Performance of Two-Dimensional ML Detector with Laser Phase Noise and Frequency Offset

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Abstract— **Simplified Maximum Likelihood (ML) detection and annular-sector (AS) detection for two dimensional constellations is shown to significantly outperform the conventional minimum Euclidean distance (MED) detector in presence of laser phase noise and frequency offset.**

Keywords-component; Maximum Likelihood detection; annular-sector detection; laser phase noise; frequency offset

I. INTRODUCTION

Recently, advanced modulation formats, such as M-ary phase shift keying (PSK), amplitude phase shift keying (APSK) and quadrature amplitude modulation (QAM), have been extensively applied for high spectral efficiencies. However, the coherent detection of phase-modulated signals suffers the impairments by additive, complex, white, Gaussian noise (AWGN) and laser phase noise. Hence, carrier phase estimation (PE) preceding the coherent detection is indispensable [1]. Currently, the decision aided, maximum likelihood (DAML) PE technique, introduced in [2], is widely accepted due to its high efficiency. Considering the finite signal to noise ratio (SNR), the residual phase reference error (PRE), which would lead to degradation to the system performance, is inevitable during data detection [3]. However, the commonly used minimum Euclidean distance (MED) detector is designed without the consideration of PRE. Hence, in [4], we have designed the maximum likelihood (ML) detector for two-dimensional carrier modulations that take into account the residual PRE.

Here, we comprehensively evaluate the performance of the ML detector with varying laser phase noise. Due to high computational complexity of exact ML detection, two simpler and reasonable approximations are tested instead. Moreover, in addition to the laser phase noise, frequency offset between transmitter and receiver oscillator is another major source of phase noise, which is unavoidable during actual application as well [5]. Since the ML detector is proposed with the consideration of PRE only, its feasibility in systems with frequency offset is worth estimating. Here, evaluation of its tolerance to frequency offset is also presented and analyzed based on numerical results.

II. ML DETECTION FOR TWO DIMENSIONAL MODULATIONS

In this paper, laser phase noise and frequency offset are considered as the dominant distortions. Hence, for simplicity, we assume perfect channel estimation and CD, PMD compensation. In this case, a canonical model of the received k-th symbol can be written as [3]

$$r(k) = m(k)e^{j(\theta(k))} + n(k) \tag{1}$$

where L represent the window length of DAML, $\sigma_n^2 = \gamma^{-1}$ is the inverse of SNR. Currently, the MED detector is widely applied during the coherent detection, which is shown as

$$\widehat{S_{MED}} = \underset{\{S_i\}}{\operatorname{argmin}}[|r - s_i|^2] \tag{2}$$

Here, r is the recovered data symbol after phase noise compensation, s_i represents the i-th transmitted symbol and $\widehat{S_{MED}}$ represents the MED detection of received symbol. However, due to the unavoidable PRE, the MED receiver is not the optimal detector in systems suffering from carrier phase noise. Taking the PRE induced by the DAML phase estimator into consideration, the exact ML decision rule for two-dimensional modulations is proposed as [4]

$$\widehat{S_{ML}} = \underset{\{A_i,\varphi_i\}}{\operatorname{argmax}} \int_{-\pi}^{\pi} \exp\left[\frac{2|r|A_i\cos(\angle r-\varphi_i-\theta)}{N_0} - \frac{\theta^2}{2\sigma_\theta^2} - \frac{A_i{}^2}{N_0}\right] \tag{3}$$

where $|r|$ and $\angle r$ are the amplitude and phase of recovered symbol after phase noise compensation. A_i and φ_i are the amplitude and phase for i-th transmitted symbol, respectively, $\tilde{\theta}$ is the PRE due to the DAML phase estimator. Considering the computational complexity, a more implementable version of the exact ML detector, $\widehat{S_{ML1}}$, is introduced as [4]

$$\widehat{S_{ML1}} = \underset{\{A_i,\varphi_i\}}{\operatorname{argmin}}\Big[\frac{(|r|-A_i)^2}{\frac{N_0}{2}} + \frac{(\angle r-\varphi_i)^2}{\frac{N_0}{2A_i|r|}+\sigma_\theta^2} + \ln\Big(\frac{N_0}{2|r|^2} + \frac{A_i}{|r|}\sigma_\theta^2\Big)\Big] \tag{4}$$

While implementing $\widehat{S_{ML1}}$, a *priori* knowledge of AWGN spectrum density N_0 and phase noise variance σ_p^2 are required, which might be unavailable in real applications. Hence, another suboptimal approximation, called annular sector (AS) detector $\widehat{S_{AS}}$, is introduced in [4], where the ring and phase detection are performed separately. For example, the AS detection for a two-ring constellation can be expressed as

$$\frac{a_k+a_{k-1}}{2} \le |r| < \frac{a_k+a_{k+1}}{2} \Rightarrow \hat{A} = a_k, \hat{\varphi} = \underset{\varphi_i\in\{\hat{A}e^{j\varphi_i}\}}{argmin} |\angle r - \varphi_i| \tag{5}$$

where $\hat{A}$ and $\hat{\varphi}$ are the suboptimal decision of received amplitude and phase, a_1 and a_2 represent the radii of first and second data ring, respectively. It has been proved that, both $\widehat{S_{ML1}}$ and $\widehat{S_{AS}}$ converge to the exact ML detector at high SNR or PRE. Therefore, in this paper, $\widehat{S_{ML1}}$ and $\widehat{S_{AS}}$ are tested in various modulation formats to show their performance comparison in terms of the laser linewidth and frequency offset tolerance. For simplicity, in the following, $\widehat{S_{ML1}}$ is called the ML detector.

