This is the accepted version of the publication Ran, J. Q., & Fu, M. W. (2016). Applicability of the uncoupled ductile fracture criteria in micro-scaled plastic deformation. International Journal of Damage Mechanics, 25(3), 289-314. Copyright © The Author(s) 2015.
DOI: https://doi.org/10.1177/1056789515578181

Applicability of the uncoupled ductile fracture criteria in micro-scaled plastic deformation

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Abstract

Micro-scaled plastic deformation, viz., microforming, has been widely used to fabricate microparts with at least two dimensions in sub-millimeter. In microforming, ductile fracture significantly affects product quality and the forming limit of materials. It happens when the deformation method, mode and sequence are not reasonable. Although extensive research on ductile fracture and ductile fracture criteria (DFC) in macroforming domain has been conducted, the applicability of these criteria in microforming scenario has not yet been extensively explored and studied. It is thus necessary to explore and address this issue to support the design of microforming process and quality control of the microformed parts. In this paper, the applicability of the uncoupled DFCs in microforming is investigated and discussed. The hybrid constitutive model developed in prior research is used to study the applicability of DFCs in microforming via finite element simulation and physical experiment. The simulation results of each criterion are compared with the upsetting experiment to determine the most suitable criterion for further experimental implementation in flanged upsetting and extrusion processes. The influence of size effect on the applicability of the uncoupled DFCs is explored and the stress-induced fracture map is used to demonstrate this effect. The applicability of DFCs in microforming is extensively revealed and analyzed and the research thus provides an in-depth understanding of the applicability of DFCs in microforming processes.

Keywords: Micro-scaled plastic deformation, Ductile fracture criteria, Stress-induced fracture map, Applicability of fracture criterion, size effect.

1. Introduction

Micromanufacturing, which includes micromachining, micro injection molding, powder injection molding, and micro-electro-mechanical-systems (MEMS) based lithography, etc., plays an important role in product miniaturization. Due to the ubiquitous trend of product miniaturization for saving of material and energy and further the ease usage and handling of products, how to efficiently fabricate micro-scaled parts and components in a large scale in tandem with this miniaturization trend poses a great challenge to manufacturing arena. As one of the promising micro-manufacturing processes, micro-scaled plastic deformation, or microforming, produces micro-scaled parts with at least two dimensions in sub-millimeter scale. This deformation process is thus widely used in many industrial clusters including consumer electronics, bio-medical, and aerospace, etc., for its high productivity, good mechanical properties and low production cost (Geiger et al., 2001; Vollertsen et al., 2003, 2008, Fu et al, 2012). Many prior researches in microforming are more focused on size effect, which is the most eluded and tantalized research issue and phenomenon as it significantly affects the deformation behavior and process performance in microforming processes and further the product quality of microformed parts. Size effect thus characterizes the unique difference between macroforming and microforming and their own unique features in each domain (Fu et al., 2012, Chan et al., 2012, 2013). Until now, many efforts have been provided to explore and investigate this effect and how it affects the deformation and process behaviors and phenomena and further influences the design of microforming process and tooling (Fu and Chan, 2014).

Generally, the plastic deformation in microforming does not exceed the ductility limit of forming materials. But if this happens, ductile fracture (DF) could occur in the process. Ductile fracture criterion (DFC) articulates the relationship between the ductility and plasticity of materials and the deformation degree in deformation process. From macro-scaled deformation perspective, many explorations and researches have been done in terms of the development of DFCs and application of the criteria in different deformation scenarios (Ayada et al., 1984, Bai et al., 2008, Brunig et al., 2008, Khan et al., 2012, Li et al., 2011, Xue et al., 2007, Liu and Fu, 2014a, 2014b). Among them, Bai et al. (2008) believed both the pressure effect and the effect of the third deviatoric stress invariant should be included in the constitutive description of material. A general form of asymmetric metal plasticity, considering both the pressure sensitivity and the Lode dependence, is postulated. A calibration method for the new metal plasticity is discussed. Hamblia et al. (2002) presented a detailed calibration of several kinds of fracture criteria. A computation methodology for identifying the critical value of these fracture criteria is developed for simulation of metal blanking process. Li et al. (2011) provided a detailed evaluation of ductile fracture criteria in macro-scaled forming and identified the reliable fracture criteria for tensile- and compression-dominant forming processes and other different stress states. Khan et al. (2012a, 2012b) presented a phenomenological fracture criterion based on the magnitude of the stress vector for 2024-T351 aluminum alloy. Wierzbicki et al. (2005; Xue, 2007) presented a panoramic study of seven fracture criteria to determine the suitability of these criteria in a particular application and conducted calibration for a given material. In addition, Liu and Fu (Liu and Fu, 2014a, 2014b) modified the Ayada criterion by considering the effect of stress triaxiality, and more importantly, the exponential effect of the equivalent plastic strain on the damage behavior, which is generally ignored in other ductile fracture criteria. The revised criterion is used in different sheet forming processes to verify and validate its efficiency. As most of the prior arts are focused on macroforming, in-depth research on the fracture behavior in micro-scaled plastic deformation has not yet been extensively conducted. The essential issues of ductile fracture and the applicability of DFCs in microforming thus need to be systematically studied.

