11th Asia-Pacific Conference on Combustion,
The University of Sydney, NSW Australia
10th -14th December 2017.

# Lewis number effect on the flame height of circulation-controlled firewhirls

**Dehai Yu[1] and Peng Zhang[1*]**

[1]Department of Mechanical Engineering, The Hong Kong Polytechnic University
Hung Hom, Kowloon, Hong Kong

## Abstract

A theoretical analysis was carried out on the circulation-controlled firewhirls, considering multiple physical effects, including the previously investigated variable physical properties and strong vortex, and with emphasis on the effect of non-unity Lewis number. By means of perturbation method, an analytical expression of the flame height, with all effects presented in explicit forms, was derived in the situation of both fuel and of oxidizer Lewis numbers being near unity. In leading order approximation, the non-unity Lewis number effect tends to change the flame height by a factor of $Le_O/Le_F$.

## 1 Introduction

Firewhirls, as a natural phenomenon widely occurring in wild and urban fire and holding potential to cause severe damages to lives and properties, have attracted numerous investigations involving both experimental and theoretical approaches in the past a few decades[1-19]. As the flame height of firewhirls increasing, the radiative heat transfer tends to facilitate the ignition remotely therefore expediting the spread of the fires. Hence, there is always a particular interest in the flame height of firewhirl [3-8, 15, 19].

In Chuah et al.'s experimental study[5], the vortical flow was set up inclined, resulting in a correspondingly inclined firewhirl. This experiment testifies that the flame height of firewhirl was determined by circulation, instead of the previously recognized buoyance [6, 11, 20], provided that the circulation is sufficiently strong. By assuming constant density and mass diffusivity, Burgers vortex, and unity Lewis numbers, Chuah et al. proposed an theoretical prediction for the flame height[5]

$$\frac{x_h}{d_0} = \frac{Pe}{16Z_{st}} \tag{1}$$

where the flame height is linearly proportional to the diameter of the fuel pool, $d_0$, to the Peclet umber, $Pe$, and inversely proportional to the stoichiometric mixture fraction, $Z_{st}$. Compared with their own experimental results, (1) always gives an underestimation on the flame height.

Klimenko and Williams[7] argued that the Burgers vortex was not strong enough to describe the actual circulation-controlled firewhirls[21]. By replacing the Burgers vortex by a strong vortex[21], Klimenko and Williams derived a revised flame height expression

$$\frac{x_h}{d_0} = \frac{2}{\alpha_v}\frac{Pe}{16Z_{st}} \tag{2}$$

where $\alpha_v$ characterizes the strong vortex, and assumes value less than 2. The equation (2) agrees well with the experimental results of Chuah et al.[5].

In the theories of both Chuah et al.[5] and Klimenko and Williams[7], density and mass diffusivity were assumed as constants. Such an assumption is rather questionable in firewhirl, where the temperature is significantly higher than that of the ambience. As a result, as temperature increases, the density decreases and so does the flow inertia, and therefore the fuel can be transported into higher altitude, rendering a larger flame height[22]. Retaining the unity-Lewis-number assumption and regarding the vortical flow as Burgers vortex, the authors carried out a theoretical analysis for the variable physical properties effect on the flame height[15]. Introducing a Howarth-Dorodnitsyn[23] like coordinate transformation, the governing equations can be converted into a simplified density- and mass diffusivity-free form. An analytically explicit flame height expression can also be obtained[15]

$$\frac{x_h}{d_0} = \left(\frac{T_m}{T_0}\right)^{2-\alpha_T}\frac{Pe}{16Z_{st}} \tag{3}$$

where $T_m$ represents the specially defined "mean temperature", assuming value higher than the fuel pool temperature $T_0$, and $\alpha_T$ characterizes the temperature dependence of mass diffusivity[24] and it is always less than 2. The flame heights predicted by equation (3) also agree well with the experimental results of Chuah et al.[5].

