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Accurate Laser Linewidth Estimation for Coherent Optical Systems using DA-ML Carrier Phase Estimator

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ABSTRACT

We propose an accurate laser linewidth estimation technique based on the decision-aided maximum likelihood carrier phase estimator (DA-ML). Its performance is verified in M-ary quadrature amplitude modulation (MQAM) within a wide range of the laser linewidths and SNR values.

Keywords: *Laser linewidth estimation, DA-ML carrier phase estimator.*

1. INTRODUCTION

The characterization of the laser phase noise due to the transmitter and receiver lasers attracts extensive investigations. Early researchers relied on the electrical spectrum analyzer for accurate estimation of laser linewidth [1]. Recently, coherent receiver-based techniques, which allows time domain digital signal processing, become more popular. Both static and dynamic estimation can be performed using either heterodyne or intradyne receivers [2,3]. Compared with these time domain techniques, machine learning-based methods, including extended Kalman and particle filtering, achieve even better estimation accuracy [4].

This paper introduces an effective laser linewidth estimator that uses the decision-aided maximum likelihood (DA-ML) carrier phase estimator [5]. Since the proposed method relies on pure computation, it can be easily applied in dynamic optical networks without any requirement of additional hardware. The estimation accuracy of our method is investigated in multiple constellations considering both additive white Gaussian noise (AWGN) and phase noise via computer simulations.

2. OPERATION PRINCIPLE

It is assumed that the channel distortions such as chromatic dispersion (CD), polarization mode dispersion (PMD), and frequency offset (FO) are fully compensated using optical devices or digital signal processing (DSP) techniques [6], so that the channel is dominated by laser phase noise. The received signal model is [6]

$$r(k) = m(k)e^{j\theta(k)} + n(k). \tag{1}$$

Here, $m(k)$ is the transmitted signal, which takes on values from the signal set $\{S_i = A(i)e^{j\phi(i)}, i = 0,1,\dots,M-1\}$ with equal probability. $A(i)$ and $\phi(i)$ denote the amplitude and phase modulation of each symbol. M denotes the number of signal points. Term $n(k)$ is a complex, Gaussian random variable with mean zero and variance N_0, where N_0 is the one-side spectrum of the AWGN. Term $\theta(k)$ denotes the laser phase noise, which is commonly modeled as a Wiener process: [7]

$$\theta(k) = \theta(k-1) + \nu(k). \tag{2}$$

Term $\{\nu(k)\}$ is a set of independent, identical distribute, Gaussian random variables with mean zero and variance

$$\sigma_p^2 = 2\pi\Delta\nu T. \tag{3}$$

T and $\Delta\nu$ denote the symbol duration and combined linewidth of the transmitter and receiver lasers, respectively. Obviously, the phase increment $\{\nu(k)\}$ is independent with the AWGN $\{n(k)\}$. Regarding the phase of the kth received symbol, we can easily find [8]

$$\angle r(k) = \phi(k) + \theta(k) + \epsilon(k), \tag{4}$$

where $\phi(k) = 2\pi i/M$, $\{i = 0,1,\dots,M-1\}$ denoets the phase modulation of kth symbol. Term $\epsilon(k)$ denotes the additive observation phase noise (AOPN). According to previous research, for high SNR value, ϵ is shown to be approximately Gaussian distributed with mean zero and variance [9]

$$\sigma_\epsilon^2 = \eta/2\gamma, \tag{5}$$

where

$$\eta = E\left[\frac{1}{|m(k)|^2}\right] = \begin{cases} 1, & M-\text{ary PSK} \\ 1.8889, & 16\text{QAM} \\ 2.2282, & 32\text{QAM} \\ 2.6854, & 64\text{QAM} \end{cases} \tag{6}$$

is the constellation penalty [7]. Term $\gamma = E_s/N_0$ dentes the symbol SNR where E_s is the average symbol energy.

