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1           **A Dynamic Equilibrium-based Damage**  
2           **Identification Method Free of Structural**  
3           **Baseline Parameters: Experimental Validation**  
4           **in a Two-dimensional Plane Structure**

5  
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20 **Abstract**

21 A damage identification method named “pseudo-excitation” (PE) approach was established  
22 previously, the principle of which resides on local examination of perturbation of structural  
23 dynamic equilibrium conditions. While showing significant sensitivity to structural damage  
24 with small sizes, the approach exhibited high vulnerability to measurement noise due to the  
25 involvement of high order derivatives of the vibration displacements in the expression of the  
26 damage index. On the other hand, several baseline parameters, for example, Young’s  
27 Modulus and Density, are of necessity for the implementation of the approach, considered  
28 as a factor that limits the practical application of the approach. A “weak” formulation of the  
29 PE approach was established to circumvent the interference from measurement noise.  
30 However, the weak formulation of two-dimensional (2D) structural component has not been  
31 developed, and the reliance of the weak formulation on baseline parameters remains an issue  
32 unsolved. In this paper, the 2D weak formulation of the PE approach was proposed by  
33 introducing a weighting function in terms of 2D Gauss function. Through an integration  
34 operation, the selected weighting function was shown able of significantly highlighting the  
35 feature of structural damage and largely suppressing noise influence. Furthermore, a  
36 statistical strategy was developed to estimate the values of baseline parameters inversely,  
37 which signifies the elimination of the dependence of PE approach on pre-obtained baseline  
38 parameters. As a proof-of-concept investigation, multi-damage in a plane structure  
39 consisting of both beam and plate components were identified by using the modified damage  
40 identification method. And a hybrid data fusion algorithm was then used to enhance the  
41 accuracy of damage detection, revealing not only the locations, but also the sizes of damaged  
42 zones.

43 **Keywords:** damage identification, dynamic equilibrium, vibration, statistical estimation,  
44 measurement noise

45

## 46 **Introduction**

47 In recent decades, structural health monitoring (SHM) techniques have been largely  
48 developed with the aim of continuous and automated structural damage evaluation. The  
49 majority of existing SHM techniques were established based on characteristics either of  
50 guided wave propagation or structural vibration. The guided-wave-based methods (using for  
51 instance Lamb waves (Gao et al. 2014; Gao et al. 2016; Ihn and Chang 2008; Liu et al. 2016;  
52 Ostachowicz et al. 2009; Su et al. 2006; Wu et al. 2015; Zhao et al. 2007;)) possess high  
53 sensitivity to structural damage with significantly small size. The accuracy of the methods,  
54 however, could be largely limited due to the complex geometries or boundary conditions of  
55 the structure under inspection. Moreover, active excitation is of necessity for the generation  
56 of wave signals, involving relatively complex experimental configurations. The vibration-  
57 based methods (Farrar et al. 2001; Fan and Qiao 2011; Joshua and Sugunaran 2017), on the  
58 other hand, can be implemented based on the changes in a number of different vibration  
59 signatures, for example, eigen-frequencies (Guo and Li 2011; Lee and Chung 2000; Salawu  
60 1997), mode shape or modal curvature (Cao et al. 2013; Kim et al. 2003; Pandey et al. 1991),  
61 electro-mechanical impedance (Giurgiutiu and Rogers 1998;), flexibility matrix (Aoki and  
62 Byon 2001; Pandey and Biswas 1994; Siddesha and Manjunath 2017; Yan and Golinval  
63 2005) and damping properties (Kawiecki 2001). In engineering applications, these vibration  
64 signatures are convenient to obtain under structural operational state, without any necessity  
65 of active excitation. The effectiveness of traditional guided-wave- or vibration-based  
66 methods, however, largely depend on the obtainment of benchmark structures and baseline  
67 signals that need to be established numerically or experimentally, which considered as a  
68 drawback that limits the efficiency and accuracy of damage identification. To circumvent  
69 such a limitation, a new method relying on the examination of local perturbation of structural  
70 dynamic equilibrium, named Pseudo-excitation (PE) approach, was proposed by the authors

71 (Xu et al. 2011; Xu et al. 2013);. Specifically, the damage index of the PE approach was  
72 derived based on the equation of motion for different types of structural components, e.g.,  
73 beam, plate or shell components. And the locations and sizes of the damaged zones can be  
74 revealed according to the singularities of the damage indices, which correspond to perturbed  
75 dynamic equilibrium conditions. The PE approach was proven capable of detecting damage  
76 with satisfactory sensitivity, without any prior knowledge from the baseline signals or  
77 benchmark structures. However, the method suffers from interference of measurement noise  
78 due to the involvement of high-order derivatives of structural vibration displacements in the  
79 expressions of the damage indices. On the other hand, some baseline parameters (Young's  
80 Modulus, density, etc.) of the inspected structure still need to be obtained before the  
81 implementation of the method, deemed as an obstacle that limit the effectiveness and  
82 efficiency of the original PE approach.

83 The "weak" formulation of the PE index for one-dimensional (1D) beam component was  
84 proposed (Xu et al. 2015a), where different expanded forms can be established to include  
85 the measurement of different mechanical quantities, e.g., vibration displacement and  
86 structural surface strains. The main task of the weak formulation is to reduce the influence  
87 of measurement noise on the accuracy of damage identification. Moreover, by assuming  
88 point-wise satisfaction of dynamic equilibrium along a beam structure, the PE approach  
89 shows potential of damage identification without any dependence on structural baseline  
90 parameters (Xu et al. 2015b). In this study, the application of the modified PE approach was  
91 extended to characterize multi-damage in 2D structures. The weak formulation of 2D  
92 damage index was established, relying on which damage features can be largely highlighted,  
93 benefiting from the effectiveness of noise reduction. In addition, the assumption of point-  
94 wise satisfaction of dynamic equilibrium was made along 2D plate structure. Thus damage  
95 can be detected without baseline parameters. Moreover, some baseline parameters of the

96 inspected structure can be inversely estimated in a statistical manner, showing promising  
97 potential in engineering practices. As a proof-of-concept investigation, multi-damage in a  
98 plane structure, containing both beam and plate components, were identified experimentally  
99 using the proposed method. A hybrid data fusion algorithm was then used to enhance the  
100 accuracy of damage detection, revealing not only the locations, but also the sizes of damaged  
101 zones.