Both variable physical properties and strong vortex tend to lengthen the flame height as indicated in (2) and (3). Because both effects are independent in physics, a simple combination of them in a theory must overshot the predictions on the flame height. Consequently, there must exist a mechanism that reduces the flame height. Inspired by the theoretical research on droplet combustion[22, 25], the authors[19] found that the discrepancy between the fuel and oxidizer mass diffusivities can be an effective loss-mechanism to the flame height, thereby a more general flame height expression was derived

$$\frac{x_h}{d_0} = \alpha_D\frac{2}{\alpha_v}\left(\frac{T_m}{T_0}\right)^{2-\alpha_T}\frac{Pe}{16Z_{st}} \tag{4}$$

where $\alpha_D = D_F/D_O$ is the ratio of fuel mass diffusivity to that of oxidizer. In Chuah et al.' experiments, the fuels adopted were methanol, ethanol, and 2-propanol, whose molecular weights are larger than that of air, therefore the fuel mass diffusivities are smaller than that of oxidizer[24], thus the values of $\alpha_D$ are less than unity, resulting in shorter flame height.

In all the above theoretical studies, the fuel and oxidizer Lewis numbers are assumed to be unity. However, in many combustion problems, such an assumption can seldom be exactly

Corresponding author. Fax: +852-27666664
E-mail address: pengzhang.zhang@polyu.edu.hk

satisfied[22, 25]. In this paper, utilizing the perturbation method and regarding the deviation of Lewis number from unity as small parameters[26], we proposed a theoretical analysis, attempting to reveal the non-unity Lewis number effect on the flame height of circulation-controlled firewhirls.

# 2 Mathematical Formulation

## 2.1 Governing Equations

Complete specification of the circulation-controlled firewhirl system requires three transport equations governing the fuel mass fraction, oxidizer mass fraction, and energy, all of which are inhomogeneous due to the presence of chemical reaction term. Considering the stoichiometry of the chemical reaction, the governing equations can be reduced to two equations, in which the reaction terms are formally removed[25, 26]. The specific mathematical forms are given as follows:

$$\begin{aligned}&\left(\tilde{\rho}\tilde{u}\frac{\partial}{\partial\tilde{x}}+\tilde{\rho}\tilde{v}\frac{\partial}{\partial\tilde{r}}\right)\left(\tilde{Y}_F+\tilde{T}\right)\\&-\frac{2Le_F}{Pe}\frac{\partial}{\partial\tilde{x}}\left[\tilde{\rho}\tilde{D}_F\frac{\partial}{\partial\tilde{x}}\left(\frac{\tilde{Y}_F}{Le_F}+\tilde{T}\right)\right]\\&-\frac{2Le_F}{Pe}\frac{1}{\tilde{r}}\frac{\partial}{\partial\tilde{r}}\left[\tilde{\rho}\tilde{D}_F\tilde{r}\frac{\partial}{\partial\tilde{r}}\left(\frac{\tilde{Y}_F}{Le_F}+\tilde{T}\right)\right]=0\end{aligned}\tag{5}$$

$$\begin{aligned}&\left(\tilde{\rho}\tilde{u}\frac{\partial}{\partial\tilde{x}}+\tilde{\rho}\tilde{v}\frac{\partial}{\partial\tilde{r}}\right)\left(\tilde{Y}_F-\tilde{Y}_O\right)\\&-\frac{2Le_F}{Pe}\frac{\partial}{\partial\tilde{x}}\left[\tilde{\rho}\tilde{D}_F\frac{\partial}{\partial\tilde{x}}\left(\frac{\tilde{Y}_F}{Le_F}-\frac{\tilde{Y}_O}{Le_O}\right)\right]\\&-\frac{2Le_F}{Pe}\frac{1}{\tilde{r}}\frac{\partial}{\partial\tilde{r}}\left[\tilde{\rho}\tilde{D}_F\tilde{r}\frac{\partial}{\partial\tilde{r}}\left(\frac{\tilde{Y}_F}{Le_F}-\frac{\tilde{Y}_O}{Le_O}\right)\right]=0\end{aligned}\tag{6}$$

in which the nondimensional quantities are defined as $\tilde{\rho}=\rho/\rho_0$, $(\tilde{r},\tilde{x})=(r,x)/r_0$, $(\tilde{u},\tilde{v})=(u,v)/u_0$, $\tilde{D}_F=D_F/D_{F0}$, $\tilde{Y}_F=Y_F$, $\tilde{Y}_O=Y_O/\sigma_{FO}$, and $\tilde{T}=c_pT/q_c$, with the quantities having index 0 referring to their values at the ground $x=0$, $\sigma_{FO}$ specifying the stoichiometry of the reaction, and $q_c$ representing the combustion heat release by consuming unit mass of fuel. The nondimensional numbers are defined as $Le_F=\lambda/\rho c_pD_F$, $Le_O=\lambda/\rho c_pD_O$, and $Pe=u_0d_0/D_{F0}$.

