MARCH 27 2017

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J. Acoust. Soc. Am. 141, 2137–2142 (2017)
https://doi.org/10.1121/1.4977751

Convergence criteria on the acoustic velocity continuity in a panel-cavity system

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(Received 19 December 2016; revised 14 February 2017; accepted 15 February 2017; published online 27 March 2017)

Modal-based acoustoelastic formulation is regarded as the cornerstone of vibro-acoustics and has been widely used for coupling analyses of structure-cavity systems. The controversy and the skepticism surrounding the acoustic velocity continuity with the surrounding vibrating structures have been persistent, calling for a systematic investigation and clarification. This fundamental issue of significant relevance is addressed in this paper. Through numerical analyses and comparisons with wave-based exact solution, an oscillating convergence pattern of the calculated acoustic velocity is revealed. Normalization of the results leads to a unified series truncation criterion allowing minimal prediction error, which is verified in three-dimensional cases. The paper establishes the fact that the modal based decomposition method definitely allows correct prediction of both the acoustic pressure and the velocity inside an acoustic cavity covered by a flexural structure upon using appropriate series truncation criteria.
[http://dx.doi.org/10.1121/1.4977751]

[NX] Pages: 2137–2142

I. INTRODUCTION

The panel-cavity system, comprising a parallelepiped acoustic cavity with a rectangular flexible panel subjected to external excitations, has been used as a benchmark problem for studying the fundamental problems in a vibro-acoustic system for more than half a century. The issue of the pressure and velocity continuity at the structure-cavity interface using modal-based method has been arousing persistent interest and long-lasting debate among researchers. This paper intends to clarify this issue of fundamental importance.

The vibration response of a cavity-backed rectangular panel was first investigated by Dowell and Voss.[1] Since then, there has been a continuous effort in improving the modeling of such system as well as its physical understanding, exemplified by the work of Pretlove,[2,3] Pretlove and Craggs,[4] Guy and Bhattacharya,[5] and Guy[6] mainly focusing on quantities like the panel vibration, acoustic pressure, acoustic velocity, and the reverberant time inside the cavity, etc. Without any doubt, the most convenient and presumably the most commonly used method is the modal-based approach using acoustic pressure (or potential) decomposition over acoustic modes of the rigid-walled cavity. Its general framework, also referred to as acoustoelasticity theory, was elegantly summarized by Dowell *et al.*[7] and Fahy.[8] This approach, however, suffers from the seemingly "flaw" in that the velocity continuity over the panel cannot be mathematically satisfied due to the use of the rigid-walled acoustic modes, expressed in Cosine functions in the case of parallelepiped cavity.[9] This problem arouses continuous interest and endless debate in the vibro-acoustic community, even up to now. The advocators of the method argue that the method allows accurate acoustic pressure and reasonable acoustic velocity prediction if a sufficient number of acoustic modes are used. Nevertheless, there are no ruling conclusions due to the lack of quantified assessment and criteria. Various techniques were also developed in an attempt to increase the calculation accuracy such as the use of extended mode shape functions for a single cavity or the coupling between two overlapped adjacent sub-cavities.[10] Meanwhile, the skepticism on the modal-based method has always been persistent as evidenced by some recent papers. For example, the deficiencies of the method based on rigid-walled modes were reiterated by Ginsberg,[11] who employed an extension of Ritz series method to the problem, and the modified formulation is found to be accurate above the fundamental rigid-cavity resonance frequency for light fluid loading. More recently, various series expressions with added terms were also proposed to accommodate the velocity continuity.[12,13]

Modal-based acoustoelastic formulation allows elegant and clear physical representation and, to the eye of many, is the cornerstone of the vibroacoustics in dealing with structure-cavity coupling problems. As originally formulated and the way it has been used in the literature, the theory applies to light fluid, leading to a weak fluid-structural coupling. The controversy and the skepticisms surrounding the velocity continuity call for a systematic investigation and clarification, which constitutes the main motivation of the present work. This issue is addressed in this paper by investigating both acoustic pressure and particle velocity predictions through comparisons between the modal-based approach and the exact solutions using a system of simple rectangular geometry. For the particle velocity

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 0001-4966/2017/141(3)/2137/6/$30.00

prediction in the vicinity of the panel, an oscillating convergence pattern is observed when the number of acoustic modes increases. Normalization of the results leads to a unified criterion allowing minimization of the prediction error, which is then verified in three-dimensional (3D) cases.

