Development of a k - ω - φ - α turbulence model based on elliptic blending and applications for near-wall and separated flows

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A new turbulence model based on elliptic blending, termed as $k-\omega-\varphi-\alpha$ model, is developed. This model uses the latest version of Wilcox's $k-\omega$ model in near wall region and changes gradually to the BL $-\overline{v^2}/k$ model elsewhere. The capabilities of present model are evaluated on near-wall and separation flows, i.e. the 2D fully developed channel flow, the asymmetric plane diffuser flow and the 2D backward-facing step flow, in comparison with available DNS and experimental data. The computational results are compared also to those from the popular SST $k-\omega$ model and the original BL $-\overline{v^2}/k$ model, and present model is more stable than the BL $-\overline{v^2}/k$ model in complex flows. The present model provides indistinguishable velocity profiles and improved turbulent kinetic energy profiles compared to the BL $-\overline{v^2}/k$ model in the channel flow, while in the separation flows tested herein, present model can obtain comparable results with the BL $-\overline{v^2}/k$ model, and both of them show improvements in some extent comparing with the SST $k-\omega$ model.

Keywords: turbulence model; $k-\omega-\varphi-\alpha$ model; elliptic blending; near-wall flow; separated flow

Nomenclature

Greek letters

α Elliptic variable
β, γ, β^*, $\beta^{*\prime}$ Turbulence model coefficients
ε, ε_h Dissipation rate and homogeneous dissipation rate
ω Specific dissipation rate
κ Von Karman constant
μ, ν Molecular dynamic and kinematic viscosity
μ_t, ν_t Turbulent dynamic and kinematic viscosity
Ω_{ij} Vorticity rate tensor
φ Wall-normal turbulent anisotropy, $\varphi=\overline{v^2}/k$
ρ Density of fluid
σ_d Turbulence model constant

$\sigma_k, \sigma_\omega, \sigma_\varepsilon, \sigma_\varphi$ Turbulent Prandtl numbers

τ_w Wall shear stress

Latin letters

C_f Skin-friction coefficient

C_p Pressure coefficient

$C_{\varepsilon 1}, C_{\varepsilon 2}, C^*_{\varepsilon 2}\ C_{\varepsilon 3}, C_{\varepsilon 4}, C_\mu, C_T, C_\eta, C_1, C_2$ Turbulence model parameters

D_k, D_k^t Turbulent diffusion of k

D_c, D_c' Cross-diffusion terms

E The 'E' term

f Elliptic relaxation function

F_b Blending function

f_k, f_ω Damping functions

f_β Additional function

G_k Production of turbulent kinetic energy

H Channel height

k Turbulent kinetic energy

L Turbulence length scale

n Turbulence model constant

p Pressure or turbulence model constant

Re_H Reynolds number based on H

Re_t Turbulent Reynolds number

Re_τ Friction velocity based Reynolds number

S Magnitude of strain rate

S_{ij} Strain rate tensor

t Physical time

T Turbulence time scale

u_i Instantaneous velocity vector

U_b Mean velocity of the bulk flow

u^+ Normalized velocity by friction velocity

u_τ Friction velocity, $u_\tau = \sqrt{\tau_w / \rho}$

$\overline{v^2}$ Velocity variance scale

x Coordinate in the stream-wise direction

y Wall distance or coordinate in the wall-normal direction

y^+ Non-dimensional wall distance

1. Introduction

Turbulent flows are common in engineering so that predicting turbulent information is important from practical and theoretical points of view. It is well known that the Navier-Stokes (N-S) equations can describe the details of turbulent motions. Although Direct Numerical Simulation (DNS) can solve the N-S equations without any simplification, its huge computing capacity even for the simple turbulent flows prevents it from being applied in real engineering problems. Generally, Reynolds-Averaged N-S equations (corresponding to RANS method) and filtered N-S equations (corresponding to Large Eddy Simulation method, LES) are often utilized in practices. These two methods have one common characteristic that the new unknowns occurring in the averaging and filtering processes should be modelled. LES shows powerful ability for turbulent flows. However, the shortcoming of LES laying in the high resolution requirements for wall boundary layers limits its application range just for problems without wall effects or low Reynolds number wall bounded flows and limited computational domains. The RANS method is a feasible method for complex turbulent flows to some extent. The famous $k-\varepsilon$ and $k-\omega$ models, and their many improved variants, have been used successfully for a lot of engineering applications. However, it is generally accepted that no single turbulence model is superiorly applicable for all kinds of turbulent flows. Many efforts are still in progress to improve the available turbulence models or provide new models.

The $\overline{v^2}-f$ model, first introduced in 1991 by Durbin[1], has become increasingly popular due to its good performance in near wall region. This model benefits from a proper velocity scale, $\sqrt{\overline{v^2}}$ instead of $\sqrt{k}$, being chosen, so that the eddy viscosity in the near wall region can be more correctly predicted without any damping function. The $\overline{v^2}-f$ model has been continually improved in its accuracy and robustness and applied to several flows[2-7]. The evolution of $\overline{v^2}-f$ model was reviewed in details by Billard and Laurence[8,9]. An important conclusion was drawn that it is difficult to give considerations to both stability and accuracy. Billard and Laurence[8,9] developed a new turbulence model (denoted as $\mathrm{BL}-\overline{v^2}/k$ model hereafter) in which the dimensionless wall-normal anisotropy $\varphi=\overline{v^2}/k$ and another dimensionless parameter, α, resulting from an elliptic equation, are used to blend the homogeneous and near-wall limiting expressions of f. This treatment effectively eliminates the stiffness coming from the

boundary condition of f. The 'E' term in the $k-\varepsilon$ model developed by Jones and Launder[10] is modified and reintroduced to retard turbulence growth in the buffer layer, and the excessive growth of the turbulent length scale in the absence of production is corrected. The $\mathrm{BL}-\overline{v^2}/k$ model has been utilized in 2D and 3D flows and proved to strike better balance between stability and accuracy than any of the previous variants[11].

So far, most of the $\overline{v^2}-f$ models stem from the $k-\varepsilon$ model. They have a major drawback associated with the wall boundary condition of ε. In practice, the value of ε on wall is generally calculated based on the value of k at the first cell adjacent to the wall, namely, $\varepsilon_w = \nu k / y_1^2$. During iterations, changing of ε_w (with changing of k) could cause numerical difficulties. For example, applying appropriate initial value of k is difficult for complex turbulent flows, so that unreasonable value of k resulting in unphysical ε_w, further leading to divergence of solution.

