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Double-diffusive laminar natural convection and entropy generation of Carreau fluid in a heated enclosure with an inner circular cold cylinder (Part II: Entropy generation)

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Abstract

In this paper, entropy generation of double-diffusive natural convection, studying Soret and Dufour effects and viscous dissipation in a heated enclosure with an inner cold cylinder filled with non-Newtonian Carreau fluid has been simulated by Finite Difference Lattice Boltzmann Method (FDLBM). This study has been conducted for certain pertinent parameters of Rayleigh number (Ra = 10^4 and 10^5), Carreau number (Cu = 1, 10, and 20), Lewis number (Le=2.5, 5 and 10), Dufour parameter (D_f=0, 1, and 5), Soret parameter (S_r=0, 1, and 5), Eckert number (Ec=0, 1, and 10), the Buoyancy ratio (N=-1, 0.1, 1), the radius of the inner cylinder (R_d = 0.1 L, 0.2 L, 0.3 L, and 0.4 L), the horizontal distance of the circular cylinder from the center of the enclosure (Ω = -0.2 L, 0 and 0.2 L), the vertical distance of the circular cylinder from the center of the enclosure (δ = -0.2 L, 0 and 0.2 L). Results indicate that the augmentation of Rayleigh number enhances different entropy generations and declines the average Bejan number. The increase in the power-law index provokes various irreversibilities to drop significantly. The rise of Soret and Dufour parameters enhance the entropy generations due to heat transfer and fluid friction. The rise of Eckert number enhances the summation entropy generations. The increase in Lewis number augments the total summation entropy generations gradually. The enhancement of the buoyancy ratio causes the summation entropy generations to increase considerably. The rise of Carreau number declines the total entropy generation gradually. The least value of the total entropy generation in the vertical position of the cylinder occurs at δ = - 0.2 L. The increase in the size of the cylinder augments the total entropy generation substantially. The minimum values of the total entropy generations are observed in the center position (δ = 0) in different horizontal positions.

Key words: Entropy, Carreau fluid, Natural convection, Mass transfer, Viscous, LBM

1 Introduction

Natural convection flow of a Newtonian fluid has been studied immensely by researchers [1-3] due to its wide applications e.g. nuclear energy, double pane windows, heating and cooling of buildings, solar collectors, electronic cooling, etc. Many studies have conducted the effect of the presence of a hot or cold body inside the enclosure on the natural convection phenomena and focused on the diverse body shapes such as circular, square and triangular cylinders [4-9]. Natural convection of non-Newtonian power-law fluids and Bingham fluids recently have been studied by some researchers [10-17]. However, natural convection of Carreau fluids in an enclosure have not been considered thus far. Carreau fluid is a special sub-class of non-Newtonian fluids in which follows the Carreau model [18]. This model was introduced in 1972 and is used extensively up to date. Carreau models have been employed to simulate various chemical, metal, molten plastics, slurries, paints, blood, etc. Some isothermal and non-isotermal problems have been studied [19-22]. The optimal design of the cited industries is obtained with precision calculation of entropy generation since it clarifies energy losses in a system evidently. Entropy generation on natural convection has been scrutinized in some researches. Ilis et al. [23] investigated entropy generation in rectangular cavities with different aspect ratios numerically. It was demonstrated that heat transfer and fluid friction irreversibility in a cavity vary considerably with the studied aspect ratios. In addition, the total entropy generation in a cavity increases with Rayleigh number, however, the rate of increase depends on the aspect ratio. El-Maghlany et al. [24] analyzed entropy generation associated with laminar natural convection in an infinite square cavity, subjected to an isotropic heat field with various intensities for different Rayleigh numbers. Mun et al.[25] studied entropy generation of natural convection in square enclosure with an inner cylinder. They scrutinized the simulations for different Rayleigh numbers, inclined angles, and Prandtl numbers. Doo et al.[26] analyzed entropy generation of natural convection in square enclosure with inner cylinder. They scrutinized the simulations for different Rayleigh numbers, the vertical position of inner cylinder, and Prandtl numbers.

Lattice Boltzmann method (LBM) has been demonstrated to be a very effective mesoscopic numerical method to model a broad variety of complex fluid flow phenomena [27-42]. This is because the main equation of the LBM is hyperbolic and can be solved locally, explicitly, and efficiently on parallel computers. However, the specific relation between the relaxation time and

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the viscosity has caused LBM not to have the considerable success in non-Newtonian fluid especially on energy equations. In this connection, Fu et al. [43-44] proposed a new equation for the equilibrium distribution function, modifying the LB model. Here, this equilibrium distribution function is altered in different directions and nodes while the relaxation time is fixed. Independency of the method to the relaxation time in contrast with common LBM provokes the method to solve different non-Newtonian fluid energy equations successfully as the method protects the positive points of LBM simultaneously. In addition, the validation of the method and its mesh independency demonstrates that is more capable than conventional LBM. Huilgol and Kefayati [45] derived the three dimensional equations of continuum mechanics for this method and demonstrated that the theoretical development can be applied to all fluids, whether they be Newtonian, or power law fluids, or viscoelastic and viscoplastic fluids. Following the study, Huilgol and Kefayati [46] developed this method for the cartesian, cylindrical and spherical coordinates. Kefayati [47] simulated double-diffusive natural convection with Soret and Dufour effects in a square cavity filled with non-Newtonian power-law fluid by FDLBM while entropy generations through fluid friction, heat transfer, and mass transfer were analysed. Kefayati [48-49] analysed double diffusive natural convection and entropy generation of non-Newtonian power-law fluids in an inclined porous cavity in the presence of Soret and Dufour parameters by FDLBM. Kefayati and Huilgol [50] conducted a two-dimensional simulation of steady mixed convection in a square enclosure with differentially heated sidewalls when the enclosure is filled with a Bingham fluid, using FDLBM. The problem was solved by the Bingham model without any regularisations and also by applying the regularised Papanatasiou model. Kefayati [51] simulated double-diffusive natural convection, studying Soret and Dufour effects and viscous dissipation in a square cavity filled with Bingham fluid by FDLBM. In addition, entropy generations through fluid friction, heat transfer, and mass transfer were studied. The problem was solved by applying the regularised Papanastasiou model.

The main aim of this study is to simulate entropy generation of double diffusive natural convection of Carreau fluid in a heated enclosure with an inner cold cylinder. The innovation of this paper is studying entropy generations in the presence of Soret and Dufour and the viscous dissipation effect on Carreau fluid for the first time. An innovative method based on LBM has been employed to study the problem numerically. Moreover, it is endeavored to express the effects of different parameters on entropy generations. The obtained results are validated with previous numerical investigations and the effects of the main parameters (Rayleigh number, Lewis number, buoyancy ratio number, Eckert number, Carreau number, Soret parameter, and Dufour parameter) are researched.

2 Theoretical formulation

The geometry of the present problem is shown in Fig. 1. The temperature and concentration of the enclosure walls have been considered to be maintained at high temperature and concentration of T_H and C_H as the circular cylinder is kept at low temperature and concentration of T_C and C_C. The lengths of the enclosure sidewalls are L where the inner cylinder center is defined by (x_c, y_c) and the radius of the cylinder is specified by R_d. The origin of Cartesian coordinates is located in the center of the cavity as depicted in the Fig.1. For the concentric cases, the cylinder center is fixed at $(x_c = 0, y_c = 0)$ in the center of the cavity. For the eccentric cases, the horizontal and vertical distances from the center are defined by Ω and δ, respectively. The cavity is filled with a Carreau fluid. The prandtl number is fixed at Pr=0.1. The Soret, and Dufour parameters also have been considered. There is no heat generation, chemical reactions, and thermal radiation. The flow is incompressible, and laminar. The density variation is approximated by the standard Boussinesq model for both temperature and concentration. The viscous dissipation in the energy equation has been analyzed in this study.

2.1 Dimensional equations

Based on the above assumptions, and applying the Boussinesq approximation, the studied equations are [47 - 57]:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{2.1}$$

$$\rho \left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} \right) = -\frac{\partial \bar{p}}{\partial \bar{x}} + \left(\frac{\partial \bar{\tau}_{xx}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{xy}}{\partial \bar{y}} \right), \tag{2.2}$$

$$\begin{aligned} \rho \left(\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} \right) = -\frac{\partial \bar{p}}{\partial \bar{y}} + \left(\frac{\partial \bar{\tau}_{yy}}{\partial \bar{y}} + \frac{\partial \bar{\tau}_{xy}}{\partial \bar{x}} \right) \\ + g\rho \left[1 + \beta_T(\bar{T} - T_C) - \beta_C(\bar{C} - C_C) \right], \end{aligned} \tag{2.3}$$

In the above equations $(\mathbf{u} = \bar{u}\mathbf{i} + \bar{v}\mathbf{j}), \bar{T}$, and $\bar{C}$, and g are the dimensional velocities, temperature, concentration, and gravity acceleration respectively. β_T and β_C are the coefficient of thermal expansion and solutal expansion, respectively as ρ is density. Now, let the pressure $\bar{p}$ be written as the sum

$\bar{p} = \bar{p}_s + \bar{p}_d$, where the static part $\bar{p}_s$ accounts for gravity alone, and $\bar{p}_d$ is the dynamic part. Thus,

$$-\frac{\partial \bar{p}_s}{\partial \bar{y}} = \rho g \cdot \tag{2.4}$$

$$\frac{\partial \bar{T}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \alpha\left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right)+\frac{1}{\rho c_p}\left[\bar{\tau}_{xx}\left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)+\bar{\tau}_{xy}\left(\frac{\partial \bar{u}}{\partial \bar{y}}+\frac{\partial \bar{v}}{\partial \bar{x}}\right)+\bar{\tau}_{yy}\left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)\right] + K_{TC}\left(\frac{\partial^2 \bar{C}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}\right) \tag{2.5}$$

α and K_{TC} are the thermal diffusivity and the thermodiffusion, respectively. c_p is the specific heat capacity at constant pressure.

$$\frac{\partial \bar{C}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{C}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{C}}{\partial \bar{y}} = D\left(\frac{\partial^2 \bar{C}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}\right)+K_{CT}\left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right) \tag{2.6}$$

D and K_{CT} are the mass diffusivity coefficient and the diffusionthermo coefficient, respectively.

