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Optimization Models for Reducing Off-Cuts of Raw Materials in Construction Site

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Abstract: More than ten billion tons of construction waste are generated every year in the world. The large volume of construction waste not only increases costs for contractors, but also poses a threat to the environment. A significant proportion of construction waste consists of off-cuts of raw materials. Therefore, to reduce construction waste, this study builds an optimization model to reduce the volume of off-cuts of raw materials. We then develop two solution methods—a mixed-integer linear programming method and a column generation method—to solve the proposed optimization model. We conduct numerical experiments to test the efficiency and applicability of our proposed model. The mixed-integer linear programming method obtains optimal solutions and is suitable for solving small-scale instances, whereas the column generation method gives high-quality solutions within seconds and is suitable for solving large-scale instances. In the large-scale instances, the column generation method reduces waste by over 10% compared to the use of two straightforward decisions rules. Our findings will help construction projects decrease material off-cuts, reduce costs, and achieve sustainable construction.

Keywords: construction waste management; sustainable construction; green construction sites; integer linear programming

MSC: 90-10

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1. Introduction

The rapid expansion of the construction industry in recent years has created a number of problems, including increased environmental pollution. Construction, demolition, and renovation activities are the largest source of solid waste in the world [1–3]. For instance, the European construction sector produces 820 million tons of construction waste (CW) each year, which accounts for 46% of the total solid waste generated in Europe [4]. Although the environmental pollution intensity of CW is low compared to other waste streams, the total environmental impact of CW is considerable because of its high volume and weight. Therefore, the management of CW is an important component of environmental protection programs. For instance, the European Union established a dedicated working group to make recommendations for CW management [5]. In 2018, the European Commission introduced a protocol to further promote and elaborate the management of CW [6,7]. The New Zealand Ministry for the Environment [8] plans to progressively increase the landfill levy of construction waste from the current NZ$10/ton to between NZ$20/ton and NZ$140/ton. The Hong Kong government and its executive arms have introduced dozens of CW management policies, including regulations, codes, and initiatives over the past decades [9].

The above measures are all aimed at reducing construction waste. In terms of sustainable development, the 3 Rs of waste management, reducing, reusing, and recycling, are the basic rules [10]. A key strategy is the implementation of measures to minimize construction waste at the design stage [11], which would contribute to the 'reducing' goal. Hong Kong

Construction Industry Council [12] has highlighted that "it is essential for construction industry to consider the strategy for management and reduction of construction waste". A significant proportion of construction waste consists of off-cuts of construction materials (e.g., steel bars or polyvinyl chloride (PVC) pipes); see Figure 1. Construction projects usually require steel bars or PVC pipes of different lengths. Therefore, they have to cut the raw materials, which are usually a standard length, into smaller pieces. Poorly thought-out cutting plans produce many off-cuts. For example, a construction project has raw steel bars that are 9000 mm long and it needs four steel bars of 2000 mm and two steel bars of 5000 mm. Cutting one raw steel bar into four steel bars of 2000 mm and two raw steel bars into two steel bars of 5000 mm will use three raw steel bars and produce 9000 mm off-cuts. Alternatively, cutting each raw steel bar into two steel bars of 2000 mm and one steel bar of 5000 mm uses only two raw steel bars and produces no off-cuts. Obviously, the second option is a better choice. Unfortunately, many construction sites do not make a material processing plan, resulting in both surplus and waste, which increase environmental pollution. Therefore, it is meaningful to help construction site managers reduce the total quantity of off-cuts and achieve sustainable construction.

Figure 1. Construction waste (source: Australian Department of Agriculture, Water and the Environment [13]).

Our research has three objectives. First, we build an optimization model to help construction projects make optimal decisions on minimizing construction waste. Second, we design two solution methods to solve the proposed mathematical model. One is a mixed-integer linear programming method that can obtain optimal solutions for the proposed mathematical model, and the other is a column generation method which can efficiently obtain approximate optimal solutions when the proposed mathematical model is very large. Third, we apply the proposed model to real-world cases to test its effectiveness and efficiency.

1.1. Literature Review

In this section, we first review studies of models and algorithms used in the field of construction waste management. Then, we review the literature on cutting stock problems.

There are three main streams of research in the field of construction waste management. The first focuses on the effects of economic incentives or equivalent punishments on CW disposal (CWD) behavior. Such schemes have been implemented in many areas around the world [14–16]. A representative scheme is the construction waste disposal charging (CWDC) scheme in Hong Kong, which commenced in December 2005 [17]. Studies have confirmed that CWDC is one of the most successful management approaches to suppressing the negative impacts of CW [9,17]. The second stream of research examines the impact of CW and CW transportation. The impacts of CW occur not only at the disposal stage but also during transportation which aroused little attention [2,4]. CW needs to be transported by heavy diesel oil powered trucks, which are a major contributor to greenhouse gas (GHG) emissions. Maués et al. [2] assess the GHG emission of the CW transportation process in the eastern Amazon. They collect data from large CW generators and CW transportation companies and quantify the carbon dioxide (CO_2) equivalent

(CO_2eq) emitted in November 2019. They find that about 0.9 million kg CO_2 was released into the atmosphere by motor vehicles powered by fossil fuels, which is a considerable contribution to global warming. Besides, heavily loaded trucks create safety issues for drivers and other road users and shorten the use life of road pavement. In addition, the transportation cost is a significant component of the overall CWD cost [1]. The third stream of research uses big data and machine learning methods to develop insights into construction waste management. Hu et al. [18] use a support vector machine (SVM)-based model to predict the amount of construction waste generated. Lu et al. [19] adopt four machine learning methods—multiple linear regression, decision tree, gray models, and artificial neural network—to forecast construction waste generation in the Greater Bay Area, China. Lu et al. [20] use a big dataset of construction waste in Hong Kong to obtain heuristic rules for the bulk densities of construction waste. Yang et al. [21] study how to use machine learning methods to deal with missing data in construction waste management. Yuan et al. [16] develop a big data probability model to estimate waste composition.