To tackle this problem, the $\overline{v^2}-f$ models based on the $k-\omega$ model are developed by a few researchers. The value of ω on wall is infinite and independent on k. This feature has been shown to have a numerical stabilizing effect. Additionally, since the ω equation does not dependent on k, a non-zero solution for ω can be obtained even when k tends to zero, thus preventing spurious relaminarization problems. The first $\overline{v^2}-f$ model based on $k-\omega$ model we found in literatures was developed by Jones in 2003[12].This model stems from the $k-\varepsilon-\overline{v^2}-f$ model by using the expression of $\varepsilon = \beta^* \omega k^n \left(\overline{v^2}\right)^{1-n}$. It has been applied for fully developed channel flow, backward-facing step flow, cavity flow and coaxial jet flow and obtained good results. Unfortunately, detailed formulations were not given and it is difficult to be followed. Taha[13] developed a $k-\omega-\overline{v^2}-f$ model from the standard $k-\omega$ model of Wilcox[14] and the $\overline{v^2}-f$ model of Lien and Kalitzin[15]. The model was applied to fully developed channel flow and the asymmetric plane diffuser flow. The results seem too rough in both flows and should be further improved. The same model was used to simulate the unsteady flows around bluff bodies by Nazari et al.[16]. It was found that this model can predict good results in unsteady flows with vortex shedding but produce poor results in steady computations. This model was also applied to evaluate the convective heat transfer around two side by side square cylinders by Mirzaei and Sohankar[17] and comparable results with experiments were obtained. Recently, Khalaji et al.[18] developed a new $k-\omega-\overline{v^2}-f$

model based on the latest version $k-\omega$ model of Wilcox[19]. This model was used for simulation of the 2D impinging jet flow and acceptable results were obtained.

So far, the $\overline{v^2}-f$ models based on $k-\omega$ model are indeed underdeveloped. In present paper, a new turbulence model (denoted as $k-\omega-\varphi-\alpha$ model later), is developed based on two latest advanced models (latest version of Wilcox's $k-\omega$ model[19] and $\mathrm{BL}-\overline{v^2}/k$ model[8,9]). This model is validated by 2D fully developed channel flow and separation flows in comparison with DNS and experimental data. It is found that present model is more insensitive to initial conditions and more stable to disturbance. As compared with the SST $k-\omega$ and $\mathrm{BL}-\overline{v^2}/k$ model, results show that in channel flow the predictions from present model are improved in some respects, and in separation flows, the results are as accurate as those from $\mathrm{BL}-\overline{v^2}/k$ model.

2. Formulation of the present model

The present $k-\omega-\varphi-\alpha$ turbulence model is developed based on the latest version of Low Reynolds Number Wilcox's $k-\omega$ model[19] (denoted as LRN $k-\omega$ model hereafter) and the $\mathrm{BL}-\overline{v^2}/k$ model. The basic idea is that the k- and ω- equations retain the formulations of the LRN $k-\omega$ model in the near wall region, and take the formulations transformed from $\mathrm{BL}-\overline{v^2}/k$ model elsewhere. The φ- and α- equations are taken directly from the $\mathrm{BL}-\overline{v^2}/k$ model. The model is described in the following.

The k- and the ε-equation of the $\mathrm{BL}-\overline{v^2}/k$ model read:

$$\frac{\partial \rho k}{\partial t}+\frac{\partial}{\partial x_i}\left(\rho u_i k\right)=G_k-\rho\varepsilon-E+\frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{2}+\sigma_{k1}\mu_t\right)\frac{\partial k}{\partial x_j}\right] \tag{1}$$

$$\frac{\partial \rho \varepsilon}{\partial t}+\frac{\partial}{\partial x_i}\left(\rho u_i \varepsilon\right)=\frac{C_{\varepsilon 1}G_k-C_{\varepsilon 2}^{*}\varepsilon}{T}+\frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{2}+\sigma_{\varepsilon}\mu_t\right)\frac{\partial \varepsilon}{\partial x_j}\right] \tag{2}$$

where $G_k=\mu_t S^2$, $S=\sqrt{2S_{ij}S_{ij}}$, $S_{ij}=0.5(u_{i,j}+u_{j,i})$. The '$E$' term

$$E=2C_{\varepsilon 3}\left(1-\alpha\right)^p\frac{k}{\varepsilon}\frac{\mu\mu_t}{\rho}\left(\frac{\partial^2 u_i}{\partial x_j\partial x_k}\right)^2 \tag{3}$$

and

$$C_{\varepsilon 2}^{*}=C_{\varepsilon 2}+\alpha^p\left(C_{\varepsilon 4}-C_{\varepsilon 2}\right)\tanh\left(\left|D_k^t/\varepsilon\right|^{\frac{3}{2}}\right) \tag{4}$$

where $D_k^t = \frac{\partial}{\partial x_j}\left(\sigma_{k1}\nu_t \frac{\partial k}{\partial x_j}\right)$.

Letting $\varepsilon = \beta^* k\omega$, the k-equation becomes

$$\frac{\partial \rho k}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i k) = G_k - \rho\beta^* k\omega - E + \frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{2} + \sigma_{k1}\mu_t\right)\frac{\partial k}{\partial x_j}\right] \tag{5}$$

Combining Eqn. (5) and letting $\gamma_1 = C_{\varepsilon 1}/(\beta^*\omega T) - 1$, $\beta_1 = [C^*_{\varepsilon 2}/(\beta^*\omega T) - 1]\beta^*$, $\sigma_{\omega 1} = \sigma_\varepsilon$, the ε-equation can be transformed to ω-equation

$$\frac{\partial \rho\omega}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i \omega) = \gamma_1 \frac{\omega}{k} G_k - \beta_1 \rho\omega^2 + D_c - D_k + \frac{\omega}{k} E + \frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{2} + \sigma_{\omega 1}\mu_t\right)\frac{\partial \omega}{\partial x_j}\right] \tag{6}$$

where the cross-diffusion term $D_c = \frac{2}{k}\left(\frac{\mu}{2} + \mu_t\sigma_{\omega 1}\right)\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}$; $D_k = (\sigma_{k1} - \sigma_{\omega 1})\frac{\omega}{k}\frac{\partial}{\partial x_j}\left(\mu_t \frac{\partial k}{\partial x_j}\right)$ represents the turbulent diffusion of the turbulence kinetic energy, as a source term occurring in the ω-equation. For simplicity, this term can be neglected under the remedy of re-adjusting some parameters, just as the baseline and SST $k-\omega$ model[20]. However, in present model, this term is kept in order to avoid adjusting many more parameters. This term will not increase any complexity of the model because its treatment is similar to that of the term D_k^t in definition of $C^*_{\varepsilon 2}$, which is important to enhance the performance of the model in defect layer.