102

### 103 **Modified PE Approach Based on “Weak” Formulation**

104 For a damaged plate structure as shown in Fig. 1, the 2D damage index of the original PE  
105 approach can be expressed as (Xu et al. 2013):

$$106 \quad DI(x, y) = D\Phi(x, y) - \rho h \omega^2 w(x, y), \quad (1a)$$

107 where

$$108 \quad \Phi(x, y) = \frac{\partial^4 w(x, y)}{\partial x^4} + 2 \frac{\partial^4 w(x, y)}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x, y)}{\partial y^4} . \quad (1b)$$

109 In the above equations,  $w(x, y)$  is the vibration displacements;  $D$  is the bending stiffness  
110 equal to  $Eh^3 / 12(1 - \nu^2)$ , where  $E$ ,  $\rho$ ,  $\nu$ ,  $h$  and  $\omega$  are the Young's Modulus, density,  
111 Poisson's ratio, thickness and angular vibration frequency of the plate, respectively. In  
112 numerical computation, Eq. (1b) can be expressed in a discrete form by using a finite  
113 difference scheme (Xu et al. 2013). Equation (1a) was derived based on the equation of  
114 motion for plate structure, indicating the local dynamic equilibrium condition of  
115 infinitesimal plate element. For an element without damage and external excitation,  
116  $DI(x, y)$  in Eq. (1a) should be zero because of the satisfaction of dynamic equilibrium  
117 condition. Where damage exists,  $DI(x, y)$  will show drastic fluctuations in magnitudes,

118 particularly at the boundaries of the damaged zone, due to the violation of the local  
119 equilibrium condition in Eq. (1a).

120

121 From the above equations, it can be seen that a variety of baseline parameters, including the  
122 Young's modulus and different parameters associated with structural geometries, should be  
123 obtained for the construction of Eq. (1a). With the purpose of eliminating the reliance of the  
124 PE approach on structural baseline parameters, the damage index in Eq. (1a) was modified  
125 to be

$$126 \quad \mathfrak{I}(x, y) = \Phi(x, y) - \lambda^* w(x, y), \quad (2a)$$

127 where

$$128 \quad \mathfrak{I}(x, y) = \frac{DI(x, y)}{D} \quad \text{and} \quad \lambda^* = \frac{\rho h \omega^2}{D}. \quad (2b)$$

129 It can be seen that all baseline parameters have been included in the expression of  $\lambda^*$ . The  
130 value of  $\lambda^*$  can be estimated in a statistical manner, which will be introduced in the  
131 following section.

132 The 1D weak formulation of PE approach was developed for the purpose of noise reduction  
133 and enabling the flexibility of measuring multiple types of mechanical quantities for damage  
134 detection, e.g., vibration displacement and surface strains (Xu et al. 2015a). In this study, the  
135 weak formulation for 2D PE approach was derived based on Eq. (2a), expressed as

$$136 \quad \bar{\mathfrak{I}}(x, y) = \int_{\Xi} \mathfrak{I}(x, y) \eta(x, y) dx dy. \quad (3a)$$

137 **Substituting Eq. (2a) into (3a) yields**

$$138 \quad \bar{\mathfrak{I}}(x, y) = \int_{\Xi} \Phi(x, y) \eta(x, y) dx dy - \lambda^* \int_{\Xi} w(x, y) \eta(x, y) dx dy. \quad (3b)$$

139 In the above equations  $\eta(x, y)$  is a weighting function;  $\Xi$  is a 2D region within which the  
140 integration was operated. **For the selection the form of  $\eta(x, y)$ , a main consideration is to**

141 effectively suppress the influence of measurement noise without sacrificing the signal  
 142 feature subject to damage. Another important consideration is to enhance the flexibility of  
 143 damage detection by enabling the measurement of multiple mechanical quantities. The latter  
 144 task has been achieved in 1D case, where the weak formulation was expanded based on  
 145 partial integration principle. For 2D case, the weak formulation in Eq. (3) is also possible to  
 146 be expanded based on principles such as the Greens's theorem. However, relevant study is  
 147 beyond the scope of this work. Therefore, the effectiveness of noise reduction is deemed as  
 148 the major criteria for the selection of  $\eta(x, y)$ . Moreover, after the form of  $\eta(x, y)$  being  
 149 fixed, the size of the integration region  $\Xi$ , needs to be adjusted to obtain optimal accuracy  
 150 of damage identification.

151

## 152 **Estimation of Structural Baseline Parameters**

153

154 In practical applications, structural damage is regarded as local event. Thus, along the  
 155 surface of the structure of interest, most areas can be assumed to be under their intact state,  
 156 which means that the value of Eq. (3b) can be assumed to be zero, i.e.,

$$157 \quad \int_{\Xi} \Phi(x, y)\eta(x, y) dx dy - \lambda^* \int_{\Xi} w(x, y)\eta(x, y) dx dy = 0. \quad (4a)$$

158 Then  $\lambda^*$  can be derived to be

$$159 \quad \lambda^* = \frac{\int_{\Xi} \Phi(x, y)\eta(x, y) dx dy}{\int_{\Xi} w(x, y)\eta(x, y) dx dy}. \quad (4b)$$

160 It can be seen that at the damaged zones, Eq. (4a) does not actually hold, and the calculated  
 161  $\lambda^*$  will differ from its actual value. Along the entire structural surface under inspection,  
 162 however, Eq. (4a) does hold and a large number of  $\lambda^*$  values can be calculated and distribute  
 163 around the actual value of  $\lambda^*$ . Thus the actual value of  $\lambda^*$  can be estimated in a statistical  
 164 way, by calculating the average of all  $\lambda^*$  values along the inspected area. Furthermore,



165 depending on the estimated  $\lambda^*$ , some baseline parameters can be calculated inversely  
166 according to Eq. (2b). For example, the Young's Modulus of the structure can be estimated  
167 when the geometric parameters and the vibration frequency of the plate are given.

168

169 For 1D structural components, i.e., beams, the baseline parameters can be estimated in an  
170 analogous way as presented by Eq. (4a). Since the details of deriving 1D weak formulation  
171 have been provided in (Xu et al. 2015a), only the expression for the inverse estimation of  
172 baseline parameters, developed in this study, is given as

$$173 \quad \lambda^* = \frac{\int_{\Xi} \Phi(x) \eta(x) dx}{\int_{\Xi} w(x) \eta(x) dx}. \quad (4c)$$

174 Equation (4c) can be easily understood since it is a regressive form of Eq. (4b).  $w(x)$  and  
175  $\eta(x)$  in Eq. (4c) are the 1D vibration displacement and weighting function;  $\Phi(x)$  equals to  
176  $d^4 w(x) / dx^4$ ;  $\Xi$  represents a 1D integration region with a selected length. It should be noted  
177 that for beam components, the baseline parameters are included in  $\lambda^*$  according to  
178  $\lambda^* = \rho S \omega^2 / EI$ , where  $S$  and  $I$  are the cross-section area and cross-sectional moment of  
179 inertia, respectively.

