

Forming limit of sheet metals in micro/meso scale plastic forming by using different failure criteria

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ABSTRACT

The ductile failure of sheet metals in micro/meso scale plastic deformation is influenced by grain and geometry size effects. Based on the forming limit experiments of copper sheet metals with different grain sizes, it is found that there is a significant reduction of forming limit with the increase of grain size under different deformation paths. To describe the size effect induced decrease of forming limit, a number of the most widely-used failure criteria and theories were employed to investigate their applicability in meso-scale plastic deformation, including the Swift/Hill criteria, Marciniak-Kuczynski model, ductile fracture criteria such as Freudenthal, Cockcroft & Latham, Ayada and Oyane models, and the Gurson-Tvergaard-Needleman model coupled with the Thomason void coalescence model (GTN-Thomason model). The applicability of these criteria and the mechanism behind them were discussed for better characterization of the failure behavior at micro/meso scale. In addition, to corroborate the developed method, meso-scale hydroforming experiments of sheet metals was conducted. The M-K model and the GTN-Thomason model are revealed to

be able to accurately predict the ultimate pressure and the height at the onset of failure by comparing to the experimental results.

Keywords: Size effect, meso-scale plastic deformation, forming limit, ductile fracture criterion, sheet metal.

1. Introduction

The plastic deformations of workpiece with at least two dimensions less than 1.0 mm and in the range of 1 to 10 mm are termed as micro- and meso- scale deformations respectively. By using these deformations, the so-called micro- and mesoforming have emerged and attracted the research focus in metal forming arena for a decade due to its significant advantages such as high productivity, good material properties and near net shape [1-3]. However, the so-called size effect has been found to play an important role in microforming and the traditional forming knowledge in macro-scale domain may not be fully valid in micro-scale scenario [4, 5]. Therefore, a lot of works have been conducted to explore and study the mechanisms and behaviors of plastic deformation at micro/meso scale [6]. The size effects of deformation behaviors in microforming process have been well explored and understood. However, the size effect of forming limit and ductile fracture (DF) behavior, which is one of the eluded and tantalized research issues and directly determines the formability of material in meso-scale forming processes, has not yet been explored thoroughly.

In recent years some researchers begin to notice and study this issue by experiments and analysis. Among them, Vollertsen et al. [7] carried out the pneumatic bulge tests of aluminum alloys. A highly irregular local distribution of strain was observed and the fracture was revealed to occur randomly within the deformation zone. They pointed out that one possible reason for these behaviors is the non-uniformity in the flow behavior of material due to the large difference of grain size. Fu and Chan [8] studied the tensile test of the annealed copper foils with different thicknesses and grain sizes. The fracture stress and strain as well as the

number of micro-voids were identified to decrease with the thickness-to-grain-size ratio (t/d). They also developed a dislocation density based model with the consideration of size effect and the prediction results were also compared with the experimental ones. Ran et al. [9, 10] investigated the DF behavior of brass material in both the micro and macro scale flanged upsetting. They found that the fracture on the flanged surface is easier to form in macro scale. The step-like shear dimples on transgranular fracture surface were also observed and the dimple size and behavior are also considered to be caused greatly by size effect. A hybrid model considering multiphase of material and size effect was further proposed to predict the fracture in microforming. Moreover, Ben Hmida et al. [11] revealed that the formability of copper foils deteriorates with the reduction of t/d based on the single point incremental forming experiments. Furushima et al. [12] focused on the fracture and free surface roughening behavior of copper foils during the uniaxial tensile test. They concluded that the fracture strain decreases with the thickness of foil specimen. The significant increase in surface roughness is also observed with the decrease of foil thickness.

On the other hand, most of the previous studies are aimed at one or several deformation conditions and there is still a lack of knowledge on how the size effect affects the forming limit and DF behavior under various deformation conditions in microforming process of sheet metals. Forming limit curve (FLC) is proved to be an efficient method to characterize the formability of sheet metals under various deformation conditions and has been widely employed in industry [13]. Therefore, revealing the FLC of sheet metals at micro/meso scale is an attractive and feasible research issue to understand the fundamental mechanism of size

effect on the micro/meso-scale failure behavior. In our previous investigations [14, 15], the forming limit experiments were conducted and the size effects on FLC were analyzed. Nevertheless, further researching efforts on the FLC predictive theories and modeling methods with the consideration of size effect are still required.

At macro scale, the modeling and improving of FLC methods have been the focus of researchers and engineers for better characterizing and predicting the necking and DF behavior of sheet metals under various proportional or non proportional deformation conditions in recent years. Bruschi et al. [16] reviewed the testing and modeling advances in the sheet metals forming field in recent years. With Regard to the modeling methods of forming limit, they pointed out that the current researches are focusing on the phenomenological characterization, physical modeling and microstructural modeling methods of the forming limit of sheet metals.