The DA-ML carrier phase estimator is first derived in [5]. Assuming slowly varying carrier phase noise, L previous decisions are used to estimate the current phase, where L is the estimation window length. With DA-ML method, the reference phasor (RP) kth symbol can be calculated as [6]

$$V(k) = U(k)^{-1} \sum_{l=k-L}^{k-1} r(l)\hat{m}^*(l), \tag{7}$$

where $U(k) = \sum_{l=k-L}^{k-1} |m(l)|^2$ is a factor used to normalize the reference phasor. Term $\hat{m}(k)$ denotes the decision of kth transmitted symbol $m(k)$. Here, the phase of the current RP $V(k)$ is recognized as the estimated phase noise $\hat{\theta}(k)$, i.e., $\angle V(k) \approx \theta(k)$. Due to the finite SNR in real applications, the application of DA-ML phase estimator would inevitably lead to a phase reference error (PRE), which is commonly defined as $\theta_e(k) = \theta(k) - \hat{\theta}(k)$. In [6], θ_e is shown, for high γ, to be approximately Gaussian distributed with mean zero and variance

$$\sigma_e^2 = \frac{2L^2+3L+1}{6L}\sigma_p^2 + \frac{\eta}{2\gamma L}. \tag{8}$$

In order to characterize the laser linewidth, we introduce an estimation variable $\omega(k)$, which is defined as

$$\omega(k) = \angle r(k) - \hat{\theta}(k) - \angle \hat{m}(k). \tag{9}$$

For high SNR, we can assume that the signal is accurately recovered through hard decisions so that $\angle \hat{m}(k) = \phi(k)$. Substituting (4) into (9), we can simply the $\omega(k)$ as

$$\begin{aligned} \omega(k) &= \phi(k) + \theta(k) + \epsilon(k) - \hat{\theta}(k) - \angle \hat{m}(k) \\ &= \theta_e(k) + \epsilon(k) \end{aligned} \tag{10}$$

Based on the definition of the PRE, we can easily find that $\theta_e(k)$ depends only on the phase noise $\theta(k)$ and DA-ML phase estimation $\hat{\theta}(k)$. As aforementioned, $\theta(k)$ consists of previos phase noise $\theta(k-1)$ and current phase increment $\nu(k)$, where both items are independent with current AWGN noise $n(k)$. Based on (7), $\hat{\theta}(k)$ is a function of the previous mesurements up to $(k-1)$th symbol, which is also independent with $n(k)$. Since $\epsilon(k)$ is defined based on the AWGN $n(k)$, we can easily find that $\theta_e(k)$ and $n(k)$ are statistically independent. From the definitation of $\omega(k)$ given in (10), we can conclude that, ω is also Gaussian distributed with mean zero and variance

$$\begin{aligned} \sigma_\omega^2 &= E[\omega^2(k)] = \sigma_e^2 + \sigma_\epsilon^2 \\ &= \frac{2L^2+3L+1}{6L}\sigma_p^2 + \frac{(L+1)\eta}{2\gamma L}. \end{aligned} \tag{11}$$

Through inverting (11), the laser phase noise variance σ_p^2 can be expressed as a function of γ, L and σ_ω^2, which is given as

$$\sigma_p^2 = \frac{6L\left(\sigma_\omega^2 - \eta\frac{L+1}{2\gamma L}\right)}{2L^2+3L+1}. \tag{12}$$

During the implementation, the DA-ML technique is applied to estimate the carrier phase of received symbol at first. Based on the definition given in (9), ω of each symbol can be easily measured through the simulation. With the measurement of $\omega(k)$, its sample variance $\text{var}[\omega] = E[\omega^2(k)]$ is calculated and recognized as an approximation of the exact variance σ_ω^2. Assuming known SNR γ, with the estimation of σ_ω^2, we can easily estimate the laser phase noise variance σ_p^2 based on (12) and further calculate the combined laser linewidth $\Delta\nu = \sigma_p^2/2\pi T$.

3. SIMULATION RESULTS AND DISCUSSIONS

Assuming perfect decision feedback, the estimation performance of the DA-ML based laser linewidth estimator is investigated. Regarding the real applications, the M-ary quadrature amplitude modulation (MQAM) with multiple amplitude levels is considered during the simulation. Here, the inverse of the normalized mean square error (NMSE), defined as

$$\text{NMSE} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\Delta\hat{\nu}_i - \Delta\nu}{\Delta\nu}\right)^2 \tag{13}$$

is used as the criterion, where $\Delta\hat{\nu}_i$ is the ith estimation of $\Delta\nu$ and N is the number of estimations used for each calculation of the NMSE.

