An Analytical Insight into Contact Acoustic Nonlinearity of Guided Ultrasonic Waves induced by A "Breathing" Crack

Kai WANG and Zhongqing SU*
Department of Mechanical Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong SAR

Contact e-mail:*zhongqing.su@polyu.edu.hk

Abstract. Analytical investigation based on a modal decomposition method and the variational principle was developed in this study, to facilitate understanding of the modulation mechanism of a "breathing" crack on propagation of Lamb waves and generation of higher-order harmonic modes. With the understanding, the "breathing" behaviour of the crack and accordingly the crack-induced Lamb wave fields when the waves traversing the crack, were elucidated. Both the linear and nonlinear features in Lamb wave signals, contributed by the "breathing" crack in a homogenous medium, was scrutinized with a two-dimensional strain scenario, in which non-propagating Lamb waves were considered to satisfy the boundary conditions at two crack surfaces. The accordingly obtained reflection and transmission indices were linked quantitatively to the crack severity, whereby the crack could be characterized. Results from the analytical investigation were compared against those from finite element methods, to observe a good agreement in between. This has demonstrated the potential of the proposed method in illustrating the contact acoustic nonlinearity of Lamb waves induced by a "breathing" crack, and the capability of evaluating crack severity in a quantitative manner.

Introduction

Among various types of defects, the cracks and particularly when they are at an embryo stage initiated from voids, stress concentration and corrosion shots, can result in structural failure without timely awareness. When subject to cyclic loads, the cracks can grow rapidly into macroscopic defects, which can lead to catastrophic consequences. To detect and estimate the crack, a rich body of non-destructive evaluation (NDE) techniques has been developed based on different mechanisms, typified as eddy-current, guided ultrasonic waves (GUWs), X-ray, *etc.* Among these NDE techniques, those based on GUWs[1-5] have attracted intensive research and development in recent years, with an attempt to make use of the appealing features of GUWs including long range of inspection, high sensitivity to damage, low energy consumption, *etc.* Lamb waves[3, 6-9] −a typical GUW propagating in a plate-like medium− have been increasingly employed to detect undersized cracks in plates or shells.

Most of these Lamb wave-based methods exploit the linear features in Lamb wave signals upon interaction with the crack, whereby site of the crack and further its severity can be estimated. However, experimental observations have accentuated that these linear methods are effective only for the crack with a scale comparable to the wavelength of a probing Lamb wave. To enhance the sensitivity of Lamb waves to the crack of small scale, the probing Lamb waves will have to be excited at high frequencies, and this, however, introduces challenges in interpreting captured Lamb wave signals because multiple wave modes are co-existent at high frequencies.

In addition, results from the linear methods could be erroneous or inaccurate, if they are used to evaluate a "breathing" crack, for example a fatigue crack. That is because the clapping behaviours at the interface of a "breathing" crack are usually not taken into consideration by these linear methods. However, in reality, the clapping behaviours exist when propagating Lamb waves traversing the crack. Under this condition, linear features of a Lamb wave signal can be influenced significantly by the clapping effect, leading to the errors in results if such an effect is not fully addressed in the linear methods.

Therefore vast effort has been directed to development of NDE approaches using nonlinear features of Lamb waves [10-17], as they exhibit higher sensitivity to small-scale cracks than those linear methods. Majority of the nonlinear approaches explore crack-induced shift of Lamb wave energy from incident frequency to other frequencies. Amongst various approaches, those based on calibrating the contact acoustic nonlinearity (CAN) evidenced in Lamb wave signals induced by a "breathing" crack[18-21] are demonstrably effective in interpreting the modulation mechanism of a "breathing" crack on propagating Lamb waves (such as higher-order harmonic generation), because CAN faithfully depicts the "breathing" behaviour of the crack. Representatively, Soldove *et al.*[18] applied a CAN model to interpret the influence of a "breathing" crack on probing waves propagation and consequent generation of higher-order harmonics. In this method the crack was defined with a specific material in which a stepwise change in material stiffness was assumed. In an extreme case when the stiffness for the compressional stress at the crack is zero, modulation of the "breathing" crack on probing waves retreats to harmonics generation of incident waves at an un-bonded interface in a one-dimensional case. Solutions to this case were provided by Richardson[22] using the characteristics of the crack surfaces in the time domain. Though helpful in understanding the influence of CAN on Lamb waves, both methods mentioned above are based on a premise that the medium is split at the site of the crack. However, such an assumption deviates from the reality in which a crack only shares part of the medium thickness.