The stress tensor for the incompressible Carreau fluids is as [19-22]

$$\bar{\tau}_{ij} = 2\,\eta(\dot{\gamma})\ S_{ij} \tag{2.7}$$

where S_{ij} is the rate of strain tensor as

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right) \tag{2.8}$$

where

$$\eta(\dot{\gamma}) = \eta_\infty + (\eta_0-\eta_\infty)\ \left[1+(\lambda\dot{\gamma})^2\right]^{(n-1)/2}\ , \qquad \dot{\gamma} = \sqrt{2S_{ij}S_{ij}} \tag{2.9}$$

where η_0 and η_∞ are the viscosities corresponding to zero and infinite viscosities, λ is the time constant and n is the power-law index where the deviation of n from unity indicates the degree of deviation from Newtonian behavior. With n$\neq$1, the constitute equation represents pseudoplastic fluid ($0 < n < 1$) and for ($n > 1$) it represents a dilatant fluid, respectively. Note that a Newtonian fluid can be recovered as a special case of the present Carreau fluid by letting n = 1 and/or $\lambda = 0$, and a power-law fluid can be obtained by assuming a large λ. The infinite shear viscosity, η_∞, is generally associated with a breakdown of the fluid, and is frequently significantly smaller ($10^3 - 10^4$ times smaller) than η_0, see [19-22, 58, 59]. So, the ratio η_∞/η_0 has been fixed at 0.001.

The flow domain is given by $\omega = (-L/2, L/2) \times (-L/2, L/2)$, and the boundary $\Gamma = \partial\omega$. It is the union of five disjoint subsets:

$$\Gamma_1 = \{(x,y), x = -L/2, -L/2 \leq y \leq L/2\}, \tag{2.10a}$$
$$\Gamma_2 = \{(x,y), x = L/2, -L/2 \leq y \leq L/2\}, \tag{2.10b}$$
$$\Gamma_3 = \{(x,y), -L/2 \leq x \leq L/2, y = -L/2\}, \tag{2.11a}$$
$$\Gamma_4 = \{(x,y), -L/2 \leq x \leq L/2, y = L/2\}, \tag{2.11b}$$
$$\Gamma_5 = \left\{(x,y), (x-x_c)^2 + (y-y_c)^2 = {R_d}^2.\right\} \tag{2.12}$$

The parameters of x_c, y_c, and R_d are the horizontal and vertical positions of the cylinder center and the radius of the cylinder; respectively.

The boundary condition for the velocity is straightforward:

$$\mathbf{u}|_{\Gamma_1} = \mathbf{u}|_{\Gamma_2} = \mathbf{u}|_{\Gamma_3} = \mathbf{u}|_{\Gamma_4} = \mathbf{u}|_{\Gamma_5} = \mathbf{0}. \tag{2.13}$$

The boundary conditions for the temperature and concentration are:

$$T|_{\Gamma_1} = T|_{\Gamma_2} = T|_{\Gamma_3} = T|_{\Gamma_4} = T_H, \quad T|_{\Gamma_5} = T_C \tag{2.14}$$

$$C|_{\Gamma_1} = C|_{\Gamma_2} = C|_{\Gamma_3} = C|_{\Gamma_4} = C_H, \quad C|_{\Gamma_5} = C_C \tag{2.15}$$

2.2 Non-dimensional equations

In order to proceed to the numerical solution of the system, the following non dimensional variables are introduced.

$$t = \frac{\bar{t}}{\left(\frac{L^2}{\alpha}\right) Ra^{-0.5}}, \quad x = \bar{x}/L, \quad y = \bar{y}/L, \quad u = \frac{\bar{u}}{\left(\frac{\alpha}{L}\right) Ra^{0.5}} \tag{2.16}$$

$$v = \frac{\bar{v}}{\left(\frac{\alpha}{L}\right) Ra^{0.5}}, \quad p_d = \frac{\bar{p}_d}{\rho\left(\frac{\alpha}{L}\right)^2 Ra}, \quad T = (\bar{T} - T_C)/\Delta T \tag{2.17}$$

$$C = (\bar{C} - C_C)/\Delta C, \quad \Delta T = T_H - T_C \quad \Delta C = C_H - C_C \tag{2.18}$$

$$\boldsymbol{\tau} = \frac{\bar{\boldsymbol{\tau}}\, L}{\eta_0 \left(\frac{\alpha}{L}\right)\, Ra^{0.5}} \tag{2.19}$$

By substitution of Eqs. (2.16) - (2.19) into Eqs. (2.1) - (2.6), the following system of non-dimensional equations is derived:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.20}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p_d}{\partial x} + \frac{Pr}{\sqrt{\mathrm{Ra}}}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right) \tag{2.21}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p_d}{\partial y} + \frac{Pr}{\sqrt{\mathrm{Ra}}}\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}\right) + \mathrm{Pr}\,(T - N\,C) \tag{2.22}$$

$$\begin{aligned} \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} &= \frac{1}{\sqrt{Ra}}\left[\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + D_f\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right)\right] \\ &+ Pr\, Ec\sqrt{Ra}\,\left[\tau_{xx}\left(\frac{\partial u}{\partial x}\right) + \tau_{xy}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \tau_{yy}\left(\frac{\partial v}{\partial y}\right)\right] \end{aligned} \tag{2.23}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \frac{1}{Le\sqrt{Ra}}\left[\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + S_r\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)\right] \tag{2.24}$$

The non-dimensional apparent viscosity is given by [21]

$$\eta(\dot{\gamma}) = \frac{\eta_\infty}{\eta_0} + (1 - \frac{\eta_\infty}{\eta_0})\ \left[1 + (Cu\dot{\gamma})^2\right]^{(n-1)/2} \tag{2.25}$$

$$\dot{\gamma} = \left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right\}^{\frac{1}{2}} \tag{2.26}$$

Hence, the stresses are:

$$\tau_{xx} = 2\eta(\dot{\gamma})\left(\frac{\partial u}{\partial x}\right)\ \ \tau_{yy} = 2\eta(\dot{\gamma})\left(\frac{\partial v}{\partial y}\right)\ \ \tau_{xy} = \eta(\dot{\gamma})\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{2.27}$$

The non-dimensional parameters for the problem are as follows:

Thermal Rayleigh number:

$$Ra = \frac{\rho\,\beta_T\,gL^3\Delta T}{\eta_0\,\alpha} \tag{2.28}$$

Prandtl number:

$$\Pr = \frac{\eta_0}{\rho\,\alpha} \tag{2.29}$$

Eckert number:

$$Ec = \frac{\left(\alpha/L\right)^2}{c_p\,\Delta T} \tag{2.30}$$

Buoyancy ratio number:

$$N = \frac{\Delta C\ \beta_T\ D}{\beta_C\ \Delta T\ \alpha} \tag{2.31}$$

Lewis number:

$$Le = \frac{\alpha}{D} \tag{2.32}$$

Dufour parameter:

$$D_f = \frac{K_{TC}\,\Delta C}{\alpha\ \Delta T} \tag{2.33}$$

Soret parameter:

$$S_r = \frac{K_{CT}\,\Delta T}{D\ \Delta C} \tag{2.34}$$

Carreau number:

$$Cu = \frac{\lambda}{\left(\frac{L^2}{\alpha}\right)\ Ra^{-0.5}} \tag{2.35}$$

3 Entropy generation

3.1 Dimensional equations

In the studied problem, the irreversibility is generated through heat transfer, fluid friction and mass transfer. As a result, the total entropy is the sum of the irreversibilities due to thermal gradients, viscous dissipation and concentration gradients as follows [23-24, 47-49, 51]:

$$\bar{S}_S = \bar{S}_F + \bar{S}_T + \bar{S}_D \ . \tag{3.1}$$

Where the entropy generations due to fluid friction ($\bar{S}_F$), heat transfer ($\bar{S}_T$), and mass transfer ($\bar{S}_D$) is calculated as follows [23-24, 47-49, 51]:

$$\bar{S}_F = \frac{\eta(\dot{\gamma})}{T_0}\left[2\left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)^2 + 2\left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)^2 + \left(\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}}\right)^2\right] \ . \tag{3.2}$$

$$\bar{S}_T = \frac{k}{T_0^2}\left[\left(\frac{\partial \bar{T}}{\partial \bar{x}}\right)^2 + \left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)^2\right] \ . \tag{3.3}$$

$$\bar{S}_D = \frac{RD}{C_0}\left[\left(\frac{\partial \bar{C}}{\partial \bar{x}}\right)^2 + \left(\frac{\partial \bar{C}}{\partial \bar{y}}\right)^2\right] + \frac{RD}{T_0}\left[\left(\frac{\partial \bar{C}}{\partial \bar{x}}\right)\left(\frac{\partial \bar{T}}{\partial \bar{x}}\right) + \left(\frac{\partial \bar{C}}{\partial \bar{y}}\right)\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)\right] , \tag{3.4}$$

