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Physarum solver: A biologically inspired method for multi-criteria sustainable supply chain network design

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Abstract In a sustainable chain network the capacity of transport links and the product flow through links are seen as design variables. We assume that a supplier has full data on the costs of the chain network maintenance, e.g. supporting manufacturing plants, shipment, storage and distribution, and the level of the carbon dioxide emission. A supplier must minimise a total cost of capacity investment and an operation cost of links yet minimise the costs asso-

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ciated with generation of unwanted byproducts, e.g. the carbon dioxide emission. The problem tackled is a multi-criteria optimisation. We proposed a novel bio-inspired algorithm for designing of sustainable supply chain networks. The algorithm employs principles of load balancing and transport optimisation of protoplasmic networks in acellular slime mould *Physarum polycephalum*. Efficiency of the algorithm is illustrated in numerical examples.

Keywords Supply chain, *Physarum*, Sustainability, Network Design, Multi-criteria decision making, Optimization

1 Introduction

Supply chain network is a vital component of production, storage, and distribution of products to customers. With the rapid development of e-commerce, customers are more than ever concerned with the on-time delivery of goods. An optimal supply chain network not only helps the company to reduce the cost, but also speeds up delivery of the products. Therefore the design of optimal supply chain network is of upmost importance (Esmaeilikia et al, 2014; Huynh and Nakamori, 2005; Nagurney, 2006, 2009; Philpott and Everett, 2001; Ramezani et al, 2014; Taleizadeh et al, 2013; Xiao et al, 2005).

A sustainability of the supply chain networks became a hot topic because our societies got concerned with the impacts of goods transportation on the climate (Dong et al, 2014; Hugo and Pistikopoulos, 2005; Nagurney et al, 2007). Thus, Hugo and Pistikopoulos (2005) combined the environmental issues with traditional economic criteria into a multi-objective problem. They used mathematical programming to tackle selection, allocation and capacity expansion of processing technologies and assignment of transportation links required to satisfy the demands at the markets. Krikke et al (2003) developed quantitative modelling to support decision-making related to the designing products and structure of a logistic network. Using this approach they designed an efficient close-loop supply chain for refrigerators. They aslo measured the environmental impacts according to the linear-energy and waste function. Che et al (2007) developed an interaction-oriented approach based on the analytic hierarchy process methodology and proportional rule, to solve the semiconductor distribution problem with multiple quantitative and qualitative criteria in the supply chain network. Tsai and Hung developed a fuzzy goal programming approach that integrates activity-based costing and performance evaluation in a value-chain structure for optimal supplier selection and flow allocation in the supply chain. Nagurney and Nagurney (Nagurney and Nagurney, 2010a; Nagurney et al, 2007) presented a mathematical model to describe the multi-criteria sustainable supply chain network design problem and employed Lagrange multiplier to deal with this problem. Although their approach is capable of designing optimal networks it is slightly complicated and time consuming.

Engineers are always looking into behaviour, mechanics, physiology of living systems to uncover novel principles of distributed sensing, information processing and decision making that could be adopted in development of future and emergent computing paradigms, architectures and implementations (Amin, 2013; Bobadilla et al, 2013; Chen and Li, 2007; Drezner and MisevičIus, 2013; Guo and Lei, 2013; Juang, 2004; Lei and Guo, 2013; Liu et al, 2007; Misevicius, 2004, 2005; Vidal et al, 2012; Wang et al, 2001; Zhang et al, 2013a). One of the most popular nowadays living computing substrates is a slime mould *Physarum Polycephalum.*

Plasmodium is a vegetative stage of acellular slime mould *P. polycephalum*, a single cell with many nuclei, which feeds on microscopic particles (Stephenson et al, 1994). When foraging for its food the plasmodium propagates towards sources of food, surrounds them, secretes enzymes and digests the food; it may form a congregation of protoplasm covering the food source. When several sources of nutrients are scattered in the plasmodium's environment, the plasmodium forms a network of protoplasmic tubes connecting the masses of protoplasm at the food sources. The network is optimal because it minimises transportation time of metabolites. The fact that Physarum optimises its protoplasmic network inspired researchers to interpret the slime mould's behaviour in terms of computation and to develop experimental laboratory prototypes and computer and mathematical models of Physarum-based algorithms and computing devices.