Motivated by this, an analytical method, aimed at understanding the modulation mechanism of a "breathing" crack on Lamb waves propagation and generation of higher-order harmonics was developed in this study. In this method, behaviours of the "breathing" crack in the time domain and the consequently generated second-order harmonics were interrogated using a modal decomposition method[23] and the variational-principle-based method[24]. A set of linear and nonlinear indices was defined, and the correlation between these indices and the crack parameters (*e.g.*, severity) was established, to describe the "breathing" crack. Results from the analytical approach were compared with those from finite element simulation.

1. Behaviours of "Breathing" Crack

When the crack closes, compressive and shear stress of propagating Lamb waves are transmitted; and when the crack opens during dilation, as shown in Fig. 1, waves are partially decoupled. These will jointly lead to different stress-strain relations at the crack interface, reflecting the "breathing" behaviour of the crack under the modulation of propagating Lamb waves. It is such a time-dependent feature of Lamb waves that induces the generation of CAN manifested in the probing Lamb waves. Such a feature was scrutinized using a modal decomposition method in this study. Furthermore, to predict the crack-induced stress/displacement fields at incident and other frequencies, a variational principle method was recalled in this study. Based on this, an analytical framework was proposed to analyse the influence of CAN induced by a "breathing" crack on Lamb wave propagation and to establish a quantitative relation between the nonlinear features in a Lamb wave signal and the crack parameters. As illustrated in Fig. 2, a set of indices was defined in this framework, to evaluate the severity of the crack.

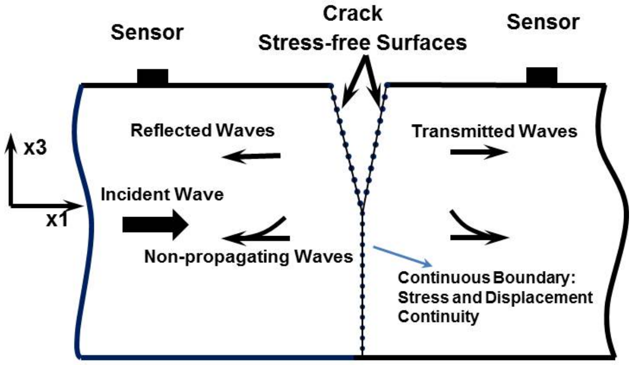


Fig. 1. Schematic of a two-dimensional infinite plate bearing a "breathing" crack when the crack is open

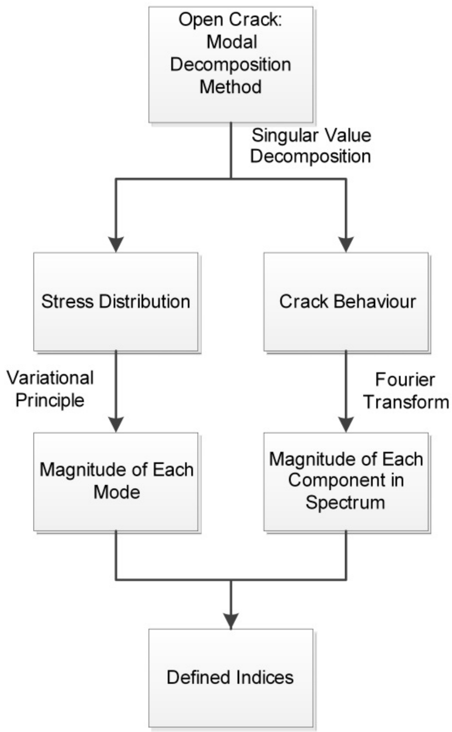


Fig. 2. Flowchart of the proposed framework to quantitatively analyze the CAN induced by a "breathing" crack

1.1 Higher-order Harmonics Generation due to "Breathing" Crack

In this framework, the crack behaviours and the duration in which the reflection and transmission of the probing Lamb waves present were first studied using a modal decomposition method in conjunction with the analysis in the time domain. When the crack is closed, Lamb waves propagate in the same fashion as in an intact plate, and the stress field is identical to that of the incident wave. In this condition, the displacement at the crack can be described by the incident displacement field. At a particular moment, denoted by t_{open}, when the tensile phase of the incident wave arrives, the crack is open and it induces wave scattering.