We can have two insights from the above studies. First, construction waste management is an important issue that draws the attention of scholars. Second, models and algorithms can have many applications in this topic. Therefore, we next summarize the methods used in cutting stock problem which is highly relevant to our research problem.

The cutting stock problem is a classical topic in operations research [22]. This type of problem usually aims to minimize cost by searching for optimal solutions [23]. Moreover, as this problem can help save materials, it is also useful in realizing sustainable and green development [24,25]. Cutting stock problem is very flexible due to practical situations. Cherri et al. [23] summarize the existing studies that solve the cutting stock problem, and they divide related studies into three categories: heuristics method, item allocation-oriented models, and cutting pattern-oriented models. In our study, we adopt heuristics method and cutting pattern-oriented models. Therefore, we review these two types of classical studies in Table 1. We can find that most of the studies will finally adopt heuristic methods to solve the problem though they initially develop a mathematical model. First fit decreasing (FFD) heuristic is an alternative for solving cutting stock problems [26]. FFD heuristic will start cutting from the longest item, and then the second longest item is considered and so on, which provides an efficient heuristic way for solving the cutting stock problem. In our study, we also list FFD heuristic as a benchmark method in Section 4 and we compare FFD with column generation to give insights. We further review the methods used to generate cutting patterns in the existing literature because cutting patterns are the basis for constructing a mixed-integer linear model to solve cutting stock problems. Ogunranti and Oluleye [27] use an integer model to minimize the off-cuts based on generated cutting patterns. They propose a pattern generation algorithm which traverses from the longest cutting length until the stock cannot be further cut to the minimum required length. Morillo-Torres et al. [28] also develop a pattern generation algorithm. They assume that there will be a maximum of two types of small items in a cutting pattern. Lomate et al. [29] illustrate that the number of cutting patterns could be in the millions and they develop a greedy algorithm to generate cutting patterns. Eshghi and Javanshir [30] also adopt greedy algorithms to generate cutting patterns. Additionally, there are also some studies that do not use cutting patterns to transform the problem into a linear problem [31] or assume that cutting patterns are known parameters [32]. For more details of cutting stock problems, please refer to Cherri et al. [23] and Delorme et al. [33], who give an exhaustive survey of methods in terms of one-dimensional cutting stock problem. In this study, we define parameters to determine the upper and lower bounds of the number of cutting patterns and introduce a binary expression which is equivalent to the number of each cutting pattern that needs to be used under each diameter. This approach helps represent all cutting patterns efficiently and provides the basis for converting the cutting stock problem to a mixed-integer linear programming formulation.

Table 1. Summary of cutting stock studies.

Study	Application	Method(s)
Gradisar et al. [34]	Clothing Industry	Bi-objective model solved by heuristics
Gradisar and Trkman [35]	General one-dimensional cutting stock problem	Heuristic procedure and branch-and-bound
Dimitriadis and Kehris [36]	Manufacturing industry	Heuristics
Cui and Yang [37]	Stock bars	Linear programming and heuristics
Gracia et al. [38]	Construction industry	Heuristics based on Genetic algorithms

As this literature review shows, optimization models can make a difference in waste management. However, there is little research in this area of construction waste management. Unlike residential or commercial waste, careful planning can reduce construction waste at the source. Therefore, we develop an optimization model for reducing the off-cuts at construction sites. Although there are many sophisticated algorithms to solve the cutting stock problem, the applications in construction management are limited. We develop models and algorithms to optimize the cutting of construction materials to reduce waste.

1.2. Objectives and Contributions

The main aim of this study is to develop models and design algorithms that optimize the cutting of construction materials and thus reduce waste. We use mathematical methods to transform our proposed model into a mixed-integer linear programming model, which can give an optimal solution, and into a column generation model, which is efficient. We summarize the theoretical and practical contributions of our study below.

(1) Theoretical contribution. The literature usually adopts heuristic algorithms to solve the cutting stock problem. In this study, we first develop a general model to minimize the off-cuts in construction sites. Additionally, we then develop two solution methods. The first is a mixed-integer linear programming model to obtain exact optimal solutions by considering all possible patterns and the proposed cutting pattern generation method is innovative. The second method is based on column generation, which deals with large scale problems. We compare the effectiveness of column generation method with two heuristics, which could provide insights of these three approaches. Using real-word cases, we demonstrate that our methods are effective and efficient.

(2) Practical contribution. The proposed optimization model and the two solution methods can be used to reduce the waste produced by cutting construction materials, e.g., steel bars and PVC pipes. Our study will help construction contractors reduce waste, save costs, and achieve sustainable and green construction targets.

The remainder of this paper is organized as follows. Section 2 presents a mathematical model for minimizing construction waste. Section 3 proposes two solution methods to solve our proposed model. Section 4 conducts numerical experiments in real-world cases that show the effectiveness and applicability of the proposed methods. Conclusions are presented in Section 5. Main symbols used in the paper are listed in Table 2.

Table 2. Symbols.

Sets	
I	Set of categories of steel bars, $I = \{1, \ldots, \lvert I \rvert\}$
J_i	Set of types of steel bars of diameter d_i, $i \in I$, $J_i = \{1, \ldots, \lvert J_i \rvert\}$
K_i	Set of cutting patterns for raw steel bars of diameter d_i, $i \in I$, $K_i = \{1, \ldots, \lvert K_i \rvert\}$
Indices	
$i \in I$	A category of steel bars
$j \in J_i$	A type for category i
$k \in K_i$	A cutting pattern for category i

Table 2. *Cont.*

Parameters	
d_i	The diameter of steel bars of category i
l_{ij}	The length of steel bars of diameter d_i and type j that the site requires
n_{ij}	Total number of steel bars of the length l_{ij} of diameter d_i and type j that the site requires
L_i	The length of raw steel bars of diameter d_i that the plant sells
Decision Variables	
v_i	Number of raw steel bars of diameter d_i to purchase from the plant
x_{ijk}	Number of steel bars of length l_{ij} that a raw steel bar of of diameter d_i and cutting pattern k will be cut into
y_{ik}	Number of raw steel bars of diameter d_i that will be cut according to pattern k

2. Model

We use steel bars as an example when building our optimization model. As steel bars have good performance in resisting tensile forces, they are widely used in the construction of reinforced concrete structures [39]. A large number of steel bars are produced every year to meet the needs of the construction industry. For example, according to the National Bureau of Statistics of China [40], China produced more than 267 million tons of steel bars in 2020. However, due to the irregular operation of workers on construction sites, wastage of steel bars is common. Therefore, we use steel bars as an example in our study of construction waste.

