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# Analysis of Metro Network Performance From a Complex Network Perspective

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**Abstract**

In this paper, the performance of metro networks is studied from a network science perspective. We review the structural efficiency of metro networks on the basis of a passenger's intuitive routing strategy that optimizes the number of transfers and the distance travelled. A new node centrality measure, called *node occupying probability*, is introduced for evaluating the level of utilization of stations. The robustness of a metro network is analyzed under several attack scenarios. Six metro networks (Beijing, London, Paris, Hong Kong, Tokyo and New York) are compared in terms of the node occupying probability and a few other performance parameters. Simulation results show that the New York metro system has better topological efficiency, the Tokyo and Hong Kong systems are the most robust under *random attack* and *target attack*, respectively.

*Keywords:* Metro network, complex network, node occupying probability, robustness

## 1. Introduction

Rapid transit systems, often called metro or subway systems, are transportation systems carrying the largest volume of commuters in major cities, and their reliability, efficiency, safety, level of comfort, convenience and accessibility are often perceived by travellers and local commuters as indicators of the quality of public transportation of the cities [1]. Major cities, due to increasing traffic demands and ever-extending city coverage, are continuously expanding their metro networks, resulting in complex subway systems that possess high station densities and intricate inter-station couplings [2]. Design and scheduling of metro systems to optimize performance have become important considerations in the development of public transportation systems. Moreover, the study of networks, under the notion of *complex networks*, has recently become popular due to the intriguing discovery of a number of universal properties in various physical and man-made networks [3, 4] as well as promising applications that have been developed in various practical fields such as communications, power systems, finance, disease control, etc. [5, 6, 7, 8, 9, 10]. Results from complex networks research are highly relevant to the study of transportation systems, especially in the provision of appropriate analytical tools for characterizing the structure of metro systems which are practical forms of networks and for understanding the operations of a complex system such as metro networks [11, 12]. Furthermore, the huge investment in this transportation infrastructure and the impact to the public certainly justify a more thorough investigation of the factors affecting performance, thus allowing a more informed planning and design for future development.

The cross-disciplinary study of subway systems from a perspective of complex networks is still relatively rare. The earliest work reported by Latora and Marchiori [13] showed that the Boston subway network exhibited a small-world property and introduced the concept of network efficiency to give useful insights into the general characteristics of real transportation networks. In the work of Derrible and Kennedy [14], most metros were found to exhibit scale-free and small-world structure. Also Angeloudis and Fisk [2] studied 20 subway networks using a 'toy' model and showed that these networks, with high connectivity and low maximum vertex degrees, provide robustness to random attacks. In the work of Lee *et al.* [15], the statistical properties of the Metropolitan Seoul subway network were analyzed, taking the passenger flow as the weight of the edge and arriving at a power-law weight distribution.

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Furthermore, Yang *et al.* [5] combined node degree and betweenness to assess the node importance, and showed that a scale-free transit network exhibited a relatively high fault tolerance to random failure but a relatively low degree of connection reliability against malicious attack. Previous works mainly apply network theory to metro systems without considering their inherent characteristics to evaluate the network topological performance. Thus, some results may not be consistent with practice. Metro networks have properties that distinguish them from other networks. For instance, metro networks are composed of one-dimensional lines of stations (nodes) and transfer points where different lines overlap to facilitate switching between lines.

Of practical relevance in the study of metro systems are *transportation efficiency* and *fault tolerance.* Transportation efficiency is a parameter determined by the system's inherent structural property that allows passengers to move from one station to another with minimum effort. Fault tolerance of a metro system is an indicator of the ability of the system to maintain its essential function when some parts of the system fail to operate normally due to component failures or intentional malicious attacks.

*Network efficiency*, denoted by $E$, was introduced by Latora [13] for evaluating the topological transportation efficiency of a metro system. In Latora's definition, $E$ is simply inversely proportional to the sum of all *shortest paths* (SP). This definition is, however, not fully consistent with the subway operation, where passengers do not necessarily choose an SP if it involves an extra number of transfers. Also, segments of some lines overlap, affecting the computation of transportation efficiency. Moreover, there has been considerable amount of prior study in evaluating the robustness or fault tolerance of a complex network [14, 16, 17, 18, 19, 20]. The node centrality has been evaluated by using the notation of degree centrality, betweenness centrality, closeness centrality, eigenvector centrality, and so on. In general, each method can reveal a particular aspect of node centrality and may be suitable for one application [21]. In assessing the robustness of a metro system suffering from random failure and malicious attack, some nodes are removed either randomly or based on node centrality. When considering the station centrality of the whole system, moreover, the traffic properties must be taken into consideration in order to describe station centrality more precisely. The effects of variations of some parameters, such as average degree, average shortest path and efficiency, on the network performance are studied.