II. THEORY AND ANALYSES

Consider a parallelepiped acoustic cavity with one of the walls covered by a vibrating plate, as shown in Fig. 1. The plate, simply supported along all four edges, is subjected to a prescribed sound pressure excitation. The rest of the cavity walls are assumed to be acoustically rigid. The acoustic field inside the cavity is described by the Helmholtz equations whereas the flexural motion of the plate is governed by the Kirchhoff equation and the damping is introduced in the model by considering complex Young's modulus and complex acoustic velocity, for the plate and the cavity, respectively.

The system is modeled using two approaches, which are briefly described below and subsequently compared and investigated through numerical analyses. Under the modal expansion framework,[7] the acoustic pressure inside the cavity p and the transversal displacement of the panel w are decomposed over the rigid-walled acoustic modes of the cavity and the *in vacuo* plate modes, respectively, namely, $p = \sum P_{nmp}\varphi_{nmp}$ and $w = \sum W_{rs}\psi_{rs}$, where P_{nmp} and φ_{nmp} are, respectively, the modal amplitude and the pressure mode shape of the cavity; W_{rs} and ψ_{rs} are the modal amplitude and displacement mode shape of the plate, respectively. φ_{nmp} and ψ_{rs} write

$$\varphi_{nmp} = \cos\left(\frac{n\pi x}{L_x}\right)\cos\left(\frac{m\pi y}{L_y}\right)\cos\left(\frac{p\pi z}{L_z}\right), \tag{1}$$

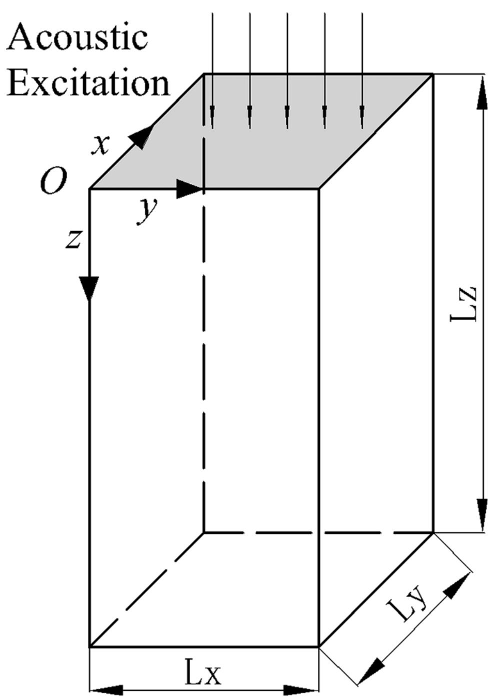

FIG. 1. The cavity-panel configuration and coordinate system.

$$\psi_{rs} = \sin\left(\frac{r\pi x}{L_x}\right)\sin\left(\frac{s\pi x}{L_y}\right), \tag{2}$$

in which n, m, and $p = 0, 1, 2,\ldots$ and r and $s = 1, 2,\ldots$. Applying the decomposition expressions in the equations of motion of the plate-cavity system and using the orthogonality property of the mode shapes, a set of linear equations with the modal amplitudes as unknowns are obtained as

$$\ddot{P}_{nmp} + j\eta_a\omega_{nmp}\dot{P}_{nmp} + \omega_{nmp}^2 P_{nmp} = -\frac{A_F}{V}\sum_{r,s} L_{nm,rs}\ddot{W}_{rs}, \tag{3}$$

$$M_{rs}\left[\ddot{W}_{rs} + j\eta_p\omega_{rs}\dot{W}_{rs} + \omega_{rs}^2 W_{rs}\right] = \rho_0 c_0^2 A_F \sum_{n,m,p} P_{nmp}\frac{L_{nm,rs}}{M_{nmp}} + Q_{rs}^E, \tag{4}$$