We consider a construction site that requires a set of different categories of steel bars. Each category is defined by a diameter. We use the set $I = \{1, \ldots, |I|\}$ to denote the categories. The diameter of the steel bars in category $i \in I$ is denoted by d_i (mm). For example, in a study of construction costs, Lee and Ahn [41] use 22 mm-diameter steel bars for reinforced concrete beams and 25 mm-diameter steel bars for reinforced concrete columns. In that case, $I = \{1,\ 2\}$, $d_1 = 22$, and $d_2 = 25$. Another example is an actual case of building a residential building in Chengdu, China. Steel bars with diameters of 12 mm, 14 mm, 16 mm, 18 mm, 20 mm, and 22 mm are used in the construction. Therefore, $I = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$, $d_1 = 12$, $d_2 = 14$, $d_3 = 16$, $d_4 = 18$, $d_5 = 20$, and $d_6 = 22$.

The construction site in our example requires different lengths of steel bars for each diameter d_i. We use set $J_i = \{1, \ldots, |J_i|\}$ to denote the types for category i. The length of steel bars of type $j \in J_i$ of category $i \in I$ is denoted by l_{ij} (mm). In the residential building example, the construction site requires 18 mm-diameter steel bars with lengths of 950 mm, 1220 mm, 1510 mm, 2050 mm, 2090 mm, 2220 mm, 2430 mm, 2450 mm, 2580 mm, 2690 mm, 2730 mm, 2740 mm, 3520 mm, 4070 mm, 4480 mm, and 7240 mm. Then, $J_4 = \{1,\ \ldots, 16\}$, $l_{4,1} = 950$, $l_{4,2} = 1220$, $l_{4,3} = 1510$, $l_{4,4} = 2050$, $l_{4,5} = 2090$, $l_{4,6} = 2220$, $l_{4,7} = 2430$, $l_{4,8} = 2450$, $l_{4,9} = 2580$, $l_{4,10} = 2690$, $l_{4,11} = 2730$, $l_{4,12} = 2740$, $l_{4,13} = 3520$, $l_{4,14} = 4070$, $l_{4,15} = 4480$, and $l_{4,16} = 7240$. For each $j \in J_i$ and each $i \in I$, a total of n_{ij} steel bars of the length l_{ij} are required. Construction sites usually buy steel bars of different diameters from steel plants based on their specific needs. For each diameter d_i of steel bars, the plant sells raw steel bars with the length L_i (mm) at certain price; $L_i \geq \max\{l_{ij},\ j \in J_i\}$, $i \in I$. The government usually regulates the length of raw steel bars and issues product standard documents. For example, China stipulates that the length of raw steel bars should be 9 m or 12 m [42], as these standardized lengths are convenient for truck transportation. When deciding how many raw steel bars of each diameter to buy from the plant, the construction project seeks to minimize the total cost while fulfilling the needs of the site. As off-cuts will be scrapped, identifying the smallest possible number of raw steel bars will also minimize the amount of construction waste.

We define the decision variable v_i as the number of raw steel bars of category $i \in I$ (each has a length L_i) that will be bought from the steel plant, and the function $F_i\left(v_i, n_{i1}, \ldots, n_{i|J_i|}\right)$ as 1 if there is a way of cutting v_i raw steel bars of length L_i into n_{i1} steel bars of length l_{i1}, ... , $n_{i|J_i|}$ steel bars of length $l_{i|J_i|}$ altogether, and 0 otherwise. Z_+ represents the set of nonnegative integers. We aim to minimize the total cost, which is equivalent to minimizing the total number of raw steel bars because the price is fixed. A mathematical programming model for the problem is

[M1]

$$\min \sum_{i \in I} v_i \tag{1}$$

subject to

$$F_i\left(v_i, n_{i1}, \ldots, n_{i|J_i|}\right) = 1,\ i \in I \tag{2}$$

$$v_i \in Z_+,\ i \in I \tag{3}$$

It is evident that the decision variables v_i for raw steel bars of different diameters can be optimized independently. In other words, we need to solve $|I|$ independent mathematical programming models:

[M2-*i*]

$$\min v_i \tag{4}$$

subject to

$$F_i\left(v_i, n_{i1}, \ldots, n_{i|J_i|}\right) = 1 \tag{5}$$

$$v_i \in Z_+ \tag{6}$$

To solve [M2-*i*], we first derive a naïve upper bound on the optimal value of v_i by requiring that each raw steel bar is cut into steel bars of only one length. For each $j \in J_i$, a raw steel bar of length L_i can be cut into $\lfloor L_i/l_{ij} \rfloor$ steel bars of length l_{ij}, where $\lfloor x \rfloor$ is the largest integer not greater than x. Therefore, to get n_{ij} steel bars of length l_{ij}, we will need $\lceil n_{ij}/\lfloor L_i/l_{ij} \rfloor \rceil$ raw steel bars, where $\lceil x \rceil$ is the smallest integer greater than or equal to x. We again take the construction of the residential building as an example. The construction project needs 40 steel bars with a diameter of 18 mm and a length of 4070 mm, and the length of the raw steel bars with a diameter of 18 mm is 9000 mm. Therefore, $L_4 = 9000$ mm, $l_{4,14} = 4070$ mm, and $n_{4,14} = 40$. We can calculate $\lfloor L_4/l_{4,14} \rfloor = 2$, i.e., a raw steel bar of 9000 mm can be cut into at most two steel bars of 4070 mm, and $\lceil n_{4,14}/\lfloor L_4/l_{4,14} \rfloor \rceil = 20$; thus, we need at least 20 raw steel bars of 9000 mm to produce 40 steel bars of 4070 mm. Hence, the construction site needs a maximum of $V_i^{\max}$ raw steel bars of diameter d_i:

$$V_i^{\max} = \sum_{j \in J} \lceil n_{ij}/\lfloor L_i/l_{ij} \rfloor \rceil \tag{7}$$

because the site can use $\lceil n_{ij}/\lfloor L_i/l_{ij} \rfloor \rceil$ raw steel bars to cut into n_{ij} steel bars of length l_{ij}, $j \in J_i$.

It is clear that using the cutting method discussed above (i.e., each raw steel bar is cut into steel bars of only one length) will produce a great deal of extra material, which in practice will be scrapped. An ideal plan for cutting the raw steel bars that produces the least amount of off-cuts is assuming that all the v_i raw steel bars are connected as a long raw steel bar of length $v_i L_i$. Based on this ideal way, we can derive a lower bound on the optimal value of v_i, denoted by $V_i^{\min}$:

$$V_i^{\min} = \left\lceil \left(\sum_{j \in J} n_{ij} l_{ij} \right) / L_i \right\rceil \tag{8}$$

In the above example, with $L_4 = 9000$ mm, $l_{4,14} = 4070$ mm, and $n_{4,14} = 40$, the lower bound is equal to 19. In sum, the optimal solution of v_i satisfies

$$V_i^{\min} \le v_i \le V_i^{\max}. \tag{9}$$

Model [M2-*i*] remains unsolvable, as the form of Constraint (5) is unknown. To express Constraint (5) mathematically, we first illustrate the meaning of "cutting pattern" using the example shown in Table 2. Suppose $L_i = 9000$ mm. J_i has four lengths: 7000 mm, 5000 mm, 4000 mm, and 2000 mm. Then, the following are a few possible cutting patterns: (i) cut a raw steel bar into one steel bar of 7000 mm and one of 2000 mm, (ii) cut a raw steel bar into one steel bar of 5000 mm and one of 4000 mm, (iii) cut a raw steel bar into one steel bar of 5000 mm and two of 2000 mm, (iv) cut a raw steel bar into two steel bars of 4000 mm, (v) cut a raw steel bar into one of steel bar of 4000 mm and two of 2000 mm, or (vi) cut a raw steel bar into four steel bars of 2000 mm.

We use set K_i to denote the cutting patterns for the raw steel bars of diameter d_i; $k = 1, \ldots, |K_i|$ denotes the cutting pattern. As we will use at most $V_i^{\max}$ raw steel bars of diameter d_i, the raw steel bars of diameter d_i will have at most $V_i^{\max}$ cutting patterns in the optimal solution. Therefore, we define $|K_i| = V_i^{\max}$. Taking the example given in Table 3, suppose we need 30 steel bars of 7000 mm, 50 of 5000 mm, 60 of 4000 mm, and 20 of 2000 mm; then, $V_i^{\max}$ equals 115 according to Equation (7). Hence, $K_i = \{1, \ldots, 115\}$. We can reformulate [M2-*i*] by defining the decision variable x_{ijk} as the number of steel bars of length l_{ij} that a raw steel bar of diameter d_i and of cutting pattern k will be cut into, and decision variable y_{ik} as the number of raw steel bars of diameter d_i that will be cut according to pattern k. The mathematical programming model for the problem is:

[M3-*i*]

$$\min \sum_{k \in K_i} y_{ik} \tag{10}$$

subject to

$$\sum_{j \in J_i} l_{ij} x_{ijk} \leq L_i,\ k \in K_i \tag{11}$$

$$\sum_{k \in K_i} y_{ik} x_{ijk} \geq n_{ij},\ j \in J_i \tag{12}$$

$$x_{ijk} \leq M y_{ik}, j \in J_i,\ k \in K_i \tag{13}$$

$$x_{ijk} \in Z_+, j \in J_i,\ k \in K_i \tag{14}$$

$$y_{ik} \in Z_+,\ k \in K_i. \tag{15}$$

Table 3. Cutting patterns.

Pattern No.	Length of Steel Bars the Site Requires (l_{ij})	Cutting Pattern
1	7000, 5000, 4000, 2000	7000 × 1, 2000 × 1
2	7000, 5000, 4000, 2000	5000 × 1, 4000 × 1
3	7000, 5000, 4000, 2000	5000 × 1, 2000 × 2
4	7000, 5000, 4000, 2000	4000 × 2
5	7000, 5000, 4000, 2000	4000 × 1, 2000 × 2
6	7000, 5000, 4000, 2000	2000 × 4

Objective Function (10) represents the total number of raw steel bars that will be cut. Therefore, we can minimize the total construction waste by minimizing Function (10). Constraints (11) ensure that the total cut length is not greater than the length of the raw steel bar, and Constraints (12) guarantee that the total number of steel bars can fulfill the requirements of the construction site. In Constraints (13), M is a large positive number that can ensure that $x_{ijk} > 0$ only when $y_{ik} > 0$. We can set M to be $\min\{n_{ij}, \lfloor L_i / l_{ij} \rfloor\}$. Constraints (14) and (15) restrict the values of the decision variables x_{ijk} and y_{ik} to be nonnegative integers.

3. Solution Method

Model [M3-*i*] is still difficult to solve because it has a large number of integer decision variables and Constraints (12) have the nonlinear term $y_{ik} x_{ijk}$. We develop two solution methods: a mixed-integer linear programming method that can obtain optimal solutions for small-scale instances, and a column generation-based heuristic that can obtain high-quality solutions for large-scale instances.

