

Sizing and grading methods with consideration of footwear styles

Ameersing Luximon, Lanyin Jiang, Yan Luximon*

**School of Design, The Hong Kong Polytechnic University, Hong Kong SAR*

* **Corresponding Author:** School of Design, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong SAR. yan.luximon@polyu.edu.hk

Abstract: Sizing and grading are very important in footwear production, directly influencing the fit and comfort of footwear. Currently, the footwear industry relies on traditional sizing and grading systems, which vary around the world. Modern measuring technologies, such as 3D scanning and modeling, are starting to be used in footwear mass production. Sizing and grading of footwear is closely related to the sizing and grading of foot. This study investigates the application of principal component analysis (PCA) in sizing and grading methods and the influence of footwear styles based on 3D foot shapes. Three sizing and grading methods were simulated and evaluated. Results show that, compared to the traditional method, the sizing and grading using PCA method provides a better modeling error, hence will result in better fit. Furthermore, the prediction error for various footwear styles are significantly different and the footwear fit near the sole could be achieved easier than instep and ankle region. This indicates that various sizing and grading rules can be applied focusing on different footwear styles in order to develop optimal sizes.

Relevance to industry: The proposed new sizing and grading method could benefit the footwear industry since it provides a better fit comparing to the traditional method. The influence of footwear styles on prediction error gives more detailed insights for manufacturers to further understand the fitting result when applying the different sizing and grading methods.

Keywords: Sizing and grading; Principal component analysis; Footwear style

1. Introduction

In its 40,000-year-long history (Trinkaus and Shang, 2008), footwear has evolved from simple foot protection to a commercial product that has aesthetic and social value (DeMello, 2009; Kawamura, 2016). As a wearable product, the fit and comfort of footwear has become one of the most important customer needs (Luximon et al., 2001). The definition of fit and comfort involves various factors in both subjective and objective aspects. One of the concepts related to footwear fit is the compatibility between the foot and the footwear (Goonetilleke et al., 2000), which can be analyzed through the dimensions and shapes of both. In order to provide good compatibility without interfering with the daily activities of the foot, footwear designers and manufacturers must consider the anatomy of the foot as well as individual variations (Luximon, 2013).

The human foot has a complex structure that varies according to many factors, such as age, gender, and ethnicity (Chan, 2013; Xiong and Zhao, 2013). Due to these variations, the sizing and grading methods (Luximon and Luximon, 2013) were developed by footwear manufacturers to reduce production costs. The manufacture of footwear starts with the shoe last (Ma and Luximon, 2013). A shoe last is a 3D mold that simulates the shape of the foot and incorporates manufacturing features. Materials are molded over and follow the shape and size of the shoe last (Ma and Luximon, 2013; Miller, 1976). A master shoe last is first made according to the design, then sizing and grading scales are applied on the master shoe last in order to manufacture other lasts in various sizes (Luximon and Luximon, 2013). The sizing and grading systems thus influence the footwear compatibility through the size and shape of the various shoe lasts.

Sizing groups the product into a series of sizes suitable for the target users, while grading is the rule representing the relationship of the sizes (Luximon et al., 2012b; Luximon and Luximon, 2013). Simple arithmetic grading is commonly used in traditional footwear sizing systems. In the

footwear industry, standard sizing systems were developed for mass production, in which foot length and foot width (and sometimes girth) are usually selected as the two basic parameters (Cheng and Perng, 2000). There are three types of grading rules: geometric, proportional, and arithmetic. Arithmetic grading is commonly used because the calculation is the simplest (Luximon and Luximon, 2013). These sizing and grading methods and foot measurements that are widely adopted in footwear production are mainly derived from devices and systems from the last century. For example, the Brannock device for foot measurement was patented in 1925, and the AKA64 system for shoe-last design was invented in 1964 (Luximon et al., 2012b; Luximon and Luximon, 2013). Since they are mostly based on empirical data in different contexts, multiple sizing and grading systems exist in different countries and regions with different standards and rules (Hinojo-Pérez et al., 2016; Texin, 2010).

In the past few decades, multiple studies were conducted on foot measurements (Witana et al., 2006), that analyzed the complicated shape of the foot (Wu et al., 2018) by employing both 2D anthropometric measurements and 3D scanning and modeling to improve the sizing and grading methods and provide better footwear fit (Bataller et al., 2001; Mauch et al., 2008). Different methods and techniques have been used to analyze foot shape. Statistical analysis tools such as simple statistics (mean, variance, standard deviation, percentile) and multivariate statistics (correlations, linear regression, factor analysis, principal component analysis) have been widely used (Goonetilleke et al., 1997; Khattree and Naik, 2018). Cluster analysis has been also used for grouping foot into different groups (Krauss et al., 2008). One of the most common tools for understanding shape variation is principal component analysis (PCA) of anthropometric measures (Brown et al., 2012; Gupta, 2014; Veitch et al., 2007) and 3D shapes (Wang et al., 2006; Amstutz et al., 2008; Luximon et al., 2012a; Luximon et al., 2012b; Stanković et al., 2018). The PCA creates a set of parameters (principal components, PCs) that are non-correlated. The principal components are linearly dependent on the original variables, which can be anthropometric variables or x, y, and

z coordinates. The vector representation of the principal components to the variables (the loading matrix) is based on the eigenvector. The first principal component provides the highest variation in the system, followed by the second according to the eigenvalues. Hence, the first few parameters can explain most of the variations in the system (Khattree and Naik, 2018). Because of this character, the PCA has been used for sizing and grading system development (Luximon et al., 2012b; Veitch et al., 2007). In terms of 3D modeling, some studies have created the template model and fit it onto the body scan data using anatomical landmarks (Allen et al., 2002; Yamazaki et al., 2013). These models are anatomically accurate and easy to parameterize the data for statistical analysis such as PCA. These techniques use deformation of the template model based on anatomical landmarks, and hence they are dependent on the number of anatomical landmarks and may be time consuming for mass production. Work still needs to be done to apply modern technologies to industrial mass production (Luximon et al., 2012b). This study focuses on the implementation of 3D modeling and principal component analysis in sizing and grading methods on a large dataset, as well as the possible influence of footwear styles on the accuracy of sizing and grading systems. This paper is somewhat an extension of Luximon et al. (2012b), and it explores more sample data, slightly different experimental protocol and considers different footwear styles.

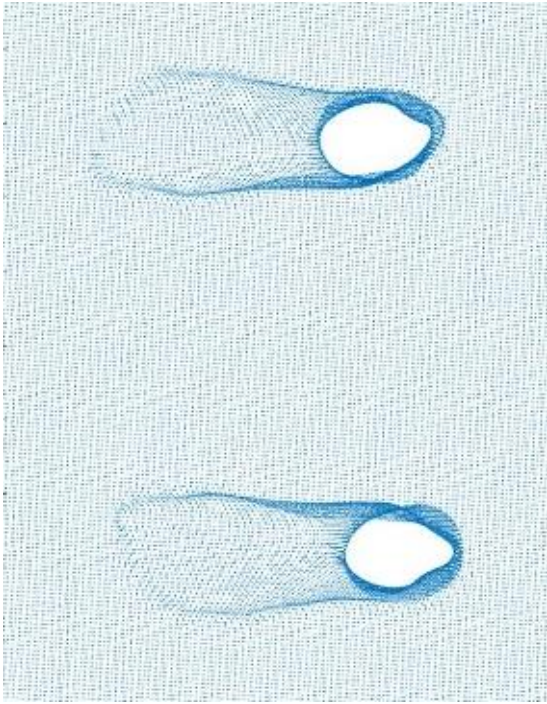
2. Method

2.1. Participants

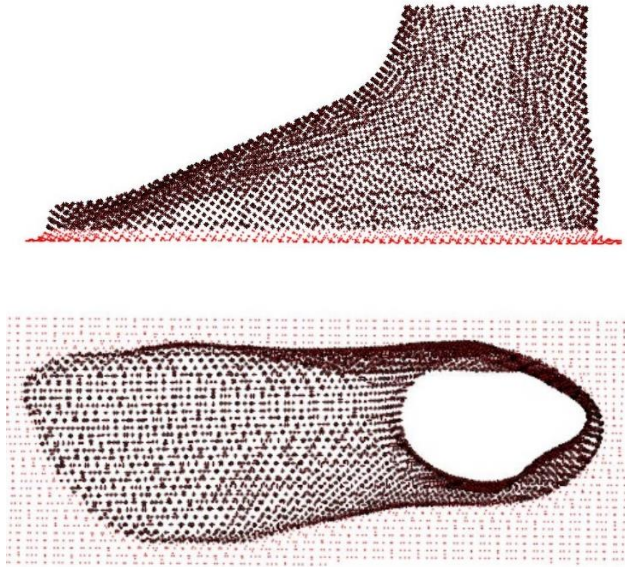
A sample size of 505 Chinese female participants between 18-60 years old were invited to take part in this study. An expert with knowledge of foot and foot disease screened all participants and selected participants without foot and lower limb illness and injuries in the study. The research ethics approval has been granted by the university ethics approval committee.