where V is the volume of the cavity; A_F the area of the vibrating panel; η_a and η_p the damping loss factor of the air and vibrating panel, respectively; ω_{nmp} and ω_{rs} the natural frequencies of the nmp acoustic mode and the rs panel mode, respectively; M_{nmp} and M_{rs} the generalized acoustic and panel modal mass, respectively; and Q_{rs}^E the generalized excitation force which can either be a point force or distributed pressure. $L_{nm,rs}$ is the modal coupling coefficient between the rs panel mode and the nmp cavity acoustic mode, defined as $L_{nm,rs} = (1/A_F)\int\varphi_{nmp}\psi_{rs}dV$. The index p is eliminated in the present configuration since the integral is calculated over the panel surface with $z = 0$. Detailed expressions of these quantities can be found in Ref. 7.

For comparisons, the same problem is also modeled to get the exact solution of the problem, referred to as wave approach, in which the acoustic modes used in the modal decomposition approach is replaced by

$$\varphi_{nm} = \cos\left(\frac{n\pi x}{L_x}\right)\cos\left(\frac{m\pi y}{L_y}\right)h_{nm}(z), \tag{5}$$

where $h_{nm}(z) = \alpha\cosh(\mu_{nm}z) + \beta\sinh(\mu_{nm}z)$. Note that the last term represents any wave propagating back and forth along the z direction, perpendicular to the panel. By satisfying the boundary conditions at $z = 0$ and $z = L_z$, α, β, and μ_{nm} can be determined. Different strategies are implemented to solve this equation set, which have been extensively discussed in literatures[5,6] so that they are not detailed here. Note that in the z direction, an imaginary wavenumber may exist. This corresponds to waves which decay exponentially along the z direction, known as evanescent waves.

In the following numerical investigations, the dimension of the cavity is set to be 0.2 m × 0.2 m × 0.5 m. A simply supported brass panel is 1.5 mm thick, located at $z = 0$. The air density is 1.29 kg/m^3; the sound speed is 343 m/s; the Young's modules of the panel is 110 × 10^9 Pa; the panel's Poisson's ratio is 0.357; the panel density is 8.9 × 10^3 kg/m^3; η_a and η_p are set to 0.001 and 0.01, respectively. A harmonic acoustic excitation is uniformly impinging on the flexible panel along the z direction. The purpose of using normal incident excitation is to simplify the modal response within the panel-cavity system, while retaining its internal physical characteristics.

A. Sound pressure

The sound pressure level (SPL) at a receiving point inside the cavity is calculated, with the external excitation pressure p_i fixed to 1 Pa. To ensure a fair comparison between the modal approach and the wave approach, the number of modes used in the transverse directions x and y are kept identical. The frequency band of interest is [0, 1000] Hz. It is well accepted that in order to ensure the correct sound pressure calculation, the truncated modal series should contain all these modes, for both the cavity and the panel, with their natural frequencies below $\alpha f_{\max}$, where $\alpha > 1$ is a margin coefficient (typically, $\alpha = 1.5$ or $\alpha = 2$) where $f_{\max}$ is the highest frequency under investigation (1000 Hz in the present case). This rule, in which α is equal to 2, is referred to as the pressure criterion in this paper. It should be pointed out that this pressure convergence criterion (even by including all lower-order modes) is not a universally accepted robust one. In some cases, especially at frequencies where the system is not very dynamic, like the anti-resonance regions between two well-separated modes, more terms may be needed. In the modal-based and wave methods used in the analyses here, the mode indices are chosen up to $n = m = p = 8$ and $r = s = 8$, which satisfy the pressure criterion. The first few lower order modes of the uncoupled cavity and the plate are tabulated in Table I. A receiving point is randomly chosen at (0.04, 0.17, 0.01) m. The SPL results are given in Fig. 2, in which the exact solution (named wave method in this paper) and the one from the modal method are compared. It can be seen that, upon using the pressure criterion, the pressure predictions by the two methods agree well, although slight differences are observable at some anti-resonance frequencies. Should the SPL be averaged within the entire cavity, these differences should disappear (not shown here). Similar observations were observed at other points inside the cavity, including those close to the vibrating panel (not shown here). Therefore, it is verified that the modal method can provide sufficient accuracy for acoustic pressure predictions everywhere throughout the cavity by using the well-established pressure criterion.