In DF arena, the DFCs can be classified into two categories, viz., coupled and uncoupled criteria. For the uncoupled DFCs, damage accumulation is formulated empirically or semi-empirically in terms of deformation state variables such as equivalent plastic strain, tensile stress and hydrostatic stress, which are most relevant to fracture initiation and propagation. Since ductility increases with hydrostatic stress in plastic deformation process, most uncoupled DFCs consider the effect of such stress. The coupled DFCs, however, incorporate damage accumulation in the constitutive equations and allow the yield surface of the deformation materials to be modified by the damage-induced density change. From implementation perspective, the latter is more difficult to be implemented numerically. In this research, the focus is to investigate the applicability of six uncoupled fracture criteria, which are widely used in macro-scaled plastic deformation. To explain their applicability in micro-scaled deformation scenarios, a hybrid constitutive model developed in prior research (Ran et al., 2013) is used in the finite element (FE) simulation and the six DFCs are implemented. The critical value of each DFC is calculated by using the hybrid constitutive model. To describe the change of flow stress in microforming process, the size factor, which represents the ratio between the number of surface grains to the total grain number in the deformation body, is determined and included in the hybrid constitutive model. The influence of size effect on fracture behavior is quantified by the stress-induced fracture map (SFM), which is a three-dimensional diagram proposed and constructed to schematically articulate the relationship between size effect and predicted fracture strain in microforming. Finally, the applicability of the six DFCs is identified by comparing the FE simulation results with the experimental ones including load-stroke curve and SFM.

2. The uncoupled ductile fracture criteria

Table 1 summarizes six widely used uncoupled DFCs, each of which is next explained in this section. Each DFC has its own critical value, and fracture occurs when the integral value is larger than or equal to its critical value. The representation of the uncoupled DFCs can be designated in the following common formulation:

$$\int_0^{\bar{\varepsilon}_f} f\left(\sigma,\bar{\varepsilon}\right) d\bar{\varepsilon} - C < 0 \qquad (1)$$

where $\bar{\varepsilon}_f$ is the fracture strain, C is the critical value, $\bar{\varepsilon}$ is the equivalent strain and σ is the corresponding stress (could be the equivalent stress or principal stress) for different fracture criteria. Eq. (1) is the representation of the uncouple DFCs. The critical value C is a material constant and usually determined by upsetting and tensile tests. For the material prepared with a specific heat treatment condition, its unique C value can be obtained.

Among the six DFCs, Cockcroft and Latham criterion is one of the pioneer criteria in ductile fracture arena (Cockcroft et al., 1968), which was proposed based on the Freudenthal criterion. It is found that the flow stress at fracture point is not affected by the shape of the necked region in tensile test, which is different from the actual experiment scenario. The Cockcroft criterion is thus developed for bulk forming and is applicable to the deformation with low stress triaxiality. The simplified form of the criterion is denoted as:

$$\int_0^{\bar{\varepsilon}_f} \bar{\sigma} \cdot \frac{\sigma_1}{\bar{\sigma}} d\varepsilon = \int_0^{\bar{\varepsilon}_f} \sigma_1 d\varepsilon = C \tag{2}$$

In Eq. (2), $\bar{\sigma}$ is the equivalent stress, σ_1 is the maximum principal stress and $\left(\frac{\sigma_1}{\bar{\sigma}}\right)$ is a non-dimensional stress-concentration factor.