In the phenomenological characterization scenario, forming limit stress diagram (FLSD) developed by Stoughton [17] has been studied a lot due to its significant advantage of strain path independency. Systematical investigations on different extensions of forming limit diagram (FLD) and FLSD were performed by Paul et al. [18, 19] based on experiments and different instability criteria to analyze the path independency and accuracy of different methods. In addition, Zhalehfar et al. [20]. also found that the FLSD is independent of changing loading paths by applying biaxial and uniaxial prestrain. Furthermore, the fracture forming limit diagram (FFLD) is also attracting considerable attention [21, 22] as some

researchers have revealed that the fracture limits the attainable deformation before necking happens in some circumstances such as the deep drawn process of part with complex structures [23] and the single-point incremental forming process [24]. Different fracture criteria were also employed to predict and discuss FFLD based on the experimental results [25-28]. Regarding to the physical modeling field, different failure models such as the Marciniak–Kuczinski (M-K) model, micromechanical damage models and continuum damage models have been employed to investigate the prediction accuracy and applicability in sheet metal forming process. Among them, Msolli et al. [29] obtained the FLC based on finite element (FE) simulation by employing the Lemaitre damage model with the consideration of anisotropy. The simulated results were then analyzed and verified by experiments in their study. By employing the Gurson-Tvergaard-Needleman (GTN) model, Abbassi et al. [30] predicted the FLC of tailor welded blank with reasonable accuracy. Uthaisangsuk et al. [31] also analyzed the FLC of steel sheet metal with GTN model and compared it with experimental results. Moreover, regarding to the microstructural analysis and modeling methods which are considered to be able to describe the intrinsic failure behavior of material, a lot of researching efforts have been performed both analytically and experimentally to reveal and understand the mechanisms of failure at different levels. In this field, Signorelli et al. [32, 33] predicted the FLD of polycrystal sheet metals by combining the M-K model and the visco-plastic self-consistent crystal-plasticity model. The effects of slip hardening, strain rate sensitivity, anisotropic etc. were induced. The predicted results were found to agree with the experimental ones for different polycrystalline materials. Wang et al. [34, 35] investigated the formability of magnesium alloy by using the elastic

visco-plastic self-consistent crystal plasticity model coupled with the M-K model. The numerical results are found to be in good qualitative agreement with the experimental observations. Wesenjak et al. [36] investigated the onset of instabilities in the microstructure of dual-phase steels by employing a decoupled sequential multiple length scale modeling approach. A good estimation of the tendencies of experimentally determined FLC was obtained by utilizing this multi-scale approach.

Although the forming limit experiments and theories have been extensively studied in macro scale, the mechanism of how the size effect affects the FLC and fracture forming limit curves (FFLC) at micro/meso scale still needs further investigation. In our previous investigations [14, 15], the forming limit experiments were conducted and the size effects on the FLC were analyzed based on the modified Oyane and GTN model. However, further discussion on the mechanism of the size effect affected necking and DF behavior of sheet metal at micro/meso scale is still required. Considering that different failure theories have been well developed at macro scale, it is thus an important issue to verify their applicability to establish a fundamental understanding of the failure mechanism at meso scale. In tandem with this, the forming limit experiments of sheet metals with different grain sizes are first analyzed. Different models including the Swift/Hill criteria, the M-K model, different uncoupled DFCs and the GTN-Thomason model are explored and applied in formability analysis and FLC/FFLC construction. The results of different models are then discussed and analyzed to study their applicability in meso-scale deformation of sheet metals. In addition, the meso-scale hydroforming experiments are also conducted to verify the applicability of the

models. This research thus provides a basis for understanding and modeling of the formability of meso scale sheet forming process, which is helpful in microforming process design and optimization to prevent the occurrence of failure and to improve the productivity.

2. Forming limit experiments and results

2.1 Experimental setup and specimen preparation

The experimental setup is shown in Fig. 1. Six different geometries of specimens (No.1 to 6) were designed based on Holmberg [37] and Marciniak tests to obtain different deformation paths. The specimens were first painted with a random pattern. After that the specimens were stretched until fracture occurs. Meanwhile a high resolution digital camera (MV-3000 UC, three megapixels resolution) was employed to take continuous images every two seconds. The limit strain was then measured according to the digital image correlation (DIC) method by analyzing the evolution of random pattern. The measuring process was described in our previous study [15] following ISO 12004-2: 2008. The experimental results of FLC at different grain size conditions were also reported in Ref. [14, 15]. To further characterize the size effect on the failure of sheet metals and to verify the applicability of different necking and fracture criteria at meso scale, the fracture limit strains were also calculated in this study. Fig. 2 demonstrates the strain distributions of deformation just before and after the DF. The major and minor strains just before the macroscopic fracture obtained by the DIC method were recorded as the fracture limit strains. A similar method was also reported in the experimental investigations conducted by Song et al. [26].

