

# Prediction of mechanical properties in bimodal nanotwinned metals with a composite structure

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## Abstract

Nanostructured face-centered cubic (fcc) metals with nanoscale twin lamellae and multiple distribution of microstructural size are proved to possess higher yield strength and good ductility. In this paper, a mechanism-based theoretical model is developed to simulate the yield strength, strain hardening, and uniform elongation of the nanotwinned composite metals with bimodal distribution of microstructural size. The mechanisms of strengthening and the failure in such bimodal nanotwinned metals are studied for evaluating the strength and ductility. A modified mean-field approach is adopted here to calculate the total stress-strain response of this kind of nanotwinned composite structures. The contribution of microcracks generated during plastic deformation has been taken into account to predict strain hardening and uniform elongation. Our simulation results indicate that the proposed model can successfully describe the mechanical properties of bimodal nanotwinned metals with a composite structure, including the yield strength and ductility. We further demonstrate that the yield strength and elongation are both sensitive to the twin spacing and the volume fraction of components. The calculations based on the proposed model agree well with the experimental results. These findings suggest that the high yield strength and

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high ductility can be achieved by optimizing the grain size and the twin spacings in the nanotwinned composite structures.

*Key Words:* Bimodal nanotwinned metal; Twin spacing; Grain Size; Yield strength; Ductility.

## **1 Introduction**

Nanostructured metals have stimulated vast interests due to their distinctive mechanical properties, for example, the superior mechanical strength compared to those of corresponding coarse-grained counterparts [1-7], which make these materials play the essential role in designing lighter and stronger structures of technological applications. Unfortunately, the improved strength in nanostructured metals is always achieved with the loss of ductility and work-hardening capability [3,4,8-13], which is similar to the traditional strengthening methods such as refining grain size, solid solution alloying and phase transformation. Therefore, how to achieve the simultaneously higher strength and ductility in metals and alloys is the essentially challenging issue in the application of the nanostructured metals. In the past ten years a great number of endeavors have been made to explore how to improve the strength with keeping good ductility and toughness. Mixing the various sizes of microstructures in nanostructured metals have been proved as an effective approach for a superior synergy in strength and ductility. For example, the nanostructured metallic materials with bi/multi-modal grain size distribution perform higher yield strength with a good ductility [14-23]. Generating the intrinsic twin boundaries in polycrystalline metals is also an alternative method to improve the tensile ductility in higher-strength nanostructured metals, such as the nanotwinned polycrystalline coppers and nanotwinned stainless steels [24-28]. More recently, a novel strategy is developed to strengthen the metallic materials by means of dynamic plastic deformation to embed the nanotwinned grains into the matrix of nano-sized grains or coarse-grains. Such nanotwinned composites with bimodal distribution of microstructural size exhibit the excellent combination of strength and ductility [29-32].

After discovering the effective methodologies in experiments for enhancing the strength-ductility synergy, a plenty of theoretical studies have been carried out to investigate the deformation mechanisms and then to predict the corresponding mechanical properties. In general, micromechanical models and finite element methods are widely utilized to simulate the stress-strain response in the composite nanomaterials such as the particle reinforced metals matrix composites and nanograins/nanotwins strengthened composite metallic materials [20-23,33-38]. For the nanostructured metals with a bimodal grain size distribution, the secant Mori-Tanaka (M-T) mean-field approach [20,21,39] and the viscoplastic self-consistent scheme [40,41] are often applied to simulate the yield strength and ductility of such composite structure, which are sensitive to the grain size distribution and volume fraction of component. For the nanotwinned polycrystalline metals, the molecular dynamic (MD) simulations are performed usually in the atomic scale to quantify the contributions of the twin boundaries (TBs) to the strength, strain hardening and toughness [42-47]. The mechanism-based theoretical models with a continuum description are presented to describe the mechanical behaviors in the nanotwinned metal. For example, the crystal plasticity models for nanotwinned copper were developed to simulate the stress-strain response as well as the fracture behaviors by finite element method [38, 48-50]. The dislocation density-based plasticity models were developed to describe the variation of strength, strain hardening and the ductility of nanotwinned metals with the twin spacing, and a scale law of the maximum strength in nanotwinned metals was explored [51-54]. However, for the nanotwinned composite metals, it remains unclear how to predict the mechanical properties as the functions of grain size, twin spacing, and volume fraction of each component. The explicit theoretical model that enables to describe the experimental results is still in lack.

In this work, a micromechanical composite model is developed to study the mechanical properties of nanotwinned composites with a bimodal distribution of microstructural size. The mechanism-based plastic model for nanotwinned metals is adopted to describe the constitutive relation of nanotwinned phase. The

strain-gradient plastic model is also presented to simulate the stress-strain relation of nano/ultrafine grained phase. In the framework of composite model, the contribution of microcracks generating during plastic deformation is taken into account in the mechanical properties of the bimodal nanotwinned composite. The calculations demonstrate that our proposed theoretical model can completely characterize the mechanical properties such as the yield strength, strain hardening, and elongation in such kind bimodal nanotwinned metal. These properties are sensitive to the microstructural size such as the twin spacing and grain size, as well as the volume fraction of each component.

## 2. Theoretical description

### 2.1 Composite model

Experiments have demonstrated that the bulk nanostructured austenitic stainless steel contains nano-sized/coarse grains with the nanoscale nanotwins embedded in micro-size grains by means of dynamic plastic deformation [30,32]. This complex microstructure can be characterized by a composite structure with a bimodal distribution of microstructures. This nanotwinned composite metal consists of polycrystalline phase and nanotwinned phase, as shown in figure 1. Motivated by this observation, we can consider the grain size in polycrystalline phase and twin spacing in nanotwinned phase as the bimodal size distribution in such nanotwinned composite metals. As a consequence, a modified mean-field approach is applied to simulate the stress-strain response of the bimodal nanotwinned metals, and the influence of bimodal size distribution must be taken into account. From the micromechanical model developed by Weng [55,56], the relationship between the hydrostatic and deviatoric strains of the constituent phases and those of the composite follows

$$\begin{aligned} \varepsilon_{kk}^{(0)} &= \frac{\alpha_0^s(\kappa_1 - \kappa_0) + \kappa_0}{c_0\alpha_0^s(\kappa_1 - \kappa_0) + \kappa_0} \bar{\varepsilon}_{kk}, \varepsilon_{ij}^{(0)'} = \frac{\beta_0^s(\mu_1 - \mu_0^s)}{c_0\beta_0^s(\mu_1 - \mu_0^s) + \mu_0^s} \bar{\varepsilon}_{ij}' - \frac{c_1\beta_0^s\mu_1}{c_0\beta_0^s(\mu_1 - \mu_0^s) + \mu_0^s} \varepsilon_{ij}^{p(1)}, \\ \varepsilon_{kk}^{(1)} &= \frac{\kappa_0}{c_0\alpha_0^s(\kappa_1 - \kappa_0) + \kappa_0} \bar{\varepsilon}_{kk}, \varepsilon_{ij}^{(1)'} = \frac{\mu_0^s}{c_0\beta_0^s(\mu_1 - \mu_0^s) + \mu_0^s} \bar{\varepsilon}_{ij}' + \frac{c_0\beta_0^s\mu_1}{c_0\beta_0^s(\mu_1 - \mu_0^s) + \mu_0^s} \varepsilon_{ij}^{p(1)} \end{aligned}, \quad (1)$$

and the mean stress components of polycrystalline phase and nanotwinned phase are given by

$$\begin{aligned}\sigma_{kk}^{(1)} &= 3\kappa_0 \frac{\kappa_1}{c_0 \alpha_0^s (\kappa_1 - \kappa_0) + \kappa_0} \bar{\varepsilon}_{kk}, \sigma_{ij}^{(1)'} = \frac{2\mu_0^s \mu_1 [\bar{\varepsilon}_{ij}' - (1 - c_0 \beta_0^s) \varepsilon_{ij}^{p(1)}]}{c_0 \beta_0^s (\mu_1 - \mu_0^s) + \mu_0^s} \\ \sigma_{kk}^{(0)} &= 3\kappa_0 \frac{\alpha_0^s (\kappa_1 - \kappa_0) + \kappa_0}{c_0 \alpha_0^s (\kappa_1 - \kappa_0) + \kappa_0} \bar{\varepsilon}_{kk}, \sigma_{ij}^{(0)'} = \frac{2\mu_0^s \{ [\beta_0^s (\mu_1 - \mu_0^s) + \mu_0^s] \bar{\varepsilon}_{ij}' - c_1 \beta_0^s \mu_1 \varepsilon_{ij}^{p(1)} \}}{c_0 \beta_0^s (\mu_1 - \mu_0^s) + \mu_0^s}.\end{aligned}\quad (2)$$

Therefore, the dilatational and deviatoric stresses and strains of the composite are connected by

$$\begin{aligned}\bar{\sigma}_{kk} &= 3\kappa_0 \left[ 1 + \frac{c_1 (\kappa_1 - \kappa_0)}{c_0 \alpha_0^s (\kappa_1 - \kappa_0) + \kappa_0} \right] \bar{\varepsilon}_{kk}, \\ \bar{\sigma}_{ij}' &= 2\mu_0^s \left\{ \left[ 1 + \frac{c_1 (\mu_1 - \mu_0^s)}{c_0 \beta_0^s (\mu_1 - \mu_0^s) + \mu_0^s} \right] \bar{\varepsilon}_{ij}' - \frac{c_1 \mu_1}{c_0 \beta_0^s (\mu_1 - \mu_0^s) + \mu_0^s} \varepsilon_{ij}^{p(1)} \right\}.\end{aligned}\quad (3)$$