2.2. Data collection and preparation

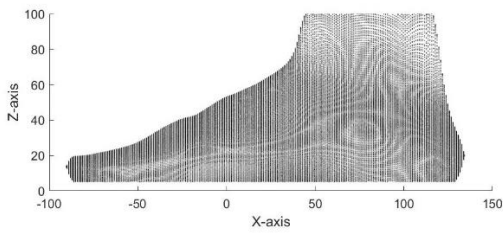
In this study, Kinect™ was used to capture 3D foot point cloud data due to its high portability and scanning efficiency (Zhao et al., 2018). Participants were required to stand straight on flat ground with equal load on both feet. The distance between two feet was about their shoulder width. Raw point cloud data retrieved from the scanning device were roughly cleaned and aligned in Artec Studio® software, ensuring that the ground plane was on the xy plane and the origin was between the left foot and the right foot. Noise points and points with large errors were removed in this step. Then, the data were exported in the STL format and imported to the Geomagic Studio® to fill small holes and repair errors such as spikes and edges. Finally, the clean data were meshed to create surface models. Since Kinect's scanning system is based on an RGB camera and an infrared depth sensor, it was not able to capture the bottom of the foot that was in contact with the ground. The STL files were processed in Matlab to remove the ground surface and recreate the foot sole. The border between the foot and the ground plane was blurred after it was fused into mesh. To recreate the foot sole shape, the exported mesh was cut 5 mm from the xy plane to remove the deformed area. A new foot sole was then computed according to the tangent of the cut edge on the foot and formed on the xy plane, and the gap between the formed sole and the upper part of the foot was filled according to the curvature (Figure 1). The mean absolute error for creating the new foot sole is 0.8mm for both left and right foot. Errors using Kinect for foot scanning has been discussed in Zhao et al. (2018).



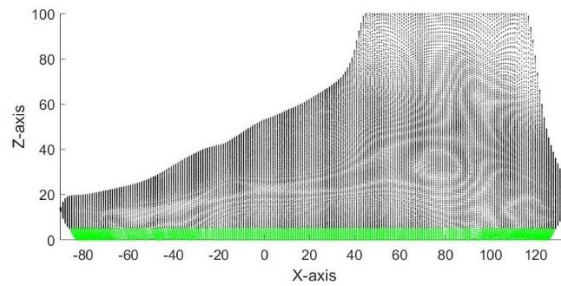
(a) Both feet scan using Kinect



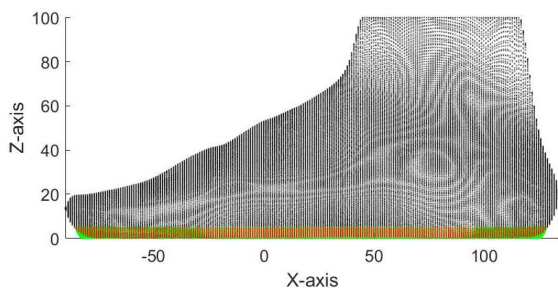
(b) Extraction of one foot and selection of the plantar part



(c) Removal of the plantar part ($< 5\text{mm}$)



(d) Modeling of the plantar part and creating foot sole



(e) Comparison of modeled and actual foot plantar region

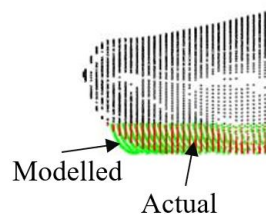


Figure 1. Creation of foot bottom (plantar) shape and error calculation

2.3. Data processing

Data processing was conducted using the Matlab program. The complete foot data were aligned firstly according to the heel centerline (Luximon et al., 2003). For further comparison and evaluation, these data were standardized and sampled. Among the 550 participants' data, the data of 450 participants were used to build the model and the data of 55 participants were used in validation of the model. Each foot model was divided into 100 sections along foot length on the x axis, with 360 points on each section. Each section was centered based on the center of the section and the points were extracted along the section based on equal angle interval (Luximon, 2001; Luximon et al., 2012b). The total number of points in each foot model is 36,000. The 3D individual foot models (sF_i , where $i = 1, \dots, 450$) developed from scanned data were used for sizing and grading analysis. sF_i has coordinates points $sP_{ij} = (sx_{ij}, sy_{ij}, sz_{ij})$ where $i = 1, \dots, 450$; $j=1, \dots, 36000$. Furthermore, the variations in PCA analysis were based on the average 3D foot model, which was calculated based on the average of the 3D foot data (36,000 points) of all 450 participants (Luximon, 2001; Luximon et al., 2012b). The average 3D foot model is given by (aF) and has coordinates points $aP_j = (ax_j, ay_j, az_j)$ where $j=1, \dots, 36000$ and ax_j is the average of 450 points sx_{ij} where $i = 1, \dots, 450$. Similarly, ay_j is the average of points sy_{ij} and az_j is the average of points sz_{ij} .

Three sizing and grading methods were simulated and evaluated in this study: anthropometric sizing and grading method (M1), sizing and grading using the PCA method (M2), and sizing and grading using anthropometric measure and PCA grading method (M3). For each sizing and grading method, the individual 3D foot shape is generated based on different sizing and grading method (mkF_i , where $k = 1, 2$ or 3 to denote model M1, M2 and M3; $i = 1, \dots, 450$). The coordinates of the points for model mkF_i is given by $mkP_{ij} = (mkx_{ij}, mky_{ij}, mkz_{ij})$ where $k = 1, 2$ or 3 ; $i = 1, \dots, 450$; $j=1, \dots, 36000$.

For the anthropometric sizing and grading method (M1), foot length and foot width were used as the two basic sizing parameters. Foot length (FL) is the maximum length from heel to toe. Foot width (FW) is the maximum width at the ball region of the foot within the 70%~80% range of foot length. For the individual foot model (sF_i), the foot length is sFL_i and width sFW_i . For the average foot model (aF), the foot length is aFL and width aFW . In order to create the foot model for M1, proportional grading was used. Since the foot to toe is along the X-axis, for proportional grading, the x coordinate values of the average foot model (aF) was scaled for length by multiplying all x values ax_j by sFL_i/aFL . All the ay_j and az_j coordinate values were scaled for width by multiplying by sFW_i/aFW . The predicted individual 3D foot model based on method M1 is $M1F_i$, where $i = 1, \dots, 450$) and has points $M1P_{ij} = (M1x_{ij}, M1y_{ij}, M1z_{ij})$ where $i = 1, \dots, 450; j = 1, \dots, 36000$.

