Inventory Sharing under Service Competition

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Abstract

Problem Description: In many markets with demand uncertainties, competing retailers may share inventories for common products that they offer consumers. This paper examines how competitors' product sharing affects their inventory and service-quality decisions. The existing literature has mainly focused on inventory sharing among independent retailers who do not compete with each other. Our research aims to fill the gap in this literature by investigating the tradeoffs of inventory sharing between retailers who directly compete for customers based on service quality. ***Methodology/Results:*** We develop a game-theoretical model in which two retailers selling a common product from the same manufacturer compete for customers by offering differentiated services together with the product. Each retailer faces stochastic demand that increases in its service quality and decreases in the competitor's service quality. When a retailer runs out of stock of the product, it may replenish its inventory directly from the manufacturer and/or request the competitor's excess inventory if they have an inventory-sharing agreement. We find that inventory sharing may *soften* or *intensify* service competition depending on the transfer price for the shared inventory. Specifically, when retailers agree to share inventory, their service levels decrease in the transfer price if their pre-season inventory levels are exogenous but are non-monotone in the transfer price if the retailers endogenously choose inventory levels. Moreover, our analysis reveals that the retailers' equilibrium inventory levels will increase in the transfer price and can be higher or lower than their levels in the case without inventory sharing. We also find that with exogenous inventory, the retailers prefer to share inventory at the highest non-moot transfer price, whereas with endogenous inventory, the retailers may prefer not to share inventory even at the optimal transfer price when the level of competition and the pre-order cost are high. Finally, we show that with service competition, inventory sharing cannot achieve full coordination under any transfer price. ***Managerial Implications:*** When deciding whether to share inventory with competitors, managers should consider not only the benefits of inventory pooling but also the strategic effect of sharing on the firms' inventory choices and service levels.

Keywords: Inventory, B2B sharing, service quality, competition, OM-marketing interface

1. Introduction

Inventory sharing (or transshipment) among retailers is prevalent in many industries such as automotive, toys, sporting goods, commodities, heavy machinery, apparel, and wholesale/retail (e.g., Comez et al. 2012; Dong and Rudi 2004; Li and Chen 2020; Park et al. 2016; Zhao et al. 2005; Zhao et al. 2020). Inventory sharing can be a potential solution to the inventory mismatch issue caused by demand uncertainties. Under inventory-sharing agreements, a retailer with insufficient inventory can purchase another retailer's excess inventory, thus avoiding expedited orders from the original manufacturers whose quick-response production may be very costly, and an overstocked retailer can sell some excess inventory at a transfer price higher than the product's salvage value. The benefits of inventory transshipment among retailers that do not directly compete with each other have been extensively studied in the operations literature (e.g., Rudi et al. 2001 and Comez et al. 2012).

In practice, we also observe inventory sharing among competing retailers. For example, many automobile dealers are often willing to transfer upon request their inventory to other local dealers that sell the same car models (see Olivares and Cachon 2009). The empirical study by Olivares and Cachon (2009) reveals inventory transfers between competing dealerships and indicates that inventory sharing has a large economic impact on the dealerships' inventory decisions. Guajardo et al. (2016) empirically show that automobile dealerships often compete with others in nearby locations by offering differentiated services. Such services may include the customer's in-store experience, warranty lengths, and after-sales services (e.g., attractiveness and comfort of the waiting areas of the service facility, free car wash/vacuuming, map updates for navigation, or free diagnostics). A higher level of services usually requires dealers to incur some additional marginal and/or fixed costs. This paper focuses on services that are associated with fixed costs; for example, in the automobile context, training salespersons for better customer experience, improvements in-store decoration/ambiance or service facility, or offering navigation updates and other software improvements typically require some fixed costs.

Besides the automobile industry, some franchisees of the same brand are also independent retailers that compete with each other and yet may agree to share their excess inventories when requested (Davis et al. 2022). Franchisees in nearby locations often offer the same products at the same prices and compete for customers by offering differentiated services, including customer service experiences or store decoration/ambiance. Offering a better customer experience or better store decorations or space often requires retailers to incur a higher fixed cost.

The Internet-based service company, WarehouseTWO, provides an online platform to facilitate inventory sharing among peer wholesaler-distributors; many manufacturers and their distributors have adopted its platform.[1] Some manufacturers, such as automobile manufacturers, also provide Intranet

[1] http://warehousetwo.com

systems connecting their retailers for inventory sharing (Zhao and Atkins 2009). Such B2B sharing has become an important part of the growing sharing economy in recent years.

The literature that studies inventory sharing among non-competitive firms has demonstrated that sharing inventory can improve profitability because of the benefits of risk pooling. Nevertheless, the literature on inventory sharing among direct competitors is scant; whether inventory sharing can improve profitability in a competitive market remains unclear. Our research aims to fill this gap by investigating the following research questions. Will inventory sharing soften or intensify service competition between retailers who face stochastic demand? How would an inventory-sharing agreement affect the retailers' inventory and service-quality decisions? Will inter-competitor inventory sharing improve the retailers' profits? Is there any transfer price that allows inventory sharing to achieve coordination as in a centralized system? To address these questions, we develop a game-theoretic model in which two retailers sell the same product from a manufacturer and compete for customers by offering differentiated services with the product. Each retailer faces demand uncertainty before the selling season; its demand increases in its service quality and decreases in the competitor's service quality. The two retailers can decide whether to share their inventory and at what transfer price; before the selling season, the retailers will order some initial inventory from the manufacturer and decide their respective service quality. When a retailer runs out of stock of the product, it can replenish its inventory directly from the manufacturer by placing a quick-response order and/or acquiring the leftover inventory from the competitor if they have agreed on sharing inventory.

We study two different scenarios with respect to the retailers' inventories: (1) Exogenous inventory levels; and (2) endogenous, strategic inventory decisions. In the first scenario, the retailers' initial pre-season inventory levels are exogenous; in the second scenario, each retailer can endogenously decide its inventory. Below, we highlight some main findings.

We start our model analysis by studying the case of exogenous pre-season inventories. We show that inventory sharing will intensify (soften) the retailers' service competition and lead to higher (lower) service quality when the transfer price for the shared inventory is low (high). The underlying reason is that, as the transfer price decreases, each retailer has an increased incentive to raise demand by improving service quality since the cost of serving excess demand by purchasing inventory from the other retailer becomes cheaper. We also find that inventory sharing would make both retailers better off when the transfer price is high and worse off otherwise. On the one hand, inventory sharing can benefit retailers by allowing risk pooling to alleviate the inventory-mismatch issue caused by demand uncertainty. On the other hand, depending on the transfer price, inventory sharing may intensify or soften service competition, leading to a higher or lower service cost. When the transfer price is low, the increased service cost will outweigh the benefit of risk pooling, making both retailers worse off. When the transfer price is intermediate, although inventory sharing leads to higher service quality, it can still make the retailers better off because the benefit from risk pooling dominates the intensified service competition. When the transfer price is high, the reduced service cost and risk pooling together make inventory sharing beneficial for retailers. In addition, the optimal transfer price that maximizes both retailers' profits is the highest non-moot transfer price, which makes inventory sharing more profitable than no-sharing but cannot achieve coordination as in a centralized system.

We have also analyzed the case where the retailers can strategically choose their pre-season inventories. First, we find that each retailer's equilibrium inventory level in the sharing case increases in the transfer price, and it is higher than that in the no-sharing case when the transfer price is high and is lower otherwise. Second, similar to the exogenous-inventory scenario, the two retailers' service quality and profits can be higher or lower in the sharing case, depending on the transfer price and the level of competition. Third, there also exists an optimal transfer price that maximizes the two retailers' profits, which could be strictly lower than the highest non-moot transfer price. Interestingly, in contrast to the case with exogenous inventory, when the retailers can strategically choose their inventory levels, inventory sharing can make both retailers worse off at any non-moot transfer price when the market competition level and the marginal pre-order cost are high. One might intuit that there should always exist an optimal transfer price to make sharing inventory strictly beneficial for both retailers since it can help mitigate demand uncertainty and also soften competition when the transfer price is high. This intuition is true only when the retailers do not compete with each other or when they do not strategically adjust their pre-season inventory levels in anticipation of sharing. However, when they can strategically choose their inventories, inventory sharing with a low transfer price would intensify service competition, as explained before; if the transfer price is high, each retailer will stock more inventories, which in turn will not only increase the inventory cost but also increase the service cost since a higher inventory can motivate each retailer to raise the service quality to compete for customers. Therefore, when competition is already intense and the pre-order cost is high, because of their strategic inventory and service responses, the retailers will not be able to benefit from inventory sharing at any transfer price and thus would not enter into a sharing agreement. Finally, similar to the exogenous-inventory case, when service competition exists, there is no transfer price that can fully coordinate the retailers since sharing inventory may help soften but cannot fully eliminate service competition between the retailers.

2. Literature Review

This research is related to the extensive literature on inventory transshipment. Many studies in this area focus on transshipment among centralized retailers. See Rudi et al. (2001) for a review of the literature on centralized transshipment. Some literature on transshipment considers decentralized retailers who do not compete with each other (e.g., Rong et al. 2010, Shao et al. 2011). Hu et al. (2007) and Rudi et al. (2001) study transshipments between two independent and non-competitive retailers to investigate the efficient transshipment price that leads to joint-profit maximization. Anupindi et al. (2001) and

Granot and Sosic (2003) focus on inventory ordering and allocation decisions among multiple independent retailers. Huang and Sosic (2010) examine two alternative approaches for inventory distribution among independent retailers, which differ in the timing of setting the transshipment price. Lee and Whang (2002) study the impacts of a secondary market where resellers can buy and sell excess inventories. Zhao et al. (2020) study inventory sharing from a behavioral perspective and show a persistent demand-side underweighting bias, which leads to understocking in an inventory-sharing system. Some other papers in this area study the transshipment between retailers with stock-out-based substitution, where a retailer facing a stock-out will request the product to be transshipped from another retailer, and if this request is rejected, the unsatisfied customer may go to another retailer with a customer overflow probability. Comez et al. (2012) examine the optimal transshipment policies for retailers with stock-out-based substitution. Zhao and Atkins (2009) show that retailers prefer transshipment if the transshipment price is high and competition is low and prefer demand substitution otherwise. Based on stock-out substitution, retailers may decline inventory transshipment since there is a chance that the unsatisfied customer will switch to the retailer with excess inventory. In these settings, there is no direct competition between the firms.

Our paper complements the above literature stream by studying inventory sharing between retailers that offer services to compete for customers directly. Note that the stock-out substitution never occurs in our setting since the retailer with a shortage can always request quick-response inventory from the manufacturer to fulfill all the demand. Moreover, the existing literature on inventory transshipment between non-competitive retailers shows that there usually exists some transfer price that can jointly maximize the total profits of the retailers and achieve coordination as in a centralized system (see Rudi et al. 2001 and Hu et al. 2007). However, our study shows that service competition between the retailers will hinder the coordination under any transfer price for inventory sharing. That is, with service competition, no transfer price can coordinate the retailers since sharing inventory may help soften but cannot fully eliminate service competition.

