

1 **A multiscale constitutive model coupled with martensitic transformation kinetics for**
2 **micro-scaled plastic deformation of metastable metal foils**

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10
11 **Abstract**

12 The mechanical behavior of metastable austenitic foils at the size scale from micron to
13 submillimeter is strongly affected by the coupling between size effect and strain-induced
14 martensitic transformation (SIMT), which remains to be a pressing issue to be explored. In
15 this research, the focus is on developing a multiscale constitutive model to reveal the
16 mechanical behavior of metastable foils and more accurately predict the size effect on SIMT.
17 In tandem with this, the martensitic transformation and hardening behavior of SUS304 foils
18 with different thicknesses and grain sizes were explored. The results figured out that the
19 SIMT is promoted by the increase in grain size and foil thickness. Furthermore, the onset and
20 end of stages II of work-hardening behavior are advanced and the work-hardening rate in
21 stage II increases faster with increasing grain size and foil thickness. The SIMT kinetic model
22 was coupled with the intermediate mixture law and the iso-work hypothesis to identify the
23 stress-strain relationship of individual austenite and martensite at the surface and interior
24 layers, which was used to construct the multiscale constitutive model. The multiscale model
25 was developed based on the framework of the surface layer model and the intermediate
26 mixture law to represent the coupling between the size effect and the SIMT. Through finite
27 element simulation by using the proposed multiscale constitutive model, the dispersion
28 hardening mechanism in micro-scaled deformation of metastable austenitic foils caused by
29 the non-homogeneous plastic deformation at the interface between austenite and martensite
30 was revealed. The multiscale model was validated via the corroboration of finite element
31 simulation with experiments and therefore can provide a robust analysis of the micro-scaled
32 deformation behavior of metastable austenitic foils.

1 **Keywords:** Multiscale constitutive model; Strain-induced martensitic transformation; Size
2 effect; Micro-scaled plastic deformation; Metastable metal foils.
3

1 **1. Introduction**

2 The increasing demand for micro-scaled products in different industrial clusters presents
3 crucial and pressing challenges in terms of the reliability and effectiveness of microforming
4 technologies [1]. Due to the size effect, the fabrication of micro-scaled parts and components
5 is still one of the bottleneck issues in the present metal forming area [2]. Recently, the size
6 effect affected deformation behaviors have been extensively explored in terms of flow stress
7 [3], ductile fracture [4], plastic anisotropy [5], and subsequent yield [6]. However, the size
8 effect in the micro-scaled deformation of metastable metal foils is rarely reported due to the
9 occurrence of the phase transformation.

10 Due to the desirable mechanical properties and corrosion resistance, metastable
11 austenitic stainless steels (ASSs) have been widely used in making miniaturized parts and
12 components, such as fuel cell bipolar plate, micro medical equipment, micro heat exchangers,
13 and MEMS [7]. However, the mechanical properties of metastable ASSs at microscale exhibit
14 significant difference with those at macroscale due to size effect, especially when there are
15 only a few grains participating in deformation. Besides, the micro-scaled mechanical
16 behavior of metastable ASSs is significantly affected by the phase transformation. The
17 transformation from the initial face-centered cubic γ -austenite phase to the body-centered
18 cubic α' -martensite can be triggered by the plastic deformation of metastable ASSs within a
19 certain temperature range, which is commonly called strain-induced martensitic
20 transformation (SIMT) [8]. The SIMT provides desirable strength and high elongation of
21 metastable ASSs but exacerbates the difficulty of predicting mechanical behavior.

22 The influence of grain size at different scales on the SIMT kinetic behavior of
23 metastable ASSs has attracted some research efforts. Varma et al. [9] and Shrinivas et al. [10]
24 first investigated the effect of grain size ranging from 53 to 285 μm on the SIMT
25 phenomenon of 304 ASSs during room temperature (RT) rolling and tension. They argued
26 that the grain refinement promotes the formation of deformation-induced martensite during
27 rolling but inhibited the SIMT during tension. Gu et al. [11] found that the onset of twinning
28 is postponed and no twin-twin intersection occurs during the torsional deformation of a
29 coarse-grained 304 ASS with a grain size of $\sim 1000 \mu\text{m}$ compared with a fine-grained 304
30 ASS with a grain size of $\sim 20 \mu\text{m}$, causing the absence of the SIMT in coarse-grained
31 materials. Matsuoka et al. [12] discussed the thermal and mechanical stability of the austenite
32 phase in Fe–16%Cr–10%Ni steels with different grain sizes ranging from 1 to 20 μm . They
33 concluded that the grain refinement enhances the austenitic thermal stability by inhibiting the

1 **martensitic** multi-variant transformation, while the **martensitic** single-variant transformation
2 of the tensile-deformed specimen leads to the grain size independence of austenitic
3 mechanical stability. However, some literature has reported that the austenitic mechanical
4 stability **decreases** with increasing grain size. Kisko et al. [13] explored the impact of grain
5 **size** ranging from 0.5 to 18 μm on the SIMT rate and nucleation sites of 204Cu steels. They
6 elucidated that the SIMT rate **increases**, and the martensite nucleation sites **are** transferred
7 from grain boundaries and twins to ε -martensite and shear bands with increasing grain size.
8 Misra et al. [14] and Challa et al. [15] figured out that the deformation mechanism of 301LN
9 ASSs **shifts** from **transformation induced plasticity (TRIP)** to **twinning induced plasticity**
10 **(TWIP)** with the decrease of grain **size** ranging from **22 to 0.32 μm** , which indicated that the
11 austenitic mechanical stability is strengthened with the reduction of grain size. Therefore, the
12 grain size effect on the mechanical stability of **austenite in** metastable alloys continues to be
13 unclear.

14 On the other hand, the size effect on the mechanical behavior of metastable ASSs **has**
15 **been** extensively studied in the micro-scaled deformation process [16, 17]. Nevertheless, the
16 effect of SIMT is seldom considered. Mao et al. [18] discussed the effect of grain **size** ranging
17 from 1 to 35 μm on the hardening behavior of a Fe–24%Ni–0.3%C steel and attributed the
18 increase of work-hardening rate to the SIMT phenomenon. Naghizadeh et al. [19] revealed
19 that coarse grains enhance the TRIP effect by promoting SIMT in 304 ASSs when the grain
20 size is less than 50 μm . In contrast, the TRIP effect becomes less pronounced by increasing
21 grain size for the steel with a grain size greater than 50 μm .

22 The modeling of macro and micro deformation behaviors of multiphase materials has
23 been developed via three main methods including the homogenization technique like the
24 self-consistent method, the crystal plasticity finite element method, and the
25 phenomenological model based on the mixture law [20-22]. Compared with the first two
26 methods, the last one is convenient for practical application, and can accurately describe and
27 represent the mechanical response of individual constituents. However, most
28 phenomenological models are applicable to macroscale rather than microscale **since the size**
29 **effect is not considered**. Perlade et al. [23] developed a physical model based on an
30 intermediate mixture law and successfully predicted the macroscopic behavior of TRIP steels.
31 A good agreement between the simulated and the experimental evolution of the stress-strain
32 partitioning coefficient proved the validity of the iso-work hypothesis in the intermediate
33 mixture law. Bouquerel et al. [24] characterized the deformation behavior of multiphase TRIP

1 steels via a Gladman-type mixture law. They confirmed the feasibility of the developed
2 model and derived the stress-strain distribution of different phases. Fu et al. [25] revised the
3 index of n in the Gladman-type mixture law to characterize the load-stress adaptability
4 between different phases in TRIP steels and related it to a function of martensitic
5 transformation rate. Dan et al. [26] proposed a phenomenological model based on the
6 **intermediate mixture law and a new** hypothesis that the strain energy density of multiphase
7 steels equals the sum of that of the individual phase. They stated that the developed model
8 could accurately describe the deformation **behaviors** of TRIP590 and DP590 multiphase
9 steels.

10 According to the previous contributions mentioned above, most works of literature are
11 limited to the investigation of SIMT under the macroscale scenario. The grain size effect on
12 SIMT has not yet been fully understood, and previous reports are unavailable to reveal the
13 interplay of geometric and grain size effects on SIMT in micro-scaled plastic deformation of
14 metastable metal foils. In this work, comprehensive research of geometric and grain size
15 effects on SIMT was carried out. The unique work-hardening behavior in micro-scaled
16 deformation of metastable metal foils was explored. To describe the size-dependent SIMT
17 and deformation response of metastable foils, a multiscale constitutive model was developed
18 by considering the interaction of size effect and martensitic transformation, and the model
19 was validated by experiment and **finite element** simulation. This study provides an in-depth
20 understanding of SIMT under multiscale and promotes innovative findings to realize the
21 microstructural control for micro-scaled deformation of metastable foils.