Here,  $c_i (i = P, T)$  is the volume fraction of the polycrystalline phase or nanotwinned phase.  $P$  indicates the polycrystalline phase, and  $T$  represents the nanotwinned phase.  $\alpha_0^s$  and  $\beta_0^s$  are the components of Eshelby's tensor for spherical inclusions, i.e.,

$$S_0^s = (\alpha_0^s, \beta_0^s), \text{ with } \alpha_0^s = \frac{1 + \nu_0^s}{3(1 - \nu_0^s)}, \beta_0^s = \frac{2(4 - 5\nu_0^s)}{15(1 - \nu_0^s)}.\quad (4)$$

The corresponding secant bulk and shear moduli of the  $i$ th phase are taken to satisfy the isotropic relations as follows

$$\kappa_i = \frac{E_i^s}{3(1 - 2\nu_i^s)}, \mu_i = \frac{E_i^s}{2(1 + \nu_i^s)},\quad (5)$$

in which the related secant Young's modulus and secant Poisson's ratio of  $i$ th phase is written as

$$E_i^s = \frac{\sigma_i}{\varepsilon_i^e + \varepsilon_i^p} = \frac{E_i}{1 + \frac{E_i \varepsilon_i}{\sigma_{flow}^i} \left( \frac{\sigma_{i11}}{\sigma_{flow}^i} \right)^{m_0 - 1}}, \quad \nu_i^s = \frac{1}{2} - \left( \frac{1}{2} - \nu_i \frac{E_i^s}{E_i} \right).\quad (6)$$

Here,  $E_i$  and  $\nu_i$  denote the Young's modulus and Poisson's ratio of the  $i$ th phase.

The flow stress  $\sigma_{flow}^i$  for polycrystalline phase and nanotwinned phase will be presented in the next section.

## 2.2 Constitutive relations

To grasp the overall stress-strain response of bimodal nanotwinned composite by using the micromechanical model, the constitutive relations of the polycrystalline

phase and nanotwinned phase in composite structures must be identified. Here, the mechanism-based strain gradient plasticity [57] is adopted to describe the stress-strain response for the constituents in bimodal nanotwinned composite. The strain rate  $\dot{\boldsymbol{\varepsilon}}$  can be decomposed into its elastic and plastic parts,

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p. \quad (7)$$

The elastic strain rate is obtained from the stress rate in the linear elastic relation as

$$\dot{\boldsymbol{\varepsilon}}^e = \mathbf{M} : \dot{\boldsymbol{\sigma}}, \quad (8)$$

where  $\mathbf{M}$  is the elastic compliance tensor. The plastic strain rate is proportional to the deviatoric stress  $\boldsymbol{\sigma}'$  based on the conventional  $J_2$ -flow theory of plasticity, given as

$$\dot{\boldsymbol{\varepsilon}}^p = \frac{3\dot{\boldsymbol{\varepsilon}}^p}{2\sigma_e} \boldsymbol{\sigma}', \quad (9)$$

where  $\sigma'_{ij} = \sigma_{ij} - \sigma_{kk}\delta_{ij}/3$  and  $\sigma_e = \sqrt{3\sigma'_{ij}\sigma'_{ij}/2}$  is the von Mises equivalent stress.

$\dot{\boldsymbol{\varepsilon}}^p$  is the equivalent plastic strain rate, expressed as

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\boldsymbol{\varepsilon}} \left[ \frac{\sigma_e}{\sigma_{flow}} \right]^{m_0}, \quad (10)$$

where  $\dot{\boldsymbol{\varepsilon}} = \sqrt{2\dot{\boldsymbol{\varepsilon}}'_{ij}\dot{\boldsymbol{\varepsilon}}'_{ij}/3}$  is the equivalent strain rate and  $\dot{\boldsymbol{\varepsilon}}'_{ij} = \dot{\boldsymbol{\varepsilon}}_{ij} - \dot{\boldsymbol{\varepsilon}}_{kk}\delta_{ij}/3$ .  $m_0$  is the rate-sensitivity exponent and  $\sigma_{flow}$  is the flow stress. Since the deformation mechanism in nano-grained/coarse-grained phase is quite different with the one in nanotwinned phase, the flow stresses in polycrystalline phase and nanotwinned need to be addressed separately.

### 2.2.1 Nanotwinned phase

The nano-scale twin lamellae in nanotwinned metals with a composite structure play an important role in strengthening the materials. The nanotwinned phase not only acts as the harder phase but also keep a good plasticity. The twin boundaries, similar to the grain boundaries, enable to provide more obstacles during dislocation motion. Here, the dislocation density in the areas of dislocation pile-up nearby the twin boundaries is present to describe the influence of the twin boundaries on the flow

stress. Therefore, a generalized Taylor relation of flow stress in the nanotwinned phase tells

$$\sigma_{flow} = \sigma_0 + M\alpha\mu b\sqrt{\rho_l + \rho_{TB}}, \quad (11)$$

where  $\alpha$ ,  $\mu$ ,  $b$  and  $M$  are the empirical constant, the shear modulus, the Burger constant and the Taylor factor, respectively.  $\sigma_0$  is the lattice friction stress.  $\rho_{TB}$  is the dislocation density in the dislocation pile-up zones nearby the twin boundaries and can be expressed as [51,58]

$$\rho_{TB} = \phi^{TB} \left( \frac{12N_0}{\pi d_G^T} + \frac{12n_p'}{\sqrt{3}\pi d_G^T} d_{TB}^{-1} \right) - \frac{3\phi_0}{\sqrt{3}} d_{TB}^{-2}, \quad (12)$$

where  $d_G^T$  is the size of grains with the twin lamellae,  $n_p'$  is the number of initial partial dislocations and  $N_0$  is the maximum number of full dislocations in grain, both of which are independent of the twin spacing.  $\phi_0$  and  $\phi^{TB}$  are the geometrical parameters.  $\rho_l$  denotes the density of dislocations in the crystal interior of grains, which can be derived from Kocks and Mecking's model [59,60], i.e.,

$$\frac{\partial \rho_l}{\partial \varepsilon^p} = M \left( \frac{k}{d_G^T} + k_1 \sqrt{\rho_l} - k_2 \rho_l \right). \quad (13)$$

Here,  $k = 1/b$ ;  $k_1 = \psi/b$ ;  $k_2 = k_{20}(\dot{\varepsilon}^p / \dot{\varepsilon}_0)^{-1/n_0}$ ,  $\psi$  is a proportionality factor,  $k_{20}$  and  $\dot{\varepsilon}_0$  are the constants, and  $n_0$  is inversely proportional to the temperature.  $\varepsilon^p$  is the plastic strain. Note that the athermal storage of dislocations and the annihilation of dislocations during the dynamic process are both involved in the K-M model.

### 2.2.2 Polycrystalline phase

The experimental studies showed that the nanograins can be generated during DPD process, and the annealing process makes the nanograins grow into micrometer-scale grains [29,31]. Thereby, the grain size in polycrystalline phase can be ranged from several tens of nanometer to several micrometers or even larger. From this perspective, we should present the flow stresses of nano-grains and coarse-grains separately.

When the grain size in the polycrystalline phase is in the nanometer-scale, the strength of the nano-grained phase will be reinforced significantly. This is originated from the increased grain boundaries blocking the motion of more lattice dislocations. Thereby, the effects of grain boundaries are taken into account during plastic deformation in the nano-grained phase. The dislocation density in the grain boundary dislocation pile-up zones (GBDPZs) is addressed to characterize the contribution of grain boundaries on the flow stress, expressed as [20]

$$\sigma_{flow}^N = \sigma_0 + M\alpha\mu b\sqrt{\rho_I + \rho_{GB}}, \quad (14)$$

where  $\rho_{GB}$  and  $\rho_I$  are the density of dislocation in the dislocation pile-up zones nearby grain boundaries and the one in the crystal interior, respectively.  $\rho_I$  can be determined from Eq. (13).  $\rho_{GB}$  is the density of dislocations in the GBDPZ, given by [51]

$$\rho_{GB} = \frac{n^{GB}\lambda^{GB}}{V_{Cell}} = k^{GB}\frac{\eta^{GB}}{b}, \quad (15)$$

where  $k^{GB} = 6d_{GBDPZ} / \phi^{GB}d_G$ ,  $d_{GBDPZ}$  is the thickness of GBDPZ,  $d_G$  is the grain size in polycrystalline phase without twins.  $n^{GB}$  is the number of dislocations around the grain boundaries and  $\phi_3$  is the geometrical factor.  $\lambda \approx \pi d_G$  is the average length of the dislocation loop for dislocations in the GBDPZ.

Due to the fact that the primary deformation mechanism in the coarse-grained phase is dominated by the intragrain dislocation-mediated interaction, and the Taylor-type relationship can be utilized to describe the flow stress which is related to the dislocation density. Therefore, the flow stress of the coarse-grained phase can be expressed as:

$$\sigma_{flow}^C = \sigma_0 + M\alpha\mu b\sqrt{\rho_I} + \sigma_b, \quad (16)$$

where  $\sigma_b$  represents the back stress. Note that the Taylor evolution law contains the isotropic strain-hardening and the back-stress-induced kinematic hardening. The back stress term in Eq. (16) can be expressed by



$$\sigma_b = M \frac{\mu b}{d_G} N_b, \quad (17)$$

where  $N_b$  is the number of dislocations blocked at the grain boundaries of the coarse grain phase, which depends on the plastic strain following the evolution law [61,62],

$$\frac{dN_b}{d\varepsilon^p} = \frac{\zeta}{b} \left(1 - \frac{N_b}{N_0^b}\right). \quad (18)$$

Here,  $\zeta$  and  $N_0^b$  are the mean spacing between slip bands and the maximum number of dislocation loops at the grain boundary in the coarse-grained metal, respectively.

