

# Automated identification and characterization of two-dimensional materials via machine learning-based processing of optical microscope images

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## Abstract

Mechanical characterization of two-dimensional (2D) materials has always been a challenging task due to their extremely small thickness. The current prevailing methods to measure the **strength** of 2D materials normally involve sophisticated testing facilities and complicated procedures of sample preparation, which are usually costly and time-consuming. In this paper, we propose a cost-effective and rapid approach to characterizing the **strength** of 2D materials by processing optical microscope images of the mechanically exfoliated 2D materials. Specifically, a machine learning-based model is developed to automate the identification of 2D material flakes of different layers from the optical microscope images, followed by the determination of their lateral size. The statistical distribution of the flakes' size is obtained and used to estimate the **strength** of the associated 2D material based on a distribution-property relationship we developed before. A case study with graphene indicates that the present machine learning-based method, as compared to the previous manual one, enhances the efficiency of characterization by more than one order of magnitude with no sacrifice of the accuracy.

## Keywords:

Image processing and recognition; Machine learning; Inverse problem; Mechanical characterization

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## 26 **1. Introduction**

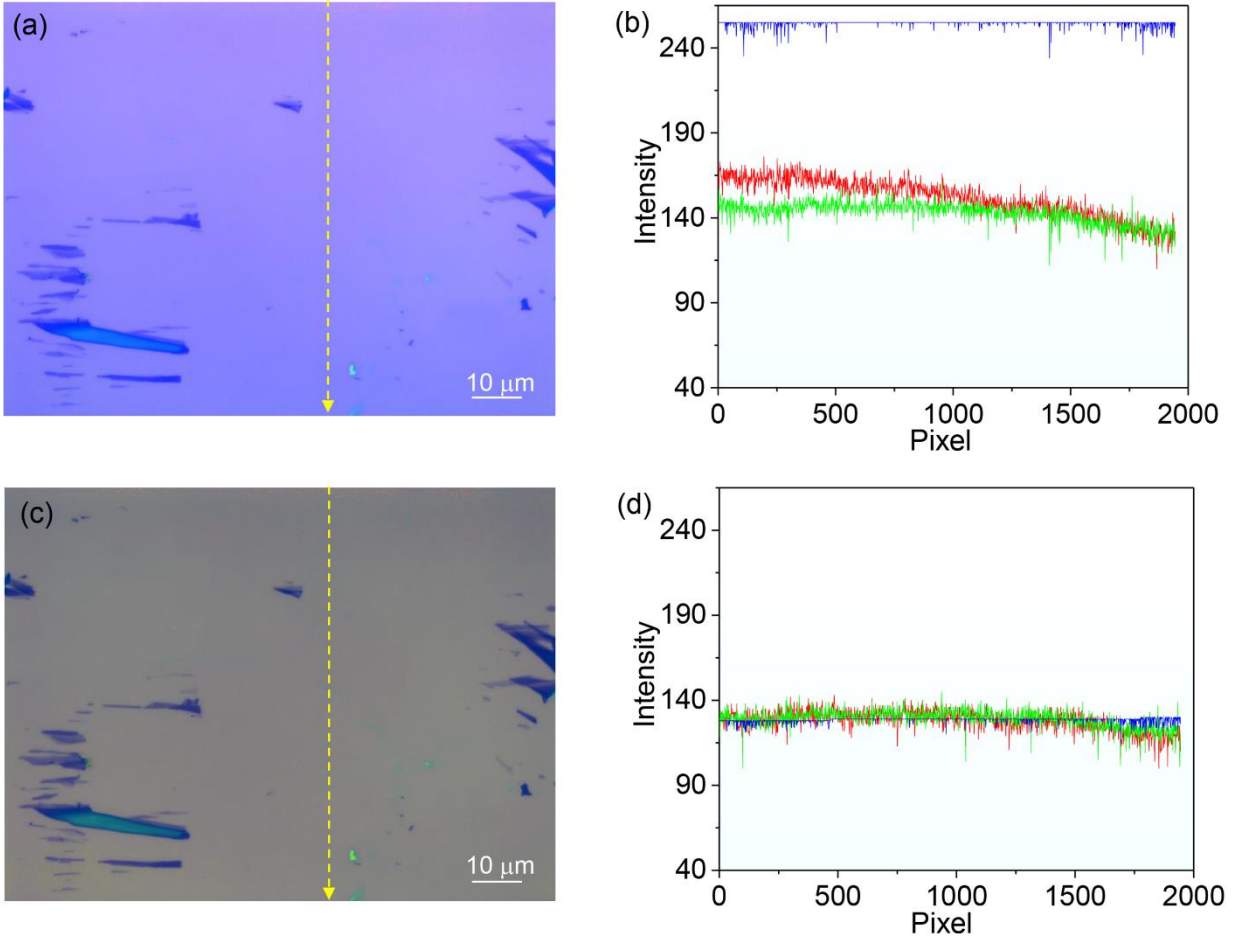
27 Mechanical properties of two-dimensional (2D) materials play important roles in almost  
28 every aspect of their applications in various fields [1, 2]. The past decade has witnessed  
29 considerable efforts and breakthroughs in the characterization of the mechanical properties of 2D  
30 materials [3]. The pioneering and probably the most commonly used method is the indentation on  
31 suspended circular 2D material films using atomic force microscopy (AFM) [2, 4-6]. Mechanical  
32 properties such as Young's modulus and strength can be determined by regression analysis of the  
33 measured force-deflection relationship under indentation with a theoretical prediction [2, 4-6].  
34 Recently, the development of micro-electromechanical systems facilitates more complex  
35 manipulations of the 2D material specimen and provides versatile platforms for *in situ* mechanical  
36 tests in a scanning electron microscope (SEM)/transmission electron microscope (TEM) [7, 8].  
37 However, these methods normally involve specialized equipment and complicated operating  
38 procedure, which are usually costly and time-consuming [9]. The associated micro-mechanical  
39 devices are also vulnerable to external loads and adverse experimental conditions [9, 10]. These  
