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# On the Capacity of Intelligent Reflecting Surface Aided MIMO Communication

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***Abstract*—Intelligent reflecting surface (IRS) is a promising solution to enhance the wireless communication capacity both cost-effectively and energy-efficiently, by properly altering the signal propagation via tuning a large number of passive reflecting units. In this paper, we aim to characterize the fundamental *capacity* limit of IRS-aided point-to-point multiple-input multiple-output (MIMO) communication systems with multi-antenna transmitter and receiver in general, by jointly optimizing the IRS reflection coefficients and the MIMO transmit covariance matrix. We consider narrowband transmission under frequency-flat fading channels, and develop an efficient *alternating optimization* algorithm to find a locally optimal solution by iteratively optimizing the transmit covariance matrix or one of the reflection coefficients with the others being fixed. Numerical results show that our proposed algorithm achieves substantially increased capacity compared to traditional MIMO channels without the IRS, and also outperforms various benchmark schemes.**

## I. Introduction

Recently, *intelligent reflecting surface (IRS)* and its various equivalents have emerged as a new and promising solution to meet the ever-increasing demand for higher-capacity communications in the fifth-generation (5G) and beyond wireless networks [1]–[4]. Specifically, IRS is a planar meta-surface equipped with a large number of *passive* reflecting elements connected to a smart controller, which is capable of inducing an independent phase shift and/or amplitude attenuation (collectively termed as "reflection coefficient") to the incident signal at each reflecting element in real-time, thereby modifying the wireless channels between one or more pairs of transmitters and receivers to be more favorable for their communications [1]. By judiciously designing its reflection coefficients, the signals reflected by IRS can be added either constructively with those via other signal paths to increase the desired signal strength at the receiver, or destructively to mitigate the co-channel interference, thus offering a new degree-of-freedom (DoF) to enhance the communication performance.

To fully exploit the new DoF brought by IRS, the *IRS reflection coefficients* (also termed as "passive beamforming") need to be carefully designed, which has been studied under various system and channel setups [5]–[8]. It is worth noting that the existing works on IRS-aided communication mainly focused on single-input single-output (SISO) or multiple-input single-output (MISO) systems with single-antenna receivers. However, there has been very limited work on *IRS-aided multiple-input multiple output (MIMO) communication* with multiple antennas at both the transmitter and the receiver, while only a couple of papers appeared recently [9], [10]. In particular, the *characterization of the capacity limit* of

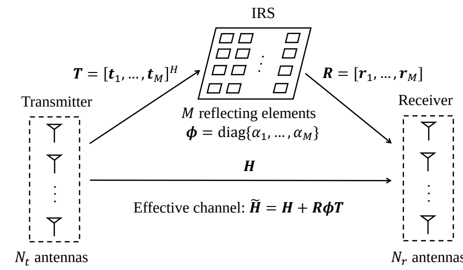


Fig. 1. Diagram of an IRS-aided MIMO communication system.

IRS-aided MIMO communication still remains open, which requires the *joint optimization of IRS reflection coefficients and MIMO transmit covariance matrix*, and thus is more challenging than the traditional MIMO channel capacity characterization [11] without the IRS reflection. Note that this problem is also more difficult to solve as compared to that in IRS-aided SISO/MISO communications with single data stream transmission only, since the MIMO channel capacity is generally achieved by transmitting *multiple data streams* in parallel (i.e., spatial multiplexing), thus the reflection coefficients need to be properly designed to optimally balance the channel gains for multiple spatial data streams so as to maximize their sum-rate. To the best of our knowledge, this problem has not been fully addressed yet (e.g., in [9], [10]), even for the point-to-point IRS-aided MIMO communication, which thus motivates this work.

In this paper, we study the joint IRS reflection coefficient and transmit covariance matrix optimization for maximizing the capacity of a point-to-point IRS-aided MIMO system with multiple antennas at both the transmitter and the receiver, as illustrated in Fig. 1. To characterize the fundamental capacity limit, we consider that perfect channel state information (CSI) of all channels involved in Fig. 1 is available at both the transmitter and the receiver by assuming that the CSI has been accurately acquired via the techniques proposed in e.g., [8], [12], [13]. Moreover, to reduce the implementation complexity of IRS,[1] we consider that the amplitude of all its reflection coefficients is fixed as the maximum value of one [1]. The capacity maximization problem is non-convex and thus difficult to solve. By exploring the structure of the MIMO capacity expression, we develop an *alternating optimization* algorithm by iteratively optimizing one of the reflection coefficients or the transmit covariance matrix with the other optimization

[1] In practice, dynamic change of the resistor load connected to each reflecting element is needed to adjust the reflection amplitude [14], which, however, is difficult to implement in real-time with separate phase-shift control.

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variables being fixed. We derive the *optimal* solution to each subproblem for optimizing one of these variables in *closed-form*, which greatly reduces the computational complexity. It is shown that the proposed algorithm is guaranteed to converge to at least a *locally optimal* solution. Numerical results are provided to validate the performance advantages of our proposed alternating optimization algorithm over other benchmark schemes with or without IRS.

## II. System Model and Problem Formulation

We consider a MIMO communication system with $N_t \geq 1$ antennas at the transmitter and $N_r \geq 1$ antennas at the receiver, as illustrated in Fig. 1, where an IRS equipped with $M$ passive reflecting elements is deployed to enhance the MIMO communication performance. Each element of the IRS is able to re-scatter the signal at the IRS with an individual reflection coefficient, which can be dynamically adjusted by the IRS controller for desired signal reflection. Specifically, let $\alpha_m \in \mathbb{C}$ denote the reflection coefficient of the $m$th IRS element, which is assumed to satisfy $|\alpha_m| = 1, \forall m = 1, ..., M$, while the phase of each $\alpha_m$ can be flexibly adjusted in $[0, 2\pi)$ [15].[2]

We assume quasi-static block-fading channels, and focus on one particular fading block where all the channels involved in Fig. 1 remain approximately constant. For the purpose of exposition, we consider narrowband transmission over frequency-flat channels. Denote $\boldsymbol{H} \in \mathbb{C}^{N_r \times N_t}$ as the complex baseband channel matrix for the direct link from the transmitter to the receiver, $\boldsymbol{T} \in \mathbb{C}^{M \times N_t}$ as that from the transmitter to the IRS, and $\boldsymbol{R} \in \mathbb{C}^{N_r \times M}$ as that from the IRS to the receiver. Let $\boldsymbol{\phi} \in \mathbb{C}^{M \times M}$ denote the diagonal reflection matrix of the IRS, with $\boldsymbol{\phi} = \operatorname{diag}\{\alpha_1, ..., \alpha_M\}$. We assume that the signal reflected by the IRS more than once is of negligible power due to the high path loss and thus can be ignored. Therefore, the effective MIMO channel matrix from the transmitter to the receiver is given by $\tilde{\boldsymbol{H}} = \boldsymbol{H} + \boldsymbol{R}\boldsymbol{\phi}\boldsymbol{T}$.