In laboratory experiments and theoretical studies it is shown that the slime mould can solve many graph theoretical problems, such as finding the shortest path (Nakagaki et al, 2000; Tero et al, 2006; Zhang et al, 2013b,d,e, 2014a,b), shortest path tree problem (Zhang et al, 2013c), influential nodes identification (Gao et al, 2013), connecting different arrays of food sources in an efficient manner (Adamatzky, 2014; Jones and Adamatzky, 2013; Nakagaki et al, 2007; Tero et al, 2008), network design (Adamatzky, 2012; Adamatzky and Alonso-Sanz, 2011; Adamatzky and Martinez, 2013; Adamatzky et al, 2011; Masi and Vasile, 2012, 2014; Tero et al, 2010). Recently, (Schön et al, 2014) proposed a novel Score-based *Physarum* Learner algorithm for learning Bayesian Network structure from data.

We explore principles of Physarum's protoplasmic network optimisation, including a continuity of cytoplasm flow during the iterative process of the optimisation and dynamic reconfiguration of the network, to design an optimal supply chain networks with minimal costs associated with the production, storage, distribution activities, along with the capacity investment and total cost of environment emissions.

The rest of this paper is organised as follows. In Section 2, we briefly introduce the sustainable supply network design model. In Section 3, based on *Physarum* solver, we propose a biologically inspired mathematical model for designing the sustainable supply chain network. In Section 4, numerical examples are used to demonstrate the practicality and flexibility of the proposed approach; we also compare our approach with the method developed in (Nagurney and Nagurney, 2010a). Results are discussed in Section 5.

2 Preliminaries

In this section, the supply chain network design model and the *Physarum* model are introduced.

2.1 The Multicriteria Sustainable Supply Chain Network Design Model (Nagurney and Nagurney, 2010b)

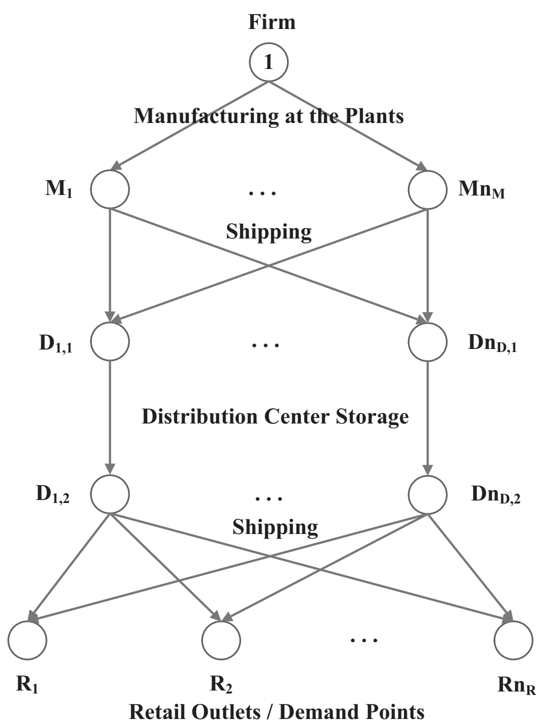

Fig. 1 The supply chain network topology

Consider the supply chain network shown in Fig. 1: a firm corresponding to node 1 aims at delivering the goods or products to the bottom level corresponding to the retail outlets. The links connecting the source node with the destination nodes represent the activities of production, storage and transportation of good or services. Different network topologies corresponds to different supply chain network problems. We assume that there exists only one path linking node 1 with each destination node, which can ensure that the demand at each retail outlet can be satisfied.

As shown in Fig. 1, the firm takes into consideration n_M manufacturers, n_D distribution centers when n_R retailers with demands $d_{R_1}, d_{R_2}, \cdots, d_{R_{n_R}}$ must be served. The node 1 in the first layer is linked with the possible n_M manufacturers, which are represented as $M_1, M_2, \cdots, M_{n_M}$. These edges in the manufacturing level are associated with the possible distribution center nodes, which are expressed by $D_{1,1}, D_{2,1}, \cdots, D_{n_D,1}$. These links mean the possible shipment between the manufacturers and the distribution centers. The links connecting $D_{1,1}, D_{2,1}, \cdots, D_{n_D,1}$ with $D_{1,2}, D_{2,2}, \cdots, D_{n_D,2}$ reflect the possible storage links. The links between $D_{1,2}, D_{2,2}, \cdots, D_{n_D,2}$ and $R_1, R_1, \cdots, R_{n_R}$ denote the possible shipment links connecting the storage centers with the retail outlets.