T_0 and C_0 are bulk temperature and bulk concentration respectively and could be calculated as

$$T_0 = \frac{T_H + T_C}{2} , \quad C_0 = \frac{C_H + C_C}{2} , \tag{3.5}$$

An important measure of the entropy field is Bejan number (Be) which is defined as the ratio between entropy generations due to heat and mass transfer irreversibilities to the total entropy generation as follow

$$\overline{Be} = \frac{\bar{S}_T + \bar{S}_D}{\bar{S}_S} \ . \tag{3.6}$$

3.2 Non-dimensional equations

The local dimensionless entropy generations with consideration to non-dimensional variables of Eqs. (2.16) - (2.18) can be acquired as follows [23-24, 47-49, 51]:

$$S_S = S_F + S_T + S_D \tag{3.7}$$

$$S_F = \Phi_I\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right] , \tag{3.8}$$

$$S_T = \left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] , \tag{3.9}$$

$$S_D = \Phi_{II}\left[\left(\frac{\partial C}{\partial x}\right)^2 + \left(\frac{\partial C}{\partial y}\right)^2\right] + \Phi_{III}\left[\left(\frac{\partial C}{\partial x}\right)\left(\frac{\partial T}{\partial x}\right) + \left(\frac{\partial C}{\partial y}\right)\left(\frac{\partial T}{\partial y}\right)\right] , \tag{3.10}$$

$$\Phi_I = \frac{\eta(\dot{\gamma})\, T_0}{k}\left(\frac{\alpha}{L\Delta T}\right)^2 Ra \ , \tag{3.11}$$

$$\Pi \ = \frac{T_0}{k}\left(\frac{\alpha}{L\Delta T}\right)^2 \tag{3.12}$$

$$\Phi_{II} = \frac{RDT_0}{kC_0}\left(\frac{\Delta C}{\Delta T}\right)^2 \tag{3.13}$$

$$\Phi_{III} = \frac{RD}{k}\left(\frac{\Delta C}{\Delta T}\right) \tag{3.14}$$

It should be mentioned that the variables of Φ_{II}, Φ_{III}, Π is taken constant and they are Φ_{II}= 0.5,Φ_{III}= 0.01, Π = 0.001 (Please see, [23-24, 47-49, 51]). The local non-dimensional Bejan number is calculated as follows:

$$Be = \frac{S_T + S_D}{S_S} \ , \tag{3.15}$$

The total dimensionless entropy generations are obtained by numerical integration of the local dimensionless entropy generation over the entire cavity volume. It is given by:

$$S_{F,\,tot} = \int\limits_{-1/2}^{1/2}\int\limits_{-1/2}^{1/2} S_F dxdy\,, \quad S_{T,\,tot} = \int\limits_{-1/2}^{1/2}\int\limits_{-1/2}^{1/2} S_T dxdy, \quad S_{D,\,tot} = \int\limits_{-1/2}^{1/2}\int\limits_{-1/2}^{1/2} S_D dxdy, \tag{3.16}$$

$$S_{S,\,tot} = \int\limits_{-1/2}^{1/2}\int\limits_{-1/2}^{1/2} S_S dxdy \ , \tag{3.17}$$

Similarity, average Bejan number can be obtained as follow

$$Be_{avg} = \int\limits_{-1/2}^{1/2}\int\limits_{-1/2}^{1/2} Be\, dxdy \ . \tag{3.18}$$

4 The numerical method

The FDLBM equations and their relationships with continuum equations have been explained in details in Huilgol and Kefayati [45-46]. Here, just a brief description about the main equations would be cited. In addition, the applied algorithm has been described and the studied problem equations in the FDLBM are mentioned.

To have the continuity and momentum equations, a discrete particle distribution function f_α is defined over a D2Q9 lattice where it should satisfy an evolution equation:

$$\frac{\partial f_\alpha}{\partial t} + \boldsymbol{\xi}_\alpha \cdot \nabla_{\mathbf{x}} f_\alpha - F_\alpha = -\frac{1}{\varepsilon\,\phi}(f_\alpha - f_\alpha^{eq}), \tag{4.1}$$

where ε is a small parameter to be prescribed when numerical simulations are considered. ϕ is the relaxation time and F is the external force.

Associated to each node is a lattice velocity vector $\boldsymbol{\xi}_\alpha$. It is defined as follows:

$$\boldsymbol{\xi}_\alpha = \begin{cases} (0,0), & \alpha = 0, \\ \sigma(\cos\theta_\alpha, \sin\theta_\alpha) & \alpha = 1,3,5,7, \\ \sigma\sqrt{2}(\cos\theta_\alpha, \sin\theta_\alpha), & \alpha = 2,4,6,8. \end{cases} \tag{4.2}$$

Here, the angles θ_α are defined through $\theta_\alpha = (\alpha - 1)\pi/4,\ \ \alpha = 1, \cdots, 8$. The constant σ has to be chosen with care for it affects numerical stability; its choice depends on the problem. The method for finding the parameter σ which satisfies the Courant-Friedrichs-Lewy (CFL) condition is described in [45-46].

The equilibrium distribution function, f_α^{eq}, is different from the conventional ones adopted by previous researchers, who normally expand the Maxwellian distribution function. In the present approach, we expand f_α^{eq} as a quadratic in terms of $\boldsymbol{\xi}_\alpha$, using the notation of linear algebra [45-46]:

$$f_\alpha^{eq} = A_\alpha + \boldsymbol{\xi}_\alpha \cdot \mathbf{B}_\alpha + (\boldsymbol{\xi}_\alpha \otimes \boldsymbol{\xi}_\alpha) : \mathbf{C}_\alpha,\ \ \alpha = 0, 1, 2, \cdots, 8. \tag{4.3}$$

Here, the scalars A_α are defined through

$$A_0 = \rho - \frac{2p}{\sigma^2} - \frac{\rho|\mathbf{u}|^2}{\sigma^2} + \frac{\tau_{xx} + \tau_{yy}}{\sigma^2}, \quad A_\alpha = 0,\ \ \alpha = 1, 2, \cdots, 8. \tag{4.4}$$

The vectors $\mathbf{B}_\alpha$ are given by

$$\mathbf{B}_1 = \frac{\rho\mathbf{u}}{2\sigma^2} = \mathbf{B}_\alpha,\ \ \alpha = 1,3,5,7; \quad \mathbf{B}_\alpha = \mathbf{0},\ \ \alpha = 0,2,4,6,8. \tag{4.5}$$

Next, the matrices $\mathbf{C}_\alpha$ are such that $\mathbf{C}_0 = 0;\ \mathbf{C}_1 = \mathbf{C}_\alpha,\ \ \alpha = 1,3,5,7;\ \mathbf{C}_2 = \mathbf{C}_\alpha,\ \ \alpha = 2,4,6,8$, where

$$\mathbf{C}_1 = \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix}, \quad C_{11} = \frac{1}{2\sigma^4}(p+\rho u^2-\tau_{xx}), \quad C_{22} = \frac{1}{2\sigma^4}(p+\rho v^2-\tau_{yy}), \tag{4.6}$$

$$\mathbf{C}_2 = \begin{bmatrix} 0 & C_{12} \\ C_{21} & 0 \end{bmatrix}, \quad C_{12} = C_{21} = \frac{1}{8\sigma^4}(\rho uv - \tau_{xy}). \tag{4.7}$$

In order to derive the macroscopic equations for an incompressible continuous medium in the presence of a body force, it has been shown that the functions F_α in (3.1) must be such that

$$\sum_{\alpha=0}^{8} F_\alpha = 0. \tag{4.8}$$

In turn, this guarantees that the conservation of mass equation is satisfied. Next, one requires that

$$\sum_{\alpha=0}^{8} F_\alpha \boldsymbol{\xi}_\alpha = \rho\mathbf{b}, \tag{4.9}$$

where $\rho\mathbf{b}$ is the body force. Thus, one choice for the set of F_α is:

$$F_0 = 0, \;\; F_1 = \frac{1}{2\sigma^2}\rho\mathbf{b}\cdot\boldsymbol{\xi}_1, \;\; F_3 = \frac{1}{2\sigma^2}\rho\mathbf{b}\cdot\boldsymbol{\xi}_3, \tag{4.10a}$$

$$F_5 = \frac{1}{2\sigma^2}\rho\mathbf{b}\cdot\boldsymbol{\xi}_5, \;\; F_7 = \frac{1}{2\sigma^2}\rho\mathbf{b}\cdot\boldsymbol{\xi}_7. \tag{4.10b}$$

$$F_\alpha = 0, \quad \alpha = 2, 4, 6, 8. \tag{4.10c}$$

One notes that $F_1 = -F_5, \; F_3 = -F_7$.