22 **2. Experimental and simulation procedure**

23 **2.1. Specimen preparation**

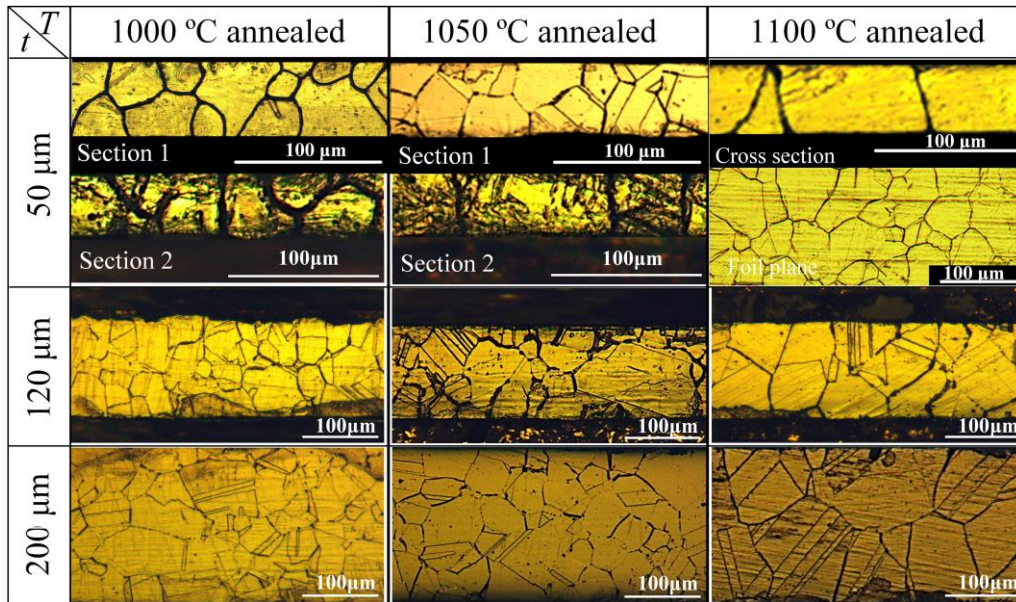
24 To explore both geometric and grain size effects on the SIMT phenomenon, the
25 commercial SUS304 foils with thicknesses of 50, 120, and 200 μm were chosen as the test
26 materials. The chemical composition is listed in Table 1.

27 Table 1 Chemical composition of SUS304 foils (in wt.%).

C	Si	Mn	P	S	Ni	Cr	Fe
0.049	0.525	1.083	0.033	0.003	8.011	18.180	Bal.

28 To achieve various grain sizes, the foils were annealed under different temperatures of
29 1000, 1050, and 1100 $^{\circ}\text{C}$, corresponding soaking durations of 1, 1, and 1.5 h in a vacuum

1 oven with a vacuum degree of 10^{-3} Pa, respectively. The foils were cooled in the vacuum
 2 oven after heating and holding. The microstructure images along the cross-section were
 3 obtained by the optical microscope after etching by the solution of 10 g oxalic acid and 200
 4 ml water, as shown in Fig. 1. The 50 μm thick foils were found to have a near-bamboo
 5 microstructure. The austenitic grain size was measured through the Heyn linear intercept
 6 method according to ASTM E112-13 standard, where twins were considered as grains. Since
 7 the grains across the thickness direction are incomplete for the 50 μm thick foil annealed at
 8 1100 $^{\circ}\text{C}$, the metallography along the foil plane was used to compute the average grain size
 9 under this condition. The average austenitic grain size of several sections containing about a
 10 hundred grains for each foil was computed to ensure the measurement accuracy, which is
 11 presented in Table 2. The ratio (t/d) of thickness (t) to austenitic grain size (d) of each foil is
 12 also listed in Table 2.



13
 14 Fig. 1. Microstructures of SUS304 foils with thicknesses of 50, 120, and 200 μm after
 15 annealing at 1000, 1050, and 1100 $^{\circ}\text{C}$.

16 Table 2 Average austenitic grain size (d) and the ratio (t/d) of specimen thickness (t) to
 17 austenitic grain size (d) under various annealing conditions.

	1000 $^{\circ}\text{C}$, 1 h		1050 $^{\circ}\text{C}$, 1 h		1100 $^{\circ}\text{C}$, 1.5 h	
t (μm)	d (μm)	t/d	d (μm)	t/d	d (μm)	t/d
50	21.3	2.35	52.6	0.95	69.1	0.72
120	28.7	4.18	60.3	1.99	72.7	1.65
200	29.5	6.78	64.8	3.09	94.8	2.11

2.2. Quasi-static uniaxial tension

To attain the flow behavior of metastable metal foils, the uniaxial tensile specimens with the thickness of 50 μm were prepared along the rolling direction according to ASTM E345-16 standard, while the uniaxial tensile specimens with the thicknesses of 120 and 200 μm were prepared along the rolling direction according to ASTM E8-08 standard. The gauge dimension of the tensile specimens is 50 \times 12.5 mm. The tensile experiments were performed at a low strain rate of $1.6 \times 10^{-3} \text{ s}^{-1}$ to avoid a temperature increase induced by the plastic deformation. The digital image correlation (DIC) was adopted for the measurement of strain in the micro-scaled deformation. To ensure the reliability of experimental results, each test was repeated three times.

The martensite phase is transformed from austenite under RT plastic deformation. The deformation temperature is lower than M_d , which is the temperature above that martensite is not produced during the plastic deformation. For 304 foils, M_d is appropriately 55 $^{\circ}\text{C}$ [27]. Therefore, the tensile tests were also performed at an elevated temperature of 85 $^{\circ}\text{C}$. Since the phase transformation takes place at RT rather than 85 $^{\circ}\text{C}$ for the tested foils, the effect of martensitic transformation was thus investigated by comparing the two scenarios. Also, to track the transformation kinetics from austenite to martensite during the micro-scaled deformation, the interrupted tensile tests at RT were conducted to different engineering strain levels including 0%, 10%, 20%, 30%, and fracture.

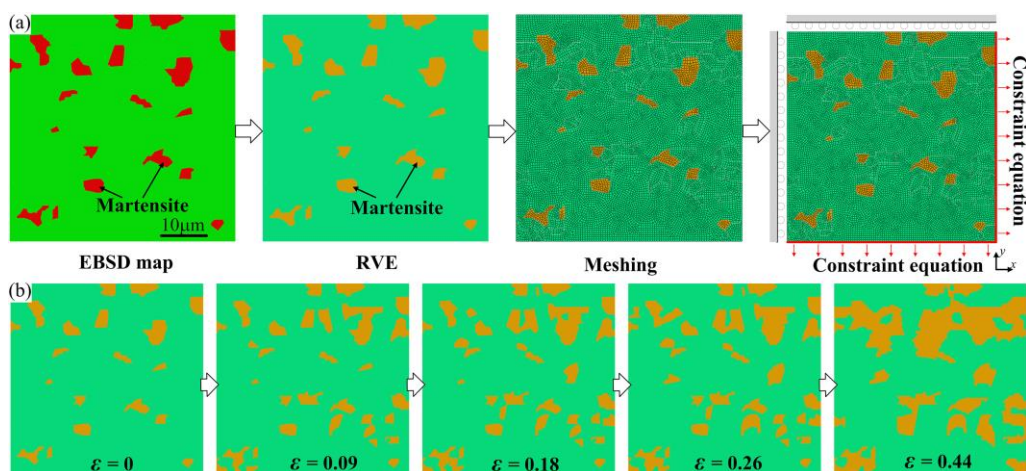
2.3. Measurement of martensite content

To evaluate the microstructure evolution of SUS304 foils at various strains, a reliable X-ray diffraction (XRD) method based on the integrated intensities of $(111)_{\gamma}$, $(200)_{\gamma}$, $(220)_{\gamma}$, $(311)_{\gamma}$, $(110)_{\alpha'}$, $(200)_{\alpha'}$, $(211)_{\alpha'}$, and $(220)_{\alpha'}$ diffraction peaks was adopted in the uniform deformation region [28]. The subscripts γ and α' represent diffraction peaks of austenite and martensite constituents, respectively. The intensity of the diffraction peak was evaluated by Rigaku D/max 2500PC XRD with $\text{CuK}\alpha$ radiation. The XRD patterns were captured by the step-scanning from 35 $^{\circ}$ to 100 $^{\circ}$ as 2θ with an increment of 0.05 $^{\circ}$. Afterward, a quantitative estimation of martensite volume fraction was computed according to the fact that the entire integrated intensity of all diffraction peaks for an individual constituent in the aggregation is proportional to the volume fraction of the phase [29].

2.4. Numerical simulation

To validate the developed multiscale constitutive model and reveal the work-hardening mechanism, the finite element simulation based on the representative volume element (RVE)

1 was performed on the platform of ABAQUS. The microstructure morphology from the
 2 electron backscatter diffraction (EBSD) map of the 120 μm thick foil with the grain size of
 3 60.3 μm was extracted by the Rhino software, and then the RVE model was established and
 4 imported in the ABAQUS software, as shown in Fig. 2 (a). The microstructure contains the
 5 hard martensite constituent, which is embedded within the soft austenite matrix. The
 6 mechanical behaviors of austenite and martensite were obtained by the multiscale constitutive
 7 model. The size of the RVE model was 50 \times 50 μm , which initially contained 8.9%
 8 martensite islands randomly distributed in the 91.1% austenite matrix. Before fracture, the
 9 martensite content gradually increased to 28.3% due to the SIMT. In this model, the
 10 constraint equations were applied to the nodes on the right and bottom sides, and the
 11 symmetrical conditions were adopted on the left and upper sides to have equal displacements.
 12 The nodes at the bottom edge had the same displacement in the y -direction while they can
 13 freely move along the x -direction, and the nodes on the right side had the same displacement
 14 in the x -direction (parallel to the tensile direction). Four-node quadrilateral elements were
 15 chosen for the plane stress analysis of two-dimensional RVE. During the tensile deformation,
 16 the volume fraction of strain-induced martensite continuously increased with plastic strain,
 17 whereas the austenite matrix, in turn, decreased, as shown in Fig. 2 (b). In the finite element
 18 model, the continuous change of martensite volume fraction with plastic strain was realized
 19 by the USDFLD subroutine, which can redefine the user-defined field variables at the
 20 integration points. Different values of field variables were set to austenite and martensite.
 21 When the equivalent plastic strain value of integration points reached the set value, the
 22 material properties in the integration points were adjusted from austenite to martensite due to
 23 the change of the field variable.



