https://doi.org/10.1016/j.ijheatmasstransfer.2017.12.080

Double-diffusive natural convection and entropy generation of Carreau fluid in a heated enclosure with an inner circular cold cylinder (Part I: Heat and Mass Transfer)

GH. R. Kefayati *, H. Tang

Department of Mechanical Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong SAR, China

Abstract

In this paper, double-diffusive natural convection, studying Soret and Dufour effects and viscous dissipation in a heated enclosure with an inner cold cylinder filled with non-Newtonian Carreau fluid has been simulated by Finite Difference Lattice Boltzmann Method (FDLBM). This study has been conducted for certain pertinent parameters of Rayleigh number (Ra = 10^4 and 10^5), Carreau number (Cu = 1, 10, and 20), Lewis number (Le=2.5, 5 and 10), Dufour parameter (D_f=0, 1, and 5), Soret parameter (S_r=0, 1, and 5), Eckert number (Ec=0, 1, and 10), the Buoyancy ratio (N=-1, 0.1, 1), the radius of the inner cylinder (R_d = 0.1 L, 0.2 L, 0.3 L, and 0.4 L), the horizontal distance of the circular cylinder from the center of the enclosure (Ω = -0.2 L, 0 and 0.2 L), the vertical distance of the circular cylinder from the center of the enclosure (δ = -0.2 L, 0 and 0.2 L). Results indicate that the increase in Rayleigh number enhances heat transfer for various studied parameters. The increase in power-law index provokes heat and mass transfer to drop gradually. The increase in the Lewis number declines the mass transfer considerably while causes heat transfer to drop marginally. The heat transfer increases with the rise of the Dufour parameter and the mass transfer enhances as the Soret parameter increases for different Rayleigh numbers. The augmentation of the buoyancy ratio number enhances heat and mass transfer. The increase in Eckert number affects heat and mass transfer; especially, at Ra = 10^5. The rise of Carreau number causes heat and mass transfer to drop gradually. The movement of the center of the cylinder from the bottom to the top side of the enclosure vertically (δ = -0.2 L, 0 and 0.2 L) decreases heat and mass transfer significantly while the effect of power-law index drops. The increase in the radius of the cylinder enhances heat and mass transfer. The alteration of the center of the cylinder horizontally (Ω) to the left and right sides enhance heat and mass transfer although this augmentation is different in various power-law indexes.

Key words: Carreau fluid, Natural convection, Mass transfer, Viscous, LBM

dissipation

1 Introduction

Analysis of natural convection in enclosures has been extensively conducted using different numerical techniques and experiments because of its wide applications and interest in engineering e.g. nuclear energy, double pane windows, heating and cooling of buildings, solar collectors, electronic cooling, and so on. The wide range of studies into this topic has led to the natural convection in a cavity to become a common benchmark among researchers in the field of CFD (Computational Fluid Dynamics). It consists of a two-dimensional cavity and the temperature of the heated section on the left is maintained at a higher temperature and the right wall is held at a lower temperature. The horizontal walls are considered to be adiabatic and the density variation is approximated by the standard Boussinesq model. The natural convection flow of a Newtonian fluid has been studied numerically by de Vahl Davis [1], Quere and de Roquefort [2], Quere [3]. Many studies have conducted the effect of the presence of isothermal bodies inside the enclosure on the natural convection phenomena and focused on the diverse body shapes, e.g. circular, square and triangular cylinders. Kim et al. [4] carried out numerical calculations for natural convection induced by a temperature difference between a cold outer square enclosure and a hot inner circular cylinder. They investigated the effect of the inner cylinder location on the heat transfer and fluid flow. Further, the location of the inner circular cylinder was changed vertically along the center-line of square enclosure. Mehrizi et al. [5] investigated a numerical study for steady-state, laminar natural convection in a horizontal annulus between a heated triangular inner cylinder and cold elliptical outer cylinder, using lattice Boltzmann method. Both inner and outer surfaces were maintained at the constant temperature and air was the working fluid. Park et al. [6] studied the natural convection induced by a temperature difference between a cold outer square enclosure and two hot inner circular cylinders. A two-dimensional solution for natural convection in an enclosure with inner cylinders was obtained using an accurate and efficient immersed boundary method. The immersed boundary method based on the finite volume method was used to handle inner cylinders located at different vertical centerline posi-

* Corresponding author: Dr. Gholamreza Kefayati (GH. R. Kefayati)
Email addresses: gholamreza.kefayati@polyu.edu.hk, gholamrezakefayati@gmail.com (GH. R. Kefayati), h.tang@polyu.edu.hk (H. Tang).

tions of the enclosure for different Rayleigh numbers. Mehrizi and Mohamad [7] utilized Lattice Boltzmann method to simulate steady-state, laminar, free convection in two-dimensional annuli between a heated triangular inner cylinder and elliptical outer cylinder. The study was performed for different inclination angles of inner triangular and outer elliptical cylinders. Mun et al. [8] conducted two-dimensional numerical simulations to investigate the natural convection heat transfer induced by the temperature difference between cold walls of the tilted square enclosure and a hot inner circular cylinder for different prandtl numbers. Seo et al. [9] conducted two-dimensional numerical simulations for the natural convection phenomena in a cold square enclosure with four hot inner circular cylinders. The immersed boundary method (IBM) was used to capture the virtual wall boundary of the four inner cylinders based on the finite volume method (FVM). Zhang et al. [10] investigated a numerical study for steady-state natural convection in a cold outer square enclosure containing a hot inner elliptic cylinder using the variational multiscale element free Galerkin method (VMEFG). In the cited studies, the fluids have been assumed to be Newtonian fluids while most materials demonstrates non-Newtonian behavior. Natural convection of non-Newtonian power-law fluids and Bingham fluids in an enclosure recently have been studied by some researchers [11-21]. However, natural convection of Carreau fluids in an enclosure have not been considered thus far. Carreau fluid is a special sub-class of non-Newtonian fluids in which follows the Carreau model [22]. This model was introduced in 1972 and has been applied extensively up to date. Carreau models have been employed to simulate various chemicals, molten plastics, slurries, paints, blood, etc. Some limited isothermal and non-isotermal problems of Carreau fluids have been studied. Shamekhi and Sadeghy [23] analyzed Lid-driven cavity flow of a purely-viscous non-Newtonian fluid obeying Carreau-Yasuda rheological model numerically using the PIM meshfree method combined with the Characteristic-Based Split-A algorithm. Results were reported for the velocity and pressure profiles at Reynolds numbers as high as 1000 for a non-Newtonian fluid obeying Carreau-Yasuda rheological model. Bouteraa et al. [24] performed a linear and weakly nonlinear analysis of convection in a layer of shear-thinning fluids between two horizontal plates heated from below. The shear-thinning behaviour of the fluid was described by the Carreau model. Shahsavari and McKinley [25] studied The flow of generalized Newtonian fluids with a rate-dependent viscosity through fibrous media with a focus on developing relationships for evaluating the effective fluid mobility. They conducted a numerical solution of the Cauchy momentum equation with the Carreau or power-law constitutive equations for pressure-driven flow in a fiber bed consisting of a periodic array of cylindrical fibers. Pantokratoras [26] considered the flow of a non-Newtonian, Carreau fluid, directed normally to a horizontal, stationary, circular cylinder. The problem was investigated numerically using the commercial code ANSYS FLUENT with a very large calculation domain in order that the flow could be considered unbounded.

Lattice Boltzmann method (LBM) has been demonstrated to be a very effective mesoscopic numerical method to model a broad variety of complex fluid flow phenomena [27-42]. This is because the main equation of the LBM is hyperbolic and can be solved locally, explicitly, and efficiently on parallel computers. However, the specific relation between the relaxation time and the viscosity has caused LBM not to have the considerable success in non-Newtonian fluid especially on energy equations. In this connection, Fu et al. [43-44] proposed a new equation for the equilibrium distribution function, modifying the LB model. Here, this equilibrium distribution function is altered in different directions and nodes while the relaxation time is fixed. Independency of the method to the relaxation time in contrast with common LBM provokes the method to solve different non-Newtonian fluid energy equations successfully as the method protects the positive points of LBM simultaneously. In addition, the validation of the method and its mesh independency demonstrates that is more capable than conventional LBM. Huilgol and Kefayati [45] derived the three dimensional equations of continuum mechanics for this method and demonstrated that the theoretical development can be applied to all fluids, whether they be Newtonian, or power law fluids, or viscoelastic and viscoplastic fluids. Following the study, Huilgol and Kefayati [46] developed this method for the cartesian, cylindrical and spherical coordinates. Kefayati [47] simulated double-diffusive natural convection with Soret and Dufour effects in a square cavity filled with non-Newtonian power-law fluid by FDLBM while entropy generations through fluid friction, heat transfer, and mass transfer were analysed. Kefayati [48-49] analysed double diffusive natural convection and entropy generation of non-Newtonian power-law fluids in an inclined porous cavity in the presence of Soret and Dufour parameters by FDLBM. Kefayati and Huilgol [50] conducted a two-dimensional simulation of steady mixed convection in a square enclosure with differentially heated sidewalls when the enclosure is filled with a Bingham fluid, using FDLBM. The problem was solved by the Bingham model without any regularisations and also by applying the regularised Papanatasiou model. Kefayati [51] simulated double-diffusive natural convection, studying Soret and Dufour effects and viscous dissipation in a square cavity filled with Bingham fluid by FDLBM. In addition, entropy generations through fluid friction, heat transfer, and mass transfer were studied. The problem was solved by applying the regularised Papanastasiou model.

The main aim of this study is to simulate double diffusive natural convection of Carreau fluid in a heated enclosure with an inner cold cylinder. The innovation of this paper is studying heat and mass transfer in the presence of Soret and Dufour and the viscous dissipation effect on Carreau fluid for the first time. An innovative method based on LBM has been employed to study the problem numerically. Moreover, it is endeavored to express the effects of different parameters on heat and mass transfer. The obtained results are validated with previous numerical investigations and the effects of the main parameters

(Rayleigh number, Lewis number, buoyancy ratio number, Eckert number, Carreau number, Soret parameter, and Dufour parameter) are researched.