Let a supply chain network be represented by a graph $G(N, L)$, where N is a set of nodes and L is a set of links. Each links in the network is associated with a cost function and the cost reflects the total cost of all the specific activities in the supply chain network, such as the transport of the product, the delivery of the product, etc. The cost related with link a is expressed by $\widehat{c}_a$. A path p connecting node 1 with a retail node shown in Fig. 1 denotes the whole activities related with manufacturing the products, storing them and transporting them, etc. Assume w_k denotes the set of source and destination nodes $(1, R_k)$ and P_{w_k} represents the set of alternative associated possible supply chain network processes joining $(1, R_k)$. We assume P means is a set of all paths joining $(1, R_k)$ and x_p is the flow of the product on path p. The following Eq. (1) must be satisfied:

$$\sum_{p \in P_{w_k}} x_p = d_{w_k}, \quad k = 1, \cdots, n_R. \tag{1}$$

Let f_a represent the flow on link a, then the following conservation flow must be met:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L. \tag{2}$$

Eq. (2) means that the inflow must be equal to the outflow on link a.

These flows can be grouped into the vector f. The flow on each link must be a nonnegative number, i.e. the following Eq. (5) must be satisfied:

$$x_p \geq 0, \quad \forall a \in L. \tag{3}$$

Suppose the maximum capacity on link a is expressed by $u_a, \forall a \in L$. It is required that the actual flow on link a cannot exceed the maximum capacity on this link:

$$\begin{aligned} f_a \leq u_a, \quad \forall a \in L, \\ 0 \leq u_a, \quad \forall a \in L. \end{aligned} \tag{4}$$

The total cost on each link, for simplicity, is represented as a function of the flow of the product on all the links (Nagurney, 2006, 2009; Nagurney and Woolley, 2010; Nagurney et al, 2002):

$$\widehat{c}_a = \widehat{c}_a(f), \quad \forall a \in L. \tag{5}$$

The total investment cost of adding capacity u_a on link a can be expressed as follows:

$$\widehat{\pi}_a = \widehat{\pi}_a(u_a), \quad \forall a \in L. \tag{6}$$

Summarily, the supply chain network design optimization problem is to satisfy the demand of each retail outlet and minimize the total cost, including the total cost of operating the various links and the capacity investments:

$$Minimize \ \sum_{a \in L} \widehat{c}_a(f) + \sum_{a \in L} \widehat{\pi}_a(u_a) \tag{7}$$

subject to constraints (1)-(4).

We also take into account the cost associated with the total amount of carbon dioxide emissions generated both in the capital phase and operation phase. The generated emissions can occur in each phase, including the manufacturing stage, storing stage, and shipping stage. Suppose $e_a(f_a)$ represents the emission-generation function on link a in the operation phase, proportional to the flow on this link. Let $\widehat{e}_a\left(u_a\right)$ denotes the emission-generation function on link a in the capital investment period. This is the function of the product flow on that link. Thus minimisation of the emission can be expressed in the following form:

$$Minimize \quad \sum_{a\in L} e_a\left(f_a\right) + \widehat{e}_a\left(u_a\right) \tag{8}$$

Combing two objectives shown in Eqs. (9) and (8), we can construct the following objective function:

$$\begin{aligned} & Minimize \quad \sum_{a\in L} c_a\left(f_a\right) + \widehat{\pi}_a\left(u_a\right) + \omega\left(\sum_{a\in L} e_a\left(f_a\right) + \widehat{e}_a\left(u_a\right)\right) \\ & s.t. \\ & \qquad f_a \leq u_a \\ & \qquad 0_a \leq u_a \end{aligned} \tag{9}$$

where ω is a nonnegative constant assigned to the emission-generation attribute. It reflects how much the firm is willing to pay for per unit of the emissions; alternatively, it can be thought of as the tax imposed by the government (Wu et al, 2006).

2.2 Physarum polycephalum

Physarum Polycephalum is a large, single-celled amoeboid organism forming a dynamic tubular network connecting the discovered food sources during foraging. The mechanism of tube formation can be described as follows. Tubes thicken in a given direction when shuttle streaming of the protoplasm persists in that direction for a certain time. There is a positive feedback between flux and tube thickness, as the conductance of the sol is greater in a thicker channel. With this mechanism in mind, a mathematical model illustrating the shortest path finding has been constructed (Tero et al, 2007).