Another stream of literature studies capacity sharing among firms. Van Mieghem (1999) studies capacity transfer after demand realization between a manufacturer and a subcontractor under different contract types and shows that only state-dependent contracts can coordinate capacity-investment decisions. Yu et al. (2015) investigate conditions under which production/service capacity sharing may benefit a set of independent firms in queueing systems. Guo and Wu (2018) study ex-ante and ex-post capacity-sharing contracts between firms with exogenous capacities and show that capacity sharing between firms with symmetric capacities softens price competition. Qin et al. (2018) study the capacity-sharing strategy under revenue-sharing contracts between firms that have fixed capacities and compete on price. They show that the firm's profit may increase or decrease in the revenue-sharing rate when the capacity is insufficient. Our paper has two key differences from this stream of literature. First, our research studies how *competing* retailers' sharing of product inventory affects their *endogenous* inventory decisions. Second, we examine how inventory sharing affects *service* competition between retailers. We find that a retailer's equilibrium inventory level increases in the transfer price of the shared inventory and can be higher or lower than the level in the no-sharing case. Moreover, each retailer's service quality can be higher or lower than that without an inventory-sharing agreement, implying that whether inventory sharing will alleviate or intensify service competition depends on the transfer price and whether inventory is a strategic decision.

Our work also relates to the literature on peer-to-peer product sharing. Some research has examined consumers' social sharing (or piracy) of information goods (e.g., Bakos et al. 1999, Besen and Kirby 1989, Johnson 1985, Liebowitz 1985, Novos and Waldman 1984, Galbreth et al. 2012, Conner and Rumelt 1991, Takeyama 1994, and Varian 2005). Recent studies have investigated the effects of consumer-to-consumer sharing of products on channels (Tian and Jiang 2018) and pricing and quality decisions (e.g., Jiang and Tian 2018). There is also research that studies the self-scheduling capacity of sharing platforms (e.g., Taylor 2016, Cachon et al. 2017, Gurvich et al. 2015), the competition between peer-to-peer marketplaces and existing markets (e.g., Seamans and Zhu 2013), or matching providers to consumers (e.g., Hu and Zhou 2015 and Allon et al. 2012). Unlike the above literature that focuses on sharing between consumers, we study business-to-business sharing between retailers that compete against each other for demand.

Another related stream of literature is horizontal outsourcing (e.g., Sappington 2005, Gayle and Weisman 2007, Cai and Chen 2011, and Jiang and Shi 2018). Spiegel (1993) shows that horizontal subcontracting between firms engaging in Cournot competition can lead to a win-win outcome. Baake et al. (1999) examine the effects of economies of scale on two competing firms' cross-supplying decisions. Some research has examined outsourcing in competitive settings with a common supplier (e.g., Cachon and Harker 2002, Shy and Stenbacka 2003, Buehler and Haucap 2006, Arya et al. 2008b, Liu and Tyagi 2011, and Wu and Zhang 2014). Arya et al. (2008a) show that an incumbent that can make essential inputs itself may outsource from a less-efficient supplier when an entrant without in-house production capability enters the market. Chen et al. (2011) study the conditions under which horizontal outsourcing occurs between two firms that sell substitute products in the presence of an outside supplier. Jiang and Shi (2018) study the effect of inter-competitor licensing of non-core technology on product innovation. Our paper complements the above literature by investigating how inventory sharing between competing retailers affects their inventory and service-quality decisions when facing demand uncertainties.

The rest of the paper is organized as follows. Section 3 describes the core model and benchmark cases. Section 4 analyzes the case with exogenous inventory levels, and Section 5 considers the case with endogenous inventory decisions. We conclude the paper in Section 6.

3. Model

In this section, we outline the model setup in Section 3.1 and present the benchmark cases as well as their results in Section 3.2.

3.1. Model Setup

Consider a market with two retailers who sell a common core product and compete for demands by offering some quality-differentiated services for the product. Both retailers can order and acquire their respective inventories of the common core product from the same manufacturer before the selling season. Let I_i denote retailer i's pre-season inventory, where $i = 1,2$. Both retailers have a marginal cost of c for the pre-season inventory of the core product; that is, they can acquire the core product at the same cost before the selling season. Each retailer's demand is uncertain before the selling season and will be realized during the selling season. When facing a shortage during the selling season, each retailer can place a quick-response order from the manufacturer at a marginal cost of $\bar{c}$. Note that the product's marginal cost for the quick-response order is higher than that of pre-season inventory (i.e., $\bar{c} > c$), e.g., due to the manufacturer's higher costs for quick-response or expedited shipping.

Each retailer will offer the core product with the necessary services to consumers. Let s_i denote retailer i's service-quality level. To provide service level s_i, retailer i needs to incur a fixed cost of ks_i^2, where $k > 0$. Both retailers offer the product at the prevailing price, p, in the market. We assume $p \geq \bar{c}$ so that during the selling season, each retailer with a shortage will always (weakly) prefer replenishing the inventory from the manufacturer and fulfilling the demand to not replenishing it.[2] Since the two retailers sell the product together with the necessary services at the prevailing retail price p, they are effectively competing for demand based on their provisions of services, which are quality-differentiated.

Retailer i's demand D_i depends on both retailers' service-quality levels and is also affected by some random factors, for $i = 1,2$. We adopt a multiplicative demand function: $D_i = \varepsilon_i \cdot d_i(s_i, s_j)$, where $\varepsilon_i \in [0,1]$ is a random variable that captures uncertainty in demand and $d_i(s_i, s_j) \geq 0$ represents the deterministic term in demand that depends on the two retailers' service-quality levels, for $i, j = 1,2$ and $i \neq j$. We assume that ε_1 and ε_2 ex ante follow the same distribution with a probability density function (pdf) of $g(\cdot)$ and a corresponding cumulative distribution function (cdf) of $G(\cdot)$ on the interval [0,1]. Note that ε_1 and ε_2 can be independent or correlated; the joint distribution of $(\varepsilon_1, \varepsilon_2)$ has a general pdf of $g_{12}(\cdot)$ and a cdf of $G_{12}(\cdot)$. Let $\mu = \int_0^1 \varepsilon_i g(\varepsilon_i) d\varepsilon_i$ denote the mean of ε_i. We assume that given the

[2] Note that if $p = \bar{c}$, each retailer is indifferent between replenishing and not replenishing its inventory from the manufacturer; if $p < \bar{c}$, each retailer will not replenish its inventory from the manufacturer. In the latter case, the retailer's equilibrium profit will still be the same as that in the case of $p = \bar{c}$. Thus, for expositional convenience and without loss of generality, we focus on $p \geq \bar{c}$.

fixed common retail price p, the deterministic demand factor is $d_i(s_i, s_j) = \alpha + \beta \cdot (s_i - s_j)$, where $\alpha > 0$ represents the base demand potential and $\beta > 0$ captures the idea that a retailer's demand increases in its service quality and decreases in the competitor's service level.[3] Note that α depends on the retail price p and other exogenous factors; for notational simplicity, all such exogenous factors are absorbed into α. We further assume $\beta \leq \bar{\beta}$ to ensure that each retailer has a non-negative profit in equilibrium, where $\bar{\beta}$ is given in the Online Appendix. Without loss of generality, we normalize α to $\frac{1}{2}$; as such, we have $d_i(s_i, s_j) = \frac{1}{2} + \beta(s_i - s_j)$ and the maximum total demand of the two retailers will be 1, i.e., $D_1 + D_2 \leq d_1(s_1, s_2) + d_2(s_1, s_2) = 1$. Note that the special case with $\beta = 0$ represents the case that the two retailers do not compete with each other. Without loss of generality, we normalize the retail price p to 1.

We adopt the linear demand function not only for analytical tractability but also because it can be rationalized by a horizontal-differentiation model where utility-maximizing consumers make purchase decisions based on the firms' differentiated services (see the detailed discussion in the Online Appendix). Our linear demand model can be a reasonable approximation or abstraction for some markets in practice. For instance, some franchisees that are independent retailers usually offer the same products at the same prices determined by the franchisor (manufacturer); they often compete with other franchisees of the same brand by offering quality-differentiated services to consumers. Similar settings have also been used in the operations and marketing literature (e.g., Chen et al., 2008).

Before the selling season, the retailers can decide whether to have an inventory-sharing agreement at some transfer price t. If they do not reach an agreement to share inventory, each retailer, when facing a shortage, can replenish its inventory only from the manufacturer by placing a quick-response order at the unit cost of $\bar{c}$. If they agree to share, then when retailer i faces a shortage and the other retailer (j) has excess inventory, besides purchasing from the manufacturer, retailer i can also buy retailer j's excess inventory at the unit transfer price t and incur an additional unit shipping cost $\delta \geq 0$ to have the inventory transferred from retailer j, where $i, j = 1,2$ and $i \neq j$. Note that inventory sharing is non-moot or useful only when the net cost for shared inventory is (weakly) lower than the quick-response order's unit cost from the manufacturer, i.e., $t + \delta \leq \bar{c}$. Otherwise, even if the retailers have an inventory-sharing agreement, each retailer with a shortage will not purchase the competitor's excess inventory. Thus, the transfer price must satisfy $t \in [0, \bar{c} - \delta]$ for a non-moot sharing agreement, and to ensure a non-empty parameter region, we assume $\delta < \bar{c}$. The transfer price may be determined by channel members (including retailers and manufacturers/suppliers) or may have been set ex ante in the market (see Davis et al. 2022; Shao et al. 2011; Zhao and Atkins 2009). We will first analyze the market

[3] Our main insights from our model remain qualitatively similar even if some other forms of demand functions are used, such as $d_i(s_i, s_j) = \alpha + \beta \cdot s_i - \beta' \cdot s_j$ with $\beta \neq \beta'$. The details of this analysis are available from authors upon request.

outcome for any (exogenous) transfer price and then explicitly analyze the optimal transfer price that maximizes both retailers' profits.

We will study two different scenarios with respect to the retailers' inventories: (1) Exogenous inventory levels; and (2) endogenous, strategic inventory decisions. The reason for studying these two scenarios is two-fold. First, each scenario fits different market situations: The exogenous inventory case suits some situations where pre-season inventory has already been ex ante committed and cannot be easily adjusted. For example, firms may have decided or received inventories before the selling season, and later the opportunity to form an ad hoc agreement for sharing inventory came along. The endogenous inventory case fits situations where pre-season inventory can be easily changed according to whether the inventory-sharing agreement has been accepted. Second, studying both scenarios can help us better delineate the effects of inventory sharing. More specifically, an inventory-sharing agreement will directly change the retailers' service-quality decisions for any given inventory level, and it will also change the retailers' inventory decisions; the change in pre-season inventory will, in turn, further influence the retailers' service-quality decisions, leading to an indirect effect of inventory sharing on service quality (see Figure 4 for a conceptual illustration of the relationship). We will study the exogenous-inventory case in Section 4 as a first step that can help us understand the direct effect of inventory sharing on service quality without its convoluting indirect effect on service quality through its effect on the retailers' strategic inventory decisions. Then, we analyze the endogenous-inventory case in Section 5 to determine how inventory sharing affects the retailers' optimal inventory levels and its overall effect on service quality. The comparison between the exogenous and endogenous inventory cases can provide insightful results. The sequence of moves in the game is as follows.

Stage 1. The two retailers decide whether to have an inventory-sharing agreement and at what transfer price $t \in [0, \bar{c} - \delta]$. Both retailers need to accept for the sharing agreement to be in effect.