Similar to the BL$-\overline{v^2}/k$ model, the 'E'term can be moved to the k-equation after multiplied by a factor of $-k/\omega$ and combined with the existing 'E' term. This treatment does not induce any change of the form of the k-equation, but the corresponding parameter, $C_{\varepsilon 3}$, should be re-calibrated. Using $\varepsilon = \beta^* k\omega$, the '$E$' term becomes

$$E = 2C_{\varepsilon 3}(1-\alpha)^p \frac{1}{\beta^*\omega}\frac{\mu\mu_t}{\rho}\left(\frac{\partial^2 u_i}{\partial x_j \partial x_k}\right)^2 \tag{7}$$

The transformed φ- and α-equations from the BL$-\overline{v^2}/k$ model read (just simply replacing ε by $\beta^* k\omega$):

$$\frac{\partial \rho\varphi}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i \varphi) = (1-\alpha^p)\rho f_w + \alpha^p \rho f_h - \frac{\varphi}{k}G_k + \frac{2}{k}\mu_t\sigma_{k1}\frac{\partial \varphi}{\partial x_j}\frac{\partial k}{\partial x_j} + \frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{2} + \sigma_\varphi \mu_t\right)\frac{\partial \varphi}{\partial x_j}\right] \tag{8}$$

$$0 = \frac{1-\alpha}{L^2} + \frac{\partial}{\partial x_j}\left(\frac{\partial \alpha}{\partial x_j}\right) \tag{9}$$

where $f_w = -\dfrac{\beta^* \omega \varphi}{2}$ and $f_h = -\dfrac{1}{T}\left(C_1 - 1 + C_2 \dfrac{G_k}{\rho \beta^* k\omega}\right)\left(\varphi - \dfrac{2}{3}\right)$, and the turbulence length scale, time scale and the turbulent viscosity are

$$L = C_L \sqrt{\frac{k}{\left(\beta^* \omega\right)^2} + C_\eta^2 \frac{\nu^{3/2}}{\left(\beta^* k\omega\right)^{1/2}}} \tag{10}$$

$$T = \sqrt{\frac{1}{\left(\beta^* \omega\right)^2} + C_T^2 \frac{\nu}{\beta^* k\omega}} \tag{11}$$

$$\mu_t = C_\mu \rho \varphi k T \tag{12}$$

The k- and ω-equations of the LRN $k-\omega$ model are

$$\frac{\partial \rho k}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho u_i k\right) = G_k - \rho f_k \beta^* k\omega + \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma_{k2}\mu_t\right)\frac{\partial k}{\partial x_j}\right] \tag{13}$$

$$\frac{\partial \rho \omega}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho u_i \omega\right) = \gamma_2 f_w \frac{\omega}{k} G_k - \rho \beta_2 \omega^2 + \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma_{\omega 2}\mu_t\right)\frac{\partial \omega}{\partial x_j}\right] + \max\left(\sigma_d \frac{\rho}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}, 0.0\right) \tag{14}$$

where $\beta_2 = \beta_0 f_\beta$, $f_\beta = \left(1 + 85\chi_\omega\right)/\left(1 + 100\chi_\omega\right)$, $\chi_\omega = \left|\Omega_{ij}\Omega_{jk}S_{ki}/\left(\beta^* \omega\right)^3\right|$, $\Omega_{ij} = 0.5(u_{i,j} - u_{j,i})$.

The decision of using LRN $k-\omega$ model in the near wall region is mainly based on two reasons. One is that the LRN model can indeed predict the turbulence quantities better than High Reynolds Number (HRN) $k-\omega$ model in the near wall region. This feature has been shown in details by Wilcox[19]. Another reason is the requirement in the Kolmogorov time scale and length scale, which are respectively used in turbulent time scale, T, and length scale, L. In HRN $k-\omega$ model, $\varepsilon = \beta^* k\omega$ is very small in the near wall region, this will result in unphysical large Kolmogorov time scale and length scale. In LRN $k-\omega$ model, this problem can be mostly eliminated because the value of k increases significantly.

In BL$-\overline{v^2}/k$ model, in order to improve the capability of the model, the ε has been replaced by $\varepsilon_h + \nu\Delta k/2$, this results in the molecular diffusion in all corresponding terms being halved. For consistency, the same treatment is performed in the LRN *k*-ω model. Namely, the dissipation term in the k- equation is replaced by $\beta^* k\omega + \nu\Delta k/2$. As predicted by Jakirlic and Hanhalic[21], after neglecting high order terms, the unique modification is that the molecular diffusion effect in all corresponding terms (in the k- and ω- equations and for the value of ω at wall boundary) are halved. The final form of the LRN *k*-ω model reads

$$\frac{\partial \rho k}{\partial t}+\frac{\partial}{\partial x_i}\left(\rho u_i k\right)=G_k-\rho f_k \beta^* k\omega+\frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{2}+\sigma_{k2}\mu_t\right)\frac{\partial k}{\partial x_j}\right] \tag{15}$$

$$\frac{\partial \rho \omega}{\partial t}+\frac{\partial}{\partial x_i}\left(\rho u_i \omega\right)=\gamma_2 f_\omega \frac{\omega}{k}G_k-\rho\beta_2\omega^2+\frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{2}+\sigma_{\omega 2}\mu_t\right)\frac{\partial \omega}{\partial x_j}\right]+\max\left(\sigma_d\frac{\rho}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j},0.0\right)$$

(16)

The two 'damping functions' are

$$f_k=\left[\beta_0/0.27+\left(\mathrm{Re}_t/8\right)^4\right]\Big/\left[1+\left(\mathrm{Re}_t/8\right)^4\right] \tag{17}$$

$$f_\omega=\left(1/9+\mathrm{Re}_t/2.61\right)/\left(1+\mathrm{Re}_t/2.61\right) \tag{18}$$

with $\mathrm{Re}_t=\rho k/(\mu\omega)$.

The 'damping function' for turbulent viscosity in the LRN $k-\omega$ model is not used in present model because the turbulent viscosity has been reduced when it is calculated by Eqn. (12). Additionally, f_ω has been modified accordingly.