180

181 The implementation process of damage identification is illustrated by a flow chart as shown  
182 in Fig. 2. It should be emphasized that given the form of weighting function, the size of  $\Xi$   
183 has a major influence on the accuracy of damage detection. A small  $\Xi$  could lead to high  
184 precision of damage detection by highlighting the local feature of damage, but probably will  
185 be associated with unacceptable noise influence because of insufficient signal averaging  
186 within  $\Xi$ . A larger  $\Xi$  could provide satisfactory noise reduction but may lead to low  
187 detection precision due to “over-smoothing” of the damage feature. Therefore, optimal

188 detection accuracy corresponds to a balance between the detection precision and the noise  
 189 reduction effect. In the following study, the relative size of  $\Xi$  are adjusted referring to the  
 190 vibration wavelengths of tested structures, i.e., for 2D case, the ratio of the side length of  $\Xi$   
 191 to the vibration wavelength. Based on a data fusion algorithm, detection signals subject to  
 192 different ratios are combined to provide the optimal detection results. The 2D data fusion  
 193 algorithm used is

$$194 \quad \bar{\mathfrak{S}}_{i,j\text{-hybrid}} = \bar{\mathfrak{S}}_{i,j\text{-arithmetic}} \cap \bar{\mathfrak{S}}_{i,j\text{-geometric}} \quad (5a)$$

195 where

$$196 \quad \bar{\mathfrak{S}}_{i,j\text{-arithmetic}} = \frac{1}{K} \sum_{L=1}^K \bar{\mathfrak{S}}_{i,j-L}, \quad (5b)$$

$$197 \quad \bar{\mathfrak{S}}_{i,j\text{-geometric}} = \sqrt[K]{\bar{\mathfrak{S}}_{i,j-1} \cdot \bar{\mathfrak{S}}_{i,j-2} \cdots \bar{\mathfrak{S}}_{i,j-L} \cdots \bar{\mathfrak{S}}_{i,j-K}}. \quad (5c)$$

198 In the above equations,  $\bar{\mathfrak{S}}_{i,j\text{-hybrid}}$  is the damage index value at point  $(i,j)$  treated by hybrid  
 199 fusion.  $\bar{\mathfrak{S}}_{i,j\text{-arithmetic}}$  and  $\bar{\mathfrak{S}}_{i,j\text{-geometric}}$  represent damage index treated by arithmetic and  
 200 geometric fusion algorithm, respectively.  $K$  is the number of the total groups of data use for  
 201 fusion, and  $L$  is the index of an individual data group.

202

## 203 **Experimental Validation**

### 204 *Setup*

205 Experimental validation was carried out to examine the reliability of the developed strategy  
 206 in identifying multiple damaged zones in a plane structure containing both beam and plate  
 207 components. The front and back views of the plane structure are shown in Fig 3(a) and (b),  
 208 respectively. The structure was made of aluminum 6061 with a density of  $2.7 \text{ kg/m}^3$  and a  
 209 Young's modulus of 68.9 GPa. The thickness of the aluminum panel is 3 mm. The structure  
 210 was fixed-supported on a testing table (NEWPORT® ST-UT2). From Fig 3(b), it can be  
 211 seen that multi-damage was introduced in both the beam and the plate components.

212 Specifically, there are four damaged zones in the three beam components (beam I, II and III),  
213 in terms of through-width notches measuring 4 mm in length and 1.3 mm in depths. In beam  
214 III there are two damaged zones. A square damaged zone was created in the plate component,  
215 the side length and depth of which are 8 mm and 1.3 mm, respectively.

216

217 The plane structure was excited on the back side using an electro-mechanical shaker  
218 (B&K® 4809) located at the excitation point as shown in Fig. 3(b), in which the **excitation**  
219 **point is 95 mm and 130 mm from the lower and the right boundary of the structure.** A  
220 scanning Doppler laser vibrometer (Polytec®PSV-400B) was used to measure the vibration  
221 displacements of the structure within the inspection regions. As shown in Fig. 3(a), the  
222 square inspection region on the plate component is  $210 \times 210 \text{ mm}^2$  in size, containing  
223  $61 \times 61$  measurement points. **It should be noted that the excitation point is not within the**  
224 **inspection region, but with a distance of 13 mm from the lower edge of the inspection region.**  
225 The inspection regions on the beam components are  $288 \text{ mm}$  in length. There are 73  
226 measurement points along the central line of each beam component.

227 

### 228 ***Damage Identification Results***

229 Under an excitation frequency of 1800 Hz, the distribution of the vibration displacements of  
230 the plate component, within the 2D inspection region, is presented in Fig. 4(a). It can be  
231 calculated that the wavelength of the vibration of the plate component, defined as  $\gamma$ , is  
232 approximately 0.21 m. Based on the measured vibration displacements, the damaged zone  
233 in the plate component was first identified by using the original formulation of the PE  
234 approach as presented in Eq. (1a). It can be understood that to construct Eq. (1a), all baseline  
235 parameters of the structure need to be known. And it is shown in Fig. 4(b) that even using  
236 the baseline parameters, the constructed signal of the damage indices is incapable of

237 reflecting any damage feature because of the severe interference from measurement noise,  
238 associated with the high-order derivatives of vibration displacements, i.e.,  $\Phi(x, y)$  in Eq.  
239 (1b). The density of the 4experimental data in Fig. 4(b) was enlarged by ten times through  
240 an interpolation algorithm before the implementation of weak formulations, aimed at  
241 improving the accuracy of damage detection.

242 The weak formulation of the PE approach, as shown in Eq. (3a), was then applied to identify  
243 the damage. The form of  $\eta(x, y)$  in Eq. (3a) was selected according to a 2D Gauss function.  
244 The 2D Gauss function possesses a highly simple form and an explicit function in signal  
245 processing. On the one hand, the 2D Gauss function is able to highlight damage feature due  
246 to its large similarity with the signal feature associated with damage, that is, large local  
247 singularities (peak values). On the other hand, the gauss function is relatively smooth that  
248 can benefit the averaging of measurement noise within the integration region. Specifically,  
249 the general form of a 2D Gauss function is

$$250 \quad \eta(x, y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2+y^2}{2\sigma^2}}, \quad (6)$$

251 where  $\sigma$  is the standard deviation of the Gauss function.  $\Xi$  in Eq. (3a) is set to be a square  
252 region, the side length of which is defined as  $d$ . It is fixed in the following analysis that  
253  $d = 10\sigma$ . The 2D Gauss function within  $\Xi$  is shown in Fig. 5, from which it can be seen  
254 that the majority of the energy of the Gauss function is included in  $\Xi$ , and at the boundaries  
255 of  $\Xi$  the values of the function vanish. The position of  $\Xi$  was then adjusted along the plate  
256 surface to scan the entire 2D inspection region, and  $\lambda^*$  was calculated according to Eq. (4b).  
257  $\lambda^*$  distributions subject to  $d/\gamma = 0.3, 0.4, 0.5$  and  $0.6$  were calculated, and the distribution  
258 subject to  $d/\gamma = 0.4$  is shown as an example in Fig. 6. From the figure, it can be seen that  
259 the distribution of  $\lambda^*$  values is relatively smooth converging to an average value, although  
260 some significant fluctuations can be observed due to the influence from measurement noise.