40 limitations bring up challenges to the fast and accurate characterization of 2D materials.  
41 Nevertheless, our earlier study revealed that the statistical distribution of the lateral size of the  
42 mechanically exfoliated 2D material flakes is intrinsically correlated to their mechanical properties  
43 specifically fracture strength [11]. This offers us a new approach to deducing the [strength](#) of 2D  
44 materials from their size distribution. This method is cost-effective and easy to be implemented  
45 since the information of size distribution can be easily acquired with an optical microscope (OM)  
46 [10]. However, the identification and size measurement of 2D materials from the OM heavily relies  
47 on a visual inspection and personal judgment, which largely limits the efficiency of this method  
48 [12]. To solve this problem, here we propose to automate this process by using image recognition  
49 and processing techniques [12-22], while the implementation of this idea still encounters some  
50 practical challenges. First, the low contrast between the 2D materials and the substrate makes the  
51 identification and classification of them difficult, especially for the monolayer ones [23].  
52 Meanwhile, the presence of the unavoidable adhesive residue and image noise increases the  
53 possibility of misidentification [12]. Moreover, 2D material flakes usually exhibit irregular shapes  
54 [1], [which further increases the difficulty of automatic identification. Recently, machine learning](#)  
55 [techniques such as support vector machine \(SVM\)\[12, 24\], clustering analysis\[13\] and](#)  
56 [convolutional neural networks \(CNN\)\[25-29\] have been applied to automate the identification and](#)

57 classification of materials using their image features. In general, unsupervised techniques like  
58 clustering analysis and CNN are more robust, but they require immense training data and higher  
59 computing resource. On the other hand, supervised machine learning like SVM deals with  
60 prelabeled training data, which not only reduces the computational expense but achieves good  
61 accuracy especially when the dimension of the feature vector is low. In view of these features, a  
62 supervised machine learning-based classification model is proposed to identify 2D materials of  
63 different layers in the light of their color features in an OM image, followed by the size  
64 measurement of the identified flakes. The whole recognition process is implemented automatically  
65 with custom codes, enabling a rapid acquisition of the statistical distribution of the flake size. In  
66 combination with a theoretical model that correlates the size distribution and the strength of the  
67 2D material, characterization of the strength of the associated 2D material can be achieved. In the  
68 rest of this paper, elaboration of this procedure will be made by using mechanically exfoliated  
69 graphene flakes as an example.