Stage 2. In the exogenous-inventory case, the two retailers simultaneously decide their respective service-quality level s_i; in the endogenous-inventory case, the two retailers simultaneously decide their respective pre-season inventory I_i and service-quality level s_i, for $i = 1,2$.[4]

Stage 3. Each retailer's demand uncertainty is realized, i.e., ε_1 and ε_2 are realized.

Stage 4. If inventory sharing was not agreed upon in Stage 1, then the retailer with a shortage will place a quick-response order from the manufacturer at marginal cost $\bar{c}$ to fulfill any excess demand. If retailers have agreed on inventory sharing in Stage 1, then the retailer with a shortage can request from the other retailer up to all the other retailer's leftover inventory at the marginal cost $t + \delta$, after which any remaining unmet demand can be replenished through quick-response orders from the manufacturer.

[4] Note that the exogenous-inventory and endogenous-inventory cases are two different settings, and the former one is *not* a sub-game of the latter one.

In addition, without loss of generality, we adopt the following tie-breaking rules: (1) When the retailers are indifferent between accepting a sharing agreement and not accepting it, they will accept it; (2) when the retailer is indifferent between placing a quick-response order from the manufacturer and purchasing the other retailer's excess inventory, it will purchase from the other retailer; (3) when the retailer is indifferent between replenishing the inventory to fulfill all its demand versus not replenishing its inventory, it will replenish the inventory. The key notations used in this paper are given in Table A.1 of the Online Appendix.

3.2. Benchmark Cases

To reveal the impacts of competition on the retailers' inventory, service quality, and sharing decisions, we first analyze two benchmark cases: (1) A centralized system, where a single firm owns both retailers and makes centralized decisions to maximize the aggregate profit of the retailers; (2) a decentralized system without competition, i.e., $\beta = 0$.

3.2.1. Centralized System

We first consider a centralized system and use superscript "*c*" on a variable to denote this case. In the centralized case, inventory sharing between the retailers is obviously beneficial and will naturally occur. Moreover, the centralized firm will set zero service quality to save cost, i.e., $s_1^c = s_2^c = 0$. We can further solve each retailer's optimal inventory level, denoted by $I_1^c = I_2^c$; the details of the analysis are provided in the Online Appendix. In this case, the transfer price is irrelevant when the centralized firm focuses on the aggregate profit. Let π^{c*} denote the optimal (expected) aggregate profit of the two retailers in this case and let $\pi_i^{c*} = \pi^{c*}/2$ denote the optimal profit of retailer i.

3.2.2. No Competition

We use superscript "*nc*" on a variable to denote the benchmark case without competition (i.e., $\beta = 0$). In this case, each retailer faces demand $D_i = \frac{1}{2}\varepsilon_i$, which is independent of both retailers' service quality. Thus, obviously, each retailer will set its service quality to zero, i.e., $s_1^{nc} = s_2^{nc} = 0$. We summarize the outcome of the no-competition case in the following Lemma 1.

Lemma 1. *When there is no competition between the two retailers (i.e., $\beta = 0$):*

(a) The retailers will choose zero service quality (i.e., $s_1^{nc} = s_2^{nc} = 0$) and prefer to share inventory at any non-moot transfer price $t \in [0, \bar{c} - \delta]$, regardless of whether the inventory is exogenously given or endogenously decided.

(b) When the inventory is exogenously given and $I_1 = I_2 \in [0,1]$, each retailer's profit is independent of transfer price t in the sharing case.

(c) When the inventory is endogenously decided, there exists an optimal transfer price t^{nc} that maximizes each retailer's profit and also achieves coordination as in the centralized system (i.e., $I_i^{nc} = I_i^c$, $\pi_i^{nc*} = \pi_i^{c*}$, for $i = 1,2$).

Lemma 1 shows that when the retailers do not compete with each other, sharing inventory is always beneficial regardless of the transfer price since it allows inventory pooling. Moreover, in the exogenous-inventory case, each retailer's chance of buying inventory from the other retailer is equal to its chance of selling excess inventory, given that the retailers' inventory levels are symmetric. Thus, the transfer cost (not including the shipping cost δ) of buying the other retailer's inventory and the profit of selling excess inventory cancel each other out, and hence the retailers' profits are independent of the transfer price. While in the endogenous-inventory case, the transfer price will affect the retailers' inventory decisions and profits. We can show that there exists an optimal transfer price that maximizes both retailers' profits. Moreover, this optimal transfer price can also achieve coordination as in the centralized system, meaning that the retailers' decisions and profits are the same as those in the centralized system. The above results are also consistent with the findings in the existing literature on inventory transshipment between non-competitive retailers (e.g., Rudi et al. 2001 and Hu et al. 2007). However, when the retailers compete on service (i.e., $\beta > 0$), as we will show in Sections 4 and 5, the impacts of inventory sharing become involved, and the above intuitions may no longer hold.

4. Exogenous Inventory

This section analyzes the case of exogenous inventory levels under service competition. Given their pre-season inventory levels, the two retailers first decide whether to share inventory and then simultaneously choose their respective service-quality levels. We focus on the symmetric setting where $I_1 = I_2 = I \in [0,1]$ and analyze the symmetric equilibrium outcome. We use "$\widehat{\ }$" over a variable to denote this case of exogenous inventory. Section 4.1 studies the subgame without an inventory-sharing agreement between the two retailers; Section 4.2 analyzes the sharing subgame for any transfer price. Section 4.3 compares the two cases and identifies the optimal transfer price in the sharing case to determine the equilibrium outcome of the full game.

4.1. No Inventory-Sharing under Exogenous Inventory

We first analyze the subgame with no inventory-sharing agreement between the retailers; the superscript "*N*" denotes this case. Each retailer chooses its service quality to maximize its expected profit:

$$\pi_i^N(s_i, I_i|s_j) = p \cdot E[\varepsilon_i \cdot d_i(s_i, s_j)] - \bar{c} \cdot E[(\varepsilon_i \cdot d_i(s_i, s_j) - I_i)^+] - k\, s_i^2 - c\, I_i, \tag{1}$$

where $d_i(s_i, s_j) = \frac{1}{2} + \beta(s_i - s_j)$, for $i, j = 1,2$ and $i \neq j$. The first term in Equation (1) is retailer i's expected revenue from selling the product to consumers; the second term is the expected cost of quick-response order in the case of shortage during the selling period; the third term is the retailer's service cost; the last term is the cost of pre-season inventory. Given $p \geq \bar{c}$, both retailers will find it profitable to replenish their inventories to fulfill all their demands. Lemma 2 presents the retailers' service decisions in the symmetric equilibrium.

Lemma 2. *Given exogenous inventories $I_1 = I_2 = I \in [0,1]$ and no inventory-sharing agreement:*

(a) There exists a unique symmetric equilibrium outcome with the retailers' service-quality levels

$$\hat{s}_1^N(I) = \hat{s}_2^N(I) = \hat{s}^N(I) = \frac{\beta(p\mu - \bar{c}\int_{2I}^{1} \varepsilon g(\varepsilon)d\varepsilon)}{2k}. \tag{2}$$

(b) $\hat{s}^N(I)$ increases in I and β, and decreases in $\bar{c}$.

(c) Retailer i's equilibrium profit $\hat{\pi}_i^N(I) = \pi_i^N(\hat{s}^N(I), I|\hat{s}^N(I))$ decreases in β, and is non-monotone in I and $\bar{c}$, for $i = 1,2$.

As Lemma 2(a) shows, in the symmetric equilibrium, each retailer offers the same service quality, obtains the same expected demand (i.e.,$d_1 = d_2 = \frac{1}{2}$ and $E[D_1] = E[D_2] = \frac{1}{2}\mu$), and earns the same profit. Lemma 2(b)-(c) shows the impacts of some key parameters (i.e., I, β, and $\bar{c}$) on the retailers' service quality and profits in the absence of an inventory-sharing agreement. As one intuits, the retailers' equilibrium service quality $\hat{s}^N(I)$ is increasing in their inventory I. A higher inventory level increases in the retailers' optimal service quality and increases their service costs, which implies more intense competition for demand between them. A lower $\bar{c}$ will also motivate both retailers to raise service quality to attract customers since the cost to replenish in-season inventory to satisfy excess demand decreases. Moreover, as each retailer's demand becomes more sensitive to service (i.e., as β increases), service competition will also intensify, leading to higher equilibrium service quality.

On the one hand, as the inventory level I increases, the probability of shortage will decrease. On the other hand, a higher I not only increases inventory costs but also induces more intense service competition and raises service costs. These two opposite effects lead to the non-monotone impact of pre-season inventory I on the retailers' profits, as shown in Lemma 2(c). One might intuit that as the marginal cost $\bar{c}$ of quick-response order increases, the retailers should be worse off. Interestingly, our result reveals that the retailer's profit can increase or decrease in $\bar{c}$ because a higher $\bar{c}$ can lower the retailers' service quality and thus reduce their service costs (see Lemma 2(b)). Moreover, as β increases, each retailer's profit will decrease due to the increased service cost in a more competitive market.

4.2. Inventory-Sharing under Exogenous Inventory

We now analyze the subgame in which the retailers agree to share their inventories at a transfer price $t \in [0, \bar{c} - \delta]$ in Stage 1. Note that if $t > \bar{c} - \delta$, the retailer with a shortage will never purchase the other retailer's excess inventory since a quick-response order from the manufacturer is cheaper. In other words, inventory sharing with $t > \bar{c} - \delta$ is moot, and the sharing case will be equivalent to the no-sharing case. Thus, in what follows, we focus on the parameter region of $0 \le t \le \bar{c} - \delta$. The *absence* of a superscript "*N*" indicates the current case that the retailers have a non-moot inventory-sharing agreement. Each retailer chooses its service quality to maximize its expected profit:

$$\pi_i(s_i, I_i | s_j, I_j) = p \cdot E[\varepsilon_i d_i] + t \cdot E[(\min\{I_i - \varepsilon_i d_i, \varepsilon_j d_j - I_j\})^+] - (t+\delta) \cdot E[(\min\{\varepsilon_i d_i - I_i, I_j - \varepsilon_j d_j\})^+]$$
$$- \bar{c} \cdot E[\left((\varepsilon_i d_i - I_i)^+ - (I_j - \varepsilon_j d_j)^+\right)^+] - k s_i^2 - c I_i, \quad \text{for } i, j = 1,2 \text{ and } i \neq j. \tag{3}$$

The first term in Equation (3) is retailer i's expected revenue from selling the product to consumers; the second term is the expected revenue from selling excess inventory to the competitor; the third term is the expected cost of purchasing the other retailer's excess inventory when a shortage occurs; the fourth term is the expected cost of the quick-response order when needed; the fifth and the last terms are the retailer's service cost and the pre-season inventory cost, respectively. We first solve the retailers' service quality for a given transfer price t in the symmetric equilibrium.