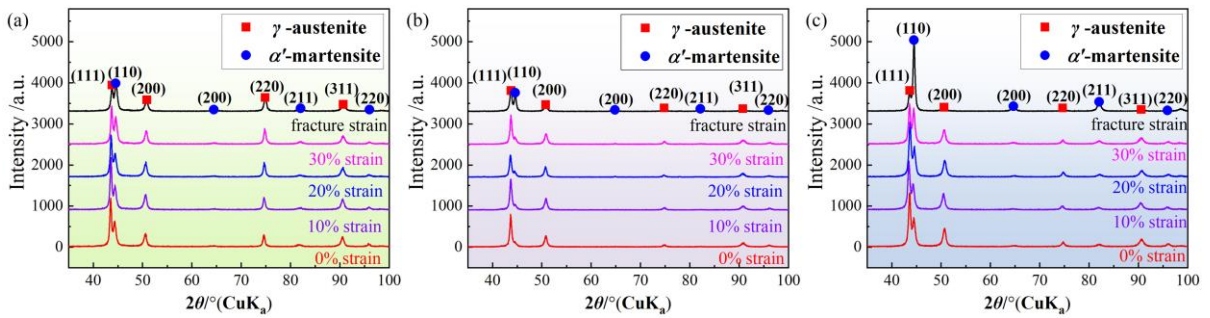
24
 25 Fig. 2. Two-dimensional RVE model of the real microstructure: (a) finite element modeling

1 process and (b) strain-induced martensitic transformation in the finite element model.

2 3. Results and discussion

3 3.1. Size effect on strain-induced martensitic transformation

4 To explore both the geometric and grain size effects on SIMT, the XRD patterns were
5 measured at diverse strain levels. Fig. 3 shows the XRD patterns of the foils annealed at
6 1000 °C as an example. It is found that the intensity of (111)_γ diffraction peak gradually
7 decreases and the intensity of (110)_{α'} diffraction peak gradually raises with the increase of
8 strain level. This phenomenon mirrors that the SIMT occurs during plastic deformation. In
9 addition, the diffraction peaks of martensite at 0% strain indicate that a small amount of
10 martensite has already been generated during the furnace cooling, which is termed as
11 annealing-induced martensitic transformation [30]. The appearance of annealing-induced
12 martensite has been proved to be related to the sensitization phenomenon during annealing
13 [31, 32], which leads to the precipitation or segregation of solute atoms and in turn increases
14 the martensite start temperature M_s in local regions. The content of annealing-induced
15 martensite was calculated using the XRD patterns. As presented in Fig. 4, the content of
16 annealing-induced martensite increases with increasing grain size and decreasing foil
17 thickness, which may be related to the influence of the size effect on sensitization.



18

19 Fig. 3. XRD patterns of the deformed specimens annealed at 1000 °C: (a) $t = 50 \mu\text{m}$, $d = 21.3$
20 μm , (b) $t = 120 \mu\text{m}$, $d = 28.7 \mu\text{m}$ and (c) $t = 200 \mu\text{m}$, $d = 29.5 \mu\text{m}$.

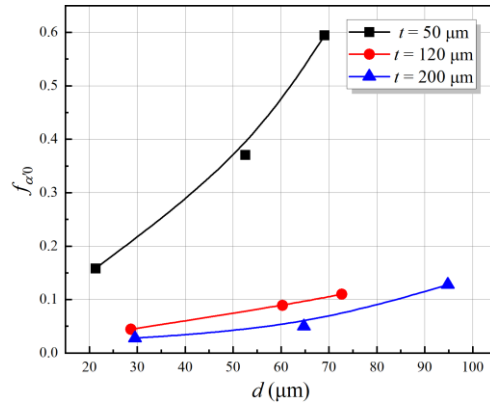
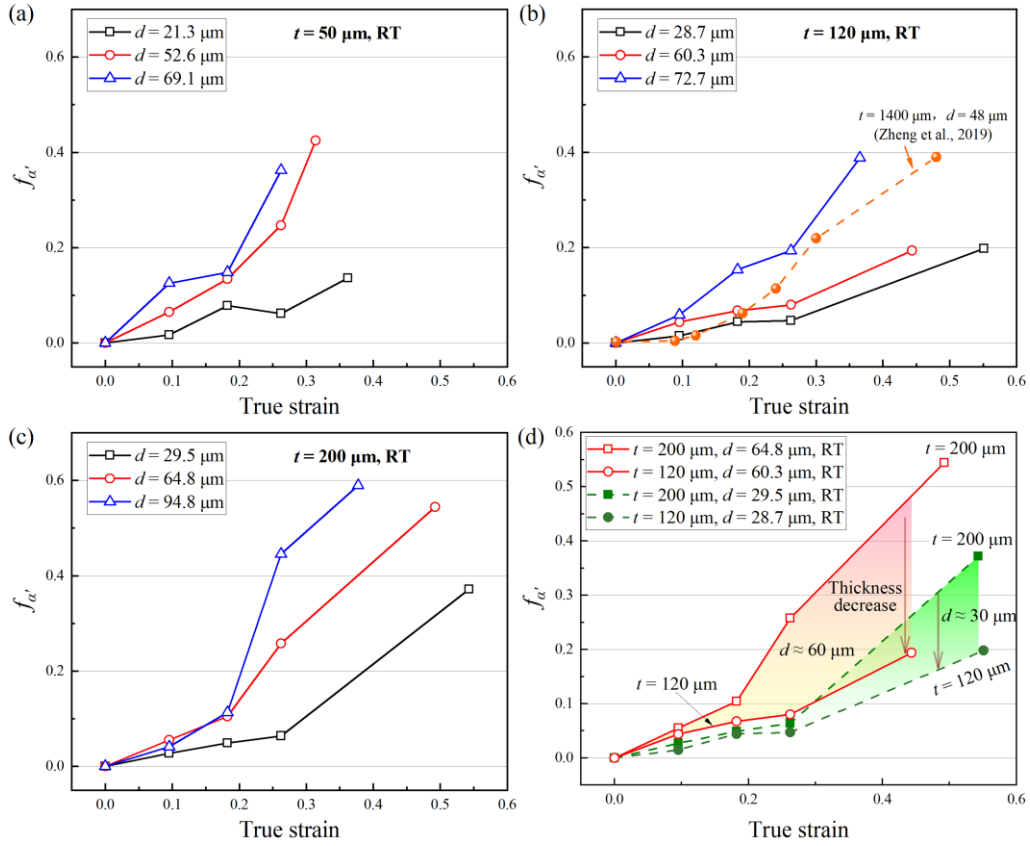


Fig. 4. Volume fraction of annealing-induced martensite $f_{\alpha'0}$ in diverse foils.

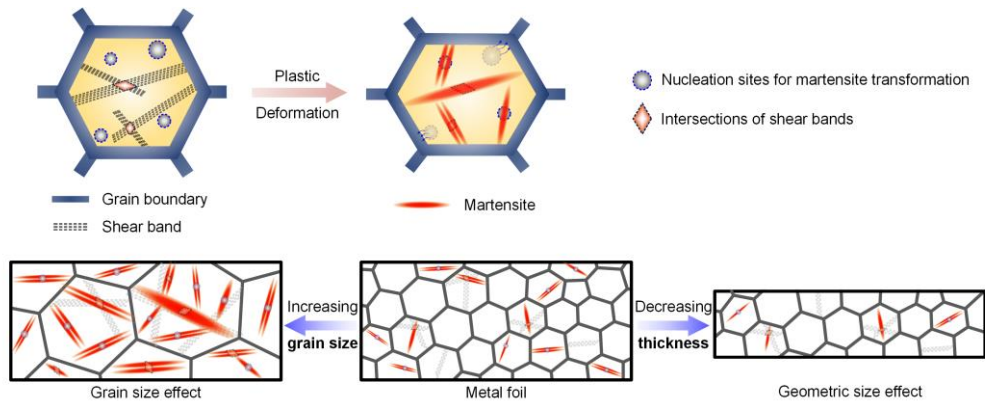
The volume fraction of strain-induced martensite was calculated by excluding the content of annealing-induced martensite, as given in Fig. 5. For a given foil thickness, the increase of grain size stimulates the generation of strain-induced martensite. In addition, the influence of foil thickness on SIMT was studied by comparing the content of deformation-induced martensite in 304 foils with identical grain size and different thicknesses. Since the 50 μm thick foils have more annealing-induced martensite, many martensitic embryos have formed before plastic deformation, which shortens the martensite nucleation process and further advances the martensite growth process. To avoid the effect of annealing-induced martensite on SIMT, the analysis of the effect of foil thickness on SIMT did not cover the 50 μm thick foils. Two sets of specimens with similar initial martensite content and grain size were selected to explore the geometric size effect on SIMT, as presented in Fig. 5 (d). It is found that the increase in foil thickness promotes the transformation from austenite to martensite, and the fine grain tends to weaken the effect of foil thickness on SIMT. Comparing the contents of strain-induced martensite in the 1400 μm thick sheet with a grain size of 48 μm [33] and the 120 μm thick foil with the grain size of 60.3 μm , the acceleration of SIMT with the increase of specimen thickness is also confirmed, as shown in Fig. 5 (b). Therefore, the increase of grain size and foil thickness can promote the SIMT of metastable metal foils during RT uniaxial tension.



1
 2 Fig. 5. Volume fraction of **strain-induced martensite** $f_{\alpha'}$ in metal foils: (a) $t = 50 \mu\text{m}$, (b) $t =$
 3 $120 \mu\text{m}$, (c) $t = 200 \mu\text{m}$ and (d) comparison **between different thicknesses specimens with**
 4 **similar initial martensite content and grain size.**

5 Fig. 6 depicts the impact mechanism of specimen thickness and grain size on SIMT. The
 6 atoms on the grain boundary are restricted by the neighboring atoms and hard to participate in
 7 the coordinated atom movement for martensitic transformation. Therefore, the grain
 8 refinement **can** strengthen the austenite matrix and hinder the shear transformation, thus
 9 delaying the trigger of SIMT [34]. In addition, the intersections of micro shear bands are
 10 acted as the dominant nucleation sites **of** martensitic embryos, where the micro shear bands
 11 include the bundles of stacking faults and deformation twins [35]. The micro shear bands
 12 near the grain boundary tend to migrate to the boundary and disappear, which results in the
 13 decrease of nucleation sites for **the** SIMT. Therefore, the SIMT is impeded by fine grains
 14 since the generation of nucleation sites is limited by grain boundaries. When the plastic strain
 15 reaches about 20%, the morphology of the formed martensite changes to a blocky irregular
 16 one due to the coalescence of martensitic embryos. Grain boundaries **hinder** the further
 17 growth of the blocky irregular martensite. It is concluded that grain **boundaries enhance** the
 18 stabilization of **austenite**, **decrease** the nucleation sites of martensite, and **hinder** the growth of

1 **martensite**, thereby inhibiting the SIMT. When the foil thickness **decreases**, the potential
 2 nucleation sites for the SIMT thereupon **decrease** due to the **reduced** fraction of grain interior.
 3 The SIMT is thus suppressed by the reduction in foil thickness. Therefore, the SIMT is
 4 dominated by the **proportion of grain boundaries**.