B. Velocity

As the main focus of the paper, the prediction accuracy of the particle velocity using the modal method is investigated. The receiving point and all other physical parameters are kept the same as in the previous pressure calculation. Since more expansion terms would be needed in the velocity calculation,[14] the number of modes used in the modal method are varied. Note that the mode variation only applies to the cavity depth direction, z, while the mode terms used in the x and y directions are kept the same. Three modal-based calculations use p up to 10, 20, and 40, respectively. Acoustic velocity u in the normal direction is obtained from $-j\omega\rho_0 u = (\partial p/\partial z)$ and the results are shown in Fig. 3, in comparison with the reference result obtained from the wave method. Compared with the reference result, it can be seen that the accuracy of the velocity prediction of the modal-based method improves as the number of modes in the z direction increases. It is not surprising that at cavity resonance frequencies, 340 and 680 Hz, the convergence is quickly achieved due to the dominating role of the corresponding rigid cavity mode at these frequencies. For the other frequencies, however, the convergence speed is slower than the case of pressure prediction (see Fig. 2 where only

TABLE I. Uncoupled resonance frequency of the system.

Plate *in-vacuo* resonance			Cavity resonance			
r	s	f_{rs} (Hz)	n	m	p	f_{nmp} (Hz)
1	1	128			—	
1	2	319.9	0	0	1	340
2	2	512	0	0	2	680
1	3	640	1	0	0	850
2	3	832	2	0	0	915.5

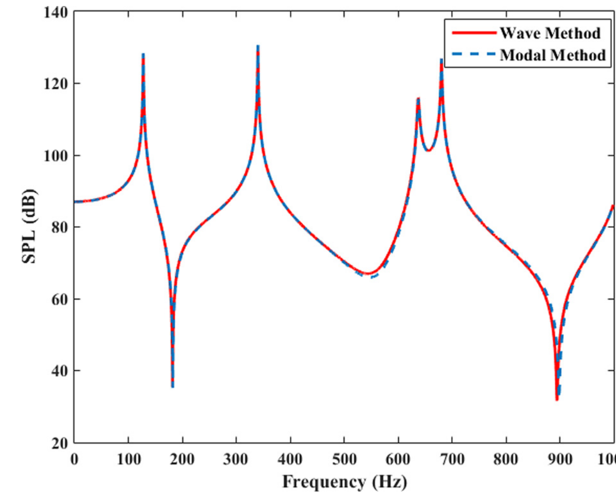

FIG. 2. (Color online) SPL predictions at point (0.04, 0.17, 0.01) m.

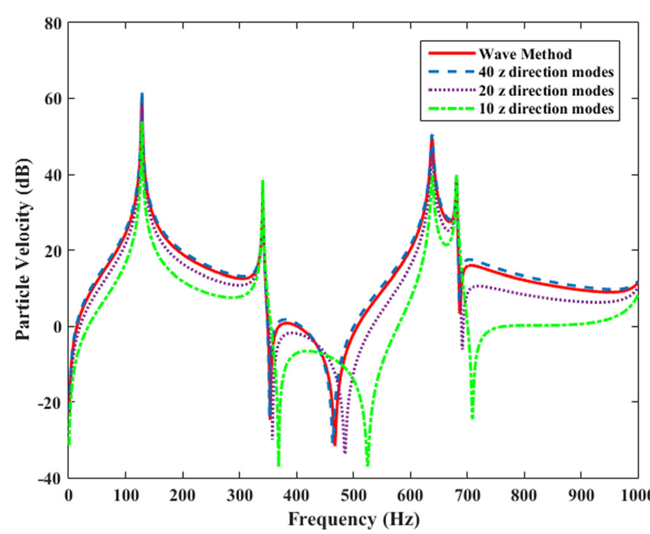

FIG. 3. (Color online) Particle velocity predictions by the wave method and modal based method: Different z-direction terms are used in the modal method.

eight z-direction terms were used). Nevertheless, upon increasing the decomposition terms, sufficient accuracy can still be achieved.