Let $\boldsymbol{x} \in \mathbb{C}^{N_t \times 1}$ denote the transmitted signal vector. The transmit signal covariance matrix is thus defined as $\boldsymbol{Q} \triangleq \mathbb{E}[\boldsymbol{x}\boldsymbol{x}^H] \in \mathbb{C}^{N_t \times N_t}$, with $\boldsymbol{Q} \succeq \boldsymbol{0}$. We consider an average sum power constraint at the transmitter given by $\mathbb{E}[\|\boldsymbol{x}\|^2] \leq P$, which is equivalent to $\operatorname{tr}(\boldsymbol{Q}) \leq P$. The received signal vector denoted as $\boldsymbol{y} \in \mathbb{C}^{N_r \times 1}$ is given by

$$\boldsymbol{y} = \tilde{\boldsymbol{H}}\boldsymbol{x} + \boldsymbol{z} = (\boldsymbol{H} + \boldsymbol{R}\boldsymbol{\phi}\boldsymbol{T})\boldsymbol{x} + \boldsymbol{z}, \quad (1)$$

where $\boldsymbol{z} \sim \mathcal{CN}(0, \sigma^2 \boldsymbol{I}_{N_r})$ denotes the independent circularly symmetric complex Gaussian (CSCG) noise vector at the receiver, with $\sigma^2$ denoting the average noise power. To reveal the fundamental capacity limit, we assume that perfect CSI is available at both the transmitter and receiver. The MIMO channel capacity is thus given by

$$C = \max_{\boldsymbol{Q}:\operatorname{tr}(\boldsymbol{Q}) \leq P, \boldsymbol{Q} \succeq \boldsymbol{0}} \log_2 \det\left(\boldsymbol{I}_{N_r} + \frac{1}{\sigma^2}\tilde{\boldsymbol{H}}\boldsymbol{Q}\tilde{\boldsymbol{H}}^H\right) \quad (2)$$

in bits per second per Hertz (bps/Hz). Note that different from the conventional MIMO channel without the IRS, i.e., $\tilde{\boldsymbol{H}} = \boldsymbol{H}$, for which the capacity is solely determined by the channel matrix $\boldsymbol{H}$, the capacity for the IRS-aided MIMO channel shown in (2) is also dependent on the IRS reflection matrix $\boldsymbol{\phi}$, since it influences the effective channel matrix $\tilde{\boldsymbol{H}}$ as well as the resultant optimal transmit covariance matrix $\boldsymbol{Q}$.

Motivated by the above, we aim to maximize the capacity of an IRS-aided MIMO channel by jointly optimizing the IRS reflection matrix $\boldsymbol{\phi}$ and the transmit covariance matrix $\boldsymbol{Q}$, subject to uni-modular constraints on the reflection coefficients and a sum power constraint at the transmitter. The optimization problem is formulated as

$$\text{(P1)} \quad \max_{\boldsymbol{\phi}, \boldsymbol{Q}} \ \log_2 \det\left(\boldsymbol{I}_{N_r} + \frac{1}{\sigma^2}\tilde{\boldsymbol{H}}\boldsymbol{Q}\tilde{\boldsymbol{H}}^H\right) \quad (3)$$

$$\text{s.t.} \quad \boldsymbol{\phi} = \operatorname{diag}\{\alpha_1, ..., \alpha_M\} \quad (4)$$

$$|\alpha_m| = 1, \quad m = 1, ..., M \quad (5)$$

$$\operatorname{tr}(\boldsymbol{Q}) \leq P \quad (6)$$

$$\boldsymbol{Q} \succeq \boldsymbol{0}. \quad (7)$$

Note that Problem (P1) is a non-convex optimization problem since the objective function can be shown to be non-concave over the reflection matrix $\boldsymbol{\phi}$, and the uni-modular constraint on each reflection coefficient $\alpha_m$ in (5) is also non-convex. Moreover, the transmit covariance matrix $\boldsymbol{Q}$ is coupled with $\boldsymbol{\phi}$ in the objective function of (P1), which makes (P1) more difficult to solve. It is worth noting that although uni-modular constraints have been considered in the designs of constant envelope precoding and hybrid analog/digital precoding at the transmitter (see, e.g., [18], [19]), the existing designs are not applicable to solving (P1) due to the different rate expressions in terms of the uni-modular variables. In the next section, we solve (P1) by exploiting its unique structure.

## III. Proposed Solution to Problem (P1)

In this section, we propose an alternating optimization algorithm for solving (P1). Specifically, we first transform the objective function of (P1) into a more tractable form in terms of the optimization variables in $\{\alpha_m\}_{m=1}^M \cup \{\boldsymbol{Q}\}$, based on which we then solve two subproblems of (P1), for optimizing respectively the transmit covariance matrix $\boldsymbol{Q}$ or one reflection coefficient $\alpha_m$ in $\boldsymbol{\phi}$ with all the other variables being fixed. We derive the *optimal* solutions to both subproblems in *closed-form*, which enable an efficient alternating optimization algorithm to obtain a *locally optimal* solution to (P1) by iteratively solving these subproblems.

### A. Alternating Optimization

In this subsection, we introduce the framework of our proposed alternating optimization for solving (P1). Our main idea is to iteratively solve a series of subproblems of (P1), each aiming to optimize one single variable in $\{\alpha_m\}_{m=1}^M \cup \{\boldsymbol{Q}\}$ with the other $M$ variables being fixed. To this end, we first provide a more tractable expression for the objective function of (P1) in (3) in terms of $\boldsymbol{Q}$ and $\{\alpha_m\}_{m=1}^M$. Note that (3) is the logarithm determinant of a linear function of $\boldsymbol{Q}$, while its relationship with $\alpha_m$'s is rather implicit. Thus, we propose to rewrite (3) as an explicit function over $\alpha_m$'s. Denote $\boldsymbol{R} = [\boldsymbol{r}_1, ..., \boldsymbol{r}_M]$ and $\boldsymbol{T} = [\boldsymbol{t}_1, ..., \boldsymbol{t}_M]^H$, with $\boldsymbol{r}_m \in \mathbb{C}^{N_r \times 1}$ and $\boldsymbol{t}_m \in \mathbb{C}^{N_t \times 1}$. Then, the effective MIMO channel can be rewritten as

[2] To characterize the capacity limit of IRS-aided MIMO systems, we assume that the phase-shift by each IRS element can be continuously adjusted, while the results of this paper can be readily extended to the practical setup with discrete phase-shift levels [16], [17].

$$\tilde{\boldsymbol{H}} = \boldsymbol{H} + \sum_{m=1}^{M} \alpha_m \boldsymbol{r}_m \boldsymbol{t}_m^H. \quad (8)$$

Notice from (8) that the effective channel is the summation of the direct channel matrix $\boldsymbol{H}$ and $M$ rank-one matrices $\boldsymbol{r}_m\boldsymbol{t}_m^H$'s each multiplied by a reflection coefficient $\alpha_m$, which is a unique structure of IRS-aided MIMO channel and implies that $\{\alpha_m\}_{m=1}^M$ should be designed to strike an optimal balance between the $M+1$ matrices for maximizing the capacity.