5
 6 Fig. 6. Schematic diagram of the mechanism of size effect on strain-induced martensitic
 7 transformation.

8 3.2. Coupled influence of size effect and SIMT on deformation behavior

9 Fig. 5 showed that the increase of grain size and foil thickness can promote the SIMT of
 10 304 foils during RT uniaxial tension. To reveal the coupled influence of size effect and
 11 size-dependent SIMT on deformation behavior, the true stress-strain responses of the foils
 12 deformed at RT and 85 °C are compared, as presented in Fig. 7. It is found the flow behavior
 13 of metastable foils presents two different patterns. Throughout the 85 °C deformation process,
 14 the specimens show an obvious size effect, i.e., the flow stress decreases with the decrease of
 15 thickness and increase of grain size. However, the stress of the specimens deformed at RT
 16 follows the typical size effect at the beginning and then exhibits noticeably increasing
 17 tendency due to the SIMT, especially when the strain exceeds 0.2. The increase of flow stress
 18 at RT is intensified with the growth of grain size and foil thickness since the SIMT is affected
 19 by both geometric and grain size effects. Moreover, the abnormal increase of flow stress of
 20 coarse-grained specimens tends to appear earlier than that of fine-grained specimens, and the
 21 subsequent flow stress is mainly controlled by the SIMT. The elongation of the foils
 22 decreases with the increase of grain size and the decrease of foil thickness regardless of the
 23 deformation temperature. This phenomenon mirrors that the elongation of metastable metal
 24 foils is still dominated by the size effect [36, 37] rather than the SIMT.

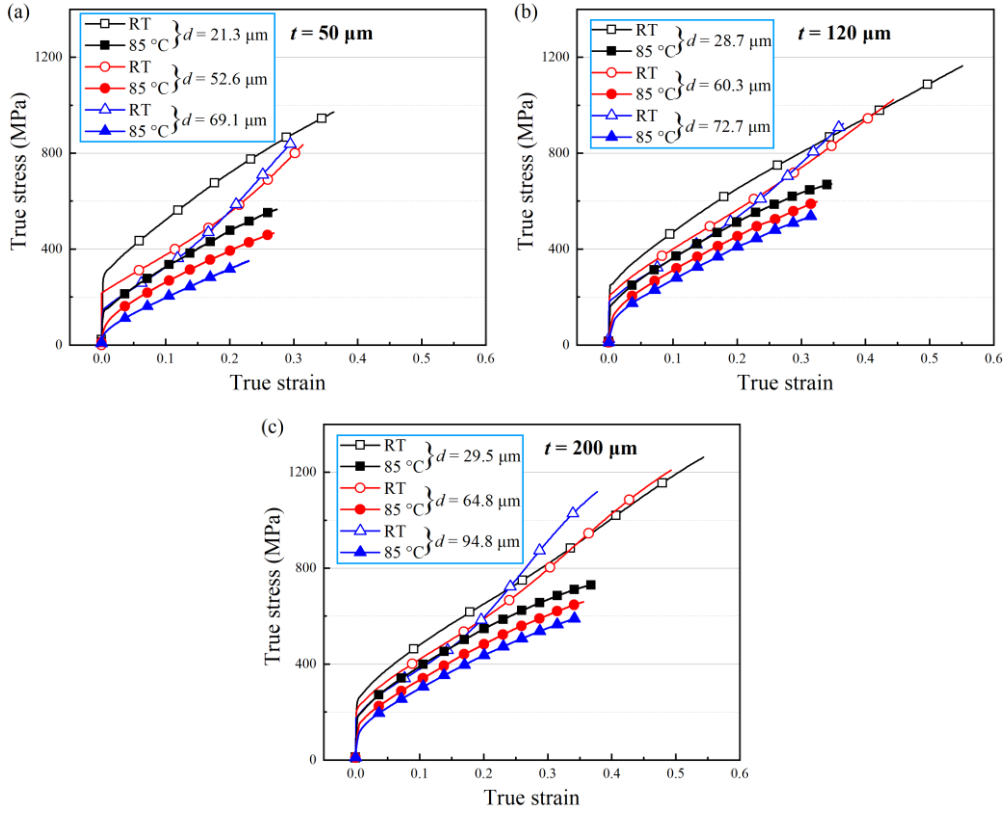


Fig. 7. True stress-strain responses of foils with various thicknesses at RT and 85 °C: (a) $t = 50 \mu\text{m}$, (b) $t = 120 \mu\text{m}$ and (c) $t = 200 \mu\text{m}$.

The typical Hall–Petch equation [38] was adopted to describe the relation of the initial yield strength σ_y and the initial austenitic grain size d :

$$\sigma_y = \sigma_0 + k_{hp}d^{-1/2} \quad (1)$$

where σ_0 and k_{hp} are material constants. Fig. 8 depicts the Hall–Petch relation for the foils deformed at RT and 85 °C. The initial yield strength of the foils deformed at RT and 85 °C increases as the initial austenitic grain size decreases. This phenomenon reflects that metastable foils with the initial annealing-induced martensite also follow the Hall–Petch relation. The macro plastic deformation of 304 foils is attributed to the onset of plastic deformation of austenite, while the martensite phase is still in the elastic stage due to the higher yield strength [39]. Therefore, the austenite phase dominates the initial yield strength of 304 foils. As the advancement of macroscopic strain, the SIMT initiates, and the martensite starts plastic deformation under the drive of applied external stress and internal stress [39], thereby improving the strength of metastable foils. Therefore, the flow behavior of metastable foils is affected by the interplay of size effect and SIMT.

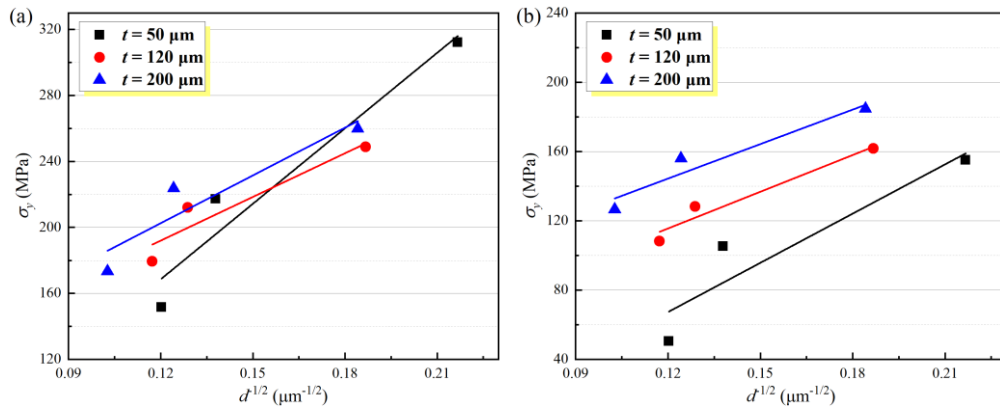
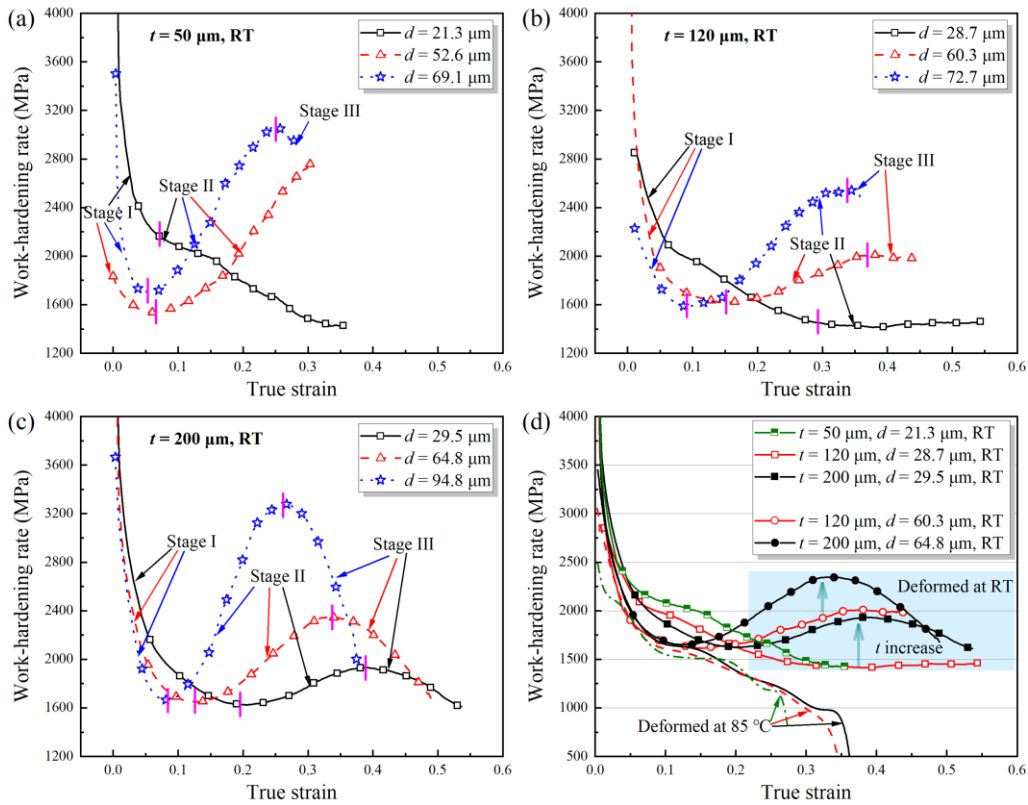


Fig. 8. Hall–Petch relation for metastable foils deformed at: (a) RT and (b) 85 °C.