Now, the transformed k- and ω- equations (Eqn. (5) and Eqn. (6)) are multiplied by a blending function, F_b, and those in the LRN $k-\omega$ model (Eqn. (15) and Eqn. (16)) are multiplied by $(1-F_b)$, then they are added together respectively to give the new k- and ω- equations:

$$\frac{\partial \rho k}{\partial t}+\frac{\partial}{\partial x_i}\left(\rho u_i k\right)=G_k-\rho\beta^{*}{}'k\omega-E+\frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{2}+\sigma_k\mu_t\right)\frac{\partial k}{\partial x_j}\right] \tag{19}$$

$$\frac{\partial \rho \omega}{\partial t}+\frac{\partial}{\partial x_i}\left(\rho u_i \omega\right)=\gamma\frac{\omega}{k}G_k-\rho\beta\omega^2+D_c'-F_bD_k+\frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{2}+\sigma_\omega\mu_t\right)\frac{\partial \omega}{\partial x_j}\right] \tag{20}$$

where the new cross-diffusion term is

$$D_c'=\left[F_b\frac{2}{k}\left(\frac{\mu}{2}+\mu_t\sigma_{\omega 1}\right)\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}+\left(1-F_b\right)\sigma_d\frac{\rho}{\omega}\max\left(\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j},0.0\right)\right] \tag{21}$$

It should be noted that the 'E' term is not multiplied by blending function because this term is actually an appendix to enhance the model performance in buffer layer. For the cross-diffusion term, there are two different opinions. Some researchers considered that the cross-diffusion term should be only activated when it is positive[19]. However, others considered that the cross-diffusion term is important even though it is negative[22]. In present model, the cross-diffusion term is divided into two parts. In the near wall region, the same treatment as Wilcox[19] is adopted, namely, the cross-diffusion term is contributed only when it is positive. It is found that such treatment is advantageous to

stability of the model. In other regions, the cross-diffusion term holds the same formulation transformed from the $\mathrm{BL}-\overline{v^2}/k$ model directly.

Let ϕ represents the five new model parameters ($\beta^{*}{}'$, σ_k, γ, β and σ_ω), ϕ_1 and ϕ_2 represent corresponding parameters in the transformed $k-\omega$ model and the LRN $k-\omega$ model, then the relation between them is

$$\phi = F_b\phi_1 + (1-F_b)\phi_2 \tag{22}$$

The parameters used in present model are listed in table 1.

The blending function should be equal to zero in the near wall region but goes to one elsewhere. It is found that α^p just satisfies this requirement. In fact, this function has been used in the φ- equation to ensure correct near wall balance. Therefore, $F_b = \alpha^p$ is selected in present model. Although the characteristic of the blending function used in present model is opposite to that of Menter's SST $k-\omega$ model, the same purpose (the $k-\omega$ model is active in the near wall region) is achieved. It is not strange because in present model the $k-\omega$ model is multiplied by $(1-F_b)$ but in the Menter's SST $k-\omega$ model the $k-\omega$ model is multiplied by F_b. It should be noted that the actuating range of the $k-\omega$ model is different in present model and the Menter's SST $k-\omega$ model. In the former, the $k-\omega$ model is just activated in the near wall region, but in the latter, the $k-\omega$ model is contributed in the whole boundary layer.

There are a few model parameters that should be modified or re-calibrated. Throughout this research 'trial-and-error' efforts were made. Most of modifications are based on the model performance in 2D fully developed channel flow with different Reynolds numbers because this flow is very simple and typical in wall-bounded flows, additionally, there are a lot of available DNS and experimental data that can be used as reference. Other modifications are based on relative complex flows, such as 2D asymmetric diffuser flow and 2D backward-facing step flow. The authors find that one of the important constants is the exponent in the tanh function in $C_{\varepsilon 2}^{*}$ definition (1.5 is used in the $\mathrm{BL}-\overline{v^2}/k$ model). After recalibration, it was observed that setting this exponent to be equal to 1.0 in present model would yield better velocity profiles in the defect layer of channel flow (see Section 4.1). At the same time, we found that the argument of the tanh function should be modified. If the absolute value of $D_k^t/(\beta^* k\omega)$ is used, just as the same as $\mathrm{BL}-\overline{v^2}/k$ model, the reattachment in the 2D diffuser flow will be delayed considerably. Scrutinizing the term $D_k^t/(\beta^* k\omega)$ we found that it has very large

negative value near the wall before the reattachment point, this will lead to $C_{\varepsilon 2}^*$ decrease significantly in the near wall region (see Section 4.2). Apparently, this behavior of $C_{\varepsilon 2}^*$ is unwanted. Consequently, the $\left|D_k^t/(\beta^* k\omega)\right|$ is replaced by $\max[D_k^t/(\beta^* k\omega),0.0]$ in present model and it can predict reattachment point more accurately. Namely, the final definition of $C_{\varepsilon 2}^*$ is

$$C_{\varepsilon 2}^* = C_{\varepsilon 2} + \alpha^p (C_{\varepsilon 4} - C_{\varepsilon 2}) \tanh\left[\max\left(\frac{D_k^t}{\beta^* k\omega}, 0\right)\right] \tag{23}$$

More attention should be paid to the exponent p, which occurs in the φ-equation, the 'E' term, the blending function F_b and coefficient $C_{\varepsilon 2}^*$. The value of p in the φ-equation has been investigated by several contributors to the elliptic blending models. Billard and Laurence [8,9] predicted that p should at least be equal to 3 in the $\mathrm{BL}-\overline{v^2}/k$ model. Mancean [23] analysed the effect of p on the simulated channel flow results. It was found that $p=1$ does not produce satisfactory results and $p=3$ yields better results than $p=2$. In present model, we found that a value at least being equal to 4 is needed not only in the φ-equation, but also in other parts, to ensure its robustness (please refer to Section 4.2 for details). Therefore, the choice of $p=4$ is used throughout the model.

Another modification is made on the parameter $C_{\varepsilon 1}$, which affects significantly the shear layer spreading rate. Wilcox[19] showed that in log-layer the relation of $\kappa^2 = \sqrt{\beta^*}(\beta/\beta^* - \gamma)/\sigma_\omega$ should be hold in $k-\omega$ model. For present $k-\omega-\varphi-\alpha$ model, in log-layer the transformed $\mathrm{BL}-\overline{v^2}/k$ model retains, and the Kolmogorov time scale is inactive, so that $\beta^*\omega T = 1$, $C_{\varepsilon 2}^* = C_{\varepsilon 2}$, and then $\beta/\beta^* = C_{\varepsilon 2} - 1$, $\gamma = C_{\varepsilon 1} - 1$, $\sigma_\omega = \sigma_{\omega 1}$, consequently, $\kappa^2 = \sqrt{\beta^*}(C_{\varepsilon 2} - C_{\varepsilon 1})/\sigma_{\omega 1}$. Using $\kappa = 0.41$ and the same values as $\mathrm{BL}-\overline{v^2}/k$ model for other constants, $C_{\varepsilon 1} = 1.456$ is obtained.