### 2.3 Contributions of microcracks

For the bimodal nanostructured metals, the nanovoids or nano/microcracks can be induced during plastic deformation in the matrix of nano-grained or ultrafine grained phase, as shown in figure 2. These nanoscale voids/cracks will affect the mechanical properties of bimodal metals significantly through modifying the stress states around the voids/cracks and the constitutive relation in the matrix phase. In the nanotwinned metals with bimodal distribution of microstructural size, the appearance of nano/microcracks in the matrix phase results in the change of overall stress and strain of polycrystalline phase. The microcrack-matrix-effective-medium approach can be utilized to model the stress and strain in the matrix phase of bimodal nanotwinned metals. Suppose the representative volume element (RVE) boundary is subjected to tractions in equilibrium with a uniform overall stress of  $\sigma^\infty$ . The average strain in a solid with nano/microcracks is consistent with regular and singular parts as

$$\bar{\boldsymbol{\varepsilon}} = \bar{\boldsymbol{\varepsilon}}^m + \bar{\boldsymbol{\varepsilon}}^c, \quad (19)$$

where  $\bar{\boldsymbol{\varepsilon}}^m$  is the matrix strain averaged over the RVE and  $\bar{\boldsymbol{\varepsilon}}^c$  is the nano/microcrack-induced variations in the overall average strain, respectively. When the matrix deform in the elastic region, the matrix strain can be written as  $\bar{\boldsymbol{\varepsilon}}^m = \mathbf{M}^m : \boldsymbol{\sigma}^\infty$ . Since the nano/microcracks generated during the plastic deformation are supposed to be parallel, the corresponding effective moduli are as follows:

$$E_1 = E_0 \left[ 1 + \frac{16(1-\nu_0^2)}{3} \rho \right]^{-1}; G_{12} = G_0 \left[ 1 + \frac{8(1+\nu_0)}{3(1-\nu_0/2)} \rho \right]^{-1}. \quad (20)$$

It should be noted from Eq. (20) that the effective moduli of the bimodal nanotwinned metals are associated with the density of microcracks in the materials. The number of these cracks will be increased during the plastic deformation, leading to that the density of nano/microcracks in the bimodal nanotwinned metals is sensitive to the applied stress or strain. From this perspective, the strain-based Weibull distribution function with respect to the plastic strain is applied to character the number of cracks. Thereby, the density of nano/microcracks can be expressed as

$$\rho = \rho_0 (1 - f_w(\varepsilon_p)) = \rho_0 [1 - \exp(-(\varepsilon_p / \varepsilon_0)^M)] \quad (21)$$

Here,  $\rho_0$  is a constant;  $f_w(\varepsilon_p)$  is the strain-based Weibull distribution function in which  $\varepsilon_0$  is the reference strain and  $M$  is the Weibull modulus. On the other hand, the existence of nano/microcracks indicates that the number of dislocations blocked at the grain boundaries or around the cracks is increased. These dislocations pile-up along the grain boundaries will lead to back stress effects on the strain hardening which must be taken into account during the plastic deformation in the nano/ultrafine-grained phase. Therefore, the constitutive relation of the nano-grained phase will contain the back stress term in the flow stress expression, which is changed into

$$\sigma_{flow} = \sigma_0 + M \alpha \mu b \sqrt{\rho} + \rho \sigma_b^*, \quad (22)$$

where  $\sigma_b^*$  is the nano/microcrack-induced back stress.

### 3. Results and discussion

We use the 316L austenitic stainless steel (316Lss) as an illustrative example to explore the relationship between mechanical properties and the size and volume fraction of microstructures in nanotwinned metals with a composite structure. A set of material parameters for 316Lss used in our simulations is extracted from the literature or fit with the experimental data [29-32], as listed in Table 1. Since the effective thickness of the TBDPZ and GBDPZ is supposed to range from 7 to 10 lattice parameters [48], we chose  $d_{GBDPZ}$  and  $d_{TBDPZ}$  of stainless steel around 3.6 nm. Under these parameters, the yield strength and ductility in the bimodal nanotwinned

316Lss are simulated based on the proposed model. Figure 3 plots the simulated stress-strain responses for the coarse-grained 316Lss and bimodal nanotwinned 316Lss with various microstructural scales. The corresponding experimental measurements for bimodal nanotwinned 316Lss with different annealing temperature are also reproduced in the figure. It is noted from the figure that the proposed micromechanical model can describe the mechanical properties of bimodal nanotwinned 316Lss successfully. The simulations agree well with the experimental data, including the yield strength, strain hardening, and uniform elongation, for different bimodal nanotwinned samples with annealing temperature of 730°C, 750°C, and 770°C. It means that the proposed model can be applicable to predict the mechanical properties of bimodal nanotwinned 316Lss for different volume fractions of components and different sizes of microstructures.

Experimental studies demonstrated that the work-hardening rate of bimodal nanotwinned 316Lss is quite sensitive to the annealing temperature [31, 32]. We plot the simulated work-hardening rate ( $\Theta=d\sigma/d\varepsilon$ ) as a function of applied true strain in figure 4(a) for various microstructural scales and volume fractions. The experimental data are also provided in the figure 4(b). From figure 4(a), it can be noticed that all curves of work-hardening rates for bimodal nanotwinned 316Lss intersect the curve of the as-received coarse-grained 316Lss. It means that with the applied strain increasing, the work-hardening rates of bimodal nanotwinned 316Lss are larger than that of the as-received coarse-grained specimens until the applied strain reaches the intersection point, and then the bimodal nanotwinned samples has a lower strain hardening rate. We can also find from figures 4(a) and 4(b) that the predictions for work-hardening rate are consistent with the experimental data. On the other hand, one can learn from figure 4(a) that the simulated work-hardening rate in the bimodal nanotwinned 316Lss decreases with the applied strain increasing, which is in accordance with experimental observations [31]. In the present model, the influences of the nano/microcracks are taken into account on plastic deformation in the bimodal nanotwinned stainless steels. These nano/microcracks could be generated in the polycrystalline phase and blocked by the nanotwin bundles, giving rise to more nano/microcracks appearing in polycrystalline matrix phase with further deformation.

Since the occurrence of nano/microcracks enables to change the stress and strain in the polycrystalline matrix phase, the nano/microcrack-induced strain will increase with the density of nano/microcrack increasing, leading to the decrease of effective strain in the polycrystalline matrix phase. As a result, the effects of the back stress in the polycrystalline matrix phase which provides the kinetic strain hardening, become faded with further deformation. Alternatively, the work-hardening rate decreases with applied strain increasing.

The contribution of the nano/microcracks on the mechanical properties in bimodal nanotwinned metals are taken into account in the proposed model, leading to the stress-strain relation in the matrix phase related to the density of microcracks. Since the Weibull distribution function is involved to describe the density of microcracks as a function of plastic strain, the simulated results are associated with the parameters in the function of microcracks density. Figure 5 shows the simulated results for the bimodal nanotwinned 316Lss with different reference density of microcracks  $\rho_0$ . One can note from figure 5(a) that the failure strain is sensitive to the reference density of microcracks. The larger the reference density is, the smaller the failure strain is. We can find it more clearly in figure 5(b) that the failure strain decreases with the reference density nonlinearly. Figure 6 further depicts the influence of the Weibull modulus in Weibull distribution function on the mechanical properties of bimodal nanotwinned 316Lss. It can be clearly noticed that the more pronounced plasticity can be reached for the larger Weibull modulus, as shown in figure 6(a). We also can note that the failure strain varies with the Weibull modulus nonlinearly. From figures 5 and 6, interestingly noted is that the various parameters in the function of microcracks density can induce to the change of only the failure strain but not the yield strength.

Being analogous to traditional composite structure, the mechanical performance of bimodal nanotwinned composite metals is closely related to the volume fraction of components. Figure 7(a) shows stress-strain relation for bimodal nanotwinned 316Lss with different volume fraction of nanotwinned phase. From the figure, clearly noted is that the yield strength is improved significantly with increasing the volume fraction  $f_{TW}$ . For example, the yield strength is enhanced from 400 MPa to 800 MPa with

$f_{TW}$  changed from 0.1 to 0.5. In addition, the failure strain of bimodal nanotwinned 316Lss is also relevant to the volume fraction of nanotwinned phase. Figure 7(b) plots the strength varied with the failure strain for different volume fraction. With increasing the volume fraction, the yield strength and ultimate strength are both increased with the failure strain decreased. The variation of yield strength is more prominent than that of ultimate strength.

Due to the fact that the predicted stress-strain responses of bimodal nanotwinned 316Lss are pronouncedly dependent on the volume fraction of nanotwinned phase and the Weibull modulus in distribution function of nano/microcrack as shown in figures. 6 and 7, we depict the work-hardening rate of bimodal nanotwinned 316Lss with different  $f_{TW}$  and various Weibull modulus  $M$  in figures 8(a) and 8(b), respectively. It can be found from figure 8(a) that the work-hardening rate of bimodal nanotwinned 316Lss is decreased with the volume fraction of nanotwinned phase increasing. With the larger volume fraction  $f_{TW}$ , the work-hardening rate decreases sharply with strain after elastic deformation, and then turns into approximately linear change with further increasing the strain. It is because that the increasing volume fraction of nanotwinned phase weakens the kinematic strain hardening originated from back stress in the polycrystalline phase, leading to the lower work-hardening rate for larger volume fraction of nanotwinned phase. From figure 8(b), we can further note that the work-hardening rate is related to the Weibull modulus. With the Weibull modulus decreasing from 10 to 2, the work-hardening capability gradually exhausted in the bimodal nanotwinned 316Lss.