Furthermore, denote $\boldsymbol{Q} = \boldsymbol{U}_Q\boldsymbol{\Sigma}_Q\boldsymbol{U}_Q^H$ as the eigenvalue decomposition (EVD) of $\boldsymbol{Q}$, where $\boldsymbol{U}_Q \in \mathbb{C}^{N_t\times N_t}$ and $\boldsymbol{\Sigma}_Q \in \mathbb{C}^{N_t\times N_t}$. Note that since $\boldsymbol{Q}$ is a positive semi-definite matrix, all the diagonal elements in $\boldsymbol{\Sigma}_Q$ are non-negative real numbers. Based on this, we define $\boldsymbol{H}' = \boldsymbol{H}\boldsymbol{U}_Q\boldsymbol{\Sigma}_Q^{\frac{1}{2}} \in \mathbb{C}^{N_r\times N_t}$, $\boldsymbol{T}' = \boldsymbol{T}\boldsymbol{U}_Q\boldsymbol{\Sigma}_Q^{\frac{1}{2}} = [\boldsymbol{t}_1',...,\boldsymbol{t}_M']^H \in \mathbb{C}^{M\times N_t}$, where $\boldsymbol{t}_m' = \boldsymbol{\Sigma}_Q^{\frac{1}{2}}\boldsymbol{U}_Q^H\boldsymbol{t}_m \in \mathbb{C}^{N_t\times 1}$. Therefore, the objective function of (P1) can be rewritten as

$$\begin{aligned} f &\overset{\Delta}{=} \log_2\det\left(\boldsymbol{I}_{N_r} + \frac{1}{\sigma^2}\tilde{\boldsymbol{H}}\boldsymbol{Q}\tilde{\boldsymbol{H}}^H\right) \quad (9)\\ &= \log_2\det\left(\boldsymbol{I}_{N_r} + \frac{1}{\sigma^2}(\boldsymbol{H}' + \boldsymbol{R}\boldsymbol{\phi}\boldsymbol{T}')(\boldsymbol{H}' + \boldsymbol{R}\boldsymbol{\phi}\boldsymbol{T}')^H\right)\\ &= \log_2\det\left(\boldsymbol{I}_{N_r} + \frac{1}{\sigma^2}\left(\boldsymbol{H}' + \sum_{i=1}^M \alpha_i\boldsymbol{r}_i\boldsymbol{t}_i'^H\right)\left(\boldsymbol{H}' + \sum_{i=1}^M \alpha_i\boldsymbol{r}_i\boldsymbol{t}_i'^H\right)^H\right)\\ &\overset{(a)}{=} \log_2\det\left(\boldsymbol{I}_{N_r} + \frac{1}{\sigma^2}\boldsymbol{H}'\boldsymbol{H}'^H + \frac{1}{\sigma^2}\sum_{i=1}^M \boldsymbol{r}_i\boldsymbol{t}_i'^H\boldsymbol{t}_i'\boldsymbol{r}_i^H\right.\\ &\left. + \frac{1}{\sigma^2}\sum_{i=1}^M\left(\boldsymbol{H}'\alpha_i^*\boldsymbol{t}_i'\boldsymbol{r}_i^H + \alpha_i\boldsymbol{r}_i\boldsymbol{t}_i'^H\boldsymbol{H}'^H + \sum_{j=1,j\neq i}^M \alpha_i\alpha_j^*\boldsymbol{r}_i\boldsymbol{t}_i'^H\boldsymbol{t}_j'\boldsymbol{r}_j^H\right)\right), \end{aligned}$$

where $(a)$ holds due to $|\alpha_m|^2 = 1, \forall m$. Note that the new objective function of (P1) shown in (9) is in an explicit form of individual reflection coefficients $\{\alpha_m\}_{m=1}^M$, which facilitates our proposed alternating optimization in the sequel.

With (9), we are ready to present the two types of subproblems that need to be solved during the alternating optimization, which aim to optimize the transmit covariance matrix $\boldsymbol{Q}$ with given $\{\alpha_m\}_{m=1}^M$ or a reflection coefficient $\alpha_m$ with given $\{\alpha_i, i\neq m\}_{i=1}^M \cup \boldsymbol{Q}$, elaborated as follows.

*1) Optimization of $\boldsymbol{Q}$ with Given $\{\alpha_m\}_{m=1}^M$:* In this subproblem, we aim to optimize the transmit covariance matrix $\boldsymbol{Q}$ with given reflection coefficients $\{\alpha_m\}_{m=1}^M$ or the effective channel $\tilde{\boldsymbol{H}}$ in (8). Note that with given $\tilde{\boldsymbol{H}}$, (P1) is a convex optimization problem over $\boldsymbol{Q}$, and the optimal $\boldsymbol{Q}$ is given by the *eigenmode transmission* [11]. Specifically, denote $\tilde{\boldsymbol{H}} = \tilde{\boldsymbol{U}}\tilde{\boldsymbol{\Lambda}}\tilde{\boldsymbol{V}}^H$ as the truncated singular value decomposition (SVD) of $\tilde{\boldsymbol{H}}$, where $\tilde{\boldsymbol{V}} \in \mathbb{C}^{N_t\times D}$, with $D = \text{rank}(\tilde{\boldsymbol{H}}) \leq \min(N_r, N_t)$ denoting the maximum number of data streams that can be transmitted over $\tilde{\boldsymbol{H}}$. The optimal $\boldsymbol{Q}$ is thus given by

$$\boldsymbol{Q}^\star = \tilde{\boldsymbol{V}}\text{diag}\{p_1^\star,...,p_D^\star\}\tilde{\boldsymbol{V}}^H, \quad (10)$$

where $p_i^\star$ denotes the optimal amount of power allocated to the $i$th data stream following the water-filling strategy: $p_i^\star = \max(1/p_0 - \sigma^2/[\tilde{\boldsymbol{\Lambda}}]_{i,i}^2, 0)$, $i = 1,...,D$, with $p_0$ satisfying $\sum_{i=1}^D p_i^\star = P$. Hence, the channel capacity with given $\{\alpha_m\}_{m=1}^M$ is $C = \sum_{i=1}^D \log_2(1 + [\tilde{\boldsymbol{\Lambda}}]_{i,i}^2 p_i^\star/\sigma^2)$.

