Forecast-Corrected Production-Inventory Control Policy in Unreliable Manufacturing Systems

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Abstract: In traditional research on production-inventory control problems with failure-prone manufacturing systems, a stationary demand process is an essential assumption. However, such a situation may not be true. This study extends the hedging-point-based production-inventory control problem into the case with non-stationary demand. The demand forecasting process is simulated and categorized into two different cases. First of all, a two-level control policy is proposed to solve the problem with a Markov modulated Poisson demand process which is often used in qualitative forecasting. Then the quantitative forecasting process using time series methods is modeled and a forecast-corrected control policy is proposed accordingly. The impact of forecasting on the system performance is then investigated. An integrated simulation and experimental design method was adopted to solve the modified optimal control problem. The results show that the proposed control policy can outperform the traditional stationary policy when the forecasting error is limited to a certain level.

Keywords: Supply Chain; Production Control; Inventory Control; Forecasting; Simulation; Optimization;

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1. Introduction

Production and inventory control problems in manufacturing systems have attracted a large number of researchers in the past three decades. The aim, in general, is to minimize the overall cost including the inventory holding/backlogging cost, production cost and transportation cost, by determining the optimal control variables, such as production speed and reordering policy. As a part of the whole supply chain, production and inventory control has a fundamental and crucial impact on the performance of the whole supply chain.

Hedging Points Policy (HPP) has been shown to be the optimum control policy for this specific problem of unreliable manufacturing systems according to the literature. Kimemia and Gershwin (1981) were the first researchers to introduce HPP for solving control problems in Flexible Manufacturing Systems (FMS). Akella and Kumar (1986) then proved the optimality of HPP and derived its optimal parameters in a single machine, single product, and a failure-prone manufacturing system.

However for production and inventory control, especially in the field of failure prone manufacturing systems, the majority of published works assumed that the demand process was stationary. In terms of stationary demand, the demand is normally viewed as a constant throughout the whole process. For instance, research on production and inventory control problems (Sethi and Zhang, 1999, Hajji et al., 2012) led to models with constant demand rate only. The demand process of industry, on the contrary, normally follows a non-stationary process. The reasons for this phenomenon can be summarized as intensive market competition, high frequency of new products, short life cycles and seasonality (Neale and Willems, 2009). All those factors have led to the fact that a stationary demand process is not realistic enough to reflect the real situation in production-inventory systems.

The impacts of non-stationary demand on inventory control policy, however, have already been investigated. Tunc et al. (2011) studied the cost of using stationary inventory policies for the condition of non-stationary demand and implied that stationary policies are preferred due to their simplicity in real cases. But using stationary inventory policy under non-stationary demand process is costly in terms of the extra inventory cost produced. The importance of the understanding the demand shapes was studied by Ramaekers and Janssens (2008). They stated that different types of demand can significantly influence the inventory management performance. Moreover, their study was based on the assumption that the mean and deviation are the same for all the distribution examined. Recently, studies on the relationship between forecasting and inventory control have been carried out, specifically on condition of non-stationary demand, by a number of researchers. Strijbosch et al. (2011) looked at the interaction between forecasting and stock control in a periodic order-up-to level inventory system. Non-stationary demand was used and the fill rate of a manufacturing system under different forecasting methods was evaluated as a performance indicator. Ali et al. (2012) discussed the impact of forecasting errors on inventory parameters. Similarly, Babai et al. (2013) investigated the relationship between forecasting accuracy and inventory performance, in terms of inventory holding and cost. In addition, order-up-to-level policy and information sharing mechanisms were applied in their supply chain model which extended the research of Ali et al. In their models, inventory policies, such as (s, S) and (R, S) were examined against non-stationary demand. Sanders and Graman (2009) proposed a method to quantify the cost of forecast errors in a warehouse environment. Their work suggested that forecasting bias had a significant influence on the organizational cost. Other similar research studies can also be found in (Syntetos et al., 2010, Warren and Chang, 2010). Our work, on the other hand, looks at the impact of non-stationary demand on production and inventory control problems.

With respect to the production and inventory control problem, there have been some studies discussing the relationship between forecasting and production-inventory control. Fildes (1992) looked at the application of forecasting on production and inventory control and how the accuracy could be improved by different forecasting methods. Similarly, a forecasting-production-inventory system was analyzed by Toktay and Wein (2001). A Martingale model of forecast evolution was used to provide forecasting updates for the forecast-corrected stock policy. However, in their research work, a reliable manufacturing system was assumed and modeled.

The existing literature reveals two major gaps in a failure-prone production-inventory control problem. First of all, non-stationary demand is normally neglected. Secondly, forecasting methods and their impacts have not been thoroughly examined and discussed. Hence, this paper incorporates forecasting and non-stationary demand into a failure-prone production-inventory control problem with lot sizing. In practice, forecasting methods can be divided into two main categories: qualitative forecasting and quantitative forecasting. Qualitative methods are based on subjective judgment. It is especially useful when the data for forecasting is not available or there is no satisfactory quantitative model to forecast demand. In terms of quantitative methods, the time series method is the most common one used in industry, and it relies on the assumption that future data follow the past patterns.