2 Theoretical formulation

The geometry of the present problem is shown in Fig. 1. The temperature and concentration of the enclosure walls have been considered to be maintained at high temperature and concentration of T_H and C_H as the circular cylinder is kept at low temperature and concentration of T_C and C_C. The lengths of the enclosure sidewalls are L where the inner cylinder center is defined by (x_c, y_c) and the radius of the cylinder is specified by R_d. The origin of Cartesian coordinates is located in the center of the cavity as depicted in the Fig.1. For the concentric cases, the cylinder center is fixed at $(x_c = 0, y_c = 0)$ in the center of the cavity. For the eccentric cases, the horizontal and vertical distances from the center are defined by Ω and δ, respectively. The cavity is filled with a Carreau fluid. The prandtl number is fixed at Pr=0.1. The Soret, and Dufour parameters also have been considered. There is no heat generation, chemical reactions, and thermal radiation. The flow is incompressible, and laminar. The density variation is approximated by the standard Boussinesq model for both temperature and concentration. The viscous dissipation in the energy equation has been analyzed in this study.

2.1 Dimensional equations

Based on the above assumptions, and applying the Boussinesq approximation, the studied equations are [47 - 57]:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{2.1}$$

$$\rho\left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{x}} + \left(\frac{\partial \bar{\tau}_{xx}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{xy}}{\partial \bar{y}}\right), \tag{2.2}$$

$$\begin{aligned}\rho\left(\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{y}} + \left(\frac{\partial \bar{\tau}_{yy}}{\partial \bar{y}} + \frac{\partial \bar{\tau}_{xy}}{\partial \bar{x}}\right)& \\ + g\rho\left[1 + \beta_T(\bar{T} - T_C) - \beta_C(\bar{C} - C_C)\right],& \end{aligned} \tag{2.3}$$

In the above equations $(\mathbf{u} = \bar{u}\mathbf{i} + \bar{v}\mathbf{j}), \bar{T}$, and $\bar{C}$, and g are the dimensional velocities, temperature, concentration, and gravity acceleration respectively. β_T and β_C are the coefficient of thermal expansion and solutal expansion, respectively as ρ is density. Now, let the pressure $\bar{p}$ be written as the sum $\bar{p} = \bar{p}_s + \bar{p}_d$, where the static part $\bar{p}_s$ accounts for gravity alone, and $\bar{p}_d$ is the dynamic part. Thus,

$$-\frac{\partial \bar{p}_s}{\partial \bar{y}} = \rho g \cdot \tag{2.4}$$

$$\frac{\partial \bar{T}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \alpha\left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right)+\frac{1}{\rho c_p}\left[\bar{\tau}_{xx}\left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)+\bar{\tau}_{xy}\left(\frac{\partial \bar{u}}{\partial \bar{y}}+\frac{\partial \bar{v}}{\partial \bar{x}}\right)+\bar{\tau}_{yy}\left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)\right] + K_{TC}\left(\frac{\partial^2 \bar{C}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}\right) \tag{2.5}$$

α and K_{TC} are the thermal diffusivity and the thermodiffusion, respectively. c_p is the specific heat capacity at constant pressure.

$$\frac{\partial \bar{C}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{C}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{C}}{\partial \bar{y}} = D\left(\frac{\partial^2 \bar{C}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}\right)+K_{CT}\left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right) \tag{2.6}$$

D and K_{CT} are the mass diffusivity coefficient and the diffusionthermo coefficient, respectively.

The stress tensor for the incompressible Carreau fluids is as [22-26]

$$\bar{\tau}_{ij} = 2\,\eta(\dot{\gamma})\;S_{ij} \tag{2.7}$$

where S_{ij} is the rate of strain tensor as

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right) \tag{2.8}$$

where

$$\eta(\dot{\gamma}) = \eta_\infty + (\eta_0-\eta_\infty)\;\left[1+(\lambda\dot{\gamma})^2\right]^{(n-1)/2}\quad,\qquad \dot{\gamma}=\sqrt{2S_{ij}S_{ij}} \tag{2.9}$$

where η_0 and η_∞ are the viscosities corresponding to zero and infinite viscosities, λ is the time constant and n is the power-law index where the deviation of n from unity indicates the degree of deviation from Newtonian behavior. With n$\neq$1, the constitute equation represents pseudoplastic fluid ($0<n<1$) and for ($n>1$) it represents a dilatant fluid, respectively. Note that a Newtonian fluid can be recovered as a special case of the present Carreau fluid by letting n

$= 1$ and/or $\lambda = 0$, and a power-law fluid can be obtained by assuming a large λ. The infinite shear viscosity, η_∞, is generally associated with a breakdown of the fluid, and is frequently significantly smaller ($10^3 - 10^4$ times smaller) than η_0, see [22, 26, 58, 59]. So, the ratio η_∞/η_0 has been fixed at 0.001.

The flow domain is given by $\omega = (-L/2, L/2) \times (-L/2, L/2)$, and the boundary $\Gamma = \partial\omega$. It is the union of five disjoint subsets:

$$\Gamma_1 = \{(x, y), x = -L/2, -L/2 \leq y \leq L/2\}, \tag{2.10a}$$
$$\Gamma_2 = \{(x, y), x = L/2, -L/2 \leq y \leq L/2\}, \tag{2.10b}$$
$$\Gamma_3 = \{(x, y), -L/2 \leq x \leq L/2, y = -L/2\}, \tag{2.11a}$$
$$\Gamma_4 = \{(x, y), -L/2 \leq x \leq L/2, y = L/2\}, \tag{2.11b}$$
$$\Gamma_5 = \left\{(x, y), (x - x_c)^2 + (y - y_c)^2 = {R_d}^2.\right\} \tag{2.12}$$

The parameters of x_c, y_c, and R_d are the horizontal and vertical positions of the cylinder center and the radius of the cylinder; respectively.

The boundary condition for the velocity is straightforward:

$$\mathbf{u}|_{\Gamma_1} = \mathbf{u}|_{\Gamma_2} = \mathbf{u}|_{\Gamma_3} = \mathbf{u}|_{\Gamma_4} = \mathbf{u}|_{\Gamma_5} = \mathbf{0}. \tag{2.13}$$

The boundary conditions for the temperature and concentration are:

$$T|_{\Gamma_1} = T|_{\Gamma_2} = T|_{\Gamma_3} = T|_{\Gamma_4} = T_H, \quad T|_{\Gamma_5} = T_C \tag{2.14}$$

$$C|_{\Gamma_1} = C|_{\Gamma_2} = C|_{\Gamma_3} = C|_{\Gamma_4} = C_H, \quad C|_{\Gamma_5} = C_C \tag{2.15}$$

2.2 Non-dimensional equations

In order to proceed to the numerical solution of the system, the following non dimensional variables are introduced.

$$t = \frac{\bar{t}}{\left(\frac{L^2}{\alpha}\right) Ra^{-0.5}}, \quad x = \bar{x}/L, \quad y = \bar{y}/L, \quad u = \frac{\bar{u}}{\left(\frac{\alpha}{L}\right) Ra^{0.5}} \tag{2.16}$$

$$v = \frac{\bar{v}}{\left(\frac{\alpha}{L}\right) Ra^{0.5}}, \quad p_d = \frac{\bar{p}_d}{\rho\left(\frac{\alpha}{L}\right)^2 Ra}, \quad T = (\bar{T} - T_C)/\Delta T \tag{2.17}$$

$$C = (\bar{C} - C_C)/\Delta C, \quad \Delta T = T_H - T_C \quad \Delta C = C_H - C_C \tag{2.18}$$

$$\boldsymbol{\tau} = \frac{\bar{\boldsymbol{\tau}}\, L}{\eta_0 \left(\frac{\alpha}{L}\right)\, Ra^{0.5}} \tag{2.19}$$

By substitution of Eqs. (2.16) - (2.19) into Eqs. (2.1) - (2.6), the following system of non-dimensional equations is derived:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.20}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p_d}{\partial x} + \frac{Pr}{\sqrt{\mathrm{Ra}}}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right) \tag{2.21}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p_d}{\partial y} + \frac{Pr}{\sqrt{\mathrm{Ra}}}\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}\right) + \mathrm{Pr}\,(T - N\,C) \tag{2.22}$$

$$\begin{aligned}\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\sqrt{Ra}}\left[\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + D_f\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right)\right] \\ +Pr\,Ec\sqrt{Ra}\,\left[\tau_{xx}\left(\frac{\partial u}{\partial x}\right) + \tau_{xy}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \tau_{yy}\left(\frac{\partial v}{\partial y}\right)\right]\end{aligned} \tag{2.23}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \frac{1}{Le\sqrt{Ra}}\left[\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + S_r\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)\right] \tag{2.24}$$

The non-dimensional apparent viscosity is given by [22]

$$\eta(\dot{\gamma}) = \frac{\eta_\infty}{\eta_0} + (1 - \frac{\eta_\infty}{\eta_0})\;\left[1 + (Cu\dot{\gamma})^2\right]^{(n-1)/2} \tag{2.25}$$

$$\dot{\gamma} = \left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right\}^{\frac{1}{2}} \tag{2.26}$$

Hence, the stresses are:

$$\tau_{xx} = 2\eta(\dot{\gamma})\left(\frac{\partial u}{\partial x}\right)\quad \tau_{yy} = 2\eta(\dot{\gamma})\left(\frac{\partial v}{\partial y}\right)\quad \tau_{xy} = \eta(\dot{\gamma})\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{2.27}$$

The non-dimensional parameters for the problem are as follows:

Thermal Rayleigh number:

$$Ra = \frac{\rho\, \beta_T\, g L^3 \Delta T}{\eta_0\, \alpha} \tag{2.28}$$

Prandtl number:

$$\mathrm{Pr} = \frac{\eta_0}{\rho\, \alpha} \tag{2.29}$$

Eckert number:

$$Ec = \frac{\left({}^{\alpha}\!/\!{}_{L}\right)^2}{c_p\, \Delta T} \tag{2.30}$$

Buoyancy ratio number:

$$N = \frac{\Delta C\ \beta_T\ D}{\beta_C\ \Delta T\ \alpha} \tag{2.31}$$

Lewis number:

$$Le = \frac{\alpha}{D} \tag{2.32}$$

Dufour parameter:

$$D_f = \frac{K_{TC}\, \Delta C}{\alpha\ \Delta T} \tag{2.33}$$

Soret parameter:

$$S_r = \frac{K_{CT}\, \Delta T}{D\ \Delta C} \tag{2.34}$$

Carreau number:

$$Cu = \frac{\lambda}{\left(\frac{L^2}{\alpha}\right)\ Ra^{-0.5}} \tag{2.35}$$

The local and the average Nusselt and Sherwood numbers at the cavity sides are as

$$Nu = \left(-\frac{\partial T}{\partial r}\right)_{r=0} +\ D_f\left(-\frac{\partial C}{\partial r}\right)_{r=0} \tag{2.36a}$$

$$Sh = \left(-\frac{\partial C}{\partial r}\right)_{r=0} +\ S_r\left(-\frac{\partial T}{\partial r}\right)_{r=0} \tag{2.36b}$$

$$Nu_{avg} = \int_{-1/2}^{1/2} Nu\, ds, \quad Sh_{avg} = \int_{-1/2}^{1/2} Sh\, ds \tag{2.36c}$$

where r denotes the unit normal direction on a specific side wall s.