## 70 **2. Methods**

### 71 **2.1 Pretreatment of optical microscope images**

72 Due to the experimental uncertainties such as non-uniform illumination, decaying of camera  
73 sensor over time and so on, the image taken by an optical microscope may not be directly used for  
74 identification and characterization. One of the most common problems is the vignetting effect  
75 induced by the lens shading [30, 31]. As shown in Fig. 1(a)-(b), the color profile of the pixels of  
76 the background varies largely with the location. This would greatly impair the accuracy of the  
77 subsequent identification of graphene flakes. To improve the image quality, a series of numerical  
78 image enhancement treatments are applied. First, a median filter is applied to the OM image to  
79 reduce noise by taking the window size of the filter as 3 pixels [10]. Next, a polynomial function  
80 is adopted to fit the color profile of the background [32]. Here, the polynomial is taken as a  
81 quadratic function, which is a good compromise between accuracy and efficiency [10]. Finally,  
82 the fitting background is subtracted from the original image, resulting in an image with significant  
83 improvement in both quality and uniformity, as displayed in Fig. 1(c)-(d). The treated image now  
84 is ready for the identification and subsequent characterization of graphene.



85

86 **Fig. 1.** Pretreatment of an OM image of mechanically exfoliated graphene. (a) A typical OM image of  
 87 graphene on SiO<sub>2</sub>/Si substrate. (b) Color profiles along the path indicated by the yellow dashed line in (a).  
 88 Curves in red, green and blue represent the intensities of the red, green and blue of the pixels, respectively.  
 89 (c) OM image of graphene after pretreatment. (d) Color profiles along the same path as indicated in (a) after  
 90 pretreatment.

