

Multiple-Plane Object Reconstruction Using Single-Pixel Digital Holography

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Abstract—We propose a method for multiple-plane object reconstruction using single-pixel digital holography. Multiple objects are placed at different axial distances from the modulator plane. Instead of using charge-coupled device (CCD) to record, we utilize single-pixel structured detection to first retrieve the holograms and then reconstruct the objects at different axial positions with a digital focusing method. In order to retrieve a high-quality digital hologram, 4-step Fourier spectrum acquisition method is used which can effectively reduce noise in the reconstruction. In addition, since images are commonly sparse in Fourier domain, it is found that only 10% Fourier spectrum coefficients can be used to retrieve high-quality digital holograms. Due to significant advantages of single-pixel imaging, e.g., imaging in conditions of low-light and non-visible wavelength, it is believed that the proposed method can effectively extend the applications of conventional digital holographic technique. Computational results are obtained and presented to demonstrate feasibility and effectiveness of the proposed method.

Keywords—digital holography, single-pixel imaging, Fourier spectrum acquisition, multiple-plane object reconstruction

I. INTRODUCTION

Holography has been developed for many years, and is considered as a promising technique in numerous applications. In a holographic setup, there are usually two beams, i.e., object beam and reference beam. The two beams interfere with each other, and form an interference pattern. In the early period, photosensitive medium, e.g., photographic plate, was used to record the intensity of interference pattern, i.e., hologram. However, there are several disadvantages, such as low efficiency and complicated procedure. By taking advantage of the rapid development of fast-speed and high-resolution optoelectronic recording devices, digital holography has attracted more and more attention. Digital holography facilitates the recovery of both amplitude and phase information by using numerical calculation. Digital holography has been applied to many research fields, e.g., surface topography [1], biological imaging [2–4], shape measurement [5, 6], optical encryption [7] and object recognition [8, 9]. However, recording device, such as CCD used in conventional digital holography, still have

several limitations. For instance, CCD camera is usually not sensitive in the condition of low light. In addition, the CCD camera usually has no capability in recording the data, when illumination source with non-visible wavelength is used.

It is recently found that single-pixel imaging (SPI) has the capability for imaging in conditions of low light and non-visible wavelength. The SPI utilizes a single-pixel detector (also called bucket detector) to collect data. Different from CCD camera which has a spatial resolution, single-pixel bucket detector has no spatial resolution. The reconstruction in the SPI can be conducted by using different algorithms. A basic method for the SPI is related to ghost imaging (GI) which is also called correlation imaging [10–15]. Correlation algorithm used in GI usually reconstructs the object with low quality, and the number of measurements is usually large. Compressive sensing (CS) is an important tool for the SPI to reconstruct objects, which can significantly reduce the number of measurements [16–19]. However, optimization methods are commonly used in the CS, which results in high computational complexity. Recently, it is demonstrated that by designing the illumination patterns in the SPI, high-quality object reconstruction can be implemented directly [20–30]. However, few work has been studied by applying the SPI into digital holography, and there is no related work in obtaining the holograms by using Fourier spectrum acquisition method.

In this paper, we propose a method for multiple-plane object reconstruction based on single-pixel digital holography. Three objects are placed at different axial distances from the modulator plane. Instead of using CCD camera to record, we utilize single-pixel structured detection to retrieve the hologram first, and then reconstruct the objects at different axial planes by digital focusing. In order to retrieve high-quality digital hologram, 4-step Fourier spectrum acquisition method is used which can effectively reduce noise in the reconstruction. Here, 10% Fourier spectrum coefficients are measured and used to retrieve the hologram. Since images are commonly sparse in Fourier domain, a small part of spectrum coefficients can be used to carry out high-quality object reconstruction. Due to significant advantages of single-pixel imaging, e.g., imaging in conditions of low-light and non-visible wavelength, it is believed that the proposed method can effectively extend the applications of

conventional digital holography. The computational results and discussion are presented to demonstrate feasibility and effectiveness of the proposed method.

II. THEORIES

A schematic setup for the proposed method is shown in Fig. 1.