The Oyane model articulates the concept of ductile fracture with four development stages, viz., micro-scaled void formation caused by dislocation pile-up, void separation decreasing due to void growth, strain concentration, and the dimple initiation on the surfaces of material (Oyane et al., 1972). For porous materials, by employing the relationship between equivalent strain $\bar{\varepsilon}$ and volumetric strain ε_v, the stress strain relationship can be represented by Eq. (3) in the following.

$$d\varepsilon_v = \frac{d\bar{\varepsilon}}{\gamma f^2}\left(\frac{\sigma_m}{\bar{\sigma}} + \alpha_0\right) \tag{3}$$

where γ is the ratio between the nominal density ρ and the constituent metal density ρ_0 of porous material. f is a function of γ and expressed as $f = \frac{1}{3}\left(1+\sqrt{\frac{\gamma}{1-\gamma}}\right)$. σ_m is the hydrostatic stress and α_0 is a material constant. The volumetric strain ε_v represents the volume change of porous materials and can be designated as the ratio between the volume of the porous material v and the volume of the constituent metal with the same weight v_0 and denoted as:

$$\varepsilon_v = ln\frac{v}{v_0} = -ln\frac{\rho}{\rho_0} = -ln\gamma \tag{4}$$

In Eq. (3), the fracture strain can be determined if the fracture occurs at a particular volumetric strain. Eq. (3) can be further formulated in the form below:

$$\int \frac{\gamma f^2}{\alpha_0} d\varepsilon_v = \int \left(\frac{1}{\alpha_0} \cdot \frac{\sigma_m}{\bar{\sigma}} + 1 \right) d\bar{\varepsilon} \tag{5}$$

As α_0, γ and f are all material constants, Eq. (5) can be written as:

$$\int_0^{\bar{\varepsilon}_f} \left(\frac{1}{\alpha_0} \cdot \frac{\sigma_m}{\bar{\sigma}} + 1 \right) d\bar{\varepsilon} = C \tag{6}$$

where C is the critical value of this criterion.

The Ayada criterion is proposed based on the Cockcroft and Oyane criteria to provide an evaluation of fracture in compression-dominant deformation, since the result predicted by the Cockcroft criterion is unsatisfactory when the strain is large and the tensile stress is small (Ayada et al., 1984). The Ayada criterion is formulated in Eq. (7).

$$\int_0^{\bar{\varepsilon}_f} \left(\frac{\sigma_m}{\bar{\sigma}} \right) d\bar{\varepsilon} = C \tag{7}$$

where σ_m is the mean stress and C is the critical value of the criterion and considered to be inversely proportional to the hardness of materials, based on the experimental results.

The Brozzo criterion in Eq. (8) is established on the basis of the Cockcroft criterion and articulates the relationship between the maximum stress and mean stress when fracture happens (Brozzo et al., 1972).

$$\int_0^{\bar{\varepsilon}_f} \frac{2\sigma_1}{3(\sigma_1 - \sigma_m)} d\bar{\varepsilon} = C \tag{8}$$

where C is the critical value of the criterion.

The Rice and Tracey model is more focused on the modeling of fracture growth of micro void with sphere shape (Rice et al., 1969). The model assumes that when initial strain is zero, the fracture growth rate is affected by stress triaxiality $\frac{\sigma_m}{\overline{\sigma}}$ and is formulated as:.

$$\int_0^{\varepsilon_f} e^{\alpha \frac{\sigma_m}{\overline{\sigma}}} = C \tag{9}$$

where α is material constant and C is the critical value of the criterion.

The last criterion to be investigated in this research is the Freudenthal criterion, which describes the influence of damage accumulation in plastic deformation process (Freudenthal, 1950). This is effectively a critical strain energy density (SED) criterion, and the integral of the equivalent stress provides the physical meaning of the energy required to initiate a crack tip per unit area. The Freudenthal criterion is formulated as follows:

$$\int_0^{\varepsilon_f} \overline{\sigma} d\overline{\varepsilon} = C \tag{10}$$

When the size effect is considered, the Freudenthal criterion can be applied in the fracture prediction of micro-scaled flanged upsetting process and the result is acceptable (Ran et al., 2013).

In this research, the applicability of the above uncoupled DFCs in micro-scaled plastic deformation is systematically explored and analyzed.

3. Research methodology and experiments

3.1 Research methodology

The purpose of this research is to study the applicability of DFCs in micro-scaled plastic deformation. This is done by comparing the difference of the prediction results in a compressive deformation process using the above six commonly used DFCs, which are widely used in macro-scaled plastic deformation. By using the conventional flow stress model and the hybrid flow stress model developed in prior research (Ran et al., 2013), the influence of size effect on the applicability of the six DFCs in ductile fracture prediction for micro-scaled deformation is considered. In addition, the prediction's deviation from each criterion is evaluated using the conventional and hybrid flow stress models is calculated and compared. A generalized formulation for describing the commonly used uncoupled DFCs is proposed and an explanation for prediction's deviation is presented. The entire research methodology is illustrated in Figure 1.