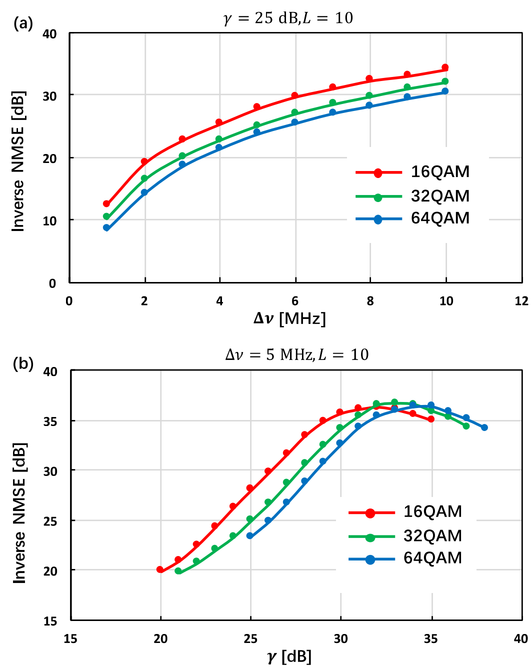

Fig. 1. Inverse NMSE versus (a). symbol SNR γ and (b). combined laser linewidth $\Delta\nu$, with symbol rate $R = 50$ GS/s.

The proposed estimator is first tested in 16, 32 and 64 QAM with various laser linewidth $\Delta\nu$ and symbol SNR γ. As shown in Fig. 1(a), we can observe a continuous performance improvement along with the increase of $\Delta\nu$ for all tested constellations. As illustrated in Fig. 1(b), there exit an optimal SNR value which leads to the best estimation performance. With small SNR, the increase of γ effectively improves the performance of the DA-ML carrier phase estimator, thereby leading to better accuracy of the laser linewidth estimation. However, when the SNR exceeds the optimal point, the variance of the estimation variable ω would be too small to be accurately estimated. In addition, the optimal SNR increases from 31 to 34 dB with the modulation index increases from 16 to 64. This is because, compared with 16 QAM, systems using advanced modulations require larger γ to achieve the same phase estimation accuracy.

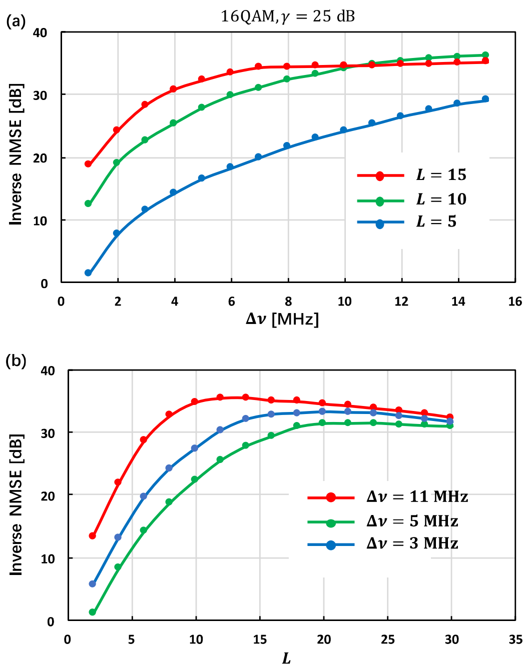

Fig. 2. Inverse NMSE versus (a). combined laser linewidth $\Delta\nu$ and (b). estimation window length L, with $R = 50$ Gs/s.

During the implementation of DA-ML carrier phase estimator, the estimation window length L is an important parameter. The effect of L on the estimation accuracy and the optimal choice of L for various laser linewidth $\Delta\nu$ are further investigated in 16QAM. As shown in Fig. 2(a), the proposed method achieves better performance at higher $\Delta\nu$ as expected. Moreover, we can observe a performance improvement of up to 10 dB with L increases from 5 to 10 and 15. The cross point of the inverse NMSE curves indicates that the optimal choice of L depends on the laser linewidth value. As shown in Fig. 2(b), there exist an optimal point for each $\Delta\nu$ as expected and the optimal L decreases from around 20 to 12 with the laser linewidth grows from 3 to 11 MHz. This is because, during the implementation of DA-ML method, a long estimation window is preferred to average out the AWGN noise at small $\Delta\nu$. In contrast, when the system suffers from high level phase noise, the long estimation window will conflict with the assumption that the phase noise within the considered estimation window is slowly varying, there by degrading the estimation performance. Hence, systems with large $\Delta\nu$ prefer a small L.

4. CONCLUSION

In this paper, an accurate laser linewidth estimation technique based on the DA-ML phase estimator is introduced and tested in multiple advanced modulations. Numerical simulations show that our technique is applicable within a wide range of symbol SNR γ and combined laser linewidth $\Delta\nu$. Moreover, the estimation performance improves with the increase of $\Delta\nu$. We further investigate the effect of the estimation window length L on the estimation performance, which facilitate the implementations of our technique.

5. ACKNOWLEDGMENTS

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