Lemma 3. *Given an inventory-sharing agreement at the transfer price* $t \in [0, \bar{c} - \delta]$ *and the pre-season inventories* $I_1 = I_2 = I \in [0,1]$:

(a) There exists a unique symmetric equilibrium outcome with service-quality levels

$$\hat{s}_1(t, I) = \hat{s}_2(t, I) = \hat{s}(t, I) = \frac{\beta(p\mu - A - B)}{2k}, \tag{4}$$

where $A = \int_0^{2I} [\int_{2I}^{4I-\varepsilon_2} (t+\delta)\varepsilon_1 g_{12}(\varepsilon_1, \varepsilon_2) d\varepsilon_1 + \int_{4I-\varepsilon_2}^{1} (\bar{c}\varepsilon_1 - \bar{c}\varepsilon_2 + (t+\delta)\varepsilon_2) g_{12}(\varepsilon_1, \varepsilon_2) d\varepsilon_1] d\varepsilon_2$ and $B = \int_{2I}^{1} (\int_0^{4I-\varepsilon_2} t\varepsilon_2 g_{12}(\varepsilon_1, \varepsilon_2) d\varepsilon_1 + \int_{4I-\varepsilon_2}^{2I} t\varepsilon_1 g_{12}(\varepsilon_1, \varepsilon_2) d\varepsilon_1 + \int_{2I}^{1} \bar{c}\varepsilon_1 g_{12}(\varepsilon_1, \varepsilon_2) d\varepsilon_1) d\varepsilon_2$.

(b) $\hat{s}(t, I)$ *increases in* β *and decreases in* t, δ, *and* $\bar{c}$; $\hat{s}(t, I)$ *increases in* I *when* $t \le \frac{\bar{c}-\delta}{2}$ *and can be non-monotone in* I *when* $t > \frac{\bar{c}-\delta}{2}$.

(c) Retailer i's equilibrium profit $\hat{\pi}_i(t, I) = \pi_i(\hat{s}(t, I), I | \hat{s}(t, I), I)$ *decreases in* β, *increases in t, and is non-monotone in I,* $\bar{c}$, *and* δ, *for* $i = 1,2$.

In the symmetric equilibrium, each retailer offers the same service quality and obtains the same expected demand and profit. Lemma 3 shows how the key parameters affect each retailer's service quality and profit in the sharing case. As shown in Lemma 3(b), an increase in the transfer price t will reduce the retailers' equilibrium service quality, alleviating competition between them. This is because a higher t makes it more profitable for each retailer to share excess inventory with the competitor, which reduces their incentive to raise service quality to compete for demand. Moreover, as the shipping cost (δ) or the marginal cost of quick-response order ($\bar{c}$) increases, the expected cost of purchasing in-season inventory to satisfy excess demand increases, which also reduces the retailers' incentive to compete for demand and leads to lower service quality. In addition, the retailer's service quality $\hat{s}(t, I)$ increases in the demand sensitivity (i.e., β) for the same reason as explained in the no-sharing case (see Lemma 2).

Recall that in the no-sharing case, the retailers' equilibrium service quality increases in their inventory level I (see Lemma 2). By contrast, in the sharing case, their service quality $\hat{s}(t, I)$ increases in I when the transfer price (t) is low but may *decrease* in I when t is high. Intuitively, with more inventory on hand, the retailers tend to have stronger incentives to compete for customers by raising service quality. However, with a sharing agreement at high t, as the inventory increases, the retailers may have less incentive to compete for demand because the inventory-acquisition cost from the competitor (in case of shortage) is high but selling excess inventory to the competitor is profitable. So, when t is high, the retailers' inventory level could have a non-monotonic impact on service quality. This result implies that, in the sharing case, a higher inventory level may intensify or alleviate service competition, depending on t and other parameters as well as the joint distribution of random demands.

As t increases, although each retailer will incur a higher cost of purchasing the competitor's excess inventory when a shortage occurs, it can also earn more profits for selling its potential excess inventory to the competitor. These negative and positive effects cancel each other out. Moreover, a higher t leads to a lower service-quality level; thus, each retailer can benefit from softened competition and enjoy a higher profit. That is, with exogenous inventory, each retailer's profit increases in the non-moot transfer price t (i.e., $t \leq \bar{c} - \delta$) due to softened competition, which indicates that the highest non-moot transfer price $t = \bar{c} - \delta$ will maximize both retailers' profits under the sharing agreement.

As the marginal cost of the quick-response order $(\bar{c})$ or the shipping cost of transferring shared inventory (δ) increases, the retailers will face a higher expected cost for purchasing in-season inventory and a lower service cost due to softened service competition. Thus, with these two opposing effects, each retailer's profit may increase or decrease in $\bar{c}$ and δ. Moreover, since the equilibrium service quality may increase or decrease in the pre-season inventory I, the retailer's profit is also non-monotone in I. Like the no-sharing case, as β increases, the equilibrium service quality increases, and each retailer's profit decreases.

4.3. Full Equilibrium Outcome under Exogenous Inventory

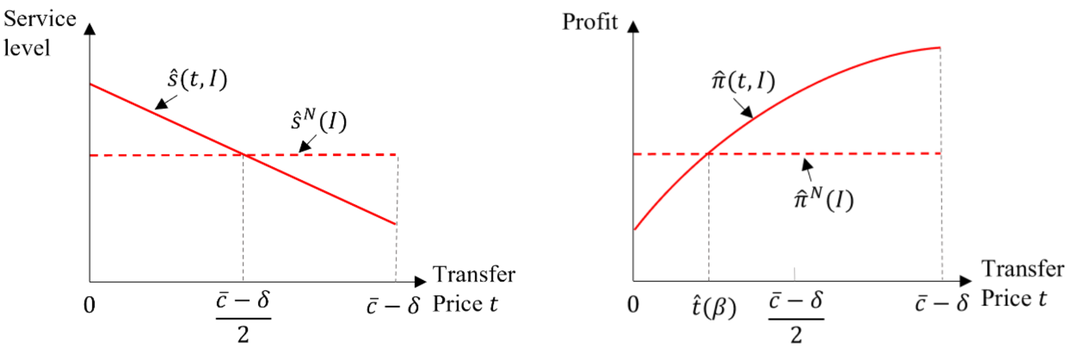
In this section, we first compare the outcomes under the sharing and no-sharing cases for any given transfer price, the results of which are summarized in Proposition 1 and Figures 1-3. Then, we characterize the optimal transfer price in the sharing case and show whether the retailers would choose to share inventory in the final equilibrium. Sections 4.1 and 4.2 have shown the impacts of transfer price t and competition level β on the retailers' service quality and profit in the sharing and no-sharing cases (see Lemmas 2 and 3). When comparing the two cases, we also focus on the impacts of t and β. We define $\widehat{\Delta}(t, I) = \hat{\pi}_i(t, I) - \hat{\pi}_i^N(I)$ as the profit difference between the two cases for given t and I.

Proposition 1. *Given exogenous inventories* $I_1 = I_2 = I \in [0,1]$, *there exists* $\hat{t}(\beta) \in [0, \frac{\bar{c}-\delta}{2})$ *(or equivalently, there exists* $\hat{\beta}(t) \in (0, \bar{\beta}(t)]$) such that:

(a) *When* $t < \frac{\bar{c}-\delta}{2}$, $\hat{s}(t,I) > \hat{s}^N(I)$; *when* $t > \frac{\bar{c}-\delta}{2}$, $\hat{s}(t,I) < \hat{s}^N(I)$.

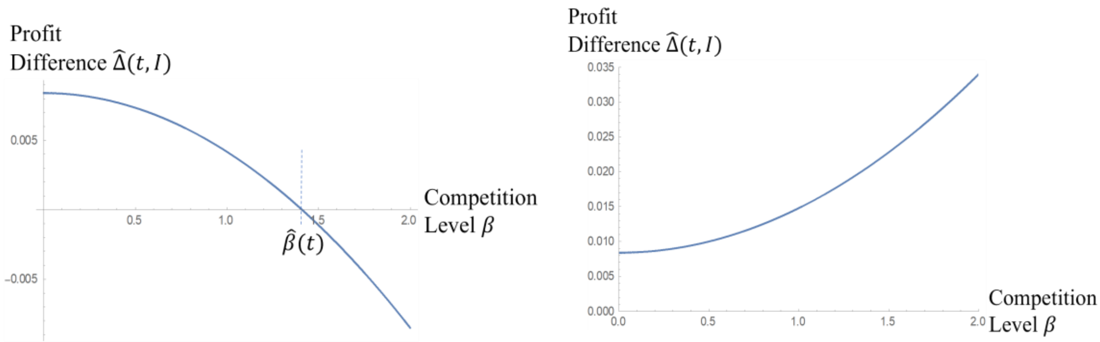
(b) *When* $t < \hat{t}(\beta)$ *(or equivalently, when* $\beta > \hat{\beta}(t)$ *and* $t < \frac{\bar{c}-\delta}{2}$*), each retailer's profit is lower in the sharing case, i.e.,* $\hat{\Delta}(t,I) < 0$; *otherwise, it is higher in the sharing case, i.e.,* $\hat{\Delta}(t,I) \geq 0$.

$\hat{\Delta}(t,I)$ *increases in* t. *Moreover, when* $t < \frac{\bar{c}-\delta}{2}$, $\hat{\Delta}(t,I)$ *decreases in* β; *when* $t \geq \frac{\bar{c}-\delta}{2}$, $\hat{\Delta}(t,I) \geq 0$ *and increases in* β. Proposition 1(a) and Figure 1(a) show that for a given transfer price t, the service quality in the sharing case is higher than that in the no-sharing case when $t < \frac{\bar{c}-\delta}{2}$ and is lower otherwise. As discussed in Lemma 3, this is because when t is high (low), each retailer has a weaker (stronger) incentive to compete for demand by adjusting service quality, leading to a lower (higher) equilibrium service-quality level. Thus, sharing inventory can *soften* service competition only when the transfer price is high enough; otherwise, it will *intensify* competition.



(a) Effect of t on Service Quality (b) Effect of t on Profit

Figure 1. Effect of Transfer Price under Exogenous Inventory



(a) When $t < \frac{\bar{c}-\delta}{2}$ (b) When $t > \frac{\bar{c}-\delta}{2}$

Figure 2. Effect of β on Profit Difference under Exogenous Inventory

Note: Figure 2 is illustrated with $g(x) = 1$ for $x \in [0,1]$ and $g_{12}(x,y) = g(x)g(y)$. Moreover, $p = 1, c = 0.3, \bar{c} = 0.5, \delta = 0.05, k = 1$, and $I = 0.3$ in both (a) and (b); $t = 0.15$ in (a) and $t = 0.35$ in (b).

As shown in Lemma 3, each retailer's profit in the sharing case increases in t, and thus the profit difference $\widehat{\Delta}(t,I)$ also increases in t. Moreover, there exists a threshold $\hat{t}(\beta)$ such that inventory sharing is more profitable only when $t > \hat{t}(\beta)$ and no-sharing is more profitable otherwise (see Proposition 1(b) and Figure 1(b)). On the one hand, inventory sharing can benefit both retailers by allowing risk pooling to cope with demand uncertainty. On the other hand, inventory sharing also affects competition. When $t > \frac{\bar{c}-\delta}{2}$, inventory sharing softens service competition and lowers service costs; so, sharing inventory benefits retailers through both risk pooling and competition alleviation. By contrast, when $t < \frac{\bar{c}-\delta}{2}$, sharing inventory intensifies competition and increases service costs, although the benefit of risk pooling still exists. More specifically, when t is moderate (i.e., $\hat{t} < t < \frac{\bar{c}-\delta}{2}$), the benefit from risk pooling outweighs the increase in service cost, and thus inventory sharing can still make both retailers better off. However, when t is very low (i.e., $t < \hat{t}(\beta)$), the increase in service cost will dominate the benefit from risk pooling, and thus inventory sharing will make both retailers worse off. Note that we can show that the threshold $\hat{t}(\beta)$ increases in β, and thus $t < \hat{t}(\beta)$ is equivalent to $\beta > \hat{\beta}(t)$, provided that we define $\hat{\beta}(t) = \bar{\beta}$ when $t \geq \frac{\bar{c}-\delta}{2}$ (see Figure 3).