*2) Optimization of $\alpha_m$ with Given $\boldsymbol{Q}$ and $\{\alpha_i, i\neq m\}_{i=1}^M$:* In this subproblem, we aim to optimize $\alpha_m$ in (P1) with given $\boldsymbol{Q}$ and $\{\alpha_i, i\neq m\}_{i=1}^M$, $\forall m \in \mathcal{M}$, where $\mathcal{M} = \{1,...,M\}$. For ease of exposition, we rewrite the objective function of (P1) in (9) in the following form with respect to each $\alpha_m$:

$$f_m \overset{\Delta}{=} \log_2\det\left(\boldsymbol{A}_m + \alpha_m\boldsymbol{B}_m + \alpha_m^*\boldsymbol{B}_m^H\right) = f, \ \forall m \in \mathcal{M}, \quad (11)$$

where

$$\begin{aligned} \boldsymbol{A}_m &= \boldsymbol{I}_{N_r} + \frac{1}{\sigma^2}\left(\boldsymbol{H}' + \sum_{i=1,i\neq m}^M \alpha_i\boldsymbol{r}_i\boldsymbol{t}_i'^H\right)\left(\boldsymbol{H}' + \sum_{i=1,i\neq m}^M \alpha_i\boldsymbol{r}_i\boldsymbol{t}_i'^H\right)^H\\ &\quad + \frac{1}{\sigma^2}\boldsymbol{r}_m\boldsymbol{t}_m'^H\boldsymbol{t}_m'\boldsymbol{r}_m^H, \ \forall m \in \mathcal{M},\\ \boldsymbol{B}_m &= \frac{1}{\sigma^2}\boldsymbol{r}_m\boldsymbol{t}_m'^H\left(\boldsymbol{H}'^H + \sum_{i=1,i\neq m}^M \boldsymbol{t}_i'\boldsymbol{r}_i^H\alpha_i^*\right), \forall m \in \mathcal{M}. \quad (12) \end{aligned}$$

Therefore, this subproblem can be expressed as

$$\begin{aligned} \text{(P1-m)} \quad \max_{\alpha_m} \ & \log_2\det(\boldsymbol{A}_m + \alpha_m\boldsymbol{B}_m + \alpha_m^*\boldsymbol{B}_m^H) \quad (13)\\ \text{s.t.} \ & |\alpha_m| = 1. \quad (14) \end{aligned}$$

Notice that $\boldsymbol{A}_m$ and $\boldsymbol{B}_m$ are both independent of $\alpha_m$. Hence, the objective function of (P1-m) can be shown to be a concave function over $\alpha_m$. Nevertheless, the uni-modular constraint in (14) is non-convex, which makes (P1-m) still non-convex. In the following, by exploiting the structure of (P1-m), we derive its *optimal* solution in *closed-form*.

### B. Optimal Solution to Problem (P1-m)

First, we exploit the structures of $\boldsymbol{A}_m$ and $\boldsymbol{B}_m$.

*Lemma 1:* For any $m \in \mathcal{M}$, $\text{rank}(\boldsymbol{A}_m) = N_r$, $\text{rank}(\boldsymbol{B}_m) \leq 1$.

*Proof:* Note from (12) that $\boldsymbol{A}_m$ is the summation of an identity matrix and two positive semi-definite matrices. Thus, $\boldsymbol{A}_m$ is a positive definite matrix with full rank. On the other hand, based on the definition of $\boldsymbol{B}_m$ in (12), we have $\text{rank}(\boldsymbol{B}_m) \leq \text{rank}(\boldsymbol{r}_m\boldsymbol{t}_m'^H) = 1$ [20]. ■

Next, by noting from Lemma 1 that $\boldsymbol{A}_m$ is of full rank and thus invertible, we rewrite the objective function of (P1-m) as

$$\begin{aligned} f_m &= \log_2\det(\boldsymbol{I}_{N_r} + \alpha_m\boldsymbol{A}_m^{-1}\boldsymbol{B}_m + \alpha_m^*\boldsymbol{A}_m^{-1}\boldsymbol{B}_m^H)\\ &\quad + \log_2\det(\boldsymbol{A}_m) \overset{\Delta}{=} f_m' + \log_2\det(\boldsymbol{A}_m). \quad (15) \end{aligned}$$

Based on (15), (P1-m) is equivalent to the maximization of $f_m' \overset{\Delta}{=} \log_2\det(\boldsymbol{I}_{N_r} + \alpha_m\boldsymbol{A}_m^{-1}\boldsymbol{B}_m + \alpha_m^*\boldsymbol{A}_m^{-1}\boldsymbol{B}_m^H)$ under the constraint in (14) by optimizing $\alpha_m$, which is addressed next.

Notice that $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m$ plays a key role in our new objective function $f_m'$, whose structure is exploited as follows. Specifically, since $\text{rank}(\boldsymbol{B}_m) \leq 1$, we have $\text{rank}(\boldsymbol{A}_m^{-1}\boldsymbol{B}_m) \leq \text{rank}(\boldsymbol{B}_m) \leq 1$. Note that for the case with $\text{rank}(\boldsymbol{A}_m^{-1}\boldsymbol{B}_m) = 0$, namely, $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m = \boldsymbol{0}$, any $\alpha_m$ with $|\alpha_m| = 1$ is an optimal solution to (P1-m), whose corresponding optimal value is thus $\log_2\det(\boldsymbol{A}_m)$. As such, we focus on the case with $\text{rank}(\boldsymbol{A}_m^{-1}\boldsymbol{B}_m) = 1$ in the next. In this case, $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m$ may be either *diagonalizable* or *non-diagonalizable*, which can be determined by the following lemma.

*Lemma 2:* $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m$ is diagonalizable if and only if $\text{tr}(\boldsymbol{A}_m^{-1}\boldsymbol{B}_m) \neq 0$.