The total average Nusselt and Sherwood numbers are as

$$Nu_{totavg} = Nu_{Lavg} + Nu_{Ravg} + Nu_{Bavg} + Nu_{Tavg} \tag{2.37a}$$

$$Sh_{totavg} = Sh_{Lavg} + Sh_{Ravg} + Sh_{Bavg} + Sh_{Tavg} \tag{2.37b}$$

In the Eqs.2.37, the subscribes of tot, L, R, B, T, avg means total, the left wall of the cavity, the right wall of the cavity, the bottom wall of the cavity, the top wall of the cavity, and average; respectively.

3 The numerical method

The FDLBM equations and their relationships with continuum equations have been explained in details in Huilgol and Kefayati [45-46]. Here, just a brief description about the main equations would be cited. In addition, the applied algorithm has been described and the studied problem equations in the FDLBM are mentioned.

3.1 The Continuity and Momentum equations

To have the continuity and momentum equations, a discrete particle distribution function f_α is defined over a D2Q9 lattice where it should satisfy an evolution equation:

$$\frac{\partial f_\alpha}{\partial t} + \boldsymbol{\xi}_\alpha \cdot \nabla_{\mathbf{x}} f_\alpha - F_\alpha = -\frac{1}{\varepsilon\, \phi}(f_\alpha - f_\alpha^{eq}), \tag{3.1}$$

where ε is a small parameter to be prescribed when numerical simulations are considered. ϕ is the relaxation time and F is the external force.

Associated to each node is a lattice velocity vector $\boldsymbol{\xi}_\alpha$. It is defined as follows:

$$\boldsymbol{\xi}_\alpha = \begin{cases} (0,0), & \alpha = 0, \\ \sigma(\cos\theta_\alpha, \sin\theta_\alpha) & \alpha = 1,3,5,7, \\ \sigma\sqrt{2}(\cos\theta_\alpha, \sin\theta_\alpha), & \alpha = 2,4,6,8. \end{cases} \tag{3.2}$$

Here, the angles θ_α are defined through $\theta_\alpha = (\alpha - 1)\pi/4,\ \alpha = 1, \cdots, 8.$ The constant σ has to be chosen with care for it affects numerical stability;

its choice depends on the problem. The method for finding the parameter σ which satisfies the Courant-Friedrichs-Lewy (CFL) condition is described in [45-46].

The equilibrium distribution function, f_α^{eq}, is different from the conventional ones adopted by previous researchers, who normally expand the Maxwellian distribution function. In the present approach, we expand f_α^{eq} as a quadratic in terms of $\boldsymbol{\xi}_\alpha$, using the notation of linear algebra [45-46]:

$$f_\alpha^{eq} = A_\alpha + \boldsymbol{\xi}_\alpha \cdot \mathbf{B}_\alpha + (\boldsymbol{\xi}_\alpha \otimes \boldsymbol{\xi}_\alpha) : \mathbf{C}_\alpha, \; \alpha = 0, 1, 2, \cdots, 8. \tag{3.3}$$

Here, the scalars A_α are defined through

$$A_0 = \rho - \frac{2p}{\sigma^2} - \frac{\rho|\mathbf{u}|^2}{\sigma^2} + \frac{\tau_{xx} + \tau_{yy}}{\sigma^2}, \quad A_\alpha = 0, \; \alpha = 1, 2, \cdots, 8. \tag{3.4}$$

The vectors $\mathbf{B}_\alpha$ are given by

$$\mathbf{B}_1 = \frac{\rho \mathbf{u}}{2\sigma^2} = \mathbf{B}_\alpha, \; \alpha = 1, 3, 5, 7; \quad \mathbf{B}_\alpha = \mathbf{0}, \; \alpha = 0, 2, 4, 6, 8. \tag{3.5}$$

Next, the matrices $\mathbf{C}_\alpha$ are such that $\mathbf{C}_0 = 0$; $\mathbf{C}_1 = \mathbf{C}_\alpha, \; \alpha = 1, 3, 5, 7$; $\mathbf{C}_2 = \mathbf{C}_\alpha, \; \alpha = 2, 4, 6, 8$, where

$$\mathbf{C}_1 = \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix}, \quad C_{11} = \frac{1}{2\sigma^4}(p + \rho u^2 - \tau_{xx}), \quad C_{22} = \frac{1}{2\sigma^4}(p + \rho v^2 - \tau_{yy}), \tag{3.6}$$

$$\mathbf{C}_2 = \begin{bmatrix} 0 & C_{12} \\ C_{21} & 0 \end{bmatrix}, \quad C_{12} = C_{21} = \frac{1}{8\sigma^4}(\rho uv - \tau_{xy}). \tag{3.7}$$

In order to derive the macroscopic equations for an incompressible continuous medium in the presence of a body force, it has been shown that the functions F_α in (3.1) must be such that

$$\sum_{\alpha=0}^{8} F_\alpha = 0. \tag{3.8}$$

In turn, this guarantees that the conservation of mass equation is satisfied. Next, one requires that

$$\sum_{\alpha=0}^{8} F_\alpha \boldsymbol{\xi}_\alpha = \rho \mathbf{b}, \tag{3.9}$$

where $\rho\mathbf{b}$ is the body force. Thus, one choice for the set of F_α is:

$$F_0 = 0,\;\; F_1 = \frac{1}{2\sigma^2}\rho\mathbf{b}\cdot\boldsymbol{\xi}_1,\;\; F_3 = \frac{1}{2\sigma^2}\rho\mathbf{b}\cdot\boldsymbol{\xi}_3, \tag{3.10a}$$

$$F_5 = \frac{1}{2\sigma^2}\rho\mathbf{b}\cdot\boldsymbol{\xi}_5,\;\; F_7 = \frac{1}{2\sigma^2}\rho\mathbf{b}\cdot\boldsymbol{\xi}_7. \tag{3.10b}$$

$$F_\alpha = 0, \quad \alpha = 2,4,6,8. \tag{3.10c}$$

One notes that $F_1 = -F_5,\;\; F_3 = -F_7$.

In this problem, the non-dimensional body force is as follows:

$$\rho\mathbf{b} = \frac{\Pr\,(T - N\,C)}{2\,\sigma^2}\;\;\mathbf{j} \tag{3.11}$$

3.2 The Energy Equation

In order to obtain the energy equation, an internal energy distribution function g_α is introduced and it is assumed to satisfy an evolution equation similar to that for f_α. Thus,

$$\frac{\partial g_\alpha}{\partial t} + \boldsymbol{\xi}_\alpha\cdot\nabla_{\mathbf{x}} g_\alpha - G_\alpha = -\frac{1}{\varepsilon\phi}(g_\alpha - g_\alpha^{eq}). \tag{3.12}$$

G_α refers to the external supply e.g. radiation in the energy equation. Here, g_α^{eq} has a monomial expansion:

$$g_\alpha^{eq} = D_\alpha + \boldsymbol{\xi}_\alpha\cdot\mathbf{E}_\alpha, \tag{3.13}$$

One way of satisfying the above is to assume, as before, that the scalars are given by $D_\alpha = D_1,\; \alpha = 1,3,5,7$, and $D_\alpha = D_2,\; \alpha = 2,4,6,8$,. In this problem, the non-dimensional parameters are obtained as follows:

$$D_0 = T, \quad D_1 = 0, \quad D_2 = 0. \tag{3.14}$$

Regarding the vectors, it is assumed that $\mathbf{E}_0 = \mathbf{0},\; \mathbf{E}_\alpha = \mathbf{E}_1,\; \alpha = 1,3,5,7; \mathbf{E}_\alpha = \mathbf{E}_2,\; \alpha = 2,4,6,8$, where

$$\mathbf{E}_1 = \frac{\left(\mathbf{u}\,T + Pr\,Ec\,\sqrt{Ra}\;\;((u\,\tau_{xx} + v\,\tau_{xy}) + (u\,\tau_{yx} + v\,\tau_{yy})) - \frac{1}{\sqrt{Ra}}\left(\frac{\partial\mathbf{T}}{\partial\mathbf{x}} + D_f\,\frac{\partial\mathbf{C}}{\partial\mathbf{x}}\right)\right)}{2\,\sigma^2}. \tag{3.15}$$

Finally, $G_\alpha = 0$.