3.2 Experiments

In this research, the multiphase alloy brass C3602 is used as the testing material. The composition of the alloy is Cu:59.0~63.0, Pb:1.8~3.7, Fe(max): 0.5, S(max):1.2, and the remainer is Zn. To obtain different microstructures of the test material, annealing was performed. The heat treatment conditions and the average grain sizes of the material after annealing are presented in Table 2.

From experimental perspective, flanged upsetting, which is one of the micro-scaled plastic deformation processes to fabricate the workpieces with different cross sections and heights, is used to examine the applicability of different DFCs in this research. The main reason to choose this forming process is that the process has a cross-shape shear band and the ductile fracture is easily occurs and is clearly observed in the shear band.

On the other hand, the micro-scaled backward extrusion is also conducted. This deformation process is used to evaluate the accuracy of the flow stress calculated using the hybrid constitutive stress model. The reason for the selection is that extrusion is widely used in industries and the micro backward extrusion is a useful compression-dominant deformation process with a very large deformation degree. By using the calculated flow stress data via the hybrid constitutive model (HCM), the simulation results of micro backward extrusion are compared with the experimental ones and the validity of HCM and each DFC can thus be investigated and revealed.

For the micro flanged upsetting process, the punch velocity is set to the minimum value of the machine of 0.01*mm/s* in such a way to ensure the strain rate does not affect the experimental result. In addition, all the specimens are compressed to 75% of the original length to facilitate the occurrence of ductile fracture. Meanwhile, the height and diameter ratio H/D of the sample is 1.5, as shown in Figure 2.

4. Criteria calibration in micro-scaled plastic deformation

4.1 Hybrid constitutive model

To explore the applicability of different DFCs, a hybrid model, which considers the influence of size effect and each phase of the multiphase alloy in micro-scaled plastic deformation, is presented in Eq. (11) (Ran et al., 2013).

$$\sigma_{total}(\varepsilon) = Mk_2\varepsilon^{n_2} + f_\alpha \alpha M \mu_\alpha b_\alpha \sqrt{\frac{C_1\varepsilon}{b_\alpha d}} + (1-f_\alpha)\alpha M \mu_\beta b_\beta \sqrt{\frac{C_2\varepsilon}{b_\beta d}}$$
$$+\lambda \cdot \left(mk_1\varepsilon^{n_1} - \left(Mk_2\varepsilon^{n_2} + f_\alpha \alpha M \mu_\alpha b_\alpha \sqrt{\frac{C_1\varepsilon}{b_\alpha d}} + (1-f_\alpha)\alpha M \mu_\beta b_\beta \sqrt{\frac{C_2\varepsilon}{b_\beta d}} \right) \right) \tag{11}$$

where $\sigma_{total}(\varepsilon)$ is the flow stress function, M is the Taylor factor, α is a phase-based particular factor of the alloy to describe the dislocation interaction. f_a is the volume fraction of α phase. μ_α and μ_b are the shear module of α and β phases. b_α and b_β are the Burgers vectors of FCC and BCC phases. d is the grain size. λ is the size factor. m is the grain orientation factor. $k_1, k_2, n_1, n_2, C_1, C_2$ are material constant. The model describes the stress contribution of each phase by using dislocation density and the influence of size effect via the size factor λ.

In this research, all the unknown coefficients in Eq. (11) are determined based on all the known coefficients and using curve fitting approach. The final form of the hybrid constitutive model is formulated in Eq. (12) (Ran et al., 2013):

$$\sigma_{total}(\varepsilon) = 587.52\varepsilon^{0.23} + 0.681\varepsilon^{0.5} \cdot d^{-\frac{1}{2}} - \lambda \cdot \left(203.52\varepsilon^{0.23} + 0.681\varepsilon^{0.5} \cdot d^{-\frac{1}{2}} \right) \tag{12}$$

The above model is used to predict the flow stress behavior in both macro- and micro-scaled deformation scenarios via considering size effect. Since this model is established using the results of micro-scaled upsetting process, it can also be used to describe the equivalent stress in other micro-scaled deformation processes.