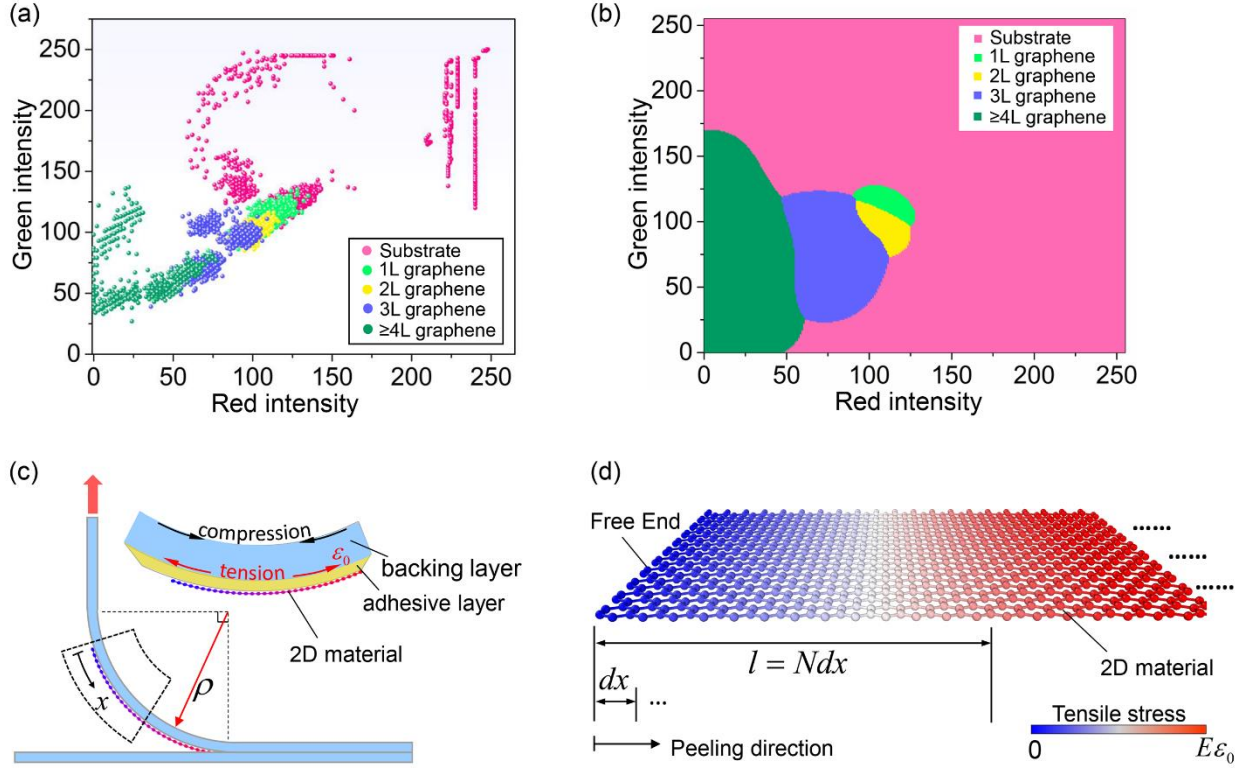
91

## 92 2.2 Identification of graphene flakes

93 To identify graphene flakes from OM images at an affordable computational cost, we use the  
 94 support vector machine (SVM), a supervised machine learning method, to classify graphene of  
 95 different layers and substrate according to their color features in the image. Define a feature vector  
 96  $x = (R, G, B)$ , where  $R, G$  and  $B$  represent the intensities of red, green and blue (RGB) of a pixel,  
 97 respectively. For an 8-bit color graph as we use,  $R, G$  and  $B$  take integers from 0 to  $(2^8 - 1) = 255$ .  
 98 First, we select a typical OM image as the training image and extract sufficient training data set

99  $(\mathbf{x}_i, y_i) = (R_i, G_i, B_i, y_i)$  from it, where  $\mathbf{x}_i$  is the feature vector of the  $i$ -th pixel in the training data  
100 set, and  $y_i$  labels the substance to which the  $i$ -th pixel pertains. For example,  $y_i = 1$  represents the  
101 substrate,  $y_i = 2$  represents monolayer graphene,  $y_i = 3$  represents bilayer graphene, *etc.* The  
102 substance labels ( $y_i$ ) in the training image are determined manually in the light of the contrast  
103 between the few-layer graphene and substrate [33]. Fig. 2(a) shows the training data points of color  
104 feature extracted from a training image. For simplicity, here the projection of the data points on  
105 the red-green (R-G) plane is shown. We can see that the red and green intensities corresponding  
106 to graphene of different layers are distributed in distinct realms on the R-G plane. Data points  
107 pertaining to the substrate exhibit the highest red and green intensities. Due to the presence of  
108 graphene and the increase of layer numbers, the intensities of both colors decrease. The complex  
109 distribution of the data points on the R-G plane makes it difficult to delimitate different realms  
110 with linear boundaries. The Gaussian kernel function is adopted to determine the nonlinear  
111 boundaries between adjacent realms (see **Supplementary material**). Meanwhile, there are some  
112 data points of adjacent realms slightly overlap near the boundaries in Fig. 2(a). These abnormal  
113 data points may result from noise or local nonuniformity. The slack variable vector is used to avoid  
114 overfitting caused by these data points (see **Supplementary material**) [34, 35]. Finally, a  
115 classification model is obtained from the training data (see Fig. 2(b)), which delimits the R-G plane  
116 into five distinct realms with each realm enclosing the R-G information corresponding to a certain  
117 type of substance such as substrate, monolayer graphene, bilayer graphene and so on. After  
118 obtaining the classification model, one can determine the substance label,  $y_i$ , for each pixel in a  
119 testing image according to its color vector  $\mathbf{x}_i$ . Then, the size of a mechanically exfoliated graphene  
120 flake, which is defined as the length along the peeling direction of exfoliation [11], can be  
121 determined from the distance between the leftmost and the rightmost pixels of the flake given that  
122 the peeling direction is horizontal.

123



124

125 **Fig. 2.** (a) The data points used to train the classification model. (b) The as-trained classification model on  
 126 the R-G plane. (c) Schematic of the peeling process during mechanical exfoliation with inset showing the  
 127 closeup near the detaching point (not in scale). (d) Schematic diagram of the graphene layer under tension  
 128 for fracture analysis.