To further quantify this observation, a term describing the velocity prediction error, is defined as $\Delta V = V_{\text{modal}} - V_{\text{wave}}$ in dB, calculated and shown in Fig. 4, in terms of different truncated series in the z direction at an arbitrarily chosen frequency of 210 Hz. It can be observed that the modal-based method quickly approaches the exact result with a relatively small but increasing number of terms, overshoots and then converges to the exact solution. The convergence, however, is not monotonous with the number of terms used, but in an oscillating manner.

A close examination of the modal expansion expression of the particle velocity allows to better understand this and eventually establish a convergence criterion. Derived from the coupling Eqs. (3) and (4), the particle velocity can be expressed as

$$v(x,y,z) = \frac{1}{\rho_0}\sum_{n,m,p}\frac{U_{nm}(x,y)}{M_{nmp}\left(-\omega^2+\omega_{nmp}^2+j\eta_a\omega\omega_{nmp}\right)} \times \frac{p\pi}{L_z}\sin\left(\frac{p\pi z}{L_z}\right), \tag{6}$$

where n, m, and p are the modal indices corresponding to the x, y, and z directions, respectively, and $U_{nm}(x, y)$ is the velocity contributions related to nm cavity modes, expressed by W_{rs},

$$U_{nm}(x,y) = -j\omega\frac{A_F}{V}\cos\left(\frac{n\pi x}{L_x}\right)\cos\left(\frac{m\pi y}{L_y}\right)\sum_{r,s}L_{nm,rs}W_{rs}. \tag{7}$$

Since only the z direction is our focus, upon fixing m and n, the above expression can then be simplified to a one-dimensional (1D) case as

$$v(z) = \sum_p \gamma_p(\omega)\sin\left(\frac{p\pi z}{L_z}\right), \tag{8}$$

with

$$\gamma_p(\omega) = \frac{1}{\rho_0}\frac{U_{nm}(x,y)}{M_{nmp}\left(-\omega^2+\omega_{nmp}^2+j\eta_a\omega\omega_{nmp}\right)}\frac{p\pi}{L_z}. \tag{9}$$

For a given frequency of interest, ω is a constant. In order to ensure a reasonable calculation accuracy, it is well accepted that the modes which need to be included in the calculation should be such that $\omega_{nmp} \gg \omega$. Therefore, γ_p can be approximated by

$$\gamma_p(\omega) \underset{p\to\infty}{\sim} \frac{1}{\rho_0}\frac{U_{nm}(x,y)}{M_{nmp}\omega_{nmp}^2}\frac{p\pi}{L_z}. \tag{10}$$

Moreover, when p is large (i.e., $p \gg \max[m, n]$) as in this paper, the modal frequency can be approximated by $\omega_{nmp} \sim c_0(p\pi/L_z)$. Under these conditions, one has

$$\gamma_p(\omega) \underset{p\to\infty}{\sim} \frac{1}{\rho_0}\frac{U_{nm}(x,y)L_z}{M_{nmp}c_0^2\pi}\frac{1}{p}. \tag{11}$$

Since M_{nmp} is independent of p for $p > 0$, $\gamma_p(\omega)$ is a decreasing function of p and satisfies $\lim_{p\to\infty}\gamma_p = 0$. Therefore, according to Abel's theorem, the series $v(z)$ should converge. Meanwhile, an oscillation behavior is expected due to the term $\sin(p\pi z/L_z)$. Therefore, the modal method should guarantee the required calculation accuracy of the particle velocity prediction, at the expense of increasing the decomposition terms up to a sufficient level, in an oscillating but converging manner.