Fig. 1. The experimental setup for forming limit tests [14].

Fig. 2. The strain distribution images onset of DF.

Cu-FRHC sheet metals with the thickness of 0.2 and 0.4 mm were used as the experimental material and annealed at different temperatures as shown in Table 1. Metallographic examinations were then performed and the microstructures of different samples are shown in Fig. 3. Uniaxial tensile tests were conducted to analyze the flow stress of the specimens with different grain sizes. As shown in Fig. 3, a significant reduction of flow stress with the increase of grain size is observed. According to the well-known surface layer model, the polycrystalline metal material is assumed to be constructed by surface and inner grains. The surface grains are less constrained than the inner ones because of the free surfaces. Hence the grain rotation and sliding can perform more easily [8] in the surface grains, which lead to the lower flow stress. With the increase of grain size, t/d decreases and the surface grains take a greater proportion. Therefore, the flow stress decreases.

The true strain-stress curves were fitted with Hollomon equation [38] and the results are shown in Table 2. The fitted results can be used in section 3 for the analytical predictions of forming limit based on different models.

Fig. 3. The true stress-strain curves of specimens with different thicknesses:

(a) 0.4 mm; (b) 0.2 mm.

Table 1. The heat treatment parameters and results [14].

Table 2. The fitted results of Hollomon equation.

2.2 Results and discussion

The forming limit results (FLC and FFLC) are shown in Fig. 4. The average necking and fracture limit strain for each type of specimen is shown in Fig. 5. Before t/d decreases to 2 or less, the significant reduction of forming limit with the decrease of t/d can be observed for each deformation condition, indicating the deterioration of the formability of sheet metals. In addition, the scatter of limit strain increases significantly for the specimens with the grain size of $166.3 \mu m$ and the thickness of 0.2 mm ($t/d < 2$). Therefore, the uncertainty of forming process increases when there is only one or two grains over the thickness.

Fig. 4. The forming limit results obtained for the specimens with different grain sizes:

(a) 0.4 mm specimens; (b) 0.2 mm specimens.

Fig. 5. The average limit strains of different specimens:

(a) The necking limit strains of 0.4 mm specimens;

(b) The necking limit strains of 0.2 mm specimens;

(c) The fracture limit strains of 0.4 mm specimens;

(d) The fracture limit strains of 0.2 mm specimens.

The strain distributions of the specimens with different grain sizes were calculated by processing the deforming patterns of specimen based on the DIC method. Some of the results are shown in Fig. 6. For the cases with the smallest grain size, the strain is uniformly distributed during the deformation until necking and strain localization starts. In contrast, the inhomogeneous distribution of strain becomes more significant for the specimens with the largest grain size. The deformation is found to localized at random spots even at the early

stage of loading as illustrated in Fig. 6 (b) and (d).

Fig. 6. The strain distributions during the experiments:

- (a) No. 3 specimen with the thickness of 0.4 mm and the grain size of 23.7 μm ;
- (b) No. 3 specimen with the thickness of 0.4 mm and the grain size of 132.2 μm ;
- (c) No. 4 specimen with the thickness of 0.2 mm and the grain size of 17.4 μm ;
- (d) No. 4 specimen with the thickness of 0.2 mm and the grain size of 166.3 μm .

In order to further investigate the fracture mechanism under different conditions, Scanning Electron Microscope (SEM) observations were carried out and the fracture surface topography is shown in Fig. 7. It can be observed that the number of micro voids decreases with the increase of grain size. Similar observations were also made by Fu et al. [8] and Furushima et al. [12]. In addition, the complex deformation of material can be observed for the conditions with t/d greater than 5. However, the significant slips of single crystal can be clearly seen near the fracture edge when t/d decreases to 3 or less.

Fig. 7. SEM observations of the fracture topography [14, 15]:

- (a) 0.4 mm; (b) 0.2 mm.

To explain the experimental observations, the widely-accepted surface layer model is used. According to the model, the surface grains take a greater proportion with the increase of grain size. Since the surface grains are less constrained, their deformation is affected by the orientation and properties of individual grain [3]. The material property thus becomes more inhomogeneous which leads to the random strain localization at weak spots as illustrated in

Fig. 6. As t/d approaches to two, the surface grains play a dominant role. Hence the significant slips of single crystals can be observed as shown in Fig. 7. In addition, considering that the dislocations are less likely to be blocked by grain boundaries in the surface grains [8], the voids are less likely to nucleate at the surface layer either. With the reduction of t/d , the inner layer becomes thinner which makes it easier for void coalescence to take place and for micro cracks to extend through the thickness direction leading to the macroscopic fracture. Therefore the FLC shifts down with the increase of grain size as revealed in Fig. 4. When t/d decreases to two or less, the inner layer can be ignored and the formability of sheet metal is determined by the surface grains. Since the surface grains are less constrained, the behaviors of different individual grains affect the ductile failure process significantly. That leads to the reduction of repeatability during the forming limit experiments. Hence the scatter of limit strain data increases significantly as shown in Fig. 5.