III. Simulation Results and Discussions

Here, Monte Carlo simulations are performed to estimate the feasibility of the proposed ML and AS detector in the systems with unknown laser linewidth and frequency offset. The performance of MED detector is simulated as a comparison. For simplicity, we assume perfect channel estimation and CD, PMD compensation. Both square QAM, like 8-square QAM and 16QAM, and circular QAM, like 8-star QAM and what we call (4, 4, 8)-16 APSK, with symbol rate of 25G/s are tested in our simulations to comprehensively evaluate its performance [4]. The (4, 4, 8) 16-APSK we applied here is a 3-ring constellation. For simplicity, we assume the uniformly spaced ring radii, with 4 signal points on the inner and middle ring and 8 signal points on the outer ring. It is introduced to compare with the widely used 16-QAM, which can be also regarded as a 3-ring constellation. We would call it 16-APSK for simplicity in the following.

The DAML method is used to compensate the phase noise, where memory length is chosen as the optimal value. According to previous investigation, the optimal memory length, L_{opt}, can be calculated as [6]

$$L_{opt} = \left\lfloor \sqrt{\left(\gamma\sigma_p^2 + 3\right)/2\gamma\sigma_p^2} \right\rfloor \tag{6}$$

In this paper, the SNR penalty is used as the criteria to evaluate the performance. With known level of phase noise variance, the required SNR is carefully chosen within a potential range determined experimentally. For each SNR, the corresponding L_{opt} is calculated and used for the DAML estimation until the given BEP is achieved.

The impact of laser phase noise on the receiver sensitivity is considered at first. As is shown in Fig. 1, the SNR penalty of systems with various modulation formats at BEP=10^{-4} is measured in reference to the varying of laser linewidth/symbol rate, $\Delta\upsilon T$. Obviously, in all considered situations, the SNR penalty increases exponentially with the increase of $\Delta\upsilon T$. Moreover, compared with their square counterparts, circular constellations tested here can achieve the same level of tolerance to $\Delta\upsilon T$ with much less SNR, due to their inherent tolerance to phase noise.

Compared with MED receiver, ML detector achieves higher receiver sensitivity in phase noise distorted channels. According to Fig. 1. (a), at laser linewidth per symbol rate $\Delta\upsilon T = 4{\times}10^{-4}$, MED detector incurs a SNR penalty of 1.85dB whereas ML detector incurs a smaller SNR penalty of 1.55dB in 8-square QAM modulated systems. Similarly, up to 0.6dB improvement can be measured at $\Delta\upsilon T = 1{\times}10^{-3}$. In 16-ary constellation cases, the observed improvement of SNR penalty is more significant. As shown in Fig. 1. (b), up to 0.5dB and1dB improvement can be observed by using ML detector in 16 APSK at $\Delta\upsilon T = 2.4{\times}10^{-4}$ and 16 QAM modulated systems at $\Delta\upsilon T = 6{\times}10^{-4}$, respectively. Generally speaking, proposed ML detector outperform conventional MED detector particularly in channels with high level laser phase noise. Additionally, among all tested cases, the most significant improvement can be measured in square 16-ary constellations, i.e. the 16 QAM. This is mainly due to its highest sensitivity to data rotation induced by the phase noise.

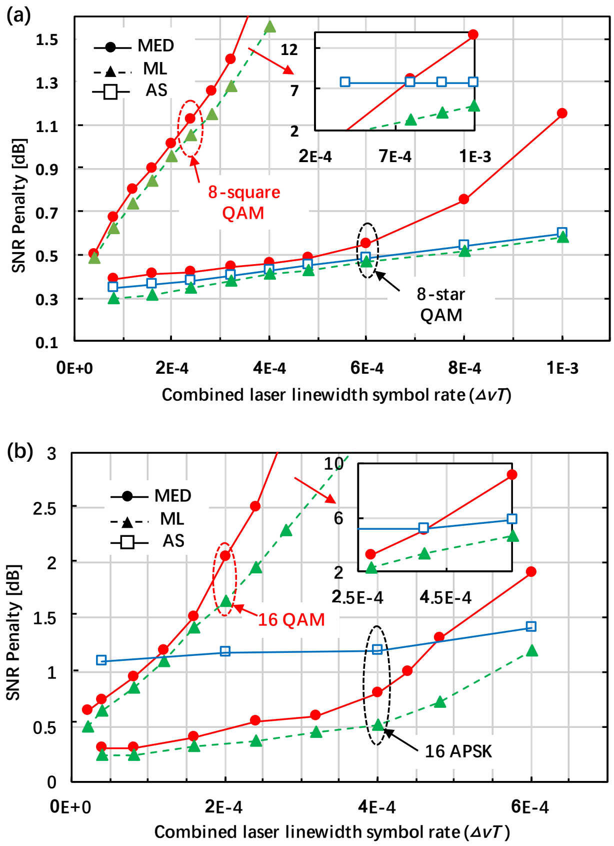

Fig. 1. Performance comparison of ML, AS and MED detectors as a function of combined laser linewidth symbol rate product at 25GS/s in (a) 8-ary modulated systems and (b) 16-ary modulated systems