In this study, qualitative and quantitative forecasting are examined in case 1 and case 2. First of all, to model the qualitative forecasting process, a two-state Markov modulated compound Poisson demand is utilized in case 1. Markov modulated compound Poisson demand, according to literatures and industrial practice, is normally generated as a result of expert judgment and empirical data (Quigley et al., 2011, Rosqvist, 2000). A two-state control policy is proposed to deal with the non-stationary demand. In case 2, for quantitative forecasting, time series methods are used to forecast real industry demand. A forecast-corrected control policy is applied. Further, the impact of forecasting methods and their accuracy in a production-inventory system are investigated based on the results from case 2. In both cases, simulation is combined with the design of experiments and linear regression meta-modeling to determine the optimal control policy.

The remainder of this paper is formed as follows: in Section 2, the research background is introduced. Section 3 lists the notations and assumptions used. The models in the two cases and their corresponding mathematical formulation are introduced in Section 4. The modified hedging point policy and details of the simulation, experimental design and linear regression meta-modeling are also provided. Section 5 presents the numerical experiments conducted. The results are discussed in Section 6 and finally**,** conclusions are made in the last section, with future work indicated.

2. Literature Review

This section shows the background information of our research. The literature on traditional failure-prone production-inventory control problem and forecasting is reviewed.

2.1 Failure-prone production and inventory control

The optimality of the Hedging Point Policy in a failure-prone manufacturing system has been proven by Akella and Kumar (1986), as mentioned earlier. They showed that under the assumption of constant demand, and for a single machine single product manufacturing system, there is a key parameter, the so-called hedging point H, existing in the optimal policy. $u(t)$ and $x(t)$ are the production rate and inventory level at time t, respectively. d represents the demand rate and u_{max} is the maximum production rate available. The traditional optimal control policy is generalized as shown in Equation 1:

$$\mathrm{u(t)} = \begin{cases} 0 & if\ x(t)\ > H \\ d & if\ x(t)\ = H \\ u_{max} & if\ x(t) < H \end{cases} \tag{1}$$

This control policy shows that the production rate adjusts its value according to the inventory level. For example, when inventory level $x(t)$ is lower than H, the maximum production rate will be adopted. In their work, the failure of the machine is modeled by a two state Markov chain. The extended work for a multi-product, single machine model was carried out by Sethi (1999) in which the hedging points for different products were determined separately. Sethi's research showed that the structure of the original hedging point policy was also valid for multi-product problem. The stochastic version of this problem was proposed by Veatch (1996) and Chen (2004) respectively. Chen's work, especially, provided the characteristics of HPP and the corresponding switching curves for both reliable and unreliable systems. The 21st century has witnessed a trend of combining production and inventory control with other problems, for instance, maintenance, replenishment and transportation. However due to the increasing complexity in the integrated problem, pure analytical analysis becomes extremely complex in solving the integrated problem (Sajadi et al., 2011). Methods such as numerical approximation, simulation and some statistical techniques were then introduced to solve the problem. In a model that considered both production and preventive maintenance rate control, Gharbi and Kenne (2000) adopted simulation, experimental design and linear regression meta-modeling. The optimal hedging point, machine switching age and maintenance age parameter were determined when the minimum inventory cost was achieved. Bouslah et al. (2013) studied the production and inventory control problem in an unreliable batch production system with constant demand. Batch size was considered together with the hedging point. By using the resolution approach, their research first approximated the structure of the control policy at a small scale. The research in this paper is inspired by the work of Gharbi (2000) and Bouslah et al. (2013) in terms of the methodology used. However, non-stationary demand is introduced into our model to make the problem more realistic. In addition, using forecasting methods enables a forecast-corrected hedging point policy to be made. Details on the modified control policy are introduced in the next section.

2.2 Forecasting methods

Qualitative forecasting, for instance the Delphi method and market research, is usually used for medium to long-term forecasting. The results are mainly based on the judgment and experience of decision makers. Compared with quantitative methods, qualitative forecasting can take more nonnumerical factors into consideration (Makridakis et al., 2008). For quantitative forecasting, time series methods are commonly used for both stationary and non-stationary demand forecasting. They are mainly based on the available historical data. Among the time series forecasting methods, Simple Moving average (SMA), Single Exponential Smoothing (SES), Holt-Winter Seasonal (HWS) method and Autoregressive Integrated Moving Average (ARIMA) are popular in both practical applications and academic research (Box et al., 2013, Bermudez, 2013). Strijbosch and Moors (2005) compared the performances of Single Exponential Smoothing, Simple Moving Average and ARIMA in an (R,S) inventory policy. Fill rate was selected instead of inventory cost to evaluate the performance in the case of non-stationary demand. These three methods were also adopted in the research of Warren and Chang (2010). Similar to the research of Strijbosch and Moors (2005), the impact of forecasting on inventory policy performance was examined and compared. However, the previous work focused more on finding the optimal forecasting parameters while the later one optimized the inventory policy. For more examples and details about the application of forecasting methods we refer readers to Syntetos et al. (2009). However, forecasting cannot produce 100% accurate results. Even for a given demand, different forecasting methods will produce different accuracies. This issue was not considered in the work of Tunc et al. (2011) and the demand was known to the system. So the cost related to inaccuracy was neglected in his research. In terms of the accuracy of forecasting, it can be measured by parameters such as Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Mean Squared Error (MSE) (Fildes, 1992, Hosoda and Disney, 2006).

In our research, WMA, ES, HWS and ARIMA are chosen to forecast the demand. The optimal parameters for each method are firstly estimated, which minimizes the prediction errors based on a given demand set. The forecast errors are calculated based on the value of MAPE in order to evaluate the accuracy of forecasting methods.

3. Notation and assumptions

This section lists all the notation and assumptions used throughout this research.