4.2 Hybrid-constitutive-model-based fracture prediction

The hybrid constitutive model in Eq. (13) is used for analysis of micro-scaled plastic deformation (Ran et al., 2013). In this research, this model is implemented to obtain the fracture critical value C for the testing material.

$$C = \int_0^{\varepsilon_f} \left(587.52\varepsilon^{0.23} + 0.681\varepsilon^{0.5} \cdot d^{-\frac{1}{2}} - \lambda \cdot \left(203.52\varepsilon^{0.23} + 0.681\varepsilon^{0.5} \cdot d^{-\frac{1}{2}} \right) \right) d\varepsilon \tag{13}$$

The physical meaning of the hybrid constitutive model is the threshold energy required to initiate the stress-induced fracture with the fracture strain. When other uncoupled DFCs such as Cockcroft-model-based and stress-triaxiality-based fracture criteria are implemented, the mean stress and the maximum principal stress are needed for calculation of the critical value of each DFC. The upsetting simulation of each scenario with a specific heat treatment condition to obtain the principal stress is conducted by using DEFORM 3D. All the upsetting simulation results are extracted and summarized into the stress-strain relationship in the form of $\sigma = k\varepsilon^n$ via curve fitting. The summarized results of the mean stress and the maximum principal stress are listed in Table 3.

By incorporating the corresponding parameters into each criterion, the critical value is thus determined. The predicted fracture strain in simple upsetting process for each case is calculated and presented in Table 4. Table 4 presents the predicted fracture strain for each scenario of the testing material with different heat treatment conditions and dimensions.

4.3 Fracture prediction with the conventional stress model (without considering size effect)

As the size factor is introduced in the hybrid-constitutive-model-based fracture model (HFM), the conventional-constitutive–model-based fracture model (CFM) is thus needed to reveal the influence of size effect by comparing with the prediction accuracy of HFM. The size factor dependent part of Eq. (11) is removed and the flow stress model without considering the size effect is described in the following (Ran et al., 2013):

$$\sigma_{total}\left(\varepsilon\right)=Mk_2\varepsilon^{n_2}+f_\alpha\alpha M\mu_\alpha b_\alpha\sqrt{\frac{C_3\varepsilon}{b_\alpha L^s}+\frac{C_1\varepsilon}{b_\alpha d}}+\left(1-f_\alpha\right)\alpha M\mu_\beta b_\beta\sqrt{\frac{C_3\varepsilon}{b_\beta L^s}+\frac{C_2\varepsilon}{b_\beta d}} \tag{14}$$

By applying Eq. (14) to each fracture criterion, the predicted fracture strain of CFM is obtained and presented in Table 5.

5. Result analysis and discussion

5.1 Comparison of the predicted fracture strain

After the predicted fracture strain is calculated, it can be compared with the actual experimental results. Figure 3 shows the actual and the predicted fracture strains using different fracture criteria in upsetting process. These fracture strains are calculated by using Eq. (13) once the critical value is obtained. In Figure 3(a), it is found that the prediction results using the Brozzo and Ayada criteria are the closest to the experimental ones with the deviation of 4.9%. Meanwhile, the Freudenthal model gives the worst performance with the deviation of 12.6% compared with the experiment. This could be caused by the different grain sizes of macro and micro-scaled specimens. To distinguish the different stress contributions arising from the grain and feature size effects, the macro and micro-scaled specimens are annealed with the same heat treatment condition. Figure 3 (b) shows the result comparison in micro scale. In the simple upsetting process with the specimen dimension of $0.5\times0.75mm$, the specimen does not have any visible macro-crack when the specimen is compressed with the reduction of 75%. The long dash line in the picture is the predicted flow stress curve if no fracture occurs in the deformation process. For the micro-scaled simple upsetting, however, the Freudenthal criterion provides the best result with the deviation of only 3.7% between the calculation and experiment. The Brozzo criterion, on the other hand, has the worst performance with the deviation of 38.2%. Therefore, the energy based criterion seems to have a better performance in analysis of micro compression deformation.

5.2 Stress-induced fracture map

The stress-induced fracture map (SFM) is a useful tool to analyze the different contribution of both the grain and feature size effects. It can be used to reveal the influence of grain and feature size effects when the size factor is the same. For the given material and using the Freudenthal fracture criterion, the SFM is established based on the fracture critical value C calculated by using Eq. 13, the billet diameter d and the predicted fracture strain ε_f, as shown in Figure 4.