On the basis of micromechanical model presented in section 2, one can note that the constitutive relation of nanotwinned phase involves the feature sizes of microstructures such as the twin spacing and grain size. Therefore, the total stress-strain response of the bimodal nanotwinned composite is sensitive to the size of twin lamellae. Figure 9(a) shows the curves of stress-strain relation of bimodal nanotwinned 316Lss with different twin spacing in nanotwinned phase. One can find that both the yield strength and ultimate strength are enhanced with the size of twin lamellae decreasing. We further plot the relationship between the strength and the failure strain for different twin spacings in figure 9(b). With twin spacing decreasing,

both the strength and failure strain are improved. From figures (7) and (9), it can be concluded that the combination performance of strength and ductility in bimodal nanotwinned metals can be controlled by tuning the volume fraction of components and feature sizes of microstructures.

#### **4. Conclusion**

In summary, a theoretical model incorporating multiple strengthening mechanisms is proposed to explore the strength and ductility of the nanotwinned metals with composite structures. The modified mean-field approach is utilized to calculate the total stress-strain relationship in which the constitutive models of nano/ultrafine grained phase and the nanotwinned phase are involved. The contribution of nano/microcracks in the bimodal nanotwinned metals are taken into account to study strain hardening and failure strain. The numerical results indicate that the proposed model can describe mechanical properties of the bimodal nanotwinned metals successfully. The predictions agree well with their experimental results. Moreover, the detailed simulations show that the mechanical properties of nanotwinned metals with composite structures are significantly dependent on the twin spacing and the volume fraction of nanotwinned phase. The present work will provide a theoretical framework to design the yield strength and ductility of nanotwinned metals with composite structures by controlling volume fraction of components and microstructural feature size.

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### Figure captions

**Figure 1.** Schematic drawings of the bimodal nanotwinned metals in the polycrystalline materials with the assumption of the composite model

**Figure 2.** Schematic drawings of cracks arising during tensile testing

**Figure 3.** Comparison of the stress–strain relationship between the experimental results [31] and simulations for bimodal nanotwinned 316Lss and coarse-grained 316Lss. The coarse grain size is 2.35  $\mu\text{m}$ . The size of

nanotwinned grain is around  $5 \mu\text{m}$  with the twin spacing of 31 nm, 26 nm, and 26 nm for annealed temperature of  $770^\circ\text{C}$ ,  $750^\circ\text{C}$ , and  $730^\circ\text{C}$ , respectively.

**Figure 4.** (a) The simulated work-hardening rate ( $\Theta=d\sigma/d\varepsilon$ ) varied with true strain and (b) the experimental measurements of strain-dependent work-hardening rate for nanotwinned samples for annealed temperature of  $730^\circ\text{C}$ ,  $750^\circ\text{C}$  and  $770^\circ\text{C}$  [31].

**Figure 5.** (a) Influence of the reference density of microcracks on the stress-strain response, (b) and the failure strain varied with the reference density  $\rho_0$ .

**Figure 6.** (a) Influence of the Weibull modulus on the stress-strain response and (b) the uniform elongation as the function of Weibull modulus M.

**Figure 7.** (a) Stress-strain response of nanotwinned composite with different volume fraction of the nanotwins and (b) the strength varied with the failure strain for different volume fraction of nanotwins.

**Figure 8.** The simulated work-hardening rate ( $\Theta=d\sigma/d\varepsilon$ ) vs. (a) true strain for different volume fraction of nanotwinned phase and (b) various Weibull modulus.

**Figure 9.** (a) Stress-strain response of nanotwinned composite with different twin spacings and (b) strength and uniform elongation for various twin spacings.

Table 1

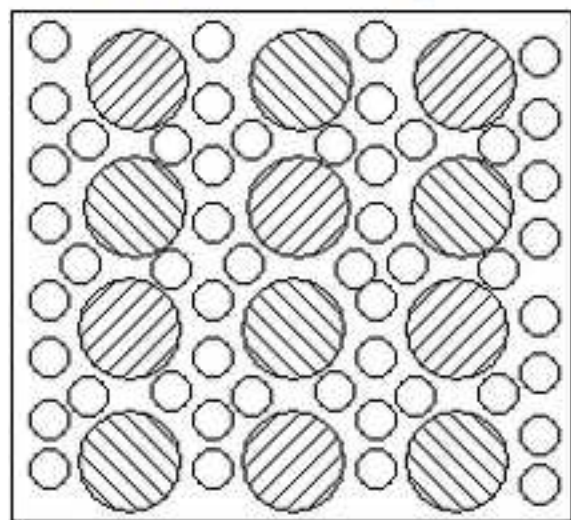
Description, symbol, and magnitude for the different parameters in the model

Parameter (Unit)	Symbol	Magnitude
Grain size (nm)	$d_G; d_G^T$	2380; 5000
Elastic modulus (GPa)	$E$	199.6
Shear modulus (GPa)	$\mu$	86
Poisson's ratio	$\nu$	0.29
Burgers vector (nm)	$b$	0.26
Taylor factor	$M$	3.06
Taylor constant	$\alpha$	0.3
Thickness of GBDPZ (nm)	$d_{GBDPZ}$	3.6
Thickness of TBDPZ (nm)	$d_{TBDPZ}$	3.6
Maximum number of dislocation loops	$N_0^b$	500
Maximum number of dislocation	$N_0$	1090
Number of initial partial dislocation	$n'_p$	321
Geometrical parameter	$\phi_0$	3.2
Dynamic recovery constant	$k_{20}$	18.5
Proportionality factor	$\psi$	0.2
Dynamic recovery constant	n	12.25
Reference strain rate (s <sup>-1</sup> )	$\dot{\epsilon}_0$	1.75
Geometric factor	$\phi^{TB}, \phi_i (i = 1, 2)$	0.5~1.5

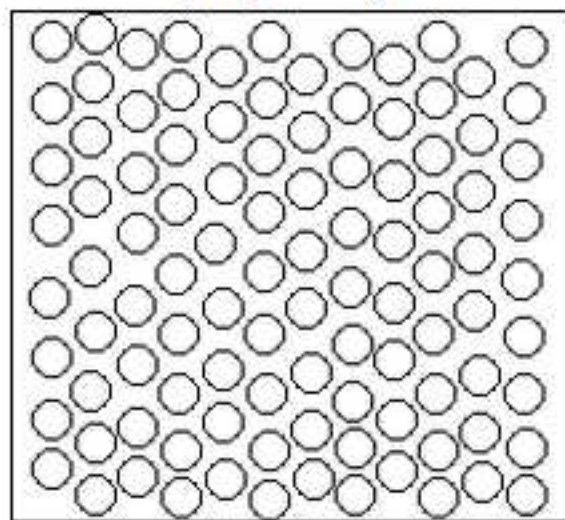


Figure1  
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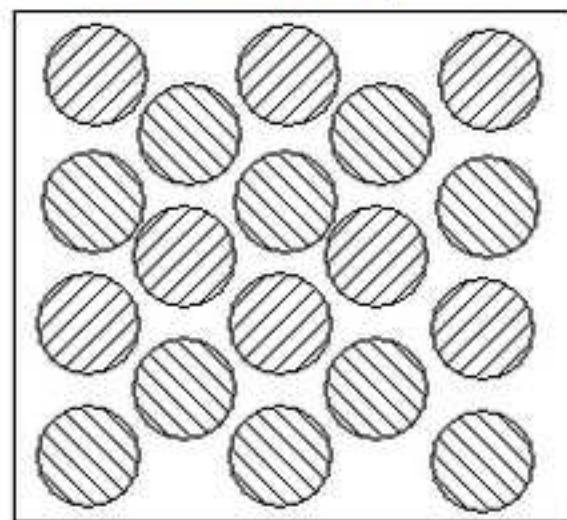
Nanotwinned composite