3.1 Notation

Table 1 Notation defined throughout this research

i	The index of batch	C_s	Setup cost ($/per batch)
j	The index of order	α	Machine in functional status
$x_i(t)$	WIP level in batch i	β	Machine is in failed status
$y(t)$	Inventory level	$u(t)$	Production rate
u_{max}	Maximum production rate	$S(t)$	Status of machine at time t
L_i	Lot size of i_{th} batch	$r_{\alpha\beta}$	Transition time from α to β
$TB(t)$	Total batch number up to time t	$r_{\beta\alpha}$	Transition time from β to α
d_j	The demand size of j_{th} order	$\lambda(t)$	Arrival rate of customer orders at time t
$df(t)$	Forecasted demand at time t	μ	Mean of demand size
$N(t)$	The number of arrived orders up to t	σ	Standard deviation of the demand size

$D(t)$	Cumulative demand from time 0 to t	θ_i	Finishing time of i^{th} batch
H	Hedging point	ξ_i	The starting time of i^{th} batch
C_h	Holding cost (per unit)	AC_c	Average cost from corrected policy
C_b	Backlogging cost (per unit)	AC_o	Average cost from original policy
PCR	Percentage cost reduction	$P(t)$	Production policy at time t
p	Mean time for high demand	q	Mean time for low demand
λ_l	Order arrival rate of low demand period	λ_h	Order arrival rate of high demand period
H_l	Low hedging point	H_h	High hedging point
L_l	Low batch size	L_h	High batch size
L_m	Medium batch size		

3.2 Assumptions

a. During the production of one batch, the batch level is simply the integral of the production rate with respect to time. The status of the machine is modeled by a two-state Markov process. The average transition time from the functional state α to failure state β and the average transition time from failure state β to the functional state α are described by two individual Poisson processes with different mean values.
b. Unmet demand is backlogged and backlog is satisfied first.
c. The production rate within one batch is assumed to be constant.
d. The replenishment of raw material is assumed to be instant and unlimited.

4. Problem Formulation

In this section, the inventory model is first introduced with respect to the mathematical formulation. Then second part presents the two modified hedging point policies and the mechanism of the control policies is elaborated in detail. Lastly, the proposed simulation, design of experiments and linear regression meta-modeling is explained.

4.1 Inventory model

A single machine, single product, failure-prone manufacturing system subject to non-stationary demand is modeled in this paper. In batch i, products are manufactured from time ξ_i with production rate $u(\tau)$ through which the Work-In-Process (WIP) batch level $y(t)$ increases accordingly. WIP products are stored beside the machine until the number of products for one batch is satisfied. The WIP inventory behavior is described in Equation 2. At time θ_i, the finished batch is sent to the inventory warehouse, and it results in a sudden increase in inventory level $x(t)$ by the specific batch size L_i which is shown in Equation 3. Customers' orders arrive at the manufacturing system and are processed in the order of First-Come-First-Served (FCFS). The corresponding number of products is delivered to the customers immediately when the order arrives. In Equation 4, the behavior of inventory level between two batches is illustrated, where $d_j$represents the demand of order j and $N(t - \xi_i)$ represents the total number of orders that arrived during time period $t - \xi_i$. The total amount of demand aggregated from the batch starting time ξ_i to time t equals to $\sum_1^{N(t-\xi_i)} d_j$.

$$y(t) = \int_0^t u(\tau)\, d\tau \quad t \in (\xi_i, \theta_i) \text{ where } y(0) = 0 \text{ and } u(\tau) \in (0, u_{max}) \tag{2}$$

$$x(\xi_{i+1}) = x(\theta_i) + L_i \quad \text{where } x(0) = x_0 \tag{3}$$

$$x(t) = x(\xi_i) - \sum_{1}^{N(t-\xi_i)} d_j \quad t \in (\xi_i, \theta_i) \tag{4}$$

Inventory surplus or backlog in each period generates inventory holding and backlog costs at the rate of $C_h$and C_b, and for each batch produced, a fixed setup cost C_s is charged. The aggregated setup cost is represented as $C_s * TB(t)$ where $TB(t)$ is the number of batches produced up to time t. Following the method used in Sethi and Zhang's work (1999), the corresponding dynamic programming cost function of the model is defined as:

$$J(x, u, \alpha) = \lim_{T\to\infty} \frac{1}{T} E \int_0^T \{C_h x^+(t) + C_b x^-(t) + C_s TB(t)\}\, dt \tag{5}$$

where $x^+(t) = max\big(0, x(t)\big)$ and $x^-(t) = max(0, -x(t))$.

The overall objective of this model is to minimize the expected long-run average cost $J(x, u, \alpha)$ by finding the optimal control policy. However solving the cost function is analytically complex, even for a simple model with constant demand, not to mention batch production (Sajadi et al., 2011, Bouslah et al., 2013). In this model, batch production and non-stationary demand are considered which significantly increases the complexity of the analytical method. Hence, we don't adopt an analytical approach to obtain the structure of optimal policy. Instead, a modified hedging point policy is employed, and the structure of the hedging point policy by (Akella and Kumar, 1986, Gharbi and Kenne, 2000, Bouslah et al., 2013) is used. This practice is also utilized in (Sajadi et al., 2011). Under the condition of non-stationary demand and batch production in this paper, it can provide a good approximation of the optimal solution when the analytical analysis is hard to be implemented.