Equations (5) and (6) satisfy the most general situation in combustion, including non-unity Lewis numbers, i.e., $Le_F\neq 1$ and $Le_O\neq 1$, and they reduce to the conservation equations for the conventional species-species and species-enthalpy coupling function, $\beta_S=\tilde{Y}_F-\tilde{Y}_O$ and $\beta_T=\tilde{Y}_F+\tilde{T}$, respectively as Lewis numbers being unity. The boundary conditions for (5) and (6) are specified as: BC(1) at $\tilde{r}=0$, $\partial\tilde{Y}_F/\partial\tilde{r}=\partial\tilde{Y}_O/\partial\tilde{r}=\partial\tilde{T}/\partial\tilde{r}=0$; BC(2) at $\tilde{r}=\infty$, $\partial\tilde{Y}_F/\partial\tilde{r}=\partial\tilde{Y}_O/\partial\tilde{r}=\partial\tilde{T}/\partial\tilde{r}=0$; BC(3a) at $\tilde{x}=0$ and $\tilde{r}\leq 1$,

$$\begin{aligned}\tilde{\rho}\tilde{u}\tilde{Y}_F-(1/Pe)\tilde{\rho}\tilde{D}_F\left(\partial\tilde{Y}_F/\partial\tilde{x}\right)&=\tilde{\rho}\tilde{u}\\\tilde{\rho}\tilde{u}\tilde{T}-(2Le_F/Pe)\tilde{\rho}\tilde{D}_F\left(\partial\tilde{T}/\partial\tilde{x}\right)&=\tilde{\rho}\tilde{u}\left(\tilde{T}_0-q_v/q_c\right)\\\tilde{\rho}\tilde{u}\tilde{Y}_O-(1/Pe)\tilde{\rho}\tilde{D}_F\left(\partial\tilde{Y}_O/\partial\tilde{x}\right)&=0\end{aligned}$$

BC(3b) at $\tilde{x}=0$ and $\tilde{r}>1$, $\partial\tilde{Y}_F/\partial\tilde{x}=\partial\tilde{Y}_O/\partial\tilde{x}=\partial\tilde{T}/\partial\tilde{x}=0$; BC(4) at $\tilde{x}=\infty$, $\tilde{Y}_F=0$, $\tilde{Y}_O=\tilde{Y}_{O,\infty}$, $\tilde{T}=\tilde{T}_\infty$. BC(1) and BC(2) are the axisymmetric and radially far field conditions, respectively. BC(3) describes the Stefan flow of fuel evaporation induced by the heat transfer from the flame. BC(4) is the axially far field condition.

Introducing a Howarth-Dorodnitsyn like density-mass-diffusivity-weighted coordinate defined by[15, 19]

$$\begin{aligned}\xi=\frac{D_{F0}}{u_0r_0}\int_0^{\tilde{x}}\tilde{\rho}^2\tilde{D}_Fdx'=\frac{2}{Pe}\int_0^{\tilde{x}}\tilde{\rho}^2\tilde{D}_Fdx',\\\eta=\int_0^{\tilde{r}}\tilde{\rho}dr'\end{aligned}\tag{7}$$

equations (5) and (6) are simplified by assuming large Peclet number, i.e., $Pe\gg 1$.

$$\left(\hat{u}\frac{\partial}{\partial\xi}+\hat{v}\frac{\partial}{\partial\eta}\right)\left(\tilde{Y}_F+\tilde{T}\right)=Le_F\frac{1}{\eta}\frac{\partial}{\partial\eta}\left[\eta\frac{\partial}{\partial\eta}\left(\frac{\tilde{Y}_F}{Le_F}+\tilde{T}\right)\right]\tag{8}$$

$$\left(\hat{u}\frac{\partial}{\partial\xi}+\hat{v}\frac{\partial}{\partial\eta}\right)\left(\tilde{Y}_F-\tilde{Y}_O\right)=Le_F\frac{1}{\eta}\frac{\partial}{\partial\eta}\left[\eta\frac{\partial}{\partial\eta}\left(\frac{\tilde{Y}_F}{Le_F}-\frac{\tilde{Y}_O}{Le_O}\right)\right]\tag{9}$$