3. FLC prediction based on different theoretical models

Various theories and models have been established to predict the FLC of sheet metals in macro scale. Some of the most widely-used models, including Swift/Hill criteria, M-K model, uncoupled ductile fracture criteria and GTN-Thomason damage model, were studied in this research to verify their applicabilities in micro-scale sheet metal forming.

3.1 Swift diffuse and Hill localized criteria

According to Hosford [39], the Swift diffuse [40] and Hill localized [41] criteria can be described as the following equation:

$$\begin{cases} \varepsilon_1^{limit} = \frac{n}{1+\rho} & -\frac{1}{2} < \rho < 0, \text{ Hill} \\ \varepsilon_1^{limit} = \frac{2n(1+\rho+\rho^2)}{(1+\rho)(2\rho^2-\rho+2)} & 0 \leq \rho \leq 1, \text{ Swift} \end{cases} \quad (1)$$

where $\rho = \frac{\varepsilon_2}{\varepsilon_1}$ represents the different loading conditions of FLD. ε_1 and ε_2 are the major and minor principal strains, respectively. After the major limit strain ε_1^{limit} is obtained, the minor limit strain $\varepsilon_2^{limit} = \rho\varepsilon_1^{limit}$ can be calculated. According to Eq. , the predicted results based on Swift and Hill criteria are solely related to the hardening coefficient n . The predicted and experimental results are shown in Fig. 8.

Fig. 8. The predicted FLCs based on Swift and Hill criteria:

(a) 0.4 mm; (b) 0.2 mm.

According to Fig. 8, the difference between the experimental and predicted results becomes more significant with the increase of grain size. This is because the Swift/Hill criteria are developed based on the homogeneous continuum hypothesis. However, it has been revealed that the influence of individual grains on the deformation and formability becomes more evident according to the SEM observations. The anisotropic and inhomogeneous deformation behaviors of micro/meso-scale material structure with only several grains have a significant influence on the failure behavior of material. Therefore, the Swift/Hill criteria may not be able to describe the intrinsic mechanism of the size effect on FLC. In order to deal with the failure behavior at meso scale, further study on the Swift/Hill criteria need to be performed to consider the anisotropic and complicated flow and necking behavior of individual grains. Moreover, the effect of grain boundary on the failure behavior also need to be reconsidered.

3.2 M-K model

According to Marciniak et al. [42], an initially geometrical non-homogeneity is assumed and shown in Fig. 9. As the deformation increases, the strain localization becomes more significant at region B. When the differences of major principal strains between regions A and B satisfy $\Delta\varepsilon_1^B \gg \Delta\varepsilon_1^A$, the localized necking is considered to occur at region B.

Fig. 9. The schematic of M-K model.

In addition, $f_0 = \frac{t_0^B}{t_0^A}$ is used to represent the initial ratio between the thicknesses of defective and flawless areas. Based on Mises yield criterion, Levy-Mises flow rule and plane stress assumption, the strain and stress states in the two regions are analyzed. The following equations can be obtained:

$$\left\{ \begin{array}{l} \frac{\varepsilon_2}{\varepsilon_1} = \frac{d\varepsilon_2}{d\varepsilon_1} = \rho \\ \bar{\varepsilon} = \frac{2}{3}\sqrt{3}\varepsilon_1\sqrt{\rho^2 + \rho + 1} \\ \eta^B = \frac{\sigma_2^B}{\sigma_1^B} = \frac{2\rho_B + 1}{2 + \rho_B} \\ \varphi^B = \frac{\bar{\sigma}^B}{\sigma_1^B} = \sqrt{1 - \eta^B + (\eta^B)^2} \\ \phi = \frac{\Delta\bar{\varepsilon}^B}{\Delta\varepsilon_1^B} = \frac{2\varphi^B}{2 - \eta^B} \\ \frac{\rho + 2}{\sqrt{3(\rho^2 + \rho + 1)}}\bar{\sigma}^A - \frac{\bar{\sigma}^B}{\varphi^B}f_0 \left(e^{\frac{\varepsilon_3^B}{\varepsilon_1^B} + \frac{\Delta\bar{\varepsilon}^B}{\phi}(-1 - \rho_B)} - e^{\frac{\sqrt{3}}{2}\frac{\rho + 1}{\sqrt{\rho^2 + \rho + 1}}\bar{\varepsilon}^A} \right) = 0 \end{array} \right. \quad (2)$$

If the incremental major principal strain $\Delta\varepsilon_1^A$ is given, $\Delta\varepsilon_1^B$ can be calculated based on Eq. by using Newton-Ranphson method. If $\Delta\varepsilon_1^B / \Delta\varepsilon_1^A \geq 7$ is satisfied, it is believed that strain localization occurs at region B and the major and minor strains of region A are recorded as

the limit strains. Otherwise another increment of $\Delta\varepsilon_1^d$ is given and the previous calculation continues. The predicted results are shown in Fig. 10.