Other constants, C_μ, C_L, C_η, $C_{\varepsilon 3}$, are retuned to provide good overall performance for near-wall and separation flows. The final values adopted in present model (most of them are extracted directly from the $\mathrm{BL}-\overline{v^2}/k$ and LRN $k-\omega$ models) are listed in Table 2 for reference.

3 Solution procedure

3.1 Numerical method

Both the present $k-\omega-\varphi-\alpha$ model and the $\mathrm{BL}-\overline{v^2}/k$ model were implemented in the FLUENT CFD code assisted by the User-Defined Function (UDF) functionality. The pressure-based segregated algorithm was used to solve the governing equations. The second order upwind scheme was applied to the convective terms in the momentum and turbulence equations. The SIMPLEC algorithm was used to deal with the velocity-pressure coupling. The least squares cell-based method was adopted to evaluate the gradients and derivatives. To facilitate convergence, the under-relaxation equations were used to control the update of computed variables at each iteration by setting appropriate under-relaxation factors. For the pressure and velocities, the default under-relaxation factors were adopted. For each variable in turbulence model, under-relaxation factor is set to 0.5.

3.2 Boundary conditions on solid wall

On solid wall, the no-slip condition is used, namely, $u_i=0$, $k=0$, $\varphi=0$, $\alpha=0$. For ε in $\mathrm{BL}-\overline{v^2}/k$ model, $\varepsilon_w=k\nu/y_1^2$ with y_1 representing the distance from the wall to the center of the first cell adjacent to the wall. For ω in present model, because the molecular diffusion has been halved, $\omega_w=3\nu/(\beta_2 y_1^2)$ is used. This is according to that used in the Meter's SST $k-\omega$ model[20], in which $\omega_w=6\nu/(\beta_2 y_1^2)$. Wilcox[19] suggested that the values of ω at 7-10 levels of grid points close to the wall should be specified to eliminate the sensitivity of the solutions on the mesh. However, it is found that this treatment is not needed in present model. The solution is not sensitive as long as $y^+<1$ is ensured at the first grid point. The reason may be that in present model the eddy viscosity is not dependent on ω directly.

3.3 Initial conditions

One of the attractive features for present $k-\omega-\varphi-\alpha$ model is its weak dependence on the initial conditions. Though this model does not demand elaborately designed initial conditions to some degree, a relative larger initial value of ω is suggested to ensure the solution more stable. Additionally, initial values of $\varphi=0.5$ and $\alpha=1.0$ are appropriate

and recommended for general problems. For the $\mathrm{BL}-\overline{v^2}/k$ model, the initial conditions given by Billard[8] are applied. The default values in Fluent code are adopted for the $\mathrm{SST}\ \ k-\omega$ model.

4 Results and discussion

The present $k-\omega-\varphi-\alpha$ turbulence model is evaluated and compared with the $\mathrm{BL}-\overline{v^2}/k$ model and the popular Meter's $\mathrm{SST}\ \ k-\omega$ model for three different test cases: the 2D fully developed turbulent channel flow, the 2D asymmetric diffuser flow and the 2D backward-facing step flow. For all simulations, the condition of $y^+<1$ at the first grid point is guaranteed.

4.1 Fully developed channel flow

The fully developed turbulent channel flow has been widely investigated using DNS method by many researchers. Many published DNS data are available for scrutinizing the behaviours of turbulence models in such simple wall-bounded flow. Lee and Moser[24] performed DNS for channel flow with friction Reynolds number (Re_τ) up to 5200, and the statistical data from their simulations are available online at http://turbulence.ices.utexas.edu. Four cases with different Re_τ (550, 1000, 2000 and 5200) are selected to validate the present turbulence model, and the calculated results using the $\mathrm{BL}-\overline{v^2}/k$ and the $\mathrm{SST}\ \ k-\omega$ models are also included for comparison.

Figure 1 shows the comparisons of normalized mean streamwise velocity profiles($u^+=u/u_\tau$). Apparently, The results from present $k-\omega-\varphi-\alpha$ model and the $\mathrm{BL}-\overline{v^2}/k$ model are almost indistinguishable and both of them yield predictions in excellent agreement with the DNS data. It is not surprise because the model coefficients in both models are carefully calibrated based on this flow. The $\mathrm{SST}\ \ k-\omega$ model under-predicts the velocity in the buffer layer and defect layer in all cases.

Figure 2 shows the comparisons of normalized turbulent kinetic energy profiles($k^+=k/u_\tau^2$). It can be found that all turbulence models under-predict k^+ in the viscous sub-layer. At $\mathrm{Re}_\tau=550$, the largest value of k^+ from the $k-\omega-\varphi-\alpha$ model is comparable with the DNS result but the $\mathrm{SST}\ \ k-\omega$ model and the $\mathrm{BL}-\overline{v^2}/k$ model have

lower values. At higher Re_τ, all models under-predict k^+. As a whole, the $k-\omega-\varphi-\alpha$ model yields better results.

The normalized turbulent viscosity profiles($\nu^+ = \nu_t / \nu$) are shown in Figure 3. It is obvious that the SST $k-\omega$ model over-predicts the turbulent viscosity in the central region of the channel. Both the $k-\omega-\varphi-\alpha$ model and the $\mathrm{BL}-\overline{v^2}/k$ model yield more reasonable turbulent viscosity profiles in the central region, thus leading to improved velocity profiles. From Figure 3, one can find that the $\mathrm{BL}-\overline{v^2}/k$ model predicts larger ν^+ than the $k-\omega-\varphi-\alpha$ model in all cases. Generally speaking, the $k-\omega-\varphi-\alpha$ model yields better ν^+ in log-layer, while the $\mathrm{BL}-\overline{v^2}/k$ model predicts better in defect layer. Additionally, the results from $k-\omega-\varphi-\alpha$ model are better in cases with small Re_τ and the results from $\mathrm{BL}-\overline{v^2}/k$ model are better in cases with higher Re_τ. It is interesting that the $k-\omega-\varphi-\alpha$ model and the $\mathrm{BL}-\overline{v^2}/k$ model yield almost indistinguishable velocity profiles despite turbulent viscosity profiles being slightly different. It is not surprising because the velocity is an integral quantity. The velocity profile depends on the global feature of the turbulent viscosity rather than the local value.