In this paper, we assume a more realistic passenger's routing strategy for establishing the possible routes taken by passengers from one station to another. Based on this routing strategy, we re-define *network efficiency* taking overlapping stations into consideration. We also propose a centrality measure, called *node occupying probability* (NOP), for evaluating realistically the level of utilization of stations. Then, the network robustness is studied by detailed simulations. Two indices, namely *disabled route ratio* (DRR) and *cost adjustment* (CA), are defined for assessing the influence of failed nodes on traffic performance. Six metro networks are studied. Simulation results show that the New York network has better topological efficiency, the Tokyo and the Hong Kong systems are the most robust under random failure and target attack, respectively.

The remainder of this paper is organized as follows. In the next section, based on a simple yet realistic passenger's routing strategy, called *passenger intuitive logic* (PIL), network efficiency is re-defined. In Section 3, the robustness of metro network is studied. In Section 4, the metro systems in a few major cities are analyzed. Finally, we summarize our main findings in Section 5.

## 2. Topological efficiency of metro systems

### *2.1. Topological properties*

A complex network with $N$ nodes can be represented as a graph $G = (V, E)$, where $V = \{v_1, v_2, ..., v_N\}$ denotes the set of nodes, and $E = \{e_1, e_2, ..., e_k\}$ denotes the set of links. A graph $G$ can be fully described by an adjacency matrix $A$, which is an $N \times N$ matrix whose entry $a_{ij}(i, j = 1, ..., N)$ equals to 1 if there exists a link between nodes $i$ and $j$, and zero otherwise. In this paper, a node is a subway station. If two stations are directly connected by a track, they are connected by a link.

### *2.2. Passenger's routing algorithm*

The route taken by a passenger moving from one station to another affects the analysis of network performance. In particular, the topological efficiency of the network is dependent upon the choice of routes by passengers. In deriving a realistic passenger's routing strategy, we make the following assumptions:

**Assumption 1:** Passengers do not have full knowledge of the metro system. They do not know the exact time taken to travel from a starting point to the destination including the time for necessary transfers. In other words, a passenger determines his route according to what he perceives as the "best" route.

**Assumption 2:** Passengers are cost-minimizing decision makers. They will choose the routes that they perceive as incurring the minimum cost.

**Assumption 3:** The impact of in-vehicle congestion is negligible, i.e., trains are not supposed to stop in the middle of their journey between stations.

In network transportation, a number of routing algorithms have been studied, such as the shortest path algorithm, the minimized degree algorithm, the traffic awareness algorithm [22], the efficient routing algorithm [23], the local routing algorithm [24], the next nearest neighbor strategy [25], the hybrid routing algorithm [26], and the local routing strategy [27]. These routing algorithms are mainly applied in communication networks, in which routes are decided by the system manager aiming to reduce congestion and improve the data transmission efficiency [28]. In road traffic, however, the choice of the route is the core part of traffic assignment [29]. Optimal design of any given system assumes the adoption of one routing strategy by passengers which defines the way a route is chosen between an origin-destination (OD) pair under a specific criterion, such as the C-Logit [30], path size logit [31], generalized nested logit [32], and cross nested logit [33]. All these strategies assume that the "perceived" travel cost of a route $C_{ij}^m$ for a passenger is expressed as a random variable consisting of a deterministic component $c_{ij}^m$ and an additive random error term $\varepsilon_{ij}^m$. Here, $c_{ij}^m$ is the travel time including in-vehicle time and transfer overhead, and $\varepsilon_{ij}^m$ is the perception error. The probability of a given path to be chosen can be represented as the probability that $C_{ij}^m$ is lower than all other routes' perceived cost, i.e.,

$$p_{ij}^n = P(C_{ij}^n \leq C_{ij}^m, m \neq n) \tag{1}$$

Most existing routing algorithms are used mainly for road traffic analysis, and are primarily focused on drivers' route choice and are not fully consistent with rail traffic [34]. In reality, passengers do not get perfect knowledge of in-vehicle time and transfer overhead, and the information passengers can obtain directly from the map is the number of stations they need to travel and the number of times they need to switch from one line to another. Passengers have varying levels of perception of the route length (station number) and transfer overhead. Furthermore, subway networks have a special structure and operational mode. First, a subway network consists primarily of one-dimensional lines along which no traffic congestion is expected. Also, routing is performed in a distributive manner, i.e., passengers choose their own routes.

### *2.3. Passengers' intuitive routing*

In our analysis, we use a simple and yet realistic routing algorithm, called *passenger intuitive logic* (PIL). Passengers' intuitions include minimizing the number of stations they need to travel through as well as the amount of transfer overhead. Thus, passengers would intuitively take a combined *shortest path* (SP) and *minimum transfer path* (MTP) approach. Here, SP corresponds to a minimum number of stations and hence minimum in-train time, but it may incur extra transfer overhead. On the other hand, MTP corresponds to the route that has the least number of transfer times, but it may not guarantee the shortest in-train time. Thus, a routing strategy based on passengers' intuition (PIL) can be conceived and represented by the following steps:

Step 1: Obtain SP $s_{ij}^{k_1}\{k_1 = 1, 2, \cdots, m_1\}$ and MTP series $m_{ij}^{k_2}\{k_2 = 1, 2, \cdots, m_2\}$ between stations $i$ and $j$ (the OD pair), where $m_1$ and $m_2$ are the number of SP and MTP connecting the OD pair, respectively.