Fig. 10. Prediction of the FLCs based on the M-K model:

(a) 0.4 mm; (b) 0.2 mm.

By optimizing the value of f_0 , the predicted FLCs are in accordance with the experimental results of different grain size conditions. However, the specimens employed in the experiments were carefully prepared during the heat treatment. The surface texture and flaw depth are the same for different grain size conditions. Therefore f_0 should also be the same under different conditions. In that case the predicted FLCs for three different grain sizes would be the same and would not be in accordance with the experimental results. On the other hand, as demonstrated by the experimental investigations, the inhomogeneous deformation of individual grains becomes more significant as the forming scale decreases to micro/meso level. The necking and strain concentration behavior of material are highly affected by the complicated local interactive deformation of only several grains. Therefore, further improvement of M-K model is needed to consider the size effect induced by the inhomogeneous deformation of individual grains. In that case, the reduction of formability, i.e., the increase of f_0 , can be modeled at different micro/meso scale deformation conditions.

3.3 Ductile fracture criteria

It is believed that ductile fracture criterion (DFC) can be used to predict the forming limit of sheet metals [25, 43]. In this study, several well-known uncoupled DFC are employed to

construct FFLD and analyze the formability of sheet metals in micro scale.

1) Freudenthal criterion

Freudenthal criterion is formulated as follows [44]:

$$\int_0^{\bar{\varepsilon}_f} \bar{\sigma} d\bar{\varepsilon} = C_1 \quad (3)$$

Based on Mises yield criterion and volume consistence condition, the effective strain is:

$$\bar{\varepsilon} = \frac{2}{3} \sqrt{3} \sqrt{\rho^2 + \rho + 1} \varepsilon_1 \quad (4)$$

Considering $\bar{\sigma} = K \bar{\varepsilon}^n$, the limit strains of fracture can be calculated as:

$$\left\{ \begin{array}{l} \varepsilon_{1f} = \sqrt{\frac{\frac{3}{4} \left(\frac{C_1 (n+1)}{K} \right)^{\frac{2}{n+1}}}{\rho^2 + \rho + 1}} \\ \varepsilon_{2f} = \rho \varepsilon_{1f} \end{array} \right. \quad (5)$$

2) Ayada criterion

Ayada et al. [45] considered the effect of hydrostatic stress $\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$ on ductile

fracture and proposed the DF criterion in the following:

$$\int_0^{\bar{\varepsilon}_f} \frac{\sigma_m}{\bar{\sigma}} d\bar{\varepsilon} = C_2 \quad (6)$$

By employing the Levy-Mises flow rule, the limit strain can be further derived for the simple

linear loading path $\rho = \varepsilon_2 / \varepsilon_1 = \text{constant}$ in the following:

$$\left\{ \begin{array}{l} \varepsilon_{1f} = \frac{3C_2}{2(\rho+1)} \\ \varepsilon_{2f} = \rho \varepsilon_{1f} \end{array} \right. \quad (7)$$

3) Cockcroft & Latham criterion

According to Cockcroft & Latham (C&L) criterion, fracture occurs when the accumulated equivalent strain modified by the maximum principal tensile stress reaches a critical value [46, 47]:

$$\int_0^{\bar{\varepsilon}_f} \frac{\sigma_1}{\bar{\sigma}} d\bar{\varepsilon} = C_3 \quad (8)$$

The limit strain of fracture can be further formulated as:

$$\begin{cases} \varepsilon_{1f} = \frac{3C_3}{2(\rho+2)} \\ \varepsilon_{2f} = \rho\varepsilon_{1f} \end{cases} \quad (9)$$

4) Oyane criterion

Oyane criterion [48] can be designated as follows:

$$\int_0^{\bar{\varepsilon}_f} \left(\frac{\sigma_m}{\bar{\sigma}} + C'_4 \right) d\bar{\varepsilon} = C_4 \quad (10)$$

This criterion also introduces the effect of hydrostatic stress and the porous plasticity theory.

The principal limit strains based on this criterion can be denoted as:

$$\begin{cases} \varepsilon_{1f} = \frac{3C_4}{2(\rho+1) + 2\sqrt{3}C'_4\sqrt{\rho^2 + \rho + 1}} \\ \varepsilon_{2f} = \rho\varepsilon_{1f} \end{cases} \quad (11)$$

Using the least square method, the FLCs based on different DFC models were constructed by fitting to the experimental data. Considering that DFCs predict the fracture limit strains instead of the necking ones, the FFLD results were employed in the calculation. The predicted FFLD results and the optimized coefficient values are shown in Fig. 11.