Polycrystal phase



Nanotwinned phase

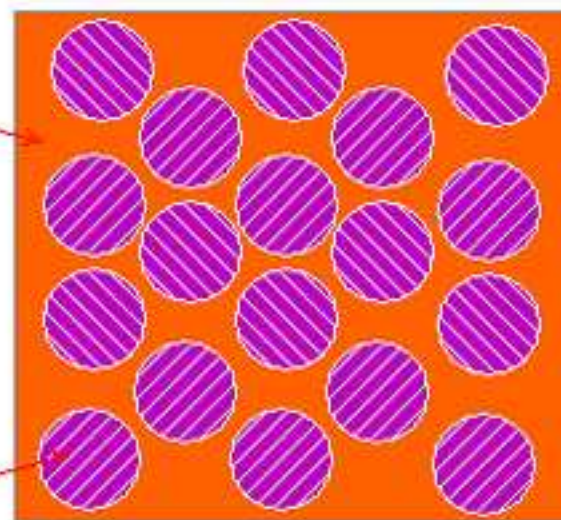


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Matrix phase



Nanotwinned phase

Figure2

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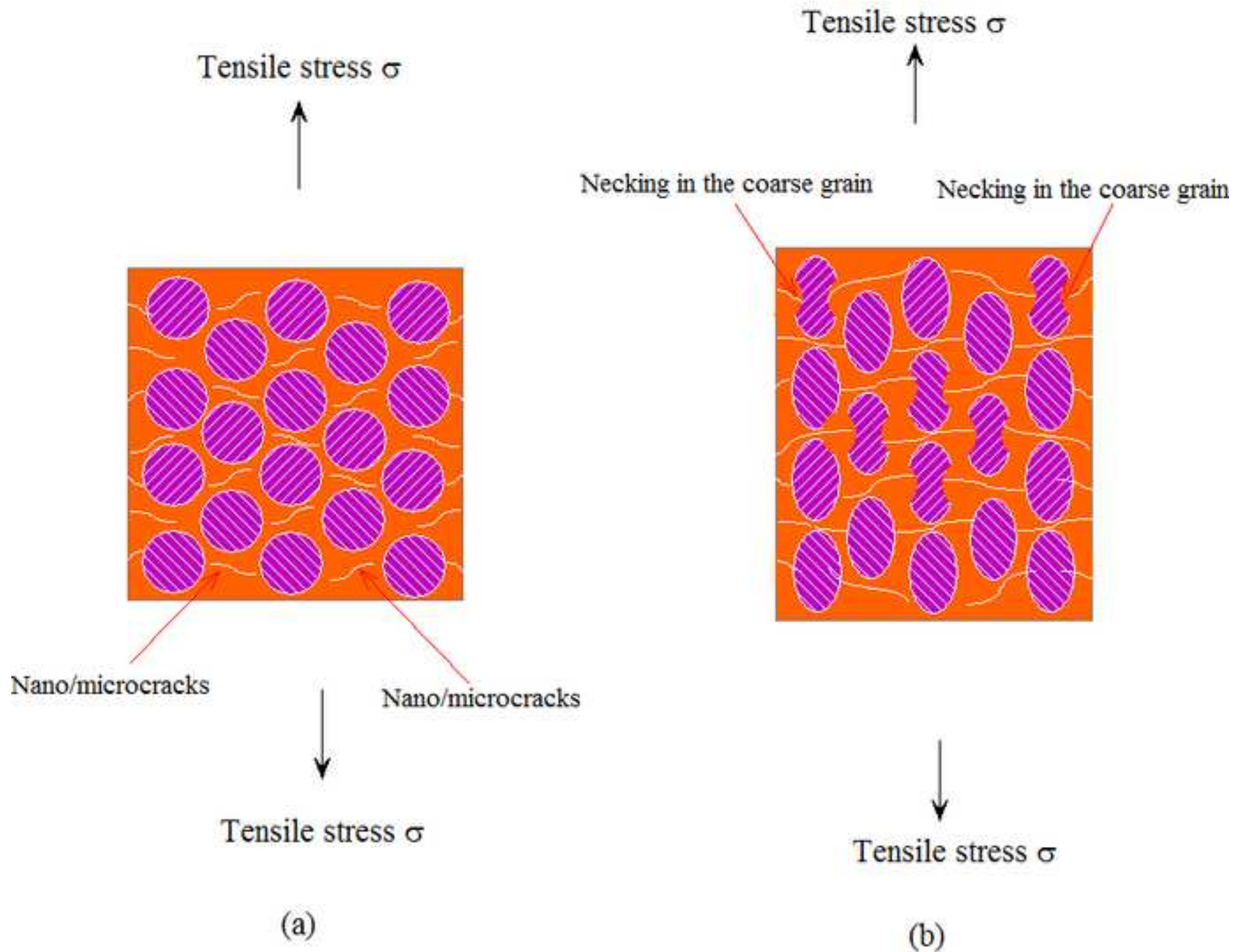


Figure3

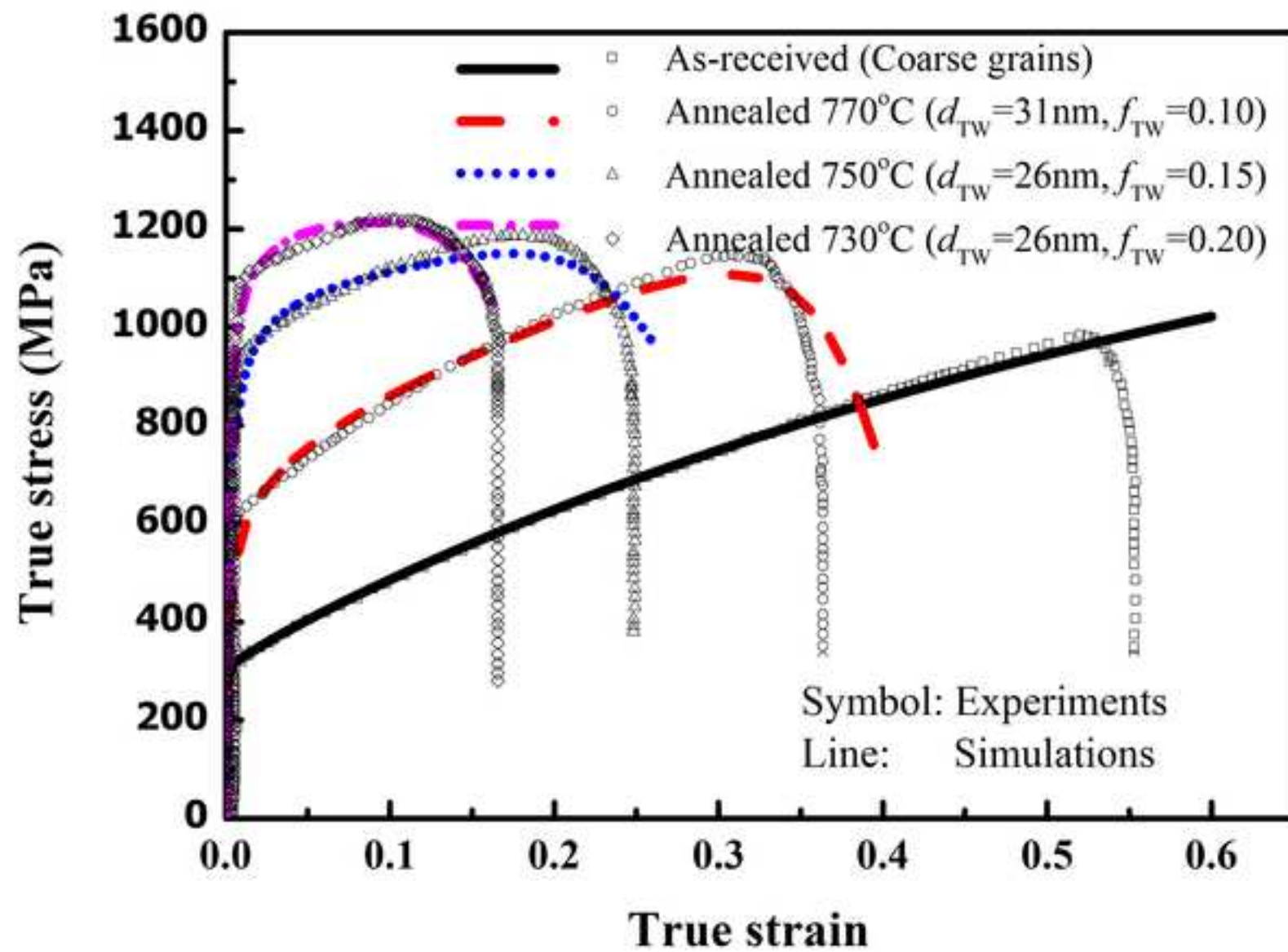
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Figure 4