*Proof:* First, since $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m$ is of rank one, we can express it as the multiplication of two vectors as $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m = \boldsymbol{u}_m\boldsymbol{v}_m^H$, where $\boldsymbol{u}_m \in \mathbb{C}^{N_r\times 1}$ and $\boldsymbol{v}_m \in \mathbb{C}^{N_r\times 1}$. Then, it follows that $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m$ is non-diagonalizable if and only if $\boldsymbol{v}_m^H\boldsymbol{u}_m = \mathrm{tr}(\boldsymbol{A}_m^{-1}\boldsymbol{B}_m) = 0$, where it becomes a *nilpotent* matrix [20]. This completes the proof of Lemma 2. ■

In the following, we investigate the two cases where $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m$ is diagonalizable or non-diagonalizable, and derive the optimal solution for each case, respectively.

*1) Case I: Diagonalizable $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m$:* First, we consider the case where $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m$ is diagonalizable, namely, its EVD exists. Since $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m$ has rank one, its EVD can be expressed as $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m = \boldsymbol{U}_m\boldsymbol{\Sigma}_m\boldsymbol{U}_m^{-1}$, where $\boldsymbol{U}_m \in \mathbb{C}^{N_r\times N_r}$, and $\boldsymbol{\Sigma}_m = \mathrm{diag}\{\lambda_m, 0, ..., 0\} \in \mathbb{C}^{N_r\times N_r}$, with $\lambda_m \in \mathbb{C}$ denoting the sole non-zero eigenvalue of $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m$. Therefore, $f'_m$ can be expressed as

$$\begin{aligned}
f'_m &= \log_2\det(\boldsymbol{I}_{N_r} + \alpha_m\boldsymbol{U}_m\boldsymbol{\Sigma}_m\boldsymbol{U}_m^{-1} \\
&\qquad + \alpha_m^*\boldsymbol{A}_m^{-1}\boldsymbol{U}_m^{-1H}\boldsymbol{\Sigma}_m^H\boldsymbol{U}_m^H\boldsymbol{A}_m) \\
&\overset{(b_1)}{=} \log_2(\det(\boldsymbol{U}_m^{-1})\det(\boldsymbol{I}_{N_r} + \alpha_m\boldsymbol{U}_m\boldsymbol{\Sigma}_m\boldsymbol{U}_m^{-1} \\
&\qquad + \alpha_m^*\boldsymbol{A}_m^{-1}\boldsymbol{U}_m^{-1H}\boldsymbol{\Sigma}_m^H\boldsymbol{U}_m^H\boldsymbol{A}_m)\det(\boldsymbol{U}_m)) \\
&\overset{(b_2)}{=} \log_2\det(\boldsymbol{I}_{N_r} + \alpha_m\boldsymbol{\Sigma}_m + \alpha_m^*\boldsymbol{U}_m^{-1}\boldsymbol{A}_m^{-1}\boldsymbol{U}_m^{-1H}\boldsymbol{\Sigma}_m^H\boldsymbol{U}_m^H\boldsymbol{A}_m\boldsymbol{U}_m) \\
&= \log_2\det(\boldsymbol{I}_{N_r} + \alpha_m\boldsymbol{\Sigma}_m + \alpha_m^*\boldsymbol{V}_m^{-1}\boldsymbol{\Sigma}_m^H\boldsymbol{V}_m), \qquad (16)
\end{aligned}$$

where $(b_1)$ holds due to $\det(\boldsymbol{A})\det(\boldsymbol{A}^{-1}) = 1$ for any invertible matrix $\boldsymbol{A}$; $(b_2)$ holds due to $\det(\boldsymbol{AB}) = \det(\boldsymbol{A})\det(\boldsymbol{B})$ for two equal-sized square matrices $\boldsymbol{A}$ and $\boldsymbol{B}$; and $\boldsymbol{V}_m \triangleq \boldsymbol{U}_m^H\boldsymbol{A}_m\boldsymbol{U}_m$ is a Hermitian matrix with $\boldsymbol{V}_m = \boldsymbol{V}_m^H$, since $\boldsymbol{A}_m$ is a Hermitian matrix according to (12). Let $\boldsymbol{\nu}_m \in \mathbb{C}^{N_r\times 1}$ denote the first column of $\boldsymbol{V}_m^{-1}$ and $\boldsymbol{\nu}_m^{'T} \in \mathbb{C}^{1\times N_r}$ denote the first row of $\boldsymbol{V}_m$. Note that it follows that $\boldsymbol{\nu}_m^{'T}\boldsymbol{\nu}_m = 1$; moreover, let $\nu_{m1}$ and $\nu'_{m1}$ denote the first element in $\boldsymbol{\nu}_m$ and $\boldsymbol{\nu}_m^{'T}$, respectively, we have $\nu_{m1} \in \mathbb{R}$ and $\nu'_{m1} \in \mathbb{R}$ since both $\boldsymbol{V}_m$ and $\boldsymbol{V}_m^{-1}$ are Hermitian matrices. Hence, (16) can be further simplified as

$$\begin{aligned}
f'_m &= \log_2\det(\boldsymbol{I}_{N_r} + \alpha_m\boldsymbol{\Sigma}_m + \alpha_m^*\boldsymbol{\nu}_m\lambda_m^*\boldsymbol{\nu}_m^{'T}) \\
&\overset{(c_1)}{=} \log_2\det(1 + \alpha_m^*\lambda_m^*\boldsymbol{\nu}_m^{'T}(\boldsymbol{I}_{N_r} + \alpha_m\boldsymbol{\Sigma}_m)^{-1}\boldsymbol{\nu}_m) \\
&\qquad + \log_2\det(\boldsymbol{I}_{N_r} + \alpha_m\boldsymbol{\Sigma}_m) \\
&= \log_2\left(\left(1+\alpha_m^*\lambda_m^* - \frac{\alpha_m^*\lambda_m^*\nu'_{m1}\alpha_m\lambda_m\nu_{m1}}{1+\alpha_m\lambda_m}\right)(1+\alpha_m\lambda_m)\right) \\
&\overset{(c_2)}{=} \log_2\left((1+\alpha_m\lambda_m)(1+\alpha_m^*\lambda_m^*) - \nu'_{m1}\nu_{m1}|\lambda_m|^2\right) \\
&= \log_2\left(1 + |\lambda_m|^2(1-\nu'_{m1}\nu_{m1}) + 2\mathfrak{Re}\{\alpha_m\lambda_m\}\right), \qquad (17)
\end{aligned}$$

where $(c_1)$ holds due to the fact that $\det(\boldsymbol{AB}) = \det(\boldsymbol{A})\det(\boldsymbol{B})$ and $\det(\boldsymbol{I}_p + \boldsymbol{CD}) = \det(\boldsymbol{I}_q + \boldsymbol{DC})$ for $\boldsymbol{C} \in \mathbb{C}^{p\times q}$ and $\boldsymbol{D} \in \mathbb{C}^{q\times p}$; $(c_2)$ holds due to $|\alpha_m|^2 = 1$.