261 Large fluctuations were deemed as outliers that distribute outside a pre-defined bandwidth  
262 (10% of the standard deviation of the entire data), and were excluded from the data set. Then  
263  $\lambda^*$  was estimated as the average value of the remaining data. The  $\lambda^*$  values subject to  
264 different  $d/\gamma$ , as selected above, were further averaged leading to an estimated value of  
265  $6.80 \times 10^6$ , around 10% larger than the actual value.

266

267 By using the estimated value of  $\lambda^*$ ,  $\bar{\mathfrak{S}}(x, y)$  was calculated according to Eq. (3b), and the  
268 distributions of  $\bar{\mathfrak{S}}(x, y)$  along the 2D inspection region subject to different  $d/\gamma$  are shown  
269 in Fig. 7(a) to (d). From the figure, it can be seen that subject to a small value of  $d/\gamma$  (i.e.,  
270 0.3), the damage feature can be well highlighted by showing a relatively exact size of the  
271 damaged zone. However, noise influence is relatively severe, causing signal disturbances at  
272 the intact region of the plate. On the other hand, the noise can be increasingly reduced along  
273 with enlarged  $d/\gamma$ , and under  $d/\gamma = 0.6$ , signal disturbances at intact region are largely  
274 suppressed. The damage size, however, is over-estimated under  $d/\gamma = 0.6$  because of the  
275 smoothing effect on the damage feature. An optimal detection accuracy can be found  
276 between  $d/\gamma = 0.4$  and 0.5, corresponding to which a balance between the detection  
277 precision and noise reduction can be achieved. The detection results associated with the four  
278 different  $d/\gamma$  were treated using hybrid data fusion algorithm as shown in Eq. (5a) to (5c),  
279 giving rise to the optimal detection result as shown in Fig. 8(a). It can be seen that the  
280 location and size of the square damaged zone are precisely revealed.

281

282 Finally, damage zones were identified in the beam components based on 1D weak  
283 formulation and the parameter estimation method as shown in Eq. (4c). One-dimensional  
284 Gauss function was selected as the weighting function, and the length of the 1D integration

285 region  $\Xi$  was fixed to be  $10\sigma$  (consistent with that in the 2D case), with  $\sigma$  being the  
286 standard deviation. According to  $d/\gamma = 0.3, 0.4, 0.5$  and  $0.6$  ( $\gamma$  here is the 1D vibration  
287 wavelength of the beam components), distributions of  $\lambda^*$  were calculated according to Eq.  
288 (4c) and then averaged, leading to the estimated value of  $\lambda^*$  being  $7.01 \times 10^6$ , which is 5%  
289 larger than the actual value. The damage identification results subject to the above different  
290  $d/\gamma$  for the three beam components were then treated using a 1D hybrid data fusion  
291 algorithm (analogous with Eq. (5a) to (5c), giving rise to optimal detection results as  
292 presented in Fig. 8(b) to (d), where the  $x$  axis corresponds to a bottom-to-top view of the  
293 beam components in Fig. 3(a) and (b). It can be seen that both the locations and sizes of the  
294 damaged zones can be clearly identified.

295

## 296 **Conclusion**

297 It was demonstrated in this study that the PE approach with its weak formulation can be  
298 effectively used to detect damage in one- and two-dimensional structural components  
299 without any prior knowledge of the baseline parameters of the structure under inspection.  
300 By assuming point-wise satisfaction of dynamic equilibrium conditions, the robustness of  
301 the PE approach can be enhanced in a statistical way. From the experimental results, it can  
302 be observed that, based on well-adjusted form and parameters of a weighting function and a  
303 hybrid data fusion algorithm, the method is able to reveal both the locations and sizes of  
304 damage with satisfactory accuracy and well-controlled measurement noise influence. A  
305 single vibration frequency was selected in the experiment for the construction of the damage  
306 indices. It can be anticipated that by using vibration displacements measured under an  
307 increased number of vibration frequencies, the detection accuracy can be further improved  
308 relying on data fusion algorithm. For the estimation of the baseline parameters, the errors  
309 are mainly attributed to measurement noise. Advanced signal processing technique should

310 be adopted in further study aimed at increasing the accuracy of parameter estimation.  
311 Furthermore, besides 2D Gauss function, the effectiveness of other forms of weighting  
312 function is worthy to be explored in future work.

313

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393 **Figure 1.** A schematic diagram of a plate structure containing a small damage zone

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395 **Figure 2.** A flow chart for the damage detection procedure for two-dimensional structural

396 components

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398 **Figure 3.** (a) Front view of the plane structure, where the inspection regions on the plate and

399 beam components are indicated by dashed red lines, and (b) back view of the plane

400 structure, indicating the locations of the damage zones and excitation point

401

402 **Figure 4.** (a) Distribution of the measured vibration displacements within the inspection

403 region on the plate component subject to vibration frequency of 1800 Hz, and (b)

404 distribution of the damage index constructed using the original PE approach

405 according to Eq. (1a) based on the vibration displacements

406

407 **Figure 5.** A two dimensional Gauss function within a square integration region  $\Xi$ , with a

408 side length of  $d = 10\sigma$

409

410 **Figure 6.** Distribution of the values of  $\lambda^*$  within the inspection region on the plate

411 component of the plane structure, subject to  $d/\gamma = 0.4$

412

413 **Figure 7.** Damage identification results constructed using Eq. (3a) based on estimated

414 value of  $\lambda^*$  within the inspection region on the plate component, subject to (a)

415  $d/\gamma = 0.3$ , (b)  $d/\gamma = 0.4$ , (c)  $d/\gamma = 0.5$  and  $d/\gamma = 0.6$

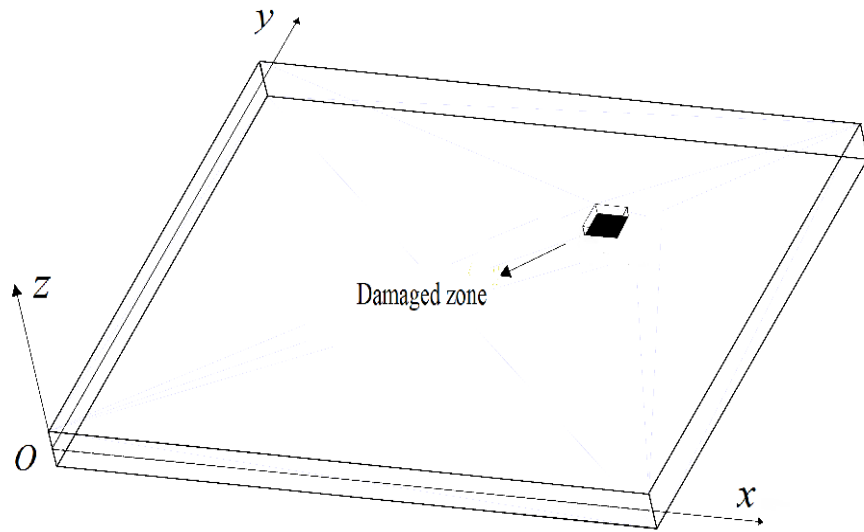
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417 **Figure 8.** Optimal damage identification results treated by data fusion algorithm for the (a)

418 plate component, (b) beam I, (c) beam II and (d) beam III, as indicated in Fig.