For sizing and grading using the PCA method (M2), the principal components (PCs) and the variances are computed first (Luximon et al., 2012b). In order to compute PCs, the 36,000 data points with x , y and z coordinates are considered as separate parameters. Thus $36,000 \times 3 = 108000$ parameter values used for PCA. In PCA analysis, the eigenvalues provide the variations in the model, while the eigenvector provides the relationship between the PCs and the 108000 parameters (PC loadings). For individual subjects, the PC loadings are PC_{il} where $i = 1, \dots, 450; l = 1, \dots, 108000$. Although all PCs can be calculated, the first few PCs provide the most variation, hence one to ten PCs were investigated to study the impact of different numbers of PCs. In this study, 10 PCs represent about 82% of the variance for left foot model and 84% for right foot model. Using PCA analysis method, each individual foot can be predicted based on the total number of PCs used. The average foot model was graded based on PC loadings for η number of PCs to create a predicted individual 3D foot model based on method M2 ($M2_{\eta}F_i$, where $i = 1, \dots, 450; \eta = 1, 2, \dots$). The details

of the calculations and equations were taken from Luximon et al. (2012b). Also, fewer parameters (less than three) are desired for practical sizing and grading, as a higher number of sizes increases the cost of production. Thus, detailed analysis for two-factor sizing and grading using PC1 and PC2 was carried out ($M2F_i$, where $i = 1, \dots, 450$), given that two-parameter sizing is commonly used in the footwear industry. The predicted individual 3D foot model based on method M2 is $M2F_i$, where $i = 1, \dots, 450$ and has points $M2P_{ij} = (M2x_{ij}, M2y_{ij}, M2z_{ij})$ where $i = 1, \dots, 450; j = 1, \dots, 36000$.

Since M2 requires 3D data, currently this method may not be widely available at the retail level. Hence, a sizing and grading method based on anthropometric measures and PCA grading (M3) was also developed. Twelve anthropometric measures representing a combination of length, width, height, and flare parameters were calculated to be used as a potential anthropometric sizing variation for the M3 method. The anthropometric measures for individual participants are A_{in} where $i = 1, \dots, 450; n = 1, \dots, 12$. Foot length is the same as FL in M1, along the x-axis. The foot width, girth, height, and flare angle at section m (where $m = 1, \dots, 100$) can be computed as shown in Figure 2. Width at section m is given by the difference between maximum and minimum y values. Height at section m is given by the maximum z value. Girth at section m is the circumference of the section curve generated by the convex hull points. The flare angle or foot curvature at section m is the angle extended from the heel to the midpoint along the y axis of section m . Therefore, width and height at 50% foot length section are calculated as foot width (FW50) and foot height (FH50). Foot width, girth, and flare are also measured at the ball region (foot within the 70%~80% range of foot length). Within this range, the width, girth, and flare angles at each cross section are calculated. Then the minimum (FWmin, FGmin, and Fmin), maximum (FWmax, FGmax, and Fmax) and average (FWav, FGav, Fav) values are calculated for width, girth, and flare angle measures. Here FWmax is the same as the foot width (FW) in M1. The relationship between the 10 PCs ($PC_{i\eta}$ where $i = 1, \dots, 450; \eta = 1, \dots, 10$) and anthropometric measures (A_{in}) was calculated to decide which anthropometric measure to use to substitute the principal component measure. The anthropometric

measurements were then used to predict the principal components ($P_{i\eta}$). Then, using the PCA loadings, the foot shape was predicted (M_3F_i , where $i = 1, \dots, 450$).

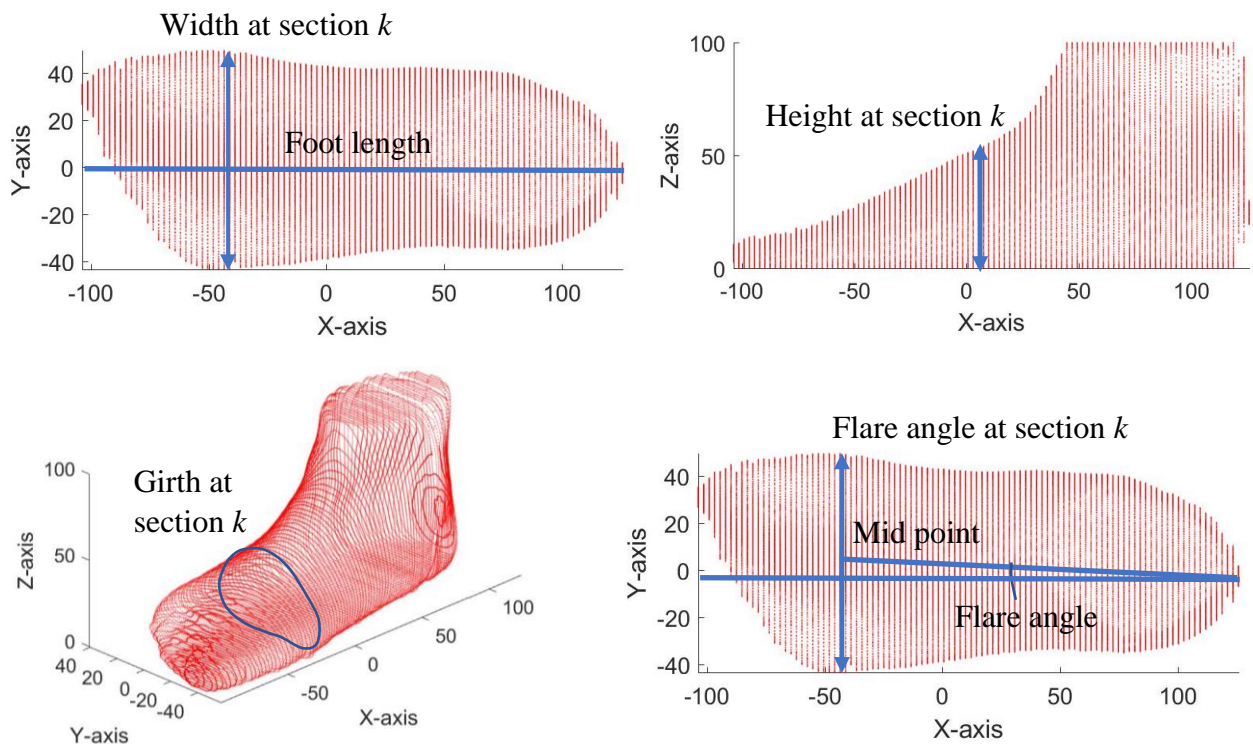
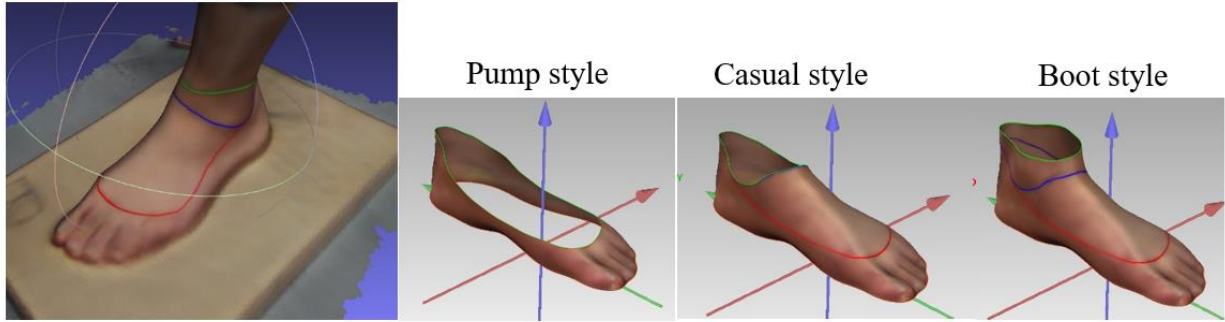