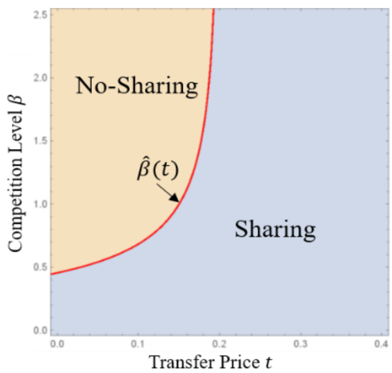


Figure 3. Sharing or No-sharing under Exogenous Inventory for a Given t

Note: This figure is illustrated with $g(x) = 1$ for $x \in [0,1]$ and $g_{12}(x,y) = g(x)g(y)$. Moreover, $p = 1$, $c = 0.3, \bar{c} = 0.5, \delta = 0.1, k = 1$, and $I = 0.4$.

Proposition 1 also shows how the profit difference $\hat{\Delta}(t,I)$ changes with β. Recall that without competition (i.e., $\beta = 0$), both retailers will offer zero service level in either sharing or no-sharing case, and sharing will always benefit the retailers due to the risk-pooling effect at any $t \in [0, \bar{c} - \delta]$ (see Lemma 1). However, when the two retailers compete on service quality (i.e., $\beta > 0$), the results will be different. As shown in Figure 2, when $t < \frac{\bar{c}-\delta}{2}$, the profit difference decreases in β and becomes negative if β is high; in contrast, when $t \geq \frac{\bar{c}-\delta}{2}$, the profit difference is always positive and increases in β. The reason is as follows. A higher β will induce a higher service level and reduce each retailer's profit in both sharing and no-sharing cases (see Lemmas 2-3). When t is low, inventory sharing further intensifies competition, making the retailers more sensitive to service competition (β); thus, the profit difference decreases and will become negative as β increases. By contrast, when t is high, inventory sharing softens competition (see Proposition 1(a)), making the retailers less sensitive to service competition; thus, sharing inventory becomes more beneficial as β increases. Therefore, for a given transfer price t, sharing inventory is *not* profitable for the retailers when competition is fierce and transfer price is low (i.e., $\beta > \hat{\beta}(t)$ and $t < \frac{\bar{c}-\delta}{2}$), as depicted in Figure 3.

Given that both retailers will fulfill all demands at price p, consumers will always benefit from intensified service competition due to the increased service quality. In the case of a moderate transfer price (i.e., $\hat{t}(\beta) < t < \frac{\bar{c}-\delta}{2}$), an inventory-sharing agreement benefits not only the retailers but also the consumers, leading to a *win-win* outcome. We summarize this result in Corollary 1 below.

Corollary 1. *Given exogenous inventories $I_1 = I_2 = I \in [0,1]$, when $\hat{t}(\beta) < t < \frac{\bar{c}-\delta}{2}$, inventory sharing increases equilibrium service quality and profits — a win-win for both consumers and retailers.*

Next, we characterize the full equilibrium in the exogenous-inventory case by identifying the optimal transfer price in the sharing case and showing whether the retailers will choose to share inventory at this optimal transfer price. We also examine whether the optimal transfer price can achieve coordination as in a centralized system. Proposition 2 summarizes these results.

Proposition 2. *Given exogenous inventories $I_1 = I_2 = I \in [0,1]$, the optimal transfer price that maximizes both retailers' profits in the sharing case is $\hat{t}^* = \bar{c} - \delta$, and both retailers prefer sharing inventory at the optimal transfer price. Moreover, when $\beta > 0$, no transfer price can achieve coordination as in the centralized system.*

With exogenous inventory levels, the two retailers' profits increase in t, and thus their most preferred transfer price is the maximum non-moot transfer price (i.e., $\hat{t}^* = \bar{c} - \delta$); sharing inventory at $\hat{t}^*$ is always more profitable than not sharing. Therefor, in the full equilibrium, the retailers will choose inventory sharing (in Stage 1). We will show in Section 5 that this result may *not* hold if the retailers strategically choose their pre-season inventory levels.

Moreover, as mentioned in Section 3.2, when the retailers are owned by a centralized firm, the optimal service quality will be zero; without competition (i.e., $\beta = 0$), there exists an optimal transfer price that can coordinate the two independent retailers (see Lemma 1). However, the existence of service competition will hinder the coordination through inventory sharing at any transfer price. When the retailers compete with each other (i.e., $\beta > 0$), their optimal service quality will always be positive at any transfer price in the sharing case. Thus, under service competition, although inventory sharing may soften competition, it cannot fully eliminate it to achieve coordination. This result contrasts the findings in our benchmark case without competition (see Lemma 1) and also those in the existing literature with non-competitive retailers (e.g., Rudi et al. 2001, Hu et al. 2007).

5. Endogenous Inventory

Section 4 has examined the effects of inventory sharing on the retailers' service-quality decisions and profits for exogenous pre-season inventory levels. This section analyzes the case of retailers endogenously choosing pre-season inventories. We use "$\sim$" over a variable to denote this scenario, in which the retailers first decide whether to agree on inventory sharing and then simultaneously make their inventory and service-quality decisions. Note that the game with simultaneous inventory and service-quality decisions fits the reality better than a sequential-move game. But even if the retailers choose their service quality after their inventory decisions, as long as the retailers cannot directly observe each other's inventory before deciding their service quality, such a sequential game will give the same equilibrium outcome as the above simultaneous-move game. We first analyze the no-sharing case in Section 5.1 and then the sharing case in Section 5.2; Section 5.3 compares the two cases and identifies the optimal transfer price in the sharing case to determine the full equilibrium outcome.

5.1. No Inventory-Sharing under Endogenous Inventory

We first analyze the subgame in which the retailers have no inventory-sharing agreement in Stage 1. We use superscript "*N*" to denote this no-sharing case. Each retailer chooses its inventory and service quality to maximize its expected profit, given by Equation (1). We focus on the symmetric equilibrium outcome and present it in Lemma 4, where $G^{-1}(\cdot)$ represents the inverse function of $G(\cdot)$, the cdf of the random demand factor (ε_i).

Lemma 4. *Given the absence of an inventory-sharing agreement:*

- *(a) There exists a unique symmetric equilibrium with $I_1 = I_2 = \tilde{I}^N$ and $s_1 = s_2 = \tilde{s}^N$, where $\tilde{I}^N$ and $\tilde{s}^N$ are given by*

$$\tilde{I}^N = \frac{G^{-1}\left(1-\frac{c}{\bar{c}}\right)}{2} \quad and \quad \tilde{s}^N = \hat{s}^N(\tilde{I}^N) = \frac{\beta\left(p\mu-\bar{c}\int_{2\tilde{I}^N}^{1}\varepsilon g(\varepsilon)d\varepsilon\right)}{2k}. \tag{5}$$

- *(b) $\tilde{I}^N$ increases in $\bar{c}$ and is independent of β.*
- *(c) $\tilde{s}^N$ decreases in $\bar{c}$ and increases in β.*
- *(d) Retailer i's equilibrium profit $\tilde{\pi}_i^N$ is non-monotone in $\bar{c}$ and decreases in β.*

In the symmetric equilibrium, the two retailers choose the same inventory and service-quality levels and also obtain the same expected demand and profit. Lemma 4 shows the impacts of competition level (β) and the unit cost of quick-response order ($\bar{c}$) on this subgame equilibrium outcome. We find that each retailer's equilibrium inventory increases in $\bar{c}$ and is independent of β. In the symmetric equilibrium, each retailer offers the same service level, and thus the deterministic demand term becomes

$d_i = \frac{1}{2}$, which is independent in β. So, the equilibrium inventory level is also independent in β. Moreover, as $\bar{c}$ increases, the retailers will stock more pre-season inventory (i.e., a higher $\tilde{I}^N$). Recall that in the exogenous-inventory case, as shown in Lemma 2, a lower $\bar{c}$ or a higher pre-season inventory level I will incentivize each retailer to raise its service quality. When the pre-season inventory is an endogenous decision, a decrease in $\bar{c}$ has both direct and indirect effects on the retailer's service quality. On the one hand, a lower $\bar{c}$ directly motivates each retailer to increase service quality, as explained in Section 4.1. On the other hand, a lower $\bar{c}$ also leads to a lower pre-season inventory level (as shown in Lemma 4(b)), which in turn will induce the retailers to lower their service quality (as explained in Lemma 2). Combining these two opposite effects, we find that the retailers' service quality will eventually decrease in $\bar{c}$. Similar to the exogenous-inventory case, the retailer's profit may increase or decrease in $\bar{c}$ since a higher $\bar{c}$ increases the retailer's cost of purchasing in-season inventory but reduces the service cost; a higher demand sensitivity to service (i.e., a higher β) will intensify service competition, leading to higher service quality and lower profits.

5.2. Inventory-Sharing under Endogenous Inventory

We now examine the subgame where the two retailers agree to share their inventories at a given transfer price $t \in [0, \bar{c} - \delta]$ in Stage 1. Note that the *absence* of a superscript "*N*" indicates the current case with inventory sharing. Each retailer chooses its inventory and service quality to maximize its expected profit, given by Equation (3). Similar to the exogenous-inventory case (discussed in Section 4.2), when $t > \bar{c} - \delta$, inventory sharing will not occur since a quick-response order from the manufacturer is cheaper, thus making the sharing case identical to the no-sharing case. Thus, for the inventory sharing case, we only focus on the non-moot transfer price $t \in [0, \bar{c} - \delta]$.

Lemma 5. *Given that the retailers agree to share inventory at the transfer price $t \in [0, \bar{c} - \delta]$:*

(a) There exists a unique symmetric equilibrium with $I_1 = I_2 = \tilde{I}(t)$ and $s_1 = s_2 = \tilde{s}(t)$, where $\tilde{I}(t)$ and $\tilde{s}(t)$ satisfy:

$$\int_0^{2\tilde{I}(t)} \left(\int_{4\tilde{I}(t)-\varepsilon_2}^{1} \bar{c} g_{12}(\varepsilon_1, \varepsilon_2) d\varepsilon_1 + \int_{2\tilde{I}(t)}^{4\tilde{I}(t)-\varepsilon_2} (t+\delta) g_{12}(\varepsilon_1, \varepsilon_2) d\varepsilon_1\right) d\varepsilon_2 +$$
$$\int_{2\tilde{I}(t)}^{1} \left(\int_{2\tilde{I}(t)}^{1} \bar{c} g_{12}(\varepsilon_1, \varepsilon_2) d\varepsilon_1 + \int_{4\tilde{I}(t)-\varepsilon_2}^{2\tilde{I}(t)} t g_{12}(\varepsilon_1, \varepsilon_2) d\varepsilon_1\right) d\varepsilon_2 = c$$

$$\text{and} \quad \tilde{s}(t) = \hat{s}(t, \tilde{I}(t)) = \frac{\beta(p\mu - A - B)}{2k},$$

in which $\hat{s}(\cdot)$ is given in Lemma 3.