419 3(a) and (b)

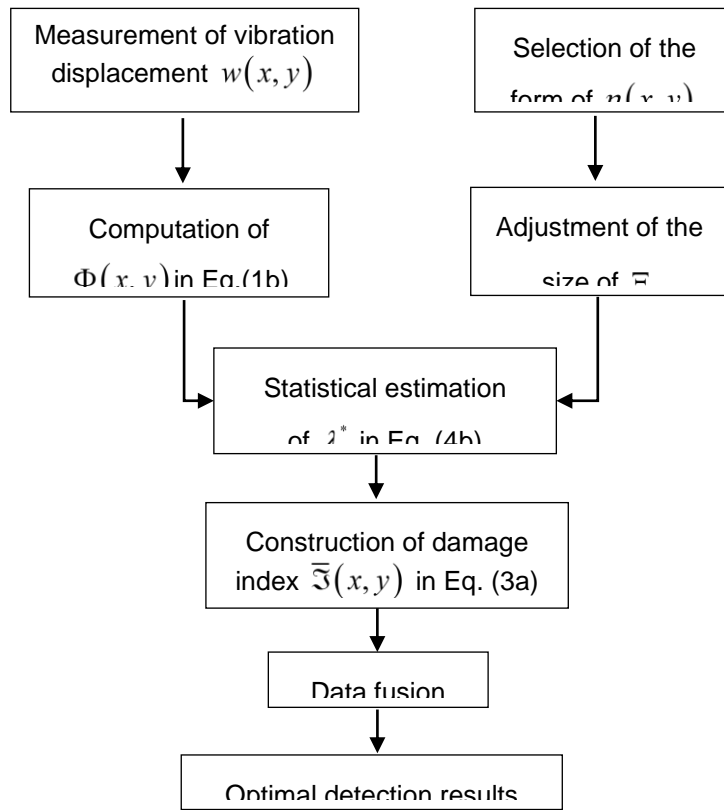
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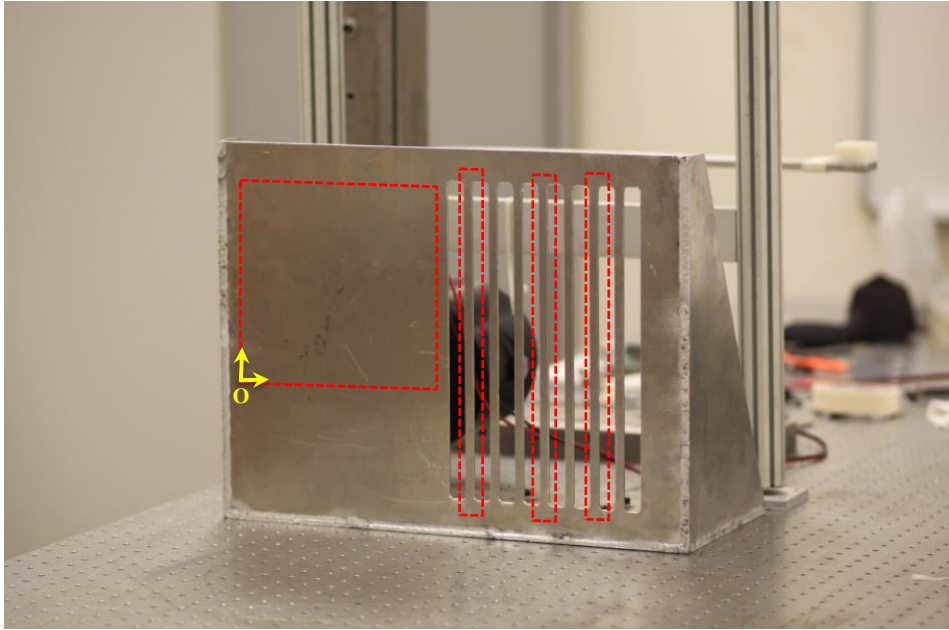
**Figure 1**

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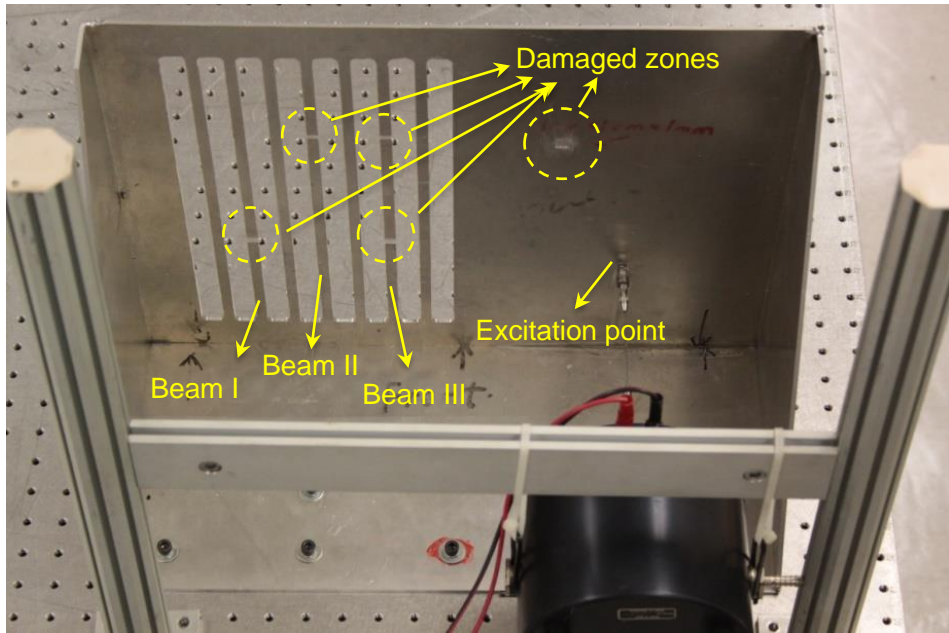
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**Figure 2**



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(a)