Figure 2. Calculation of anthropometric measures at one section

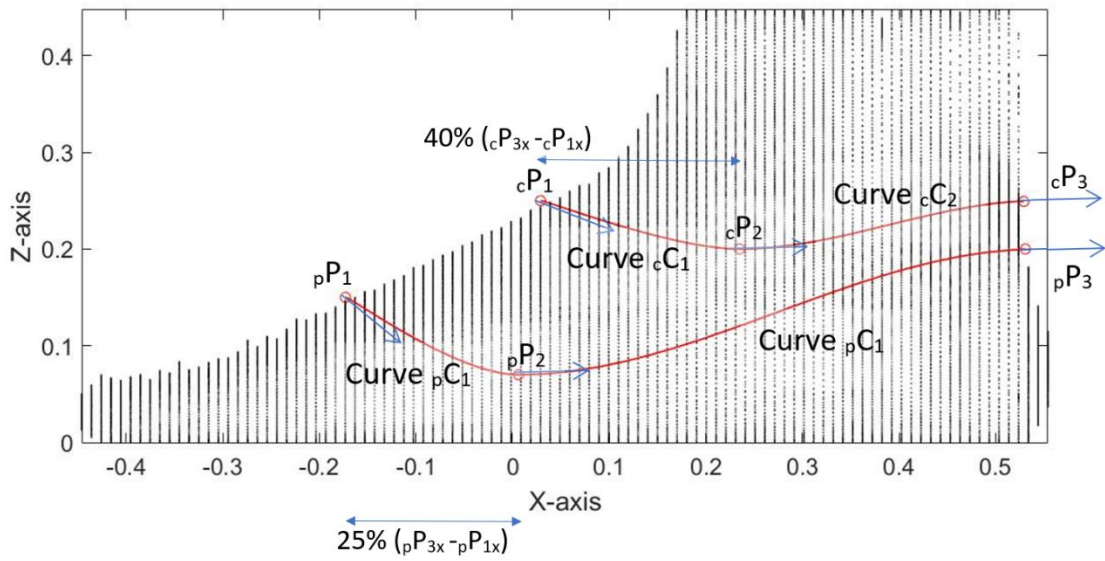
In order to decide on the accuracy of the three different prediction methods, Cartesian distance error was calculated between the actual 3D foot data (sF_i) and the predicted foot data (M_kF_i). For participant i , the error at point M_kP_{ij} is the shortest cartesian distance between the points M_kP_{ij} and sP_{ij} and is given by M_kE_{ij} where $k = 1, 2$ or 3 ; $i=1, \dots, 450$; $j=1, \dots, 36000$. For the model error, the mean of the errors at point M_kP_{ij} is given by M_kE_i where $k = 1, 2$ or 3 ; $i=1, \dots, 450$. The predicted errors M_kE_i calculated from the three methods indicate the accuracy of the sizing and grading system.

In order to study the influence of footwear styles on fit, the foot data were also divided by three different topline heights (low, medium, and high) to represent the coverage of pump style, casual style, and boot style shoes separately. The regions of three styles are shown in Figure 3. In order

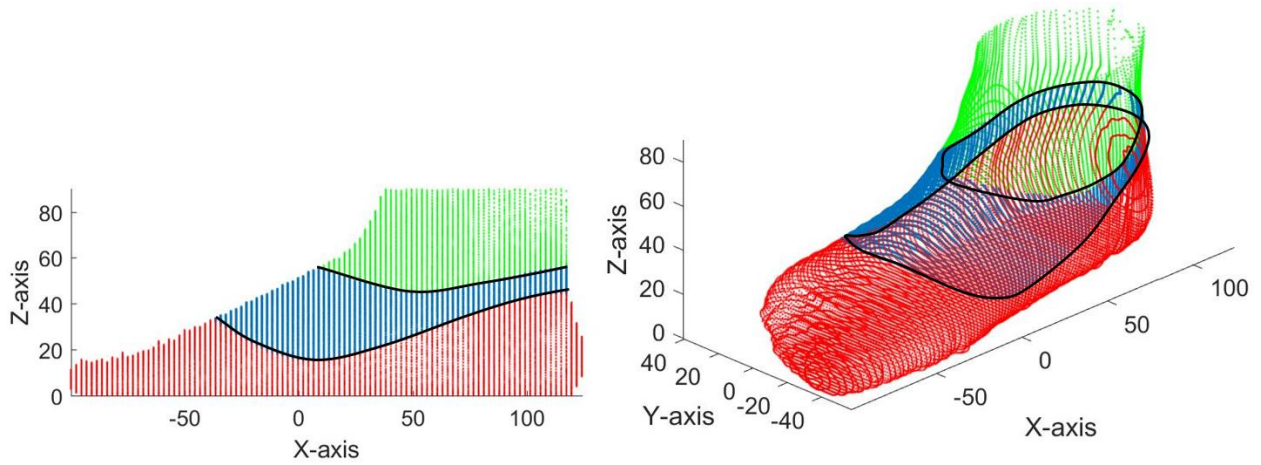
to know the location of the style on foot, a test sample of 10 participants was used. They wore different style of sample footwear (pump, casual and boot) and the topline curve was drawn using red ink for pump shoe and blue ink for casual shoe and green ink for one type of boot (Figure 3a). Based on the test the three shoe-styles were modeled using spline curves (Figure 3b). Figure 3b represents a normalized foot model and the shoe-styles based on the normalized foot. In order to model the style on the xz plane, the y-values are not considered. The pump style curve is modeled using 3 points (pP_1 , pP_2 and pP_2) and the tangent vector at the three points. The normalized z values of pP_1 , pP_2 and pP_2 are $pP_{1z}=0.15$, $pP_{2z}=0.07$ and $pP_{3z}=0.2$. X-value of pP_1 is the minimum x of the foot data point at pP_{1z} , let denote by pP_{1x} . X-value of pP_3 is the maximum x of the foot data point at pP_{3z} , let denote by pP_{3x} . X-value of pP_2 is 25% proportional distance between pP_{1x} and pP_{3x} as shown in Figure 3b. The tangent vectors at points pP_1 , pP_2 and pP_2 are $[0.1 \ 0 \ 0]$, $[1 \ 0 \ 0]$ and $[1 \ 0 \ 0]$ respectively. The casual shoe style curve is modeled using 3 points (cP_1 , cP_2 and cP_2) and the tangent vector at the three points. The normalized z values of cP_1 , cP_2 and cP_2 are $cP_{1z}=0.25$, $cP_{2z}=0.2$ and $cP_{3z}=0.25$. X-value of cP_1 is the minimum x of the foot data point at cP_{1z} , let denote by cP_{1x} . X-value of cP_3 is the maximum x of the foot data point at cP_{3z} , let denote by cP_{3x} . X-value of cP_2 is 40% proportional distance between cP_{1x} and cP_{3x} as shown in Figure 3b. The tangent vectors at points cP_1 , cP_2 and cP_2 are $[0.1 \ 0 \ 0]$, $[1 \ 0 \ 0]$ and $[1 \ 0 \ 0]$ respectively. The boot style is represented by foot data below $z = 90$ mm. Only the errors within the coverage regions were taken into consideration in the analysis of different styles. The comparison of errors among the three methods within the style-specific regions indicated the fit situation in the corresponding styles.



(a) footwear style drawn on foot and then extracted



(b) Model of footwear styles using spline curves



(c). Footwear style specific regions

Figure 3. Footwear style specific regions

3. Data analysis and results

3.1. The descriptive statistics of participants

The average age of the 450 Chinese female participants used for model building was 24.64 years old, with an average height of 159.60 cm and an average weight of 53.08 kg. The average age of the 55 Chinese female participants used for model validation was 24.34 years old, with an average height of 159.14 cm and an average weight of 52.12 kg. The descriptive statistics of all participants are presented in Table 1.

Table 1. Descriptive statistics of participants

		Age	Height (cm)	Weight (kg)
Data for model building (n=450)	Mean	24.64	159.60	53.08
	Median	21	160	52
	SD	8.28	5.36	9.23
	Min.	18	145	37
	Max.	60	176	130
Data for model validation (n=55)	Mean	24.36	159.14	52.12
	Median	22	160	51
	SD	6.18	4.57	6.72
	Min.	18	145	40
	Max.	47	170	74

3.2. Anthropometric sizing and grading method (M1)

Table 2 shows the errors calculated in anthropometric sizing and grading method (M1). The anthropometric sizing and grading method had mean errors of 2.01 mm and 2.15 mm for the left and right foot, median errors of 1.84 mm and 1.95 mm, and standard deviations of 0.84 mm and 0.94 mm. The results of the left and right foot in M1 were found to be statistically significant ($z = -2.97$, $p = 0.003$) using the Wilcoxon signed rank test.