3.3 The Concentration Equation

In order to obtain the concentration equation, an internal concentration distribution function h_α is introduced and it is assumed to satisfy an evolution equation similar to that for f_α. Thus,

$$\frac{\partial h_\alpha}{\partial t} + \boldsymbol{\xi}_\alpha \cdot \nabla_{\mathbf{x}} h_\alpha - H_\alpha = -\frac{1}{\varepsilon\phi}(h_\alpha - h_\alpha^{eq}). \tag{3.16}$$

Here, h_α^{eq} has a monomial expansion:

$$h_\alpha^{eq} = M_\alpha + \boldsymbol{\xi}_\alpha \cdot \mathbf{N}_\alpha, \tag{3.17}$$

One way of satisfying the above is to assume, as before, that the scalars are given by $M_\alpha = M_1,\ \alpha = 1,3,5,7$, and $M_\alpha = M_2,\ \alpha = 2,4,6,8,$. In this problem, the non-dimensional parameters are obtained as follows:

$$M_0 = C, \quad M_1 = 0, \quad M_2 = 0. \tag{3.18}$$

Regarding the vectors, it is assumed that $\mathbf{N}_0 = \mathbf{0},\ \mathbf{N}_\alpha = \mathbf{N}_1,\ \alpha = 1,3,5,7; \mathbf{N}_\alpha = \mathbf{N}_2,\ \alpha = 2,4,6,8$, where

$$\mathbf{N}_1 = \frac{\left(\mathbf{u}\, C - \frac{1}{Le\sqrt{Ra}}\left(\frac{\partial \mathbf{C}}{\partial \mathbf{x}} + S_r \frac{\partial \mathbf{T}}{\partial \mathbf{x}}\right)\right)}{2\,\sigma^2} \tag{3.19}$$

Finally, $H_\alpha = 0$.

3.4 Algorithm

The main equations of the discrete particle distribution function, the internal energy distribution function, the internal concentration distribution function are solved by the splitting method. Hence, the equations can be separated into two parts. The first one is the streaming section which is written as

$$\frac{\partial f_\alpha}{\partial t} + \boldsymbol{\xi}_\alpha \cdot \nabla_{\mathbf{x}} f_\alpha - F_\alpha = 0. \tag{3.20}$$

$$\frac{\partial g_\alpha}{\partial t} + \boldsymbol{\xi}_\alpha \cdot \nabla_{\mathbf{x}} g_\alpha = 0. \tag{3.21}$$

$$\frac{\partial h_\alpha}{\partial t} + \boldsymbol{\xi}_\alpha \cdot \nabla_{\mathbf{x}} h_\alpha = 0. \tag{3.22}$$

Eqs.(3.20), (3.21), and (3.22) have been solved with FDM and the following equations are used.

$$\begin{aligned} f_\alpha^{n+1}(i,j) = f_\alpha^n(i,j) - \frac{\Delta t}{2\Delta x}\xi_\alpha(i)\left[f_\alpha^n(i+1,j) - f_\alpha^n(i-1,j)\right] \\ - \frac{\Delta t}{2\Delta y}\xi_\alpha(j)\left[f_\alpha^n(i,j+1) - f_\alpha^n(i,j-1)\right] + \\ \frac{\Delta t^2}{2\Delta x^2}{\xi_\alpha}^2(i)\left[f_\alpha^n(i+1,j) - 2f_\alpha^n(i,j) + f_\alpha^n(i-1,j)\right] + F_\alpha(i)\Delta t + \\ \frac{\Delta t^2}{2\Delta y^2}{\xi_\alpha}^2(j)\left[f_\alpha^n(i,j+1) - 2f_\alpha^n(i,j) + f_\alpha^n(i,j-1)\right] + F_\alpha(j)\Delta t\ , \end{aligned} \tag{3.23}$$

and

$$\begin{aligned} g_\alpha^{n+1}(i,j) = g_\alpha^n(i,j) - \frac{\Delta t}{2\Delta x}\xi_\alpha(i)\left[g_\alpha^n(i+1,j) - g_\alpha^n(i-1,j)\right] \\ - \frac{\Delta t}{2\Delta y}\xi_\alpha(j)\left[g_\alpha^n(i,j+1) - g_\alpha^n(i,j-1)\right] + \\ \frac{\Delta t^2}{2\Delta x^2}{\xi_\alpha}^2(i)\left[g_\alpha^n(i+1,j) - 2g_\alpha^n(i,j) + g_\alpha^n(i-1,j)\right] + \\ \frac{\Delta t^2}{2\Delta y^2}{\xi_\alpha}^2(j)\left[g_\alpha^n(i,j+1) - 2g_\alpha^n(i,j) + g_\alpha^n(i,j-1)\right] \end{aligned} \tag{3.24}$$

and

$$\begin{aligned} h_\alpha^{n+1}(i,j) = h_\alpha^n(i,j) - \frac{\Delta t}{2\Delta x}\xi_\alpha(i)\left[h_\alpha^n(i+1,j) - h_\alpha^n(i-1,j)\right] \\ - \frac{\Delta t}{2\Delta y}\xi_\alpha(j)\left[h_\alpha^n(i,j+1) - h_\alpha^n(i,j-1)\right] + \\ \frac{\Delta t^2}{2\Delta x^2}{\xi_\alpha}^2(i)\left[h_\alpha^n(i+1,j) - 2h_\alpha^n(i,j) + h_\alpha^n(i-1,j)\right] + \\ \frac{\Delta t^2}{2\Delta y^2}{\xi_\alpha}^2(j)\left[h_\alpha^n(i,j+1) - 2h_\alpha^n(i,j) + h_\alpha^n(i,j-1)\right] \end{aligned} \tag{3.25}$$

In Eqs.(3.23), (3.24), and (3.25), we have put

$$\xi_\alpha(i) = \boldsymbol{\xi}_\alpha \cdot \mathbf{i}, \quad \xi_\alpha(j) = \boldsymbol{\xi}_\alpha \cdot \mathbf{j}, \quad F_\alpha(i) = \mathbf{F}_\alpha \cdot \mathbf{i}, \quad F_\alpha(j) = \mathbf{F}_\alpha \cdot \mathbf{j}. \tag{3.26}$$

The second part is the collision section which is as follows:

$$\frac{\partial f_\alpha}{\partial t} = -\frac{1}{\phi}(f_\alpha(\mathbf{x}, t) - f_\alpha^{eq}(\mathbf{x}, t)), \tag{3.27}$$

$$\frac{\partial g_\alpha}{\partial t} = -\frac{1}{\phi}(g_\alpha(\mathbf{x}, t) - g_\alpha^{eq}(\mathbf{x}, t)). \tag{3.28}$$

$$\frac{\partial h_\alpha}{\partial t} = -\frac{1}{\phi}(h_\alpha(\mathbf{x}, t) - h_\alpha^{eq}(\mathbf{x}, t)). \tag{3.29}$$

Eqs.(3.27), (3.28), and (3.29) can be solved by using the Euler method and the choice of ϕ is taken as the time step (Δt). That is

$$\frac{f_\alpha(\mathbf{x}, t + \Delta t) - f_\alpha(\mathbf{x}, t)}{\Delta t} = -\frac{1}{\phi}(f_\alpha(\mathbf{x}, t) - f_\alpha^{eq}(\mathbf{x}, t)), \tag{3.30}$$

$$\frac{g_\alpha(\mathbf{x}, t + \Delta t) - g_\alpha(\mathbf{x}, t)}{\Delta t} = -\frac{1}{\phi}(g_\alpha(\mathbf{x}, t) - g_\alpha^{eq}(\mathbf{x}, t)), \tag{3.31}$$

$$\frac{h_\alpha(\mathbf{x}, t + \Delta t) - h_\alpha(\mathbf{x}, t)}{\Delta t} = -\frac{1}{\phi}(h_\alpha(\mathbf{x}, t) - h_\alpha^{eq}(\mathbf{x}, t)), \tag{3.32}$$

from which one obtains

$$f_\alpha(\mathbf{x}, t + \Delta t) = f_\alpha^{eq}(\mathbf{x}, t), \tag{3.33}$$

and

$$g_\alpha(\mathbf{x}, t + \Delta t) = g_\alpha^{eq}(\mathbf{x}, t). \tag{3.34}$$

$$h_\alpha(\mathbf{x}, t + \Delta t) = h_\alpha^{eq}(\mathbf{x}, t). \tag{3.35}$$

The numerical procedures are summarised below.

Initial stage

(a) Initial conditions for all macroscopic quantities including the boundary points are given. The initial values of $f_\alpha^{0,eq}$, $g_\alpha^{0,eq}$, and $h_\alpha^{0,eq}$ including the boundary points are determined. These are used as initial values to start the calculation.

Streaming stage

(b) With f_α, g_α, and h_α at time t (including the boundary points) known, intermediate values f_α^I, g_α^I, and h_α^I are calculated by solving Eqs.(3.23), (3.24), and (3.25) respectively.
(c) Using these f_α^I, g_α^I, and h_α^I, the corresponding macroscopic quantities $(u_I, v_I, p_I, T_I, C_I)$ for all interior grid points are calculated.
(d) The boundary conditions for the macroscopic level are then set as in any finite difference method.
(e) Using the macroscopic quantities thus determined over the complete domain including the boundary points, the corresponding $f_\alpha^{I,eq}$, $g_\alpha^{I,eq}$, and $h_\alpha^{I,eq}$ are obtained, including all of the boundary points.

Collision stage

(f) Due to Eqs.(3.33), (3.34), and (3.35) the collision step is completed by setting the new value at time $t + \Delta t$. Since each set of macroscopic quantities will map uniquely to an equilibrium distribution function and vice versa, the macroscopic quantities thus obtained are, in fact, the values at time $t + \Delta t$, i.e., $(u, v, p, T, C)_{t+\Delta t} = (u_I, v_I, p_I, T_I, C_I)$.

(g) Time marching proceeds by repeating steps (b)-(f).

4 Code validation and grid independence

Finite Difference Lattice Boltzmann Method (FDLBM) scheme is utilized to simulate laminar double diffusive natural convection in a heated enclosure with an inner cold cylinder that is filled with a Carreau fluid in the presence of Soret and Dufour parameters and the viscous dissipation in the energy equation. The prandtl number is fixed at Pr=0.1. This problem is investigated at different Rayleigh numbers of (Ra = 10^4 and 10^5), Carreau number (Cu = 1, 10, and 20), buoyancy ratio number (N = 0.1, 1, and -1), Lewis number (Le = 2.5, 5, and 10), power-law index (n = 0.2 - 1.8), Eckert number (Ec = 0, 1, and 10), the radius of the inner cylinder (R_d = 0.1 L, 0.2 L, 0.3 L, and 0.4 L), Soret parameter (S_r = 0, 1, 5), and Dufour parameter (D_f =

0, 1, 5). An extensive mesh testing procedure was conducted to guarantee a grid independent solution. Seven different mesh combinations were explored for the case of Ra = 10^5, Cu = 1, N = 0.1, Le = 2.5, n = 1.4, Ec = 0, R_d = 0.2 L, D_f = 0 and S_r = 0. The average Nusselt and Sherwood numbers on the hot wall have been studied. It was confirmed that the grid size (200*200) ensures a grid independent solution as portrayed by Table.1. The running time for the grid size (200*200) is 3851 seconds. The applied code for the fluid flow and heat transfer is validated by the study of Zhang et al. [10] in the Fig.2 at Ra=10^5 and Pr = 0.71 for the case of cooled enclosure with a heated cylinder in the center of the cavity. FDLBM is applied for double diffusive natural convection of power-law and Bingham fluids recently [47-51] which demonstrates the accuracy of the utilized code for different non-Newtonian fluids properly.

