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Chromatic Dispersion Monitoring by Extended Kalman Filter for Coherent Optical OFDM Systems

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Abstract—**This paper proposes to monitor the total accumulated chromatic dispersion (CD) by using extended Kalman filter (EKF) for coherent optical orthogonal frequency-division multiplexing (CO-OFDM) systems. Accurate estimation performance can be achieved by only one OFDM training symbol. Simulations results show that the proposed EKF method can quickly converge to the exact dispersion value within 40 subcarriers. Besides, the influence of inter-carrier interference (ICI) induced by the carrier frequency offset (CFO) and laser phase noise (LPN) on the CD monitoring performance is also investigated. Even with the appearance of 3 GHz CFO and 200 kHz combined laser linewidth (CLW) and without compensation, the mean square error (MSE) and bit error rate (BER) of the proposed method vary little, which illustrate its good tolerance to the ICI.**

Keywords—Chromatic dispersion (CD); extended Kalman filter (EKF); coherent optical orthogonal frequency-division multiplexing (CO-OFDM)

I. INTRODUCTION

Coherent optical orthogonal frequency-division multiplexing (CO-OFDM) has been an attractive technology for long-haul optical communication systems. CO-OFDM takes the advantages of 'coherent detection' and 'OFDM' modulation, which show high spectral efficiency due to the orthogonally overlapped subcarriers [1] and the ease of compensating the chromatic dispersion (CD) in the electrical domain via digital signal processing (DSP) [2]. Compared with utilizing optical dispersion compensation fiber (DCF) [3], Bragg gratings [4] and optical resonators [5], the electronic dispersion compensation (EDC) has several advantages. First, the chromatic dispersion in long-haul optical links varies temporally, the EDC can be realized by simply adjusting the parameters of the compensation algorithm without any changes to outside plant [6]. Second, it reduces the first-installed cost by elimination of optical compensators and supporting amplifiers, which is required for optical loss induced by the optical dispersion compensation [7]. In addition, the deployment and reconfiguration as each channel discovers and optimizes its own dispersion is simplified. Besides, the linear channel impairments caused by optical filters can also be reduced [8].

EDC can be mainly classified into two categories: compensation at the transmitter (pre-distortion) and compensation at the receiver (post compensation). The electronic pre-distortion (EPD) operates at the transmitter which creates an optical signal that has already propagated through a 'virtual DCF', in order to ensure the received signal undistorted [6]. However, EPD requires a feedback path from receiver to transmitter and a modulator with dual drives for longer systems, so that it cannot compensate for the rapid variations caused by thermal drift, vibration, optical network switching, and polarization rotation [7]. The electronic post compensation (EPC) [9] can be applied for the fibers with rapid variations. Maximum likelihood sequence estimation (MLSE) combined with optical duo-binary modulation can compensate for dispersions up to 4500 ps/nm [10]. For longer distances, EDC at the receiver combined with optical single sideband (OSSB) works well, since the optical phase is translated to an electrical phase signal by the photodiode, whereas in a double sideband system, the two optical sidebands destroy its direct relationship [11-12].

In this paper, the extended Kalman filter is proposed for monitoring the total accumulated chromatic dispersion. The CD can be almost completely compensated without knowing the information of transmission distance and dispersion parameter of the fiber. Simulations including the MSE and BER performance investigation of the proposed method with or without the influence of ICI caused by the CFO and LPN are carried out.

II. SIGNAL MODEL

The n-th received time domain OFDM sample $r(n)$ with the appearance of carrier frequency offset (CFO), laser phase noise (LPN) and the chromatic dispersion (CD) can be expressed as [1]:

$$r(n) = e^{j(\theta + 2\pi\varepsilon n/N)} \cdot \frac{1}{N} \sum_{k=-N/2}^{N/2-1} X(k) e^{j2\pi kn/N} e^{j\Phi_D(k)} + w(k) \quad (1)$$

$$\Phi_D(k) = \pi\lambda^2 / c \cdot D_t \cdot f_k^2 \quad (2)$$

where θ is the LPN and ε is the CFO normalized by the OFDM subcarrier spacing. $X(k)$ represents the transmitted signal on the k-th subcarrier in the frequency domain. $w(k)$ is the complex additive white Guassian Noise (CAWGN) and

N is the Fast Fourier Transform (FFT) size. $\Phi_D(k)$ is the phase dispersion of each subcarrier owing to the fiber chromatic dispersion, where λ is the wavelength of the laser diode, c is the speed of light, D_t is the total accumulated chromatic dispersion in units of ps/nm and f_k is the frequency of the k-th subcarrier. Here the nonlinearity impact on CO-OFDM systems is not considered.