Based on (17), (P1-m) is equivalent to maximizing $\mathfrak{Re}\{\alpha_m\lambda_m\}$ under the constraint in (14) when $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m$ is diagonalizable, for which we have the following proposition.

*Proposition 1:* If $\mathrm{tr}(\boldsymbol{A}_m^{-1}\boldsymbol{B}_m) \neq 0$, the optimal solution to (P1-m) is given by $\alpha_m^{\star\mathrm{I}} = e^{-j\arg\{\lambda_m\}}$. The optimal value of (P1-m) is thus given by $f_m^{\star\mathrm{I}} = \log_2(1 + |\lambda_m|^2(1-\nu'_{m1}\nu_{m1}) + 2|\lambda_m|) + \log_2\det(\boldsymbol{A}_m)$.

*Proof:* Since $\mathfrak{Re}\{\alpha_m\lambda_m\} \leq |\alpha_m\lambda_m| = |\lambda_m|$, where the inequality holds with equality if and only if $\arg\{\alpha_m\} = -\arg\{\lambda_m\}$, the proof of Proposition 1 is completed. ■

*2) Case II: Non-Diagonalizable $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m$:* Next, consider the case where $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m$ is non-diagonalizable. In this case, we express it as $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m = \boldsymbol{u}_m\boldsymbol{v}_m^H$, where $\boldsymbol{u}_m \in \mathbb{C}^{N_r\times 1}$, $\boldsymbol{v}_m \in \mathbb{C}^{N_r\times 1}$, and $\boldsymbol{v}_m^H\boldsymbol{u}_m = \boldsymbol{u}_m^H\boldsymbol{v}_m = \mathrm{tr}(\boldsymbol{A}_m^{-1}\boldsymbol{B}_m) = 0$ according to Lemma 2. To exploit the structure of $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m$ in this case, we first provide the following lemma for $\boldsymbol{u}_m$ and $\boldsymbol{v}_m$.

*Lemma 3:* $\boldsymbol{I}_{N_r} + \alpha_m\boldsymbol{u}_m\boldsymbol{v}_m^H$ is an invertible matrix, whose inversion is given by $(\boldsymbol{I}_{N_r} + \alpha_m\boldsymbol{u}_m\boldsymbol{v}_m^H)^{-1} = \boldsymbol{I}_{N_r} - \alpha_m\boldsymbol{u}_m\boldsymbol{v}_m^H$.

*Proof:* Lemma 3 follows from the Sherman-Morrison-Woodbury formula [20], which states that for an invertible matrix $\boldsymbol{A} \in \mathbb{C}^{N_r\times N_r}$ and two vectors $\boldsymbol{a} \in \mathbb{C}^{N_r\times 1}$ and $\boldsymbol{b} \in \mathbb{C}^{N_r\times 1}$, $\boldsymbol{A} + \boldsymbol{ab}^H$ is invertible if and only if $1 + \boldsymbol{b}^H\boldsymbol{A}^{-1}\boldsymbol{a} \neq 0$, and the inversion is given by $(\boldsymbol{A} + \boldsymbol{ab}^H)^{-1} = \boldsymbol{A}^{-1} - \frac{\boldsymbol{A}^{-1}\boldsymbol{ab}^H\boldsymbol{A}^{-1}}{1+\boldsymbol{b}^H\boldsymbol{A}^{-1}\boldsymbol{a}}$. Based on this, by replacing $\boldsymbol{A}$ with $\boldsymbol{I}_{N_r}$, we have $1 + \alpha_m\boldsymbol{v}_m^H\boldsymbol{u}_m = 1 \neq 0$, thus $\boldsymbol{I}_{N_r} + \alpha_m\boldsymbol{u}_m\boldsymbol{v}_m^H$ is an invertible matrix, with inversion $\boldsymbol{I}_{N_r} - \alpha_m\boldsymbol{u}_m\boldsymbol{v}_m^H$. ■

Based on the results in Lemma 3, $f'_m$ can be rewritten as

$$\begin{aligned}
f'_m &= \log_2\det\left(\boldsymbol{I}_{N_r} + \alpha_m\boldsymbol{u}_m\boldsymbol{v}_m^H + \alpha_m^*\boldsymbol{A}_m^{-1}\boldsymbol{v}_m\boldsymbol{u}_m^H\boldsymbol{A}_m\right) \\
&\overset{(d_1)}{=} \log_2\det(\boldsymbol{I}_{N_r} + \alpha_m^*(\boldsymbol{I}_{N_r} - \alpha_m\boldsymbol{u}_m\boldsymbol{v}_m^H)\boldsymbol{A}_m^{-1}\boldsymbol{v}_m\boldsymbol{u}_m^H\boldsymbol{A}_m) \\
&\qquad + \log_2\det(\boldsymbol{I}_{N_r} + \alpha_m\boldsymbol{u}_m\boldsymbol{v}_m^H) \\
&\overset{(d_2)}{=} \log_2\det(\boldsymbol{I}_{N_r} + \alpha_m^*(\boldsymbol{I}_{N_r} - \alpha_m\boldsymbol{u}_m\boldsymbol{v}_m^H)\boldsymbol{A}_m^{-1}\boldsymbol{v}_m\boldsymbol{u}_m^H\boldsymbol{A}_m) \\
&\overset{(d_3)}{=} \log_2\det(\boldsymbol{A}_m(\boldsymbol{I}_{N_r} + \alpha_m^*(\boldsymbol{I}_{N_r} - \alpha_m\boldsymbol{u}_m\boldsymbol{v}_m^H) \\
&\qquad \times \boldsymbol{A}_m^{-1}\boldsymbol{v}_m\boldsymbol{u}_m^H\boldsymbol{A}_m)\boldsymbol{A}_m^{-1}) \\
&\overset{(d_4)}{=} \log_2\det(\boldsymbol{I}_{N_r} + \alpha_m^*\boldsymbol{v}_m\boldsymbol{u}_m^H - \boldsymbol{A}_m\boldsymbol{u}_m\boldsymbol{v}_m^H\boldsymbol{A}_m^{-1}\boldsymbol{v}_m\boldsymbol{u}_m^H) \\
&\overset{(d_5)}{=} \log_2\det(\boldsymbol{I}_{N_r} - (\boldsymbol{I}_{N_r} - \alpha_m^*\boldsymbol{v}_m\boldsymbol{u}_m^H)\boldsymbol{A}_m\boldsymbol{u}_m\boldsymbol{v}_m^H\boldsymbol{A}_m^{-1}\boldsymbol{v}_m\boldsymbol{u}_m^H) \\
&\qquad + \log_2\det(\boldsymbol{I}_{N_r} + \alpha_m^*\boldsymbol{v}_m\boldsymbol{u}_m^H) \\
&\overset{(d_6)}{=} \log_2\det(\boldsymbol{I}_{N_r} - \boldsymbol{A}_m^{-1}\boldsymbol{v}_m\boldsymbol{u}_m^H(\boldsymbol{I}_{N_r} - \alpha_m^*\boldsymbol{v}_m\boldsymbol{u}_m^H)\boldsymbol{A}_m\boldsymbol{u}_m\boldsymbol{v}_m^H) \\
&\overset{(d_7)}{=} \log_2\det(\boldsymbol{I}_{N_r} - \boldsymbol{A}_m^{-1}\boldsymbol{v}_m\boldsymbol{u}_m^H\boldsymbol{A}_m\boldsymbol{u}_m\boldsymbol{v}_m^H), \qquad (18)
\end{aligned}$$