From the above analyses, it can be surmised that a larger number of modes may improve the accuracy for particle velocity, but not necessarily in a monotonous manner. Owing to the oscillating feature of the convergence curve shown above, it is desirable then to find the suitable number of modes to be used, with which the prediction error can be locally minimal. On the other hand, it goes without saying that the so-called criterion shall also depend on the distance of the observation point from the vibrating plate. To further investigate this, ΔV is calculated for different z coordinates, with results shown in Fig. 5. One can observe that, for all z

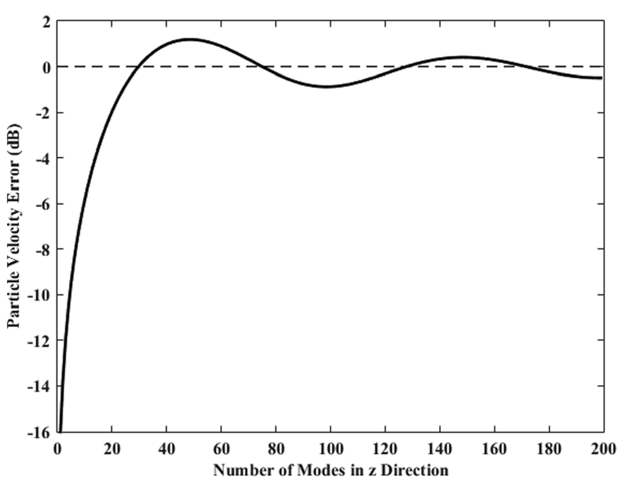

FIG. 4. Velocity prediction error at point (0.04, 0.17, 0.01) m with respect to the number of modes in the z direction.

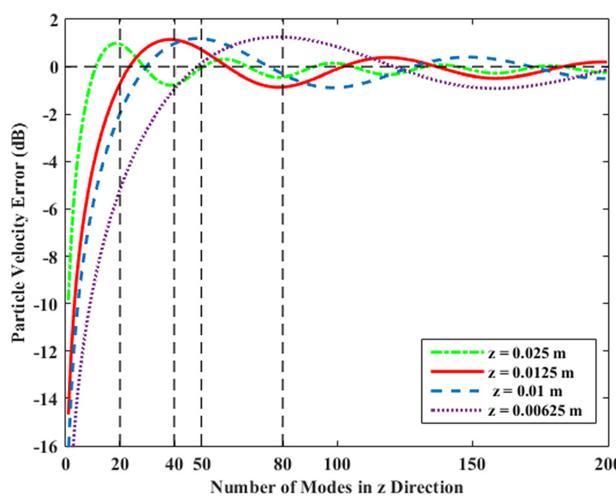

FIG. 5. (Color online) Velocity prediction error with respect to the number of modes in the z direction.

values, all ΔV curves exhibit a similar variation trend with respect to z, as described above. However, the convergence becomes increasingly slower as the observation point gets closer to the vibrating plate (when z gets smaller), along with a larger oscillation period. For the smallest z analyzed ($z = 0.00625$ m), for example, it requires 80 z-direction modes for ΔV to approach zero.

The oscillating nature of the convergence curves suggest that, for a given distance from the panel z, it should be possible to employ a small number of p terms to get the local minimum ΔV. The so-called truncation criterion, if it exists, should depend on the relationship between the number of modes in the z direction p and the coordinate z. To establish this relationship, a generalized mode number G is defined to connect the wavelength of mode p, $\lambda_p = 2L_z/p$, and the coordinate z, as

$$G = \frac{z}{\frac{\lambda_p}{2}} = \frac{pz}{L_z}. \tag{12}$$

Using this definition, different curves shown in Fig. 6 are normalized with respect to G and the results are shown in