Table 2. Error results for anthropometric sizing and grading method (M1)

	Left foot (mm)			Right foot (mm)		
	Bootstrap 95% Confidence Interval			Bootstrap 95% Confidence Interval		
		Lower	Upper		Lower	Upper
Mean	2.01	1.94	2.08	2.15	2.07	2.23
Median	1.84	1.80	1.90	1.95	1.90	2.00
SD	0.84	0.70	1.00	0.94	0.82	1.07
Min.	0.91			0.81		
Max.	9.32			8.28		

3.3. Sizing and grading using the PCA method (M2)

Table 3 shows the errors in sizing and grading using the PCA method (M2). The more PCs that were adopted in M2, the smaller the errors became. Results indicate that M2 is more accurate than the traditional anthropometric method when using two or more PCs, with smaller mean errors, median errors, and standard deviations. Table 4 shows the detailed descriptive statistics of two principal components in M2. Compared to the errors in M1, two principal components in M2 achieved mean errors of 1.48 mm for the left foot and 1.45 mm for the right foot, median errors of 1.41 mm for the left foot and 1.39 mm for the right foot, and standard deviations of 0.35 mm for the left foot and 0.36 mm for the right foot. The comparison of errors between the left and right foot in M2 showed that there is no statistically significant difference ($z = 1.56$, $p = 0.118$) using the Wilcoxon signed rank test. The difference of errors between M1 and M2 was found to be statistically significant for the left foot ($z = -13.65$, $p = 0.000$) and the right foot ($z = -14.09$, $p = 0.000$) using the Wilcoxon signed rank test. The performance of M2 is better than M1, indicating that the PCA system can provide a better fit than the traditional anthropometric measurement system.

The variance of principal components in M2 was also calculated. The explained variance for the first principal component (PC1) was 50.37% for the left foot and 46.28% for the right foot. The second highest variance (PC2) was 16.53% for the left foot and 16.70% for the right foot. Thus, the first two PCs can account for 66.91% variance for the left foot and 62.99% variance for the right foot. The variance contributions of PC3 were 4.50% and 5.30%, left foot and right foot respectively.

Table 3. Error results for sizing and grading using PCA method (M2)

Number of PC(s)	Left foot (mm)					Right foot (mm)				
	Mean	Median	SD	Min.	Max.	Mean	Median	SD	Min.	Max.
1	1.79	1.65	0.58	0.91	5.56	1.77	1.62	0.61	0.99	6.08
2	1.48	1.41	0.35	0.89	3.62	1.46	1.40	0.36	0.80	3.49
3	1.40	1.33	0.33	0.77	3.94	1.34	1.30	0.28	0.77	3.24
4	1.33	1.27	0.34	0.65	4.02	1.24	1.20	0.30	0.62	2.74
5	1.15	1.11	0.28	0.66	3.92	1.15	1.11	0.25	0.66	2.75
6	1.10	1.06	0.26	0.66	3.27	1.08	1.04	0.23	0.66	2.91
7	1.06	1.03	0.22	0.64	3.01	1.03	1.01	0.21	0.66	2.70
8	1.02	0.98	0.21	0.64	2.82	0.98	0.96	0.20	0.63	2.70
9	0.97	0.93	0.19	0.61	2.82	0.94	0.92	0.18	0.63	2.58
10	0.93	0.91	0.17	0.60	2.34	0.93	0.90	0.17	0.58	2.20

Table 4. Error results of M2 with two principal components

	Left foot (mm)			Right foot (mm)		
	Bootstrap 95% Confidence Interval			Bootstrap 95% Confidence Interval		
		Lower	Upper		Lower	Upper
Mean	1.48	1.44	1.51	1.46	1.42	1.49
Median	1.41	1.40	1.40	1.40	1.40	1.40
SD	0.35	0.31	0.40	0.36	0.31	0.41
Min.	0.89			0.80		
Max.	3.62			3.49		

3.4. Sizing and grading using anthropometric measures and PCA grading method (M3)

In sizing and grading using anthropometric measures and PCA grading method (M3), correlation and regression analysis was conducted on 10 principal components and 12 anthropometric

measurements, including FL, FW50, FH50, FWmax, FWmin, FWav, FGmax, FGmin, FGav, Fmax, Fmin, and Fav. The simple statistics of the 12 anthropometric measures are listed in Table 5. Since two-component PCA presented satisfying results in M2, the first two principal components and their highly correlated parameters were selected for foot sizing and grading in M3. The Spearman's rank correlations of 22 parameters of left and right foot data are shown in Table 6 and Table 7 respectively. The numbers 1-12 on the axes represent anthropometric measurements and 13-22 are principal components. For the left foot data, PC1 is highly correlated to Fmax (0.982), Fmin (0.984), and Fav (0.984), and PC2 is highly correlated to FL (-0.871). For the right foot data, the results are similar. PC1 is highly correlated to Fmax (0.991), Fmin (0.988), and Fav (0.990), and PC2 is highly correlated to FL (-0.897). Fmax, Fmin, and Fav provided similar results for both the left and right foot, and the correlation coefficients among them are greater than 0.992. Fav and FL were selected as two foot measurements to predict a new PC1 (NPC1) and a new PC2 (NPC2) in M3. The regression equations of these parameters are shown in Table 8. NPC1 and NPC2 were then used to predict 3D foot shape. Errors were then calculated between the predicted foot shape and the actual 3D foot data.

Table 5. Anthropometric measures of participants

	Left foot					Right Foot				
	Mean	Median	SD	Min.	Max.	Mean	Median	SD	Min.	Max.
FL (mm)	233.66	232.82	9.81	198.30	263.20	232.90	232.25	9.60	201.98	266.07
FWmax (mm)	90.14	89.83	5.21	74.59	110.60	87.56	87.54	4.99	73.40	111.58
FWmin (mm)	83.59	83.37	5.27	66.67	103.18	81.62	81.25	5.06	66.27	104.23
FWav (mm)	87.90	87.47	5.09	71.70	108.06	85.30	85.09	4.89	70.57	108.76
FW50 (mm)	82.76	82.35	5.41	68.33	101.13	81.15	80.87	5.18	68.34	101.14
FH50 (mm)	56.87	56.55	4.94	46.26	94.87	55.78	55.68	4.53	45.14	76.80
FGmax (mm)	208.95	208.37	11.57	176.60	253.34	203.69	202.92	11.18	172.79	257.04
FGmin (mm)	188.93	188.34	11.60	153.61	231.90	185.53	184.89	11.26	151.05	235.28
FGav (mm)	201.43	200.52	11.35	167.48	246.30	196.57	195.88	11.01	164.42	249.12
Fmax (°)	1.69	1.74	2.97	-8.94	11.00	-0.14	-0.29	3.02	-11.89	8.61
Fmin (°)	1.01	1.22	2.78	-9.41	9.90	-0.74	-0.83	3.15	-12.99	8.07
Fav (°)	1.39	1.51	2.91	-9.17	10.54	-0.45	-0.55	3.11	-12.50	8.45

Table 6. Correlation coefficients of anthropometric measures and principal components (left foot)