On the other hand, the simulation results indicate the advantage of AS detector in systems suffering from high level phase noise. Compared with MED receiver, AS receiver incurs a higher requirement to the SNR at relatively low level of laser phase noise, particularly for the square constellations. However, with increasing $\Delta\upsilon T$, we can observe a continuous decrease of the performance gap between MED and AS receiver. Finally, it will outperform the MED receiver at $\Delta\upsilon T = 7.8{\times}10^{-4}$, $\Delta\upsilon T = 4{\times}10^{-4}$ and $\Delta\upsilon T = 4.7{\times}10^{-4}$ for 8-square QAM, 16QAM and 16 APSK, respectively. After that, the AS detector will gradually converge to the ML one at higher level of laser phase noise. As for the 8-star QAM case, an increasing performance improvement can be measured compared with the MED receiver over the entire selected SNR range. It is mainly due to the high radii ratio between two separate rings of the 8-star QAM which is set to the optimal value $r_2/r_1 = 2.4$ [4].

To investigate the frequency offset tolerance at a given laser linewidth, $\Delta v = 1{\times}10^{6}$ Hz, the SNR penalty of multiple constellations using MED, ML and AS detection methods is measured as a function of frequency offset symbol rate product, ΔfT, in Fig. 2. As illustrated, obviously, circular constellation achieves better tolerance compared with their square counterparts under same level frequency offset induced phase noise. Considering the performance comparison of different receivers, the ML method achieves the best tolerance to frequency offset under all considered circumstance and the performance improvement increases exponentially with the increase of ΔfT. The AS detector, instead, leads to higher requirement of SNR for low level of frequency offset. Similarly, the performance gap decreases with the increase of frequency offset and finally outperform its MED counterpart at $\Delta fT = 1.75{\times}10^{-2}$ for 8-sqaure QAM, $\Delta fT = 4.7{\times}10^{-3}$ for 16 QAM and $\Delta fT = 2.8{\times}10^{-3}$ for 16 APSK modulated systems. Finally, at relatively high level of ΔfT, the AS detector will converge to the ML detector.

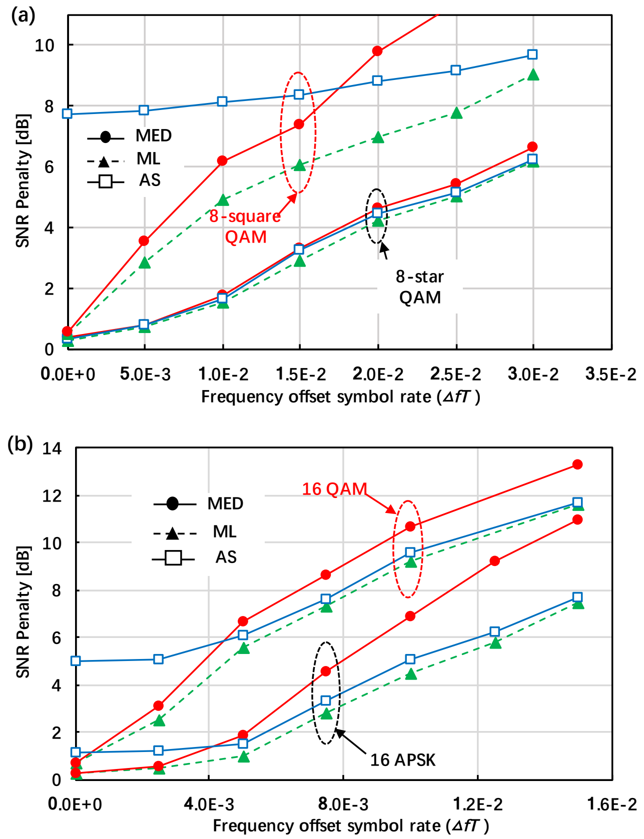

Fig. 2. Performance comparison of ML, AS and MED detectors for fixed laser linewidth as a function of frequency offset/symbol rate at 25GS/s in (a). 8-ary modulated systems and (b). 16-ary modulated systems

IV. Conclusion

In spite of the required explicit knowledge of the channel parameters: AWGN variance and PRE variance, the ML detector is shown to achieve the best performance compared with the AS detector and the conventional MED detector in the presence of varying laser phase noise and frequency offset. At high level SNR or phase noise induced by laser linewidth and frequency offset, AS detector will outperform the conventional MED detector and finally converges to ML detector at high level of phase noise and frequency offset. Overall, ML detector is more robust in real applications with inevitable phase rotation when the channel parameters are known. If not, the AS detector would be an effective alternative particularly in systems with strong phase rotation.

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