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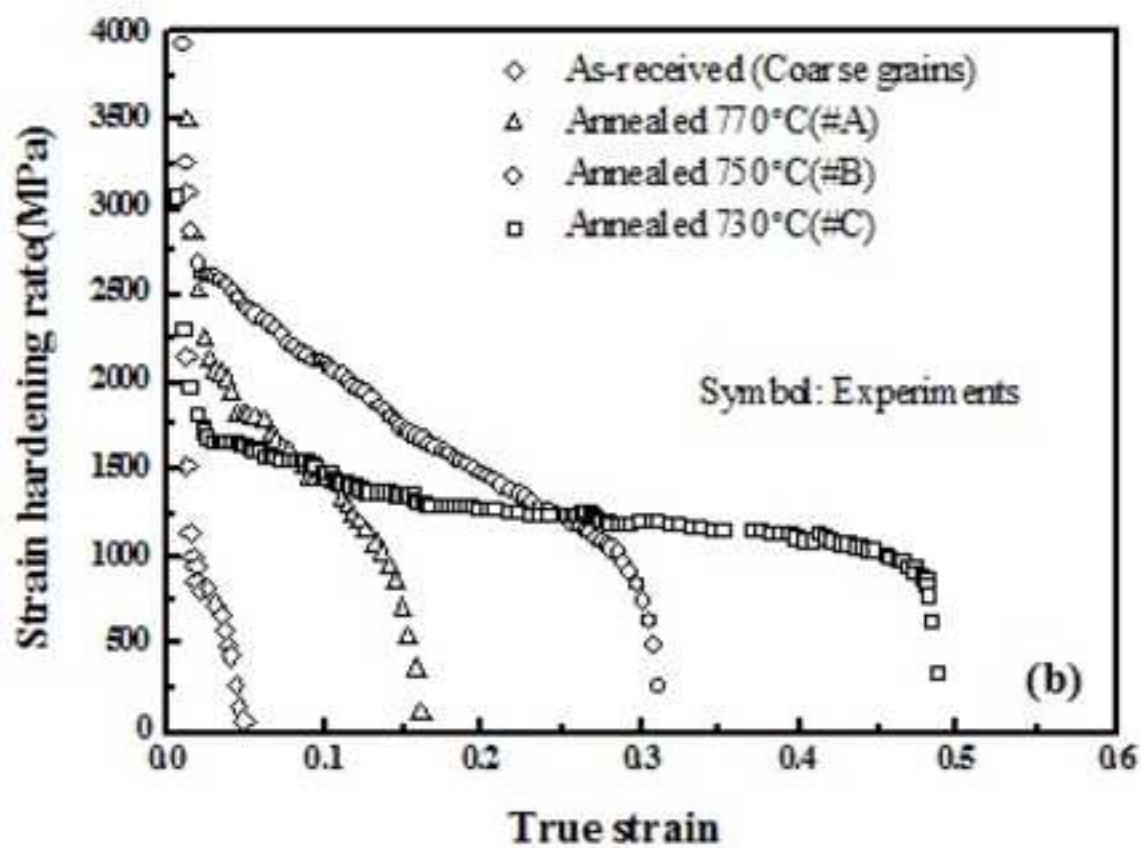
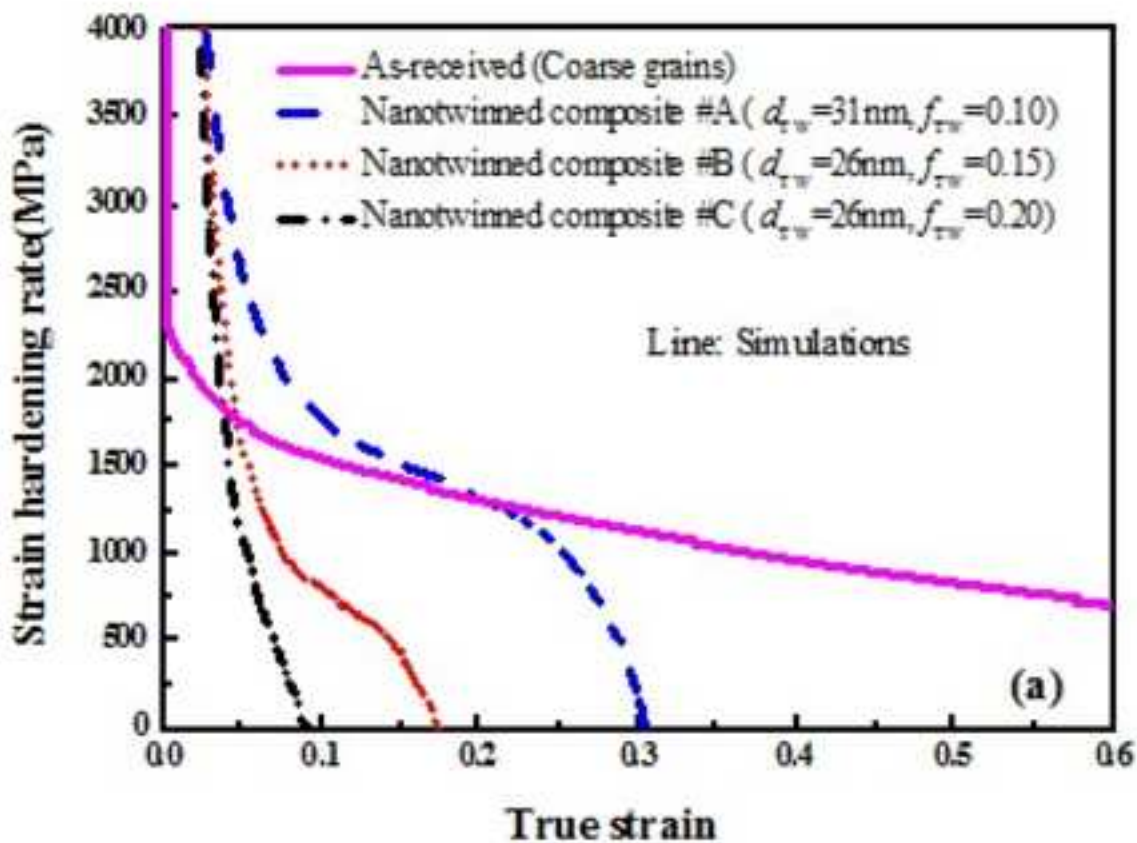


Figure5

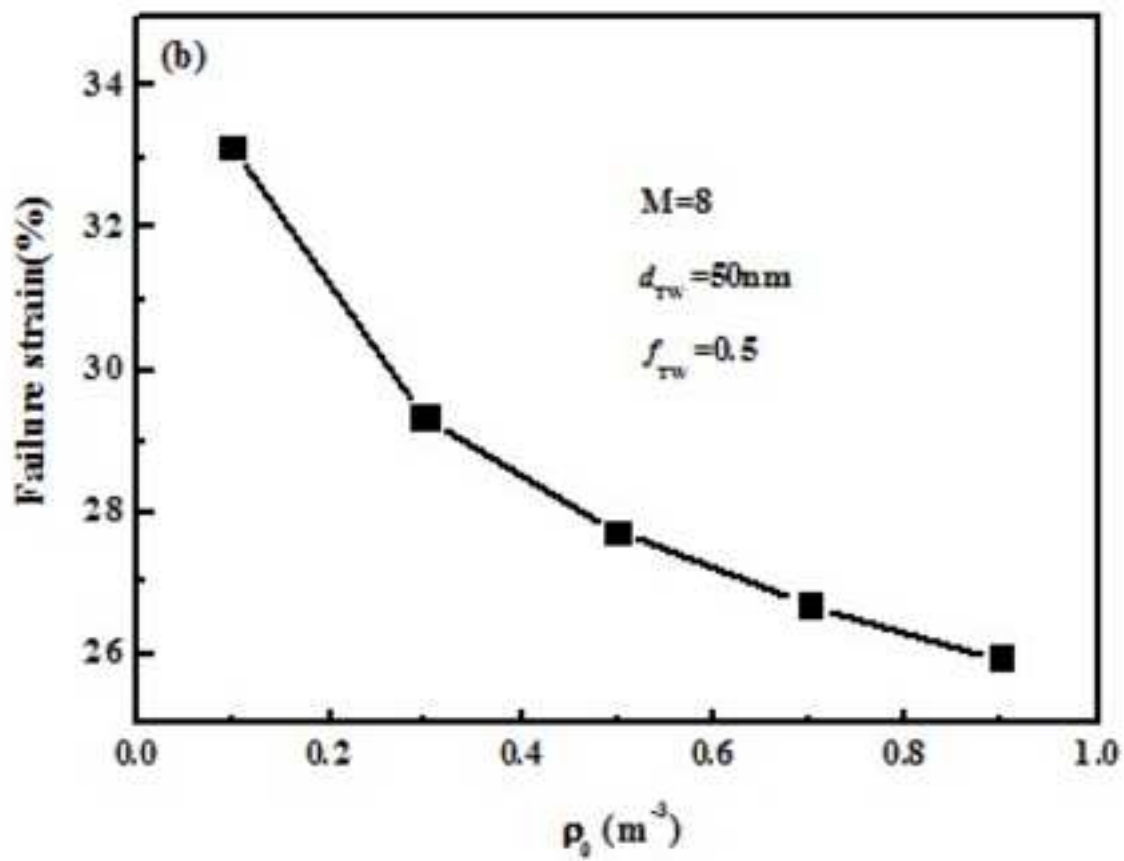
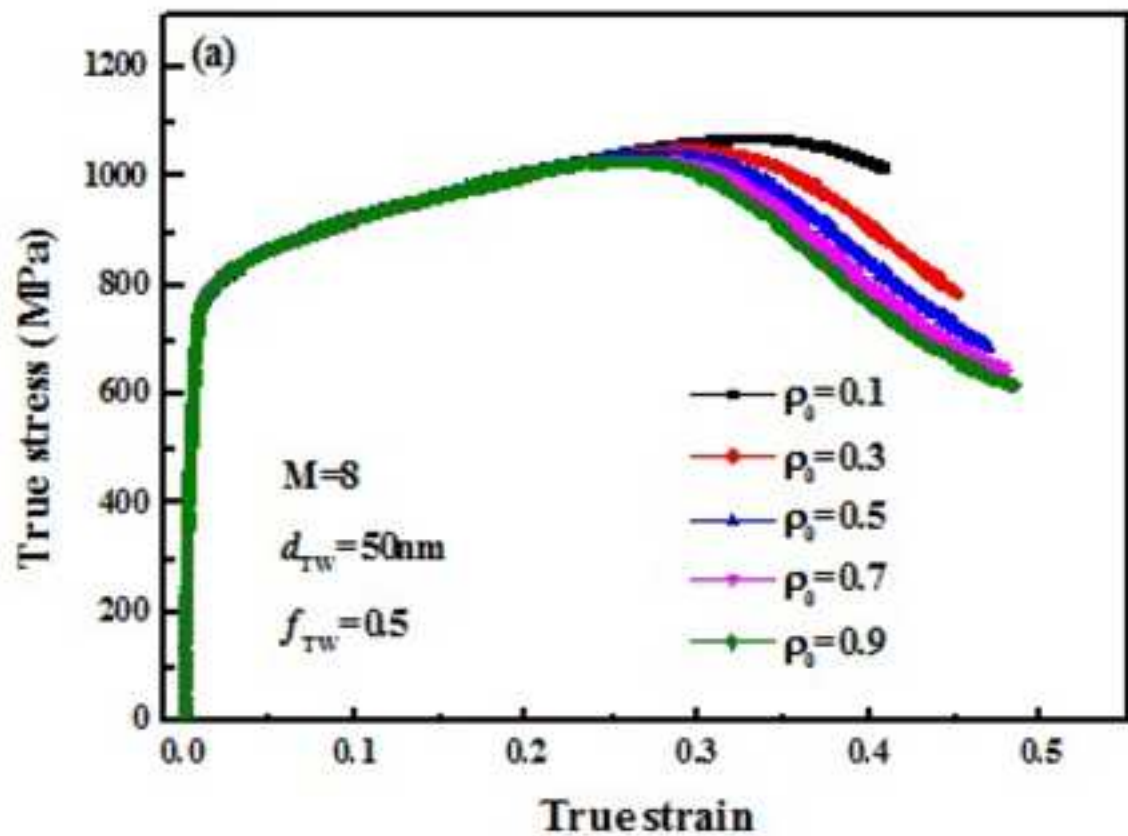
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Figure6