To further describe the morphological transformation of the flow behavior of 304 foils under the interaction of size effect and SIMT, the corresponding work-hardening rates were computed and depicted in Fig. 9. The 200 μm thick foils present a work-hardening abnormality at RT similar to the previous work [40], which can be characterized as three stages. In stage I, the hardening rate drops sharply due to the initiation of dislocation glide. With the occurrence of SIMT, the work-hardening behavior is transferred from stage I to II. The work-hardening rate in stage II increases rapidly due to the dominant hardening effect of martensitic transformation. When the martensite volume fraction is close to saturation, the martensitic transformation rate drops rapidly, causing a fast decline of the work-hardening rate. The work-hardening behavior is transferred from stage II to III. The work-hardening behaviors of the 50 and 120 μm thick specimens are similar to that of the 200 μm thick specimens. Due to the restricted SIMT for the 50 and 120 μm thick specimens with fine grains, the weakening role of dislocation annihilation surpasses the combined hardening effect of SIMT and dislocation accumulation, resulting in a decrease of work-hardening rate in stage II and the absence of stage III.



1
 2 Fig. 9. Work-hardening behavior of 304 foils under diverse conditions: (a) $t = 50 \mu\text{m}$, (b) $t =$
 3 $120 \mu\text{m}$, (c) $t = 200 \mu\text{m}$ and (d) comparison between different thicknesses specimens with
 4 similar grain size.

5 The stain range of three stages and the magnitude of the increase in work-hardening rate
 6 are closely related to foil thickness and grain size. There is a clear trend that the strain range
 7 of stage I is shrunken with increasing grain size, i.e., the work-hardening behavior of
 8 coarse-grained foils during the deformation process is transferred from stage I to II earlier.
 9 The trend is caused by the fact that the increase of grain size accelerates the onset of SIMT. In
 10 addition, the work-hardening rate in stage II increases more rapidly with increasing grain size
 11 for a certain thickness and reaches a peak value prematurely. The early saturation of
 12 martensite in coarse-grained specimens leads to the shrinkage of stage II. The result confirms
 13 that the martensitic transformation rate is promoted by increasing grain size, in turn, resulting
 14 in an earlier onset and end of stage II and a faster increase in the work-hardening rate in stage
 15 II. Since the increase of geometric size can promote SIMT, the effect of foil thickness on the
 16 working-hardening rate is similar to the effect of grain size. As shown in Fig. 9 (d), the
 17 increase of foil thickness facilitates the work-hardening rate and advances the onset and end
 18 of stage II. The contribution of SIMT is gradually strengthened with the increase of foil
 19 thickness, resulting in the transformation of strain-hardening mode in stage II from decline

1 for the 50 μm thick foil to nearly constant for the 120 μm thick foil, and then to rise for the
2 200 μm thick foil. Comparing the hardening behavior of metal foils deformed at RT, the
3 work-hardening at 85 $^{\circ}\text{C}$, which is only affected by the size effect, exhibits typical patterns
4 due to the absence of SIMT.

5 **4. Modeling and verification**

6 **4.1. Multiscale constitutive model considering size effect and SIMT**

7 **4.1.1. Martensitic transformation kinetic model**

8 The martensite constituent in the metal foils **consists** of two types of martensite, viz.,
9 annealing-induced and strain-induced martensite, **which** can be formulated as:

$$10 \quad f_M = f_{\alpha'0} + f_{\alpha'} \quad (2)$$

11 where f_M , $f_{\alpha'0}$, and $f_{\alpha'}$ are the **volume fractions of the total, annealing-induced, and**
12 **strain-induced martensite**, respectively.

13 **Some martensitic transformation kinetic models have been derived to reflect the volume**
14 **fraction of strain-induced martensite $f_{\alpha'}$ in metastable ASSs, including the Olson and Cohen**
15 **model [35], Gompertz model [41], and Logistic model [42]. Compared with the first two**
16 **models, the Logistic model considers both the saturation of deformation-induced martensite**
17 **and the physical process of SIMT, which is expressed as [42]:**

$$18 \quad f_{\alpha'} = \frac{f_s}{1 + \exp(-\beta(\varepsilon - \varepsilon_m))} \quad (3)$$

19 where f_s is the **saturation volume fraction of martensite**, β characterizes the martensitic
20 transformation rate, ε is the **true plastic strain**, and ε_m is the strain value corresponding to 50%
21 of the saturated level of martensite. The work-hardening rate drops rapidly in the later period
22 of plastic deformation, **implying that the rate of SIMT decreases and the content of**
23 **deformation-induced martensite gradually saturates. Therefore, the value of f_s was replaced**
24 **by the content of martensite after the fracture in the fitting process.**

25 **4.1.2. Constitutive models of austenite and martensite**

26 Since the SIMT in the plastic deformation of SUS304 foils occurs at room temperature,
27 the microstructure does not undergo dynamic recrystallization, and the grain number thus
28 does not change during tension. The austenitic grain size gradually **decreases** with the
29 decrease of the austenitic volume. Assuming that the austenitic grain has a spherical shape
30 **[24]**, the austenitic volume in the initial state and deformation process can be expressed as:

$$1 \quad f_{\gamma 0} = \frac{\pi}{6} d^3 \times N \quad (4)$$

$$2 \quad f_{\gamma}(\varepsilon) = \frac{\pi}{6} d_{\gamma}^3(\varepsilon) \times N \quad (5)$$

3 In Eq. (4), $f_{\gamma 0}$, d , and N denote the initial austenitic volume fraction, grain size, and grain
 4 number, respectively. In Eq. (5), $f_{\gamma}(\varepsilon)$ and $d_{\gamma}(\varepsilon)$ represent the instantaneous austenitic volume
 5 and grain size during plastic deformation, respectively. The instantaneous austenitic grain size
 6 can then be designated as:

$$7 \quad d_{\gamma}(\varepsilon) = d \times \left(\frac{f_{\gamma}(\varepsilon)}{f_{\gamma 0}} \right)^{1/3} \quad (6)$$

8 where $f_{\gamma 0}$ and $f_{\gamma}(\varepsilon)$ can be calculated based on the content of martensite.

9 The plastic flow behavior of the austenitic phase in the metal foils can be characterized
 10 using Kocks–Mecking formalism, which can be used to describe the size effect on the
 11 mechanical deformation of metals and is written as follows [43]:

$$12 \quad \sigma_{\gamma} = \sigma_{0\gamma} + \alpha M G_{\gamma} b_{\gamma} \sqrt{\rho} \quad (7)$$

$$13 \quad \frac{d\rho}{Md\varepsilon_{\gamma}} = \frac{1}{b_{\gamma} d_{\gamma}(\varepsilon)} + \frac{k}{b_{\gamma}} \sqrt{\rho} - f\rho \quad (8)$$

14 where σ_{γ} and ε_{γ} are the true stress and plastic strain of austenite, respectively, $\sigma_{0\gamma}$ represents
 15 the lattice friction stress and the solid solution strengthening, α is a material parameter
 16 representing dislocation interactions, M is the Taylor factor, G_{γ} is the shear modulus for
 17 face-centered cubic, b_{γ} is the Burgers vector for face-centered cubic, and ρ is the forest
 18 dislocation density.

19 Eq. (8) reflects the variation of dislocation density with the true plastic strain of
 20 austenite. The first term on the right represents the contribution of grain boundaries to the
 21 geometrically necessary dislocations. The second term denotes the athermal storage of
 22 dislocation. The third one is the dislocation annihilation caused by dynamic recovery. k and f
 23 reflect the dislocation storage and annihilation rates, respectively.

24 The flow stress dependence of grain size has been well expressed by the Hall–Petch
 25 relation [38] in the following:

$$26 \quad \sigma_{\gamma} = \sigma_{0\gamma} + k_{hp} d^{-1/2} \quad (9)$$

27 Comparing Eqs. (7) and (9), the initial dislocation density ρ_0 can be designated as a function
 28 of grain size d as below [44]:

$$\rho_0 = k_d d^{-1} \quad (10)$$

The combination of Eqs. (7), (8), and (10) is adopted to represent the constitutive behavior of austenite. The material parameters including k , f , and k_d needed to be fitted, and $\sigma_{0\gamma}$ in Eq. (7) can be theoretically estimated from an empirical equation [45]:

$$\sigma_{0\gamma} \text{ (MPa)} = 68 + 354 \text{wt.\%C} + 20 \text{wt.\%Si} + 3.7 \text{wt.\%Cr} \quad (11)$$

Since $\sigma_{0\gamma}$ represents the initial stress associated with the presence of lattice friction and the solid solution strengthening, it was assumed that its value is not affected by the surface effect. In other words, the magnitude of $\sigma_{0\gamma}$ in the interior is the same as that at the surface layer.