In this problem, the non-dimensional body force is as follows:

$$\rho\mathbf{b} = \frac{\mathrm{Pr}\,(T - N\,C)}{2\,\sigma^2} \;\; \mathbf{j} \tag{4.11}$$

4.2 The Energy Equation

In order to obtain the energy equation, an internal energy distribution function g_α is introduced and it is assumed to satisfy an evolution equation similar to that for f_α. Thus,

$$\frac{\partial g_\alpha}{\partial t} + \boldsymbol{\xi}_\alpha \cdot \nabla_{\mathbf{x}} g_\alpha - G_\alpha = -\frac{1}{\varepsilon\phi}(g_\alpha - g_\alpha^{eq}). \tag{4.12}$$

G_α refers to the external supply e.g. radiation in the energy equation. Here, g_α^{eq} has a monomial expansion:

$$g_\alpha^{eq} = D_\alpha + \boldsymbol{\xi}_\alpha \cdot \mathbf{E}_\alpha, \tag{4.13}$$

One way of satisfying the above is to assume, as before, that the scalars are given by $D_\alpha = D_1,\ \alpha = 1,3,5,7$, and $D_\alpha = D_2,\ \alpha = 2,4,6,8,$. In this problem, the non-dimensional parameters are obtained as follows:

$$D_0 = T, \quad D_1 = 0, \quad D_2 = 0. \tag{4.14}$$

Regarding the vectors, it is assumed that $\mathbf{E}_0 = \mathbf{0},\ \mathbf{E}_\alpha = \mathbf{E}_1,\ \alpha = 1,3,5,7; \mathbf{E}_\alpha = \mathbf{E}_2,\ \alpha = 2,4,6,8,$ where

$$\mathbf{E}_1 = \frac{\left(\mathbf{u}\,T + Pr\,Ec\,\sqrt{Ra}\ \left((u\,\tau_{xx} + v\,\tau_{xy}) + (u\,\tau_{yx} + v\,\tau_{yy})\right) - \frac{1}{\sqrt{Ra}}\left(\frac{\partial \mathbf{T}}{\partial \mathbf{x}} + D_f\,\frac{\partial \mathbf{C}}{\partial \mathbf{x}}\right)\right)}{2\,\sigma^2}. \tag{4.15}$$

Finally, $G_\alpha = 0$.

4.3 The Concentration Equation

In order to obtain the concentration equation, an internal concentration distribution function h_α is introduced and it is assumed to satisfy an evolution equation similar to that for f_α. Thus,

$$\frac{\partial h_\alpha}{\partial t} + \boldsymbol{\xi}_\alpha \cdot \nabla_{\mathbf{x}} h_\alpha - H_\alpha = -\frac{1}{\varepsilon\phi}(h_\alpha - h_\alpha^{eq}). \tag{4.16}$$

Here, h_α^{eq} has a monomial expansion:

$$h_\alpha^{eq} = M_\alpha + \boldsymbol{\xi}_\alpha \cdot \mathbf{N}_\alpha, \tag{4.17}$$

One way of satisfying the above is to assume, as before, that the scalars are given by $M_\alpha = M_1,\ \alpha = 1,3,5,7$, and $M_\alpha = M_2,\ \alpha = 2,4,6,8,$. In this problem, the non-dimensional parameters are obtained as follows:

$$M_0 = C, \quad M_1 = 0, \quad M_2 = 0. \tag{4.18}$$

Regarding the vectors, it is assumed that $\mathbf{N}_0 = \mathbf{0},\ \mathbf{N}_\alpha = \mathbf{N}_1,\ \alpha = 1,3,5,7; \mathbf{N}_\alpha = \mathbf{N}_2,\ \alpha = 2,4,6,8,$ where

$$\mathbf{N}_1 = \frac{\left(\mathbf{u}\,C - \frac{1}{Le\sqrt{Ra}}\left(\frac{\partial \mathbf{C}}{\partial \mathbf{x}} + S_r \frac{\partial \mathbf{T}}{\partial \mathbf{x}}\right)\right)}{2\,\sigma^2} \tag{4.19}$$

Finally, $H_\alpha = 0$.

4.4 Algorithm

The main equations of the discrete particle distribution function, the internal energy distribution function, the internal concentration distribution function are solved by the splitting method. Hence, the equations can be separated into two parts. The first one is the streaming section which is written as

$$\frac{\partial f_\alpha}{\partial t} + \boldsymbol{\xi}_\alpha \cdot \nabla_{\mathbf{x}} f_\alpha - F_\alpha = 0. \tag{4.20}$$

$$\frac{\partial g_\alpha}{\partial t} + \boldsymbol{\xi}_\alpha \cdot \nabla_{\mathbf{x}} g_\alpha = 0. \tag{4.21}$$

$$\frac{\partial h_\alpha}{\partial t} + \boldsymbol{\xi}_\alpha \cdot \nabla_{\mathbf{x}} h_\alpha = 0. \tag{4.22}$$

Eqs.(4.20), (4.21), and (4.22) have been solved with FDM and the following equations are used.

$$\begin{aligned} f_\alpha^{n+1}(i,j) = f_\alpha^{n}(i,j) - \frac{\Delta t}{2\Delta x}\xi_\alpha(i)\left[f_\alpha^n(i+1,j) - f_\alpha^n(i-1,j)\right] \\ - \frac{\Delta t}{2\Delta y}\xi_\alpha(j)\left[f_\alpha^n(i,j+1) - f_\alpha^n(i,j-1)\right] + \\ \frac{\Delta t^2}{2\Delta x^2}{\xi_\alpha}^2(i)\left[f_\alpha^n(i+1,j) - 2f_\alpha^n(i,j) + f_\alpha^n(i-1,j)\right] + F_\alpha(i)\Delta t + \\ \frac{\Delta t^2}{2\Delta y^2}{\xi_\alpha}^2(j)\left[f_\alpha^n(i,j+1) - 2f_\alpha^n(i,j) + f_\alpha^n(i,j-1)\right] + F_\alpha(j)\Delta t\ , \end{aligned} \tag{4.23}$$

and

$$
\begin{aligned}
g_\alpha^{n+1}(i,j) = g_\alpha^n(i,j) - \frac{\Delta t}{2\Delta x}\xi_\alpha(i)\left[g_\alpha^n(i+1,j) - g_\alpha^n(i-1,j)\right] \\
- \frac{\Delta t}{2\Delta y}\xi_\alpha(j)\left[g_\alpha^n(i,j+1) - g_\alpha^n(i,j-1)\right] + \\
\frac{\Delta t^2}{2\Delta x^2}{\xi_\alpha}^2(i)\left[g_\alpha^n(i+1,j) - 2g_\alpha^n(i,j) + g_\alpha^n(i-1,j)\right] + \\
\frac{\Delta t^2}{2\Delta y^2}{\xi_\alpha}^2(j)\left[g_\alpha^n(i,j+1) - 2g_\alpha^n(i,j) + g_\alpha^n(i,j-1)\right]
\end{aligned} \tag{4.24}
$$

and

$$
\begin{aligned}
h_\alpha^{n+1}(i,j) = h_\alpha^n(i,j) - \frac{\Delta t}{2\Delta x}\xi_\alpha(i)\left[h_\alpha^n(i+1,j) - h_\alpha^n(i-1,j)\right] \\
- \frac{\Delta t}{2\Delta y}\xi_\alpha(j)\left[h_\alpha^n(i,j+1) - h_\alpha^n(i,j-1)\right] + \\
\frac{\Delta t^2}{2\Delta x^2}{\xi_\alpha}^2(i)\left[h_\alpha^n(i+1,j) - 2h_\alpha^n(i,j) + h_\alpha^n(i-1,j)\right] + \\
\frac{\Delta t^2}{2\Delta y^2}{\xi_\alpha}^2(j)\left[h_\alpha^n(i,j+1) - 2h_\alpha^n(i,j) + h_\alpha^n(i,j-1)\right]
\end{aligned} \tag{4.25}
$$

In Eqs.(4.23), (4.24), and (4.25), we have put

$$
\xi_\alpha(i) = \boldsymbol{\xi}_\alpha \cdot \mathbf{i}, \quad \xi_\alpha(j) = \boldsymbol{\xi}_\alpha \cdot \mathbf{j}, \quad F_\alpha(i) = \mathbf{F}_\alpha \cdot \mathbf{i}, \quad F_\alpha(j) = \mathbf{F}_\alpha \cdot \mathbf{j}. \tag{4.26}
$$

The second part is the collision section which is as follows:

$$
\frac{\partial f_\alpha}{\partial t} = -\frac{1}{\phi}(f_\alpha(\mathbf{x},t) - f_\alpha^{eq}(\mathbf{x},t)), \tag{4.27}
$$

$$
\frac{\partial g_\alpha}{\partial t} = -\frac{1}{\phi}(g_\alpha(\mathbf{x},t) - g_\alpha^{eq}(\mathbf{x},t)). \tag{4.28}
$$

$$
\frac{\partial h_\alpha}{\partial t} = -\frac{1}{\phi}(h_\alpha(\mathbf{x},t) - h_\alpha^{eq}(\mathbf{x},t)). \tag{4.29}
$$

Eqs.(4.27), (4.28), and (4.29) can be solved by using the Euler method and the choice of ϕ is taken as the time step (Δt). That is

$$\frac{f_\alpha(\mathbf{x}, t+\Delta t) - f_\alpha(\mathbf{x}, t)}{\Delta t} = -\frac{1}{\phi}(f_\alpha(\mathbf{x}, t) - f_\alpha^{eq}(\mathbf{x}, t)), \tag{4.30}$$

$$\frac{g_\alpha(\mathbf{x}, t+\Delta t) - g_\alpha(\mathbf{x}, t)}{\Delta t} = -\frac{1}{\phi}(g_\alpha(\mathbf{x}, t) - g_\alpha^{eq}(\mathbf{x}, t)), \tag{4.31}$$

$$\frac{h_\alpha(\mathbf{x}, t+\Delta t) - h_\alpha(\mathbf{x}, t)}{\Delta t} = -\frac{1}{\phi}(h_\alpha(\mathbf{x}, t) - h_\alpha^{eq}(\mathbf{x}, t)), \tag{4.32}$$

from which one obtains

$$f_\alpha(\mathbf{x}, t+\Delta t) = f_\alpha^{eq}(\mathbf{x}, t), \tag{4.33}$$

and

$$g_\alpha(\mathbf{x}, t+\Delta t) = g_\alpha^{eq}(\mathbf{x}, t). \tag{4.34}$$

$$h_\alpha(\mathbf{x}, t+\Delta t) = h_\alpha^{eq}(\mathbf{x}, t). \tag{4.35}$$

The numerical procedures are summarised below.