Afterward, the crack interface behaves the same as in the case in which Lamb waves traverse a fully opened notch with the same depth. In this method, the stress or displacement fields can be deemed as the superposition of those of the propagating and non-propagating wave modes. Then, the boundary conditions at the crack, when it is open, are

$$\begin{matrix}\sigma_{11}\\ \sigma_{13}\end{matrix} = \begin{cases}\sum_N b_N \sigma_{11}^N \\ \sum_N b_N \sigma_{13}^N\end{cases} = \begin{cases}0 \\ 0,\end{cases} \text{ stress free at the right surface of crack} \tag{1}$$

$$\begin{matrix}\sigma_{11}\\ \sigma_{13}\end{matrix} = \begin{cases}b_{Inc}\sigma_{11}^{Inc} + \sum_N b_{-N}\sigma_{11}^{-N} \\ b_{Inc}\sigma_{13}^{Inc} + \sum_N b_{-N}\sigma_{13}^{-N}\end{cases} = \begin{cases}0 \\ 0,\end{cases} \text{ stress free at the left surface of crack} \tag{2}$$

$$\sum_N b_N u^N = b_{Inc}u^{Inc} + \sum_N b_{-N}u^{-N}, \text{ displacement continuity at continuous boundary} \tag{3}$$

$$\begin{matrix}\sum_N b_N \sigma_{11}^N \\ \sum_N b_N \sigma_{33}^N \\ \sum_N b_N \sigma_{13}^N\end{matrix} = \begin{cases}b_{Inc}\sigma_{11}^{Inc} + \sum_N b_{-N}\sigma_{11}^{-N} \\ b_{Inc}\sigma_{33}^{Inc} + \sum_N b_{-N}\sigma_{33}^{-N} \\ b_{Inc}\sigma_{13}^{Inc} + \sum_N b_{-N}\sigma_{13}^{-N}.\end{cases} \text{ stress continuity at continuous boundary} \tag{4}$$

In the above, N is an index of propagating and non-propagating modes, and $-N$ indicates the modes propagating opposite to the incident wave. σ^N and u^N denote the stress and displacement fields induced by the N^{th} wave mode, respectively. Coefficient b_N is the unknown complex magnitude to be correlated with the magnitude of the incident wave which is denoted by b_{Inc}. σ^{Inc} and u^{Inc} are the stress and displacement fields induced by the incident wave, respectively.

Upon solving Eqs. (1) – (4) using a singular value decomposition method, the magnitude of each mode can be obtained and therefore the displacement and stress fields at the crack can be depicted as follows:

$$\begin{aligned} &u_1^-\left(x_3,t\right) = b_{Inc}u^{Inc} + \sum_N b_{-N}u^{-N},\ u_1^+\left(x_3,t\right) = \sum_N b_N u^N, \\ &\sigma_{11}^{Crack-} = b_{Inc}\sigma_{11}^{Inc} + \sum_N b_{-N}\sigma_{11}^{-N}, \quad \sigma_{11}^{Crack+} = \sum_N b_N\sigma_{11}^N, \\ &\sigma_{13}^{Crack-} = b_{Inc}\sigma_{13}^{Inc} + \sum_N b_{-N}\sigma_{13}^{-N}, \quad \sigma_{13}^{Crack+} = \sum_N b_N\sigma_{13}^N,\end{aligned} \tag{5}$$

where σ^{Crack+} (σ^{Crack-}) is the stress field at the crack induced by the incident wave and the reflected waves (transmitted waves). u_1^+ and u_1^- denote the in-plane displacements of points on the right (transmitted) and left (reflected) stress-free surface of crack, respectively. Setting the gap between the two crack surfaces as zero gives the moment, denoted by t_{close} when the crack closes and reflection/transmission suspends. The gap can be depicted as:

$$\Delta = u_1^+\left(x_3,t_{close}\right) - u_1^-\left(x_3,t_{close}\right) = 0. \tag{6}$$