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III. EXTENDED KALMAN FILTERING ALGORITHM

Suppose only the chromatic dispersion and AWGN appears in the system and the first OFDM symbol is known and regarded as the training symbol. Then to estimate the total accumulated chromatic dispersion, D_t, a state-space model is employed as follows,

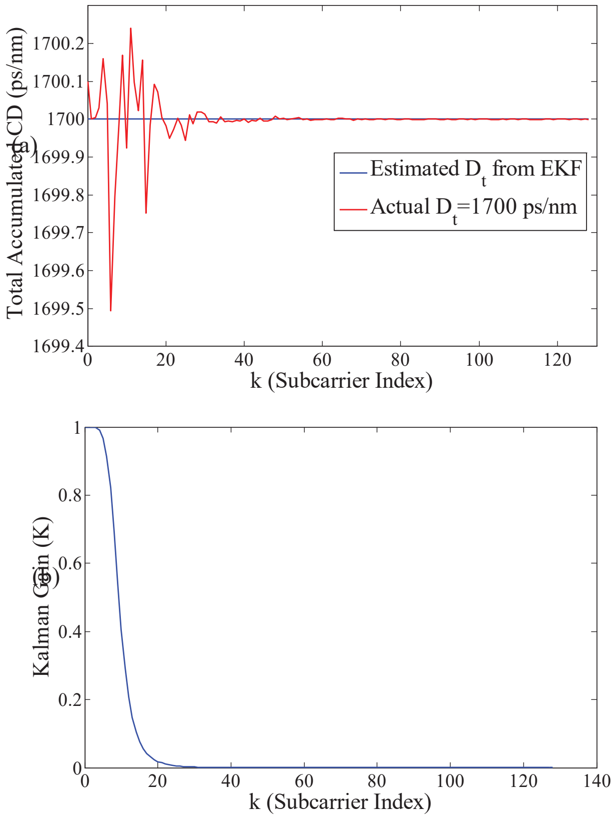

Fig. 1. (a) Estimated total accumulated chromatic dispersion using EKF; (b) Kalman gain of the total accumulated CD, at 5 dB SNR.

The state equation is given by

$$D_t(k) = D_t(k-1) \tag{3}$$

The measurement equation is given by

$$\hat{R}(k) = X(k)e^{j\pi\lambda^2/c\cdot D_t(k)\cdot f_k^2} + w(k) \tag{4}$$

where $k = 0,1,2,\cdots,N-1$ and $w(k)$ is CAWGN with the variance of σ^2. $\hat{R}(k)$ is the received sample on the k-th subcarrier in the frequency domain, which is obtained by taking the N-point FFT of $r(k)$.

Here, based on the proposed model, we apply the extended Kalman filter (EKF) [15] with the following steps,

1) Initialization: σ^2, $P(0)$, $\hat{D}_t(0)$, $k=0$,

$$g(x) = x \tag{5}$$

2) Recursive update for $k = 1,2,\cdots,N-1$ (N is the FFT size).