129

### 130 2.3 Mechanical property characterization

131 Our previous study indicated that the size distribution of mechanically exfoliated graphene  
 132 flakes is not entirely random but follows certain statistical patterns [11, 36]. In fact, the flake size  
 133 is determined by the fracture behavior of graphene, which is synergistically controlled by the  
 134 external load and its intrinsic mechanical properties. Considering the brittle nature of graphene,  
 135 we assume it follows the Weibull strength theory. Therefore, the survival (no fracture) probability  
 136 of a graphene flake with a size of  $\Delta x$  under stress  $\sigma$  is given by [37]

137

$$P(\Delta x, \sigma) = \exp\left(-\frac{\Delta x}{l_0} \left(\frac{\sigma}{\sigma_0}\right)^\alpha\right) \quad (1)$$

138 where  $\alpha$  is the Weibull modulus,  $l_0$  is the reference size with  $\sigma_0$  being the corresponding  
139 Weibull characteristic strength. Since the survival probability of material should be independent  
140 of the selection of characteristic parameters,  $l_0\sigma_0^\alpha$  in Eq. (1) should be constant for a given material  
141 [38]. Without loss of generality, in this paper, we take  $l_0 = 1 \mu\text{m}$  [11]. Due to the bending of the  
142 Scotch tape at the peeling edge, the lower side of its backing layer undergoes tensile strain  
143  $\varepsilon_0 = t_B\sqrt{\gamma b / 2K}$  (see Fig. 2(c)), where  $t_B$  is the thickness of the tape backing layer;  $\gamma$ ,  $b$  and  $K$   
144 are the adhesion energy, width and bending stiffness of the tape respectively. By taking  
145  $\gamma = 181 \text{ N}\cdot\text{m}^{-1}$  [39],  $b = 19 \text{ mm}$ ,  $t_B = 0.08\text{mm}$ , and  $K = 0.6526 \text{ N}\cdot\text{mm}^2$  [11], it is estimated that  
146  $\varepsilon_0 = 12.9\%$ . Such tension is transferred to the graphene via the adhesive layer, resulting in non-  
147 uniform tension of the graphene layer. Consider a graphene part with a size of  $l$  as shown in Fig.  
148 2(d). It is composed of  $N$  infinitesimal segments each with a size of  $dx$ . The tensile stress in each  
149 segment can be deemed uniform. The probability of attaining a graphene flake with size larger  
150 than  $l$  equals the probability of survival of  $N$  consecutive segments during the exfoliation, i.e.  
151  $P(\text{size} \geq l) = \prod_{i=1}^N P(dx, \sigma_i)$ , where  $\sigma_i$  is the stress of the  $i$ -th segment. The probability of obtaining  
152 graphene flakes with a length smaller than  $l$  can be thus is given by

$$153 \quad P(\text{size} \leq l) = 1 - P(\text{size} \geq l) = 1 - \prod_{i=1}^N P(dx, \sigma_i) \quad (2)$$

154 By taking the derivative of Eq.(2) with respect to  $l$ , the probability density, which represents  
155 the size distribution of mechanically exfoliated graphene flakes, is given by [11]

$$156 \quad p(l) = \frac{dP(\text{size} \leq l)}{dl} = l_0^{-1}\eta^\alpha \left[ 1 - \exp(-\bar{l}) \right]^\alpha \exp\left( -(\beta l_0)^{-1} \eta^\alpha \int_0^{\bar{l}} [1 - \exp(-x)]^\alpha dx \right) \quad (3)$$

157 where  $\bar{l} = l/l_0$ ,  $\eta = E\varepsilon_0/\sigma_0$ ,  $\beta = \sqrt{G_A/Et_A t}$ . Here,  $E$  and  $t$  represent Young's modulus and  
158 thickness of the graphene, respectively;  $G_A$  and  $t_A$  represent the shear modulus and thickness of  
159 the adhesive layer in the tape, respectively. The parameters  $\alpha$ ,  $\beta$ ,  $\eta$  in Eq. (3) can be determined  
160 by regression analysis using the size distribution obtained by the machine learning-based method

161 above. Taking  $E = 1$  TPa [2, 5] and  $\varepsilon_0 = 12.9\%$ , the Weibull characteristic strength  $\sigma_0$  can be  
162 derived from  $\eta$ . The expectation of the strength can be then determined through [40, 41]