5.3 Stress induced fracture map using different fracture criteria

By using the predicted fracture strain presented in Tables 4 and 5, the SFM based on the hybrid model and the conventional model can thus be constructed. Figure 5 shows the SFM comparison between experiment and simulation. The experimental results represented by red star have the similar shape with the SFM of the Freudenthal criterion, as shown in Figure 5(a). It indicates that the Freudenthal criterion can be used for both the macro- and micro-scaled fracture prediction and provides relatively accurate results.

Figure 5(b-i) also shows the SFMs of other five uncoupled DFCs. Although the values of the actual fracture strain are the same, the SFM shapes of the actual fracture strain are different based on the Oyane, Ayada and Rice & Tracey DFCs. The tendencies of different SFMs will be explained in Section 5.4.

Figures 6, 7 and 8 present the deviation between the predicted and the actual fractures for each DFC. These three figures are used to evaluate the influence of size factor in fracture prediction. In Figure 6, both the HFM and CFM show a good result in macro-scaled plastic deformation. However, the deviation of HFM is less than 11% in micro-scaled plastic deformation process, while the deviation of CFM is 20.77%~31.61% for the scenarios with different heat treatment conditions. The different deviation indicates the influence of size effect must be considered if the Freudenthal fracture criterion is used in micro-scaled scenario.

In addition, Figure 7 shows the deviation between the experimental result and the calculation ones based on Brozzo and C&L criteria. The predicted results with and without considering size effect are the same, which means the size effect does not directly affect the result of the predicted fracture strain. When the specimen dimension is less than 0.5mm, the predicted result has an over 40% deviation compared with the experimental results. Thus, these two fracture criteria are not suitable for the analysis and prediction of fracture in micro-scale.

Furthermore, the deviation percentage of the rest three fracture criteria including Ayada, Oyane and Rice & Tracey criteria are shown in Figure 8. These three criteria have one thing in common: the fracture prediction result by CFM is even more accurate than the result by HFM. The reason for this will be explained in Section 5.4.

5.4 The generalized fracture model formulation

Among various DFCs, the uncoupled criteria consider the damage accumulation in deformation process. Meanwhile, the coupled criteria assume that the most ductile fracture is caused by void nucleation, accumulation and growth, which further lead to macro-scaled fracture. The coupled criteria introduce the damage factor D to simulate the void growth in plastic deformation process. As most of these criteria use tensile test to determine the critical value, their application in prediction of tensile-dominant deformation is acceptable. However, their efficiency in compression-dominant deformation is unsatisfactory.

Void-growth-based fracture criteria such as GTN and McClintock DFCs are not well applicable to compression-dominant deformation. The main reason is that the fracture initiation and growth in tensile test is caused by both the void growth and shear stress concentration. In compression-dominant deformation, however, void can hardly exist inside the specimen. Unlike the tensile-dominant deformation, when the brittle phase in the multiphase metal is broken due to compression stress, the formed void will immediately be filled up by its surrounded material. Thus, the existence of ductile fracture in the compression-dominant deformation is mainly caused by shear stress concentration.

For the conventional DFCs listed in Table 1, the critical value is a key factor to evaluate the existence of ductile fracture. As mentioned in Eq. (1), the integral of the strain related function changes in deformation process and becomes the critical value C when the strain value reaches the fracture strain, as shown in Eq. (15).

$$F\left(\varepsilon_f\right)=C \tag{15}$$

In Eq. (15), $F(\varepsilon)$ is the damage value function, ε_f is the fracture strain and C is the critical value. For each fracture criterion, its simplified form can be written as a strain related exponent function.

In tensile-dominant deformation, stress triaxiality is critical to determine whether void growth or shear stress concentration has a major contribution to fracture initiation. In compression deformation, stress triaxiality and non-dimensional stress-concentration factor, which is presented in Cockcroft & Latham model, are considered as the two important factors which affect the damage value function $F(\varepsilon)$. Table 6 presents the generalized formulations of different DFCs. Most of the uncoupled DFCs can be represented by the integral of stress triaxiality, non-dimensional stress-concentration factor and mean stress. The damage value function is described by Eq. (16)

$$F = \int \mu(\varepsilon)^{n_1} \cdot \eta(\varepsilon)^{n_2} \cdot {\sigma_m}^{n_3} d\varepsilon \tag{16}$$

In Eq. (16), $\mu(\varepsilon)$ is the stress concentration factor equal to $\frac{\sigma_1}{\bar{\sigma}}$, $\eta(\varepsilon)$ is the stress triaxiality and designated as $\frac{\sigma_m}{\bar{\sigma}}$. This equation reveals the physical meaning of the damage value function: the fracture initiation is caused by stress concentration and affected by the contribution of void growth and the magnitude of deformation force.