As said earlier, the reflection/transmission of the probing Lamb wave is induced during crack opening and absent otherwise. This phenomenon can be treated as the scenario in which the crack-induced stress field in an open crack case is modulated by a periodic window function,

$$f\left(t\right) = \begin{cases}1, & t_{open} < t < t_{close} \\ 0, & t_{close} < t < t_{open} + T,\end{cases} \tag{7}$$

where T is the period of the incident wave. Therefore, the stress field induced by the "breathing" crack in the reflection, can be obtained as

$$\begin{aligned}\sigma_{11}^{ref} &= \sigma_{11}^{ref}\left(x_3\right)\mathrm{e}^{iwt} \times f\left(t\right) \\ \sigma_{13}^{ref} &= \sigma_{13}^{ref}\left(x_3\right)\mathrm{e}^{iwt} \times f\left(t\right).\end{aligned} \tag{8}$$

In the above, σ^{ref} represent the crack-induced stress field in the reflection, of which the distribution is to be analysed in the following. The corresponding spectrum of the crack-induced stress field can be obtained using the convolution between the incident wave period function e^{iwt} and a window function $f(t)$. In the spectrum, the magnitude of each component can be ascertained. To summarize, the above process can be illuminated in Fig. 3.

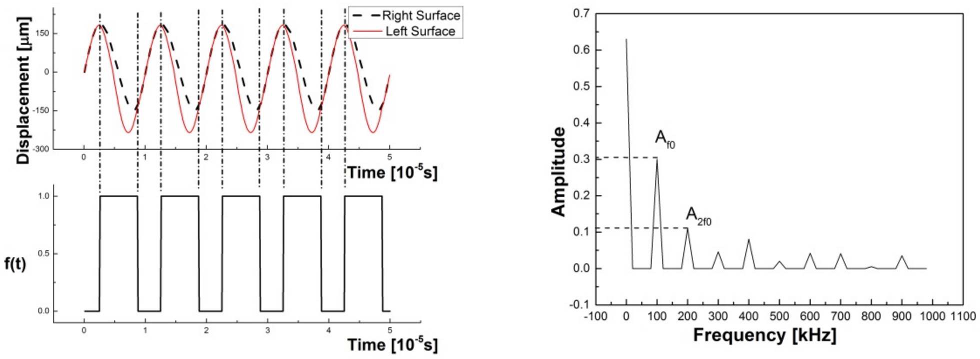


Fig. 3. (a) Modulating function based on Eq. (7); and (b) Spectrum of the crack-induced stress field

1.2 Calculation of Magnitudes of Propagating Wave Modes at Incident and Double Frequencies

In order to achieve an insight into the consequently generated propagating harmonics at incident and double frequencies in the reflection/transmission wave fields, a method based on the variational principle was employed. In this method, the crack-induced stress field, *i.e.* the deviation of stress field when the crack is open from that in an intact plate, was deemed as an additional excitation source to induce wave reflection/transmission. With knowing the magnitudes of the crack-induced stress source at incident and double frequencies (A_{f_0} and A_{2f_0}, respectively), the crack-induced wave source can be yielded as

$$\begin{aligned} \sigma^{ref-f_0} &= A_{f_0}\sigma^{ref}(x_3)e^{iwt} = A_{f_0}\left(\sigma^{Crack-} - b_{Inc}\sigma^{Inc}\right)e^{iwt} \\ \sigma^{ref-2f_0} &= A_{2f_0}\sigma^{ref}(x_3)e^{i2wt} = A_{2f_0}\left(\sigma^{Crack-} - b_{Inc}\sigma^{Inc}\right)e^{i2wt}. \end{aligned} \tag{9}$$

In the above, σ^{ref-f_0} and σ^{ref-2f_0} denote the crack-induced stress source at incident and double frequencies, respectively. $\sigma^{Crack-} - b_{Inc}\sigma^{Inc}$ represents the cross-thickness distribution of the crack-induced stress source, and it accounts for the magnitude of each wave mode generated by that source. A representative cross-thickness distribution of the crack-induced stress source is shown in Fig. 4.