Fig. 11. Prediction of the FLCs based on different DFCs:

- (a) The results of 0.4 mm specimens based on the Freudenthal and Ayada criteria;
- (b) The results of 0.4 mm specimens based on the C&L and Oyane criteria;
- (c) The results of 0.2 mm specimens based on the Freudenthal and Ayada criteria;
- (d) The results of 0.2 mm specimens based on the C&L and Oyane criteria.

It is revealed that the DFCs can describe the FFLCs of specimens with different thicknesses and grain sizes by adjusting the value of parameters. By doing this, the reduction of formability of material can be characterized in an easy and simply way. According to the experimental investigations, the DF behavior at micro/meso scale is determined by the localized interactive deformation of only a few individual grains. However, the intrinsic mechanism of the reduction of forming limit at micro/meso scale was not included in the DFCs. How to describe the size effect on the traditional DFCs thus is an important issue considering that the traditional DFCs have been well-developed based on various mechanisms.

Furthermore, by comparing to the experimental results, it can be observed that the Freudenthal, C&L and Ayada criteria can characterize the FFLD with acceptable accuracy. However, the accuracy for different loading conditions could be different according to those models. For example, all the three criteria tend to under-estimate the fracture limit strains for the bi-stretching condition. On the other hand, the Oyane criterion can be used to describe the limit strains on the left side of FFLD. However, the prediction accuracy decreases when the criterion is applied on the right side of FFLD.

It should be noted that the predicted results based on DFCs are different from the ones calculated according to Swift/Hill criteria or M-K model. The DFCs describe the fracture condition while the Swift/Hill criteria and M-K model predict the limit strain onset of localized necking. It has been revealed that the plastic deformation ceases outside of the necking zone when the localized necking begins. After that the deformation at the necking region progresses under plane-strain condition until fracture [25] happens.

3.4 GTN-Thomason model

Zhang et al. [49] and Pardoen et al. [50] found that GTN-Thomason model can provide a complete description of all the stages of void nucleation, growth and coalescence, and can also associate DF to the micromechanical parameters directly. Therefore, the model has been attracting considerable attention of many researchers.

The well-known strain-controlled void nucleation model of Needleman [51] can be formulated as follows:

$$\begin{cases} \dot{f}_{nucleation} = A \dot{\bar{\epsilon}}_m^p \\ A = \frac{f_n}{s_N \sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{\bar{\epsilon}_m^p - \epsilon_N}{s_N}\right)^2\right) \end{cases} \quad (12)$$

The nucleation rate is assumed to follow the normal distribution of the equivalent plastic strain $\bar{\epsilon}_m^p$ of base material. f_n is the nucleation speed coefficient, s_N and ϵ_N are the standard deviation and mean value of the normal distribution, respectively.

The void growth can be described using the classical GTN model [52] in the following:

$$\Phi = \left(\frac{\sigma_v}{\sigma_y} \right)^2 + 2q_1 f^* \cosh \left(-\frac{3}{2} \frac{q_2 \sigma_m}{\sigma_y} \right) - (1 + q_3 f^{*2}) = 0 \quad (13)$$

$\sigma_v = \sqrt{\frac{3}{2} S_{ij} S_{ij}}$ is the von Mises equivalent stress, $\sigma_m = \frac{1}{3} \sigma_{kk}$ is the hydrostatic stress,

$S_{ij} = \sigma_{ij} - \sigma_m \delta_{ij}$ is the deviatoric component of Cauchy stress, δ_{ij} is the Kronecker delta,

σ_y is the equivalent stress of the base material, q_1 , q_2 and q_3 are the coefficients

introduced. f^* is the effective void volume fraction and defined as:

$$f^* = \begin{cases} f & f < f_c \\ f_c + \frac{f_u - f_c}{f_f - f_c} (f - f_c) & f \geq f_c \end{cases} \quad (14)$$

f^* is employed to describe the fast loss of load carrying capacity due to the coalescence of voids. f_c represents the void volume fraction at the onset of coalescence. $f_u = 1/q_1$ is the void volume fraction when the stress of material equals zero. f_f is the final void volume fraction when failure occurs.

Both the void growth and nucleation contribute to the increase of void volume fraction f :

$$\dot{f} = \dot{f}_{growth} + \dot{f}_{nucleation} \quad (15)$$

The growth of existing voids can be formulated as:

$$\dot{f}_{growth} = (1-f) \dot{\epsilon}_{kk}^p \quad (16)$$

where $\dot{\epsilon}_{kk}^p$ is the hydrostatic component of the plastic strain rate.