Figure 4 shows the comparisons of normalized Reynolds stress profiles ($-\overline{u'v'}^+ = -\overline{u'v'}/u_\tau^2$). It can be found that all turbulence models predict good Reynolds stress profiles compared to DNS data for all Re_τ cases.

In the $\mathrm{BL}-\overline{v^2}/k$ model, a functional $C_{\varepsilon 2}^*$ coefficient, instead of a constant, is adopted (Eqn.(4)). This modification enables the $C_{\varepsilon 2}^*$ to take a smaller value where the ratio D_k^t/ε is significant (for example, in the defect layer of channel flow), thus improving the turbulent viscosity and velocity predictions(see Figure 3 and Figure 1). In the definition of $C_{\varepsilon 2}^*$, a tanh function is used. The exponent of the variable (i.e. D_k^t/ε) for the tanh function affects variation of $C_{\varepsilon 2}^*$ significantly, further affecting other quantities. The $C_{\varepsilon 2}^*$ profiles computed using present $k-\omega-\varphi-\alpha$ model with different exponents (1.0 and 1.5) for channel flow at $\mathrm{Re}_\tau = 550$ are illustrated in Figure 5(a). As a reference, the result from the $\mathrm{BL}-\overline{v^2}/k$ model is also included. Meanwhile, the corresponding velocity profiles are compared in Figure 5(b). Clearly, the $k-\omega-\varphi-\alpha$ model with exponent of 1.5 yields similar $C_{\varepsilon 2}^*$ to the $\mathrm{BL}-\overline{v^2}/k$ model. However, it under-predicts the

velocity in defect layer apparently. For the $k-\omega-\varphi-\alpha$ model with exponent of 1.0, the $C_{\varepsilon 2}^{*}$ begins to decrease earlier, but the velocity in the defect layer is improved.

4.2 2D asymmetric diffuser flow

The 2D asymmetric diffuser flow (denoted as diffuser flow hereafter) has several desirable features which make it a good test case for validation of turbulence models. This problem was studied experimentally by Buice and Eaton[25,26],and being an issue of the 8th ERCOFTAC/IAHR/COST Workshop on Refined Turbulence modelling. The available experimental data can be obtained directly from the Web[27].

Kaltenbach et al.[28] simulated this flow using LES. Apsley and Leschziner[29] calculated this flow using different advanced RANS turbulence models. Laurence et al.[30] studied this flow using their robust $\overline{v^2}-f$ model. Iaccarino[31] presented a detailed and careful comparison of the simulations performed using different commercial CFD codes. It has been shown that this flow can be simulated well by LES, but it is particularly challenging for RANS models.

The detailed geometry of the diffuser can be found in several references[25,26,29] and here only a sketch is shown in Figure 6. The $x-$axis takes the stream-wise direction and the origin of the $x-$axis is located at the intersection of the tangents to the straight and inclined walls. The computational domain ranges from $x=-11H$ to $x=77H$. The boundary conditions are also shown in Figure 5. On all walls, no-slip boundary condition is used. On the outlet, the pressure outlet condition is adopted. On the inlet, conditions obtained from a separate simulation of fully developed 2D channel flow using the same geometry and turbulence model, are applied. The Reynolds number of the flow, Re_H, based on the centreline velocity in the inlet channel and the channel height H, is 20000. After a mesh sensitivity research, the total number of computational cells of 169600 is used because it can be considered fine enough to obtain grid independent solutions (halving the grid cell size leading to 2% of enlargement of the recirculation zone).

It should be noted that the distributions of the physical quantities (velocity, turbulent kinetic energy, Reynolds stress, et al.) at inlet may be different for different turbulent models although keeping the same Reynolds number. These quantities at inlet are shown in Figure 7 (show only one half due to symmetry). Because the inlet profiles are extracted from fully developed channel flow, the comparisons among these three models are similar to the results in previous section. For example, the inlet velocity

profiles are very similar for the $k-\omega-\varphi-\alpha$ model and the $\mathrm{BL}-\overline{v^2}/k$ model, but the turbulent kinetic energy profiles are different. The SST $k-\omega$ under-predicts the velocity near the center of the channel.

In Figure 8 computed skin friction coefficients (defined as $C_f = 2\tau_w/(\rho U_b^2)$ along the top and bottom walls are compared against the experimental data[27]. It can be found that on the bottom wall, the $k-\omega-\varphi-\alpha$ model can obtain better result than the other two, and on the top wall, the $\mathrm{BL}-\overline{v^2}/k$ model obtains better prediction. The points with $C_f = 0$ on the bottom wall represent the separation and reattachment points, which are given in Table 3. It is demonstrated that the SST $k-\omega$ model predicts a far too early separation point. This is consistent with the results obtained by other researchers[8,9,29,31]. The $\mathrm{BL}-\overline{v^2}/k$ model predicts both the separation and reattachment points slightly later and the $k-\omega-\varphi-\alpha$ model predicts the best separation and reattachment points compared with experiment.

The distributions of the pressure coefficients $C_p = 2(p-p_{ref})/(\rho U_b^2)$ on the top and bottom walls are almost identical. Therefore, only the comparison of the computational and experimental C_p along the bottom wall is shown in Figure 9. It is illustrated that all the three turbulence models overestimate C_p in redeveloping zone and the $k-\omega-\varphi-\alpha$ model predicts the largest one. The differences between the $k-\omega-\varphi-\alpha$ model and the $\mathrm{BL}-\overline{v^2}/k$ model are not significant.

Figure 10 predicts the normalized streamwise velocity profiles at different sections. Overall, the SST $k-\omega$ model gives better results in the recirculation region. It should be noted that the $k-\omega-\varphi-\alpha$ model and the $\mathrm{BL}-\overline{v^2}/k$ model obtain similar results on the whole. Both models under-predict the velocity in the region near the top wall. As noted by Laurence et al. that the pressure field is very sensitive to the recirculation bubble and affects the bulk of the flow[30].It can be seen that the better the pressure prediction, the better the velocity profiles in this region. In the redeveloping region, the differences among three models are very slight.