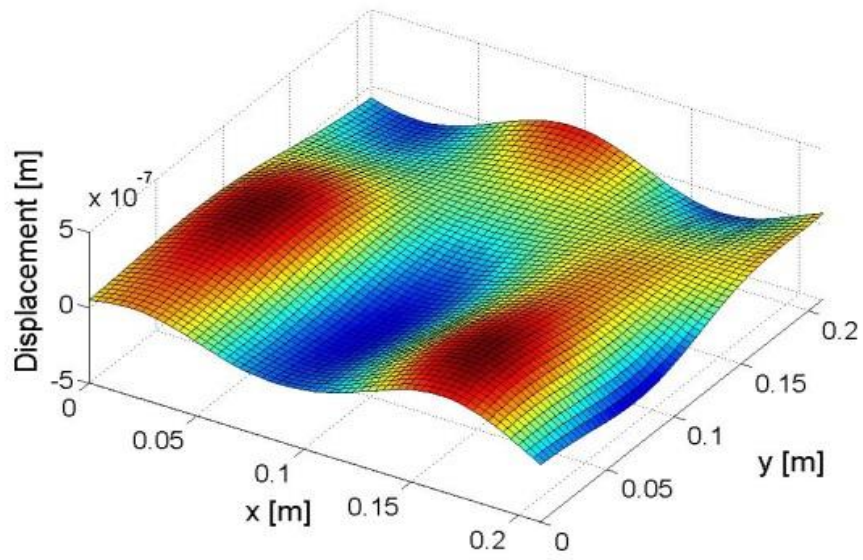


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(b)

Figure 3

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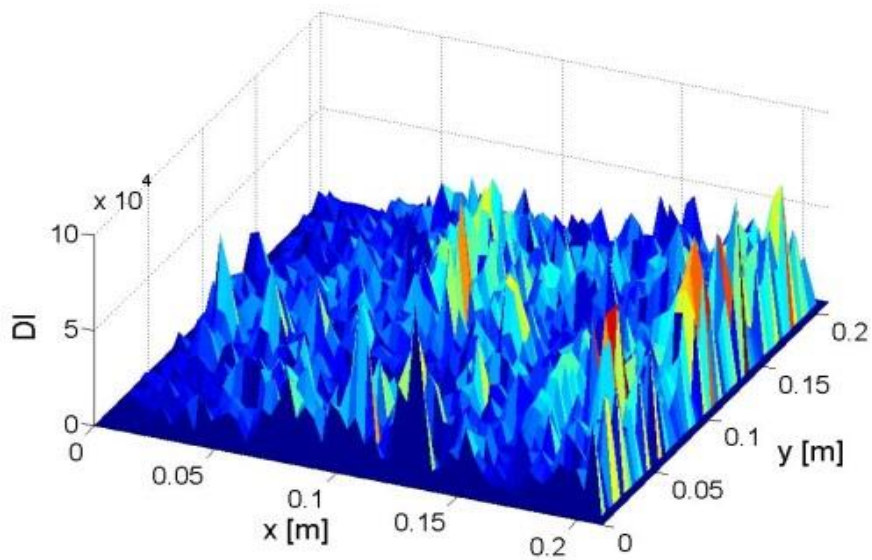
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(a)



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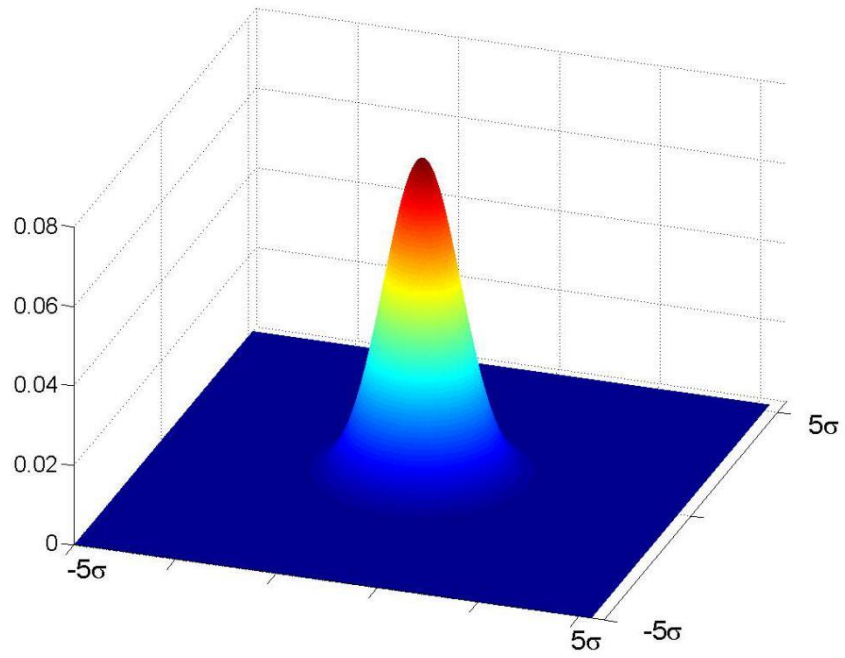
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(b)

Figure 4

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**Figure 5**

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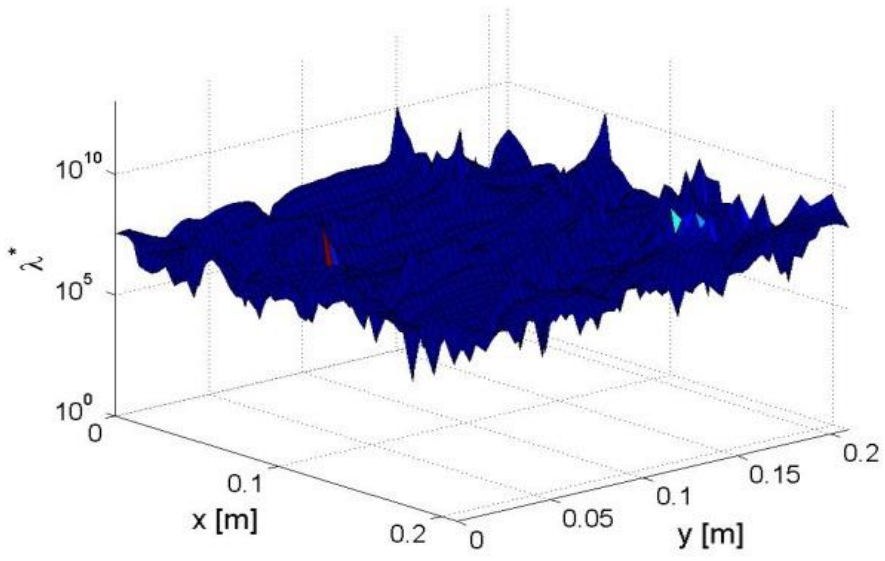
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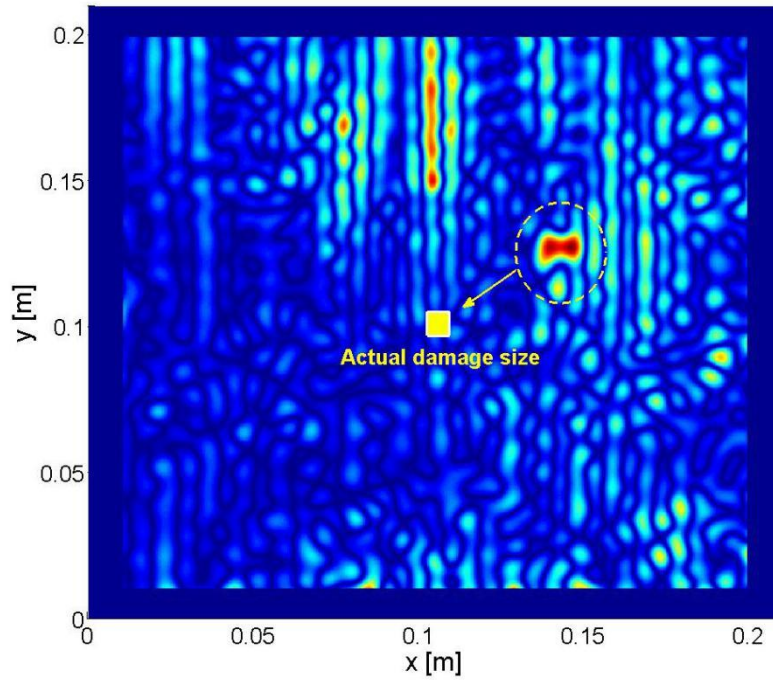
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**Figure 6**