According to the Thomason model [53], the void coalescence can be described as follows:

$$\begin{cases} \frac{\sigma_{zz}}{\sigma_y} = (1 - \chi^2) \left[0.1 \left(\frac{1}{\chi} - 1 \right)^2 + 1.2 \sqrt{\frac{1}{\chi}} \right] \\ \chi = \left(\frac{3}{2} f \lambda_0 \left(\frac{3}{2} e_{zz} \right) \right)^{1/3} \end{cases} \quad (17)$$

σ_{zz} is the major principal stress applied on the unit cell. e_{zz} is the major principal strain.

λ_0 is a fitting coefficient denoting the spatial distribution of micro voids. Based on Besson's research [54], void coalescence starts when χ reaches the critical value of $\chi_c = 0.9$.

In this research, the GTN-Thomason model was implemented into ABAQUS/Explicit via the user material subroutine VUMAT [55].

FEM simulation based on the GTN-Thomason model can then be conducted. In order to realize the accurate description of material behavior, the parameters of the established model need to be determined. According to the previous researches [14, 56, 57], some of these parameters can be given in Table 3. On the other hand, the other two parameters f_n and λ_0 can be obtained by analyzing the deformation behavior in the uniaxial tensile test simulation according to Abassi et al. [30].

Table 3. The parameters of the GTN-Thomason model.

The uniaxial tensile test was first simulated and the engineering strain-stress curve was measured accordingly. The numerical curve was then compared with the experimental ones to determine the parameter values. The highest point of the experimental curve was first recorded. Considering that the highest point can be treated as the start of diffusive instability,

the void coalescence did not begin at that moment. It is further noted that most of the material parameters are identified and given in Table 3, hence the stress at the highest point is solely related to f_n . Therefore, f_n can be determined on the coincidence of the simulated and experimental stresses at the highest point. The λ_0 can then be optimized by minimizing the difference between the simulated and experimental results at the failure point. A sudden drop of the strain-stress curve takes place at the failure point, which reveals the fracture after void coalescence. The calculated parameters for different grain size conditions are shown in Table 4. The simulated and experimental results are presented in Fig. 12. There is a good agreement between the FE predicted and experimental curves.

Fig. 12. Comparison of the simulated and experimental engineering strain-stress curves.

Table 4. The fitted values of f_n and λ_0 .

Upon the determination of parameters, the FE model was established to realize different loading paths as shown in Fig. 13 by applying different displacement conditions in x and y directions. The void coalescence starts when $f = f_c$ and followed by the fracture onset of $f = f_f$. By reading the major and minor strain history data of the necking spot based on the FE results, the necking FLCs were determined following the procedure of ISO 12004-2: 2008. The fracture FLCs were also recorded at the onset of $f = f_f$. The predicted results are compared with the experimental ones and shown in Figs. 14 and 15.

Fig. 13. The FE models of different loading paths.

Fig. 14. The predicted forming limit results of the specimens with the thickness of 0.4 mm

based on the GTN-Thomason model:

(a) Grain size: 23.7 μm ; (b) Grain size: 58.9 μm ; (c) Grain size: 132.2 μm .

Fig. 15. The predicted forming limit results of the specimens with the thickness of 0.2 mm based on the GTN-Thomason model:

(a) Grain size: 17.4 μm ; (b) Grain size: 35.2 μm .

On the other hand, it is found that the predicted FLCs and FFLCs agree with both the necking and fracture experimental results respectively. Therefore, the applicability of GTN-Thomason model is validated. By using the parameters given in Table 4, the reduction of forming limit with the increase of grain size is well predicted by introducing different parameter values. The increase of λ_0 with the grain size actually reflects the increasing tendency of coalescence, which is in accordance with the experimental observations and the previous analysis based on the surface layer model. Therefore, the GTN-Thomason model can not only predict the FLCs under different grain size conditions, but also describe the intrinsic mechanism of how the size effect affects the fracture behavior of sheet metals. Nevertheless, how the size effect affects the void evolution progress still need to be discussed to quantify the parameter values under different geometry and grain size conditions. In our previous study [14], a preliminary research work was conducted based on the surface layer model. However, further research is necessary to include the effect of the inhomogeneous deformation of individual grains on the void evolution such as anisotropy, shear stress, grain boundary, etc.

In addition, the condition of the sheet metals with the thickness of 0.2 mm and the grain size

of $166.3 \mu\text{m}$ was not discussed in the current analysis. This is because that the individual grains of surface layer play a dominant role in that condition since there is only one or two grain over the thickness. The ductile fracture behavior of void nucleation, growth and coalescence may not be the major failure mode. The fracture behavior is more affected by the single crystal deformation and failure. Therefore, the condition was not discussed in this research and future research is needed to address this issue.

4. Hydroforming experiments and analysis

In order to further verify the above-described methods in micro/meso scale forming process of sheet metals, the hydroforming of copper sheet metals was conducted as a case study.