Step 2: Let $L_s$ denote the length of SP and $L_m$ denote the length of MTP. Thus, $\varepsilon = L_m - L_s$ is the length difference between the two routes. Also, let $C_s$ denote the minimum transfer count of SP, and $C_m$ denote the transfer count of MTP. Then, $\gamma = C_s - C_m$ is the transfer count difference. Here, two assumptions are made. (1) When $\varepsilon \geq \lambda$, passengers will not take MTP into consideration, where $\lambda$ is the route length divergence threshold. (2) When $\gamma \geq \xi$, passengers will not take SP into consideration, where $\xi$ is the route transfer count divergence threshold. In this paper, $\lambda = 7$ and $\xi = 3$. Intuitively, more passengers prefer the path with fewer transfer counts. Thus, the choice of the type of routing path can be determined by the following empirical probabilities:

$$P_{\mathrm{MTP}} = \left(1 - \frac{\varepsilon^2}{\lambda^2}\right)^{\frac{1}{2}} \left(1 - \frac{(\gamma - \xi)^2}{\xi^2}\right) \text{ where } \varepsilon \in [0, \lambda], \gamma \in [0, \xi]; \text{and } P_{\mathrm{SP}} = 1 - P_{\mathrm{MTP}} \tag{2}$$

where $P_{\mathrm{MTP}}$ is the probability of taking a minimum transfer path, and $P_{\mathrm{SP}}$ is the probability of taking a shortest path. We can see from this empirical equation that $P_{\mathrm{MTP}}$ increases with the increase of $\gamma$ and decrease of $\varepsilon$, as illustrated in Fig. 1. If SP is chosen, go to step 3 for determining the specific path. Otherwise, go to step 4.

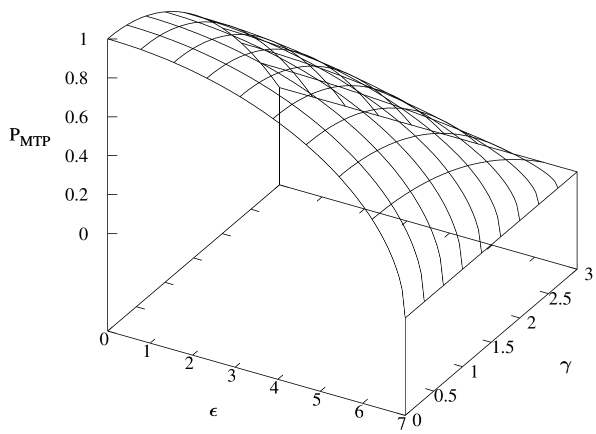



Figure 1: Intuitive routing decision probability $P_{\mathrm{MTP}}$ (probability of choosing the route with a minimum transfer count) versus transfer count difference $\gamma$ and path length difference $\epsilon$, with route transfer count divergence threshold $\xi = 3$ and route length divergence threshold $\lambda = 7$.

Step 3: Choose one route from $s_{ij}^{k_1}$ with the following probability:

$$f(x_i) = \begin{cases} \frac{1}{n_{\min}} & \text{where } R_{x_i} = n_{\min} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where $R_{x_i}$ denotes the number of transfer times of path $x_i$ and $n_{\min}$ denotes the number of paths with minimum transfer time.

Step 4: Choose one route randomly from $m_{ij}^{k2}$.

### 2.4. Metro topological efficiency

Efficiency $E$, introduced by Latora and Marchiori [13], is a measure of effectiveness of information exchange over the network. Denoted as $\epsilon_{ij}$, the *efficiency of transfer* from nodes $i$ to $j$ is taken as being inversely proportional to the shortest path length $d_{ij}$, i.e.,

$$\epsilon_{ij} = \frac{1}{d_{ij}} \quad \forall i, j \tag{4}$$

and the *network efficiency* $E$ is defined as

$$E(G) = \frac{1}{N(N-1)} \sum_{i \neq j} \epsilon_{ij} = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{d_{ij}} \tag{5}$$

This network efficiency measures how fast a piece of information can be transported through a network. However, it is not fully consistent with the metro system in two aspects. First, as analyzed before, passengers do not only focus on the shortest path but also the minimum transfer overhead. Second, when segments of some lines overlap, the transportation efficiency will be altered. To overcome these inconsistencies, we propose a new measure, namely, *metro topological efficiency* (MTE). Specifically, if node $u$ and its neighbor node $v$ are connected by multiple edges, we scale the link connecting the two nodes by a factor $w_{uv}$

$$w_{uv} = \frac{1}{n} \tag{6}$$

where $n$ is the number of edges connecting the stations. These scaled links are used to compute the "scaled" shortest path length $r_{ij}$ Then, MTE is defined as

$$\mathrm{MTE} = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{r_{ij}} \tag{7}$$

where $r_{ij}$ is the length of the scaled path selected by PIL.