5 Results and discussion

5.1 Effects of Rayleigh number, and Power-law index on fluid flow, heat and mass transfer

Fig.3 illustrates the isotherms, isoconcentrations and streamlines for different Rayleigh numbers at Cu = 1, N = 0.1, n = 1, Le = 2.5, Ec = 0, R_d = 0.2 L, D_f = 0, and S_r = 0. As the Rayleigh number increases, the movements of the isotherms between the cold cylinders and hot walls ameliorate significantly and they become progressively curved. Moreover, the gradient of temperature on the hot wall augments with the rise of Rayleigh number. In fact, it occurs while the thermal boundary layer thickness on the side walls decreases with increasing Rayleigh number. The streamlines exhibit that the convection process has been enhanced by the growth of Rayleigh numbers as the second circulations at the bottom of the cavity at Ra = 10^4, which makes the main circulation weak, is removed at Ra = 10^5. The isoconcentration demonstrates a different manner where the increase in Rayleigh number drops the gradient of isoconcentrations on the hot walls. It depicts that mass transfer drops significantly with the rise of Rayleigh number.

Fig.4 shows the isotherms, isoconcentrations and streamlines for different power-law indexes at Cu = 1, N = 0.1, Ra = 10^5, Le = 2.5, Ec = 0, R_d = 0.2 L, D_f = 0, and S_r = 0. As the power-law index increases, the movements of the isotherms between the cold cylinders and hot walls declines significantly. Moreover, the gradient of temperature on the hot wall augments with the drop of power-law index. In fact, it occurs while the thermal boundary layer thickness on the side walls decreases with the drop of the power-law index. The streamlines exhibit that the convection process has been enhanced by the decrease of power-law index. The isoconcentration shows that the increase in power-law index drops the isoconcentration movement between the hot walls and the cold cylinder. It depicts that mass transfer drops substantially with the rise of power-law index.

Fig.5 indicates the vertical velocity, the local Nusselt and sherwood numbers on the left wall have been studied for different power-law indexes at Cu = 1, Ra = 10^5, N = 0.1, Le = 2.5, Ec = 0, R_d = 0.2 L, D_f =0, and S_r=0. Generally, it displays that the vertical velocity for different power-law indexes has the maximum values close to the sidewalls while the minimum magnitudes are observed close to the clod cylinder. The vertical velocity component is essentially negligible in the presence of the cold cylinder. The amplitudes of the vertical velocity magnitude on the top and bottom sides of the cavity do indeed drop with augmentation of power-law index regularly. The local Nusselt and sherwood numbers on the hot wall show sinuously behavior. They demonstrate that the maximum Nusselt and Sherwood numbers appear at Y = 0.2. Further, the increase in power-law index declines the local Nusselt and sherwood numbers at $Y < 0.5$ while they enhance due to the rise of the power-law index at $Y > 0.5$.

Fig.6 indicates the average Nusselt and Sherwood numbers for different power-law indexes and Rayleigh numbers at Cu = 1, N = 0.1, Le = 2.5, Ec = 0, R_d = 0.2 L, D_f = 0, and S_r = 0. It demonstrates that the average Nusselt numbers drop significantly as the power-law index enhances gradually in different Rayleigh numbers. In addition, it shows that the rise of Rayleigh number in various power-law indexes enhances the average Nusselt number, although this enhancement drops noticeably with the increase in the power-law index. The average Sherwood number declines due to the rise of power-law index for different power-law indexes. It shows that the average sherwood number at Ra=10^5 for n=0.2 and 0.4 is higher than Ra = 10^4, but at n¿0.6 the average Sherwood number is higher at Ra = 10^4 compared to the Ra = 10^5 and this trend strengthens with the rise of power-law index steadily.

5.2 Effects of Lewis number on fluid flow, heat and mass transfer

Fig.7 illustrates the isoconcentrations for different Lewis numbers and Rayleigh numbers at Cu = 1, n = 1, N = 0.1, Le = 2.5, Ec = 0, R_d = 0.2 L, D_f = 0, and S_r = 0. The contours exhibit that the movement of the isoconcentrations from the cold cylinder to the hot walls diminish with the increase in Lewis numbers for multifarious Rayleigh numbers. The pattern confirms that mass transfer declines with the rise of Lewis number considerably. Table.2 demonstrates that the average Nusselt number decreases slightly with the rise of Lewis number in the both studied Rayleigh numbers. In addition, the average Sherwood number falls considerably when the Lewis number enhances in various Rayleigh numbers.

5.3 Effects of Buoyancy ratio on fluid flow, heat and mass transfer

Fig.8 displays the isotherms, streamlines and the isoconcentrations for different buoyancy ratios at $Ra = 10^5$, n = 1, Le = 2.5, Ec = 0, $R_d = 0.2$ L, $D_f =$ 0, and $S_r = 0$. The comparison between the isotherms demonstrates the rise of the buoyancy ratio from N = -1 to 1 causes the gradient of the isotherms increase significantly. Hence, the pattern clarifies that the augmentation of buoyancy ratio enhances heat transfer. Moreover, the trend is observed in isoconcentrations as they incline to the hot wall and their gradient augments noticeably. As a result, mass transfer similar to heat transfer is improved by the increase in buoyancy ratio. The shapes of the streamlines in different buoyancy ratios can prove the cited result in the isotherms and isoconcentrations properly. At N = -1, a secondary vortex close to the cold cylinder is observed and moreover another core circulation in the streamline is generated at N=-1. In addition, it is evident that the second core circulation and the secondary circulation close to the cold cylinder disappears as buoyancy ratio enhances from N = -1 to 0.1. The main circulation becomes stronger when the buoyancy ratio increases from N = 0.1 to 1. Table.3 demonstrates that the average Nusselt and Sherwood numbers augment considerably with the enhancement of the buoyancy ratio.

5.4 Effects of Soret parameter on fluid flow, heat and mass transfer

Fig.9 presents the Soret effects on the isotherms, streamlines and the isoconcentrations at $Ra = 10^5$, n = 1, Le = 2.5, $R_d = 0.2$ L, Ec = 0, and $D_f = 0$. It is clear that there is no considerable alteration into isotherms with the addition of the Soret parameter. In fact, the marginal alteration as a result of the soret parameter on the temperature and increasing it provokes the buoyancy force to rise and augment vortexes. The phenomenon is utterly understandable while the Soret parameter is added to the concentration equations and affects this part significantly. The isoconcentrations confirm the issue properly where they change noticeably as the Soret parameter increases from $S_r = 0$ to 1. In fact, the isoconcentrations become more comfortable and the gradients of them in the cavity augment vastly. As the main force term with the alteration of concentration can influence streamlines, it is evident that the vortexes expand due to the growth of the Soret parameter. Table.4 clarifies the influence of the Soret parameter on the average Nusselt and Sherwood numbers for different Soret parameters at $Ra = 10^5$, n = 1, Le = 2.5, $R_d = 0.2$ L, Ec =

0, and $D_f = 0$. It displays that the average Nusselt number increases slightly with the enhancement of the Soret parameter in different Rayleigh numbers. The average Sherwood number increases considerably when the Soret number rises for various Rayleigh numbers.

5.5 Effects of Dufour parameter on fluid flow, heat and mass transfer

Fig.10 presents the Dufour effects on the isotherms, streamlines and the iso-concentrations at $Ra = 10^5$, n = 1, Le = 2.5, Ec = 0, R_d = 0.2 L,and S_r = 0. The addition of the Dufour parameter to the energy equation causes the isotherms to change considerably and the gradient of temperatures on the vertical walls rises. As the Dufour parameter increases from D_f = 0 to 1, two main circulations of isotherms with the values of T = 0 close to the cold cylinder are generated. The isotherms exhibit that the gradient of temperature and therefore heat transfer augments significantly with the presence of the Dufour parameter. This trends continue on the isotherms at D_f = 5 where a circulation of the isotherm of T = 1.1 is generated on the bottom of the cavity and the maximum value of T = 1.2 is appeared in the isotherms. Two secondary circulations are generated in the bottom of the cavity are generated as the Dufour number increases from D_f = 0 to 1. The rise of Dufour from D_f = 1 to 5, the secondary circulation becomes stronger and third and fourth circulations in the corners and the bottom of the cavity, respectively are created due to the rise of Dufour parameter. The isoconcentrations show that the convection of mass transfer strengthens with the augmentation of the Dufour parameter. In fact, the gradient of isoconcentrations enhances as the Dufour number rises. Table.5 clarifies the influence of the Dufour parameter on the average Nusselt and Sherwood numbers for different Rayleigh numbers at n = 1, N = 0.1, Le = 2.5, Ec = 0, R_d = 0.2 L and S_r = 0. It is clear that the average Nusselt number rockets up for different Rayleigh numbers as the Dufour parameter enhances. Further, the average sherwood number increases gradually with the rise of the Dufour parameter.