On the other hand, the constitutive model established by Rodriguez et al. [46] was used to simulate the mechanical behavior of martensite in metal foils:

$$\sigma_{\alpha'} = \sigma_{0\alpha'} + \alpha M G_{\alpha'} \sqrt{b_{\alpha'}} \sqrt{\frac{1 - \exp(-Mf \varepsilon_{\alpha'})}{fL}} \quad (12)$$

where $\sigma_{\alpha'}$ and $\varepsilon_{\alpha'}$ are the true stress and plastic strain of martensite, $\sigma_{0\alpha'}$ represents the yield strength of the martensite phase, $G_{\alpha'}$ is the shear modulus for body-centered cubic, $b_{\alpha'}$ is the Burgers vector for body-centered cubic, and L is considered as the martensitic lath width. The yield strength of martensite in the interior, $\sigma_{0\alpha'i}$, is not influenced by the surface effect, which can be computed by the carbon content of the material by Eq. (14) [47]:

$$\sigma_{0\alpha'i} \text{ (MPa)} = 461 + 1310 (\text{wt.\%C})^{1/2} \quad (13)$$

In Eq. (12), L can be calculated by assuming that the austenite and martensite constituents have the same grain number [24]. Nevertheless, the increase of lath number rather than lath size of martensite in austenitic grains dominates the increase in martensite constituent. Therefore, L is considered as the dislocation mean free path in the martensite phase in this research. In the constitutive model of the martensite phase, the yield strength $\sigma_{0\alpha's}$ at the surface layer, L , and f are the fitted parameters to accurately determine the stress-strain behavior of martensite.

4.1.3. Modeling coupled mixture law with surface layer model

The stress-strain mixture law has been widely used to describe the macroscopic deformation behavior of dual-phase steels and even multiphase steels because of its simplicity and applicability. A stress mixture law was proposed by Gladman et al. [48] using a power-law volume fraction function designated as:

$$\sigma(\varepsilon) = (1 - f_2^n) \sigma_1(\varepsilon) + f_2^n \sigma_2(\varepsilon) \quad (14)$$

1 where $\sigma(\varepsilon)$, $\sigma_1(\varepsilon)$, and $\sigma_2(\varepsilon)$ are the stresses of the overall, soft phase, hard phase, respectively.
 2 The parameter f_2 denotes the volume fraction of the hard phase. The difference in strain
 3 between different phases is included in the value of n , which is usually set to be 1 or 2.
 4 However, the parameter n is related to the strength ratio of austenite to martensite and the
 5 martensitic transformation rate during the plastic deformation of metastable ASS, which
 6 makes the value of n relevant to plastic strain [25]. An intermediate mixture law [49] is more
 7 appropriate for describing the mechanical behavior of metastable ASSs than the
 8 Gladman-type mixture law, which is expressed as follows:

$$9 \quad \begin{aligned} \sigma(\varepsilon) &= (1 - f_M(\varepsilon))\sigma_\gamma(\varepsilon_\gamma) + f_M(\varepsilon)\sigma_{\alpha'}(\varepsilon_{\alpha'}) \\ \varepsilon &= (1 - f_M(\varepsilon))\varepsilon_\gamma + f_M(\varepsilon)\varepsilon_{\alpha'} \end{aligned} \quad (15)$$

10 where $\sigma_\gamma(\varepsilon_\gamma)$ and $\sigma_{\alpha'}(\varepsilon_{\alpha'})$ are the stresses of austenite and martensite, respectively, ε_γ and $\varepsilon_{\alpha'}$ are
 11 the strains of austenite and martensite when the overall strain is ε , respectively. An iso-work
 12 hypothesis [50], i.e., the mechanical work increment is taken equally in each constituent
 13 regardless of the material state, is utilized to avoid the arbitrary stress-strain distribution
 14 between two phases, which is expressed as follows:

$$15 \quad \sigma_\gamma(\varepsilon_\gamma)d\varepsilon_\gamma = \sigma_{\alpha'}(\varepsilon_{\alpha'})d\varepsilon_{\alpha'} \quad (16)$$

16 The combination of Eqs. (15) and (16) is used to represent the allocation of strain and stress
 17 in austenite and martensite at a certain macro strain.

18 Considering that the surface and interior grains have different mechanical properties, the
 19 kinetic model of SIMT and the constitutive models of austenite and martensite were
 20 substituted into the intermediate mixture law with the iso-work hypothesis to fit the overall
 21 stress-strain curves of metastable ASSs almost completely composed of the surface or interior
 22 layers. The constitutive parameters of austenite and martensite at the surface or interior layers
 23 were thus determined. However, most metal foils are composed of the surface and interior
 24 grains, whose flow stress is denoted as [51]:

$$25 \quad \sigma(\varepsilon) = \eta\sigma_s(\varepsilon) + (1 - \eta)\sigma_i(\varepsilon) \quad (17)$$

26 where $\sigma_s(\varepsilon)$ and $\sigma_i(\varepsilon)$ are the flow stresses of the surface and interior layers, respectively. η
 27 represents the proportion of the surface layer, which can be denoted as:

$$28 \quad \eta = \frac{N_s}{N} = \frac{d}{t} \cdot \frac{2(w+t-2d)}{w} \quad (18)$$

29 where N is the total number of grains, N_s is the surface grain number, and w is the foil width.

30 The volume fraction and constitutive model of austenite and martensite are expressed in

1 vector forms:

$$\mathbf{f} = \begin{pmatrix} 1 - f_M(\varepsilon) \\ f_M(\varepsilon) \end{pmatrix}$$

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_\gamma(\varepsilon_\gamma) \\ \sigma_{\alpha'}(\varepsilon_{\alpha'}) \end{pmatrix} = \begin{pmatrix} \sigma_{0\gamma} + \alpha M G_\gamma b_\gamma \sqrt{\rho(\varepsilon_\gamma)} \\ \sigma_{0\alpha'} + \alpha M G_{\alpha'} \sqrt{b_{\alpha'}} \sqrt{\frac{1 - \exp(-Mf \varepsilon_{\alpha'})}{fL}} \end{pmatrix} \quad (19)$$

3 Assuming that martensite is evenly distributed in the material, the martensite volume fraction
4 of the surface and interior layers is the same. According to the intermediate mixture law, the
5 flow stress of the surface layer is obtained:

$$\sigma_s(\varepsilon) = \mathbf{f}^T \boldsymbol{\sigma}_s \quad (20)$$

7 Similarly, the flow stress of the interior layer is calculated as follows:

$$\sigma_i(\varepsilon) = \mathbf{f}^T \boldsymbol{\sigma}_i \quad (21)$$

9 By substituting Eqs. (20) and (21) into Eq. (17), the multiscale model is constructed as below:

$$\sigma(\varepsilon) = \eta \mathbf{f}^T \boldsymbol{\sigma}_s + (1 - \eta) \mathbf{f}^T \boldsymbol{\sigma}_i \quad (22)$$

11 **The constructing process for the multiscale constitutive model is also described in Appendix**
12 **A in the form of a flowchart.**

13 **4.2. Mechanical behaviors of austenite and martensite**

14 **To verify the validity of the multiscale model, the mechanical responses of austenite and**
15 **martensite at the surface and interior layers need to be determined first.** The flow stresses of
16 austenite and martensite at the surface layer can be derived from the experimental data of the
17 120 μm thick foil with the grain size of 60.3 μm and the 200 μm thick foil with the grain size
18 of 94.8 μm , which are entirely composed of the surface layer ($t/d \approx 2$). The stress-strain
19 curves and the martensite volume fractions obtained by the Logistic model of these two sets
20 of foil were substituted into the intermediate mixture law **with** the iso-work hypothesis to fit
21 the constitutive parameters (Eqs. (7), (8), (10), and (12)) of the surface austenite and
22 martensite. On the other hand, the experimental data of the 1400 μm thick foil with the grain
23 size of 48 μm [33], which is almost entirely composed of the interior layer ($t/d \approx 29$), was
24 chosen to achieve the constitutive parameters of the interior austenite and martensite. Since
25 the martensite constituent is still in the stage of elastic deformation when austenite begins to
26 yield, the elastic stage of martensite was thus added in the fitting process. The elastic
27 modulus of martensite **was** assumed to be consistent with that of austenite, which **was** set as

1 199 GPa. Table 3 lists the input parameters for the calibration.

2 Table 3 Input parameters for the constitutive models of austenite and martensite.

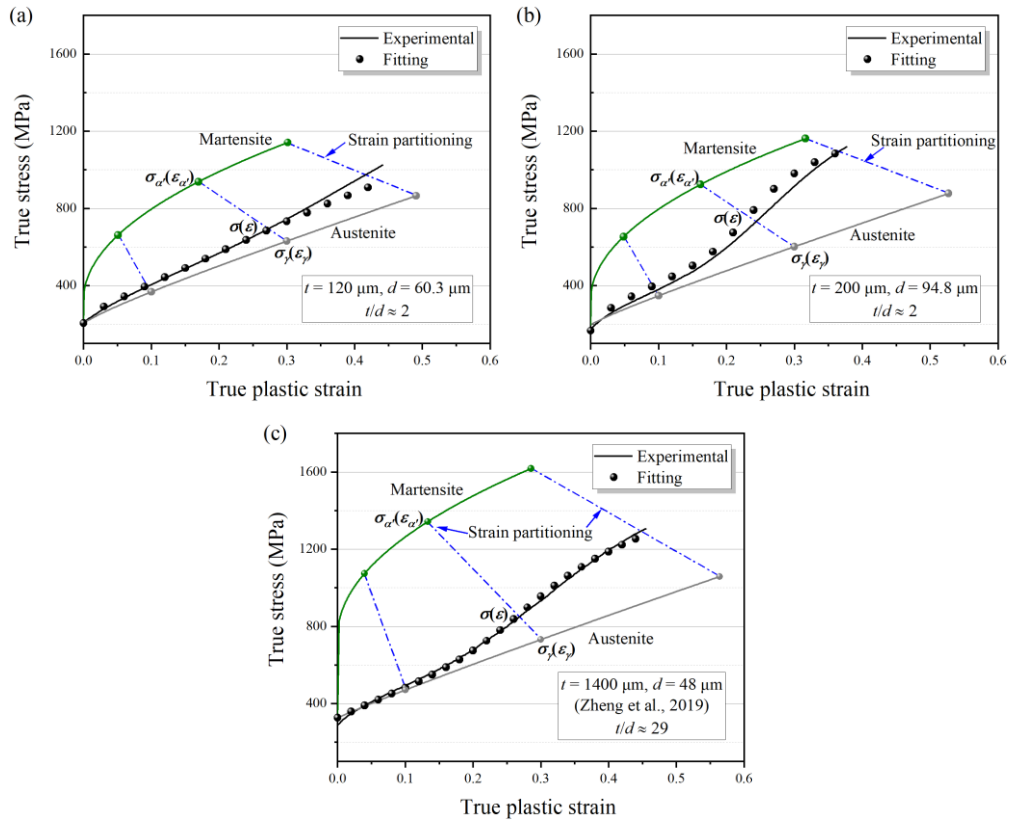
Parameters	Value	Source
$\sigma_{0\gamma}$ (MPa)	163	Eq. (11)
$\sigma_{0\alpha'}$ (MPa)	750	Eq. (13)
α	0.4	
M	3	
G_γ (MPa)	72000	
$G_{\alpha'}$ (MPa)	78500	Ref. [24]
b_γ (mm)	2.58×10^{-7}	
$b_{\alpha'}$ (mm)	2.48×10^{-7}	