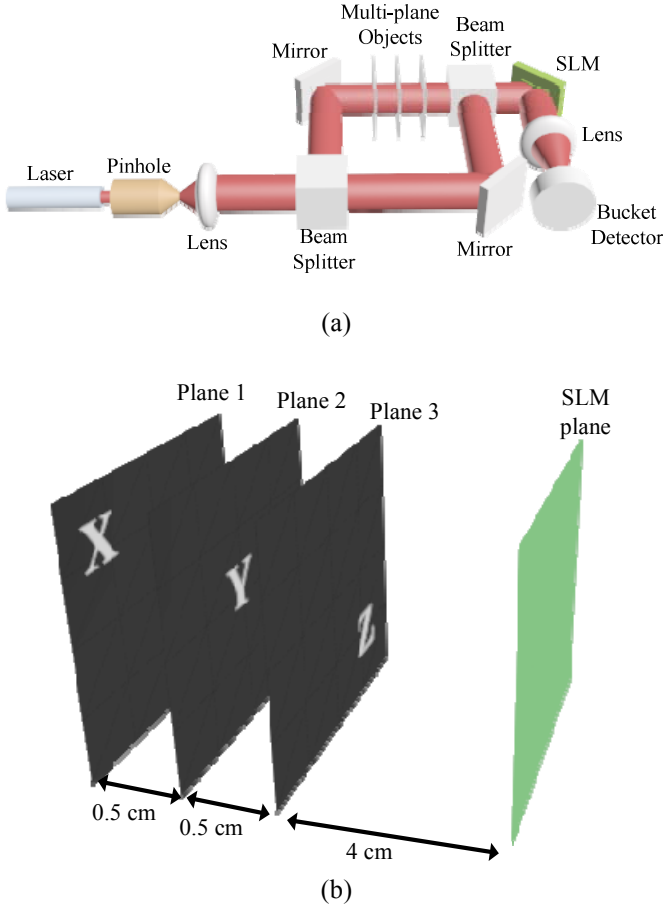


Fig. 1. (a) Schematic setup. SLM: spatial light modulator, and (b) multiple-plane objects placed at different axial distances.

As shown in Fig. 1(a), a laser beam is collimated by a pinhole and a lens. There are two beams after passing through a beam splitter. In the beam path containing objects, the beam is sequentially modulated by three objects which are located at different axial positions. The two beams interfere with each other just before the SLM plane. The interference pattern is modulated by the SLM, and the light reflected by the SLM is collected by a single-pixel bucket detector without spatial resolution. Figure 1(b) shows three different objects placed at different axial positions. The wavefront $O[m, n]$ modulated by an object can be described by

$$O[m, n] = IFT \left(FT \{ U[m, n] \} ST[p, q] \right), \quad (1)$$

where FT and IFT respectively denote Fourier transform and inverse Fourier transform, $[m, n]$ and $[p, q]$ respectively denote coordinates of spatial domain and Fourier domain, $U[m, n]$ represents an object, and $ST[p, q]$ denotes transfer function which can be expressed as

$$ST[p, q] = \exp \left[jk_0 z \sqrt{1 - \frac{(p\Delta_{kx})^2}{k_0^2} - \frac{(q\Delta_{ky})^2}{k_0^2}} \right], \quad (2)$$

where $j = \sqrt{-1}$, wave number $k_0 = 2\pi/\lambda$, λ denotes wavelength, z denotes axial distance between the object plane and the SLM plane, and Δ_{kx} and Δ_{ky} denote frequency resolution corresponding to sampling periods Δ_x and Δ_y . The relationship between $(\Delta_{kx}, \Delta_{ky})$ and (Δ_x, Δ_y) is described by

$$\Delta_{kx} = \frac{2\pi}{M\Delta_x} \text{ and } \Delta_{ky} = \frac{2\pi}{N\Delta_y}, \quad (3)$$

where M and N respectively denote the dimensions in horizontal and vertical directions.

In the proposed method, the interference pattern is modulated by a series of sinusoidal patterns sequentially embedded in the SLM. Hence, the hologram H just before the SLM plane can be retrieved by taking advantage of Fourier spectrum acquisition method. The sinusoidal pattern used in Fourier spectrum acquisition method can be expressed as

$$P_\psi = a + b \cos \left(2\pi \frac{p}{M} m + 2\pi \frac{q}{N} n + \psi \right), \quad (4)$$

where a denotes a constant which is equivalent to average intensity of the object, b denotes a scale factor which is a constant, and ψ denotes the phase which respectively has four values, i.e., $0, \pi/2, \pi$ and $3\pi/2$. Hence, 4-step Fourier spectrum acquisition method can be expressed as

$$\begin{aligned} B_\psi &= \sum_m \sum_n H[m, n] P_\psi \\ &= \sum_m \sum_n H[m, n] \left[a + b \cos \left(2\pi \frac{p}{M} m + 2\pi \frac{q}{N} n + \psi \right) \right], \end{aligned} \quad (5)$$

where B_ψ denotes single-pixel value corresponding to a given phase. After four single-pixel values, i.e., $B_0, B_{\pi/2}, B_\pi$ and $B_{3\pi/2}$ are obtained, each coefficient can be constructed by

$$G(p, q) = \frac{1}{2b} \left[(B_0 - B_\pi) + j(B_{\pi/2} - B_{3\pi/2}) \right]. \quad (6)$$

After the hologram just before the SLM is retrieved by using the aforementioned approach, free-space wave propagation principle [31–34] is applied to recover each object at different axial positions using digital focusing.