4.2 Case 1: HPP with qualitative forecasting

In order to model the non-stationary demand, we firstly use compound Poisson process demand to model the intermittent arrival of orders which is forecast by expert judgment. The size of demand d_i follows a normal distribution of parameters μ and σ. The Poisson process models the arrival of orders with parameter $\lambda(t)$ which changes with time. Then a two-state Markov chain is combined with the compound Poisson process to model the mean time for high demand and low demand periods with their respective values of p and q. The cumulated demand $D(t)$ can be expressed in Equation 6 and $N(t)$ is the number of orders that arrived up to time t, following a Poisson process with an arrival rate of λ. At the same time, the demand size of order j, d_j follows a normal distribution with mean μ and standard deviation σ.

$$D(t) = \sum_{1}^{N(t)} d_j \tag{6}$$

This modeling method has been widely used in the academic literature because of its simplicity and practicality [Shang, 2012]. According to the theory of the compound Poisson process, the expected value for $D(t)$ is calculated as follows:

$$E[D(t)] = E[N(t)]E[d] = t\lambda(t)E[d_j] = t\mu\lambda(t) \tag{7}$$

To cope with the proposed non-stationary demand, a modified HPP is developed accordingly. In terms of production rate, as can be seen, when the inventory level $x(t)$ is larger than the sum of a hedging point H, the production rate will be reduced to zero. And when $x(t)$ is smaller than H but larger than the difference between H and L_i, the production rate will be set to the value of the forecasted demand $df(t+1)$ in next period, which equals to $\lambda t\mu$. Lastly, if the inventory level is smaller than $H - L_i$, then the maximum production rate will be applied to the production system. For hedging point H and lot size L_i, when $\lambda(t) = \lambda_l$, the set of (H_l, L_h) is applied where a relatively larger hedging point and smaller lot size is implemented. A combination of (H_h, L_l) is used instead for the condition of low demand. It is inspired by the theory of Rossi and Lodding (2012) in which they stated that a small lot size helps minimize the amplification of demand fluctuation. As explained above, the following equation illustrates the control policy proposed.

$$P(t) = \begin{cases} u(t) = \begin{cases} 0 & when\ x(t) \geq H \\ df(t+1) & when\ H - L_i \leq x(t) < H \\ u_{max} & when\ x(t) < H - L_i \end{cases} \\ (H, L_i) = \begin{cases} (H_l, L_h) & if\ \lambda(t) = \lambda_l \\ (H_h, L_l) & if\ \lambda(t) = \lambda_h \end{cases} \end{cases} \tag{8}$$

4.3 Case 2: HPP with quantitative forecasting

In this case, the time series forecasting method is used to forecast the non-stationary demand. In order to take the forecast demand into consideration, a forecast-corrected hedging policy is proposed. Similar to the previous control policy, the forecast-corrected control policy also consists of three parts, but the demand forecast is added into the control policy. The hedging point is corrected according to the demand quantity forecast for the next time period $t+1$, $df(t+1)$. This work classifies the demand data into three levels, high, medium and low. The classification is decided according to historical data. It means that if one specific forecast demand is higher than one third of the recorded demand data in the past, then this demand data is viewed as high demand and a small lot size L_s is used for production. Similarly, a forecast demand that is lower than one third but higher than two thirds of historical data is classified as medium demand. Correspondingly, a medium lot size L_m will be applied. In the paper, all the optimal values of the hedging point and high/medium/low lot sizes are determined simultaneously. The forecast-corrected control policy can be constructed as shown below.

$$u(t) = \begin{cases} 0 & when\ x(t) \geq H + df(t+1) \\ df(t+1) & when\ H + df(t+1) - L_i \leq x(t) < H + df(t+1) \\ u_{max} & when\ x(t) < H + df(t+1) - L_i \end{cases} \tag{9}$$

4.3 Proposed methodology

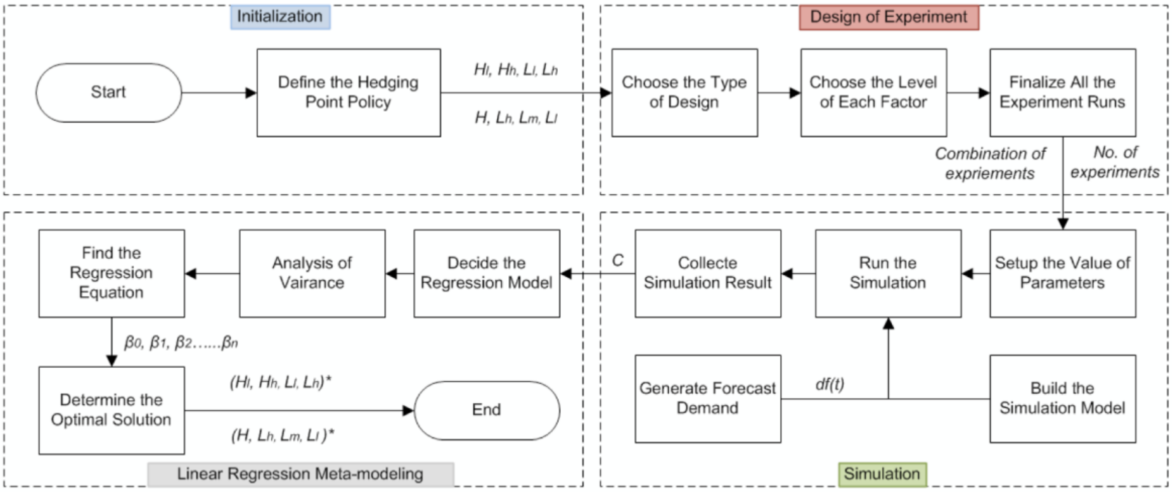
The whole process mainly consists of 4 stages: initialization stage, design of experiment, the simulation and linear regression meta-modeling stage as shown in Figure 1. In the initialization stage, the proposed hedging point policy and forecast demand are generated. Especially the forecast demand in case 2, which uses the proposed time series methods. The data are then stored separately and later are employed as the input to simulation process. For instance, for 3-period WME and SES, the following equations are applied to generate the forecast, where n_t is the forecast for period t and m_t is the actual demand for period t:

$$n_{t+1} = \alpha m_t + \beta m_{t-1} + \gamma m_{t-2} \qquad \alpha, \beta, \gamma \in [0,1] \; and \; (\alpha + \beta + \gamma) = 1 \tag{10}$$

$$n_{t+1} = \alpha m_t + (1 - a) n_t \quad \alpha \in [0,1] \tag{11}$$

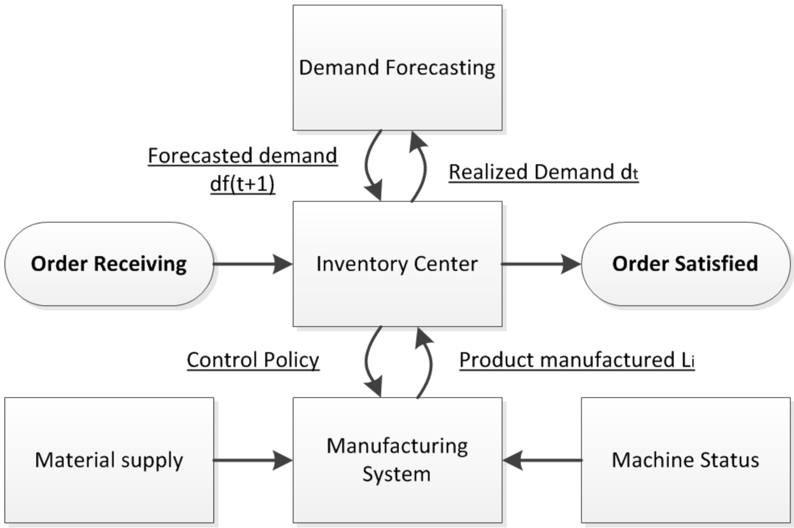
The optimal values of α, β and γ are decided by minimizing the MAPE for the whole forecasting period. In terms of ARIMA and HWS, the forecasting is implemented by using statistical software $R^{®}$.

Figure 1: The flow of simulation, experimental design and **linear regression meta-modeling**



The design of experiment stage provides a formal plan to change the value of various parameters in the simulation model. To be more specific, in this research, the Box-Wilson Central Composite Designs (CCD) method is employed which is commonly used in DOE (Myers et al., 2009). The simulation model is based on the proposed manufacturing system. The discrete event simulation software ARENA® is used to execute the simulation process. In the simulation stage, first of all, demand for the next period is forecast based on information of historical demand. Orders are received and the inventory is reduced accordingly to satisfy the demand. A control policy is adjusted based on information on the demand forecast and current inventory level. Then, the manufacturing system produces products according to the defined control policy. At the end of the simulation, the value of the average inventory cost for the whole simulation time is calculated and recorded. The simulation process is illustrated in Figure 2.

Figure 2 The flowchart of simulation model



Lastly, the linear regression meta-modeling helps determine the relationship between control factors and the response. After finishing all the simulation experiments, the values of the simulated average inventory cost under different control policies are recorded. The linear regression meta-modeling uses the values of the inventory cost as the response and the optimal control variables in each policy can be obtained as a result. In this research, a second-degree regression model is used to find the relationship between the cost and the control variables, as shown in Equation 12. x_i represents the control variable, and β_i is the coefficient to be estimated from simulation results. n is the number of control variables in the design, and in this paper it equals 4 in both cases; namely H_l, H_h, L_l, L_h.

$$C = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{j=1}^{n-1} \sum_{i=j+1}^{n} \beta_{ij} x_i x_j + \sum_{i=1}^{n} \beta_{ii} x_i^2 + \varepsilon \quad i, j = 0, 1, 2, \dots, n; \ i < j \tag{12}$$

5. Numerical Experiments

This section mainly looks at the numerical study process. First of all, case 1 is examined using the Markov modulated compound Poisson process. The second case with historical demand data and forecast-corrected hedging point policy is investigated next. The impact of forecasting methods and accuracy is discussed.

5.1 Case 1: two-level control policy

In this scenario, modified two-level hedging point policy is adopted to control the production-inventory system. There are 4 control variables in total which are H_l, H_h, L_l, L_h. However in order to avoid a situation in which H_h and L_h are smaller than H_l and L_l, two extra variables R_H and R_L are introduced and defined for the hedging point and lot size as follows:

$$\begin{cases} R_H = H_l / H_h \\ R_L = L_l / L_h \end{cases} \quad where\ 0 \le R_H, R_L \le 1 \tag{13}$$

Each variable is divided into 5 levels. According to the theory of CCD, a single experiment with 25 combinations of the factors is generated as a result, consisting of 16 full factorial points, 8 axial points and 1 central point. The range of each control variable is presented as shown in Table 2.