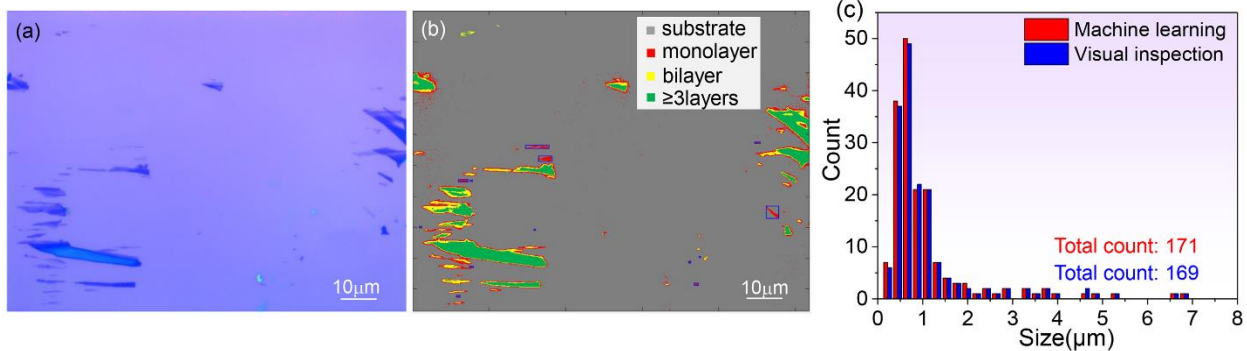
$$163 \quad \langle \sigma_s \rangle = \left( \frac{l}{l_0} \right)^{-\frac{1}{\alpha}} \sigma_0 \Gamma \left( 1 + \frac{1}{\alpha} \right) \quad (4)$$

164 where  $\Gamma(\cdot)$  is the Gamma function. Although the above characterization is implemented on  
165 graphene, the characterization method can be extended to other 2D materials, provided that they  
166 fracture in a brittle manner and the theoretical model is applicable. Moreover, the essence of our  
167 method is to apply the correlation between mechanical properties (strength) and the structural  
168 feature (flake size) to deduce the associated property. This concept can be further generalized to  
169 the characterization of mechanical properties other than strength as long as the correlation can be  
170 revealed. It is also worth noting that the above characterization is based on a quasi-static exfoliation  
171 process. Higher peeling velocity results in higher tensile stress in the 2D material layer and smaller  
172 as-exfoliated flakes. Likewise, reducing the peeling angle below  $90^\circ$  also has a similar effect on  
173 the flake size. Consideration of these effects requires more sophisticated mechanics models [42,  
174 43].

### 175 **3. Results and discussions**

176 Fig. 3(a) shows a typical OM image of mechanically exfoliated graphene with a resolution of  
177  $2592 \times 1944$  pixels corresponding to actual dimensions of  $115 \times 86 \mu\text{m}$ . Fig. 3(b) shows the  
178 identification results of the graphene flakes. In line with the theoretical model in which the 2D  
179 material is assumed as uniform in thickness, here we only focus on the monolayer graphene flakes  
180 which are enclosed by their bounding boxes in Fig. 3(b). The size of these flakes is around a couple  
181 of microns.