As mentioned in Section 2, the exponent p in present model is a vital factor. Inappropriate value will lead to divergent solution and ultimately blow up. To show the effect of p on the model performance, five cases with different p value are tested, while keeping the same meshes, boundary conditions and initial conditions. The detailed information of each case and the testing results are listed in Table 4. The exponent p

appears in the φ-equation, the 'E' term, the blending function F_b and coefficient $C_{\varepsilon 2}^*$. In each case, $p=3$ is assigned to one equation while $p=4$ is assigned to other three equations. Results show that $p=4$ is needed in the φ-equation, the 'E' term and the blending function F_b. The requirement of p in coefficient $C_{\varepsilon 2}^*$ is not so rigorous. For the φ-equation, $p=3$ works well in the BL$-\overline{v^2}/k$ model but fails in present $k-\omega-\varphi-\alpha$ model. The reason may be that these two models predict turbulent dissipation rate ε differently, especially in the near wall region, thus changing the behaviour of φ and resulting in larger p required to ensure the homogeneous term of φ vanishing in the near wall balance. In the BL$-\overline{v^2}/k$ model, the main purpose of moving the 'E' term from the ε-equation to the k-equation is to tackle the numerical problem caused by handling this term explicitly in the ε-equation. However, after being transformed to the ω form, this benefit seems vanish. It is obviously that this term will be handled explicitly in the k-equation again (see Eqn. (7)). In fact, convergence problem is indeed encountered (in Case 2). Increasing the exponent p in this term can tackle this problem in some degree. In the 'E' term a factor $(1-\alpha)^p$ is invoked to restrict this term to be active only in the near wall region. In flows with separation, small p may not entirely prevent this term into the recirculation zone and leads to numerical difficulty. Actually, during the simulations performed on the 2D validation cases in present study, this term did not raise any numerical problem when $p=4$ is used. The requirement of $p=4$ in the blending function F_b is due to the cross-diffusion term (Eqn. (21)), in which $F_b=\alpha^p$ is used to ensure the $k-\omega$ model to be active in near wall region and the BL$-\overline{v^2}/k$ model to be active elsewhere. In the $k-\omega$ model used in present $k-\omega-\varphi-\alpha$ model, the cross-diffusion term is active only when it is positive; in the BL$-\overline{v^2}/k$ model, the cross-diffusion term is always active. Negative value of the cross-diffusion term in the near wall region may result in numerical difficulties. The value of p in coefficient $C_{\varepsilon 2}^*$ has less effect on the model. This is because in $C_{\varepsilon 2}^*$, α^p is used to ensure that the $C_{\varepsilon 2}^*$ does not decrease considerably near the wall. Virtually, $C_{\varepsilon 2}^*$ cannot decrease dramatically because D_k^t/ε is very small near the wall after the max function in $C_{\varepsilon 2}^*$ being used. Therefore, the impact of p in this term is weak.

Figure 11 shows the $C_{\varepsilon 2}^*$ contours and streamlines computed from present model with different definitions of $C_{\varepsilon 2}^*$. One uses $\max[D_k^t/(\beta^* k\omega),0.0]$ in the argument of the

tanh function (Eqn.(23)), another uses $\left|D_k^t/(\beta^* k\omega)\right|$. The results computed from the $\mathrm{BL}-\overline{v^2}/k$ model are included for reference. It can be seen that the argument $\max[D_k^t/(\beta^* k\omega),0.0]$ can yield results close to those from the $\mathrm{BL}-\overline{v^2}/k$ model. However, the argument $\left|D_k^t/(\beta^* k\omega)\right|$ results in excessive decrease of $C_{\varepsilon 2}^*$ near the bottom wall, thus inducing much larger recirculation bubble and delaying the flow reattachment significantly.

4.3 2D backward-facing step flow

The backward-facing step flow (abbreviated as step flow later) is another case most frequently selected as a test case for turbulence model validation due to its geometrical simplicity and abundant flow behaviours, such as strong flow separation, recirculation and reattachment[22,32,33]. The experiment of Jovic and Drive[34,35] is most frequently used as reference. In their experiment, the expansion ratio is 1.2 and the Reynolds number, Re_H, based on the step height and mean bulk velocity of the inlet channel, is 5000. Following this experiment, Le et al.[36] performed a DNS research with $\mathrm{Re}_H = 5100$. Even though the Re_H is different slightly, good agreements were achieved.

Following the schematic of the wind tunnel used in the experiment of Jovic and Drive[32], the backward-facing step configuration is symmetric about the centreline of the channel, so that only half of the tunnel is employed as the computational domain. The sketch of the geometry and boundary conditions of the backward-facing step model is shown in Figure 12. The step height is H, and the channel height is $5H$ in the inlet section and $6H$ after the step, yielding an expansion ratio of 1.2. The overall computational domain ranges from $x=-10H$ to $x=20H$, with the step located at $x=0$. The symmetry boundary condition is used at the centreline of the channel. On walls, no-slip boundary condition is employed. On the outlet, the pressure outlet condition is adopted. On inlet, some researchers used either the DNS data[32] or the experimental data[33] in their simulations. For present model, the DNS and experimental data cannot be directly used because the profile of α cannot be properly extracted from DNS or experimental data. Consequently the quantities extracted from a separate simulation of fully developed 2D channel flow using the same geometry and turbulence model are applied. A grid dependency study was performed to ensure mesh independent solutions being obtained. The total number of computational cells finally used is 118400.

The inlet profiles are shown in Figure 13. The same characteristics as that in the diffuser flow can be found.

The calculated skin friction coefficient C_f along the bottom wall is compared against the experimental result of Jovic and Drive[34] in Figure 14. It can be found that the $k-\omega-\varphi-\alpha$ model and the $\mathrm{BL}-\overline{v^2}/k$ model can yield better C_f than $\mathrm{SST}\ k-\omega$ model. As the whole, the difference between the $k-\omega-\varphi-\alpha$ model and $\mathrm{BL}-\overline{v^2}/k$ model is insignificant. An interesting feature is that the $k-\omega-\varphi-\alpha$ model can predict the best peak value of C_f in the recirculation zone. The location of reattachment point is usually used to justify the ability of turbulence model for this type flow. The $k-\omega-\varphi-\alpha$ model, the $\mathrm{BL}-\overline{v^2}/k$ model and the $\mathrm{SST}\ k-\omega$ model give $6.68H$, $6.56H$ and $7.0H$ respectively. They are all larger than the experimental result of $6.0H$ and DNS result of $6.23H$. On the whole, the departure of the $\mathrm{SST}\ k-\omega$ model is the largest.

Figure 15 illustrates the pressure coefficient (C_p) distribution along the wall. It is demonstrated that all three model under-predict C_p in the upstream channel but over-predict C_p in the re-developing zone. An interesting feature is that the results predicted by $k-\omega-\varphi-\alpha$ model and $\mathrm{BL}-\overline{v^2}/k$ model are almost undistinguishable. The result of $\mathrm{SST}\ k-\omega$ model has larger deviation.