The corresponding boundary conditions are simplified to BC(I) at $\eta=0$, $\partial\tilde{Y}_F/\partial\eta=\partial\tilde{Y}_O/\partial\eta=\partial\tilde{T}/\partial\eta=0$; BC(II) at $\eta\to\infty$, $\partial\tilde{Y}_F/\partial\eta=\partial\tilde{Y}_O/\partial\eta=\partial\tilde{T}/\partial\eta=0$; BC(III-a) at $\xi=0$ and $\eta\leq 1$, $\tilde{Y}_F=\tilde{Y}_{F0}$, $\tilde{T}=\tilde{T}_0-q_v/q_c$, $\tilde{Y}_O=0$; BC(III-b) at $\xi=0$ and $\eta>1$, $\tilde{Y}_F=0$, $\tilde{T}=\tilde{T}_\infty$, $\tilde{Y}_O=\tilde{Y}_{O,\infty}$. The detailed derivation has been presented in[19].

Considering strong vortex effect, the vortical flow are characterized by the following stream function[19]

$$\psi=\begin{cases}s(\xi)\eta^{\alpha_v'}, & \eta<\eta_c\\\eta_c^{\alpha_v'-\alpha_v}s(\xi)\eta^{\alpha_v}, & \eta\geq\eta_c\end{cases}\tag{10}$$

in terms of which the velocity components can be determined as

$$\hat{u}=\begin{cases}\alpha_v'\eta^{\alpha_v'-2}s(\xi), & \eta<\eta_c\\\alpha_v\eta_c^{\alpha_v'-\alpha_v}s(\xi)\eta^{\alpha_v-2}, & \eta\geq\eta_c\end{cases}\tag{11}$$

$$\hat{v}=\begin{cases}-s'(\xi)\eta^{\alpha_v'-1}, & \eta<\eta_c\\-\eta_c^{\alpha_v'-\alpha_v}s'(\xi)\eta^{\alpha_v-1}, & \eta\geq\eta_c\end{cases}\tag{12}$$

where $\eta_c$ represents the radius of the vortex core, and $\alpha_v'$ and $\alpha_v$ characterize the vortical flow inside and outside the vortex core, respectively.

Defining the stream function coordinate as

$$\chi=\frac{\alpha_v}{2}\xi,\qquad\zeta=\sqrt{2\psi}\tag{13}$$

equations (8) and (9) are converted into

$$\frac{\partial}{\partial\chi}\left(\tilde{Y}_F+\tilde{T}\right)=Le_F\frac{1}{\zeta}\frac{\partial}{\partial\zeta}\left[\zeta\frac{\partial}{\partial\zeta}\left(\frac{\tilde{Y}_F}{Le_F}+\tilde{T}\right)\right]\tag{14}$$

$$\frac{\partial}{\partial\chi}\left(\tilde{Y}_F-\tilde{Y}_O\right)=Le_F\frac{1}{\zeta}\frac{\partial}{\partial\zeta}\left[\zeta\frac{\partial}{\partial\zeta}\left(\frac{\tilde{Y}_F}{Le_F}-\frac{\tilde{Y}_O}{Le_O}\right)\right]\tag{15}$$

in which the convection and diffusion transports are completely separated. Correspondingly, the boundary conditions become: BC(i) at $\zeta=0$, $\partial\tilde{Y}_F/\partial\zeta=\partial\tilde{Y}_O/\partial\zeta=\partial\tilde{T}/\partial\zeta=0$; BC(ii) at $\zeta\to\infty$, $\partial\tilde{Y}_F/\partial\zeta=\partial\tilde{Y}_O/\partial\zeta=\partial\tilde{T}/\partial\zeta=0$; BC(iii-a) at $\chi=0$ and $\zeta\leq 1$, $\tilde{Y}_F=\tilde{Y}_{F0}$, $\tilde{T}=\tilde{T}_0'$, $\tilde{Y}_O=0$; BC(iii-b) at $\chi=0$ and $\zeta>1$, $\tilde{Y}_F=0$, $\tilde{T}=\tilde{T}_\infty$, $\tilde{Y}_O=\tilde{Y}_{O,\infty}$.

## 2.2 Perturbation Solution

In the problems with distinct fuel and oxidizer (non-unity) Lewis numbers, an exact solution to the fuel and oxidizer mass fractions as well as temperature requires the division of the whole domain into two regions, namely, the fuel region and the oxidizer region. The governing equations must be solved separately to yield general solutions being valid in either region, in terms of which the flame location as well as the flame temperature can be determined by matching the solutions at the flame location [22, 25]. However, the three-dimensionality of the firewhirl system invalidates the above approach for arbitrary non-unity Lewis number.