3 Table 4 summarizes the fitted constitutive parameters via the global optimization
4 algorithm, and the identified mechanical behaviors of austenite and martensite at the surface
5 and interior layers are presented in Fig. 10. The flow stress of the individual phase
6 demonstrates that the martensite has a high yield strength than the austenite matrix, which
7 causes a significant increase in flow stress with plastic strain. Compared with the interior
8 austenite, the surface austenite has a higher dislocation annihilation rate since the dislocations
9 are easy to slip out of the free surface. A more obvious difference is that the surface austenite
10 has less initial dislocation density, which can be attributed to the fact that the mechanical
11 properties of surface grains are rather similar to a single crystal and the grain boundary
12 strengthening is negligible [51]. This quasi-single crystalline status leads to a lower
13 deformation resistance of the surface austenite. On the other hand, the yield strength of the
14 martensite constituent significantly **decreases** at the surface layer, and the surface martensite
15 has a larger dislocation mean free path than the interior one. The less dislocation density at
16 the surface layer may bring about a reduction in the dislocation density in the transformed
17 martensite at the surface, resulting in lower yield strength and a larger dislocation mean free
18 path. According to the proposed multiscale constitutive model, the given macro stress-strain
19 point (ε, σ) corresponds to two micro stress-strain points, i.e., the point $(\varepsilon_\gamma, \sigma_\gamma)$ for austenite
20 and the point $(\varepsilon_{\alpha'}, \sigma_{\alpha'})$ for martensite, as shown in Fig. 10. The difference of stress-strain
21 responses in austenite and martensite constituents implies a must for partitioning the
22 stress-strain data between different constituents.

1

Table 4 Fitted parameters in the multiscale model considering size effect and SIMT.

Phase	Parameters	Value (Surface)	Value (Interior)
Austenite	k	0.009	0.009
	f	0.055	0.043
	$k_d (\times 10^6 \text{ mm}^{-1})$	0.2	2.582
Martensite	σ_0 (MPa)	327	—
	f	0.002	0.0017
	L (mm)	0.003	0.0025



2

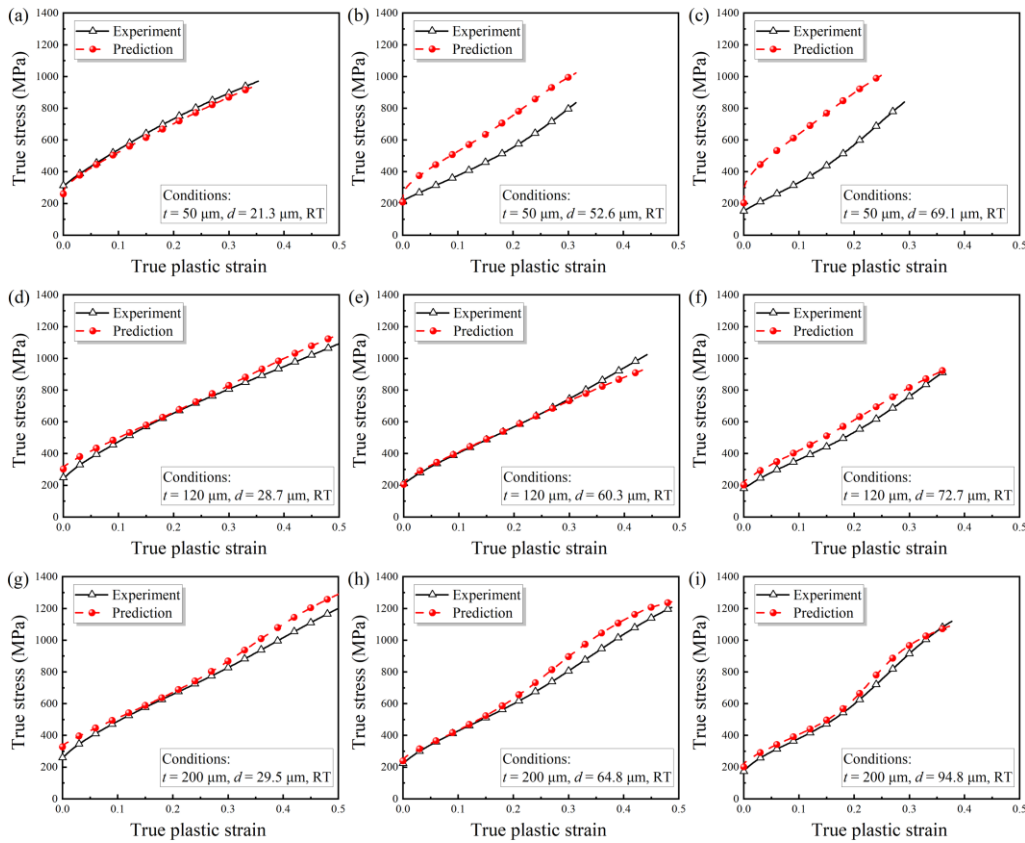
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4 Fig. 10. Stress-strain responses and strain partitioning for austenite and martensite at different
5 material conditions: (a) $t = 120 \mu\text{m}$, $d = 60.3 \mu\text{m}$, (b) $t = 200 \mu\text{m}$, $d = 94.8 \mu\text{m}$ and (c) $t =$
6 $1400 \mu\text{m}$, $d = 48 \mu\text{m}$ [33].

7 4.3. Verification of multiscale constitutive model

8 Due to the coupled impact of the size effect and the SIMT, the instability of 304 foil is
9 significantly aggravated. The experimental data under all material conditions were used to
10 confirm the validity of the established multiscale model. The comparison between the
11 experimental flow stress curves and the ones predicted by the multiscale constitutive model
12 is presented in Fig. 11. The results demonstrated that the multiscale constitutive model

1 considering martensitic transformation and size effect is valid for characterizing the
 2 deformation behavior of metastable metal foils with micron-sized grains at $t/d \geq 2$. It is also
 3 found that the flow stress of quasi-single crystalline state ($t/d \leq 1$) is overestimated by the
 4 multiscale model, as shown in Fig. 11 (b~c). The multiscale model is proposed based on the
 5 surface layer model, and the surface layer model does not consider the scenario of $t/d < 2$.
 6 Furthermore, the solid solution strengthening is restricted due to the severe sensitization in
 7 the annealing process for the 50 μm thick foils with the grain sizes of 52.6 and 69.1 μm ,
 8 resulting in an obvious decrease in flow stress. In addition, the deformation behavior of the
 9 quasi-single crystalline is strongly dependent on the individual grain size, shape, and
 10 orientation, which is not considered in the multiscale constitutive model.

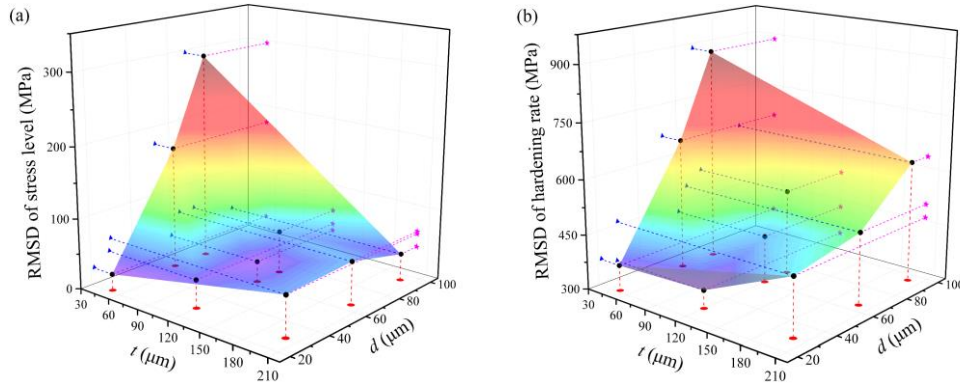


11
 12 Fig. 11. Model validation by comparing the experimental results and the predicted ones for
 13 metal foils with different states: (a~c) $t = 50 \mu\text{m}$, (d~f) $t = 120 \mu\text{m}$ and (g~i) $t = 200 \mu\text{m}$.

14 Meanwhile, the root mean square deviations (RMSD) between the experimental data and
 15 the predictive results in terms of stress level and work-hardening rate were used to evaluate
 16 the applicability of the developed multiscale constitutive model:

$$17 \quad \text{RMSD} = \sqrt{\frac{\sum_{i=1}^n (\hat{x}_i - x_i)^2}{n}} \quad (23)$$

1 where \hat{x}_i and x_i are the predicted and experimental results, respectively. The values of
 2 RMSD under different material conditions are shown in Fig. 12. It is observed that the
 3 disparity between the experimental curves and predictive ones is aggravated with reducing
 4 specimen thickness and rising grain size. In addition, the prediction deviation reaches the
 5 maximum value at $t/d \leq 1$, which remains to be further studied.