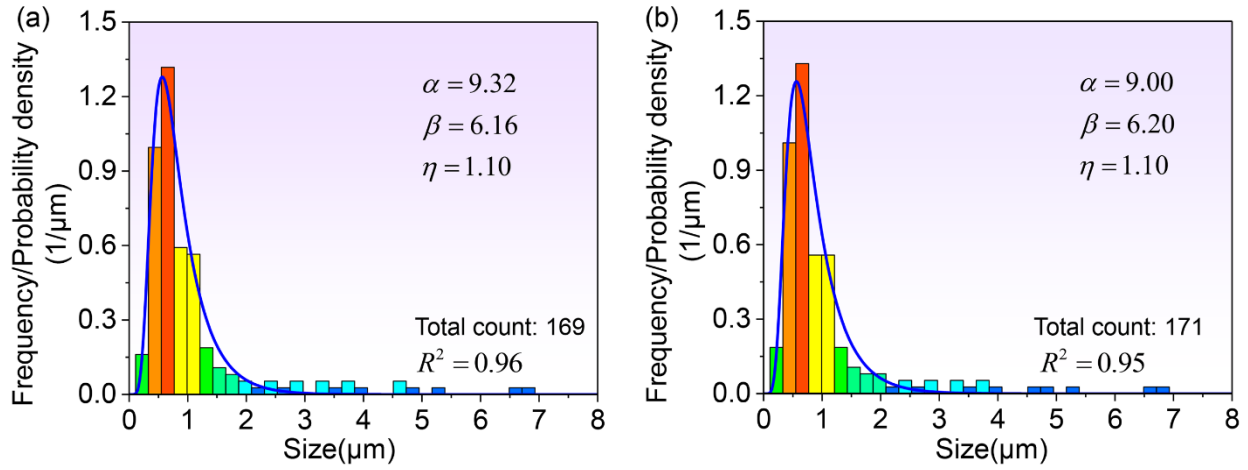
182  
 183 **Fig. 3.** Results of graphene identification and characterization. (a) A typical OM image for testing; (b)  
 184 identification results of graphene flakes corresponding to the test image shown in (a), the blue rectangle  
 185 represents the bounding box of each identified monolayer graphene flake; (c) statistics of size of monolayer  
 186 graphene flakes obtained from machine learning-based recognition and visual inspection, respectively. The  
 187 interval of flake size, 0.22  $\mu\text{m}$ .

188 Automatic identification and size measurement are performed on 18 OM images. 171  
 189 monolayer graphene flakes are identified in total. The histogram in Fig. 3(c) shows the statistics  
 190 of the size of these identified graphene flakes in comparison to that obtained from manual visual  
 191 inspection. The size of the graphene flakes exhibits an asymmetrical distribution about its mean  
 192 value. Results obtained by machine learning-based identification show no significant difference  
 193 from those obtained by visual inspection. A more quantitative comparison shown in Table 1  
 194 indicates that the proposed machine learning-based method can achieve comparable recall and  
 195 precision as the manual inspection does. However, the processing time used in this method is 136  
 196 s, which is one order of magnitude less than that spent by the manual method, implying that the  
 197 efficiency of identification of graphene and size measurement is greatly improved.

198 **Table 1.** Comparison of the performance between the proposed machine learning-based method and the  
 199 manual inspection-based method

	Identified count	Correct count	Actual number	Recall	Precision	Time consumed (second)
Machine learning	171	168	170	98.8%	98.2%	136
Manual inspection	169	169	170	99.4%	100%	~1800

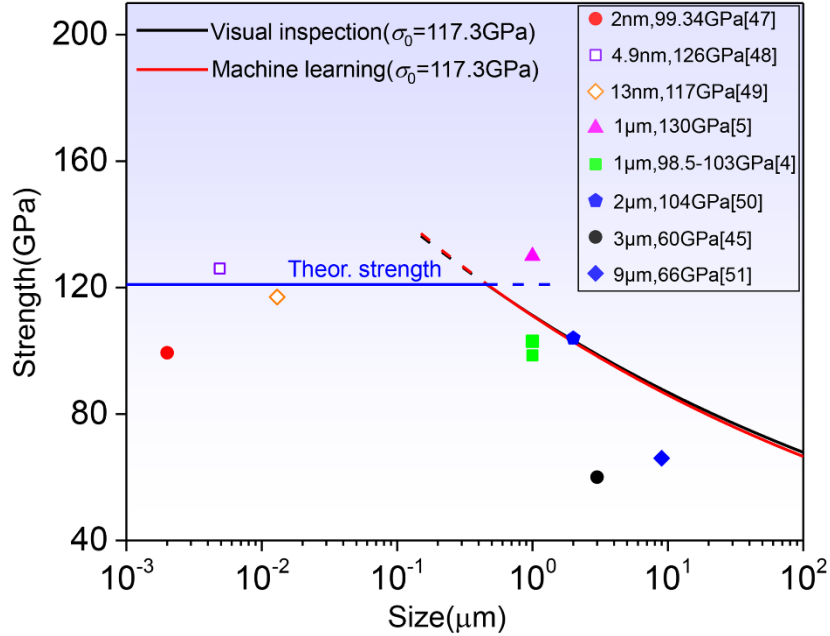
200 *Notes:* Identified count refers to the number of monolayer graphene flakes identified; Correct count refers  
 201 to the number of the identified monolayer graphene flakes that are indeed monolayer; Actual number refers  
 202 to the actual number of monolayer graphene flakes existing in the images, which is determined by cross-  
 203 checking of the machine learning-based and manual inspection methods; Recall is the ratio of the Correct  
 204 count to the Actual number; Precision is the ratio of the Correct count to the Identified count [44].