If both fuel and oxidizer Lewis numbers are near unity, equations (14) and (15) can be approximately solved by means of perturbation expansion. Following Chung and Law's approach[26], we define the Lewis number weighted coupling function by

$$\beta_S = \frac{\tilde{Y}_F}{Le_F} - \frac{\tilde{Y}_O}{Le_O}, \qquad \beta_T = \frac{\tilde{Y}_F}{Le_F} + \tilde{T} \tag{16}$$

which become the conventional coupling functions as Lewis numbers are unity. In the absence of unity Lewis number, the coupling functions defined by (16) have better mathematical property than conventional coupling function, since the latter are smooth across the flame whereas the former is continuous but not smooth[22, 26]. For near-unity Lewis numbers, we can introduce two small quantities,

$$\left(1 - \frac{1}{Le_i}\right) = l_i, \qquad i = F, O \tag{17}$$

in terms of which the distributions of fuel and oxidizer mass fractions and temperature can be expanded as

$$\tilde{Y}_F = \tilde{Y}_F^0 + l_F \tilde{Y}_F^1 + \cdots \tag{18}$$

$$\tilde{Y}_O = \tilde{Y}_O^0 + l_F \tilde{Y}_O^1 + \cdots \tag{19}$$

$$\tilde{T} = \tilde{T}^0 + l_F \tilde{T}^1 + \cdots \tag{20}$$

Substituting (18)-(20) into (16) gives the expansion of Lewis-number-weighted coupling functions, and further substitution of the latter into (14) and (15), gives the leading order equation, describing the conservation of the leading order Lewis-number-weighted coupling function, being valid in the whole field

$$\frac{\partial \beta_T^0}{\partial \chi} - Le_F \frac{1}{\zeta} \frac{\partial}{\partial \zeta}\left(\zeta \frac{\partial \beta_T^0}{\partial \zeta}\right) = 0 \tag{21}$$

$$\frac{\partial \beta_S^0}{\partial \chi} - Le_F \frac{1}{\zeta} \frac{\partial}{\partial \zeta}\left(\zeta \frac{\partial \beta_S^0}{\partial \zeta}\right) = 0 \tag{22}$$

The boundary conditions are given by: LBC(1) at $\zeta = 0$, $\partial\beta_S^0/\partial\zeta = \partial\beta_T^0/\partial\zeta = 0$; LBC(2) at $\zeta \to \infty$, $\partial\beta_S^0/\partial\zeta = \partial\beta_T^0/\partial\zeta = 0$; LBC(3a) at $\chi = 0$ and $\zeta \le 1$, $\beta_S^0 = \tilde{Y}_{F0}/Le_F$, $\beta_T^0 = \tilde{Y}_{F0}/Le_F + \tilde{T}_0'$; LBC(3b) at $\chi = 0$ and $\zeta > 1$, $\beta_S^0 = -\tilde{Y}_{O,\infty}/Le_O$, $\beta_T^0 = \tilde{T}_\infty$.

## 2.3 Flame Height

The flame contour expression can be determined by the equating (24) to zero, implying the complete consumption of fuel and oxidizer

$$\left(\frac{\tilde{Y}_{F0}}{Le_F} + \frac{\tilde{Y}_{O,\infty}}{Le_O}\right) \int_0^\infty J_0(\omega\zeta_f) J_1(\omega) \exp(-Le_F \omega^2 \chi_f) d\omega = \frac{\tilde{Y}_{O,\infty}}{Le_O} \tag{25}$$

The flame height is the highest point on the flame contour:,

$$\left(\frac{\tilde{Y}_{F0}}{Le_F} + \frac{\tilde{Y}_{O,\infty}}{Le_O}\right) \int_0^\infty J_1(\omega) \exp(-Le_F \omega^2 \chi_h) d\omega = \frac{\tilde{Y}_{O,\infty}}{Le_O} \tag{26}$$

where $\chi_h$ represents the flame height in stream function coordinate. The left-hand side of (26) can be integrated exactly, yielding

$$\int_0^\infty J_1(\omega) \exp(-Le_F \omega^2 \chi_h) d\omega = 1 - \exp\left(-\frac{1}{4Le_F\chi_h}\right) \tag{27}$$