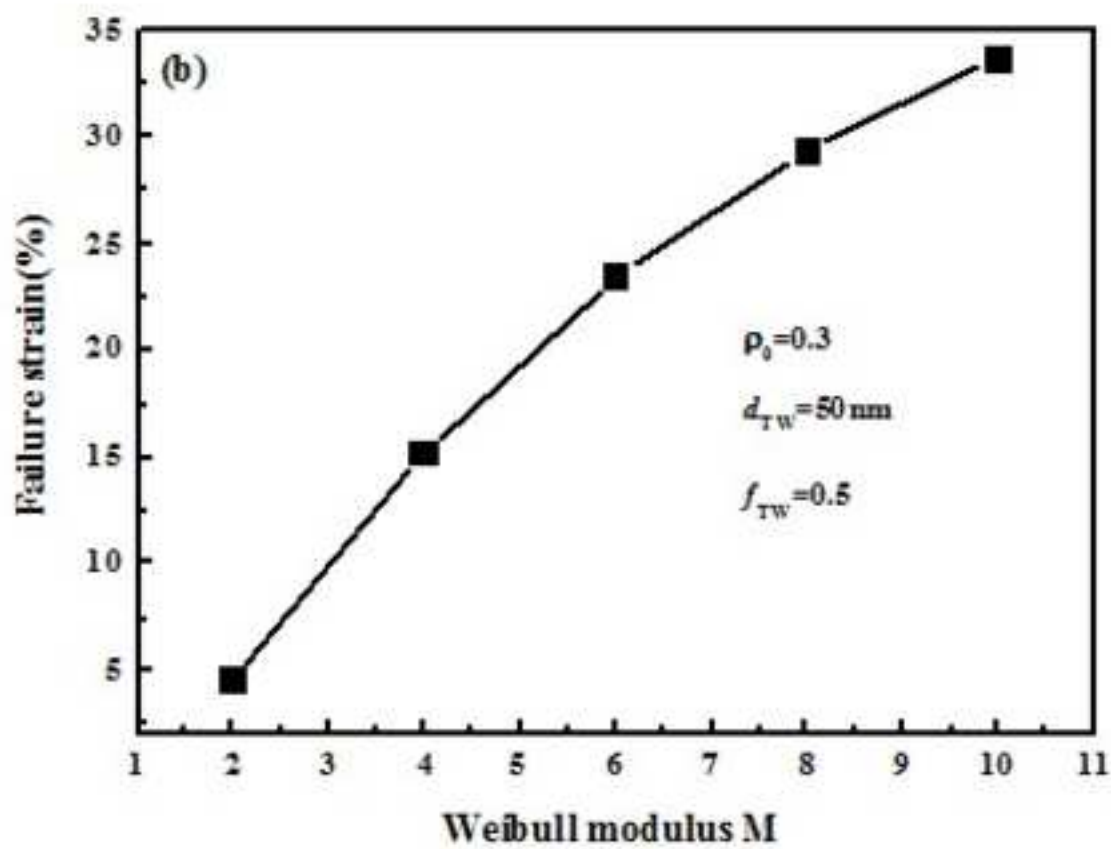
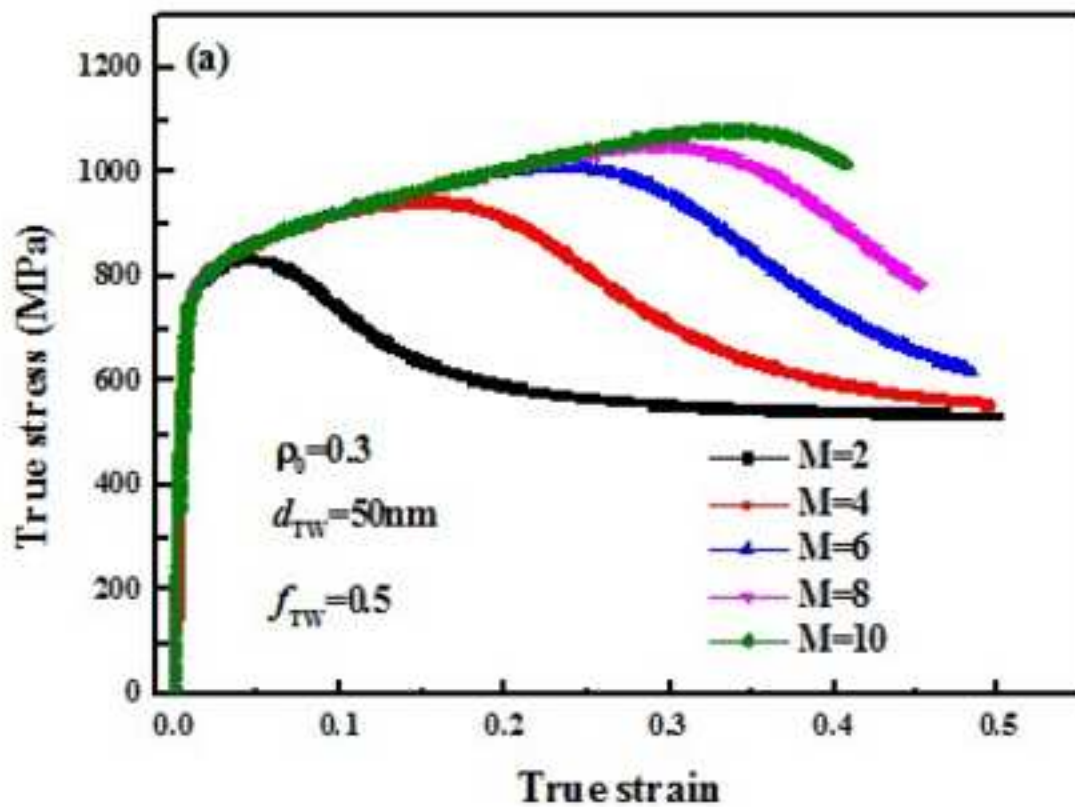
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Figure 7