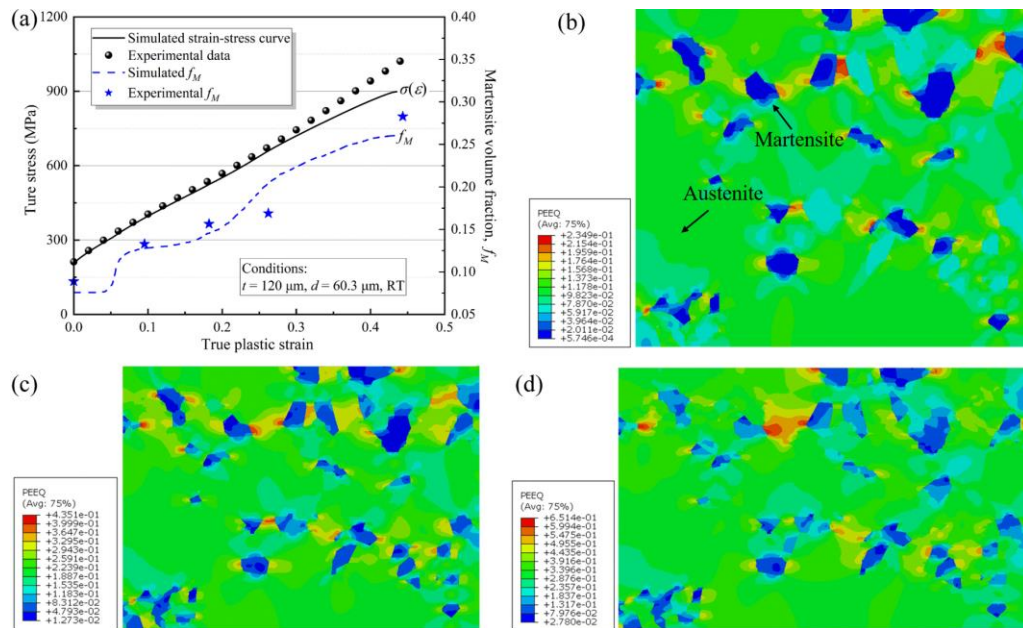


6
 7 Fig. 12. **Root mean square deviations (RMSD)** analysis of the experiment results and the
 8 predictive ones for (a) flow stress and (b) work-hardening rate.

9 In addition, the **mechanical behaviors of austenite and martensite** were used to
 10 numerically simulate the uniaxial tension process to **verify the validation of the model and**
 11 **further** reveal the hardening mechanism during the deformation. Fig. 13 depicts the
 12 simulation results of metal foil with the thickness of 120 μm and the grain size of 60.3 μm . It
 13 is seen from Fig. 13 (a) that the work-hardening is intensified **by the continuous increase of**
 14 **martensite content with plastic strain. The overall stress-strain curve obtained by simulation**
 15 **coincides well with the experimental one, which verifies the validation of the multiscale**
 16 **constitutive model.**

17 Fig. 13 (b~d) present the **microscopic strain distribution of austenite and martensite**
 18 **mixture at the macroscopic equivalent plastic strains of 0.1, 0.2, and 0.3, respectively.** It is
 19 found that the specimen is deformed heterogeneously and the deformation is concentrated **in**
 20 the austenite matrix. Whereas, the martensite islands (blue area) are considered as hard
 21 nondeformable dispersions embedded **within** the soft austenite matrix. The heterogeneous
 22 plastic deformation at the interface between austenite and martensite is formed to maintain
 23 the compatibility between austenite and martensite, as shown with red regions in Fig. 13
 24 (b~d). The **geometrically necessary** dislocations are generated at the **austenite and martensite**
 25 interface to accommodate the heterogeneous plastic deformation, which strengthens the
 26 kinetic constraints to the matrix and leads to a dispersion hardening effect [52]. As the
 27 deformation advances, the martensite volume fraction **increases** and the martensite phase

1 starts to plastically deform to accommodate the shape change, which further enhances the
 2 dispersion hardening effect and in turn increases the work-hardening rate.



3
 4 Fig. 13. Simulation for verifying the multiscale constitutive model and revealing the
 5 hardening mechanism: (a) comparison between simulation and experiment and (b~d)
 6 microscopic strain distributions at different macroscopic levels: (b) $\varepsilon = 0.1$, (c) $\varepsilon = 0.2$ and (d)
 7 $\varepsilon = 0.3$.

8 5. Conclusions

9 In this research, the size-dependence strain-induced martensitic transformation (SIMT)
 10 and unique hardening behavior in micro-scaled deformation of metastable austenitic foils
 11 were explored. To further understand the unique deformation behavior, a multiscale
 12 constitutive model considering the coupled influence of size effect and SIMT was developed.
 13 The work-hardening mechanism of austenite and martensite aggregation was thus revealed
 14 through finite element simulation and physical examination. The main conclusions are
 15 summarized in the following:

16 (1) A small amount of martensite is produced during the annealing, which is attributed to
 17 the sensitization phenomenon that increases the martensite start temperature M_s in local
 18 regions. The SIMT is impeded by the decrease of grain size and foil thickness because the
 19 nucleation and growth of martensite are restricted by the increase of grain boundary.

20 (2) The flow behavior of metastable foils at room temperature is affected by the
 21 interplay of size effect and SIMT. The martensitic transformation rate is promoted by
 22 increasing grain size and foil thickness, in turn, resulting in an earlier onset and end of stage

1 **II and a faster** increase in the work-hardening rate **in stage II**.

2 (3) The austenite and martensite phases at the interior layer have greater deformation
3 assistance and work-hardening rate than those at the surface layer due to the surface effect.
4 The established multiscale constitutive model considering the coupled influence of
5 martensitic transformation and size effect is validated to be able to characterize and predict
6 the plastic deformation behavior of metal foils **with micron-sized grains** at $t/d \geq 2$.

7 (4) The dispersion hardening effect of metastable foils is caused by the heterogeneous
8 plastic deformation of austenite and martensite aggregation. With the increase of plastic strain,
9 the martensite **volume fraction increases and the martensite phase starts to plastically deform**
10 **to accommodate the shape change, which further enhances the dispersion hardening effect**
11 **and in turn increases the work-hardening rate**.

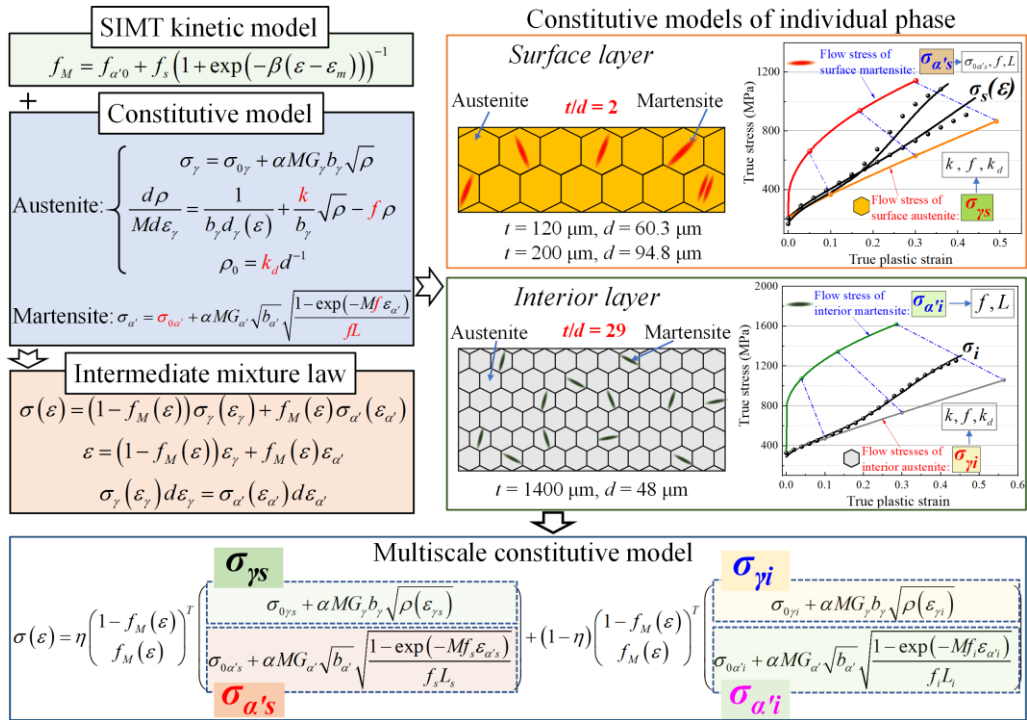
12 (5) The validity of the developed multiscale constitutive model is verified by physical
13 experiment and **finite element** simulation. The model represents the correlation among
14 martensitic transformation kinetics, size effect, and dislocation density-based constitutive
15 description of each phase, and can be used for controlling and tailoring of microstructure in
16 micro-scaled deformation of metastable foils.

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21 **Appendix A. Constructing process for the multiscale constitutive model**

22 Fig. A1 gives a detailed constructing process for the multiscale constitutive model
23 coupling the strain-induced martensitic transformation with the size effect. First, the suitable
24 kinetic model of strain-induced martensitic transformation, the constitutive models of
25 austenite and martensite, and the intermediate mixture law were evaluated and selected.
26 Subsequently, the phase transformation kinetic model and the constitutive models of austenite
27 and martensite were imported into the intermediate mixture law to fit the overall stress-strain
28 curves of metastable austenite stainless steels almost completely composed of the surface or
29 interior layer grains. The constitutive parameters of austenite and martensite at the surface
30 and interior layers were thus determined. Finally, the multiscale constitutive model was
31 constructed by coupling the surface layer model, the intermediate mixture law, the phase
32 transformation kinetic model, and the constitutive models of individual phases at the surface
33 and interior layers.



1
2
3

Fig. A1 Development framework of the multiscale constitutive model.

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