2.1) Calculate the Kalman Gain $K(k)$:

$$K(k) = P(k-1)H^*(k)\left[H(k)P(k-1)H^*(k)+\sigma^2\right]^{-1} \tag{6}$$

$$\begin{aligned} H(k) &= \left.\frac{\partial \hat{R}(k)}{\partial D_t}\right|_{D_t=\hat{D}_t(k-1)} \\ &= j\angle\left\{e^{j\pi\lambda^2/c\cdot f_k^2}\right\}X(k)e^{j\pi\lambda^2/c\cdot \hat{D}_t(k-1)\cdot f_k^2} \end{aligned} \tag{7}$$

2.2) Update the estimate:

$$\hat{D}_t(k) = g\left\{\hat{D}_t(k-1) + I_{update}(k)\right\} \tag{8}$$

$$I_{update}(k) = \Re\left\{K(k)\left[\hat{R}(k) - X(k)e^{j\pi\lambda^2/c\cdot\hat{D}_t(k-1)\cdot f_k^2}\right]\right\} \tag{9}$$

2.3) Calculate the estimation error variance:

$$P(k) = \left[1-K(k)H(k)\right]P(k-1). \tag{10}$$

After $N-1$ times of recursions, $\hat{D}_t(N-1)$ is used as the estimated total accumulated chromatic dispersion, $\hat{D}_t$. In (7), after the differentiation, the multiplied factor of $H(k)$ should be $j\pi\lambda^2 f_k^2/c$. However, since the subcarrier frequency f_k is quite large, the corresponding $H(k)$ is also quite large. Finally, after the recursions, $H(N-1)$ will almost be infinity and the Kalman Gain $K(k)$ will almost be zero, which lead to the infeasibility of the extended Kalman filter.

Since when calculating the $e^{j\pi\lambda^2/c\cdot\hat{D}_t(k-1)\cdot f_k^2}$, the exponential part $\left\{j\pi\lambda^2\hat{D}_t(k-1)f_k^2/c\right\}$ is actually within the range of $[-\pi,\pi)$, here we solve the above problem by taking the angle operation of $\left\{e^{j\pi\lambda^2/c\cdot f_k^2}\right\}$, which is regarded as the first order derivative of $e^{j\pi\lambda^2/c\cdot\hat{D}_t(k-1)\cdot f_k^2}$ with respect to $\hat{D}_t(k-1)$.

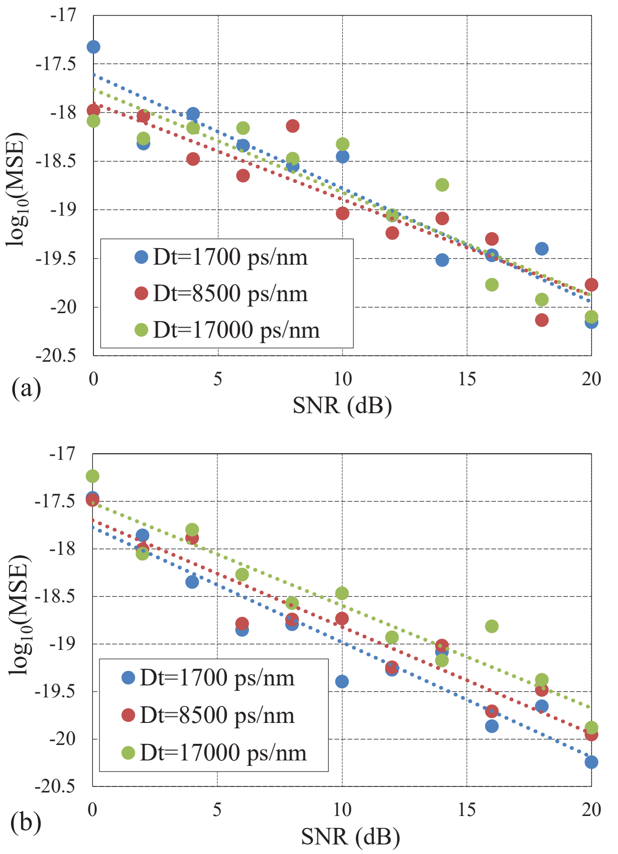

Fig. 2. The MSE of D_t versus SNR (E_s/N_0) (a) without the appearance of CFO and LPN; (b) with 3 GHz CFO and 200 kHz combined laser linewidth, w/o compensation.