Regarding the deviation of DFCs presented in Section 5.3, it can be explained by the general formulations presented in Table 6. In Table 6, the stress triaxiality of Freudenthal criteria is the only one which is inversely proportional to damage value function. According to the deformation of multiphase alloy, the decrease of stress triaxiality means that the main contribution to fracture initiation comes from shear stress concentration. In upsetting experiment, it is found that when the specimen size becomes smaller, the stress triaxiality in macro-scaled deformation is smaller than that in micro-scaled deformation, as shown in Figure 9(a). The equation of Freudenthal criterion in Table 6 can thus be re-designated in the following:

$$F = \int \frac{1}{\eta_{\lambda-dep}} \cdot \sigma_m d\varepsilon_{\lambda-dep} = \int \frac{1}{\dfrac{\sigma_m}{587.52\varepsilon^{0.23} + 0.681\varepsilon^{0.5} \cdot d^{-\frac{1}{2}} - \lambda \cdot \left(203.52\varepsilon^{0.23} + 0.681\varepsilon^{0.5} \cdot d^{-\frac{1}{2}}\right)}} \cdot \sigma_m d\varepsilon_{\lambda-dep}$$

$$F = \int \frac{1}{\eta_{\lambda-ind}} \cdot \sigma_m d\varepsilon_{\lambda-ind} = \int \frac{1}{\dfrac{\sigma_m}{587.52\varepsilon^{0.23} + 0.681\varepsilon^{0.5} \cdot d^{-\frac{1}{2}}}} \cdot \sigma_m d\varepsilon_{\lambda-ind} \tag{17}$$

where $\varepsilon_{\lambda-dep}$ and $\eta_{\lambda-dep}$ are the equivalent strain and stress triaxiality which consider the influence of size effect, while $\varepsilon_{\lambda-ind}$ and $\eta_{\lambda-ind}$ are the equivalent strain and stress triaxiality without considering size effect.

The first equation of Eq. (17) is the damage value function considering size effect, but the second one does not consider this effect. In micro scale, it is obvious that the stress triaxiality considering size effect $\eta_{\lambda-dep}$ is larger than the one which does not consider size effect $\eta_{\lambda-ind}$. To obtain the same damage value *F*, the predicted fracture strain $\varepsilon_{\lambda-dep}$, which considers size effect, needs to be larger than $\varepsilon_{\lambda-ind}$. The predicted fracture strain is thus closer to the experimental results.

The stress triaxiality in Ayada, Oyane and Rice & Tracey criteria, on the other hand, are all proportional to the damage value function. The influence of size factor makes the calculation result even more deviated from the experimental result. Taking Ayada criterion as an instance, which can be re-designated in Eq. (18) as follows:

$$F=\int \eta_{\lambda-dep} d\varepsilon_{\lambda-dep}=\int \frac{\sigma_m}{587.52\varepsilon^{0.23}+0.681\varepsilon^{0.5}\cdot d^{-\frac{1}{2}}-\lambda\cdot\left(203.52\varepsilon^{0.23}+0.681\varepsilon^{0.5}\cdot d^{-\frac{1}{2}}\right)} d\varepsilon_{\lambda-dep}$$
$$F=\int \eta_{\lambda-ind} d\varepsilon_{\lambda-ind}=\int \frac{\sigma_m}{587.52\varepsilon^{0.23}+0.681\varepsilon^{0.5}\cdot d^{-\frac{1}{2}}} d\varepsilon_{\lambda-ind} \quad (18)$$

As mentioned above, $\eta_{\lambda-dep}$ is larger than $\eta_{\lambda-ind}$, $\varepsilon_{\lambda-dep}$ needs to be smaller than $\varepsilon_{\lambda-ind}$ to obtain the same damage value and thus makes $\varepsilon_{\lambda-dep}$ more deviated from the experimental result. This conclusion reveals why the fracture prediction result by using the conventional constitutive model is better than the one using the hybrid constitutive model when Ayada, Oyane and Rice & Tracey DFCs are used.