III. RESULTS AND DISCUSSION

In this study, computational work is conducted to show validity of the proposed method, and a schematic setup for the proposed method is shown in Fig. 1. Each of the images has 512×512 pixels. The objects are separated from each other with an axial distance of 0.5 cm . The axial distance from the SLM plane to the nearest object plane is 4.0 cm . It is assumed that the illumination source has a wavelength of 632.8 nm , and pixel size of SLM is set as $4.65 \mu\text{m}$.

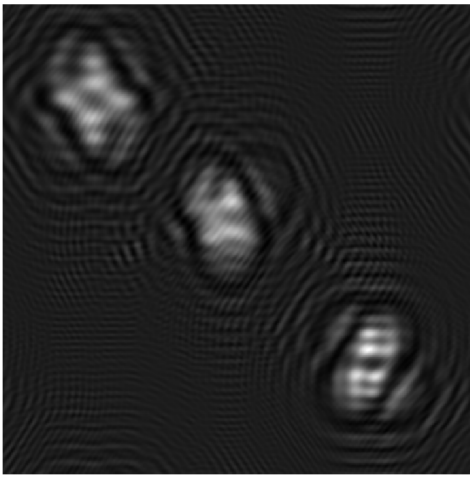
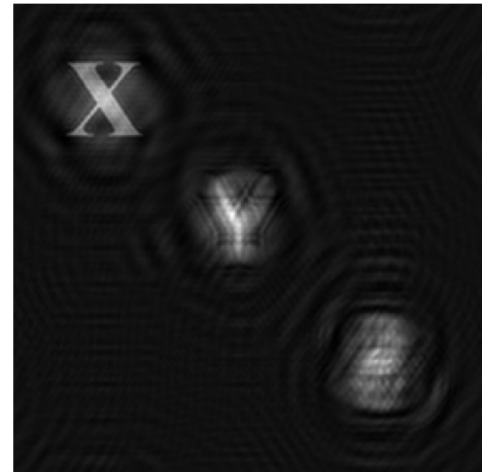


Fig. 2. A hologram retrieved just before the SLM by using Fourier spectrum acquisition method with only 10% Fourier spectrum coefficients in the SPI.

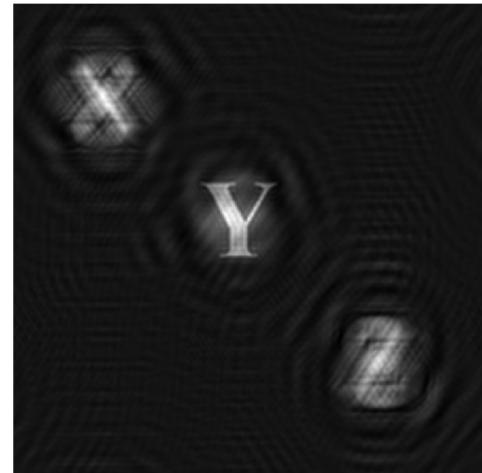
Figure 2 shows a hologram retrieved by using Fourier spectrum acquisition method with only 10% Fourier spectrum coefficients in the SPI. In this case, digital hologram just before the SLM plane has been retrieved by taking advantage of Fourier spectrum acquisition method, and a series of single-pixel intensity measurements have been used to retrieve digital hologram just before the SLM. To verify whether the retrieved hologram can be further used to reconstruct the three objects located at different axial planes, free-space back wave propagation [31–34] is numerically carried out and object recovery results are shown in Fig. 3.

As shown in Fig. 3, digital hologram retrieved by using the proposed method can be effectively used to reconstruct the three objects located at different axial planes. When the axial distance is chosen to be 5.0 cm for free-space back propagation, only the letter X can be clearly observed in the reconstructed image, as shown in Fig. 3(a). Focal distance of the second object is 4.5 cm , and only the letter Y can be clearly observed as illustrated in Fig. 3(b) when we set the axial distance as 4.5 cm for object reconstruction. Similarly, the letter Z can be

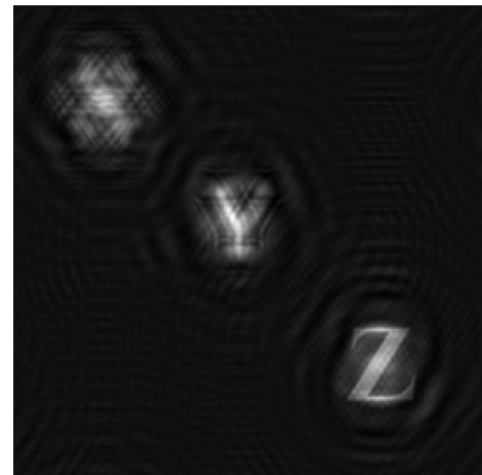
reconstructed by using a correct focal distance, i.e., 4.0 cm , as seen in Fig. 3(c).