The predictions of the normalized streamwise velocity profiles at locations of $-3.12H$, $4H$, $6H$, $10H$, $15H$ and $19H$ are shown in Figure 16. Generally speaking, results from three turbulence models are in good agreement with experimental data. Again, the results predicted by present $k-\omega-\varphi-\alpha$ model and the $\mathrm{BL}-\overline{v^2}/k$ model are almost undistinguishable.

To investigate the influence of initial condition on the model performance, the permitted maximum under relaxation factor of turbulent quantities for solution convergence is studied based on backward-facing step flow. In all cases tested here, uniform distribution of initial physical quantities is adopted. The computational meshes, the boundary conditions, the discrete schemes, the initial values of turbulent quantities and the under relaxation factor for pressure and momentum are the same and only the initial velocity and the under relaxation factor for turbulent quantities could be changed. The initial pressure is set to zero and the initial values of turbulent quantities are computed from the area-weighted average values at inlet. Four different initial velocities, 0, $0.25U_i$, $0.5U_i$ and U_i (where U_i represents the area-weighted average velocity at inlet) are

considered. In each case, computation begins with under relaxation factor (same factors are used for all turbulent quantities) of 0.9. If solution diverges, the under relaxation factor decreases with interval of 0.1 and computation is re-initialized and begins again. This cycle continues until the solution converges. The permitted maximum under relaxation factor in each case is tabulated in Table 5. It can be found that both the $k-\omega-\varphi-\alpha$ model and the $\mathrm{BL}-\overline{v^2}/k$ model can yield convergent solution when the initial value of velocity is 0 and the maximum under relaxation factor permitted is 0.7 and 0.5 respectively. When the initial velocity is set to be $0.25U_i$, the maximum under relaxation factor permitted is 0.2 for the $\mathrm{BL}-\overline{v^2}/k$ model and 0.7 for the $k-\omega-\varphi-\alpha$ model. When the initial velocity is set to be $0.5U_i$ or U_i, convergent solution can not be obtained with the $\mathrm{BL}-\overline{v^2}/k$ model even the under relaxation factor decreases to 0.1(smaller values are not tried further). However, for present $k-\omega-\varphi-\alpha$ model, convergent solution can be achieved with relatively large under relaxation factor of 0.8. These results show that present $k-\omega-\varphi-\alpha$ model is more insensitive to initial conditions.

The stability of the model is also studied by introducing a disturbance to a converged solution, and the effect of the disturbance on the subsequent solution convergence is evaluated. For simplicity, the disturbance is introduced only to the turbulent kinetic energy by changing k from the converged value to $2k_i, 3k_i, 4k_i, \cdots$ (k_i represents the average k at inlet) in whole computational domain. It is found that the solution diverges when k becomes $3k_i$ for the $\mathrm{BL}-\overline{v^2}/k$ model, but for the $k-\omega-\varphi-\alpha$ model the solution begins to diverge until k becomes $14k_i$. This means that the $k-\omega-\varphi-\alpha$ model can bear larger disturbance and has better stability.

Conclusions

A new $k-\omega-\varphi-\alpha$ turbulence model is developed based on elliptic blending and its capabilities are tested on the channel flow, the diffuser flow and the step flow. The calculated results are validated against available DNS and experimental data, and the current model is also compared with the $\mathrm{SST}\ k-\omega$ model and the $\mathrm{BL}-\overline{v^2}/k$ model. Our results lead to the following conclusions:

(1) The present $k-\omega-\varphi-\alpha$ model is more insensitive to initial conditions and more stable than the $\mathrm{BL}-\overline{v^2}/k$ model.(2) The present model shows improvements on turbulent kinetic energy profiles in the channel flow.

(3) The present model yields indistinguishable velocity profiles with the $\mathrm{BL}-\overline{v^2}/k$ model in the channel flow although the turbulent viscosity profiles have difference. Both of the present model and the $\mathrm{BL}-\overline{v^2}/k$ model predict better velocity profiles and turbulent viscosity profiles than the $\mathrm{SST}\ \ k-\omega$ model.

(4) The locations of separation and reattachment points both in diffuser flow and step flow predicted by present model and the $\mathrm{BL}-\overline{v^2}/k$ model are more accurate than those by the $\mathrm{SST}\ \ k-\omega$ model.

(5) Generally speaking, present model can provide comparable results to the $\mathrm{BL}-\overline{v^2}/k$ model in separation flows. For example, the skin friction coefficient, pressure coefficient and velocity profiles computed from these two models are similar.

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Table 1. Parameters used in present model.

Table 2. Model constants.

Table 3. Locations of separation (Sep.) and reattachment (Reatt.) points in diffuser flow.

Table 4. Values of p in different cases.

Table 5. The permitted maximum under relaxation factor in cases with different initial velocities.

Table 1. Parameters used in present model.

ϕ	$\beta^{*}{}'$	σ_k	γ	β	σ_ω
ϕ_1	0.09	1.0	$C_{\varepsilon 1}/(\beta^{*}\omega T)-1$	$\beta^{*}[C^{*}_{\varepsilon 2}/(\beta^{*}\omega T)-1]$	0.667
ϕ_2	$0.09\, f_k$	0.6	$0.52\, f_\omega$	$\beta_0 f_\beta$	0.5

Table 2. Model constants.

$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	$C_{\varepsilon 3}$	$C_{\varepsilon 4}$	σ_φ	C_μ	C_T	C_L
1.456	1.83	4.3	0.4	1.0	0.20	4	0.160
C_η	C_1	C_2	p	σ_d	β^{*}	β_0	
65	1.7	0.9	4.0	0.125	0.09	0.0708	

Table 3. Locations of separation (Sep.) and reattachment (Reatt.) points in diffuser flow.

Model	Sep.	Reatt.
Exp.	$7.34\,H$	$28.9\,H$
SST $k-\omega$	$2.65\,H$	$28.6\,H$
BL $-\overline{v^2}/k$	$8.86\,H$	$29.9\,H$
$k-\omega-\varphi-\alpha$	$7.89\,H$	$29.1\,H$

Table 4. Values of p in different cases.

Case No.	p				Converge?
	φ-equation	'E' term	F_b	$C^*_{\varepsilon 2}$	
1	3	4	4	4	No
2	4	3	4	4	No
3	4	4	3	4	No
4	4	4	4	3	Yes
5	4	4	4	4	Yes

Table 5. The permitted maximum under relaxation factor in cases with different initial velocities.

$U_{initial}$	Under Relaxation factor	
	BL $-\overline{v^2}/k$	$k-\omega-\varphi-\alpha$
0	0.5	0.7
0.25 U_i	0.2	0.7
0.5U_i	<0.1	0.8
U_i	<0.1	0.8