Initial stage

(a) Initial conditions for all macroscopic quantities including the boundary points are given. The initial values of $f_\alpha^{0,\,eq}$, $g_\alpha^{0,\,eq}$, and $h_\alpha^{0,\,eq}$ including the boundary points are determined. These are used as initial values to start the calculation.

Streaming stage

(b) With f_α, g_α, and h_α at time t (including the boundary points) known, intermediate values f_α^I, g_α^I, and h_α^I are calculated by solving Eqs.(4.23), (4.24), and (4.25) respectively.
(c) Using these f_α^I, g_α^I, and h_α^I, the corresponding macroscopic quantities $(u_I, v_I, p_I, T_I, C_I)$ for all interior grid points are calculated.
(d) The boundary conditions for the macroscopic level are then set as in any finite difference method.
(e) Using the macroscopic quantities thus determined over the complete domain including the boundary points, the corresponding $f_\alpha^{I,\,eq}$, $g_\alpha^{I,\,eq}$, and $h_\alpha^{I,\,eq}$ are obtained, including all of the boundary points.

Collision stage

(f) Due to Eqs.(4.33), (4.34), and (4.35) the collision step is completed by setting the new value at time $t + \Delta t$. Since each set of macroscopic quantities will map uniquely to an equilibrium distribution function and vice versa, the macroscopic quantities thus obtained are, in fact, the values at time $t + \Delta t$, i.e., $(u, v, p, T, C)_{t+\Delta t} = (u_I, v_I, p_I, T_I, C_I)$.

(g) Time marching proceeds by repeating steps (b)-(f).

5 Code validation and grid independence

Finite Difference Lattice Boltzmann Method (FDLBM) scheme is utilized to simulate entropy generation of laminar double diffusive natural convection in a heated enclosure with an inner cold cylinder that is filled with a Carreau fluid in the presence of Soret and Dufour parameters and the viscous dissipation in the energy equation. The Prandtl number is fixed at Pr=0.1. This problem is investigated at different parameters of Rayleigh number (Ra = 10^4 and 10^5), Carreau number (Cu = 1, 10, and 20), Lewis number (Le=2.5, 5 and 10), Dufour parameter (D_f=0, 1, and 5), Soret parameter (S_r=0, 1, and 5), Eckert number (Ec=0, 1, and 10), the Buoyancy ratio (N=-1, 0.1, 1), the radius of the inner cylinder (R_d = 0.1 L, 0.2 L, 0.3 L, and 0.4 L), the horizontal distance of the circular cylinder from the center of the enclosure (Ω = -0.2 L, 0 and 0.2 L), the vertical distance of the circular cylinder from the center of the enclosure (δ = -0.2 L, 0 and 0.2 L). An extensive mesh testing procedure was conducted to guarantee a grid independent solution. Seven different mesh combinations were explored for the case of Ra= 10^5, Cu=1, N=0.1, Le=2.5, n=1.4, R_d = 0.2 L, Ec=0, D_f=0 and S_r=0. The total summation of entropy generations and the average Bejan number have been studied. It was confirmed that the grid size (200*200) ensures a grid independent solution as portrayed by Table.1. For the validation of the entropy generation study in Fig.2, the local summation entropy generation (S_S) for the case of Ra=10^5 and Pr=0.71 has been compared with the study of Mun et al. [25] which demonstrates a good agreement. FDLBM is applied for double diffusive natural convection and entropy generation of power-law and Bingham fluids recently [47-51] which demonstrates the accuracy of the utilized code for the problem properly.

6 Results and discussion

6.1 Effects of Rayleigh number on entropy generation

Fig.3 displays the Rayleigh number effects on entropy generations due to heat transfer (S_T), fluid friction (S_F), mass transfer (S_D),summation entropy generation (S_S), and the local Bejan number (Be) at n=0.2, Le=2.5, R_d = 0.2 L, Cu=1, Ec=0, N=0.1, D_f =0, and S_r=0. It demonstrates that S_T increases generally, but this augmentation is not uniform in the cavity nad in some places we face a decrease in the S_T. For example, it is observed that minor values are appeared the bottom side of the cold cylinder. However, some places e.g. the bottom side of the cavity shows the S_T augments in a higher rate compared other parts. However, the minimum values of S_T are observed in both studied Rayleigh numbers in the corners of the enclosure. The S_F demonstrates that the high values are observed on the sidewalls and the surface of the cylinder in both Rayleigh numbers. Generally, the S_F increases considerably with the rise of Rayleigh number where the minimum value bottom of the cylinder becomes smaller due to the increase in Rayleigh number. In fact, high values are replaced with the low values at Ra = 10^5 close to the cylinder on the bottom section of the cavity. The entropy generation due to mass transfer (S_D) enhances generally with the rise of Rayleigh number; although, this augmentation is less than S_F. It shows the gradient of S_D on the cylinder enhances with the rise of Rayleigh number. In addition, the entropy generation due to mass transfer in bottom of the cylinder diminishes as Rayleigh number enhances while close to the bottom wall of the enclosure a high value is generated at Ra = 10^5. Interestingly, the increase in Rayleigh number, causes a low value section of S_D is generated on the top section of the enclosure which proves that the rise of Rayleigh number decreases the irreversibility due to mass transfer. The contour of the total entropy generations exhibits that the high values are concentrated around the cylinder and sections which are immensely close to the sidewalls. At Ra = 10^4, the high values of S_S form a mushroom shape while at Ra = 10^5, the bottom side of the cylinder has a low value. However, two small slices of high values on the bottom side of the cavity are obtained at Ra = 10^5. In addition, the rise of Rayleigh number does not affect the total entropy generation significantly where the values of S_S = 5 has covered the top side at Ra = 10^5. However, two small stuck parts on the top side of the enclosure is present at Ra = 10^5. Table. 2 shows that the entropy generations due to fluid friction and heat and mass transfer increases as Rayleigh number rises. The highest rate of the increase in the entropy generations due to the increase in Rayleigh number is observed in the SF where this augmentation was approximately 33 times. But, the total entropy generation enhances nearly three times as the Rayleigh number increase from Ra = 10^4 to 10^5. The average Bejan number decreases significantly by 50 percent with the rise of

Rayleigh number.

6.2 *Effects of power-law index on entropy generation*

Fig.4 displays the power-law index effects on entropy generations due to heat transfer (S_T), fluid friction (S_F), and mass transfer (S_D) at Ra = 10^5, Le=2.5, Cu=1, R_d = 0.2 L, Ec=0, N=0.1, D_f =0, and S_r=0. It shows that the S_T becomes weak slightly as the power-law index increases. The bottom side of the enclosure can demonstrate the effect of power-law index on the S_T clearly where the high values of S_T=20 and 40 at n=0.2 disappears at n=1.8. However, the increase in power-law index does not influence the top section of cylinder in S_T. Table 2 confirms the trend of the contours of S_T where shows the ST declines gradually for the both studied Rayleigh numbers with the rise of power-law index. The enhancement of power-law index from n = 0.2 to 1 enhances the values in the contours of S_F marginally where the S_F = 20 can distinguish this pattern. But, the values of S_F declines from n = 1 to 1.8. Table 2 also demonstrates that the S_F at Ra = 10^5 increases from n = 0.2 to 1, but it drops from n = 1 to 1.8. Table 2 shows the SF at Ra= 10^4 exhibits a different manner where decreases gradually as the power-law index enhances. It displays that the values i the contours of S_D weakens steadily as the power-law index enhances. The bottom wall of the enclosure demonstrates that the S_D = 10 and 20 disappear with the rise of power-law index. It should also be noted that the low value on the top of the enclosure is not affected by the change of the power-law index. Table 2 shows that the SD declines gradually with the rise of power-law index in the both studied Rayleigh numbers.