(a)



(b)



(c)

Fig. 3. Reconstructed objects obtained by using a focal distance of (a) 5.0 cm , (b) 4.5 cm , and (c) 4.0 cm .

IV. CONCLUSIONS

We have proposed a method for recovering multiple objects located at different axial planes using single-pixel digital holography. Fourier spectrum acquisition method is utilized to retrieve high-quality digital hologram. It is demonstrated that the retrieved digital hologram can be used to recover multiple-plane objects. It is believed that the proposed method can resolve the problems existing in conventional digital holographic method, and applications of digital holography can be effectively extended.

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REFERENCES

- [1] J. Kühn, F. Charrière, T. Colomb, E. Cuche, F. Montfort, Y. Emery, P. Marquet, and C. Depeursinge, "Axial sub-nanometer accuracy in digital holographic microscopy," *Meas. Sci. Technol.*, Vol. 19, pp. 074007, May 2008.
- [2] P. Marquet, B. Rappaz, P. J. Magistretti, E. Cuche, Y. Emery, T. Colomb, and C. Depeursinge, "Digital holographic microscopy: a noninvasive contrast imaging technique allowing quantitative visualization of living cells with subwavelength axial accuracy," *Opt. Lett.*, vol. 30, pp. 468–470, March 2005.
- [3] B. Kemper and G. von Bally, "Digital holographic microscopy for live cell applications and technical inspection," *Appl. Opt.*, vol. 47, pp. A52–A61, February 2008.
- [4] B. Rappaz, F. Charrière, C. Depeursinge, P. Magistretti, and P. Marquet, "Simultaneous cell morphometry and refractive index measurement with dual-wavelength digital holographic microscopy and dye-enhanced dispersion of perfusion medium," *Opt. Lett.*, vol. 33, pp. 744–746, April 2008.
- [5] S. Seebacher, W. Osten, and W. Jüptner, "Measuring shape and deformation of small objects using digital holography," *Proc. SPIE*, vol. 3479, pp. 104–115, July 1998.
- [6] G. Pedrini, P. Fröning, H. Tiziani, and F. M. Santoyo, "Shape measurement of microscopic structures using digital holograms," *Opt. Commun.*, vol. 164, pp. 257–268, June 1999.
- [7] B. Javidi and T. Nomura, "Securing information by use of digital holography," *Opt. Lett.*, vol. 25, pp. 28–30, January 2000.
- [8] B. Javidi and E. Tajahuerce, "Three-dimensional object recognition by use of digital holography," *Opt. Lett.*, vol. 25, pp. 610–612, May 2000.
- [9] A. Nelleri, U. Gopinathan, J. Joseph, and K. Singh, "Three dimensional object recognition from digital Fresnel hologram by wavelength matched filtering," *Opt. Commun.*, vol. 259, pp. 499–506, March 2006.
- [10] T. B. Pittman, Y. H. Shih, D. V. Strekalov, and A. V. Sergienko, "Optical imaging by means of two-photon quantum entanglement," *Phys. Rev. A*, vol. 52, no. 5, pp. R3429–R3432, November 1995.
- [11] D. V. Strekalov, A. V. Sergienko, D. N. Klyshko, and Y. H. Shih, "Observation of two-photon "ghost" interference and diffraction," *Phys. Rev. Lett.*, vol. 74, no. 18, May 1995, Art. ID 3600.
- [12] A. Gatti, E. Brambilla, M. Bache, and L. A. Lugiato, "Ghost imaging with thermal light: comparing entanglement and classical correlation," *Phys. Rev. Lett.*, vol. 93, no. 9, Aug. 2004, Art. ID 093602.
- [13] A. Valencia, G. Scarcelli, M. D'Angelo, and Y. Shih, "Two-photon ghost imaging with thermal light," *Phys. Rev. Lett.*, vol. 94, no. 6, Feb. 2005, Art. ID 063601.
- [14] J. H. Shapiro, "Computational ghost imaging," *Phys. Rev. A*, vol. 78, no. 6, Dec. 2008, Art. ID 061802R.