(b) $\tilde{I}(t)$ increases in $\bar{c}$, t, and δ; $\tilde{I}(t)$ is independent of β.

(c) $\tilde{s}(t)$ is non-monotone in $\bar{c}$, t, and δ; $\tilde{s}(t)$ increases in β.

(d) Retailer i's equilibrium profit $\tilde{\pi}_i(t)$ is non-monotone in $\bar{c}$, t, and δ and decreases in β.

Lemma 5 shows how the key parameters affect each retailer's pre-season inventory and service decisions as well as the profitability when the retailers have a non-moot inventory-sharing agreement. Similar to the no-sharing case, each retailer's inventory level increases as the marginal cost ($\bar{c}$) of quick-response order increases and is independent of β in the symmetric equilibrium. Moreover, in the sharing case, each retailer's optimal pre-season inventory also depends on the transfer price (t) and the unit shipping cost (δ) of shared inventory. As t increases, each retailer's pre-season inventory will increase because selling excess inventory to the competitor becomes more profitable while purchasing the competitor's inventory in the case of shortage becomes more expensive. Thus, each retailer has more incentives to stock a higher level of pre-season inventory to reduce the chance of shortages. This result is also consistent with the findings in the existing literature on inventory transshipment among non-competitive retailers (e.g., Rudi et al. 2001). Furthermore, as δ increases, each retailer will also stock more pre-season inventory since the cost of purchasing inventory from the other retailer increases.

Recall that in the sharing case with exogenous inventory, as shown in Lemma 3, as the transfer price t increases, service competition between the retailers is alleviated, and the retailers' service quality decreases. However, when the retailers can endogenously choose their pre-season inventory levels, their service quality $\tilde{s}(t)$ may increase or decrease in t. The underlying reason is as follows. On the one hand, a higher t can lower the retailer's service quality for a given (exogenous) level of inventory via its direct effect, as shown by Lemma 3. On the other hand, a higher t can also increase each retailer's inventory level (see Lemma 5(b)), which in turn may increase or decrease the retailer's service quality (as explained in Lemma 3(b)). Thus, overall, these direct and indirect effects can lead to the equilibrium service quality's non-monotonic dependence on the transfer price (see Figure 4 for a conceptual illustration).

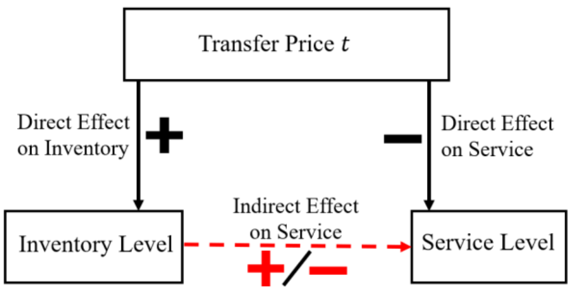


Figure 4. Direct and Indirect Effects of Inventory Sharing

The above result suggests that, unlike the exogenous inventory case, a higher transfer price does not necessarily alleviate service competition and can *intensify* competition when the retailers strategically adjust their pre-season inventories. Moreover, in the exogenous inventory case, Lemma 3 shows that a higher inventory-transfer shipping cost (δ) or a higher cost of quick-response order ($\bar{c}$) can lead to lower service quality. However, when the retailers strategically choose their inventory levels, the effects of δ and $\bar{c}$ on the equilibrium service quality are also non-monotone for the similar reason that an increase in δ or $\bar{c}$ will raise the retailers' inventory levels, which in turn affects the equilibrium service quality. Furthermore, the impact of the competition level (β) on the retailer's equilibrium service quality and profit is the same as that in the exogenous-inventory case: A higher β leads to higher service quality and lower profits.

Recall that, with exogenous pre-season inventory, each retailer's profit increases in the transfer price t because of softened competition and reduced service cost (see Lemma 3). Again, this result will not necessarily hold when the retailers strategically choose their inventory levels. As t increases, the retailers' inventory levels will increase, which will directly raise their inventory cost but also increase their sales revenue. Overall, each retailer's profit first increases and then decreases as its inventory level increases (or as t increases), given that the service quality remains unchanged. However, the retailers' strategic adjustment of inventory levels will alter their optimal service quality and indirectly influence their profits. That is, a higher t will raise each retailer's pre-season inventory and may increase or decrease service quality, leading to a higher or lower service cost. Thus, depending on other parameters and the joint distribution of random demand factors (ε_1 and ε_2), the retailers can be better off or worse off as the transfer price increases, as shown in Lemma 5(d). Lemma 5(d) also shows that each retailer's equilibrium profit is non-monotone in the unit shipping cost of transferring inventory (δ) and the marginal cost of quick-response order ($\bar{c}$) because these parameters affect not only the retailers' profit margin but also their inventory and service quality decisions.

5.3. Full Equilibrium Outcome under Endogenous Inventory

We now compare the sub-equilibrium outcomes in the sharing and no-sharing cases and then characterize the optimal transfer price in the sharing case and show whether and when the retailers will choose to share inventory at the optimal transfer price. Proposition 3 shows the comparison of the retailers' inventory levels in the two cases, which holds for any general joint distribution of demands.

Proposition 3. *When retailers endogenously choose their inventory levels, there exists $\tilde{t}_I$ such that $\tilde{I}(t) > \tilde{I}^N$ when $t > \tilde{t}_I$, and $\tilde{I}(t) < \tilde{I}^N$ when $t < \tilde{t}_I$.*

Proposition 3 shows that an inventory-sharing agreement can increase or decrease each retailer's pre-season inventory, depending on the transfer price t, as illustrated in Figure 5(a). Specifically, when $t > \tilde{t}_I$, the retailers have incentives to stock more pre-season inventories in the sharing case than in the no-sharing case since they could, with some probability, sell excess inventory to the other retailer at the high transfer price. By contrast, when $t < \tilde{t}_I$, the retailers tend to stock a lower level of inventory in the sharing case because the retailer with a shortage has a chance to buy the competitor's excess inventory at a relatively low transfer price

(a) Effect of t on Inventory (b) Effect of t on Service Quality

Figure 5. Effect of t under Endogenous Inventory

Note: These plots are illustrated with $g(x) = 1$ for $x \in [0,1]$ and $g_{12}(x, y) = g(x)g(y)$. Moreover, $p = 1$, $\bar{c} = 0.5$, $k = 1$, $\beta = 1$, and $\delta = 0.03$ in both (a) and (b); $c = 0.3$ in (a) and $c = 0.41$ in (b).

Next, we will compare the retailers' equilibrium service quality and profits in the sharing and no-sharing cases. The comparisons are analytically intractable to characterize for general joint distributions of random demands. However, we can study the special case where the uncertain demand factors ε_1 and ε_2 are independent and follow the same uniform distribution on $[0,1]$; this allows us to analytically characterize the comparison of service quality and profit for any given transfer price. Hereafter, our analysis will assume a uniform distribution. We further define $\widetilde{\Delta}(t) = \tilde{\pi}_i(t) - \tilde{\pi}_i^N$ as the profit difference between the sharing and no-sharing cases and present the comparison in Proposition 4.

Proposition 4. *Suppose that ε_1 and ε_2 are independent and follow the same uniform distribution on* $[0,1]$. *When the retailers endogenously choose their inventory levels, there exist $\tilde{t}_s$ and $\tilde{\beta}(t)$ such that:*

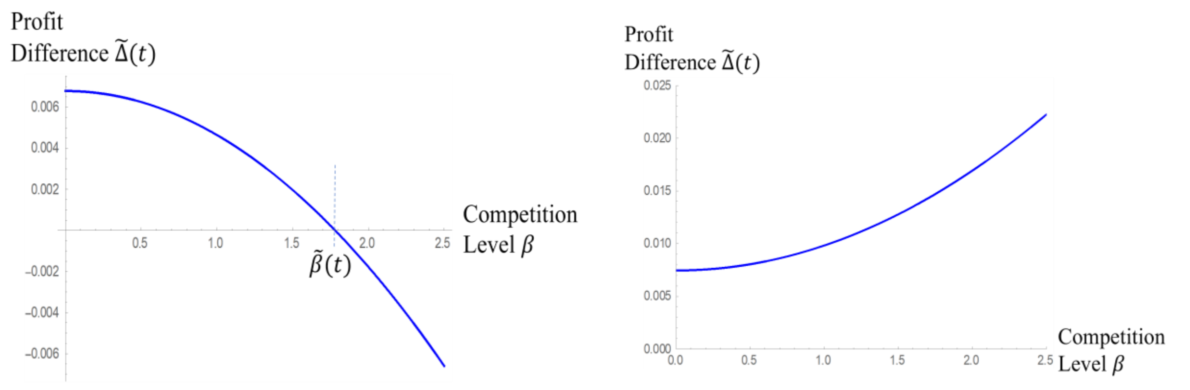
(a) When $\tilde{t}_s < t \le \bar{c} - \delta$, $\tilde{s}(t) < \tilde{s}^N$; and when $t < \tilde{t}_s$, $\tilde{s}(t) > \tilde{s}^N$.

(b) When $\beta > \tilde{\beta}(t)$ and $t < \tilde{t}_s$, each retailer's profit is lower in the sharing case, i.e., $\tilde{\Delta}(t) < 0$; otherwise, each retailer's profit is higher in the sharing case, i.e., $\tilde{\Delta}(t) \ge 0$.[5]

(c) Moreover, when $t < \tilde{t}_s$, $\tilde{\Delta}(t)$ decreases in β; when $t \ge \tilde{t}_s$, $\tilde{\Delta}(t) \ge 0$ and increases in β.

Recall (from Lemma 5) that the retailers' service quality in the sharing case (i.e., $\tilde{s}(t)$) is non-monotone in the transfer price t under the endogenous inventory case, which contrasts the exogenous inventory case. However, similar to the exogenous inventory case, we compare service quality in the sharing and no-sharing cases under endogenous inventory; we find that the service quality is lower in the sharing case when the transfer price is high and is higher otherwise. This result is depicted in Figure 5(b). Moreover, in Proposition 4(a), the condition $\tilde{t}_s < t \le \bar{c} - \delta$ is valid if and only if the marginal cost c is low, i.e., $c \le \tilde{c} = \frac{3r^2 - 5r\delta + 2\delta\sqrt{r\delta}}{3r - 3\delta}$. That is, if the marginal cost c is high enough, then $\tilde{s}(t) > \tilde{s}^N$ will hold for any transfer price $t \in [0, \bar{c} - \delta]$.

[5] Note that in Proposition 4, we define $\tilde{\beta}(t) = \bar{\beta}$ when $t \ge \tilde{t}_s$.



(a) Effect of β on Profit Difference when $t < \tilde{t}_s$ (b) Effect of β on Profit Difference when $t > \tilde{t}_s$

Figure 6. Effect of β under Endogenous Inventory

Note: These plots are illustrated with $g(x) = 1$ for $x \in [0,1]$ and $g_{12}(x, y) = g(x)g(y)$. Moreover, $p = 1$, $c = 0.3, \bar{c} = 0.5, \delta = 0.1, k = 1$ in both (a) and (b); $t = 0.05$ in (a) and $t = 0.35$ in (b).