## 3. Metro system robustness analysis

### 3.1. Station centrality evaluation

A fundamental problem in network science is to evaluate the relative importance of the role a node plays in a network, and this helps understand the structural characteristic of the network. A number of measures have been proposed for this purpose, such as degree centrality [35], betweenness centrality [36], closeness centrality, node occupying probability [37] and so on. Degree centrality describes a node's importance in a local scale, while betweenness and closeness can reflect a node's importance in a global scale. However, they are all based on the shortest path routing, which ignores the inherent characteristics of subway networks. Here, we propose a parameter called *node occupying probability* (NOP), denoted as $I(i)$, to describe the importance of subway station $i$:

$$I(i) = \frac{\sum\limits_{u \subset S} \sum\limits_{w \subset S} \rho_{uw}(i)}{N(N-1)} u \neq w \tag{8}$$

where $S$ is the set of nodes, $N$ is the total number of nodes, and $\rho_{uw}(i) = 1$ if the path from nodes $u$ to $v$ passes through node $i$ under PIL; otherwise $\rho_{uw}(i) = 0$. NOP can directly reflect how busy a station is and the value of it is proportional to the number of routes that pass through this station, thus indicating the influence of this station to the network traffic.

### 3.2. Robustness Assessment

The subway network is an important infrastructure in any modern city, and its resilience is crucial for maintaining the essential transportation function in the events of component failures and malicious attacks. There have been a number of network robustness studies over the past few years [14, 16, 17, 18, 19, 20]. The structural robustness is especially relevant to establishing the reliability of the network [38], as well as in other applications [39, 40]. The relationship between topological structure and robustness is thus important in implementing safety management and planning.

#### 3.2.1. Attack model

Subway accidents may be caused by natural malfunctionings or intentional malicious attacks [41]. Because of the uncertainly of these causes, we classify failures into two categories, namely, *random failure* (RF) and *target attack* (TA). For implementing RFs, we randomly select some nodes and delete them from the network. Moreover, for implementing TAs, and for comparison purposes, we delete nodes that have the highest NOP and compare the results with deleting nodes that have the highest betweenness.

#### 3.2.2. Performance indicators

In previous studies of network robustness, several indicators were used to evaluate the network performance in the events of failures, e.g., degree variation, characteristic path variation, clustering coefficient variation, network efficiency variation, and so on. Our purpose in this study, however, is to unfold the relationship between the network topological structure and the subway network transportation performance. One aspect is the impact of attacks on the subway performance leading to the removal of all routes connecting an OD pair and the removal of the lowest-cost route. Thus, we use two parameters to evaluate the subway robustness, namely, *disabled route ratio* (DRR) and *cost adjustment* (CA). DRR is defined as the ratio of the number of disabled routes when some nodes are removed from the network to the total number of possible routes, i.e.,

$$\mathrm{DRR} = \frac{N_d}{N(N-1)} \tag{9}$$

where $N_d$ is the number of disconnected OD pairs. Also, CA is defined as the total cost adjustment, i.e.,

$$\mathrm{CA} = \frac{\sum c_{ij}^f}{\sum c_{ij}} \tag{10}$$

where $c_{ij}^f$ is the cost of the route between nodes $i$ and $j$ after some nodes are attacked (deleted), and $c_{ij}$ is the cost of the route between nodes $i$ and $j$ before the attack. All pairs of nodes $i$ and $j$ are considered as long as there is still at least one route between nodes $i$ and $j$ after the removal of some nodes. Thus, CA effectively reveals the extent of added cost when some nodes are removed. Also, we will analyze the critical fraction of removed vertices $f$ for dysfunctioning the whole network under RA and TA. In this paper, a network is said to be collapsed or dysfunctioned if over 90% of OD pairs are disconnected.

Table 1: Basic data of metro system scale (as of 2016)

| City | Number of stations | Number of lines |
|---|---|---|
| Beijing | 285 | 17 |
| London | 361 | 13 |
| Paris | 293 | 15 |
| Hong Kong | 86 | 10 |
| Tokyo | 207 | 13 |
| New York | 365 | 23 |

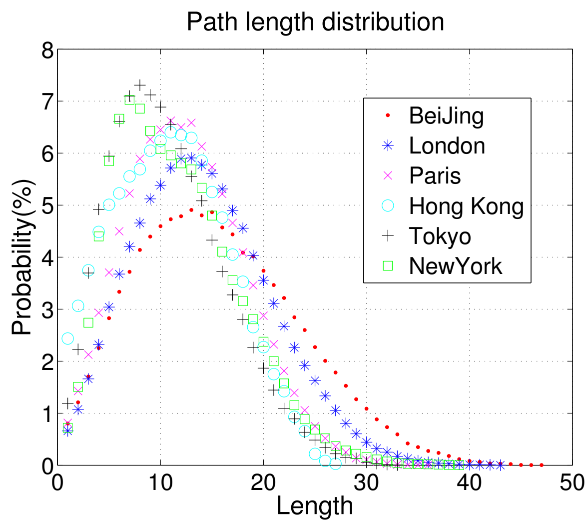



Figure 2: Path length distribution

## 4. Simulation results

The metro systems in Beijing, London, Paris, Hong Kong, Tokyo and New York are studied in this paper. Basic information of these metro systems are listed in Table 1. All data are obtained from the respective official websites.