Table 2 The five values of the control variables in the CCD

H_h	$\boldsymbol{R_H}$	$\boldsymbol{L_h}$	$\boldsymbol{R_L}$
4000	0.98	3500	0.98
3000	0.74	2750	0.74
2000	0.5	2000	0.5
100	0.26	1250	0.26
0	0.02	500	0.02

For the purpose of our investigation, the key parameters in the simulation stage are defined in Table 3. Simulation is replicated 4 times in each experimental run for both case 1 and 2.

Table 3 The value of key parameters

$\boldsymbol{C_h}$	**0.1 $/unit/period**	$\boldsymbol{r_{\alpha\beta}}$	**10** $\boldsymbol{periods}$	$\boldsymbol{\lambda_h}$	**5**
$\boldsymbol{C_b}$	2 $/unit/period	$r_{\beta\alpha}$	0.8 $periods$	λ_l	2
$\boldsymbol{C_s}$	200 $/batch	p	50 $periods$	μ	50 unit
$\boldsymbol{u_{max}}$	300 unit/period	q	30 $periods$	σ	5 unit

After the simulation, using the values of the control variables in Table 2 and the simulation parameters in Table 3, the data obtained is used to find the corresponding regression function and the full quadratic function is given in Equation 14. Minitab® is used to obtain the regression function during this process. R-sq and R-Sq (adj) are 95.70% and 95.25% respectively.

$$\begin{aligned} Cost = {} & 1436 + 0.317 * H_h - 798 * R_H - 0.141 * L_h - 4375 * R_L \\ & + 0.000196\, H_h\text{^}2 + 5447\, R_H\text{^}2 - 0.000154\, H_h * L_h - 1.434\, H_h \\ & * R_L + 0.502\, L_h * R_L - 0.0012 R_H * L_h + 15 R_H * R_L + 859\, R_L\text{^}2 \\ & + 0.000051\, L_h\text{^}2 \end{aligned} \quad (14)$$

The optimal combination of the four control variables can be estimated by solving the first order optimality conditions dC/dH=0.

$$(H_h, R_H, L_h, R_L)^* = (1818,\ 0.469,\ 1248,\ 0.584)$$

This gives the optimal combination using the auxiliary variables R_H and R_L. So Equation (13) gives the optimal combination of the original control variables as follows:

$$(H_h, H_l, L_h, L_l)^* = (1818, 852, 1248, 729)$$

The minimum average inventory cost is 171.56, and the contour plots for the average inventory cost between H_h, R_H and L_h, R_L are provided separately in Figures 3(a) and 3(b).

Figure 3(a): Contour plot between H_h and R_H

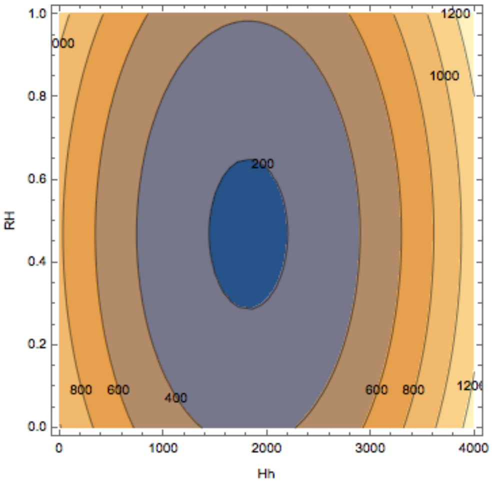
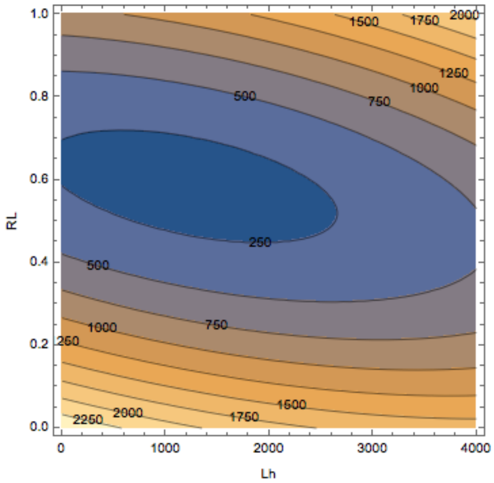


Figure 3(b): Contour plot between L_h and R_L



5.2 Case 2: forecast-corrected control policy

Tunc (2011) utilized four categories of demand that can be summarized as stationary, erratic, sinusoidal and life-cycle demand. Each demand is generated manually based on a certain distribution. However, in order to simulate the real demand situation and test the corresponding performance of the system, we select two real-world non-stationary demand data sets out of an industrial case study. They represent a type of demand pattern that can often be observed in real cases. The two demand sets are illustrated in Figure 4(a) and Figure 4(b). As can be seen, both demand sets show non-stationarity, but demand set 2 also has seasonality. So in terms of choosing forecasting method, apart from Simple Moving Average (SMA), Single Exponential Smoothing (SES) and Autoregressive Integrated Moving Average (ARIMA), the Holt-Winters Seasonal (HWS) method is also chosen for forecasting demand set 2.

As mentioned in the previous section, there are also four control variables H, L_h, L_m and L_l which represents the hedging point and three lot size values for different demand rates. The integrated simulation, design of experiments and linear regression meta-modeling is again employed to obtain the optimal control policy in both demand sets. The whole numerical experiments consist of 9 different scenarios. For demand set 1, there are four scenarios in total, which consist of the scenarios with ARIMA, WMA, SES and the one with stationary control policy. While for demand set2, an extra scenario with HWS is added. The repeated procedure are not shown in detail.

The experimental design and the result of demand set 1 using the ARIMA method is presented as an example.