3.1. Mixed-Integer Linear Programming Method

In our first method, we reformulate model [M3-i] into an integer linear optimization model. Note that an upper bound for y_{ik}, $k \in K_i$, is $\min\left\{|K_i|, \max_{j \in J_i} n_{ij}\right\}$. The upper bound $|K_i|$ is valid because $v_i \le V_i^{\max}$ and $\sum_{k \in K_i} y_{ik} = v_i$. Additionally, the upper bound $\max_{j \in J_i} n_{ij}$ is valid because if $y_{ik} > \max_{j \in J_i} n_{ij}$, we can reset $y_{ik} \leftarrow \max_{j \in J_i} n_{ij}$ and the resulting solution is still feasible. Therefore, we define a parameter:

$$Q_i = \left\lfloor \log_2 \min\left\{|K_i|, \max_{j \in J_i} n_{ij}\right\}\right\rfloor \tag{16}$$

and define intermediate binary decision variables z_{ikq}, $q = 0, 1, \ldots, Q_i$. Then, instead of using general integer decision variable y_{ik} to represent the number of raw steel bars of diameter d_i that will be cut according to pattern k, we can use binary decision variables z_{ikq}:

$$y_{ik} = 2^0 z_{ik0} + 2^1 z_{ik1} + \ldots + 2^{Q_i} z_{ikQ_i} = \sum_{q=0}^{Q_i} 2^q z_{ikq},\ k \in K_i. \tag{17}$$

Equation (17) can be viewed as a binary representation of a number. For example, if $\min\left\{|K_i|, \max_{j \in J_i} n_{ij}\right\}$ equals 5, then Q_i equals 2. Therefore, $z_{ik0} = 1$, $z_{ik1} = 0$, $z_{ik2} = 1$, and $y_{ik} = 2^0 \times 1 + 2^1 \times 0 + 2^2 \times 1 = 5$. Then, Constraints (12) can be rewritten as

$$\sum_{k \in K_i} \sum_{q=0}^{Q_i} 2^q z_{ikq} x_{ijk} \ge n_{ij},\ j \in J_i. \tag{18}$$

Constraints (18) are still nonlinear. We can define intermediate decision variables u_{ijkq} and Constraints (18) are equivalent to

$$\sum\nolimits_{k \in K_i} \sum_{q=0}^{Q_i} 2^q u_{ijkq} \ge n_{ij},\ j \in J_i \tag{19}$$

$$u_{ijkq} \le z_{ikq} x_{ijk},\ j \in J_i, k \in K_i, q = 0, \ldots, Q_i \tag{20}$$

Note that an upper bound on x_{ijk} is $X_{ij}^{\max} = \min\left\{\lfloor L_i / l_{ij} \rfloor, n_{ij}\right\}$. Thus, Constraints (20) are equivalent to

$$u_{ijkq} \le \min\left\{x_{ijk}, X_{ij}^{\max} z_{ikq}\right\},\ j \in J_i, k \in K_i, q = 0, \ldots, Q_i. \tag{21}$$

It is evident that Constraints (21) can be written as the following linear constraints

$$u_{ijkq} \le x_{ijk},\ j \in J_i, k \in K_i, q = 0, \ldots, Q_i \tag{22}$$

$$u_{ijkq} \le X_{ij}^{\max} z_{ikq},\ j \in J_i, k \in K_i, q = 0, \ldots, Q_i. \tag{23}$$

Now, we have a mixed-integer linear programming model [M4-i]:

$$\min \sum\nolimits_{k \in K_i} y_{ik} \tag{24}$$

subject to Constraints (11), (13)–(15), (17), (19), (22)–(23).

3.2. Column Generation Method

To solve large-scale instances, we apply a column generation method. As we define $|K_i| = V_i^{\max}$, the scale of the integer decision variables will be very large when the construction site requires many steel bars of different types (i.e., $|J_i|$ and n_{ij} are big numbers); this makes the MILP model in Section 3.1 hard to solve. Therefore, we reformulate Model [M3-*i*] into a column generation model that can solve large-scale instances in our second method.

Column generation is formulated through a linear programming (LP) relaxation of the restricted master problem (RMP) and a subproblem. The LP relaxation of the RMP is solved by Simplex and the subproblem is used to determine whether the unconsidered variables can improve the LP relaxation of the RMP. The unconsidered variables that can make the reduced cost of the LP relaxation of the RMP less than 0 are added to the LP relaxation of the RMP. In large-scale instances, most columns never enter the basic matrix (e.g., most patterns in K_i are never used in our study), and we can therefore reduce the scale by not generating any or by generating only a small number of these unused columns. In other words, finding an initial set of feasible basic patterns is easy in our problem (e.g., we can let each pattern consist of only one length). Therefore, starting from an easily identified initial solution, the key task is to find other cutting patterns that can improve our objective function.

We use the set $\hat{K}_i$ to denote columns that are already generated, and each column represents a cutting pattern. For each pattern $k \in \hat{K}_i$, a raw steel bar can be cut into $\hat{x}_{ijk}$ steel bars of length l_{ij}. Note that the values of $\hat{x}_{ijk}$ are known. We reformulate [M3-*i*] into an RMP and a subproblem in the column generation model. The LP relaxation of the RMP is

[M5-LP-RMP-*i*]

$$\min \sum\nolimits_{k \in \hat{K}_i} y_{ik} \tag{25}$$

subject to

$$\sum\nolimits_{k \in \hat{K}_i} \hat{x}_{ijk} y_{ik} \geq n_{ij},\ j \in J_i \tag{26}$$

$$y_{ik} \geq 0, k \in \hat{K}_i. \tag{27}$$

We relax the integer constraints of y_{ik} in [M5- LP-RMP-*i*]. Since the value of $\hat{x}_{ijk}$ are known, [M5-LP-RMP-*i*] is a linear programming problem and can be solved efficiently. We denote by λ_j the values of the dual variables of Constraints (26). The subproblem is built to check if there exist other optimal patterns to [M5-LP-RMP-*i*]:

[M5-Subproblem-*i*]

$$\min 1 - \sum\nolimits_{j \in J_i} \lambda_j x_{ij,|\hat{K}_i|+1} \tag{28}$$

subject to

$$\sum\nolimits_{j \in J_i} l_{ij} x_{ij,|\hat{K}_i|+1} \leq L_i \tag{29}$$

$$x_{ij,|\hat{K}_i|+1} \in Z_+,\ j \in J_i. \tag{30}$$

Equation (28) is the reduced cost of [M5-LP-RMP-*i*]. If the optimal value of Equation (28) is less than 0, we find a better pattern, $|\hat{K}_i| + 1$, and a raw steel bar can be cut into $\hat{x}_{ij,|\hat{K}_i|+1}$ steel bars of length l_{ij}. Then, the pattern $|\hat{K}_i| + 1$ is added to $\hat{K}_i$. [M5-Subproblem-*i*] is an integer programming problem with $|J_i|$ decision variables. We repeat to solve [M5-RMP-*i*] and [M5-Subproblem-*i*] until the optimal value of [M5-Subproblem-*i*] is greater than 0. Finally, we have the set $\hat{K}_i$ that includes all of the cutting patterns in the optimal solution to [M5-LP-RMP-*i*].

As we relax the integer constraints of y_{ik} in [M5-LP-RMP-*i*], we solve the following integer programming problem to obtain the final value of y_{ik}:

[M6-*i*]

$$\min \sum\nolimits_{k \in \hat{K}_i} y_{ik} \tag{31}$$

subject to

$$\sum\nolimits_{k \in \hat{K}_i} \hat{x}_{ijk} y_{ik} \geq n_{ij},\ j \in J_i \tag{32}$$

$$y_{ik} \in Z_+, k \in \hat{K}_i. \tag{33}$$

In the above model, the values of $\hat{x}_{ijk}$ are known. Therefore, [M6-*i*] has high computing efficiency. The whole column generation method is illustrated in Figure 2.

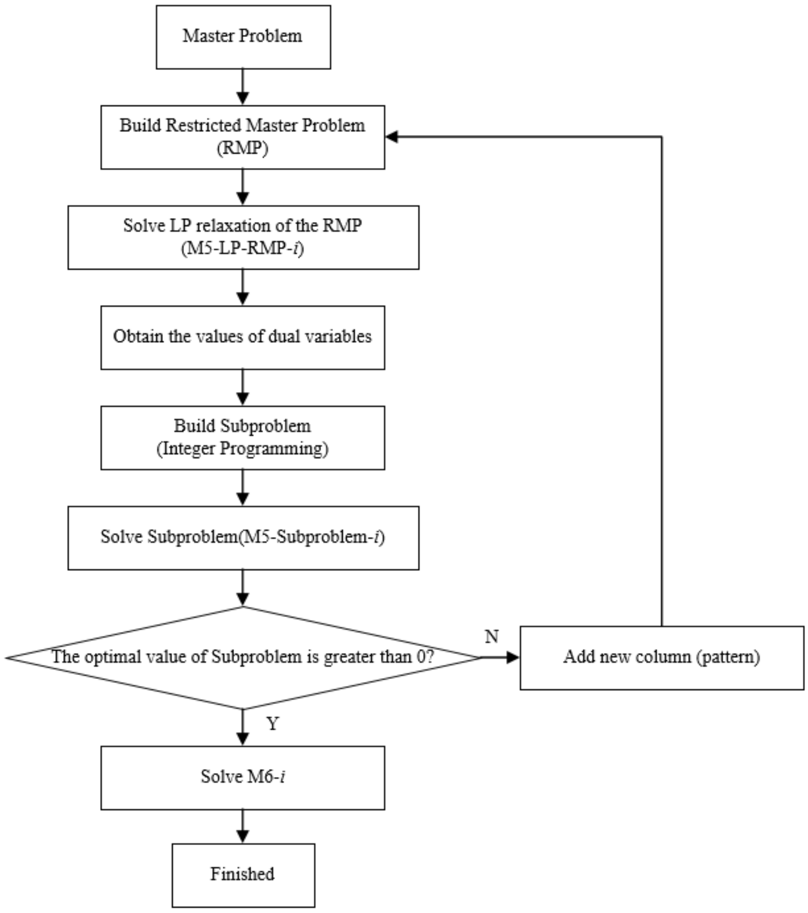

Figure 2. Column generation method.

4. Numerical Experiments

In this section, we report the results of numerical experiments that test the effectiveness and efficiency of the proposed mixed-integer linear programming and column generation methods. The experiments are run on a laptop computer equipped with 2.60 GHz of Intel Core i7 CPU and 16 GB of RAM, and models are solved using CPLEX Python API 20.1.0.

4.1. Data

The data for our experiment are drawn from a real-world case: a residential building in Chengdu, China. As shown in Table 4, we collect data on the number of 12 mm and 18 mm steel bars required for the construction of Floors 4 to 7. The project requires 24 different lengths of 12 mm steel bars and 16 different lengths of 18 mm diameter steel bars. The steel plant sells raw steel bars of different diameters that are all 9000 mm in length.

Table 4. Data for the required numbers of steel bars of diameter 12 mm and diameter 18 mm.