### *4.1. Topological efficiency evaluation*

We first simulate each metro system to conduct exhaustive search of all paths between any two nodes. For each system, we perform simulations for 100 realizations of PIL routing. The average path length and the average transfer count are listed in Table 2. We see that the Tokyo metro has the minimum average path length and lowest average transfer count. Figure 2 shows the average path length distribution of each network, from which we observe that the path length basically follows a Gaussian distribution, i.e., $f(x) = a \exp\left(-\frac{x-b}{c}\right)^2$, and the value of the three parameters are also shown in Table 2. We should emphasize that the curves shown in Fig. 2 may deceptively look like a Poisson type distribution due to the absence of physical data in the negative x-axis.

The values of MTE, based on weighted edges and PIL routing as explained in Section 2.4, are computed. Table 3 lists the values of MTE for the metro networks under study. As expected, these metro systems have topological efficiency of below 1, i.e., less efficient that the fully connected network. This is obvious because the number of existing edges for every network are far fewer than the theoretical maximum number $Q_t = \frac{N(N-1)}{2}$. Using MTE, we can compare the topological efficiencies of different metro systems thereby identifying the topological structure that would better support traffic flow in the network. In this respect, the New York metro system is found to be more *topologically efficient* than the others. We should emphasize that topological efficiency does not provide a comprehensive assessment of efficiency which would necessitate consideration of multiple operational factors including dynamic load demands, in-train congestion time, transfer time, and allocation of resource such as

Table 2: Statistics and distribution parameters fitting $f(x) = a\exp(-\frac{x-b}{c})^2$

| City | Average path length | Average transfer count | a | b | c |
|---|---|---|---|---|---|
| Beijing | 15.60 | 1.68 | 4.881 | 14.22 | 12.17 |
| London | 14.11 | 1.89 | 5.903 | 13.24 | 9.825 |
| Paris | 12.45 | 1.85 | 6.736 | 11.84 | 8.652 |
| Hong Kong | 11.03 | 1.77 | 6.496 | 10.34 | 9.486 |
| Tokyo | **10.74** | **1.62** | 7.121 | 9.514 | 8.313 |
| New York | 11.56 | 2.59 | 6.642 | 10.17 | 8.934 |

Table 3: Metro topological efficiency (MTE) of metro networks

| City | Beijing | London | Paris | Hong Kong | Tokyo | New York |
|---|---|---|---|---|---|---|
| MTE | 0.0976 | 0.1175 | 0.1177 | 0.1519 | 0.1490 | **0.1906** |

frequency of train departure and carrying capacity. In this preliminary attempt of application-oriented study, we focus on network topology and its relevant parameters for practical assessment.

### *4.2. Robustness assessment*

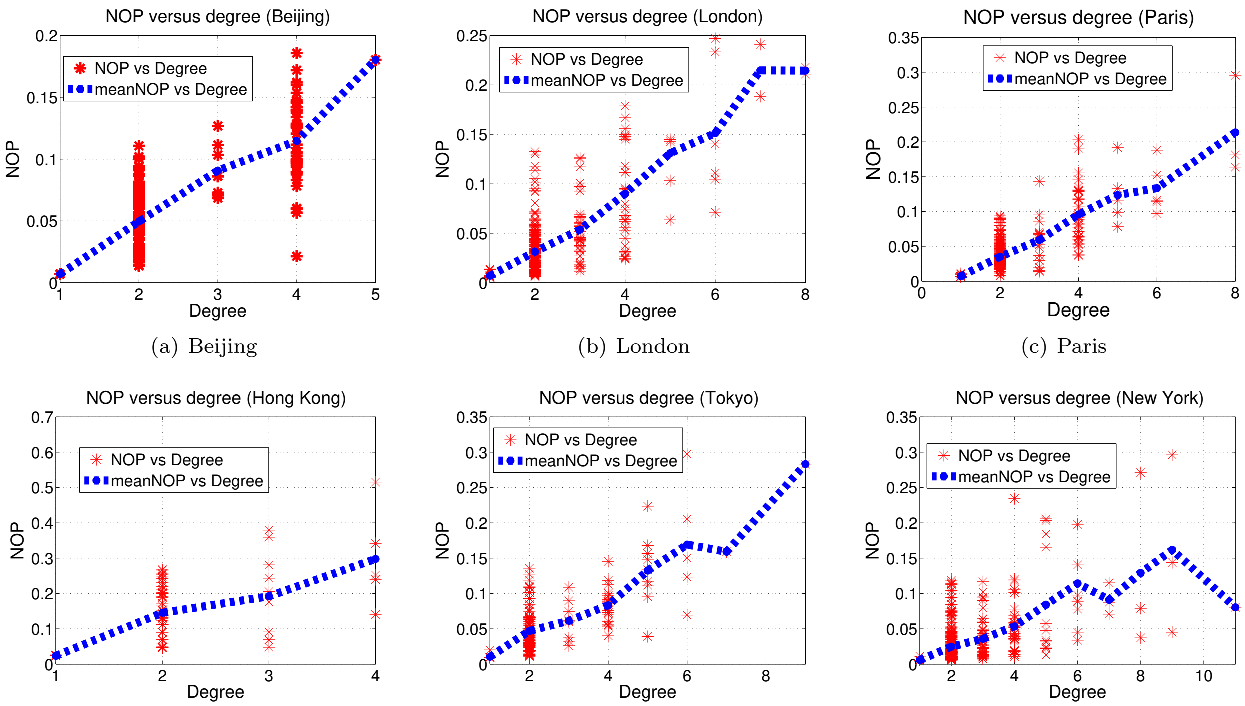
We now focus on the node centrality of each metro network. Figure 3 shows that NOP is related to the degree $D$ of the node. Nodes with a higher degree tend to have a higher mean value of NOP. We list the top 10 stations in Table 5 according to their values of NOP and betweenness. We observe that the station centrality is different using the two kinds of centrality measure. We will see that NOP is more suited for metro networks.