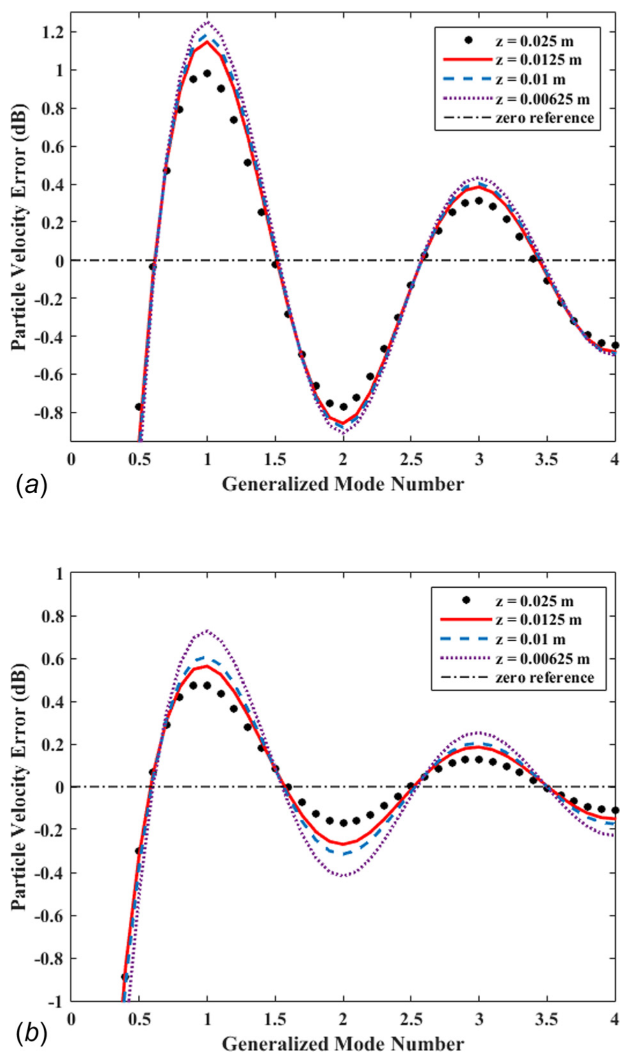

FIG. 6. (Color online) Velocity prediction error with respect to the generalized mode number G: (a) Uncoupled non-resonance frequency at 210 Hz; (b) uncoupled resonance frequency at 128 Hz.

Fig. 6(a). It is clear that the normalized curves show a highly consistent trend for all z values considered. One can observe that ΔV approaches local maximum values at every integer of G (1, 2, 3,…). The first oscillation point starts at $G = 1$. Deriving from Eq. (12), this corresponds to $z = 1/2\ \lambda_p$, which is the half acoustic wavelength. The local minima of ΔV are obtained roughly at the middle of two extreme values, starting from $G = 1.5$, followed by 2.5, 3.5, etc. Taken the first minima as an example, $G = 1.5$ corresponds to $z = 3/4\ \lambda_p$. Note that p is the highest mode index that is included in the calculation. Therefore, to minimize the accurate acoustic velocity prediction error, a rule of thumb would be to increase the number of acoustic modes in the z direction, until reaching the one with its 3/4 wavelength falling into z. In another word, for a given distance from the vibrating plate, all the lower-order modes in the cavity depth direction whose 3/4 wavelength is larger than that distance should be used in the series decomposition to ensure a good prediction accuracy for the particle velocity.

Mindful of the possible dependence of the aforementioned on the frequency, the above proposed truncation criterion is checked for one of the plate resonances frequencies at 128 Hz, with results shown in Fig. 6(b). Once again, the normalized ΔV curves show an identical variation trend as the previous non-resonance cases, which leads to exactly the same conclusions in terms of velocity convergence criterion. Nevertheless, it is found that the oscillation amplitude of the ΔV curves at the resonance frequency is somehow smaller than that of the non-resonance one. On all accounts, the proposed criterion on oscillating convergence seems to apply to all frequencies.

As a final check, Fig. 7 compares the velocity prediction results using the proposed truncation criterion with $G = 1.5$ and the wave method in the 3D configuration. According to Eq. (12), $G = 1.5$ results in 120 z-direction modes for $z = 0.00625$ m in Fig. 7(a) and 30 z-direction modes for $z = 0.025$ m in Fig. 7(b). While according to the pressure criterion, the number of z-direction modes is 8 for both cases. The result obtained with the pressure criterion is also added for reference. It is worth recalling that the use of only pressure criterion would not be enough to guarantee the velocity calculation, although the use of a larger number of modes is definitely helpful. The proposed velocity convergence criterion, however, results in a significant improvement to the particle velocity prediction. Additionally, comparisons between Figs. 7(a) and 7(b) also show that the proposed criterion holds well for different calculation point positions with different z coordinates.