Based on HFM and the data obtained in the previous section, the FE simulations for micro-scaled flanged upsetting and backward extrusion are conducted in a finite element software system called DEFORM 3D, and the results are presented in Figures 10, 11, 12 and 13. In Figure 10, 12 and 13, there is an obvious deviation between the simulation and experimental results. This is mainly caused by the simulation setting in DEFORM. In simulation process, the distorted meshes should be considered to be invalid and deleted when it reaches the critical value C. When some fracture criteria are used, some of these distorted meshes will “disappear” before they reach the critical value *C*, as the billet will be re-meshed when massive distortion occurs. The deviation is thus caused in between the simulation and experimental results. In Figure 11, as no major cracks exist, the simulation and experimental result are very close.

In Figure 10, it is found that the Freudenthal fracture criterion is the only applicable DFC that can predict fracture in macro-scaled flanged upsetting. For other fracture criteria, as the damage value does not reach its own critical value, no fracture initiates in these scenarios, and their load stroke curves are almost the same. In Figure 11, the simulation result of the micro-scaled flanged upsetting shows that no fracture exists by applying all the DFCs. As the load-stroke curve is pretty close to the experimental result, the Freudenthal criterion is thus considered to be the most suitable DFC for analysis of micro-scaled flanged upsetting process.

In Figures 12 and 13, all the load-stroke curves of backward extrusion using different DFCs have the same trend with the actual experimental results. When the stroke is higher than 0.4*mm*, the deviation between the load obtained by simulation and the one obtained from the actual experiment begins to increase. For both the as-received and the annealed samples at $750^{\circ}C$, the load-stroke curves based on the Freudenthal and Rice & Tracey DFCs are close to the experimental results. The "valley" on the load stroke curve of the as-received samples by using the Freudenthal fracture criterion may be caused by the remeshing in the course of finite element simulation.

6. Conclusions

The applicability of six most widely used fracture criteria in micro-scaled plastic deformation is examined and calibrated. Each DFC together with the hybrid constitutive model is implemented for fracture prediction in both the macro- and micro-scaled deformation scenarios. Through simulation and experiment, the following conclusions are made:

(1) The Freudenthal criterion is the most suitable criterion for analysis of ductile fracture in compression-dominant deformation processes in both the macro and micro scales among the investigated six criteria. In micro-scaled plastic deformation, the deviation between the predicted fracture strain and the experimental one is 4.23%~10.77% when the influence of size effect is considered. But it is 20.77%~31.61% without considering the size effect. This indicates that the influence of size effect is significant in micro-scaled deformation. In addition, the SFM generated based on the Freudenthal fracture model has a similar trend with the SFM constructed using the actual experimental data. Furthermore, the two verification experiments, i.e. the flanged upsetting and backward extrusion, have shown that the simulation results by using the Freudenthal fracture model are closed to the actual experimental results. In macro-scaled plastic deformation, the simulation result also shows that the Freudenthal fracture criterion is the most applicable criterion that can predict the ductile fracture in compression-dominant plastic deformation well.

(2) In compression-dominant deformation, the applicability of Cockcroft & Latham and Brozzo criteria is limited. In micro-scaled forming, the deviation of fracture strain between the actual experimental results and the predicted ones by using the two fracture criteria is 22.99% ~ 43.08%. The Ayada, Rice and Tracey, and Oyane criteria are able to predict the ductile fracture in both the macro and micro scales when the constitutive model without considering size effect is used. In micro-scaled forming, the fracture strain deviation between the experimental results and the ones predicted by the three fracture criteria with the built-in size-effect-dependent fracture constitutive model is 30.28%~56.15%, while the deviation in

using the three criteria with the constitutive model without considering size-effect is 19.71%~48.46%. It is obvious that introducing the size factor into the three DFCs makes the prediction results even more deviation from the experiment ones.

(3) By using the Freudenthal criterion, only the flow stress curve is needed for fracture prediction. Other DFCs, however, need principal stress to calculate the fracture critical value *C*. The Freudenthal DFC is thus the most convenient one for fracture prediction in micro-scaled deformation.

(4) A generalized representation of the six uncoupled fracture model is presented in this research. The general formulation is established to explain why the fracture model without considering size effect provides a better prediction than the one considering size effect when the Ayada, Rice and Tracey, Oyane criteria are used.

Acknowledgements

The authors would like to thank the funding support to this research from the General Research Fund of Hong Kong Government under the project of 515012 (B-Q33F) and the projects of G-YK10, G-YM93, G-U923 and G-UB59 from The Hong Kong Polytechnic University.

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