Diameter (d_i)	**Types of Steel Bars (j)**	**Length of Steel Bars the Site Requires (l_{ij})**	**Total Number of Steel Bars the Site Requires (n_{ij})**
12 mm	1	7860	2
	2	7460	4
	3	7220	4
	4	7060	4
	5	4950	2
	6	4920	8
	7	4420	8
	8	3860	4
	9	3560	4
	10	3390	4
	11	3260	4
	12	3160	2
	13	3120	4
	14	2850	4
	15	2810	12
	16	2360	10
	17	2060	12
	18	1960	6
	19	1710	2
	20	1690	2
	21	1520	2
	22	1390	2
	23	1360	4
	24	760	4
18 mm	1	7240	4
	2	4480	2
	3	4070	4
	4	3520	2
	5	2740	4
	6	2730	2
	7	2690	4
	8	2580	2
	9	2450	4
	10	2430	2
	11	2220	2
	12	2090	4
	13	2050	4
	14	1510	4
	15	1220	2
	16	950	2

Therefore, we know that in our experiment, $I = \{1, 2\}$, $d_1 = 12$, $d_2 = 18$, $L_1 = 9000$, $L_2 = 9000$, $J_1 = \{1, \ldots, 24\}$, and $J_2 = \{1, \ldots, 16\}$. The values of l_{ij} and n_{ij} can be obtained from Table 4 (e.g., $l_{1,1} = 7860$ and $n_{1,17} = 12$).

4.2. Computational Analysis

We examine the effectiveness of the proposed mixed-integer linear programming method using the subsets of J_1 and J_2. Specifically, for the 12 mm diameter steel bars, we consider 2, 4, 6, and 8 different lengths required by the construction site, and for the 18 mm diameter steel bars, we consider 3, 5, 7, and 9 different lengths required by the construction site. We thus have eight instances to solve. The solutions and the CPU times for each instance are shown in Table 5. The CPU time increases rapidly with the number of types of steel bars because a larger J_i means more decision variables and constraints in the model. Therefore, the proposed mixed-integer linear programming method is suitable for obtaining optimal solutions for small-scale instances, but it is not a good choice for solving large-scale problems because of the CPU time requirements. From the results in Table 5, we can also find that when there are more than six types of steel bars, the CPU time exceeds 600 s, which is not feasible in practical applications. Hence, we suggest that using six types as the quantitative number for dividing large-scale and small-scale problems. That is, cases with less or equal to six types of steel bars should be solved by the mixed-integer linear programming and cases with more than six types of steel bars should be solved by the column generation method.

Table 5. Data for the required numbers of steel bars of diameter 12 mm and diameter 18 mm.

Diameter	Types of Steel Bars	Solutions (Cutting Pattern × Number of Raw Steel Bars, i.e., $\{\sum_{j\in J} l_{ij}\hat{x}_{ijk}\}\times y_{ik}$)	CPU Time (s)
d_1	$\{1,2\}$	$\{7860\times 1\}\times 2$, $\{7460\times 1\}\times 4$	0.05
	$\{1,2,3,4\}$	$\{7860\times 1\}\times 2$, $\{7460\times 1\}\times 4$, $\{7220\times 1\}\times 4$, $\{7060\times 1\}\times 4$	0.30
	{1,2,3,4,5,6}	$\{7860\times 1\}\times 2$, $\{7460\times 1\}\times 4$, $\{7220\times 1\}\times 4$, $\{7060\times 1\}\times 4$, $\{4950\times 1\}\times 2$, $\{4920\times 1\}\times 8$	1.73
	{1,2,3,4,5,6,7,8}	N.A.	>600
d_2	$\{1,2,3\}$	$\{7240\times 1\}\times 4$, $\{4480\times 1,\ 4070\times 1\}\times 2$, $\{4070\times 2\}\times 1$	0.09
	$\{1,2,3,4,5\}$	$\{7240\times 1\}\times 4$, $\{4480\times 2\}\times 1$, $\{4070\times 2\}\times 2$, $\{3520\times 1,\ 2740\times 2\}\times 2$	1.09
	{1,2,3,4,5,6,7}	$\{7240\times 1\}\times 4$, $\{4480\times 2\}\times 1$, $\{4070\times 2\}\times 2$, $\{3520\times 1,\ 2740\times 2\}\times 2$, $\{2730\times 2,\ 2690\times 1\}\times 1$, $\{2690\times 3\}\times 1$	33.11
	{1,2,3,4,5,6,7,8,9}	N.A.	>600

Next, we assess the computational efficiency of the column generation method. The example shown in Table 4 can be solved within 1 s, as shown in Table 6. According to the results, there are 21 and 14 cutting patterns in the optimal solutions for steel bars with diameters of 12 mm and 18 mm, respectively. Therefore, our results can help guide workers on construction sites to minimize total cost and reduce waste.

To further test the computational efficiency of the column generation method, we generate instances with 50, 100, 200, 500, and 1000 different lengths of steel bars (i.e., $|J_i| \in \{50,\ 100,\ 200,\ 500,\ 1000\}$). For each $|J_i|$, we randomly generate 10 instances, each of which involves randomly generated lengths and randomly generated demand for each length of steel bars. The average CPU times over the ten instances are shown in Table 7. The largest instances with $|J_i| = 1000$ can be solved in less than 13 s on average. Therefore, the column generation method shows excellent performance in computational efficiency and is efficient enough for practical purposes.

Table 6. Data for the required numbers of steel bars of diameter 12 mm and diameter 18 mm.