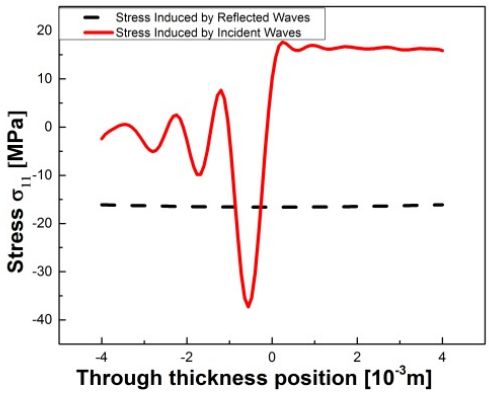


Fig. 4. Cross-thickness distribution of the crack-induced stress field at the crack

In this study, the infinite plate bearing a crack was deemed as two semi-infinite parts tied together via the continuous boundary at the site of crack, and the crack-induced stress field with the above distribution was applied at the boundaries of the two parts left and right to the crack, serving as an additional excitation source. Using a method based on variational principle, the generated wave modes can be obtained as follows:

$$[b_n]=\left[R_{nm}\right]\bullet\frac{1}{h}\int_{-h}^{h}\left(u_1^m\overline{T}_1+u_3^m\overline{T}_3\right)dx_3, \tag{10}$$

$$\left[R\right]=\left[\frac{1}{h}\int_{-h}^{h}\left(u_1^m\overline{\sigma}_{11}^n+u_3^m\overline{\sigma}_{13}^n\right)dx_3\right]^{-1} \tag{11}$$

In the above, $[b_n]$ is the vector of the magnitude of each wave mode generated by the crack-induced stress source. $\overline{T}_1=\overline{\sigma}_{11}^{ref-f_0}\left(or\ \overline{\sigma}_{11}^{ref-2f_0}\right)$ and $\overline{T}_3=\overline{\sigma}_{13}^{ref-f_0}\left(or\ \overline{\sigma}_{13}^{ref-2f_0}\right)$. u_1^m and u_3^m denote the in-plane and out-of-plane displacement fields of the m^{th} mode at the incident (or double) frequency, respectively. $\overline{\sigma}^n$ is the conjugate of the stress field of the n^{th} mode. h is the half-thickness of the plate.

Though both propagating and non-propagating modes existent, the non-propagating modes transfer no energy and decay exponentially along the propagating path, and thus only the propagating modes continue propagation. The displacement fields of the crack-generated propagating modes at incident and double frequencies, with the respective magnitude $b_1^{f_0}$ and $b_1^{2f_0}$ obtained via Eq. (10), can be described by

$$\begin{aligned}&U_1^{f_0}=b_1^{f_0}u_1^{f_0}\left(x_3\right),U_3^{f_0}=b_1^{f_0}u_3^{f_0}\left(x_3\right),\\&U_1^{2f_0}=b_1^{2f_0}u_1^{2f_0}\left(x_3\right),U_3^{2f_0}=b_1^{2f_0}u_3^{2f_0}\left(x_3\right),\end{aligned} \tag{12}$$

where $U_1^{2f_0}$ represents the in-plane displacement induced by a crack-related propagating wave at double frequency, $U_1^{f_0}$ denotes the in-plane displacement induced by the incident wave. $u_1^{f_0}\left(x_3\right)$($u_1^{2f_0}\left(x_3\right)$) and $u_3^{f_0}\left(x_3\right)$ ($u_3^{2f_0}\left(x_3\right)$) are the mode shape functions for the in-plane and out-of-plane displacements at incident (double) frequency, respectively. The results show that the crack-generated harmonics at double frequency in the reflection and transmission packets are identical, because the crack-induced stress fields at double frequency are equivalent in both parts of the plate. This implies that, in principle, characterization of the crack using the second-order harmonic acquired by either "pulse-echo" or "pitch-catch" sensing configuration is equally feasible, with comparable precision.

With the accordingly obtained magnitudes of the crack-generated propagating modes, a set of dimensionless damage indices was proposed, to be linked to the severity of the fatigue crack by

$$NI=\frac{U_1^{2f_0}}{U_1^{f_0}}=\frac{b_1^{2f_0}u_1^{2f_0}\left(x_3\right)}{b_{Inc}u_1^{f_0}\left(x_3\right)},\ \ LI^R=\frac{b_1^{f_0}}{b_{Inc}}, \tag{13}$$

where NI denotes a nonlinear index, which is the ratio of magnitude of the reflected propagating-mode at double frequency to the magnitude of the incident wave. LI^R represents a linear index, which is the ratio of magnitude of the reflected propagating-mode at incident frequency to the magnitude of the incident wave. Using the same method, the linear index $LI^T=b_1^{T-f_0}/b_{Inc}$ with respect to the transmitted waves can be obtained by substituting the stress field in the reflection with that in the transmission. In particular, in the case that the sensor is on the upper surface of the plate, the nonlinear damage index can be evaluated at the upper surface $x_3=h$, this yielding

$$NI = \frac{U_1^{2f_0}}{U_1^{f_0}} = \frac{b_1^{2f_0} u_1^{2f_0}\left(h\right)}{b_{Inc} u_1^{f_0}\left(h\right)}. \tag{14}$$

2. Validation of Analytical Method

For proof-of-concept, an aluminium plate, measuring 8 mm in thickness and 1000 mm in length, was meshed and analysed by ABAQUS®/EXPLICIT, to validate the above analytical method. The properties of the aluminium plate are listed in Table 1.