The hydroforming experimental setup is shown in Fig. 16. Three sets of dies with different channel feature dimensions were designed and fabricated by wire electric discharge machining (WEDM), as shown in Fig. 17. The die was first assembled in the center of die holder plate, on top of which the copper specimen was then placed. After that the four pieces of apparatus were tightly clamped by eight threaded bolts for reliable sealing. The forming pressure is provided by a Maximator pump (MHU-M111-G500-2-4500).

Copper sheets with the thickness of 0.2 mm were employed in the hydroforming experiments. The pressure was first increased at the constant rate of 0.2 MPa/s until fracture occurred. After that the profile of formed specimens were measured using the KEYENCE KS-1100 laser measurement system with the resolution of $0.05 \mu\text{m}$. The heights of the hydroformed

specimens using Die No. 1 is likely to exceed the measurable scope of this system (-1 to 1 mm), hence a micrometer caliper with the accuracy of $10\ \mu\text{m}$ was adopted in that case. The laser measurement results are shown in Fig. 18. Each condition was repeated three times. After that both the hydroforming pressure and the height of channels at the onset of failure were averaged and calculated.

Fig. 16. The hydroforming experimental setup.

Fig. 17. Geometric dimensions of the hydroforming dies.

Fig. 18. Measurement of the hydroformed specimens.

The ultimate height and pressure were predicted based on the GTN-Thomason model. FE simulations of the hydroforming process were conducted using ABAQUS/Explicit with VUMAT subroutine. The predictive results of Swift/Hill criteria, M-K model and the uncoupled DFCs were also obtained based on the FE simulations without the subroutine. The results are shown in Fig. 19.

Fig. 19. Comparison of experimental and analytical results:

- (a) The ultimate pressure predicted according to the Swift/Hill criteria, M-K model and
GTN-Thomason model;
- (b) The ultimate height predicted according to the Swift/Hill criteria, M-K model and
GTN-Thomason model;
- (c) The ultimate pressure predicted according to different uncoupled DFCs;
- (d) The ultimate height predicted according to different uncoupled DFCs.

It can be observed that both the GTN-Thomason and M-K model predictions are in accordance with the experimental results. The M-K model can describe the reduction of forming limit with the increase of grain size by adjusting the coefficient f_0 which depicts the flaw depth of sheet metal. The mechanism established based on the M-K model of the necking behavior affected by the size effect at micro/meso scale thus needs further discussion to construct the relation between f_0 and the scale factor. Furthermore, by employing the GTN-Thomason model, the reduction of forming limit with the increase of grain size can be predicted by increasing the parameter λ_0 which depicts the tendency of coalescence. Hence the size effect on forming limit can be both well-estimated and reasonably-explained according to the GTN-Thomason model. Although a preliminary model has been developed in our previous study [14], further investigation is still necessary to discuss the anisotropy, the shear fracture behavior, the role of grain boundary etc. which are important issues have not been discussed in this work.

The Swift/Hill model tends to overestimate the formability of sheet metal with the increase of grain size. These observations have a good agreement with the analysis made based on the FLC prediction results, i.e., the swift/Hill model cannot describe the size effect affected forming limit by solely considering the localized or diffusive necking. The effect of the interactive deformation of individual grains on the plastic flow and necking behavior at micro/meso scale need to be considered.

In addition, the uncoupled DFCs also tends to overestimate the ultimate forming height and

pressure. This is because the uncoupled DFCs cannot describe the effect of damage accumulation and solely determines the fracture limit strain. Hence the uncoupled DFCs cannot describe the effect of grain and geometry size on the necking behavior while the DF occurs quickly following localized necking in the necking zone during the hydroforming process. On the other hand, there is also a significant difference between the predicted FFLC based on these DFCs and the experimental FFLC. As illustrated in Fig. 11, the uncoupled DFCs tends to overestimate the FFLC near the plane strain deformation condition, this could also lead to the overestimation of hydroformed height and pressure onset of failure.

5. Conclusions

Based on the forming limit experiments of sheet metals with different grain sizes, how the size effect affects the ductile failure of sheet metals was analyzed. Different theoretical methods were then employed to predict the FLCs under different load paths. It is found that the FLC shifts down with the increase of grain size. When the grain size approaches to the thickness of sheet metal, i.e., there is only one or two grains over the thickness direction in the workpiece, the repeatability of experiments becomes much worse and the highly scattered results were observed. The applicability of Swift/Hill criteria, M-K model, ductile fracture criteria and GTN-Thomason model is validated and corroborated by experiments. It is revealed that M-K and GTN-Thomason models can describe the size effect affected FLC under different grain size conditions. In addition, the GTN-Thomason model can explain the deterioration of formability caused by the increasing tendency of void coalescence with the increase of grain size. In addition, the meso-scale channels with different geometric

dimensions were formed as validation experiments. The maximum height and the pressure at the onset of failure were measured. By comparing the experimental results with the predictions made according to different models, the GTN-Thomason model and M-K model are found to be promising methods which can predict the failure of material with a satisfactory accuracy.

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