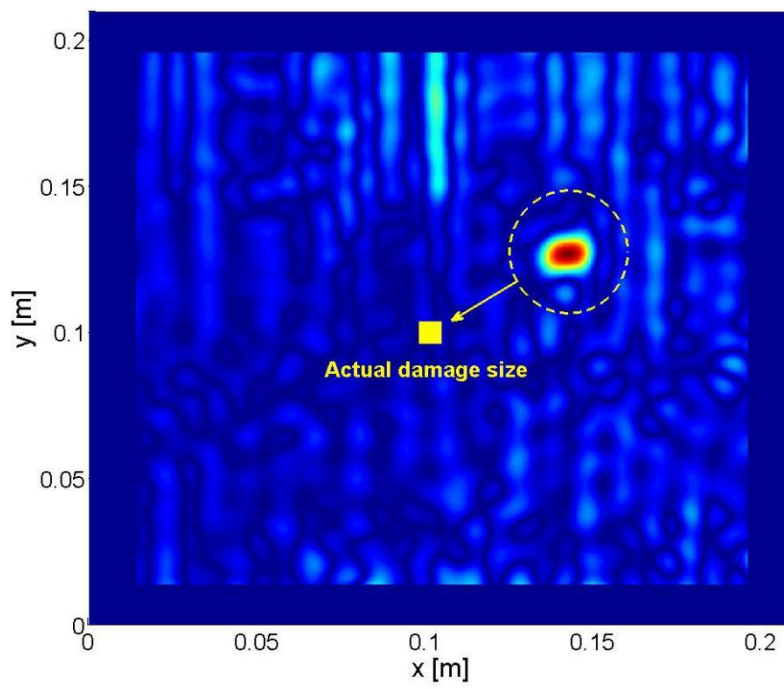


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(a)



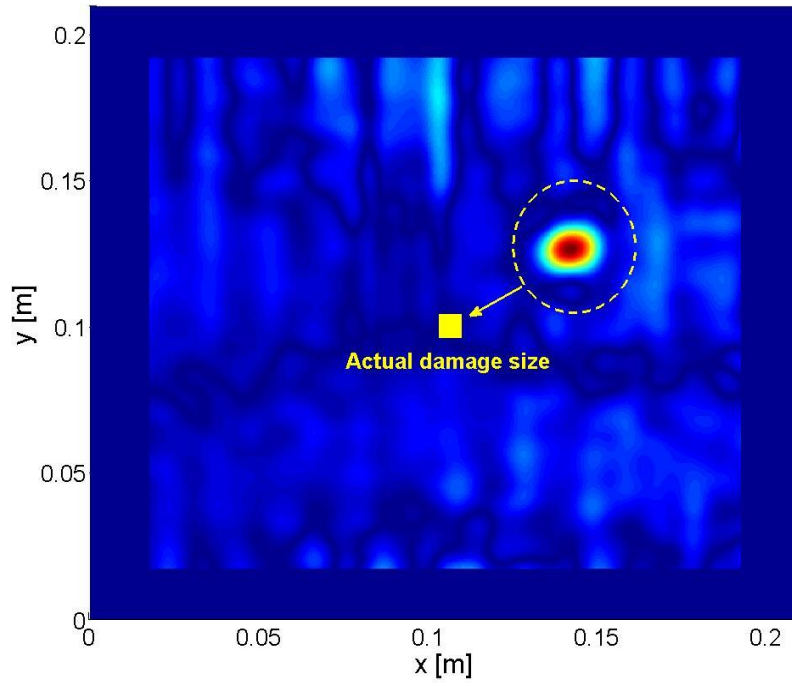
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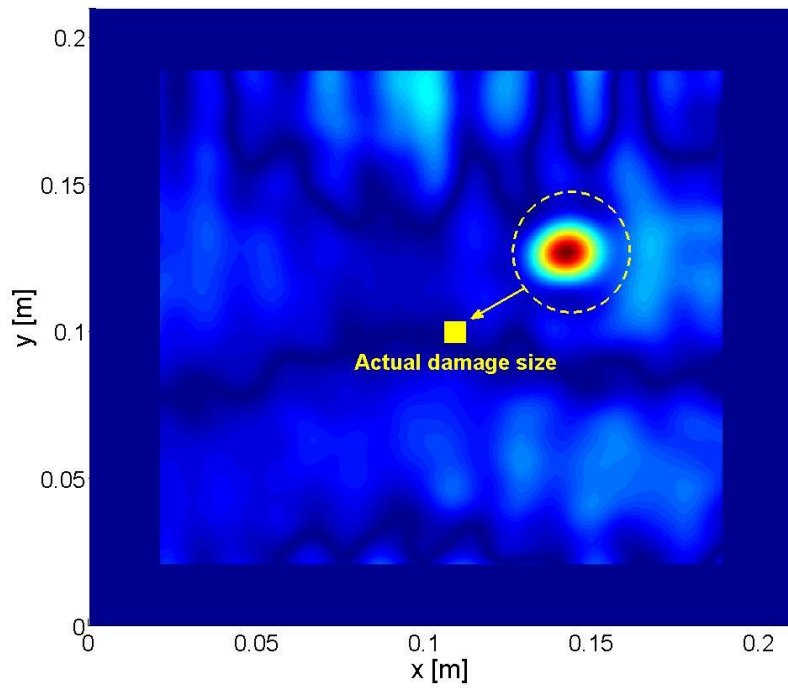
(b)