5.6 Effects of Carreau number (Cu) on fluid flow, heat and mass transfer

Fig.11 illustrates the Carreau number effects on the isotherms, streamlines and the isoconcentrations at $Ra = 10^5$, n = 1.4, Le = 2.5, Ec = 0, N = 0.1, R_d = 0.2 L, and $S_r = D_f = 0$. The increase in the Carreau number causes the isotherms to change slightly and the gradient of temperatures to drop. As the Carreau number increases from Cu = 1 to 10, the core of the streamline circulation is broken two cores and demonstrates the drop of the convection process. This trend continues at Cu = 20, where the streamline weakens. The isoconcentrations show that the convection of mass transfer becomes weak with the augmentation of the Carreau number. In fact, the gradient of isoconcentrations declines as the Carreau number rises. Table.6 clarifies the influence of the Carreau number on the total average Nusselt and Sherwood numbers for different Rayleigh numbers at n = 1.4, N = 0.1, Le = 2.5, Ec = 0, and $S_r = D_f = 0$. It is clear that the average Nusselt number decreases marginally for different Rayleigh numbers as the Carreau number enhances. Further, the average sherwood number drops gradually with the rise of the Carreau number.

5.7 Effects of Eckert number on fluid flow, heat and mass transfer

Fig.12 shows the Eckert number effects on the isotherms, streamlines, and isoconcentrations at Ra = 10^5, n = 1, Le = 2.5, N = 0.1, R_d = 0.2 L, $D_f = 0$, and $S_r = 0$. It demonstrates that the curve shape of the isotherms increases slightly as the Eckert number increases from Ec = 0 to 1 and therefore it results in increase of heat transfer. When the Eckert number increases from Ec = 1 to 10, two main circulations close to sidewalls are generated in the isotherms withe values of T = 1 and 1.1 which demonstrates the rise of heat transfer considerably. The effect of Eckert number on the streamline was demonstrated by the form and the strength of the streamlines central circulation. As the Eckert number increases, the maximum value of the streamline enhances and the width of the streamline to improves by increasing in the Eckert number. The figure shows that the isoconcentrations display different behavior where moves toward the heated walls in a slower trend as the Eckert number enhances. Table.7 exhibits that the influence of the Eckert number on the total average Nusselt and Sherwood numbers for different Rayleigh numbers at n = 1, N = 0.1, Le = 2.5, Ec = 0, and $S_r = D_f = 0$. It is clear that the average Nusselt number rises marginally at Ra = 10^4, but it augments considerably at Ra = 10^5. The average sherwood number increases barely due to the increase in the Eckert number at Ra = 10^4, but; it declines substantially at Ra = 10^5.

5.8 Effects of the vertical distance of the cylinder from the center on fluid flow, heat and mass transfer

Fig.13 shows the vertical distance of the cylinder from the center (δ) on the isotherms, streamlines, and isoconcentrations at Ra = 10^5, n= 1, Le = 2.5, R_d = 0.2 L, Ec = 0, N = 0.1, $D_f = 0$, and $S_r = 0$. It demonstrates that the curve shape of the isotherms increases significantly as the cylinder position changes from δ =0 to -0.2 L and therefore it results in increase of heat transfer. But, the gradient of isotherms declines slightly from δ =0 to 0.2 L. The effect of the vertical position on the streamline was demonstrated by the form and the strength of the streamlines circulations. As the cylinder moves toward the bottom side of the enclosure, another small circulation close to the bottom wall is generated in the same direction of the main vortex. Hence, this second small circulation improve the convection process. The figure shows that the isoconcentrations behave the same pattern as the isotherms move toward the heated walls in a slower trend as the δ enhances. Table.8 exhibits that the influence of the vertical position on the total average Nusselt and Sherwood numbers for Ra=10^5 at n = 1, N = 0.1, Le = 2.5, Ec = 0, and $S_r = D_f = 0$. It is clear that the average Nusselt and sherwood numbers drop substantially as the vertical position increases from δ = -0.2 L to 0.2 L. This table also indicates that the increase in the power-law index causes the average Nusselt and Sherwood numbers to drop in different positions; however, the effect of power-law index is different in various vertical positions. In fact, it demonstrates that the influence of power-law index on the average Nusselt and Sherwood numbers drops gradually when the cylinder moves from δ = -0.2 L to 0.2 L.

5.9 Effects of the radius of the inner cylinder on fluid flow, heat and mass transfer

Fig.14 shows the radius of the inner cylinder (R_d) on the isotherms, streamlines, and isoconcentrations at Ra = 10^5, n = 1, Le = 2.5, Ec = 0, N = 0.1, D_f = 0, and S_r = 0. It demonstrates that the gradient of isotherms on the cylinder increases considerably as the radius of the cylinder augments. This trend confirms that the the convection process improves as the radius of the cylinder rises. When the radius increases from R_d = 0.1 L to 0.3 L, two main circulations in the core of the main vortex are generated in the streamlines which demonstrates the rise of the convection considerably. The figure shows that the isoconcentrations concentrations augment considerably with the rise of the cylinder radius. Table.9 exhibits that the effect of the cylinder radius on the total average Nusselt and Sherwood numbers at Ra = 10^5, n = 1, N = 0.1, Le = 2.5, Ec = 0, and $S_r = D_f = 0$. It is clear that the average Nusselt and Sherwood numbers enhance significantly as the radius of the cylinder augments in different power-law indexes. For different radii, the increase in power-law index declines the average Nusselt and Sherwood numbers steadily.

5.10 Effects of the horizontal distance of the cylinder on fluid flow, heat and mass transfer

Fig.15 shows the horizontal distance of the cylinder from the center (Ω) on the isotherms, streamlines, and isoconcentrations at Ra = 10^5, n = 1, Le =

2.5, Ec = 0, N = 0.1, R_d = 0.2 L, D_f = 0, and S_r = 0. It demonstrates that the gradient of isotherms on the cylinder increases when the cylinder moves to the left and right sides horizontally. The streamlines demonstrate that two small circulations in the corners in the opposite directions of the main vortex are generated. The figure displays that the isoconcentrations shows the same behavior as the isotherms where their concentration on the cylinder augments with the movement of the cylinder horizontally. Table.10 exhibits that the influence of the horizontal position on the total average Nusselt and Sherwood numbers for Ra=10^5 at n=1, N=0.1, Le=2.5, Ec=0, and $S_r = D_f = 0$. It is clear that the average Nusselt and sherwood numbers drop as the horizontal position moves from the center. This table also indicates that the increase in the power-law index causes the average Nusselt and Sherwood numbers to drop in different positions. In addition, the highest effect of the power-law index among the studied cases in the horizontal distances is observed at $\Omega =$ 0.

6 Concluding Remarks

Double diffusive natural convection of Carreau fluid in a heated enclosure with an inner cold cylinder in the presence of Soret and Dufour parameters as well as viscous dissipation has been analyzed by Finite Difference Lattice Boltzmann method (FDLBM). This study has been conducted for the pertinent parameters in the following ranges: Rayleigh number (Ra = 10^4 and 10^5), Carreau number (Cu = 1, 10, and 20), Lewis number (Le=2.5, 5 and 10), Dufour parameter (D_f=0, 1, and 5), Soret parameter (S_r=0, 1, and 5), Eckert number (Ec=0, 1, and 10), the Buoyancy ratio (N=-1, 0.1, 1), the radius of the inner cylinder (R_d = 0.1 L, 0.2 L, 0.3 L, and 0.4 L), the horizontal distance of the circular cylinder from the center of the enclosure (Ω = -0.2 L, 0 and 0.2 L), and the vertical distance of the circular cylinder from the center of the enclosure (δ = -0.2 L, 0 and 0.2 L). The main conclusions of the present investigation can be summarized as follows:

- Heat and mass transfer enhances with augmentation of Rayleigh number in different studied parameters.
- The average Nusselt and Sherwood numbers demonstrate that the heat and mass transfer decline with the rise of the power-law index in various studied parameters.
- The increase in Rayleigh number causes the effect of power-law index to decline.
- It was found that the rise of Lewis number declines the mass transfer substantially, but provokes the heat transfer to drop marginally.
- The enhancement of the buoyancy ratio increases heat and mass transfer considerably.

- Generally, the heat transfer increases with the rise of the Dufour parameter significantly while the mass transfer rises marginally.
- The mass transfer enhances substantially as the Soret parameter increases for different Rayleigh numbers.
- The enhancement of the Carreau number declines heat and mass transfer slightly.
- The Eckert number has a marginal effect on Ra = 10^4 while the rise of Eckert number at Ra = 10^5 declines heat and mass transfer steadily.
- The vertical movement of the cylinder from the bottom of the cavity to the top side declines heat and mass transfer.
- It was found that the effect of power-law index drops gradually as the vertical position rises from δ = -0.2 L to 0.2 L.
- The enhancement of the cylinder radius increases heat and mass transfer considerably.
- It was observed that the highest effect of power-law index occurs at R_d = 0.2 L while the least impact is obtained at R_d = 0.4 L.
- The movement of the cylinder horizontally in the positive and negative directions enhance heat and mass transfer. It was observed that the positive and negative horizontal movement in the same distance (Ω = -0.2 L and 0.2 L) has nearly the same effect on heat and mass transfer.
- The heat and mass transfer declines gradually as the power-law index enhances for different horizontal positions. However, it shows that the highest effect of the power-law indexes on heat and mass transfer is obtained in the center of the enclosure compared to other horizontal positions.

References

[1] G. de Vahl Davis, Natural convection of air in a square cavity: a bench mark numerical solution, Int. J. Numer. Meth. Fluids., 3 (1983) 249-264.

[2] P. L. Quere, Accurate solutions to the square thermally driven cavity at high Rayleigh number, Computers and Fluids, 20 (1991) 29-41.

[3] P. L. Quere, T. A. de Roquefort, Computation of natural convection in two dimensional cavities with Chebyshev polynomials. J. Comput. Phys., 57 (1985) 210-228.

[4] B.S. Kim, D.S. Lee, M.Y. Ha, H.S. Yoon, A numerical study of natural convection in a square enclosure with a circular cylinder at different vertical locations, International Journal of Heat and Mass Transfer 51 (2008) 1888-1906.

[5] A. A. Mehrizi, K. Sedighi, M. Farhadi, M. Sheikholeslami, Lattice Boltzmann simulation of natural convection heat transfer in an elliptical-triangular annulus, International Communications in Heat and Mass Transfer 48 (2013) 164-177.

[6] Yong Gap Park, Man Yeong Ha, Changyoung Choi, Jaehyun Park, Natural convection in a square enclosure with two inner circular cylinders positioned at different vertical locations, International Journal of Heat and Mass Transfer 77 (2014) 501-518.