III. CONCLUDING REMARKS

The prevailing conclusion of the present paper is the confirmation that the modal-based decomposition method, as formulated in the classical work of Dowell and Fahy, allows correct prediction of both the acoustic pressure and the acoustic velocity inside an acoustic cavity covered by a flexural structure upon using appropriate series truncation criterion. The acoustic pressure prediction using the modal method can be sufficiently accurate, throughout the cavity

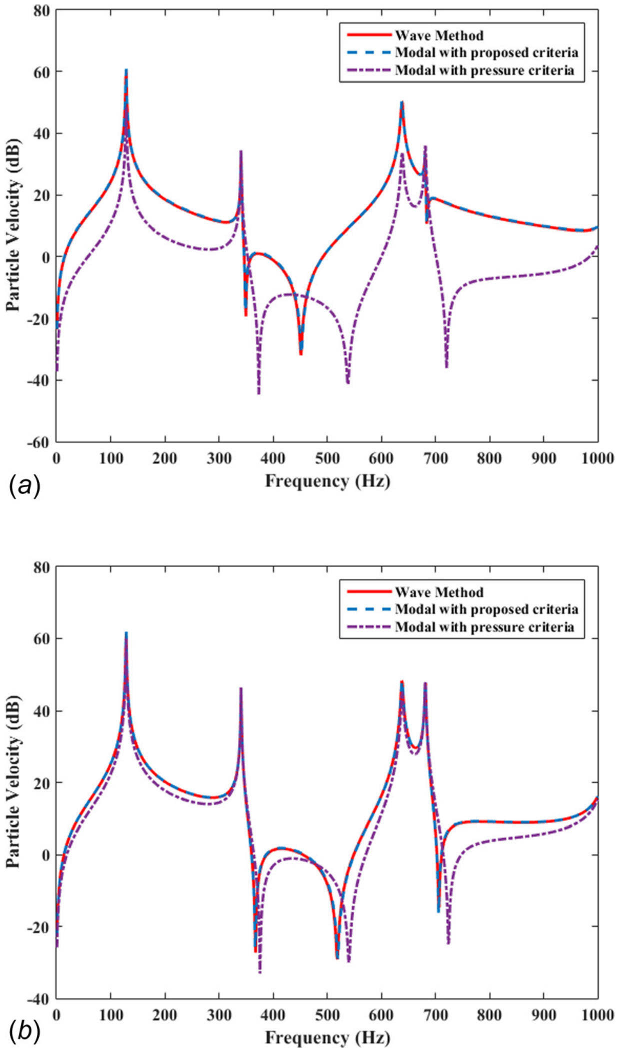

FIG. 7. (Color online) Particle velocity predictions: (a) $z = 0.00625$ m; (b) $z = 0.025$ m.

including vibrating interface as long as a sufficient number of cavity modes (prescribed by the pressure convergence criterion) are used, in agreement with the common understanding reported in the literature. The conventionally used pressure criterion, however, cannot guarantee the velocity prediction accuracy, especially when the calculation point is close to the vibrating structure, due to the inherent weakness of the modal shape functions. Nevertheless, numerical studies reveal an oscillating convergence pattern of the particle velocity when the decomposition terms in the cavity depth direction increases. More specifically, for a given calculation point, the calculated particle velocity using the modal approach first monotonously approaches to the exact value with a relatively small but increasing number of terms, overshoots and then converges to the exact solution in an oscillating manner, starting roughly from the generalized mode number $G = 1$. For a given distance from the vibrating plate, the modal series in the cavity depth direction should be truncated up to $G = 1.5$, 2.5, 3.5,…, etc. Explained using the series decomposition theories and verified in both 1D and 3D configurations, this so-called velocity truncation criterion suggests to use all these lower-order modes in the cavity depth direction, whose 3/4 wavelengths are larger than the distance between the calculation point and the vibrating plate, to ensure a good prediction accuracy for the particle velocity. Therefore, when both the pressure criterion and the proposed velocity convergence rule are satisfied, a fast convergence of the particle velocity can be achieved.

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