Substitution of (27) into (26) gives

$$\left(\frac{\tilde{Y}_{F0}}{Le_F} + \frac{\tilde{Y}_{O,\infty}}{Le_O}\right)\left[1 - \exp\left(-\frac{1}{4Le_F\chi_h}\right)\right] = \frac{\tilde{Y}_{O,\infty}}{Le_O} \tag{28}$$

Since the firewhirl is circulation-controlled, we can assume large flame height, i.e., $4Le_F\chi_h \gg 1$, and have

$$\exp\left(-\frac{1}{4Le_F\chi_h}\right) \sim 1 - \frac{1}{4Le_F\chi_h} \tag{29}$$

Substituting of (29) into (28) and recalling the definition of stoichiometric mixture fraction[22],

$$Z_{st} = \frac{\tilde{Y}_{O,\infty}}{\tilde{Y}_{F0} + \tilde{Y}_{O,\infty}} \tag{30}$$

the flame height expression can be derived as

$$\chi_h = \frac{Le_O}{Le_F} \frac{1}{4Z_{st}} \tag{31}$$

where all terms of order $O(l_F)$ and higher are neglected in leading order approximation. Inverting the stream function coordinates to density-mass-diffusivity-weighted coordinates, and then the latter to physical coordinates, the flame height expression becomes

$$\frac{x_h}{d_0} = \frac{2}{\alpha_v} \frac{Le_O}{Le_F} \left(\frac{T_m}{T_0}\right)^{2-\alpha_T} \frac{Pe}{16Z_{st}} \tag{32}$$

Equation (32) reveals three mechanisms that can change flame height. The strong vortex effect tends to intensify the axial stretching effect of the vortical flow, therefore increasing the flame height. The variable physical properties effect resolves the decreasing of density due to high temperature, resulting in the fuel being more readily transported to higher altitude, i.e., lengthening the flame height. The non-unity Lewis number effect modifies the flame height according to the specific combustion system. For those using hydrocarbons with large molecular weight, the fuel Lewis number is usually larger than that of air, thus the effect of non-unity Lewis number tends to decrease the flame height.

More generally, the flame contours can be plotted by numerically soling (25) with respect to various pairs of Lewis numbers, as shown in Fig. 1. It is seen that the decreasing of fuel Lewis number tends to expand the flame contour, particularly in axial direction, while the oxidizer Lewis number exhibits the inverse effect on the flame contour. The Lewis numbers can in physics be interpreted as the ratio of thermal diffusivity to mass diffusivity. The smaller Lewis number of fuel than that of oxidizer implies that the fuel has larger mass diffusivity than the oxidizer, due to which the fuel has higher capability to be transported to larger altitude, i.e., extending the flame contour in axial direction. In opposite situation, i.e. Lewis number of oxidizer being lower than that of fuel, the larger mass diffusivity of oxidizer tends to squeeze flame contour, especially in the axial direction because the flame end close to the fuel pool is anchored at the rim of fuel pool due to flame sheet approximation.

In a special situation of equal Lewis numbers, the flame contours are presented in Fig 1(right), which are only moderately different from each other. It testifies that (32) that the flame height tends to rely on the variation of Lewis numbers by their ratio, which, according to respective influence of fuel and oxidizer Lewis numbers on the flame contour, must be $Le_O/Le_F$. In addition, the flame shape with larger Lewis numbers tend to be slimmer than that with lower Lewis numbers. The physical reason can be interpreted as follows. The larger Lewis numbers for both fuel and oxidizer mean lower mass diffusivities so that the axial convection tends to be more dominant than diffusion, resulting in a more stretched flame shape in the axial direction. Similarly, lower Lewis numbers for fuel and oxidizer mean enhanced mass diffusion towards all the directions, which tends to counteract the axial convection and to make flame shape stouter.

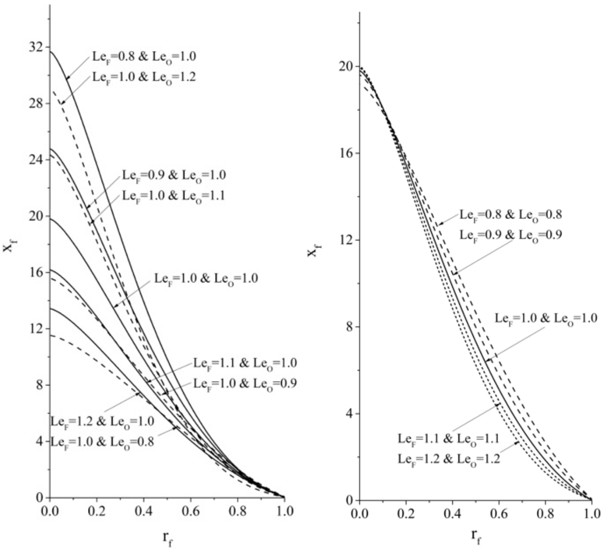


Fig 1. Flame contours with various pairs of Lewis numbers.