Table 1. Properties of the aluminum plate used in validation

Density (kg/m^3)	E (GPa)	υ	c_L (m/s)	c_T (m/s)
2660	71.8	0.33	6324	3185

For the discussed crack along the plate thickness, the symmetric propagating mode (S_0) was chosen as the incident wave to make use of its higher sensitivity to this type of crack than an antisymmetric mode, because the in-plane displacement dominates the signal energy. The incident five-cycle Hanning-windowed sinusoidal tone bursts were produced by applying the cross-thickness displacement field of pure S_0 mode at incident frequency. To model the “breathing” crack in the plate, the stress-free surfaces of the crack were defined as a seam crack, and a contact-pair interaction between the two crack surfaces was applied.

Applied with the short-time Fourier Transform (STFT) analysis, the time-frequency spectra of the captured Lamb waves can be obtained. Fig. 5 representatively shows the spectra when crack depth is 50% and 75% of the plate thickness. From the spectra, each wave mode can be distinguished and their respective magnitudes can be extracted to calculate the indices using Eqs. (13) and (14).

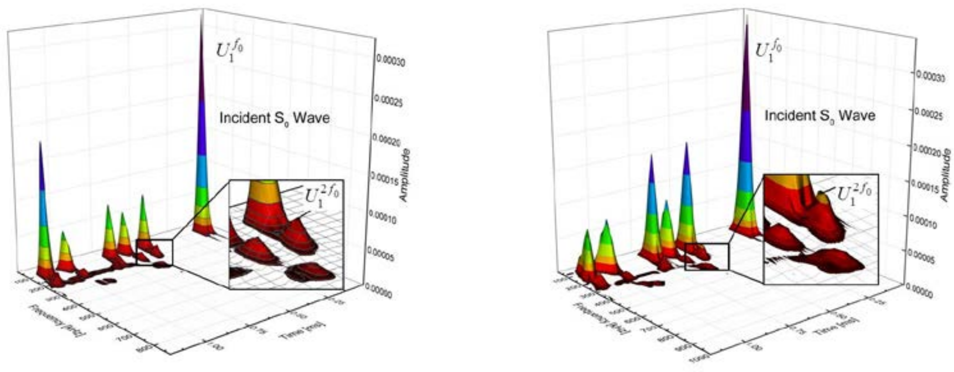
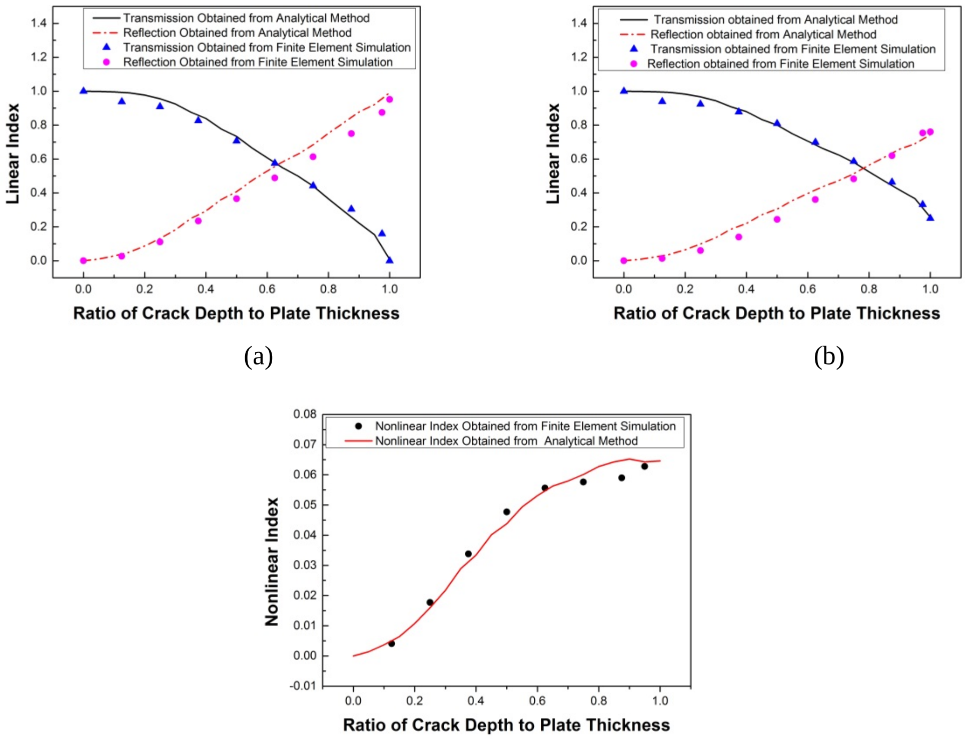