Fig.5 displays the power-law index effects on entropy generations due to summation entropy generation (S_S), and the local Bejan number (Be)at Ra = 10^5, Le=2.5, Cu=1, Ec=0, R_d = 0.2 L, N=0.1, D_f =0, and S_r=0. The contours of the total entropy generation show that the values of S_S declines slightly as the power-law index enhances. Table 2 indicates that the total summation of entropy generation decreases gradually when the power-law index enhances. The local Bejan number clarifies that the sections of high values enlarges slightly. Table 2 confirms this trend at Ra = 10^5 where the average Bejan number rises as the power-law index increases. The same pattern for the average Bejan number at Ra = 10^4 where the average Bejan number drops due to the increase in the power-law index.

6.3 *Effects of Lewis number on entropy generation*

Fig.6 displays the Lewis number effects on entropy generations due to heat transfer (S_T), fluid friction (S_F), mass transfer (S_D),summation entropy generation (S_S), and the local Bejan number (Be) at n=1, Ra = 10^5, Cu=1, Ec=0, R_d = 0.2 L, N=0.1, D_f =0, and S_r=0. It demonstrates that the local entropy generation due to heat transfer and fluid friction do not affect considerably as the Lewis number enhances. But, the local entropy generation due to mass transfer shows a different distribution with the rise of the Lewis number where the gradient and values of S_D around the cylinders increase considerably. In addition, the summation of entropy generation (S_S) shows a marginal increase in values as the Lewis number enhances. The local Bejan number shows a marginal changes with the rise of Lewis number where the maximum values in the bottom of the cavity diminishes due to the increases in the entropy generation of mass transfer. Table 3 shows that the increase in Lewis number declines the ST and SF slightly while the SD increases considerably with the enhancement of Lewis number. The summation entropy generation enhances gradually with the rise of Lewis number in both Rayleigh numbers. In addition, the augmentation of Lewis number causes the average Bejan number marginally to drop and increase at Ra = 10^4 and 10^5, respectively.

6.4 Effects of Buoyancy ratio on entropy generation

Fig.7 displays the Buoyancy ratio effects on entropy generations due to heat transfer (S_T), fluid friction (S_F), mass transfer (S_D),summation entropy generation (S_S), and the local Bejan number (Be) at n=1, Ra = 10^5, Cu=1, R_d = 0.2 L, Ec=0, Le = 2.5, D_f =0, and S_r=0. It demonstrates that the local entropy generation due to heat transfer changes considerably as the buoyancy ratio increases from N=-1 to 0.1 where the values augment considerably and the values of S_T and their gradients around cylinder augments and two sections of high values are observed on the bottom side of the cavity. The augmentation of S_T continues from N = 0.1 to 10, but the shape is the same as N=0.1. Table 4 demonstrates that the ST increases gradually as the buoyancy ratio enhances from N = -1 to 1. The local entropy generation due to fluid friction has changed utterly as the buoyancy ratio increases from N = -1 to 0.1 where the maximum values are formed close to the cylinder at N = 1. The S_F augments considerably in the contours at N= 1. Table 4 shows the average entropy generation due to fluid friction increases substantially when the buoyancy ratio rises from N=-1 to 1. The local entropy generation due to mass transfer strengthens as the buoyancy ratio increases where the values of S_D in bottom side of the cavity can distinguish the enhancement clearly. Table 4 indicates that the SD enhances in the both studied Rayleigh numbers with the rise of the buoyancy ratio from N =-1 to 1. The local contours of the summation entropy generation demonstrates that the increase in buoyancy ratio augments the values around the cylinder although small sections with low values bottom of the cylinder are generated. Table 4; clearly, shows that the SS increases significantly as the buoyancy ratio enhances. The local Bejan number displays that the rise of buoyancy ratio from N = -1 to 0.1 diminishes the high values in the middle of the cavity considerably. The decreases in the local Bejan number continues at N=1; although, this reduction is not extensive. Table 4 confirms this results where the average Bejan number decreases considerably at Ra = 10^5. In addition, the average Bejan number in the Rayleigh number of Ra = 10^4 drops slightly as the buoyancy ration enhances from N = -1 to 1.

6.5 *Effects of Soret parameter on entropy generation*

Fig.8 displays the Soret parameter effects on entropy generations due to heat transfer (S_T), fluid friction (S_F), mass transfer (S_D),summation entropy generation (S_S), and the local Bejan number (Be) at n=1, Ra = 10^5, Cu=1, N=0.1, R_d = 0.2 L, Ec=0, Le = 2.5, and D_f =0. It shows that the local entropy generation due to heat transfer becomes stronger around the cylinder as the Soret parameter increases. However, it demonstrates that the S_T in the bottom and top side of the enclosure does not alter significantly. The local entropy generation due to fluid friction increases slightly when the Soret parameter enhances. The highest impact of the soret parameter is observed on the S_D where the distribution of the entropy generation alters completely. In fact, the high values around the cylinders changes to low magnitudes. In addition, the maximum values of the S_D drops considerably; however, the size of sections with the average values of S_D = 15 to 20 enlarge significantly. In addition, the total entropy generation contours demonstrate the low values of S_S on the top of the cavity diminishes and replaced with higher magnitudes. Hence, it is observed the S_S strengthens with the rise of Soret parameter generally. The local Bejan number demonstrates that the increase in Soret parameter causes the sections with high values in the center of the cavity to expand. The main reason for the enhancement of the local Bejan number around the cylinders is the drop of the S_D as the Soret parameter increases. Table 5 shows that the entropy generations due to heat and fluid flow augment slightly as the Soret parameter increases. In addition, the table displays that the S_D and S_S decreases as the Soret parameter rises from Sr =0 to 1, but drops from Sr = 1 to 5. The average Bejan number also augments as the Soret parameter enhances.

6.6 *Effects of Dufour parameter on entropy generation*

Fig.9 displays the Dufour parameter effects on entropy generations due to heat transfer (S_T), fluid friction (S_F), mass transfer (S_D),summation entropy generation (S_S), and the local Bejan number (Be) at n=1, Ra = 10^5, Cu=1, $R_d = 0.2$ L, Ec=0, Le = 2.5, and S_r=0. It demonstrates that the S_T enhances considerably with the rise of Dufour number where the maximum values were observed on the top and bottom of the cylinders and close to the sidewalls. However, it shows the low values of S_T are obtained in the corners of the enclosure same as the absence of Dufour number. The contour of S_F shows a substantial increase as the Dufour parameter enhances. In addition, the distribution of the irreversibility due to fluid friction changes where the left and right sides of the cylinder have low values in contrast with the $D_f = 0$. Further, two sections with high values on the top and bottom side of the enclosure are generated at $D_f = 5$ while low values are observed at $D_f = 0$. The contour of S_D demonstrates that a big section of low value in the bottom of the enclosure is generated with the rise of Dufour parameter. In addition, a high value section which is stick to the bottom of cylinder is generated by the rise of Dufour parameter. The contours of total summation of entropy generation in different Dufour parameters exhibits that iireversibility augments considerably. In addition, the uniform shape of the entropy changes to discrete parts with the enhancement of Dufour numbers. It shows that the increase in Dufour parameter enhances the maximum parts of the local Bejan number. The main reason of the trend is because of the enhancement of entropy generation due to heat transfer considerably. Table 6 indicates that different entropy generations and the average Bejan number enhance slightly with the increase in Soret parameter in various Rayleigh numbers. Table 6 reveals that different entropy generations; especially entropy generation due to heat transfer, rise significantly. Moreover, the average Bejan number increases when the Dufour parameter enhances.

6.7 Effects of Eckert number on entropy generation

Fig.10 displays the Eckert number effects on entropy generations due to heat transfer (S_T), fluid friction (S_F), mass transfer (S_D),summation entropy generation (S_S), and the local Bejan number (Be) at n=1, Ra = 10^5, Cu=1, Ec=0, $R_d = 0.2$ L, Le = 2.5, D_f =0, and S_r=0. As it was predicted, the main effect of Eckert number was observed in the entropy generation due to heat transfer. It can be seen that the two maximum value sections of S_T close to the cylinder are generated by the rise of Eckert number. However, two low values parts next to the cylinder is created. Moreover, the S_T strengthens in sidewalls of the enclosure as the Eckert number rises. The contour of S_F displays that the high values sections next to the cylinders and the sidewalls become stronger when the Eckert numbers rises. However, the S_D weakens slightly due to the increase of Eckert number. The Fig exhibits that the rise of Eckert number enhances the summation of entropy generation moderately. It was found from the obtained contours that the local Bejan number declines slightly in the presence of the Eckert number. Table 7 demonstrates that the Eckret number has a minor influence on entropy generations. However, the trend of the table shows that the total entropy generations of heat transfer enhance gradually for the both studied Rayleigh numbers as the Eckert number enhances. In addition the entropy generation due to the fluid friction and the summation entropy generation at Ec = 10 are more than the Eckert number Ec = 10; although drops slightly from Ec =0 to 1.

6.8 Effects of Carreau number on entropy generation

Fig.11 displays the Carreau number effects on entropy generations due to heat transfer (S_T), fluid friction (S_F), mass transfer (S_D),summation entropy generation (S_S), and the local Bejan number (Be) at n=1.4, Ra = 10^5, Ec=0, R_d = 0.2 L, Le = 2.5, D_f =0, and S_r=0. The local entropy generations due to heat transfer demonstrates that the S_T weakens slightly as the Carreau number rises. But the contour of S_F becomes weak significantly in the presence of the Carreau number.On the other hand, the local entropy generation due to mass transfer drops marginally due to the enhancement of the Carreau number. It was observed that the summation of entropy generations especially around the cylinder drop as the Carreau number enhances. It was also found that the local Bejan number with high values expand in the result of the increase in the Carreau number. The main reason of the pattern is the decline of the local entropy generation due to the fluid friction. Table 8 shows that the total entropy generation due to fluid friction declines substantially as the Carreau number rises. In addition, other entropy generations icluding the summation of entropy generations decrease moderately. However, the average Bejan number enhances gradually with the rise of the Carreau parameter.