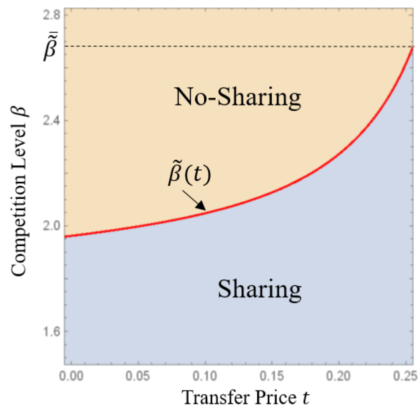


Figure 7. Equilibrium outcome under endogenous inventory

Note: This figure is illustrated with $g(x) = 1$ for $x \in [0,1]$ and $g_{12}(x, y) = g(x)g(y)$. Moreover, $p = 1$, $c = 0.46$, $\bar{c} = 0.5$, $k = 1$, and $\delta = 0.25$.

As Proposition 4 shows, similar to the exogenous inventory case, the retailers' profits in the sharing case can be higher or lower than those in the no-sharing case, depending on the level of competition and the transfer price. Moreover, similarly, when the transfer price is low (i.e., $t < \tilde{t}_s$), the profit difference $\hat{\Delta}(t)$ decreases in β and becomes negative when β is high; when the transfer price is high (i.e., $t \geq \tilde{t}_s$), the profit difference is always positive and increases in β (see Figure 6). But unlike the exogenous inventory case, this result is driven by the tradeoffs between the inventory-pooling effect and the *strategic* effect of inventory-sharing on the retailers' inventory and service decisions. According to Proposition 4, for a given transfer price t, sharing inventory is *not* profitable for the retailers when competition is fierce and transfer price is low (i.e., $\beta > \tilde{\beta}(t)$ *and* $t < \tilde{t}_s$), which is depicted in Figure 7. Note that the notation $\bar{\tilde{\beta}}$ in Figure 7 will be explained in Proposition 5 below.

Finally, we characterize the optimal transfer price in the sharing case and examine whether the retailers will share inventory at the optimal transfer price and whether the optimal transfer price can achieve coordination as in the centralized system. Proposition 5 summarizes the full equilibrium of the endogenous-inventory case.

Proposition 5. *Suppose that* ε_1 *and* ε_2 *are independent and follow the same uniform distribution on* $[0,1]$. *When the retailers endogenously choose their inventory levels:*

(a) There exists an optimal transfer price $\tilde{t}^* \in [0, \bar{c} - \delta]$ *that maximizes both retailers' profits in the sharing case.*

(b) There exist $\bar{\bar{\beta}}$ *and* $\tilde{c}$ *such that the retailers prefer not sharing inventory (even at the optimal transfer price* $\tilde{t}^*$*) if* $\beta > \bar{\bar{\beta}}$ *and* $c > \tilde{c}$ *and prefer sharing at the optimal transfer price* $\tilde{t}^*$*otherwise.*

(c) Moreover, if $\beta > 0$, *there does not exist any transfer price that allows inventory sharing to achieve coordination as in the centralized system.*

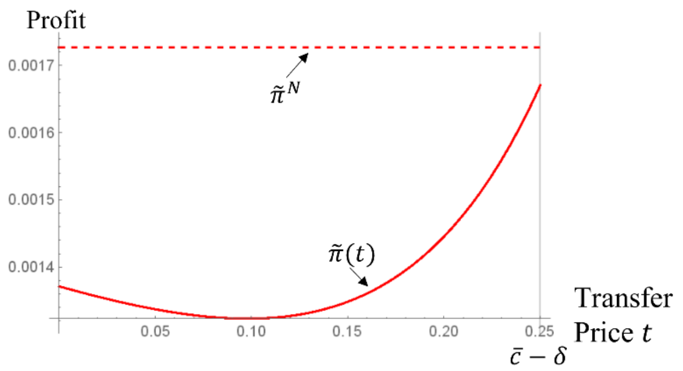


Figure 8. Profit under Endogenous Inventory when $\beta > \bar{\bar{\beta}}$ and $c > \tilde{c}$

Note: This figure is illustrated with $g(x) = 1$ for $x \in [0,1]$ and $g_{12}(x, y) = g(x)g(y)$. Moreover, $p = 1$, $c = 0.46, \bar{c} = 0.5, \delta = 0.25, k = 1,$ and $\beta = 2.8$.

Intuitively, one might expect that there always exists some optimal transfer price that makes the inventory-sharing agreement strictly beneficial to both retailers since inventory pooling can help mitigate demand uncertainty and may also sometimes soften service competition. This intuition holds true when the retailers do not compete with each other (i.e., $\beta = 0$) or when their pre-season inventory levels are exogenous (see Lemma 1 and Proposition 2). Specifically, without competition (i.e., $\beta = 0$), because of the risk pooling benefit, the retailers prefer to share inventory for any non-moot transfer price; with exogenous inventory, as shown in Section 4.3, the retailers always prefer to share inventory by choosing the maximum non-moot transfer price (i.e., $\hat{t}^* = \bar{c} - \delta$).

However, when the retailers can strategically choose their pre-season inventories, their profits in the sharing case do not necessarily increase in t (see Lemma 5), and thus the optimal transfer price could be strictly lower than $\bar{c} - \delta$. More importantly, even under the optimal transfer price, inventory sharing may make the retailers worse off. In other words, it is possible that sharing inventory will hurt the retailers for any non-moot transfer price t, including the optimal transfer price; under such circumstances, the retailers would rather not enter into an inventory-sharing agreement. This counterintuitive result is depicted in Figures 7 and 8, whose conditions are provided in Proposition 5. The intuition is as follows. An inventory-sharing agreement with a low transfer price will intensify service competition and is detrimental to the retailers, regardless of whether the inventory is exogenously given or endogenously dertermined. When retailers can strategically choose their inventories, an inventory-sharing agreement with a *high* transfer price may also be harmful because a high transfer price will induce each retailer to stock more pre-season inventories, which will not only increase the inventory cost but also potentially increase service competition. Thus, when competition is already intense (i.e., β is large) and the pre-order cost (c) is high, because of their strategic inventory and service responses, the retailers will not be able to benefit from inventory sharing, even though sharing inventory has a risk-pooling benefit ex post.

Moreover, similar to the exogenous inventory case, with service competition in the market, inventory sharing with any transfer price will not be able to coordinate the two retailers like in the centralized system. In a centralized system, the optimal service quality is zero; by contrast, when the two retailers are independent and $\beta > 0$, their service quality will be positive even with a sharing agreement. Since the service quality cannot be coordinated, coordination cannot be achieved regardless of whether inventory decisions can be coordinated. The results of Proposition 2 and Proposition 5 indicate that when $\beta > 0$, inventory sharing may help soften competition but cannot fully eliminate it to achieve coordination, regardless of whether inventory is exogenous or endogenous. This result differs from the findings in our benchmark case without competition (see Lemma 1) and also those in the existing literature with non-competitive retailers (e.g., Rudi et al. 2001, Hu et al. 2007).

Our results can provide some plausible explanations and managerial insights for competing retailers' inventory-sharing strategies. In practice, inventory sharing is common under some circumstances but not under others. First, our results show that a higher transfer price leads to higher pre-season inventories by the retailers. Second, our results indicate that the intensity of competition, the transfer price, and whether firms strategically adjust pre-season inventories based on their sharing agreement are important driving factors for the market outcome. In the automobile industry, we anecdotally observe that some car dealerships that are close by (in the same city) tend not to share inventories, whereas those that are farther away (e.g., in surrounding cities) tend to share inventories. This is consistent with our qualitative findings. One might consider the distance between dealerships as a measure of the level of direct competition (β) between them. Our results predict that if competition between the dealerships is not very intense (e.g., their locations are far from each other), then sharing inventory can be mutually beneficial to the dealerships, and we should observe more inventory-sharing. We acknowledge that the locations of and the distance between the dealerships may affect not only competition but also other factors, such as transfer costs between dealerships. Moreover, there may exist other factors that influence the retailers' inventory-sharing decisions. However, it is beyond the scope of this paper to explicitly model the firms' locations and other factors; we leave it for future research to explore such models to gain additional insights.

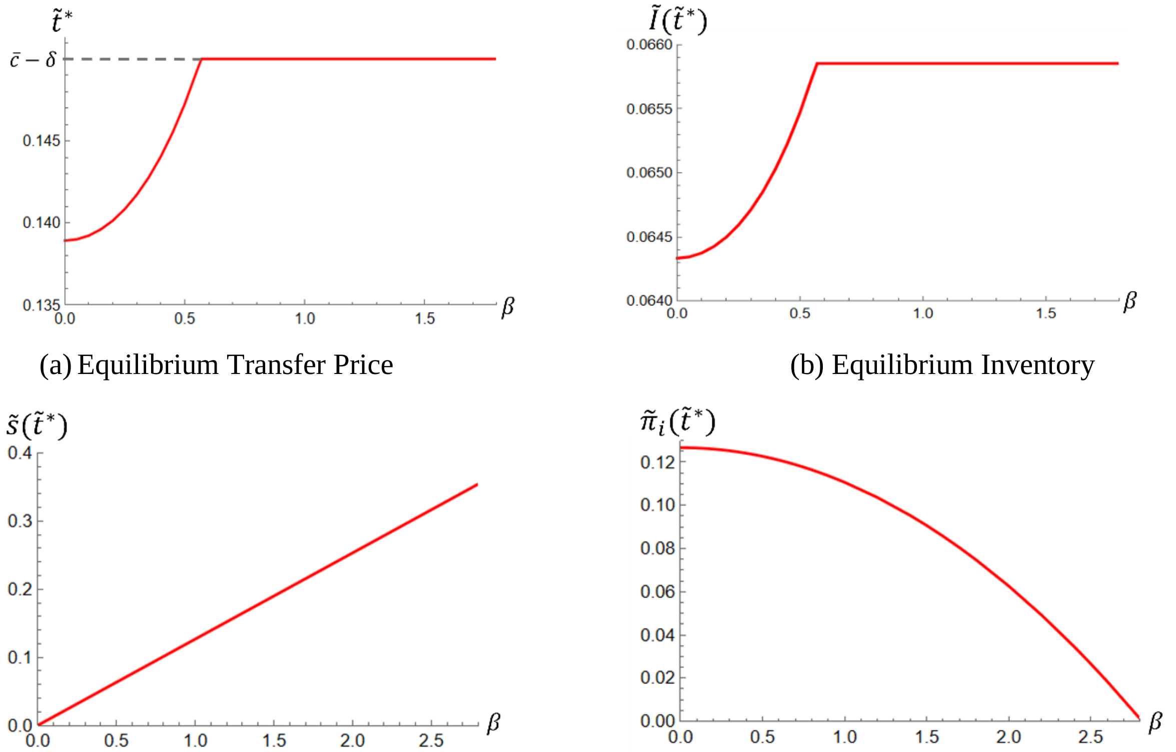
5.4. Numerical Analysis

Given that it is intractable to explicitly solve the optimal transfer price in the endogenous-inventory case, to study comparative statics, we resort to numerical analysis to solve $\tilde{t}^*$ and the corresponding full equilibrium outcome. In particular, we consider the case in which the uncertain demand factors ε_1 and ε_2 are independent and follow a uniform distribution on $[0,1]$. We focus on examining how the level of competition (i.e., β) affects the equilibrium. To do so, we fix other parameters and numerically solve the full equilibrium for different values of β. We have conducted the numerical analysis for many different parameter regions; here, we present the results in Figures 9 and 10, using a representative example with $k = 1$, $p = 1$, $c = 0.45$, and $\bar{c} = 0.5$.