where $(d_1)$ can be derived similarly as $(c_1)$ via Lemma 3; $(d_2)$ holds since $\log_2\det(\boldsymbol{I}_{N_r} + \alpha_m\boldsymbol{u}_m\boldsymbol{v}_m^H) = \log_2\det(1 + \alpha_m\boldsymbol{v}_m^H\boldsymbol{u}_m) = 0$; $(d_3)$ can be derived in a similar manner as $(b_1)$ and $(b_2)$ by noting that $\boldsymbol{A}_m$ is invertible; $(d_4)$ holds since $|\alpha_m|^2 = 1$; $(d_5)$ can be derived similarly as $(d_1)$; $(d_6)$ follows from $\det(\boldsymbol{I}_p + \boldsymbol{CD}) = \det(\boldsymbol{I}_q + \boldsymbol{DC})$ and $\log_2\det(\boldsymbol{I}_{N_r} + \alpha_m^*\boldsymbol{v}_m\boldsymbol{u}_m^H) = \log_2\det(1 + \alpha_m^*\boldsymbol{u}_m^H\boldsymbol{v}_m) = 0$; and $(d_7)$ holds since $\boldsymbol{u}_m^H\boldsymbol{v}_m = 0$, and consequently $\boldsymbol{A}_m^{-1}\boldsymbol{v}_m\boldsymbol{u}_m^H\alpha_m^*\boldsymbol{v}_m\boldsymbol{u}_m^H\boldsymbol{A}_m\boldsymbol{u}_m\boldsymbol{v}_m^H$ becomes an all-zero matrix.

It is worth noting from (18) that when $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m$ is non-diagonalizable, $f'_m$ is *independent* of $\alpha_m$. Therefore, we have the following proposition.

*Proposition 2:* If $\mathrm{tr}(\boldsymbol{A}_m^{-1}\boldsymbol{B}_m) = 0$, any $\alpha_m$ with $|\alpha_m| = 1$ is an optimal solution to (P1-m). The optimal value of (P1-m) is thus given by $f_m^{\star\mathrm{II}} = \log_2\det(\boldsymbol{A}_m - \boldsymbol{B}_m^H\boldsymbol{A}_m^{-1}\boldsymbol{B}_m)$.

*Proof:* The first half of Proposition 2 follows directly from (18). The second half of Proposition 2 can be derived as $f_m^{\star\mathrm{II}} = \log_2\det(\boldsymbol{I}_{N_r} - \boldsymbol{A}_m^{-1}\boldsymbol{v}_m\boldsymbol{u}_m^H\boldsymbol{A}_m\boldsymbol{u}_m\boldsymbol{v}_m^H) + \log_2\det(\boldsymbol{A}_m) = \log_2\det(\boldsymbol{A}_m - \boldsymbol{v}_m\boldsymbol{u}_m^H\boldsymbol{A}_m\boldsymbol{u}_m\boldsymbol{v}_m^H) = \log_2\det(\boldsymbol{A}_m - \boldsymbol{B}_m^H\boldsymbol{A}_m^{-1}\boldsymbol{B}_m)$, by noting that $\boldsymbol{A}_m^{-1}\boldsymbol{B}_m = \boldsymbol{u}_m\boldsymbol{v}_m^H$ holds. This thus completes the proof of Proposition 2. ■

Based on Proposition 2, we set $\alpha_m^{\star\mathrm{II}} = 1$ as the optimal solution to (P1-m) in this case without loss of optimality.

*3) Summary of the Optimal Solution to Problem (P1-m):* To summarize, the optimal solution to (P1-m) is given by

$$\alpha_m^{\star} = \begin{cases} e^{-j\arg\{\lambda_m\}}, & \text{if } \mathrm{tr}(\boldsymbol{A}_m^{-1}\boldsymbol{B}_m) \neq 0 \\ 1, & \text{otherwise.} \end{cases} \quad (19)$$

The corresponding optimal value of (P1-m) is given by

$$f_m^{\star} = \begin{cases} f_m^{\star \mathrm{I}}, & \text{if } \mathrm{tr}(\boldsymbol{A}_m^{-1}\boldsymbol{B}_m) \neq 0 \\ f_m^{\star \mathrm{II}}, & \text{otherwise.} \end{cases} \quad (20)$$

### C. Overall Algorithm

With the optimal solution to (P1-m) derived above, we are ready to complete our proposed alternating optimization algorithm for solving (P1). Specifically, we first randomly generate $L > 1$ sets of $\{\alpha_m\}_{m=1}^M$ with $|\alpha_m| = 1, \forall m$ and phases of $\alpha_m$'s following the uniform distribution in $[0, 2\pi)$. By obtaining the optimal transmit covariance matrix $\boldsymbol{Q}$ for each set of $\{\alpha_m\}_{m=1}^M$ according to (10) as well as the corresponding channel capacity, we select the set with maximum capacity as the initial point. The algorithm then proceeds by iteratively solving the two subproblems presented in Section III-A, until convergence is reached. Note that since we have obtained the *optimal* solution to every subproblem, *monotonic convergence* of the proposed algorithm is guaranteed, since the algorithm yields non-decreasing objective value of (P1) over the iterations, which is also upper-bounded by a finite capacity. Moreover, since the objective function of (P1) is differentiable and all the variables $\{\alpha_m\}_{m=1}^M$ and $\boldsymbol{Q}$ are not coupled in the constraints, any limit point of the iterations generated by the proposed algorithm satisfies the Karush-Kuhn-Tucker (KKT) condition of (P1) [21]. By further setting the convergence criteria as that the objective function of (P1) cannot be further increased by optimizing any variable in $\{\alpha_m\}_{m=1}^M \cup \boldsymbol{Q}$, the proposed algorithm is guaranteed to converge to at least a *locally optimal* solution of (P1). Finally, the complexity of the proposed algorithm can be shown to be $\mathcal{O}(N_r N_t(M + \min(N_r, N_t))L + ((3N_r^3 + 2N_r^2 N_t + N_t^2)M + N_r N_t \min(N_r, N_t))I)$ with $I$ denoting the number of outer iterations, which is polynomial over $N_r$, $N_t$, and $M$.