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(c)



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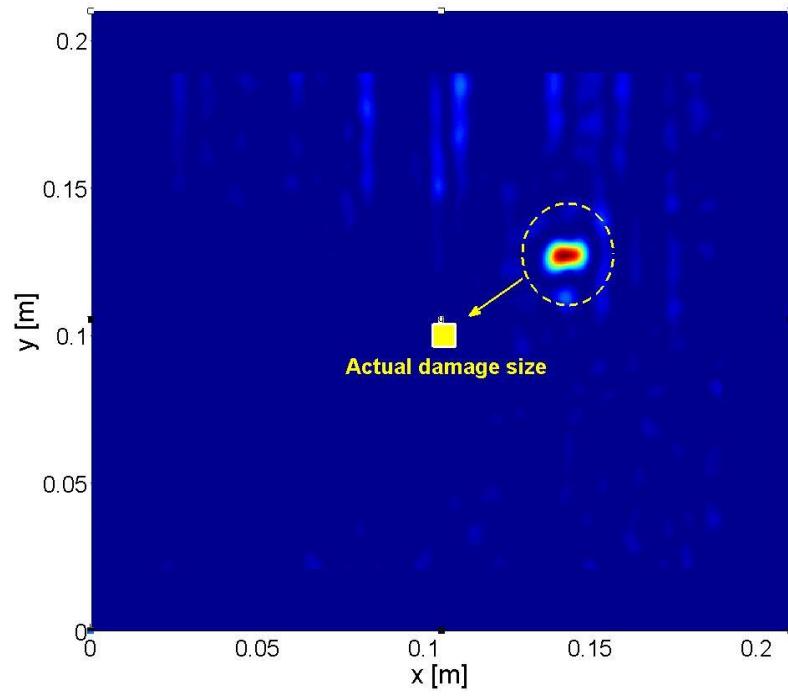
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(d)

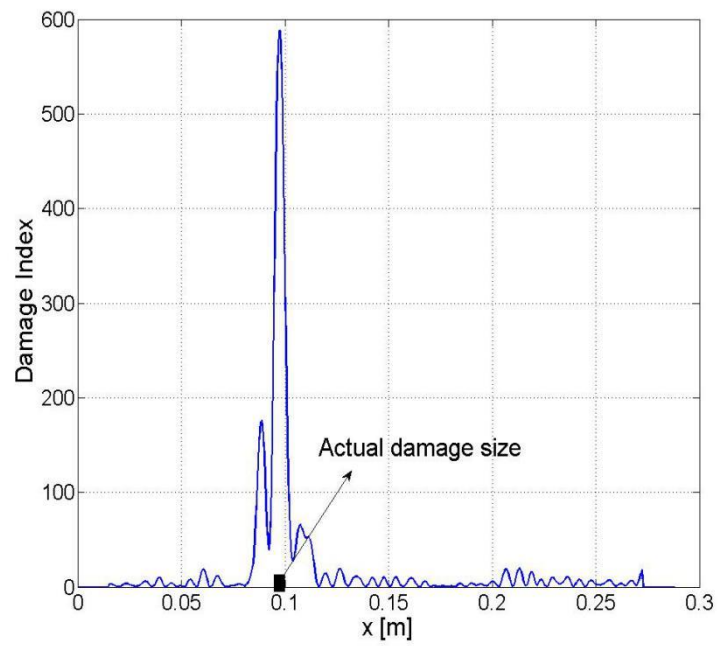
Figure 7



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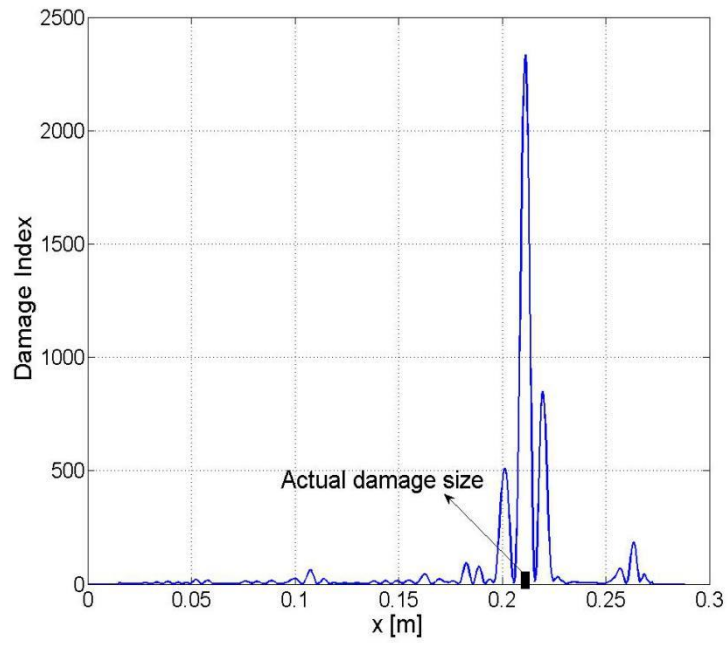
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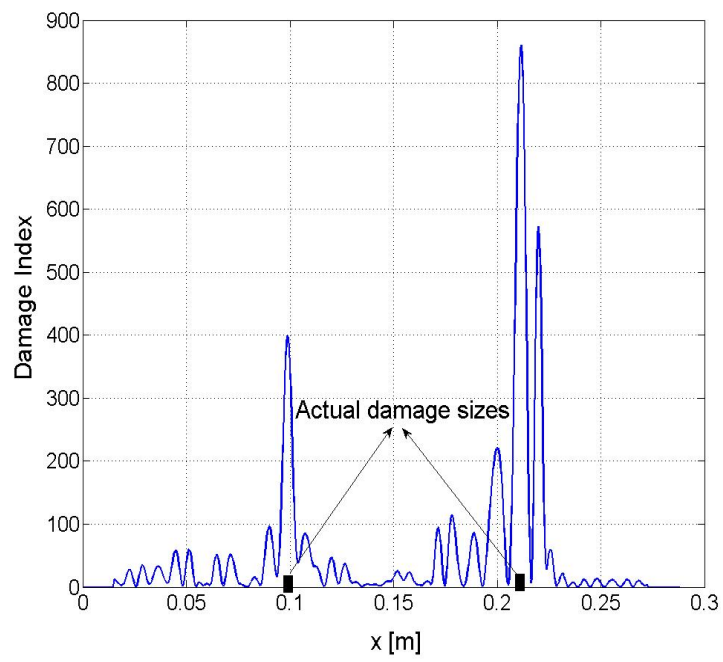
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(c)



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(d)

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**Figure 8**

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