## 3 Conclusions

This paper presents a theoretical analysis to extend the existing theories on the circulation-controlled firewhirls to non-unity Lewis number. Utilizing the perturbation method, an analytical expression of flame height is derived in the situation of Lewis number being near unity. The results show that both strong vortex and variable physical properties effects tend to lengthen the flame height, but the non-unity Lewis number effect can reduce the flame height of firewhirls.

## 5 Acknowledgment

This work was supported by the Hong Kong RGC/GRF (operating under contract numbers PolyU 152217/14E and 152651/16E) and partly by the Hong Kong Polytechnic University (G-UA2M and G-YBGA).

## References

[1] F. Battaglia, K.B. McGrattan, R.G. Rehm, H.R. Baum, Combust. Theory Modelling 4 (2000) 123-138.

[2] F. Battaglia, R.G. Rehm, H.R. Baum, Phys. Fluids, 12 (2000) 2859-2867.

[3] K.H. Chuah, G. Kushida, Proc. Combust. Inst. 31 (2007) 2599-2606.

[4] K.H. Chuah, K. Kuwana, K. Saito, Combust. Flame 156 (2009) 1828-1833.

[5] K.H. Chuah, K. Kuwana, K. Saito, F.A. Williams, Proc. Combust. Inst. 33 (2011) 2417-2424.

[6] H.W. Emmons, S.-J. Ying. The fire whirl. Symposium (International) on Combustion, 1967. p. 475-488.

[7] A.Y. Klimenko, F.A. Williams, Combust. Flame 160 (2013) 335-339.

[8] K. Kuwana, S. Morishita, R. Dobashi, K.H. Chuah, K. Saito, Proc. Combust. Inst. 33 (2011) 2425-2432.

[9] K. Kuwana, K. Sekimoto, K. Saio, F.A. Williams, Y. Hayashi, H. Masuda, Aiaa Journal 45 (2007) 16-19.

[10] K. Kuwana, K. Sekimoto, K. Saito, F.A. Williams, Fire Safety Journal 43 (2008) 252-257.

[11] J. Lei, N. Liu, K. Satoh, Proc. Combust. Inst. 35 (2015) 2503-2510.

[12] J. Lei, N.A. Liu, L.H. Zhang, K. Satoh, Combust. Flame 162 (2015) 745-758.

[13] J.A. Lei, N.A. Liu, L.H. Zhang, H.X. Chen, L.F. Shu, P. Chen, Z.H. Deng, J.P. Zhu, K. Satoh, J.L. de Ris, Proc. Combust. Inst. 33 (2011) 2407-2415.

[14] F.A. Williams, Progress in Energy and Combustion Science 8 (1982) 317-354.

[15] D. Yu, P. Zhang, Proc. Combust. Inst. (2017) 3097-3104.

[16] K. Zhou, N. Liu, K. Satoh, Fire Safety Science 10 (2011) 681-691.

[17] K. Zhou, N. Liu, L. Zhang, K. Satoh, Fire Technology 50 (2014) 1573-1587.

[18] R. Zhou, Z.N. Wu, J. Fluid Mech. 583 (2007) 313-345.

[19] D. Yu, P. Zhang, Combust. Flame 182 (2017) 36-47.

[20] P.B. Sunderland, J.E. Haylett, D.L. Urban, V. Nayagam, Combust. Flame 152 (2008) 60-68.

[21] A. Klimenko, Theoretical and Computational Fluid Dynamics 14 (2001) 243-257.

[22] C.K. Law, Combustion Physics, Cambridge University Press, 2006.

[23] P.M. Chung, Advances in Heat Transfer 2 (1965) 109-270.

[24] E.L. Cussler, Diffusion: mass transfer in fluid systems, Cambridge University Press, 2009.

[25] F. Williams, Combustion Theory 2nd. The Benjamin/ Cummings, 1985.

[26] S.H. Chung, C.K. Law, Combust. Sci. and Tech. 37 (1984) 21-46.