## IV. Numerical Results

In this section, we provide numerical results to examine the performance of our proposed algorithm. We consider a Rayleigh fading model for all channels involved, where each channel coefficient is an independent CSCG random variable with zero mean and variance denoting the path loss, namely, $[\boldsymbol{H}]_{i,j} \sim \mathcal{CN}(0, \beta_{\mathrm{D}}), \forall i, j$, $[\boldsymbol{T}]_{i,j} \sim \mathcal{CN}(0, \beta_{\mathrm{TI}}), \forall i, j$, and $[\boldsymbol{R}]_{i,j} \sim \mathcal{CN}(0, \beta_{\mathrm{IR}}), \forall i, j$, where $\beta_{\mathrm{D}}$, $\beta_{\mathrm{TI}}$, and $\beta_{\mathrm{IR}}$ respectively denote the corresponding distance-dependent path loss modeled by $\beta = \beta_0(d/d_0)^{-\bar{\alpha}}$, with $\beta_0 = -30$ dB denoting the path loss at the reference distance $d_0 = 1$ m; $d$ denoting the link distance; and $\bar{\alpha}$ denoting the path loss exponent. The distances for the direct link, the transmitter-IRS link, and the IRS-receiver link are set as 600.0833 m, 598.0033 m, and 10.3923 m, respectively, and the corresponding path loss exponents are set as $\bar{\alpha}_{\mathrm{D}} = 3.5$, $\bar{\alpha}_{\mathrm{TI}} = 2.2$, and $\bar{\alpha}_{\mathrm{IR}} = 2.8$, respectively. We set $N_t = N_r = 4$, $P = 30$ dBm, and $\sigma^2 = -90$ dBm. For the proposed algorithm, we set the number of random initializations as $L = 100$, and the convergence threshold in terms of the relative increment in the objective value as $\epsilon = 10^{-5}$.

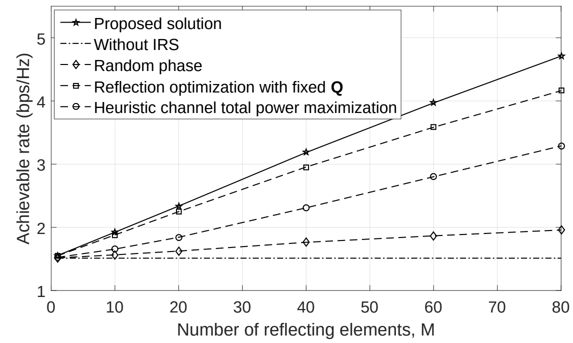


Fig. 2. Performance of IRS-aided MIMO communication.

We compare the performance of our proposed alternating optimization algorithm with the following benchmark schemes: **1) Without IRS**: Obtain the channel capacity in (2) by optimizing $\boldsymbol{Q}$ with given $\tilde{\boldsymbol{H}} = \boldsymbol{H}$; **2) Random phase**: Randomly generate $\{\alpha_m\}_{m=1}^M$ with $|\alpha_m| = 1, \forall m$ and phases of $\alpha_m$'s following independent uniform distribution in $[0, 2\pi)$. Obtain the channel capacity in (2) by optimizing $\boldsymbol{Q}$ with given $\{\alpha_m\}_{m=1}^M$; **3) Reflection optimization with fixed $\boldsymbol{Q}$**: We first obtain the optimal (capacity-achieving) transmit covariance matrix $\boldsymbol{Q}$ for the direct channel $\boldsymbol{H}$; then, we apply our proposed algorithm to optimize $\{\alpha_m\}_{m=1}^M$ with $\boldsymbol{Q}$ fixed; **4) Heuristic channel total power maximization**: We propose a heuristic approach to maximize the channel total power by maximizing its lower bound, which is given by $\|\tilde{\boldsymbol{H}}\|_F^2 = \sum_{i=1}^{N_r}\sum_{j=1}^{N_t} |[\boldsymbol{H}]_{i,j} + \sum_{m=1}^M \alpha_m r_{mi} t_{mj}^*|^2 \geq |\tilde{h}^d + \sum_{m=1}^M \alpha_m \tilde{h}_m^r|^2$, where $\tilde{h}^d \triangleq \sum_{i=1}^{N_r}\sum_{j=1}^{N_t} [\boldsymbol{H}]_{i,j}$ and $\tilde{h}_m^r \triangleq \sum_{i=1}^{N_r}\sum_{j=1}^{N_t} r_{mi} t_{mj}^*$. The optimal $\{\alpha_m\}_{m=1}^M$ that maximizes this lower bound can be easily shown to be $\alpha_m = e^{j(\arg\{\tilde{h}^d\} - \arg\{\tilde{h}_m^r\})}, \forall m$. Note that this scheme is the optimal solution to the SISO case with $N_t = N_r = 1$.

We show in Fig. 2 the achievable rate versus the number of reflecting elements $M$ for the proposed algorithm and the benchmark schemes. All results are averaged over 100 independent channel realizations. It is observed that all the schemes with IRS outperform that without the IRS, and the performance gain increases with $M$; moreover, benchmark schemes 3) and 4) with IRS outperform the random phase scheme. It is also observed that our proposed algorithm achieves the best performance among all schemes at all values of $M$. Particularly, the proposed algorithm outperforms benchmark scheme 4) with transmit covariance matrix $\boldsymbol{Q}$ optimized only based on the direct MIMO channel, which shows the necessity of jointly optimizing $\boldsymbol{Q}$ and the IRS reflection coefficients.

## V. Conclusions

This paper studied the capacity maximization for IRS-aided MIMO communication via joint IRS reflection and transmit covariance matrix optimization. An alternating optimization algorithm was proposed to find a locally optimal solution by iteratively optimizing one optimization variable with the others being fixed, for which the optimal solutions were derived in closed-form. It was shown via numerical results that our proposed algorithm achieves superior rate performance over various benchmark schemes with or without IRS.

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