Table 4: Critical number $N_c$ and fraction $f$ of removed nodes (leading to 90% of OD pairs disconnected) under random attack (RA) and target attack (TA)

| City | Beijing | | London | | Paris | | Hong Kong | | Tokyo | | New York | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $N_c$ | $f$ | $N_c$ | $f$ | $N_c$ | $f$ | $N_c$ | $f$ | $N_c$ | $f$ | $N_c$ | $f$ |
| RA | 63 | 22.11% | 78 | 21.61% | 74 | 25.26% | 25 | 29.08% | **66** | **31.88%** | 105 | 28.77% |
| TA based on NOP | 20 | 7.02% | 25 | 6.93% | 19 | 6.48% | **16** | **18.60%** | 16 | 7.73% | 42 | 11.51% |
| TA based on betweenness | 22 | 7.72% | 25 | 6.93 % | 20 | 6.83% | **18** | **20.93%** | 18 | 8.0% | 48 | 13.15% |

To evaluate the network robustness, we remove vertices from the network 1) randomly; 2) in order of descending NOP; 3) in order of descending betweenness. Figures 4 and 5 show DRR and CA versus the number of attacks under RA and TA, respectively. Table 4 lists the critical fraction of removal under all three methods of attack. We see that in all cases, DRR increases with the number of nodes removed (attacked) rapidly at the beginning and saturates before the network collapses. Furthermore, there is a critical removal point where CA reaches its maximum value. We also observe that the metro networks are more robust under RA, and that TA based on the order of descending NOP can disrupt the metro network more rapidly than TA based on order of descending betweenness. Overall, the Tokyo metro network is most robust under RA (the critical removal fraction being 31.88% of $N$). Nevertheless, the Hong Kong metro is most robust against TA than the others (the critical removal fraction being 18.60% of $N$).

Our key message here is that the choice of appropriate measure for assessment of metro networks is crucial in evaluating a metro system's robustness. In particular, NOP is found to be more practical and highly indicative of the importance of a metro station, and its use in formulating attacks is expected to result in more severe damages to the system. Thus, evaluating robustness against removal of higher NOP nodes would more truly reflect the system's ability in maintaining its performance under possible intentional attacks.




(a) Beijing (b) London (c) Paris

(d) Hong Kong (e) Tokyo (f) New York

Figure 3: Node occupying probability (NOP) versus node degree

## 5. Conclusion

In this paper, the topological properties of metro networks are studied. Based on a realistic passenger's routing algorithm, the concept of network efficiency is re-defined for better consistency with metro systems. A new node centrality measure based on the utilization of the node is proposed to assess the node centrality. This new centrality measure provides more realistic assessment of centrality. The metro network robustness is studied under random attack and target attack. Performance of the network under attack is assessed in terms of disabled route ratio and cost adjustment. Comparison is made among a few selected metro systems (i.e., Beijing, London, Paris, Hong Kong, Tokyo and New York). It is shown that the New York metro network has better topological efficiency, the Tokyo and Hong Kong metro networks are most robust under random and target attacks, respectively. All networks under study possess better robustness under random attacks than target attacks. The method proposed in this paper can be used to provide a handy analytical basis on which to plan and design metro networks. For developed metro networks, the concept of node occupying probability is useful in assessing the relative importance of stations as well as vulnerability of the network under possible attack. Our work here aims to assess the topological efficiency of real metro networks. Moreover, finding better structure for metro networks requires consideration of multiple operational factors including dynamic load demands, in-train congestion time, transfer time, and allocation of resource such as frequency of train departure and carrying capacity. These dynamic operational factors will be considered in our future work.