	FL	FWmax	FWmin	FWav	FW50	FH50	FGmax	FGmin	FGav	Fmax	Fmin	Fav
FWmax	0.478**											
FWmin	0.369**	0.842**										
FWav	0.460**	0.978**	0.927**									
FW50	0.457**	0.816**	0.625**	0.772**								
FH50	0.336**	0.489**	0.446**	0.496**	0.356**							
FGmax	0.500**	0.965**	0.846**	0.957**	0.810**	0.620**						
FGmin	0.408**	0.837**	0.975**	0.917**	0.643**	0.530**	0.878**					
FGav	0.485**	0.945**	0.919**	0.975**	0.764**	0.599**	0.978**	0.947**				
Fmax	0.089	0.121*	-0.061	0.071	0.245**	0.250**	0.145**	-0.021	0.090			
Fmin	0.088	0.127**	-0.020	0.089	0.250**	0.263**	0.154**	0.015	0.110*	0.992**		
Fav	0.090	0.126**	-0.045	0.080	0.249**	0.258**	0.151**	-0.006	0.100*	0.998**	0.997**	
PC1	0.153**	0.150**	-0.025	0.102*	0.275**	0.267**	0.173**	0.017	0.122**	0.982**	0.984**	0.984**
PC2	-0.871**	-0.636**	-0.612**	-0.651**	-0.530**	-0.474**	-0.676**	-0.656**	-0.691**	0.065	0.051	0.058
PC3	-0.119*	0.317**	0.416**	0.363**	0.255**	0.421**	0.389**	0.435**	0.406**	-0.084	-0.072	-0.080
PC4	0.389**	-0.150**	-0.319**	-0.212**	-0.052	0.006	-0.142**	-0.301**	-0.204**	0.019	-0.011	0.009
PC5	-0.133**	-0.005	-0.007	-0.008	-0.071	0.554**	0.113*	0.061	0.093*	0.050	0.049	0.051
PC6	-0.020	0.092	0.052	0.082	0.240**	-0.011	0.093*	0.050	0.075	0.057	0.049	0.053
PC7	0.056	-0.242**	-0.190**	-0.228**	-0.314**	-0.262**	-0.283**	-0.218**	-0.258**	-0.126**	-0.136**	-0.133**
PC8	-0.007	-0.197**	-0.171**	-0.193**	-0.220**	0.145**	-0.162**	-0.133**	-0.150**	-0.051	-0.025	-0.041
PC9	-0.017	0.060	0.001	0.045	-0.115*	0.032	0.016	-0.020	0.005	0.015	-0.007	0.006
PC10	-0.019	-0.017	-0.018	-0.024	0.012	-0.063	-0.012	-0.009	-0.013	-0.097*	-0.080	-0.093*

Note: ** p < 0.01 * p < 0.05

Table 7. Correlation coefficients of anthropometric measures and principal components (right foot)

	FL	FWmax	FWmin	FWav	FW50	FH50	FGmax	FGmin	FGav	Fmax	Fmin	Fav
FWmax	0.513**											
FWmin	0.392**	0.869**										
FWav	0.490**	0.980**	0.940**									
FW50	0.491**	0.794**	0.619**	0.756**								
FH50	0.338**	0.463**	0.458**	0.481**	0.341**							
FGmax	0.528**	0.965**	0.869**	0.960**	0.794**	0.593**						
FGmin	0.426**	0.848**	0.975**	0.921**	0.625**	0.539**	0.888**					
FGav	0.513**	0.948**	0.927**	0.975**	0.753**	0.587**	0.981**	0.949**				
Fmax	-0.077	-0.115*	-0.032	-0.111*	-0.281**	-0.211**	-0.133**	-0.047	-0.125**			
Fmin	-0.079	-0.115*	-0.016	-0.104*	-0.283**	-0.203**	-0.130**	-0.030	-0.116*	0.996**		
Fav	-0.079	-0.117*	-0.023	-0.109*	-0.283**	-0.208**	-0.132**	-0.038	-0.121**	0.998**	0.999**	
PC1	-0.103*	-0.120*	-0.022	-0.111*	-0.288**	-0.203**	-0.133**	-0.037	-0.122**	0.991**	0.988**	0.990**
PC2	-0.897**	-0.615**	-0.570**	-0.624**	-0.526**	-0.422**	-0.647**	-0.613**	-0.659**	-0.019	-0.027	-0.023
PC3	-0.065	0.384**	0.463**	0.417**	0.299**	0.597**	0.478**	0.506**	0.486**	-0.058	-0.053	-0.055
PC4	0.314**	-0.109*	-0.290**	-0.176**	-0.012	0.131**	-0.080	-0.263**	-0.150**	0.008	-0.017	-0.006
PC5	0.209**	0.077	0.024	0.053	0.121*	-0.473**	-0.025	-0.031	-0.032	0.048	0.044	0.047
PC6	0.031	0.146**	0.116*	0.133**	0.189**	-0.194**	0.079	0.067	0.077	0.021	0.011	0.016
PC7	0.077	-0.064	-0.025	-0.054	-0.118*	-0.195**	-0.087	-0.046	-0.070	0.001	-0.009	-0.004

PC8	0.021	-0.183**	-0.153**	-0.178**	-0.329**	0.134**	-0.168**	-0.139**	-0.159**	0.013	0.026	0.021
PC9	-0.037	0.114*	0.116*	0.119*	0.009	0.100*	0.087	0.094*	0.093*	-0.093*	-0.114*	-0.105*
PC10	0.001	0.075	0.046	0.068	0.092	-0.023	0.054	0.035	0.047	-0.053	-0.047	-0.050

Note: ** p < 0.01 * p < 0.05

Table 8. Regression equations of left and right foot

Regression equations	
Left foot	NPC1 = $403.98 \times Fav + 141.08$, $R^2 = 0.9814$ NPC2 = $-67.79 \times FL + 15766$, $R^2 = 0.7945$
Right foot	NPC1 = $404.87 \times Fav - 535.99$, $R^2 = 0.9735$ NPC2 = $-63.49 \times FL + 14806$, $R^2 = 0.7680$

Table 9 shows the errors calculated using anthropometric sizing and PCA grading (M3). Similar to M2, results using two principal components are more accurate than when using one. Table 10 includes the details of errors in two principal components in M3, with mean errors of 1.55 mm and 1.50 mm, median errors of 1.47 mm and 1.43 mm, and standard deviations of 0.38 mm and 0.39 mm for the left and right foot, respectively. The left and right foot results in M3 were found to be statistically significant ($z = 2.53$, $p = 0.011$) using the Wilcoxon signed rank test.

Table 9. Error results for sizing and grading using anthropometric measures and PCA grading method (M3)

Number of PC(s)	Left foot (mm)					Right foot (mm)				
	Mean	Median	SD	Min.	Max.	Mean	Median	SD	Min.	Max.
1	1.81	1.68	0.59	0.89	5.55	1.78	1.65	0.61	0.99	6.02
2	1.55	1.47	0.38	0.85	3.66	1.50	1.43	0.39	0.80	4.03

Table 10. Error results of M3 with two principal components

	Left foot (mm)				Right foot (mm)			
		Bootstrap 95% Confidence Interval			Bootstrap 95% Confidence Interval			
		Lower	Upper		Lower	Upper		
Mean	1.55	1.51	1.58	1.50	1.47	1.54		
Median	1.47	1.50	1.50	1.43	1.40	1.50		

SD	0.38	0.34	0.43	0.39	0.34	0.44
Min.	0.85			0.80		
Max.	3.66			4.03		

Compared to M1 and M2, the mean and median errors in M3 are much smaller than in M1 and slightly larger than in M2 with two principal components. Results in the Wilcoxon signed rank test showed that M3 were significantly different from M1 for the left foot ($z = -11.95, P = 0.000$) and the right foot ($z = -13.11, P = 0.000$). M3 were also found to be significantly different from M2 for the left foot ($z = -8.39, P = 0.000$) and the right foot ($z = -5.52, P = 0.000$). Figure 4 shows an overview of the error distribution in M1, M2, and M3. It can be seen from the graph that results in M2 and M3 are very close and have smaller variations than M1. In M2 and M3 the maximum error values are also much smaller. The results indicate that M3, using average foot flare and foot length as anthropometric measurements and PCA grading rule, demonstrates a reliable performance. The footwear fit of this system is much better than the traditional system and is close to the PCA method. The regional error plot for three methods is shown in Figure 5. Based on Tables 2, 4 and 10 it can be seen that the standard deviation of the errors for the method M1 is larger, which is evident from Figure 5.