[7] A. A. Mehrizi, A. A. Mohamad, Effect of the Inclination Angle and Eccentricity on Free Convection Heat Transfer in EllipticalTriangular Annuli: A Lattice Boltzmann Approach, Numerical Heat Transfer, Part A: Applications 68 (2015) 17-43.

[8] Gi Su Mun, Jeong Hoon Doo, Man Yeong Ha, Thermo-dynamic irreversibility induced by natural convection in square enclosure with inner cylinder. Part-I: Effect of tilted angle of enclosure, International Journal of Heat and Mass Transfer 97 (2016) 1102-1119.

[9] Young Min Seo, Yong Gap Park, Minsung Kim, Hyun Sik Yoon, Man Yeong Ha, Two-dimensional flow instability induced by natural convection in a square enclosure with four inner cylinders. Part I: Effect of horizontal position of inner cylinders, International Journal of Heat and Mass Transfer 113 (2017) 1306-1318.

[10] Ping Zhang, Xiaohua Zhang, Jiheng Deng, Laizhong Song, A numerical study of natural convection in an inclined square enclosure with an elliptic cylinder using variational multiscale element free Galerkin method, International Journal of Heat and Mass Transfer 99 (2016) 721-737.

[11] Osman Turan, Nilanjan Chakraborty, Robert J. Poole, Laminar natural convection of Bingham fluids in a square enclosure with differentially heated side walls, J. Non-Newtonian Fluid Mech. 165 (2010) 901-913.

[12] GH. R. Kefayati, Simulation of non-Newtonian molten polymer on natural convection in a sinusoidal heated cavity using FDLBM, Journal of Molecular Liquids 195 (2014) 165-174.

[13] Osman Turan, Robert J. Poole, Nilanjan Chakraborty, Aspect ratio effects in laminar natural convection of Bingham fluids in rectangular enclosures with differentially heated side walls. J. Non-Newt. Fluid Mech., 166 (2011) 208-230.

[14] Osman Turan, Nilanjan Chakraborty, Robert J. Poole, Laminar Rayleigh-Benard convection of yield stress fluids in a square enclosure. J. Non-Newt. Fluid Mech., 171-172 (2012) 83-96.

[15] R.R. Huilgol, G.H.R. Kefayati, Natural convection problem in a Bingham fluid using the operator-splitting method, Journal of Non-Newtonian Fluid Mechanics 220 (2015) 22-32.

[16] I. Karimfazli, I. A. Frigaard, A. Wachs, A novel heat transfer switch using the yield stress, Journal of Fluid Mechanics 783 (2015) 526-566.

[17] G.B. Kim, J.M. Hyun, H.S. Kwak, Transient buoyant convection of a power law non-Newtonian fluid in an enclosure, International Journal of Heat and Mass Transfer 46 (2003) 3605-3617.

[18] M. Lamsaadi, M. Naimi , M. Hasnaoui, M. Mamou, Natural convection in a vertical rectangular cavity filled with a non- Newtonian Power law Fluid and subjected to a horizontal temperature gradient, Numerical Heat Transfer, Part A 49 (2006) 969-990.

[19] O. Turan, A. Sachdeva, N. Chakraborty, R. J. Poole, Laminar natural convection of power-law fluids in a square enclosure with differentially heated side walls subjected to constant temperatures, Journal of Non-Newtonian Fluid Mechanics 166 (2011) 1049-1063.

[20] L. Khezzar, D. Siginer, I. Vinogradov, Natural convection of power law fluids in inclined cavities, International Journal of Thermal Sciences 53 (2012) 8-17.

[21] M. H. Matin, I. Pop, S. Khanchezar, Natural convection of power-law fluid between two-square eccentric duct annuli, Journal of Non-Newtonian Fluid Mechanics 197 (2013) 11-23.

[22] P. J. Carreau, Rheological equations from molecular network theories, Trans Soc Rheol 16 (1972) 99-127.

[23] A. Shamekhi, K. Sadeghy, Cavity flow simulation of Carreau-Yasuda non-Newtonian fluids using PIM meshfree method, Applied Mathematical Modelling 33 (2009) 4131-4145.

[24] M. Bouteraa, C. Nouar1, E. Plaut, C. Metivier, A. Kalck, Weakly nonlinear analysis of Rayleigh Benard convection in shear-thinning fluids: nature of the bifurcation and pattern selection, J. Fluid Mech. 767 (2015) 696-734.

[25] S. Shahsavari, G. H. McKinley, Mobility of Power-law and Carreau Fluids through Fibrous Media, Physical review E, Statistical, nonlinear, and soft matter physics 92 (2015) 063012.

[26] A. Pantokratoras, Steady flow of a non-Newtonian Carreau fluid across an unconfined circular cylinder, Meccanica 51 (2016) 1007-1016.

[27] GH. R. Kefayati, SF Hosseinizadeh, M Gorji, H Sajjadi, Lattice Boltzmann simulation of natural convection in an open enclosure subjugated to water/copper nanofluid, International Journal of Thermal Sciences 52 (2012) 91-101.

[28] A. A. Mehrizi, M. Farhadi, S. Shayamehr, Natural convection flow of Cu-Water nanofluid in horizontal cylindrical annuli with inner triangular cylinder using lattice Boltzmann method, International Communications in Heat and Mass Transfer 44 (2013) 147-156.

[29] A. A. Mehrizi, M. Farhadi, H. H. Afroozi, S. Shayamehr, Lattice Boltzmann simulation of natural convection flow around a horizontal cylinder located beneath an insulation plate, Journal of Theoretical and Applied Mechanics 51 (2013) 729-739.

[30] GH. R. Kefayati, Effect of a magnetic field on natural convection in an open cavity subjugated to Water/Alumina nanofluid using lattice Boltzmann method International Communications in Heat and Mass Transfer 40 (2013) 67-77.

[31] GH. R. Kefayati, Simulation of ferrofluid heat dissipation effect on natural convection at an inclined cavity filled with kerosene/cobalt utilizing the Lattice Boltzmann method, Numerical Heat Transfer, Part A: Applications 65 (2014) 509-530.

[32] A. A. Mehrizi, M. Farhadi, K. Sedighi, M. A. Delavar, Effect of fin position and porosity on heat transfer improvement in a plate porous media heat exchanger, Journal of the Taiwan Institute of Chemical Engineers 44 (2013) 420-431.

[33] H. R. Ashorynejad, A. A. Mohamad, M. Sheikholeslami, Magnetic field effects on natural convection flow of a nanofluid in a horizontal cylindrical annulus using Lattice Boltzmann method, Int. J. Therm. Sci. 64 (2013) 240-250.

[34] H. Sajjadi, M. Gorji, G.H.R. Kefayati, D.D. Ganji, Lattice Boltzmann simulation of turbulent natural convection in tall enclosures using Cu/Water nanofluid, Numer. Heat Transf., Part A 62 (2012) 512-530.

[35] H. R. Ashorynejad, B. Hoseinpour, Investigation of different nanofluids effect on entropy generation on natural convection in a porous cavity, European Journal of Mechanics B/Fluids 62 (2017) 86-93.

[36] H. Sajjadi, M. Salmanzadeh, G. Ahmadi, S. Jafari, Turbulent indoor airflow simulation using hybrid LES/RANS model utilizing Lattice Boltzmann method, Computers and Fluids 150 (2017) 66-73.

[37] GH. R. Kefayati, Double-diffusive mixed convection of pseudoplastic fluids in a two sided lid-driven cavity using FDLBM, Journal of the Taiwan Institute of Chemical Engineers 45 (2014) 2122-2139.

[38] H. R. Ashorynejad, K. Javaherdeh, Investigation of a waveform cathode channel on the performance of a PEM fuel cell by means of a pore-scale multi-component lattice Boltzmann method, Journal of the Taiwan Institute of Chemical Engineers, 66 (2016) 126-136.

[39] GH. R. Kefayati, Mesoscopic simulation of magnetic field effect on natural convection of power-law fluids in a partially heated cavity, Chemical Engineering Research and Design 94 (2015) 337-354.

[40] H. Sajjadi, M. Salmanzadeh, G. Ahmadi, S. Jafari, Simulations of indoor airflow and particle dispersion and deposition by the lattice Boltzmann method using LES and RANS approaches, Building and Environment 102 (2016) 1-12.

[41] GH. R. Kefayati, Simulation of magnetic field effect on non-Newtonian blood flow between two-square concentric duct annuli using FDLBM, Journal of the Taiwan Institute of Chemical Engineers 45 (2014) 1184-1196.

[42] K. Bamdad, H. R. Ashorynejad, Inverse analysis of a rectangular fin using the lattice Boltzmann method, Energy Conversion and Management, 97 (2015) 290-297.

[43] S.C. Fu, W.W.F. Leung, R.M.C. So, A Lattice Boltzmann Based Numerical Scheme for Microchannel Flows, Journal of Fluids Engineering, 131 (2009) 081401-081412.

[44] S.C. Fu, R.M.C. So, W.W.F. Leung, Linearized-Boltzmann-type-equation-based finite difference method for thermal incompressible flow, Computers and Fluids 69 (2012) 67-80.

[45] R. R. Huilgol, GH. R. Kefayati, From mesoscopic models to continuum mechanics: Newtonian and non-Newtonian fluids, Journal of Non-Newtonian Fluid Mechanics 233 (2016) 146-154.

[46] R. R. Huilgol, GH. R. Kefayati, A particle distribution function approach to the equations of continuum mechanics in Cartesian, cylindrical and spherical coordinates: Newtonian and non-Newtonian Fluids, Journal of Non-Newtonian Fluid Mechanics (2017) (Accepted).

[47] GH. R. Kefayati, FDLBM simulation of entropy generation in double diffusive natural convection of power-law fluids in an enclosure with Soret and Dufour effects, International Journal of Heat and Mass Transfer 89 (2015) 267-290.

[48] GH. R. Kefayati, Simulation of double diffusive natural convection and entropy generation of power-law fluids in an inclined porous cavity with Soret and Dufour effects (Part I: Study of fluid flow, heat and mass transfer), International Journal of Heat and Mass Transfer 94 (2016) 539-581.