- [15] Y. Bromberg, O. Katz, and Y. Silberberg, "Ghost imaging with a single detector," *Phys. Rev. A*, vol. 79, no. 5, May 2009, Art. ID 053840.
- [16] D. L. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, Apr. 2006.
- [17] E. J. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 489–509, Feb. 2006.
- [18] M. F. Duarte, M. A. Davenport, D. Takhar, J. N. Laska, T. Sun, K. F. Kelly, and R. G. Baraniuk, "Single-pixel imaging via compressive sampling," *IEEE Signal Process. Mag.*, vol. 25, no. 2, pp. 83–91, Mar. 2008.
- [19] W. L. Chan, K. Charan, D. Takhar, K. F. Kelly, R. G. Baraniuk, and D. M. Mittleman, "A single-pixel terahertz imaging system based on compressed sensing," *App. Phys. Lett.*, vol. 93, no. 12, Sep. 2008, Art. ID 121105.
- [20] Z. Zhang, X. Ma, and J. Zhong, "Single-pixel imaging by means of Fourier spectrum acquisition," *Nat. Comm.*, vol. 6, no. 6225, Feb. 2015.
- [21] L. Bian, J. Suo, X. Hu, F. Chen, and Q. Dai, "Efficient single pixel imaging in Fourier space," *J. Opt.*, vol. 18, no. 8, Jul. 2016, Art. ID 085704.
- [22] B. L. Liu, Z. H. Yang, X. Liu, and L. A. Wu, "Coloured computational imaging with single-pixel detectors based on a 2D discrete cosine transform," *J. Mod. Opt.*, vol. 64, no. 3, pp. 259–264, Sep. 2016.
- [23] J. Huang, D. Shi, K. Yuan, S. X. Hu, and Y. J. Wang, "Computational-weighted Fourier single-pixel imaging via binary illumination," *Opt. Express*, vol. 26, no. 13, pp. 16475–16559, Jun. 2018.
- [24] H. Z. Jiang, S. G. Zhu, H. J. Zhao, B. J. Xu, and X. D. Li, "Adaptive regional single-pixel imaging based on the Fourier slice theorem," *Opt. Express*, vol. 25, no. 13, pp. 15118–15130, May 2017.
- [25] H. D. Ren, S. M. Zhao, and J. Gruska, "Edge detection based on single-pixel imaging," *Opt. Express*, vol. 26, no. 5, pp. 5501–5511, Feb. 2018.
- [26] B. J. Xu, H. Z. Jiang, H. J. Zhao, X. D. Li, and S. G. Zhu, "Projector-defocusing rectification for Fourier single-pixel imaging," *Opt. Express*, vol. 26, no. 4, pp. 5005–5017, Feb. 2018.
- [27] Z. B. Zhang, S. J. Liu, J. Z. Peng, M. H. Yao, G. A. Zheng, and J. G. Zhong, "Simultaneous spatial, spectral, and 3D compressive imaging via efficient Fourier single-pixel measurements," *Optica*, vol. 5, no. 3, pp. 315–319, Mar. 2018.
- [28] Z. B. Zhang, S. M. Jiao, M. H. Yao, X. Li, and J. G. Zhong, "Secured single-pixel broadcast imaging," *Opt. Express*, vol. 26, no. 11, pp. 14578–14591, May 2018.
- [29] Z. B. Zhang, X. Y. Wang, G. A. Zheng, and J. G. Zhong, "Fast Fourier single-pixel imaging via binary illumination," *Sci. Rep.*, vol. 7, Sep. 2017, Art. ID 12029.
- [30] Y. Xiao, L. N. Zhou, and W. Chen, "Fourier spectrum retrieval in single-pixel imaging," *IEEE Photon. J.*, Accepted and In press, 2019.
- [31] J. W. Goodman, *Introduction to Fourier Optics*, 2nd ed., New York, McGraw-Hill, 1996.
- [32] W. Chen, B. Javidi, and X. Chen, "Advances in optical security systems," *Adv. Opt. Photon.*, vol. 6, no. 2, pp. 120–155, April 2014.
- [33] W. Chen and X. Chen, "Object authentication in computational ghost imaging with the realizations less than 5% of Nyquist limit," *Opt. Lett.*, vol. 38, no. 4, pp. 546–548, February 2013.
- [34] W. Chen and X. Chen, "Focal-plane detection and object reconstruction in the noninterferometric phase imaging," *J. Opt. Soc. Am. A*, vol. 29, no. 4, pp. 585–592, March 2012.