Figure 4(a): Demand Set 1

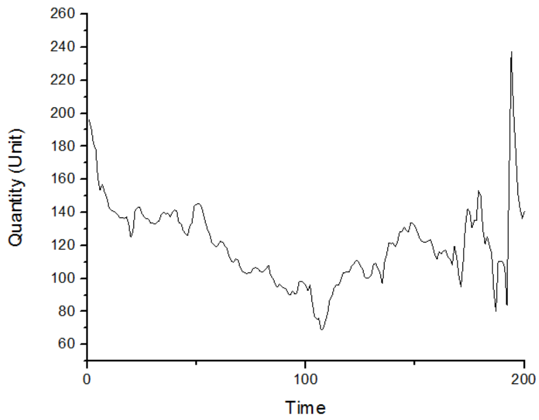


Figure 4(b): Demand Set 2

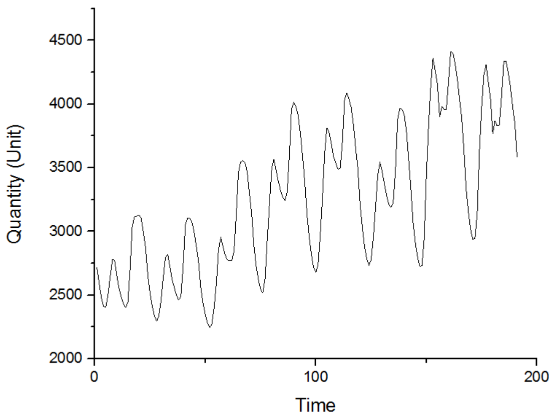


Table 4: The experimental range of control variables

H	L_h	R_1	R_2
800	300	0.98	0.98
600	250	0.74	0.74
400	200	0.5	0.5
200	150	0.26	0.26
0	100	0.02	0.02

Similarly, in order to avoid the situation in which $L_h < L_m$ and $L_m < L_l$, $R_1$and R_2 are employed and defined as $\frac{L_m}{L_h}$ and $\frac{L_l}{L_m}$.

After the optimization, the optimal control variables in the proposed policy are determined as $(371, 235, 0.482, 0.92)$ under the given set of parameters, and the optimal inventory cost is determined as 270.9. It means that the optimal hedging point is 371 while the optimal lot size for high demand, medium demand and low demand should be set to 235, 117 and 111 respectively. The results of other scenarios are obtained with the same methodology. The results are used in the following section to analyze the impact of forecasting accuracy on the production-inventory system.

6. Impact of forecasting accuracy and proposed policy on production-inventory system

Table 5: List of Abbreviations

PCR	Percentage Cost Reduction
AC	Average Cost
MAPE	Mean Average Percentage Error
ARIMA	Autoregressive Integrated Moving Average
ES	Exponential Smoothing
WMA	Weighted Moving Average
HWS	Holt-Winters

In this section, the impact of forecasting and forecast-corrected hedging policy control policy is examined first. The abbreviations used in this section are listed in Table 5. The relationship between the forecasting errors and cost reduction is then investigated by analyzing the results from all the scenarios in case 2. A parameter called Percentage Cost Reduction (PCR) is used to measure the cost differences between different scenarios. AC_c and AC_o represent the Average Cost (AC) with forecast-corrected control policy and original control policy respectively. This parameter helps compare the simulation results generated from forecast-corrected scenarios against the results from the scenarios with original control policy in Equation 1.

$$PCR = \frac{AC_o - AC_c}{AC_c} * 100 \quad (15)$$

First of all, we start an investigation on the accuracy of each forecasting methods. As illustrated in Figure 5, the value of MAPE varies and describes the accuracy of forecasting method. In addition, different demand sets produce various MAPE values when the same forecasting method is applied. For instance, ARIMA yields a value of MAPE around 4 in Demand set 1 and a value about 2 in Demand set 2. A similar situation also occurs in the scenarios with ES and WMA. It shows that the pattern of the demand set has a significant impact on the forecasting performance. In terms of a specific demand set, demand set 2 for example; ARIMA can produce the most accurate prediction of the future demand compared with the other two methods. Especially the MAPE value of WMA and Holt-Winter are two times more than the value in ARIMA. In demand set 1, ARIMA is also the best method followed by WMA.

Figure 5: The relationship between forecasting methods and MAPE

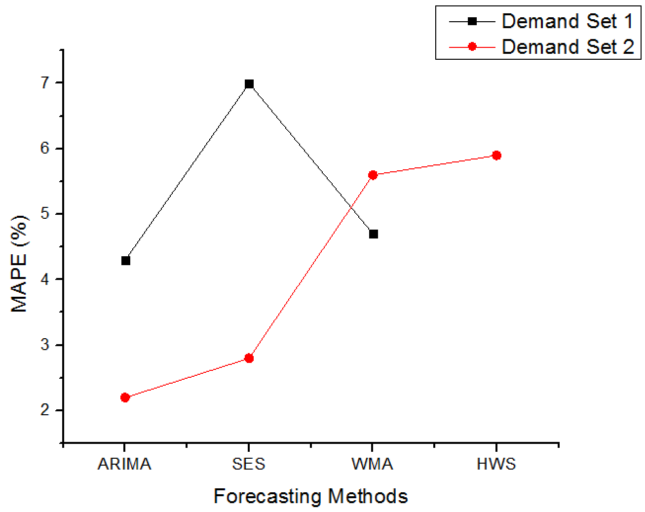
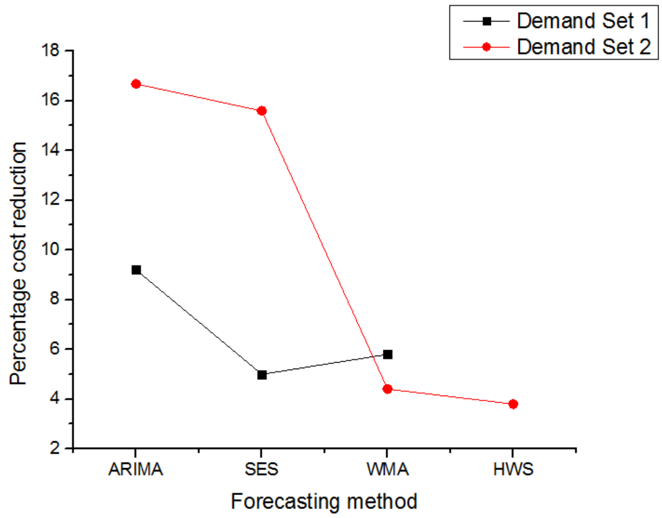