Fig. 5. Spectra of the captured signals when the ratio of crack depth to thickness is: (a) 50%; and (b) 75%

The accordingly obtained correlation between the defined indices and crack parameters was established, in Fig. 6, compared with the results obtained from the analytical method, to observe good coincidence. It is demonstrated in the Fig. 6(a) and 6(b) that the “breathing” crack exerts influence on the linear features of the Lamb waves when the severity of the “breathing” crack reaches a certain extent, indicating that it is significant to consider the “breathing” behaviour for the linear methods to guarantee the accuracy of results. Furthermore, Fig. 6(c) demonstrates that the severer a crack is the larger the defined nonlinear index it will be. From the monotonous correlation shown in Fig. 6(c), conclusion can be drawn that the NI can be used to represent the severity of the crack damage.



(c)

Fig. 6. Linear and nonlinear indices against ratio of crack depth to the plate thickness: (a) linear index for the open crack; (b) and (c) the respective linear and nonlinear indices for the "breathing" crack

3. Discussions and Concluding Remarks

In this paper, an analytical method was presented, aimed at interpreting the modulation mechanism of a "breathing" crack on Lamb waves and the generation of crack-induced second harmonic. Using the method, the influence of the "breathing" crack in a two-dimensional scenario on the linear features of Lamb waves and CAN induced by the "breathing" crack were illustrated. Based on a modal decomposition method and the variational principle, behaviours of the "breathing" crack and the generation of higher-order harmonics were studied. The method defined a set of linear and nonlinear indices. The correlation between the indices and crack parameters was developed to represent the severity of fatigue crack in a quantitative manner. The proposed method was validated by a finite element method. The comparison showed that it is of vital necessity to take into account of the influence of the "breathing" behaviours in the linear methods. Based on the results obtained using the proposed method, it is possible to evaluate the severity of the crack in a quantitative manner by measuring the nonlinear features of the probing wave signals.

Acknowledgements

This project is supported by the Hong Kong Research Grants Council via General Research Funds (No. 523313), and by the Hong Kong Innovation and Technology Commission via an Innovation and Technology Fund (ITF) (No. ITS/058/14).

References

[1] Y. Lu, L. Ye, Z. Su, and C. Yang, “Quantitative assessment of through-thickness crack size based on Lamb wave scattering in aluminium plates,” Ndt & E International, 41, 59-68 (2008).

[2] V. Dayal and V. K. Kinra, “Leaky Lamb waves in an anisotropic plate. II: Nondestructive evaluation of matrix cracks in fiber-reinforced composites,” The Journal of the Acoustical Society of America, 89, 1590-1598 (1991).

[3] D. N. Alleyne and P. Cawley, “The interaction of Lamb waves with defects,” Ultrasonics, Ferroelectrics, and Frequency Control, IEEE Transactions on, 39, 381-397 (1992).

[4] Z. Su and L. Ye, [Identification of damage using Lamb waves: from fundamentals to applications], Springer Science & Business Media, 48(2009).

[5] C. Tirado and S. Nazarian, “Impact of damage on propagation of Lamb waves in plates,” Nondestructive Evaluation Techniques for Aging Infrastructures & Manufacturing,267-278 (1999).

[6] C. Eisenhardt, L. J. Jacobs, and J. Qu, “Experimental Lamb wave spectra of cracked plates,” REVIEW OF PROGRESS IN QUANTITATIVE NONDESTRUCTIVE EVALUATION: Volume 19,343-350 (2000).