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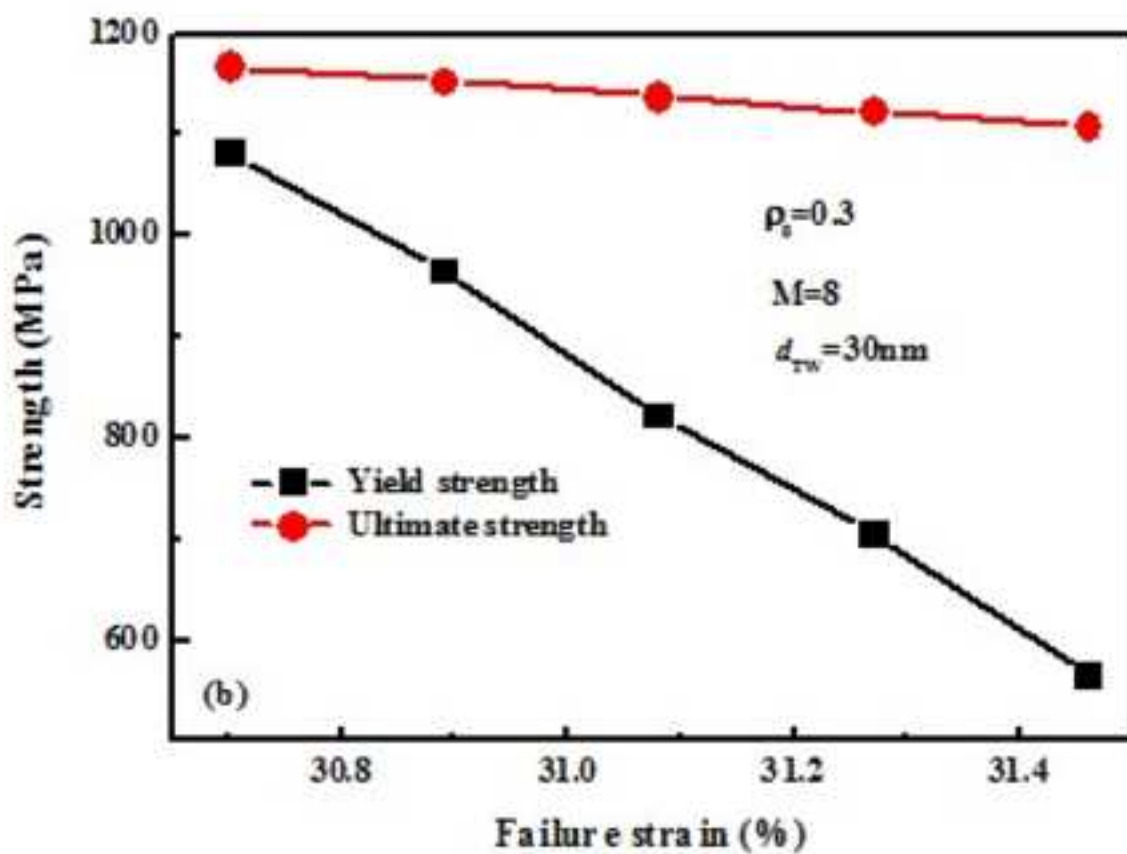
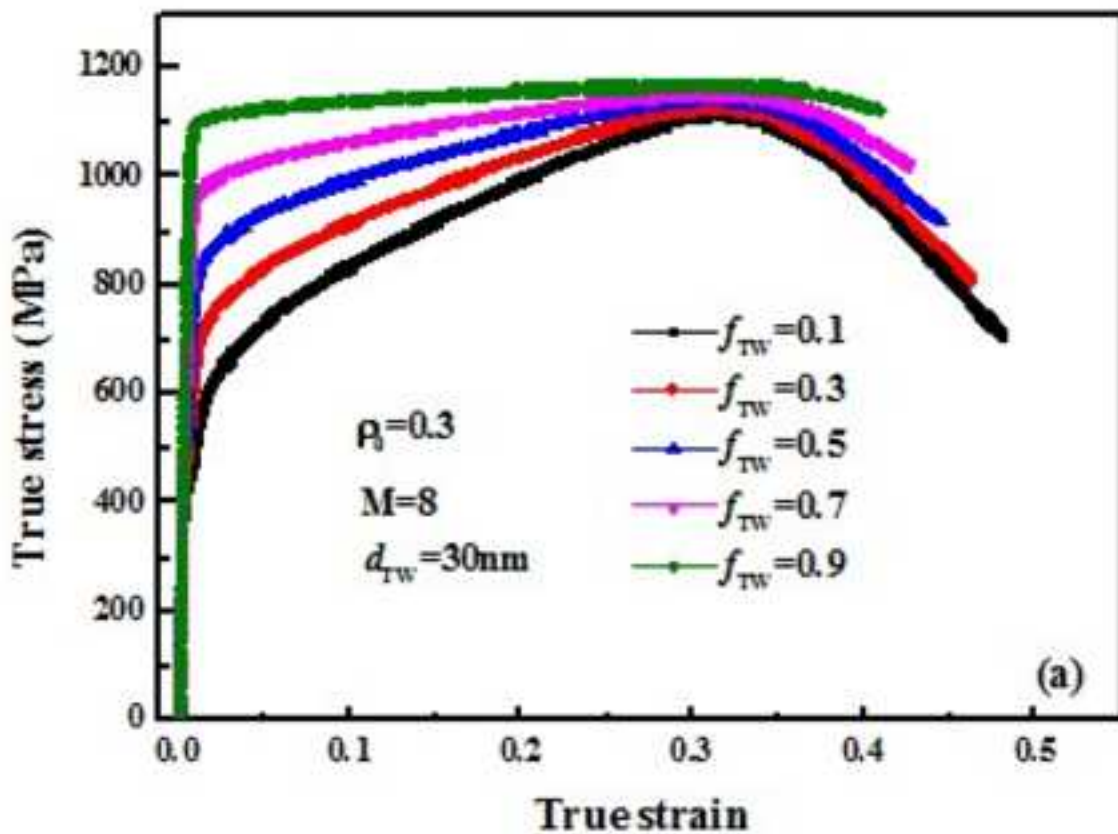


Figure8

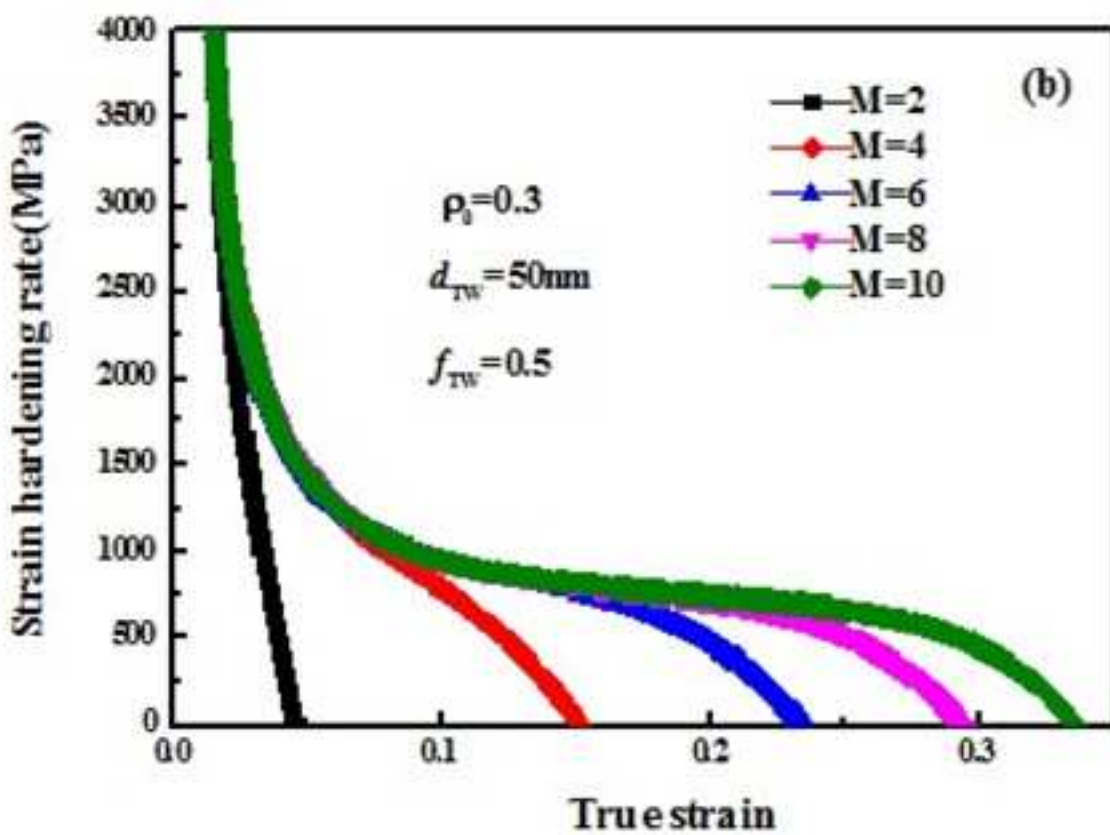
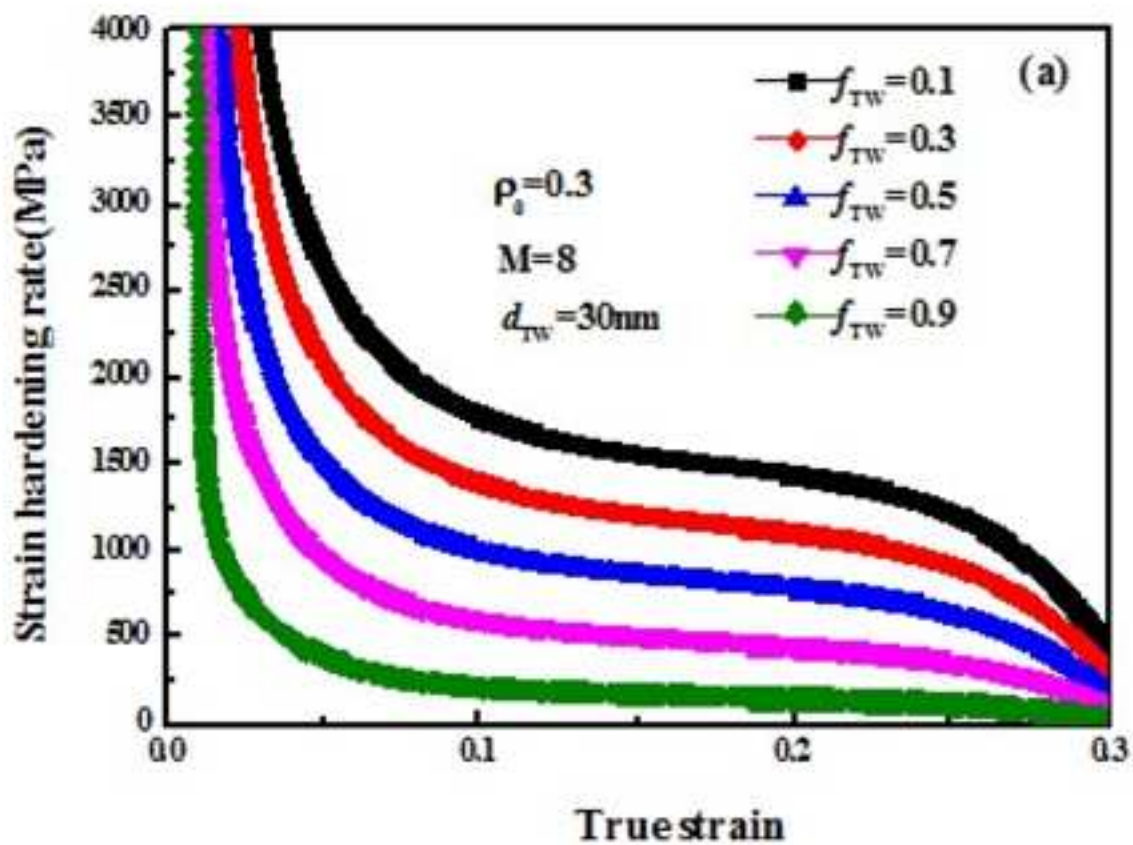
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Figure9

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