Figure 6 presents the overall performance of the production and inventory control system which is measured by the average inventory cost. The vertical axis represents the percentage cost reduction when compared with the cost in the stationary scenario.

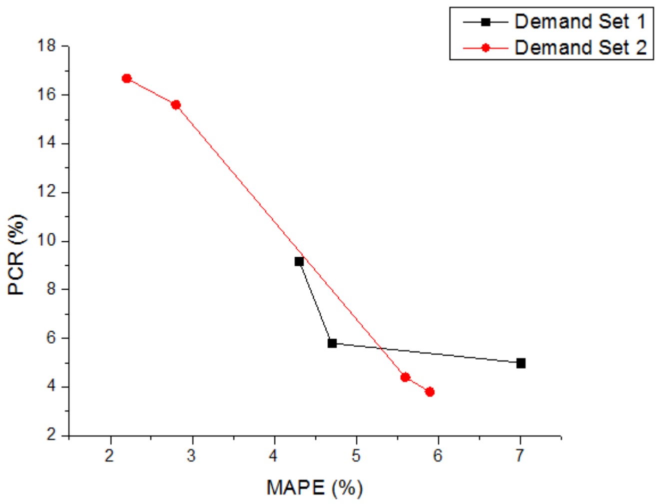
The results show that cost has been reduced by using the proposed control policy and forecasting techniques in Demand sets 1 and 2. Take Demand set 2 as an example; ARIMA and SES can help reduce the average inventory cost by around 17 and 16 percent respectively. For WMA and HWS, despite their relatively high forecasting errors, still can reduce the average inventory cost by around 4 percent.

Figure 6: The relationship between forecasting methods and PCR



In order to further explain the differences between the demand sets, the relationship between percentage cost reduction and forecasting errors is examined and shown in Figure 7. Regardless of the forecasting technique, the relationship between the percentage cost reduction and MAPE is inverse. To be more specific, the higher the MAPE is, the lower the percentage cost reduction will be. So if the forecasting errors exceed a certain value, non-stationary control policy cannot provide a better performance with the original policy.

Figure 7: The relationship between MAPE and PCR



7. Conclusions

The traditional hedging-point-based production and inventory control problem normally assumes a stationary demand process. However, stationary demand is not found in the real world. Questions on the influence of using stationary control policy when the demand is non-stationary in production and inventory control problem need to be answered. Little work has been reported in terms of the integration of forecasting and production-inventory control considering a failure-prone manufacturing system, non-stationary demand process and lot sizing.

In our study, we modeled both the quantitative and qualitative forecasting processes of non-stationary demand. The Markov-modulated compound Poisson process was utilized first and a corresponding two-level control policy is proposed. Two sets of real world demand data were employed together with time series forecasting to simulate qualitative forecasting process. The forecast-corrected hedging point policy used was modified from the traditional hedging point policy. A large number of simulations and experiments have been conducted; the experiments were used to estimate the optimal values of the various control variables, and they proved that the proposed forecast-corrected method can result in a better performance than the traditional stationary control policy under the condition of non-stationary demand. However, when the forecasting errors are large enough due to the pattern of demand and forecasting methods, stationary control policy is preferred instead of non-stationary policy since it can already provide a good approximation to the non-stationary control.

This study helps us gain some significant insights about which policy should be chosen when the forecasting inaccuracy is known to a company. The main contributions of this paper can be concluded as a) Investigating the impact of forecasting on the performance of a manufacturing system subject to failure and non-stationary demand. b) Providing a solution to determine the optimum control policy under the condition of non-stationary demand and batch production. Managers in manufacturing companies are also recommended to utilize a non-stationary production-inventory control policy because it leads to a better performance in terms of cost. However, managers should understand under what types of conditions they can use non-stationary control policy. The impact of forecasting errors on the systems should be given attention by managers. In addition, they should be cautious about the choice of forecasting methods according to the type of demand process, since different forecasting methods generate different accuracy. Limitation also exists in our study in which once the demand behavior changed, the optimal setting should be determined again using the same simulation model.

The current research can be further developed in a few potential directions. Extending the model to multi-product and multi-machine introduces more complexity but at the same time, the model becomes closer to reality. A second direction is to integrate the dynamic lot sizing scheduling problem to the current model. Other potential directions include investigating the impacts of other factors on the system as well as coordinating other supply chain activities, such as transportation and replenishment, into the model.

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