[49] GH. R. Kefayati, Simulation of double diffusive natural convection and entropy generation of power-law fluids in an inclined porous cavity with Soret and Dufour effects (Part II: Entropy generation), International Journal of Heat and Mass Transfer 94 (2016) 582-624.

[50] GH. R. Kefayati, R.R. Huilgol, Lattice Boltzmann Method for simulation of mixed convection of a Bingham fluid in a lid-driven cavity, International Journal of Heat and Mass Transfer 103 (2016) 725-743.

[51] GH. R. Kefayati, Double-diffusive natural convection and entropy generation of Bingham fluid in an inclined cavity, International Journal of Heat and Mass Transfer 116 (2018) 762-812.

[52] M. Hatami, D. Song, D. Jing, Optimization of a circular-wavy cavity filled by nanofluid under the natural convection heat transfer condition, International Journal of Heat and Mass Transfer 98 (2016) 758-767.

[53] J. Zhou, M. Hatami, D. Song, D. Jing, Design of microchannel heat sink with wavy channel and its time-efficient optimization with combined RSM and FVM methods, International Journal of Heat and Mass Transfer 103 (2016) 715-724.

[54] W. Tang, M. Hatami, J. Zhou, D. Jing, Natural convection heat transfer in a nanofluid-filled cavity with double sinusoidal wavy walls of various phase deviations, International Journal of Heat and Mass Transfer 115 (2017) 430-440.

[55] M. Hatami, Nanoparticles migration around the heated cylinder during the RSM optimization of a wavy-wall enclosure, Advanced Powder Technology 28 (2017) 890-899.

[56] M. Hatami, D. Jing, Optimization of wavy direct absorber solar collector (WDASC) using Al2O3-water nanofluid and RSM analysis, Applied Thermal Engineering 121 (2017) 1040-1050.

[57] M. Hatami, J. Zhou, J. Geng, D. Song, D. Jing, Optimization of a lid-driven T-shaped porous cavity to improve the nanofluids mixed convection heat transfer, Journal of Molecular Liquids 231 (2017) 620-631.

[58] R. B. Bird, R. Amstrong, O. Hassager, 1987 Dynamics of Polymeric Liquids. Wiley-Interscience.

[59] R. Tanner, 2000 Engineering Rheology. Oxford University Press.

Nomenclature

b Body force
C Concentration
c Lattice speed
c_p Specific heat capacity at constant pressure
Cu Carreau number
D Mass diffusivity
D_f Dufour parameter
E Eckert number
F External forces
f_α Density distribution functions for the specific node of α
f_α^{eq} Equilibrium density distribution functions for the specific node of α
g_α Internal energy distribution functions for the specific node of α
g_α^{eq} Equilibrium internal energy distribution functions for the specific node of α
g Gravity
h_α Internal concentration distribution functions for the specific node of α
h_α^{eq} Equilibrium internal concentration distribution functions for the specific node of α
k Thermal conductivity
K_{TC} Thermodiffusion coefficient
K_{CT} Diffusionthermo coefficient
L Length of the cavity
Le Lewis number
n Power-law index
N Buoyancy ratio
Nu Nusselt number
p Pressure
Pr Prandtl number
R Gas constant
Ra Rayleigh number
R_d Radius of the inner circular cylinder
S Rate of strain tensor
Sh Sherwood number
S_r Soret parameter
T Temperature
t Time
x, y Cartesian coordinates
x_c, y_c The horizental and vertical positions of the cylinder center
u Velocity in x direction
v Velocity in y direction

Greek letters

β_T Thermal expansion coefficient
β_C Solutal expansion coefficient
ϕ Relaxation time
τ Shear stress
ξ Discrete particle speeds
Δx Lattice spacing
Δt Time increment
α Thermal diffusivity
ρ Density of fluid
η Dynamic viscosity
η_0 Zero shear viscosity
η_∞ Infinite shear viscosity
ψ Stream function value
λ Time constant

Subscripts

avg Average
B Bottom
C Cold
c Center
d Dynamic
H Hot
L Left
x, y Cartesian coordinates
α Specific node
R Right
s Static
T Thermal, Top
tot Total
D Solutal

Table 1
Grid independence study at $Ra = 10^5$, Ec=0, Le = 2.5, Cu = 1, n=1.4, $R_d = 0.2$ L, S_r=D_f=0, and N=0.1

Mesh size	Nu_{avg}	Sh_{avg}
150*150	7.25302	5.25061
160*160	6.98147	5.0261
170*170	6.83729	4.92607
180*180	6.53107	4.82507
190*190	6.48082	4.68931
200*200	6.46430	4.65322
210*210	6.46430	4.65322

Table 2
Effects of the Lewis number (Le) in different Rayleigh numbers on the average Nusselt and Sherwood numbers at Ec=0, R_d = 0.2 L, n = 1, $S_r = D_f$ = 0, and N = 0.1

	Le = 2.5	Le = 5	Le = 10
Ra = 10^4			
Nu_{avg}	6.32672	6.32557	6.32372
Sh_{avg}	6.12838	4.71141	2.40221
Ra = 10^5			
Nu_{avg}	6.74342	6.69025	6.631487
Sh_{avg}	5.22323	2.90277	0.84160

Table 3
Effects of the Buoyancy ratio (N) on Nusselt and Sherwood numbers at Ec = 0, R_d = 0.2 L, n = 1, $S_r = D_f = 0$, and Le = 2.5

	N = -1	N = 0.1	N = 1
Ra = 10^4			
Nu_{avg}	6.24547	6.32672	6.50048
Sh_{avg}	5.70908	6.12838	6.95650
Ra = 10^5			
Nu_{avg}	5.26790	6.74342	8.42131
Sh_{avg}	2.12484	5.22323	9.06288

Table 4
Effects of the Soret Parameter (S_r) in different Rayleigh numbers on the average Nusselt and Sherwood numbers at Ec = 0, n = 1, R_d = 0.2 L, D_f = 0, Le = 2.5 and N = 0.1

	$S_r = 0$	$S_r = 1$	$S_r = 5$
Ra = 10^4			
Nu_{avg}	6.32672	6.32715	6.32907
Sh_{avg}	6.12838	12.5888	38.41383
Ra = 10^5			
Nu_{avg}	6.74342	6.77066	6.885083
Sh_{avg}	5.22323	13.04610	44.45110

Table 5
Effects of the Dufour Parameter (D_f) in different Rayleigh numbers on the average Nusselt and Sherwood numbers at Ec = 0, n = 1, $R_d = 0.2$ L, $S_r = 0$, Le = 2.5 and $N = 0.1$

	$D_f = 0$	$D_f = 1$	$D_f = 5$
Ra = 10^4			
Nu_{avg}	6.32672	12.7761	38.1211
Sh_{avg}	6.12838	6.2104	7.18031
Ra = 10^5			
Nu_{avg}	6.74342	15.6556	64.2592
Sh_{avg}	5.22323	7.002	9.7643

Table 6
Effects of the Carreau number (Cu) in different Rayleigh numbers on the average Nusselt and Sherwood numbers at n = 1,$D_f = S_r = 0$, $R_d = 0.2$ L, Le = 2.5 and $N = 0.1$

	Cu = 1	Cu = 10	Cu = 20
Ra = 10^4			
Nu_{avg}	6.3136	6.2747	6.2661
Sh_{avg}	6.0684	5.8736	5.82703
Ra = 10^5			
Nu_{avg}	6.4643	5.9135	5.7678
Sh_{avg}	4.65322	3.5135	3.2108

Table 7
Effects of the Eckert number (Ec) in different Rayleigh numbers on the average Nusselt and Sherwood numbers at n = 1,D_f= S_r = 0, R_d = 0.2 L, Le = 2.5 and N = 0.1

	Ec = 0	Ec = 1	Ec = 10
Ra = 10^4			
Nu_{avg}	6.32672	6.32613	6.32311
Sh_{avg}	6.12838	6.12871	6.13194
Ra = 10^5			
Nu_{avg}	6.74342	6.7993	7.77011
Sh_{avg}	5.22323	5.18179	4.9484

Table 8
Effects of the vertical distance of the cylinder from the center (δ) in different Rayleigh numbers on the average Nusselt and Sherwood numbers at Ra = 10^5, Ec = 0, n = 1, R_d = 0.2 L, $D_f = S_r$ = 0, Le = 2.5 and N = 0.1

	δ = -0.2 L	$\delta = 0$	δ = 0.2 L
Nu_{avg}			
n = 0.2	9.5758	7.6841	6.7109
n = 1	8.7147	6.7434	6.4147
n = 1.8	8.2681	6.2813	6.3089
Sh_{avg}			
n = 0.2	11.8797	7.3928	3.5086
n = 1	9.7887	5.2232	3.3512
n = 1.8	8.6585	4.2871	3.3308

Table 9
Effect of the radius of the cylinder (R_d) on the isotherms, isoconcentrations, and streamlines at Ra = 10^5, Ec = 0, n = 1, $D_f = S_r = 0$, Le = 2.5 and N = 0.1

	R_d = 0.1 L	R_d = 0.2 L	R_d = 0.3 L	R_d = 0.4 L
Nu_{avg}				
n = 0.2	4.6290	7.6841	11.6423	21.0487
n = 1	3.9883	6.7434	10.9166	20.9642
n = 1.8	3.5541	6.2813	10.7467	20.9260
Sh_{avg}				
n = 0.2	4.0953	7.3928	11.4780	21.2673
n = 1	3.0081	5.2232	9.4212	20.9494
n = 1.8	2.3491	4.2871	8.9834	20.7817

Table 10
Effects of the horizontal distance of the cylinder from the center (Ω) in different Rayleigh numbers on the average Nusselt and Sherwood numbers at Ra = 10^5, Ec = 0, n = 1, D_f = S_r = 0, R_d = 0.2 L, Le = 2.5 and N = 0.1

	Ω = -0.2 L	Ω = 0	Ω = 0.2 L
Nu_{avg}			
n = 0.2	8.6068	7.6841	8.5912
n = 1	8.1150	6.7434	8.1148
n = 1.8	7.7990	6.2813	7.8024
Sh_{avg}			
n = 0.2	7.8309	7.3928	7.8179
n = 1	7.0087	5.2232	7.0157
n = 1.8	6.5602	4.2871	6.5678