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Table 5: Top 10 robust stations under target attacks (removal of nodes) according to order of descending NOP and betweenness

| Rank | Beijing | | London | | Paris | |
|---|---|---|---|---|---|---|
| | NOP | Betweenness | NOP | Betweenness | NOP | Betweenness |
| 1 | Jintailu | Xizhimen | Waterloo | Bank | Chatelet | Chatelet |
| 2 | Xizhimen | Chegongzhuang | King's Cross St. Pancras | Waterloo | Madeleine | Madeleine |
| 3 | Dawanglu | Chaoyangmen | Green Park | Green Park | Concorde | Gare De Lyon |
| 4 | Hujialou | Jintailu | Bank | King's Cross St. Pancras | Gare De Lyon | Pyramides |
| 5 | Liuliqiao | Shaoyaoju | Westminster | Westminster | Saint Lazare | Concorde |
| 6 | Junshibowuguan | Dawanglu | Baker Street | Euston | Republique | Invalides |
| 7 | Zhichunlu | Wangjingxi | Euston | Stratford | Montparnasse Bienvenue | Saint Lazare |
| 8 | Jiaomenxi | Baishiqiaonan | Stratford | Barker Street | Invalides | La Motto Picquet Grenelle |
| 9 | Songjiazhuang | Junshibowuguan | Finchley Road | Finchley Road | La Motto Picquet Grenelle | Montparnasse Bienvenue |
| 10 | Chegonzhuang | Zhichunlu | Bond Street | Willesden Junction | Pyramides | Republique |

| Rank | Hong Kong | | Tokyo | | New York | |
|---|---|---|---|---|---|---|
| | NOP | Betweenness | NOP | Betweenness | NOP | Betweenness |
| 1 | Kowloon Tong | Kowloon Tong | Kasuga | Kasuga | 125th Street | Seventh Avenue |
| 2 | Prince Edward | Tai Wai | Otemachi | Otemachi | Seventh Avenue | 125th Street |
| 3 | Tai Wai | Prince Edward | Shinjuku | Iidabashi | Atlantic Avenue Barclays Center | Atlantic Avenue barclays Center |
| 4 | Mei Foo | Lok Fu | Iidabashi | Shinjuku | Queens Plaza | Queens Plaza |
| 5 | Mongkok | Mei Foo | Ichigaya | Ichigaya | 59th Street Columbus Circle | 59th Street Columbus Circle |
| 6 | Lok Fu | Wong Tai Sin | Hibiya | Shinjuku San Chome | 36th Street | DeKalb Avenue |
| 7 | Shamshuipo | Mongkok | Kudanshita | Hibiya | DeKalb Avenue | Jackson Heights Roosevelt Avenue |
| 8 | Cheungshawan | Mongkok East | Nipponbashi | Kudanshita | Jackson Heights Roosevelt Avenue | 36th Street |
| 9 | Wong Tai Sin | Diamond Hill | Shinjuku San Chome | Akebonobashi | Prospect Avenue | Prospect Avenue |
| 10 | Yaumatei | Hung Hom | Monzen Nakacho | Monzen Nakacho | 86th Street | Elmhurst Avenue |


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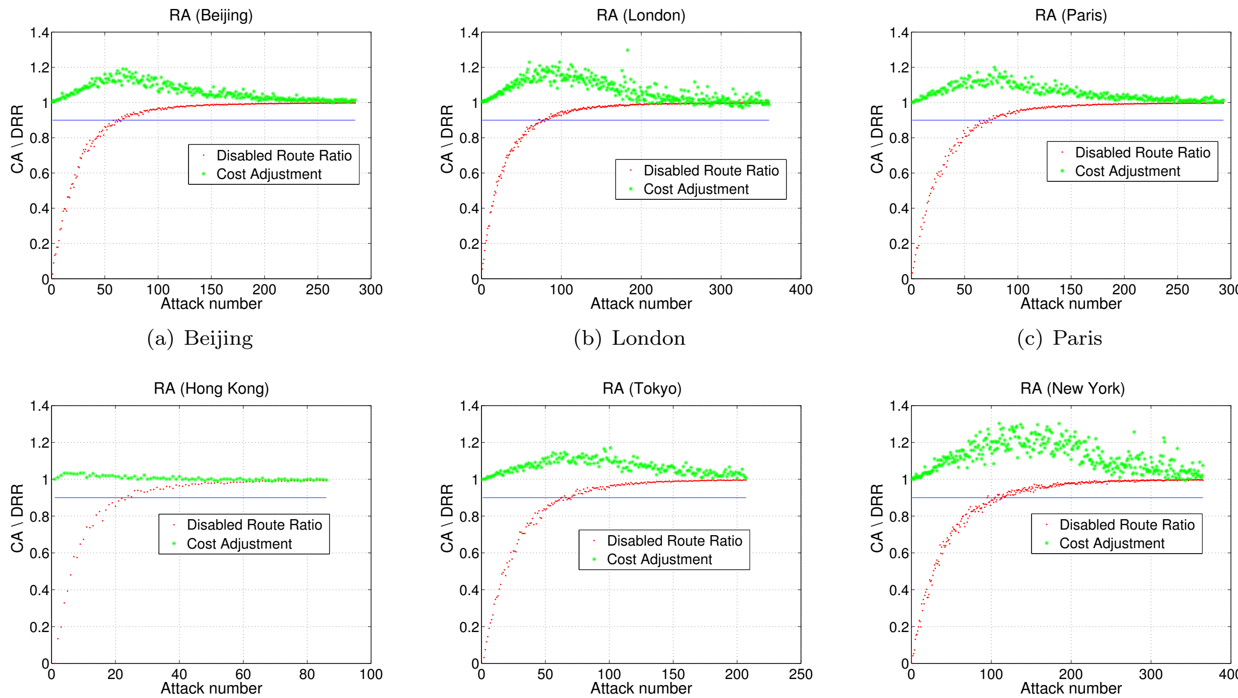