206

207 **Fig. 4.** The size distribution of monolayer graphene flakes obtained by (a) manual visual inspection and (b)  
 208 machine learning-based automatic characterization method. The frequency density is obtained by dividing  
 209 the count of each bar shown in Fig 3(c) by the product of total count and the bar width. The blue curves are  
 210 the regression curves based on the theoretical expression (Eq. (3)).

211 Based on the statistical distribution shown in Fig. 3(c), the frequency density, which can be  
 212 deemed as the measured probability density in practice, is calculated and shown in Fig. 4, followed  
 213 by regression analysis using Eq. (3). The high  $R^2$  (coefficient of determination) in both cases  
 214 indicates that Eq. (3) describes the probability density of the flake size very well. The values of  $\alpha$   
 215 and  $\eta$  determined by the regression analysis allow us to derive the characteristic strength  $\sigma_0$ .  
 216 Thus, the expectation of the strength of graphene, as given by Eq. (4), is plotted in Fig. 5. It can  
 217 be seen that the results obtained by machine learning-based characterization and visual inspection  
 218 are quite close, and both in reasonable agreement with the data reported in the literature, as  
 219 illustrated by the scatter points. However, some data points exhibit a relatively significant deviation  
 220 from our prediction. For example, the black circle in Fig. 5 indicates that for a graphene monolayer  
 221 with a size of 3  $\mu\text{m}$ , the strength is about 60 GPa [45], which is far below our prediction. This may  
 222 be attributed to the edge defects induced by focused ion beam (FIB) in the sample preparation  
 223 process. These defects would largely affect the strength of the test specimen, as demonstrated by  
 224 the authors with molecular dynamics simulation [45]. In view of this fact, we can conclude that  
 225 our characterization results are consistent with those reported in the literature. The validity of the  
 226 proposed machine learning-based method is confirmed.



227

228 **Fig. 5.** The expected strength of graphene as a function of flake size obtained using the present machine  
 229 learning-based method in comparison with that by the prior visual inspection-based method. The strength-  
 230 size curves are capped by the theoretical strength of graphene (121GPa) obtained from the *ab initio*  
 231 calculation [46]. The scatter symbols represent the results reported in the literature [45, 47-51]. The hollow  
 232 symbols pertain to the computational results while the solid symbols represent the experimental results.

233

## 234 4. Conclusion

235 In summary, we reported an efficient approach to characterizing the thickness, size, and  
 236 strength of mechanically exfoliated 2D materials. A machine learning-based classification model  
 237 is developed to identify 2D material flakes with different thicknesses from their optical microscope  
 238 images, followed by the measurement of the flake size using image processing techniques. This  
 239 procedure is automated, enabling us to quickly identify a sufficient number of 2D material flakes  
 240 and obtain the statistical distribution of their size which, in the light of a theoretical model we  
 241 developed before, can be applied to decipher the strength of the 2D materials. A case study on  
 242 graphene proved that our automated method could achieve comparable accuracy as the manual  
 243 visual inspection does in obtaining the size distribution of graphene flakes, while the efficiency is  
 244 enhanced by more than one order of magnitude. The strength of graphene characterized by our  
 245 method shows good agreement with those reported in the literature. Our work establishes a new  
 246 paradigm of incorporating mechanics modeling and machine learning to achieve high-efficiency

247 material characterization. The method developed in this paper can also be extended to other 2D  
248 materials and is believed to provide valuable references to the high-throughput characterization of  
249 2D materials in various fields.

## 250 **Declaration of competing interest**

251 The authors declare that they have no conflict of interest.

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255 YBXP).

## 256 **Appendix A. Supplementary data**

257 Details on the machine learning technique can be found in the Supplementary material related  
258 to this article.

259

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