IV. SIMULATION RESULTS

In our simulation, the length of transmitted Pseudo-Random Binary Sequence (PRBS) is $2^{15}-1$, which is mapped onto 16-ary quadrature amplitude modulation (QAM). The OFDM signal is transferred to the time domain by taking 256-point inverse fast Fourier transform (IFFT), with 128 effective subcarriers and 32-sample cyclic prefix (CP) added in front of each OFDM symbol. The sample rate is 25 GSa/s, with 64 OFDM symbols transmitted. One OFDM symbol is used for carrier frequency offset (CFO) estimation and five subcarriers are used for pilot-aided laser phase noise (LPN) estimation. Thus the effective bit rate is 47.3 Gbps (= $25G\,Sa/s\times\log_2 16\times 123/256\times 63/64$).

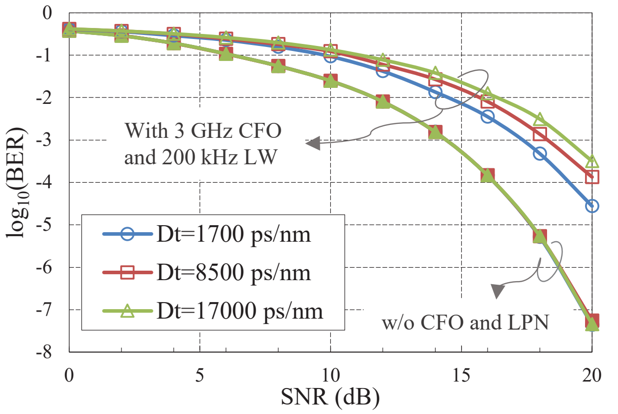

Fig. 3. BER versus SNR under different total accumulated chromatic dispersion values with and without the appearance of CFO and LPN.

In Fig. 1, we apply the proposed EKF to monitor the total accumulated CD of 1700 ps/nm, which is selected as the dispersion parameter being 17 ps/(nm-km) and transmission distance being 100 km, at 5 dB SNR (E_s/N_0). Fig. 1(a) shows the acquisition process of the extended Kalman filter. As can be seen, the EKF can quickly converge a steady state within 40 subcarriers, where the actual D_t is located. The Kalman gain which illustrates the fast tracking characteristic of EKF is shown in Fig. 1(b). The Kalman gain $K(k)$ decides the update rate of $\hat{D}_t(k)$ according to the new measurement $\hat{R}(k)$ at k-th subcarrier. In Fig. 1, the Kalman gain decreases quickly and finally approach a sufficient small value, which illustrates that the EKF is almost in a steady state.

We use the mean square error (MSE) given by $\sum_{k=1}^{N_i}\left(\left(D_t-\hat{D}_t\right)_k\right)^2/N_i$, to investigate the CD estimation performance of the proposed EKF, where N_i (=20000) is the number of simulation runs and k is the iteration index. Fig. 2 shows the MSE of the proposed EKF method versus SNR under different accumulated CD with and without the appearance of CFO and LPN. As can be seen, the MSE of the proposed method is quite small which is even below 10^{-17} . Besides, with the increase of D_t , the MSE varies little, even with the appearance of 3 GHz CFO and 200 kHz combined laser linewidth (LW), which illustrates the stable estimation performance of the proposed method.

Fig. 3 shows the BER performance of the proposed method versus SNR under different values of D_t , with and without CFO and LPN, which is compensated by using traditional algorithms in [2] and [3], separately. It can be seen from Fig. 3 that the BER curves of $D_t = 1700\,ps/nm$, $8500\,ps/nm$, and $17000\,ps/nm$ are the same, while with 3 GHz CFO and 200 kHz combined LW, there are small differences between the BERs. This may because the ICI introduced by the CFO and LPN affects the estimation performance and the residual CD exists in the signal. In addition, the BER with CFO and LPN is larger than that of without them, since the compensation algorithms in [13] and [14] may have estimation errors.

V. CONCLUSION

We proposed to monitor the total accumulated chromatic dispersion by using the extended Kalman filter. The MSE and BER performances were investigated which verify the feasibility and stability of the proposed algorithm. Also, the simulation results show that the proposed method is robust to the inter-carrier interference induced by carrier frequency offset and laser phase.

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