[7] M. Lowe, P. Cawley, J. Kao, and O. Diligent, “Prediction and measurement of the reflection of the fundamental anti-symmetric Lamb wave from cracks and notches,” REVIEW OF PROGRESS IN QUANTITATIVE NONDESTRUCTIVE EVALUATION: Volume 19,193-200 (2000).

[8] M. Koshiba, S. Karakida, and M. Suzuki, “Finite-element analysis of Lamb wave scattering in an elastic plate waveguide,” IEEE transactions on sonics and ultrasonics, 31, 18-25 (1984).

[9] J. Paffenholz, J. W. Fox, X. Gu, G. S. Jewett, S. K. Datta, and H. A. Spetzler, “Experimental and theoretical study of Rayleigh-Lamb waves in a plate containing a surface-breaking crack,” Research in Nondestructive Evaluation, 1, 197-217 (1990).

[10] J. H. Cantrell, “Quantitative assessment of fatigue damage accumulation in wavy slip metals from acoustic harmonic generation,” Philosophical Magazine, 86, 1539-1554 (2006).

[11] C. Zhou, M. Hong, Z. Su, Q. Wang, and L. Cheng, “Evaluation of fatigue cracks using nonlinearities of acousto-ultrasonic waves acquired by an active sensor network,” Smart Materials and Structures, 22, 015018 (2012).

[12] C. Bermes, J.-Y. Kim, J. Qu, and L. J. Jacobs, “Nonlinear Lamb waves for the detection of material nonlinearity,” Mechanical Systems and Signal Processing, 22, 638-646 (2008).

[13] J. Pei and M. Deng, “Assessment of fatigue damage in solid plates using ultrasonic lamb wave spectra,” Ultrasonics Symposium, 2008. IUS 2008. IEEE,1869-1872 (2008).

[14] J.-Y. Kim, L. J. Jacobs, J. Qu, and J. W. Littles, “Experimental characterization of fatigue damage in a nickel-base superalloy using nonlinear ultrasonic waves,” The Journal of the Acoustical Society of America, 120, 1266-1273 (2006).

[15] K. Dziedziech, L. Pieczonka, P. Kijanka, and W. J. Staszewski, “Enhanced nonlinear crack ‐ wave interactions for structural damage detection based on guided ultrasonic waves,” Structural Control and Health Monitoring, (2016).

[16] A. Klepka, W. Staszewski, R. Jenal, M. Szwedo, J. Iwaniec, and T. Uhl, “Nonlinear acoustics for fatigue crack detection–experimental investigations of vibro-acoustic wave modulations,” Structural Health Monitoring, 11, 197-211 (2012).

[17] G. Zumpano and M. Meo, “Damage localization using transient non-linear elastic wave spectroscopy on composite structures,” International Journal of Non-Linear Mechanics, 43, 217-230 (2008).

[18] I. Y. Solodov, N. Krohn, and G. Busse, “CAN: an example of nonclassical acoustic nonlinearity in solids,” Ultrasonics, 40, 621-625 (2002).

[19] J. Chen, D. Zhang, Y. Mao, and J. Cheng, “Contact acoustic nonlinearity in a bonded solid–solid interface,” Ultrasonics, 44, Supplement, e1355-e1358 12/22/ (2006).

[20] S. Biwa, S. Nakajima, and N. Ohno, “On the Acoustic Nonlinearity of Solid-Solid Contact With Pressure-Dependent Interface Stiffness," Journal of Applied Mechanics, 71, 508-515 (2004).

[21] I. Y. Solodov, “Nonlinear NDE using contact acoustic nonlinearity (CAN),” Ultrasonics Symposium, 1994. Proceedings., 1994 IEEE,1279-1283 vol.2 (1994).

[22] J. M. Richardson, “Harmonic generation at an unbonded interface—I. Planar interface between semi-infinite elastic media,” International Journal of Engineering Science, 17, 73-85 (1979).

[23] M. Castaings, E. Le Clezio, and B. Hosten, “Modal decomposition method for modeling the interaction of Lamb waves with cracks,” The Journal of the Acoustical Society of America, 112, 2567-2582 (2002).

[24] P. J. Torvik, “Reflection of Wave Trains in Semi ‐ Infinite Plates,” The Journal of the Acoustical Society of America, 41, 346-353 (1967).