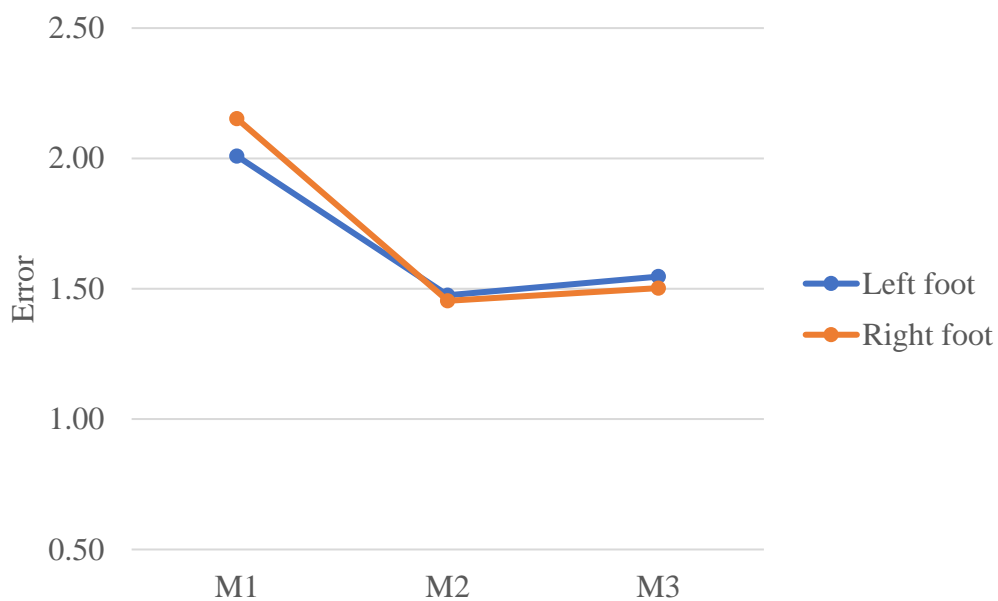


Figure 4. Overview of the results of M1, M2 and M3

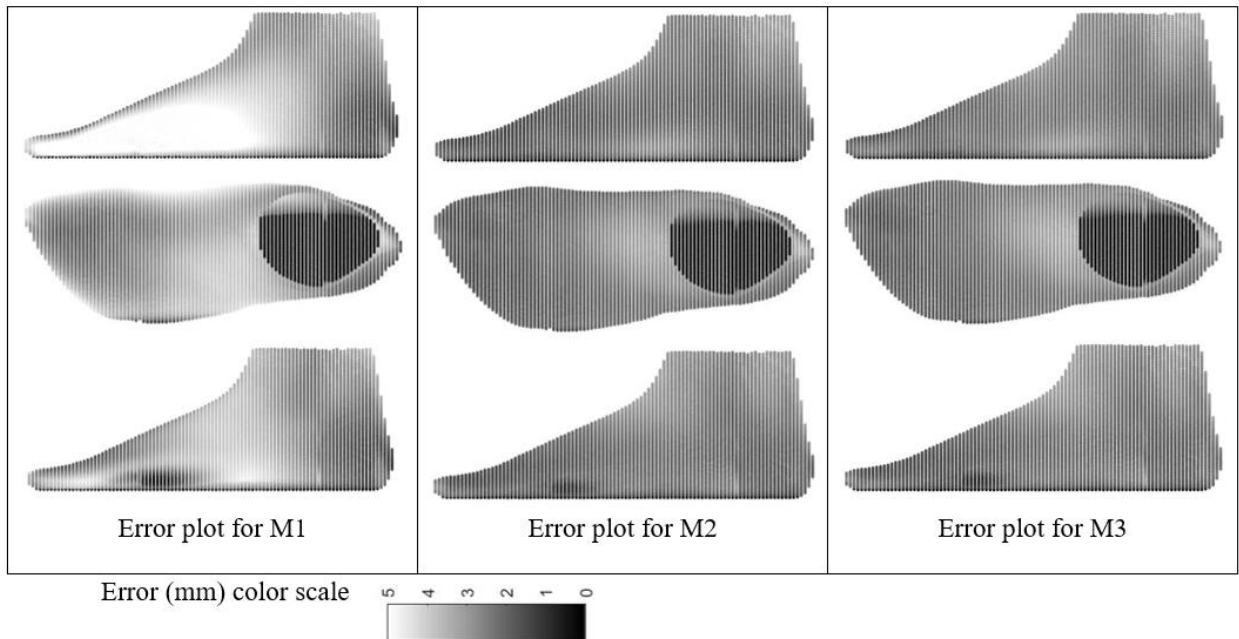


Figure 5. Error plots for M1, M2 and M3

3.5. Comparisons among the footwear styles

In order to determine the influence of footwear styles on prediction errors, pump, casual, and boot styles were compared for each method. The error results of three footwear styles for all three methods are shown in Table 11. Figure 6 and Figure 7 plot the overview of three sizing and grading methods for the three styles. The Kruskal-Wallis test was applied to the three styles for each method and results showed significant differences ($P < 0.001$) among styles for all three methods. Further Wilcoxon signed rank testing showed that three styles were all significantly different to each other ($P < 0.001$) for each method. The variations of the errors among each style follow a very similar pattern in all three methods (Figure 6 and Figure 7). The pump style, with the lowest topline and the least coverage, showed the smallest errors. The boot style showed the largest errors. The error increase is larger between the pump style and the casual style than that between the casual style and boot style.

Table 11. Error results for three footwear styles

Methods	Styles	Left foot (mm)					Right foot (mm)				
		Mean	Median	SD	Min.	Max.	Mean	Median	SD	Min.	Max.
M1	Pump	1.55	1.38	0.70	0.57	6.11	1.68	1.49	0.85	0.58	8.85
	Casual	1.91	1.74	0.87	0.70	8.15	2.08	1.85	1.01	0.72	9.14
	Boot	2.03	1.84	0.87	0.86	9.30	2.19	1.98	0.98	0.80	8.63
M2	Pump	1.08	1.04	0.20	0.66	2.08	1.06	1.03	0.23	0.59	2.49
	Casual	1.32	1.26	0.29	0.74	2.95	1.31	1.25	0.32	0.67	3.25
	Boot	1.47	1.41	0.34	0.86	3.52	1.46	1.39	0.36	0.77	3.49
M3	Pump	1.13	1.09	0.24	0.72	2.36	1.10	1.06	0.25	0.63	2.48
	Casual	1.39	1.33	0.33	0.72	3.09	1.36	1.29	0.35	0.71	3.62
	Boot	1.55	1.48	0.38	0.83	3.33	1.51	1.43	0.39	0.80	4.05

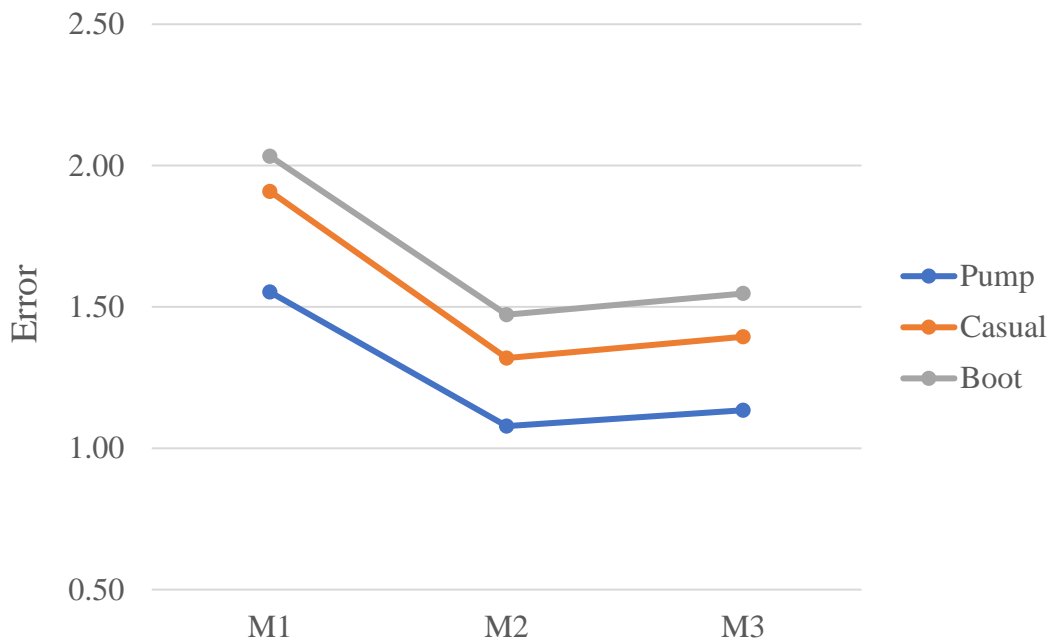


Figure 6. Error comparison of the footwear styles for each method (left foot)