6.9 Effects of the vertical distance of the cylinder from the center on entropy generation

Fig.12 displays the vertical distance of the cylinder from the center (δ) effects on entropy generations due to heat transfer (S_T), fluid friction (S_F), mass transfer (S_D),summation entropy generation (S_S), and the local Bejan number (Be) at n=1, Ra = 10^5, Cu=1, R_d = 0.2 L, Ec=0, Le = 2.5, D_f =0, and S_r=0. It demonstrates that the entropy generation due to heat transfer drops considerably as the location moves from δ = -0.2 L to 0. But, the movement of the cylinder to the top side enhances S_T significantly. Moreover, it was observed that at δ =-0.2 L, the half of the enclosure has a low value of S_T =1 which is the main reason for the low entropy generation of heat transfer. The same trend occurs at S_F where the local entropy due to fluid friction strengthens substantially when the cylinder moves from the bottom side to the center. This pattern can be seen clearly at $S_F = 20$ as it enlarges considerably from δ = -0.2 L to 0. However, the S_F declines slightly from $\delta = 0$ to 0.2 L; especially, the bottom side of the cylinder. The local entropy generation due to mass transfer demonstrates evidently that the movement of the cylinder from bottom side to the center declines slightly, but rises marginally in the top side. The local summation entropy generation exhibits that the values enhance when the cylinder moves from the bottom to the top side.It shows that the distribution of the entropy generation alters utterly with the change of the cylinder position. However, it clarifies that the highest values are observed around the cylinder and close to the side walls. The local Bejan number demonstrates that at δ = -0.2 L, the top half section includes low values because the entropy generations due to heat and mass transfer are minimum. Generally,it was found, the highest values of the local Bejan number are around the cylinder. Table 9 indicates that various entropy generations declines gradually as the power-law indexes enhance while the average Bejan number increases in different vertical positions. It can be seen that at n=0.2, the total entropy generation (SS) drops with the rise of the cylinder center. But, the total entropy generation (SS) enhances slightly when the cylinder moves from bottom to the top side of the enclosure. It was also observed the highest average Bejan number is observed in the center position.

6.10 Effects of the radius of the inner cylinder from the center on entropy generation

Fig.13 displays the radius of the cylinder (R_d) effects on entropy generations due to heat transfer (S_T), fluid friction (S_F), mass transfer (S_D),summation entropy generation (S_S), and the local Bejan number (Be) at n=1, Ra = 10^5, Cu=1, Ec=0, Le = 2.5, D_f =0, and S_r=0. It can be observed that the increase in the size of the cylinder enhance the total entropy generation due to heat transfer gradually where the low value on the top of the cavity in the size of R_d=0.1 L is removed with the augmentation of the cylinder size. It shows that the local entropy generation due to fluid friction becomes strong as the size of the cylinder rises from R_d = 0.1 L to 0.3 L. But, at R_d =0.4 L,the S_F declines vastly in the local contour. The values in the local entropy generations due to mass transfer augment gradually as the size of the cylinder rises. The local summation entropy generation shows that the maximum values of the entropy generations decline as the size od the cylinder enhances; however, the values of the total entropy generation in the cavity enhance considerably. The local Bejan number augments clearly as the size of the cylinder rises. It exhibits that the entropy generation due to fluid friction becomes weak with the rise of cylinder compared to entropy generations due to heat and mass transfer. Table 10 displays that the increase in the power-law index in various sizes declines different entropy generations. In different power-law indexes, the enhancement of the cylinder size increases the ireversibilities due to heat and mass transfer as well as the summation entropy generation. However, the entropy generation due to fluid friction has a non-uniform manner against the size of the cylinder. In addition, the highest average Bejan number is observed in the size of d=0.3 L.

6.11 Effects of the horizontal distance of the cylinder from the center on entropy generation

Fig.14 displays the horizontal distance of the cylinder from the center (Ω) effects on entropy generations due to heat transfer (S_T), fluid friction (S_F), mass transfer (S_D),summation entropy generation (S_S), and the local Bejan number (Be) at n=1, Ra = 10^5, Cu=1, Ec=0, R_d = 0.2 L, Le = 2.5, D_f =0, and S_r=0. The local entropy generations due to heat and mass transfer drop in the center compared to the sidewall positions of the cylinder. But, the local entropy generation due to fluid friction strengthens considerably in the center position compared to the sidewall positions. The local total entropy generation demonstrates that the highest entropy generation is observed in the center position. The local Bejan number demonstrates that the high rate of the average Bejan number is present around the cylinder and low values are far from the cylinder. Table 11 shows that the lowest entropy generations are observed at n=1.8. In different power-law indexes, the highest different entropy generations are observed in the center position; although, the lowest average Bejan number is in this position.

7 Concluding Remarks

Entropy generation of double diffusive natural convection of Carreau fluid in a cavity in the presence of Soret and Dufour parameters as well as viscous dissipation has been analyzed by Finite Difference Lattice Boltzmann method (FDLBM). This study has been conducted for the pertinent parameters in the following ranges: Rayleigh number (Ra = 10^4 and 10^5), Carreau number (Cu = 1, 10, and 20), Lewis number (Le=2.5, 5 and 10), Dufour parameter (D_f=0, 1, and 5), Soret parameter (S_r=0, 1, and 5), Eckert number (Ec=0, 1, and 10), the Buoyancy ratio (N=-1, 0.1, 1), the radius of the inner cylinder (R_d = 0.1 L, 0.2 L, 0.3 L, and 0.4 L), the horizontal distance of the circular cylinder from the center of the enclosure (Ω = -0.2 L, 0 and 0.2 L), the vertical distance of the circular cylinder from the center of the enclosure (δ = -0.2 L, 0 and 0.2 L). The main conclusions of the present investigation can be summarized as follows:

- The enhancement of Rayleigh number augments different irreversibilities and the highest level of growth is observed at the entropy generation due to fluid friction.
- Bejan number declines significantly with the augmentation of Rayleigh number which demonstrates a jump in the irreversibility due to fluid friction.
- The enhancement of power-law index declines different entropy generations steadily while the average Bejan number rises gradually.
- The increase in the buoyancy ratio enhances the entropy generations due to heat and mass transfer, fluid friction and causes the average Bejan number to augment.
- The rise of Lewis number enhances the entropy generation due to mass transfer for different studied parameters whereas the entropy generations due to heat transfer and fluid friction decrease by the growth of the Lewis number.
- The average Bejan number decreases and enhances when the Lewis number increases at Ra = 10^4, and 10^5; respectively.
- In the absence of the Soret and Dufour parameters ($S_r = D_f = 0$), the entropy generations due to heat and mass transfer as well as the summation of entropy generation decrease slightly as the power-law index enhances for multifarious Rayleigh numbers.
- In the absence of the Soret and Dufour parameters ($S_r = D_f = 0$), the increase in the power-law index enhances the average Bejan number gradually in various Rayleigh numbers.
- The addition of Dufour parameter enhances different total entropy generations; especially the fluid friction, for the studied Rayleigh numbers.
- The total summation of entropy generation enhances with the augmentation of the Dufour parameter while the average Bejan number drops at Ra = 10^4 and enhances at Ra = 10^5 with the rise of the Dufour parameter.
- The addition of Soret parameter augments the total entropy generation due to fluid friction and heat transfer. Further, the average Bejan number decreases with the increase in Soret parameter in various Rayleigh numbers.
- The addition of Eckert number in a low value (Ec =1) decrease the total summation entropy generation slightly, but at high values (Ec=10) can result in the enhancement of the total entropy generations considerably.
- The addition of Carreau number causes the total entropy generation to drop gradually.
- It was found the lowest entropy generation in the vertical positions is in the close position to the bottom side (δ=-0.2L) in the power-law index of n=1 and 1.8. But, the entropy generations at n=0.2 are nearly the same for δ=-0.2L and 0.
- I was observed the increase in the size of the cylinder enhances the total entropy generations gradually.
- The highest entropy generations in the studied horizontal positions was observed in the center position in various power-law indexes.