Diameter	Types of Steel Bars	Solutions (Cutting Pattern × Number of Raw Steel Bars, i.e., $\{\sum_{j\in J} l_{ij}\hat{x}_{ijk}\}\times y_{ik}$)	CPU Time (s)
d_1	$\{1, \ldots, 24\}$	$\{7860 \times 1\} \times 2$, $\{7460 \times 1,\ 1520 \times 1\} \times 4$, $\{7220 \times 1\} \times 4$, $\{7060 \times 1,\ 1690 \times 1\} \times 4$, $\{4950 \times 1,\ 3860 \times 1\} \times 2$, $\{4920 \times 1,\ 2360 \times 1,\ 1390 \times 1\} \times 2$ $\{4920 \times 1,\ 3560 \times 1\} \times 4$ $\{4920 \times 1,\ 2810 \times 1\} \times 2$ $\{4420 \times 2\} \times 4$ $\{3860 \times 2\} \times 1$ $\{3390 \times 2,\ 2060 \times 1\} \times 2$ $\{3260 \times 2,\ 2360 \times 1\} \times 1$ $\{3260 \times 1,\ 2850 \times 2\} \times 2$ $\{3160 \times 2,\ 2360 \times 1\} \times 1$ $\{3120 \times 1,\ 2810 \times 2\} \times 4$ $\{2850 \times 1,\ 2810 \times 2\} \times 1$ $\{2360 \times 3,\ 1710 \times 1\} \times 2$ $\{2060 \times 4,\ 760 \times 1\} \times 2$ $\{2060 \times 2,\ 1960 \times 1\} \times 1$ $\{1960 \times 4,\ 760 \times 1\} \times 1$ $\{1360 \times 6,\ 760 \times 1\} \times 1$	0.40
d_2	$\{1, \ldots, 16\}$	$\{7240 \times 1,\ 1510 \times 1\} \times 4$, $\{4480 \times 2\} \times 1$, $\{4070 \times 2\} \times 2$, $\{3520 \times 2,\ 950 \times 2\} \times 1$, $\{2740 \times 3\} \times 1$, $\{2740 \times 1,\ 2730 \times 2\} \times 1$, $\{2740 \times 1,\ 2050 \times 3\} \times 1$, $\{2690 \times 3\} \times 1$, $\{2690 \times 1,\ 2580 \times 2\} \times 1$, $\{2450 \times 3\} \times 1$, $\{2450 \times 1,\ 2430 \times 2\} \times 1$, $\{2220 \times 2,\ 2090 \times 2\} \times 1$, $\{2090 \times 2,\ 2050 \times 2\} \times 1$, $\{1510 \times 1,\ 1220 \times 6\} \times 1$	0.23

Table 7. Data for the required numbers of steel bars of diameter 12 mm and diameter 18 mm.

Number of Types of Steel Bars	Average CPU Time (s)
50	1.75
100	2.34
200	3.41
500	6.93
1000	12.51

We further compare the quality of the solutions obtained using the column generation method with two heuristics, which can help examine the effectiveness of the column generation method. One is the one-length-per-raw-bar heuristic: exactly $|J_i|$ patterns are considered, and each pattern cuts the raw steel bar into steel bars of one length. The other is a greedy heuristic, which tries to cut a raw steel bar into as many steel bars as possible while prioritizing longer steel bars. This is elaborated in Algorithm 1 below:

Algorithm 1: Greedy heuristic

Initialize: Number of steel bars of length l_{ij} to cut: $\theta_{ij} \leftarrow n_{ij}$
Number of raw steel bars that have been used $\Gamma \leftarrow 1$
Remaining length of the current raw steel bar $\gamma \leftarrow L_i$
While true:
While true:
boolCannotCutAnyMore = true
For $j = |J_i|, \ldots, 1$: // we prioritize longer steel bars
If $\theta_{ij} \geq 1$ and $\gamma \geq l_{ij}$
$\theta_{ij} \leftarrow \theta_{ij} - 1$, $\gamma \leftarrow \gamma - l_{ij}$ // cut a steel bar of type j
boolCannotCutAnyMore = false
Break
If boolCannotCutAnyMore:
If $\theta_{ij} = 0$ for all $j \in J_i$, return;
Else:
Set $\Gamma \leftarrow \Gamma + 1$, $\gamma \leftarrow L_i$
Break;

We again generate instances with 50, 100, 200, 500, and 1000 different lengths of steel bars with random lengths and demand. We perform 10 experiments for each $|J_i|$ and present the average optimal objective values obtained by these three methods. The results are shown in Table 8. Compared to the other two methods, we can see that the column generation method performs better. In particular, the column generation method can reduce costs and waste by over 10% relative to the solutions produced by the other two methods.

Table 8. Comparison results the three methods.

Number of Types of Steel Bars	Average Optimal Objective Value by Column Generation	Average Optimal Objective Value by One-Length-per-Raw-Bar Heuristic	Average Optimal Objective Value by Greedy Heuristic	Percentage of Cost Reduction by Column Generation Compared with One-Length-per-Raw-Bar Heuristic	Percentage of Cost Reduction by Column Generation Compared with Greedy Heuristic
50	436.10	517.60	509.80	15.75%	14.46%
100	830.70	976.10	959.20	14.90%	13.40%
200	1837.20	1968.60	1934.10	6.67%	5.01%
500	4707.10	4964.90	4873.70	5.19%	3.42%
1000	9452.00	9886.00	9711.50	4.39%	2.67%

5. Conclusions

This study proposes an optimization model to reduce the quantity of off-cuts produced by construction projects and develops two methods to solve the proposed model. The solution methods are proven to be effective and efficient in real-world examples. For small-scale instances, the mixed-integer linear programming method is a good choice, as it can give optimal solutions. For large-scale instances, the column generation method is more suitable, as it can give high-quality solutions within seconds. We also develop two heuristic based algorithms to compare with column generation method. Numerical experiments show that the column generation method can reduce waste by over 10% in some cases compared with the other two heuristic methods. Although our research takes steel bars as an example, our optimization models can be applied to any one-dimensional building materials, such as PVC and wood, because these materials need to be cut according to the construction requirements after they are transported to the construction site. Our findings provide an option for reducing leftovers and can help construction contractors decrease material costs and achieve sustainable and green construction.

This study is not without limitations. We only consider reducing the number of used raw materials by adopting the optimal cutting pattern. We do not consider the transportation process from factory to construction site, which will influence the availability of different sizes of raw materials. For example, some trucks cannot deliver steel bars over a certain length limit or transporting longer steel bars will incur extra cost. Therefore, future research can take this problem into account and develop an optimization model considering the whole system.

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