Recall that in the sharing case with exogenous inventory, a higher transfer price is always more beneficial for the retailers, and thus they always prefer the maximum non-moot transfer price (i.e., $\hat{t}^* = \bar{c} - \delta$) in the equilibrium. By contrast, with endogenous inventory, the transfer price affects not only the retailers' service competition but also their pre-order inventories, and thus a higher transfer price will not necessarily benefit the retailers. As shown in Figure 9(a), the optimal transfer price $\tilde{t}^*$ can be lower than $\bar{c} - \delta$ when β is small, and it reaches the maximum non-moot level (i.e., $\tilde{t}^* = \bar{c} - \delta$) when β is large. Moreover, we also observe that the optimal transfer price $\tilde{t}^*$ weakly increases in β. This is because as β increases, service competition becomes more fierce, and the retailers would prefer a higher transfer price in order to alleviate service competition; and when β is high enough, the retailers will prefer the maximum non-moot transfer price.

Given that $t = \tilde{t}^*$, we illustrate each retailer's full equilibrium inventory ($\tilde{I}(\tilde{t}^*)$) and service quality ($\tilde{s}(\tilde{t}^*)$) in Figure 9 (b)-(c). As shown in Lemma 5, for any given transfer price t, $\tilde{I}(t)$ is independent of β and increases in t, and the service quality $\tilde{s}(t)$ increases in β and is non-monotone in t. Our numerical results indicate that the full equilibrium inventory and service quality, $\tilde{I}(\tilde{t}^*)$ and $\tilde{s}(\tilde{t}^*)$, both increase in β since the optimal price $\tilde{t}^*$ tends to increase in β.

Moreover, we have also analyzed the impact of shipping cost (δ) by fixing the other parameters. Through numerical studies, we observe that the optimal transfer price ($\tilde{t}^*$) and the full equilibrium inventory ($\tilde{I}(\tilde{t}^*)$) both decrease in δ (see Figure A.1 in Part F of the Online Appendix). As δ increases, the cost of getting the competitor's excess inventory becomes higher, the non-moot region of the transfer price shrinks, and thus the optimal transfer price becomes lower. Interestingly, the result that the full equilibrium inventory ($\tilde{I}(\tilde{t}^*)$) decreases in δ directly contrasts the previous Lemma 5, which shows that for a fixed t, retailers will stock more inventory as δ increases. On the one hand, a higher δ incentivizes the retailers to stock more pre-season inventory, given the ordering cost from the competitor increases. On the other hand, a higher δ also leads to a lower optimal transfer price, which tends to reduce the retailers' inventory levels since selling excess inventory is less profitable. Overall, the combination of these two opposing effects leads to the negative impact of δ on the equilibrium inventory.



(a) Equilibrium Transfer Price

(b) Equilibrium Inventory

(c) Equilibrium Service

(d) Equilibrium Profit

Figure 9. Numerical Analysis

Note: This figure is illustrated with $g(x) = 1$ for $x \in [0,1]$ and $g_{12}(x, y) = g(x)g(y)$. Moreover, $k = 1$, $p = 1$, $c = 0.45$, $\bar{c} = 0.5$, and $\delta = 0.35$.

Finally, the retailer's full equilibrium profit in the sharing case (i.e., $\tilde{\pi}_i(\tilde{t}^*)$) and the profit difference between sharing and non-sharing cases (i.e., $\tilde{\Delta}(\tilde{t}^*) = \tilde{\pi}_i(\tilde{t}^*) - \tilde{\pi}_i^N$) are illustrated in Figure 9(d) and Figure 10, respectively. Recall that for a given transfer price t, the retailer's profit in the sharing case (i.e., $\tilde{\pi}_i(t)$) decreases in β (see Lemma 5), and the profit difference (i.e., $\tilde{\Delta}(t)$) increases in β when t is high and decreases in β when t is low. Our numerical analysis indicates that at the optimal transfer price $\tilde{t}^*$, the retailer's equilibrium profit in the sharing case (i.e., $\tilde{\pi}_i(\tilde{t}^*)$) still decreases in β, and the profit difference (i.e., $\widetilde{\Delta}(\tilde{t}^*) = \tilde{\pi}_i(\tilde{t}^*) - \tilde{\pi}_i^N$) may decrease or increase in β.

Particularly, when δ is small, the optimal transfer price $\tilde{t}^*$ will be high (see Figure A.1 in the Online Appendix), and so $\widetilde{\Delta}(\tilde{t}^*)$ may increase in β. By contrast, when δ is large, $\tilde{t}^*$ will be low, and thus, $\widetilde{\Delta}(\tilde{t}^*)$ decreases in β. Moreover, $\widetilde{\Delta}(\tilde{t}^*)$ could be negative when β is large, which contrasts the result that inventory sharing at the optimal transfer price is always beneficial under exogenous inventories. The fundamental reason is that when retailers can strategically choose their inventories, inventory sharing with a low transfer price will intensify service competition, and sharing with a high transfer price will induce the retailers to stock more inventory, resulting in higher inventory costs and potentially intensifying service competition. Thus, when competition is already intense (i.e., β is large), it is likely that the retailers may not benefit from inventory sharing at any transfer price.

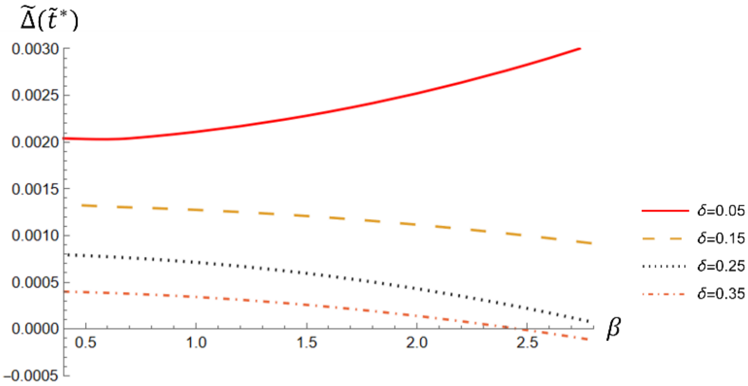


Figure 10. Equilibrium Profit Difference

Note: This figure is illustrated with $g(x) = 1$ for $x \in [0,1]$, $g_{12}(x, y) = g(x)g(y)$. Moreover, $k = 1$, $p = 1$, $c = 0.45$, and $\bar{c} = 0.5$.

6. Conclusion

Competitive retailers sometimes share product inventories to cope with uncertain demand. However, the tradeoffs of inventory sharing in a competitive market are not well understood in the existing literature. In this paper, we develop a game-theoretic model to study inventory sharing between two retailers that directly compete for demands through their provision of services. We examine the effects of inventory sharing on the retailers' optimal service quality, inventories, and profits. We first examine the scenario in which retailers have exogenous pre-season inventory levels. Then, we study the scenario in which retailers can strategically adjust their inventories after their inventory-sharing decision.

We find that though inventory sharing can benefit retailers through risk pooling, it may intensify or soften service competition, depending on the transfer price and the level of market competition. With exogenous inventory levels, when the transfer price is low, inventory sharing will *intensify* service competition, leading to high service quality and making both retailers worse off if the increased service cost outweighs the benefit of risk pooling. When the transfer price is high, inventory sharing will *soften* competition, and both retailers will choose low service quality and become better off with a non-moot inventory-sharing agreement. Moreover, the retailers always prefer inventory sharing at the maximum non-moot transfer price.

With endogenous pre-season inventories, the effect of the transfer price on equilibrium service quality becomes non-monotone because the transfer price not only directly affects service quality but also affects the retailers' inventory decisions, which will also alter the retailers' optimal service quality. We show that if the retailers strategically choose their inventory levels, inventory sharing will lead to higher inventory levels when the transfer price is high and lower inventory levels otherwise. Interestingly, different from the exogenous-inventory case, both retailers will become worse off with inventory-sharing for any non-moot transfer price when market competition and the pre-order cost are high. This is because an inventory-sharing agreement with a low transfer price will intensify service competition, whereas an inventory-sharing agreement with a high transfer price will induce each retailer to stock more pre-season inventories, which will not only increase the inventory cost but also potentially increase service competition. Under such circumstances, the retailers will choose not to enter into an inventory-sharing agreement. Furthermore, we find that with service competition in the market, there does not exist any transfer price that allows inventory sharing to achieve coordination like a centralized system because inventory sharing cannot fully eliminate service competition regardless of whether inventory is exogenous or endogenous.

Our results offer some useful managerial insights. In many market industries, inventory sharing among retailers is usually an effective mechanism to cope with the inventory-mismatch problem caused by uncertain demand and to help soften competition. However, our study shows that in a market where both service quality and inventory are the retailers' strategic decisions, even though inventory sharing can alleviate the inventory-mismatch problem, it may increase or decrease pre-season inventory levels, intensify or soften service competition, and may or may not improve the retailers' profits. The results depend on the terms of the inventory-sharing agreement (e.g., the transfer price), the level of market competition, and the pre-order unit cost. The retailers' optimal service quality and pre-season inventory decisions are significantly affected by the inventory-sharing agreement. Because of their strategic responses to anticipated sharing, under some conditions, the retailers may have no net benefit from an inventory-sharing agreement. Even in a situation with exogenous inventory levels, inventory sharing at a low transfer price can intensify competition, inducing both retailers to spend more on services to compete for customers. In that situation, the retailers can benefit from inventory sharing only if they reach an agreement at a relatively high transfer price.

We conclude by discussing some possible directions for future research. First, in our setting, the manufacturer plays a non-strategic role. It may be useful, though mathematically even more challenging, for future research to study inventory sharing among retailers in a supply-chain setting to investigate how upstream manufacturers/suppliers are affected by the downstream firms' inventory-sharing agreement. Second, we have focused on service competition. It may also be interesting to study a more general competitive setting in which retailers compete on both service and price to understand how inventory sharing affects the retailers' joint pricing and service decisions. Again, such exploration will be analytically challenging and may require simplifications of the model frameworks. Third, our model has focused on two symmetric retailers. Future research may consider asymmetric retailers with different cost structures or market demands and examine how the results are affected by the difference between the retailers. These research directions can potentially generate additional insights and deserve separate studies that are beyond the scope of the current paper. Lastly, our research offers some testable hypotheses about the inventory-sharing marketplace. For example, our results predict that firms are more likely to share inventories with competitors when their customers are less sensitive to services or the cost for pre-season inventories is not too high. When firms share inventories, a higher transfer price tends to lead to higher pre-season inventories and may lead to higher or lower service levels in the market. We hope our research motivates empirical researchers with suitable data to test our predictions in the future.

Acknowledgement

Both authors contributed equally. X. Guo has received research support from the Research Grants Council of Hong Kong (RGC Reference No. 15501820). The authors thank the MSOM review team for helpful suggestions that have significantly improved the manuscript. They also thank participants at the POMS Annual Conference, CSAMSE Annual Conference, POMS-HK Annual Conference, and seminar participants at Fudan University for helpful comments on earlier versions of the manuscript.

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