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Table 1
Grid independence study at $Ra = 10^5$, Ec=0, $R_d = 0.2$ L, Le = 2.5, Cu = 1, n=1.4, S_r=D_f=0, and N=0.1

Mesh size	SS	Be_{avg}
150*150	28.1504	0.5181
160*160	27.9025	0.5021
170*170	27.6393	0.4951
180*180	27.4158	0.4881
190*190	27.3908	0.4680
200*200	27.3472	0.4501
210*210	27.3472	0.4501

Table 2
Different entropy generations in various Rayleigh numbers and power-law indexes at Ec = 0,D_f= S_r=0, R_d = 0.2 L, Le=2.5 and N=0.1

	ST	SF	SD	SS	Be_{avg}
Ra = 10^4					
n=0.2	6.3671	0.7639	3.5322	10.6632	0.7809
n=0.4	6.3357	0.6744	3.4703	10.4804	0.7922
n=0.6	6.3152	0.5968	3.4283	10.3404	0.8031
n=0.8	6.3010	0.5267	3.3984	10.2261	0.8131
n=1	6.2907	0.4637	3.3760	10.1305	0.8223
n=1.2	6.2828	0.4076	3.3587	10.0491	0.8304
n=1.4	6.2767	0.3577	3.3449	9.9792	0.8376
n=1.6	6.2717	0.3134	3.3335	9.9187	0.8439
n=1.8	6.2676	0.2743	3.3240	9.8659	0.8495
Ra = 10^5					
n=0.2	8.4255	13.3706	6.2282	28.0243	0.3869
n=0.4	8.2489	14.1741	6.0354	28.4584	0.3926
n=0.6	8.0654	14.8410	5.8313	28.7378	0.3997
n=0.8	7.8845	15.2706	5.6267	28.7817	0.4088
n=1	7.7186	15.3824	5.43603	28.5370	0.4203
n=1.2	7.5744	15.1942	5.2682	28.0368	0.4342
n=1.4	7.4527	14.7695	5.1250	27.3472	0.4501
n=1.6	7.3513	14.1738	5.0044	26.5296	0.4674
n=1.8	7.2669	13.4616	4.9030	25.6315	0.4858

Table 3
Effects of the Lewis number (Le) on different entropy generations in different Rayleigh numbers at Ec=0, R_d = 0.2 L, n = 1, S_r=D_f=0, and N=0.1

	Le = 2.5	Le = 5	Le = 10
Ra = 10^4			
ST	6.2907	6.2896	6.2877
SF	0.4637	0.4567	0.4449
SD	3.3760	3.7878	4.6837
SS	10.1305	10.5342	11.4163
Be_{avg}	0.8223	0.8178	0.8130
Ra = 10^5			
ST	7.7186	7.6692	7.6160
SF	15.3824	14.7758	14.2254
SD	5.43603	7.1269	9.0520
SS	28.5370	29.5719	30.8935
Be_{avg}	0.4203	0.4222	0.4239

Table 4
Effects of the Buoyancy ratio (N) on different entropy generations in different Rayleigh numbers at Ec=0, n = 1, R_d = 0.2 L, Le = 2.5,D_f=0, and S_r=0

	N=0.1	N=1	N=-1
Ra = 10^4			
ST	6.2907	6.4677	6.2084
SF	0.4637	1.4398	0.000812
SD	3.3760	3.7739	3.1691
SS	10.1305	11.6815	9.3783
Be_{avg}	0.8223	0.7521	0.8846
Ra = 10^5			
ST	7.7186	9.0314	6.2869
SF	15.3824	32.3129	0.71382
SD	5.43603	6.7836	3.6568
SS	28.5370	48.1280	10.6575
Be_{avg}	0.4203	0.3319	0.7447

Table 5
Effects of the Soret parameter (S_r) on different entropy generations in different Rayleigh numbers at Ec=0, n = 1, R_d = 0.2 L, Le = 2.5, D_f=0, and N=0.1

	$S_r = 0$	$S_r = 1$	$S_r = 5$
Ra = 10^4			
ST	6.2907	6.2910	6.2927
SF	0.4637	0.4658	0.4745
SD	3.3760	3.2484	3.3879
SS	10.1305	10.0053	10.1550
Be_{avg}	0.8223	0.8228	0.8255
Ra = 10^5			
ST	7.7186	7.7445	7.8532
SF	15.3824	15.7818	17.4999
SD	5.43603	4.3048	5.7688
SS	28.5370	27.8311	31.1219
Be_{avg}	0.4203	0.4266	0.5082

Table 6
Effects of the Dufour parameter (D_f) on different entropy generations in different Rayleigh numbers at Ec=0, n = 1, Le = 2.5, R_d = 0.2 L, S_r=0, and N=0.1

	D_f=0	D_f=1	D_f=5
Ra = 10^4			
ST	6.2907	6.4126	34.3796
SF	0.4637	0.5330	1.6032
SD	3.3760	3.4102	3.84970
SS	10.1305	10.3558	39.8326
Be_{avg}	0.8223	0.8185	0.7769
Ra = 10^5			
ST	7.7186	9.8518	118.03276
SF	15.3824	29.6132	46.1777
SD	5.43603	6.1126	7.1888
SS	28.5370	45.5776	171.3992
Be_{avg}	0.4203	0.4335	0.5990

Table 7
Effects of the Eckert number (Ec) on different entropy generations in different Rayleigh numbers at D_f=0, n = 1, Le = 2.5, R_d = 0.2 L, S_r=0, and N=0.1

	Ec=0	Ec=1	Ec=10
Ra = 10^4			
ST	6.2907	6.2909	6.3380
SF	0.4637	0.4640	0.4673
SD	3.3760	3.3761	3.3770
SS	10.1305	10.1311	10.1823
Be_{avg}	0.8223	0.8218	0.8174
Ra = 10^5			
ST	7.7186	7.7097	12.0678
SF	15.3824	15.3058	15.7789
SD	5.43603	5.4118	5.2482
SS	28.5370	28.4273	33.0949
Be_{avg}	0.4203	0.4165	0.4213

Table 8
Effects of the Carreau number(Cu) on different entropy generations in different Rayleigh numbers at Ec=0, D_f=0, n = 1.4, R_d = 0.2 L, Le = 2.5,S_r=0, and N=0.1

	Cu=1	Cu=10	Cu=20
Ra = 10^4			
ST	6.2767	6.2373	6.2288
SF	0.3577	0.1112	0.07186
SD	3.3449	3.2487	3.2262
SS	9.9792	9.5972	9.5269
Be$_{avg}$	0.8376	0.8671	0.8731
Ra = 10^5			
ST	7.4527	6.9121	6.7695
SF	14.7695	5.6493	3.9045
SD	5.1250	4.4753	4.2975
SS	27.3472	17.0367	14.9715
Be$_{avg}$	0.4501	0.5528	0.5978

Table 9
Effects of the vertical distance of the cylinder from the center (δ) on different entropy generations at Ra = 10^5, Ec=0, D_f=0, R_d = 0.2 L, n = 1, Le = 2.5,S_r=0, and N=0.1

	δ=-0.2 L	δ=0	δ=0.2 L
n = 0.2			
ST	9.9135	8.4255	9.4128
SF	11.0906	13.3706	11.6653
SD	7.0517	6.2282	6.4258
SS	28.0559	28.0243	27.5038
Be_{avg}	0.3036	0.3869	0.3847
n = 1			
ST	9.2557	7.7186	8.9462
SF	10.4543	15.3824	13.6346
SD	6.2581	5.43603	5.8904
SS	25.9682	28.5370	28.4713
Be_{avg}	0.3615	0.4203	0.4164
n = 1.8			
ST	8.8935	7.2669	8.6196
SF	8.5386	13.4616	12.4801
SD	5.7758	4.9030	5.4598
SS	23.2080	25.6315	26.5595
Be_{avg}	0.4699	0.4858	0.4797

Table 10
Effect of the radius of the inner cylinder (R_d) on different entropy generations at Ra = 10^5, Ec=0, D_f=0, n = 1, R_d = 0.2 L, Le = 2.5,S_r=0, and N=0.1

	R_d = 0.1 L	R_d = 0.2 L	R_d = 0.3 L	R_d = 0.4 L
n = 0.2				
ST	6.0189	8.4255	11.8295	20.9512
SF	9.4786	13.3706	14.5286	8.7013
SD	4.7651	6.2282	7.4064	11.1330
SS	20.2626	28.0243	33.7646	40.7855
Be_{avg}	0.2886	0.3869	0.4379	0.3989
n = 1				
ST	5.5678	7.7186	11.1291	20.8397
SF	10.5777	15.3824	13.0013	4.6182
SD	4.3131	5.43603	6.5063	10.9538
SS	20.4586	28.5370	30.6367	36.4118
Be_{avg}	0.3331	0.4203	0.4669	0.4293
n = 1.8				
ST	5.1756	7.2669	10.9248	20.7973
SF	9.7920	13.4616	9.1324	2.9721
SD	3.8892	4.9030	6.2354	10.8634
SS	18.8568	25.6315	26.2927	34.6327
Be_{avg}	0.4170	0.4858	0.5215	0.4516

Table 11
Effects of the horizontal distance of the cylinder from the center (Ω) on different entropy generations at Ra = 10^5, R_d = 0.2 L, Ec=0, D_f=0, n = 1, Le = 2.5,S_r=0, and N=0.1

	Ω=-0.2 L	Ω=0	Ω=0.2 L
n = 0.2			
ST	9.7970	8.4255	9.7828
SF	9.5683	13.3706	9.5283
SD	6.5213	6.2282	6.4979
SS	25.8866	28.0243	25.8090
Be_{avg}	0.3952	0.3869	0.3950
n = 1			
ST	9.4149	7.7186	9.4142
SF	10.8030	15.3824	10.8265
SD	6.0994	5.43603	6.09417
SS	26.3173	28.5370	26.3349
Be_{avg}	0.4456	0.4203	0.4456
n = 1.8			
ST	9.0931	7.2669	9.0973
SF	10.1295	13.4616	10.1606
SD	5.7430	4.9030	5.7434
SS	24.9656	25.6315	25.0013
Be_{avg}	0.4994	0.4858	0.4995