 1 = d_1/d_2$, the slope of the curves at the high SNR region (i.e., the sensing gain) becomes smaller and is given by $D(h) = hd_2 \times (N - K + 1) = 3h$. The results conclude that to achieve the highest sensing gain with the minimum transmitted power, we should adjust the transmission power of each CR node such that the average received SNR $\bar{\gamma}_2$ at the FC from each CR node satisfies $\log\bar{\gamma}_2/\log\bar{\gamma}_1 = h = d_1/d_2$.

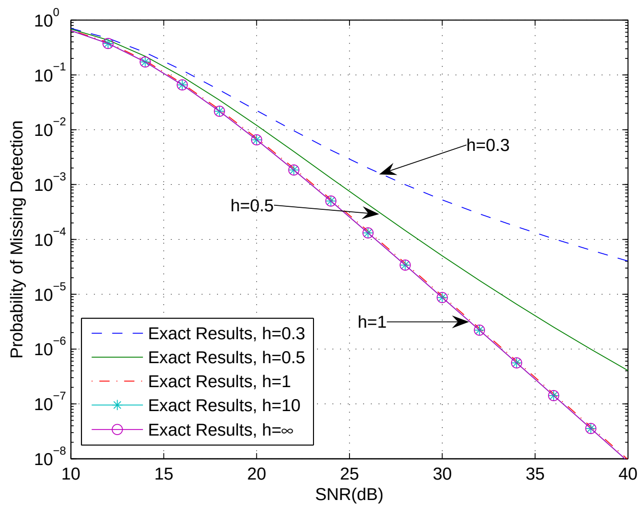

Fig. 6. Exact overall probability of missing detection P_M versus the average received SNR $\bar{\gamma}_1$ at the CR node under the K-out-of-N rule for different values of the log-average-SNR ratio (LASR) h. $P_F = 0.05$, $K = 3$, $N = 5$ and $d_2 = d_1 = 1$.

C. The non-homogeneous scenario

Finally, we pay our attention to the non-homogeneous scenario when $P_F = 0.05$, $K = 2$ and $N = 5$. For the SNR coefficients, we set $u_1^{(1)} = 0.1$, $u_1^{(2)} = 0.5$, $u_1^{(3)} = 1$, $u_1^{(4)} = 1.5$, and $u_5^{(5)} = 1.9$, and suppose that $u_2^{(1)} = u_2^{(2)} = u_2^{(3)} = u_2^{(4)} = u_2^{(5)} = 1$. Figure 7 plots the exact overall probability of missing detection (44) versus the average SNR $\bar{\gamma}_1$ at the CR node for different values of h. Similar to the homogeneous scenario, the sensing performance with $h = 10$ is almost the same as the one

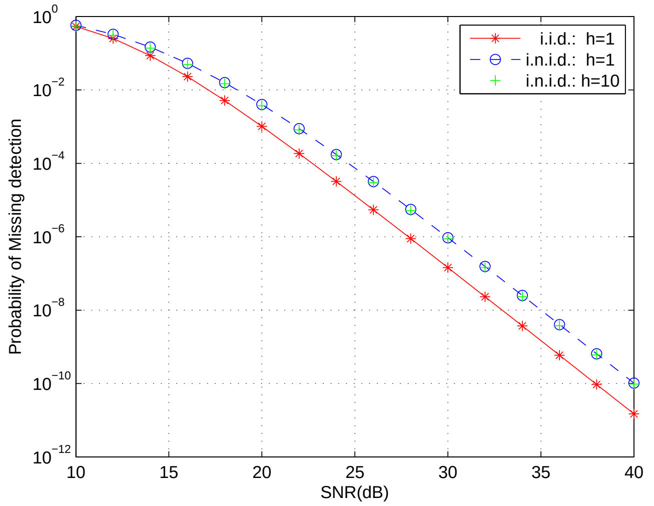

Fig. 7. Exact overall probability of missing detection P_M versus the average received SNR $\bar{\gamma}_1$ for non-homogeneous scenario under the K-out-of-N rule for different values of the log-average-SNR ratio (LASR). $P_F = 0.05$, $K = 2$, $N = 5$ and $d_2 = d_1 = 1$.

with $h = 1$. For comparison, Figure 7 also plots the exact overall probability of missing detection (11) under the homogeneous case. As expected, it can be seen from this figure that the sensing performance under the homogeneous situation outperforms the one under the non-homogeneous situation, although the two situations are shown to have the same sensing gain.

VI. CONCLUSION

In this paper, we have analyzed the asymptotic sensing performance of a cooperative cognitive radio (CR) network under the K-out-of-N decision rule. By introduce the "log-average-SNR ratio" (LASR denoted by h) that relates the average SNR of the CR-node-FC (CR-FC) link ($\bar{\gamma}_2$) and that of the primary-user-CR-node (PU-CR) link ($\bar{\gamma}_1$), we have derived analytically the sensing gain and the coding gain of the CR network when the average SNR of the PU-CR link is large. Our results have shown that when $K = 1$, i.e., the "Or rule" is applied, the sensing gain of the network is the largest and is N times that of a single composite PU-CR-FC link. Furthermore, to achieve the highest sensing gain with the minimum transmitted power, we should adjust the transmission power of each CR node such that the average received SNR at the FC from each CR node ($\bar{\gamma}_2$) satisfies $\log \bar{\gamma}_2 / \log \bar{\gamma}_1 = h = d_1/d_2$, where d_1 is the sensing gain of each CR detector and d_2 is the diversity gain of the fading channel between each CR node and the FC.

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