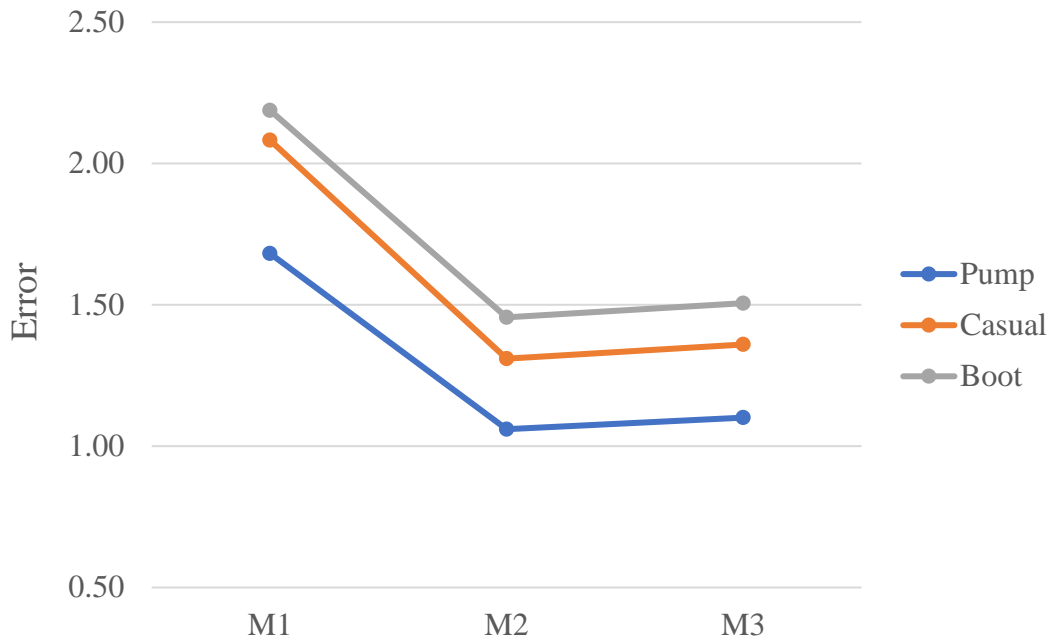


Figure 7. Error comparison of the footwear styles for each method (right foot)

3.6. Validation

The data of 55 participants was used to validate the model and the results of errors are displayed in Table 12 and Table 13. In general, the differences of means between model building and validation test are less than 0.12 mm for all methods and footwear styles.

Table 12. Error comparison between model building and validation for three methods

Methods	Model building				Validation			
	Left foot (mm)		Right foot (mm)		Left foot (mm)		Right foot (mm)	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
M1	2.01	1.84	2.15	1.95	1.95	1.73	2.15	1.94
M2	1.48	1.41	1.45	1.39	1.59	1.51	1.53	1.43
M3	1.55	1.47	1.50	1.43	1.65	1.53	1.59	1.53

Table 13. Error comparison between model building and validation for three footwear styles

Methods	Styles	Model building				Validation			
		Left foot (mm)		Right foot (mm)		Left foot (mm)		Right foot (mm)	
		Mean	Median	Mean	Median	Mean	Median	Mean	Median
M1	Pump	1.55	1.38	1.68	1.49	1.44	1.27	1.66	1.46

	Casual	1.91	1.74	2.08	1.85	1.81	1.58	2.07	1.85
	Boot	2.03	1.84	2.19	1.98	1.97	1.71	2.18	2.00
M2	Pump	1.08	1.04	1.06	1.03	1.16	1.14	1.11	1.09
	Casual	1.32	1.26	1.31	1.25	1.42	1.38	1.37	1.29
	Boot	1.47	1.41	1.46	1.39	1.59	1.52	1.53	1.46
M3	Pump	1.13	1.09	1.10	1.06	1.18	1.14	1.16	1.11
	Casual	1.39	1.33	1.36	1.29	1.46	1.38	1.43	1.40
	Boot	1.55	1.48	1.51	1.43	1.65	1.53	1.59	1.57

4. Discussion and conclusion

The sizing and grading system determine the shape and size of the footwear at an early stage of footwear mass production. Footwear sizing and grading is determined by the size of shoe-last, a 3D mold for making shoes. Although the shoe-last is not exactly same as the 3D foot shape, it is influenced by the 3D foot shape in order to provide a good footwear fit. In footwear industry, a model or master shoe-last is created in European size 36 or 37 for women and European size 40 or 41 for men. The model shoe-last is graded based on length and width (or girth) to create different sizes. However, the sizing and grading system based on linear measurements of length and width are outdated. Many studies have shown that the foot curvature is a better parameter to consider for foot sizing. Furthermore, with the development of 3D scanners and computerized system there is a need to reconsider foot sizing and grading since footwear fit influences foot illnesses and sometime foot injuries.

This study provides a new approach to sizing and grading methods, combining modern foot modeling techniques with traditional methods based on scientific analysis. PCA was adopted to develop the new sizing and grading method in this study. Three sizing and grading methods were discussed, including the anthropometric method using foot length and foot width (M1), the PCA method (M2), and the combination of anthropometric measures and PCA grading (M3). The M2 performed the best of the three methods, with a more than 25% reduction in error compared to the M1. The M3, using anthropometric measurements instead of computed principal components as a

practical way for application, also gives satisfying results that are rather close to the M2 (mean difference around 0.1 mm). In other words, the M3 method enables sizing and grading without the use of 3D scanners and is useful for reducing the cost of foot measurement without sacrificing the fit.

Previous studies on the PCA of sizing and grading have indicated the importance of foot flare in foot shape modeling (Luximon et al., 2012b; Xiong et al., 2008). In the M3 method, the first two principal components, which are highly correlated to foot flare and foot length, can represent 66.09% and 62.53% variance of all the data for the left and right foot respectively. The prediction using foot flare and foot length in the M3 method gives satisfying results that are close to the M2 method, which proves the importance of foot flare in foot sizing and grading. On the other hand, the principal components related to foot length and foot width (the second and the third components) only represented less than 20% variance. The lack of representativeness in the parameters in traditional anthropometric methods could result in the fit problems of current sizing and grading systems.

The styles of the footwear also influence the accuracy of sizing and grading method. From the results, the errors near the foot sole are smaller than in the upper regions of the foot, indicating that the variance of foot shape is larger at the instep or ankle regions than the regions near the sole and toes. However, considering the structure of the foot, the instep and ankle regions often require more space for daily activities compared to the sole. The compatibility calculated based on the standing foot 3D foot models may not be representative enough for actual fit and comfort. Footwear with a higher top line and more coverage tend to provide less fit. Hence, there is a need for lacing and adjustments with boots and dress shoes, while pump style shoes may not require any fastening mechanisms. This is evident for all type of sizing and grading methods. Different sizing and grading rules can be applied focusing on different footwear styles in order to develop optimal sizes. Further research and experiments need to be conducted to evaluate the sizing and grading methods on

subjective feelings about footwear fit and comfort. Finally, research on other factors in comfort and fit, such as material, activity level, and design types, needs to be carried out.

Acknowledgement

This work was supported by the General Research Fund from Hong Kong University Grants Committee [grant number GRF 15403414].

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