(a) Beijing (b) London (c) Paris

(d) Hong Kong (e) Tokyo (f) New York

Figure 4: DRR and CA versus the attack node number under random attack (RA)

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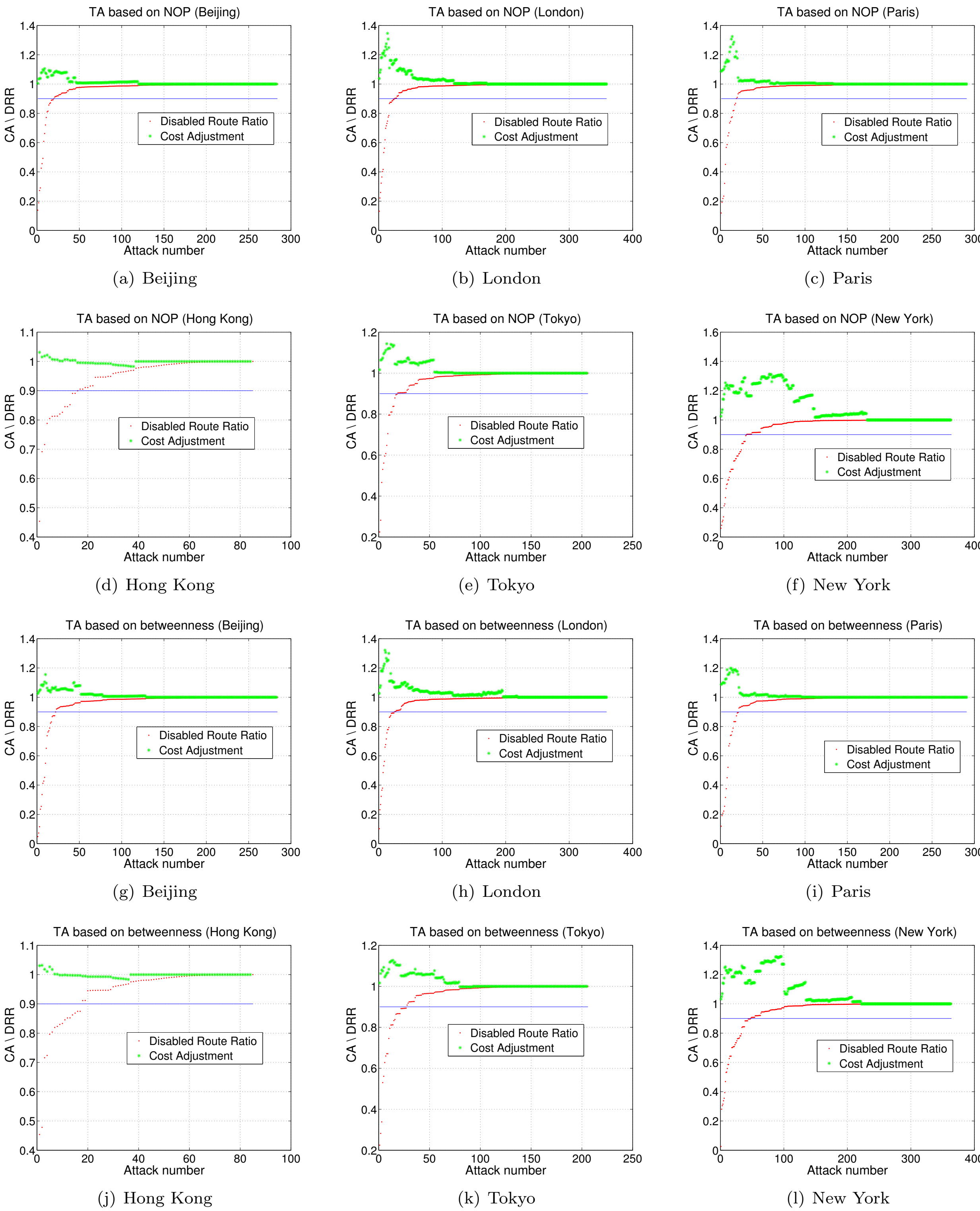


(a) Beijing (b) London (c) Paris
(d) Hong Kong (e) Tokyo (f) New York
(g) Beijing (h) London (i) Paris
(j) Hong Kong (k) Tokyo (l) New York

Figure 5: DRR and CA versus attack node number under (a)–(f) target attack (TA) according to the order of descending NOP; and (g)–(l) target attack according to the order of descending betweenness.

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