Suppose the shape of the network formed by the *Physarum* is represented by a graph, in which a plasmodial tube refers to an edge of the graph and a junction between tubes refers to a node. Two special nodes labeled as N_1, N_2 act as the starting node and ending node respectively. The other nodes are labeled as N_3, N_4, N_5, N_6 etc. The edge between nodes N_i and N_j is M_{ij}. The parameter Q_{ij} denotes the flux through tube M_{ij} from node N_i to N_j. Assume the flow along the tube is approximated by Poiseuille flow. Then the flux Q_{ij} can be expressed as:

$$Q_{ij} = \frac{D_{ij}}{L_{ij}}(p_i - p_j) \tag{10}$$

where p_i is a pressure at a node N_i, D_{ij} is a conductivity of a tube M_{ij}, and L_{ij} is its length.

By assuming that the inflow and outflow must be balanced, we have:

$$\sum Q_{ij} = 0 \quad (j \neq 1, 2) \tag{11}$$

For the source node N_1 and the sink node N_2 the following holds:

$$\sum_i Q_{i1} + I_0 = 0 \tag{12}$$

$$\sum_i Q_{i2} - I_0 = 0 \tag{13}$$

where I_0 is the flux flowing from the source node and I_0 is a constant value here.

In order to describe such an adaptation of tubular thickness we assume that the conductivity D_{ij} changes over time according to the flux Q_{ij}. An evolution of $D_{ij}(t)$ can be described by the following equation:

$$\frac{d}{dt}D_{ij} = f(|Q_{ij}|) - \gamma D_{ij} \tag{14}$$

where γ is a decay rate of the tube. The equation implies that a conductivity becomes nil if there is no flux along the edge. The conductivity increases with the flux. The f is monotonically increasing continuous function satisfying $f(0) = 0$.

Then the network Poisson equation for the pressure can be obtained from Eq. (10-13) as follows:

$$\sum_i \frac{D_{ij}}{L_{ij}}(p_i - p_j) = \begin{cases} +1 & for \quad i = 1, \\ -1 & for \quad j = 2, \\ 0 & otherwise. \end{cases} \tag{15}$$

By setting p_2=0 as a basic pressure level, all p_i can be determined by solving Eq. (15) and Q_{ij} can also be obtained.

In this paper, $f(Q) = |Q|$ is used because $f\left(|Q_{ij}|\right) = |Q|, \gamma = 1$, the *Physarum* converges to the shortest path with a high rate (Tero et al, 2007). With the flux calculated, the conductivity can be derived, where Eq. (16) is used instead of Eq. (14), adopting the functional form $f(Q) = |Q|$.

$$\frac{D_{ij}^{n+1} - D_{ij}^{n}}{\delta t} = |Q| - D_{ij}^{n+1} \tag{16}$$

3 Proposed Method

In this section, we employ the Physarum model to solve the supply chain network design problem. Generally speaking, there are two sub-problems to address:

- In the shortest path finding model, there is only one source node and one ending node. However, in the sustainable supply chain network design problem, there are more than one retail outlets.
- In the sustainable supply network design problem, the objective is to minimize the total cost, including the operation cost, the capacity investment cost, and the emission-generation cost. We need to modify the classical *Physarum* to achieve this objective.

3.1 One Source Many-Sinks Physarum Model

In the original model constructed by Tero et al. in Ref. (Tero et al, 2007), there is one starting node and one ending node. In the sustainable supply chain network design problem, as shown in Fig. 1, there are R_{n_R} retail outlets in demand of the goods or products. From the left to the right, from the top to the bottom, we can label these nodes as shown in this figure. In order to solve the one source multi-sinks' model in the supply chain network design, the following Eq. (17) is derived:

$$\sum_i \frac{D_{ij}}{L_{ij}}(p_i - p_j) = \begin{cases} +\sum\limits_{i=1}^{n_R} d_{R_i} & for\ j = 1 \\ -d_{R_j} & for\ j = R_1, R_2, \cdots R_{n_R} \\ 0 & otherwise \end{cases} \tag{17}$$

where $j = 1$ means that $\sum\limits_{i=1}^{n_R} d_{R_i}$ units of goods are distributed from the firm to the other manufacturing facilities, $j = R_{n_R}$ denotes n_R retail outlets are in need of d_{R_j} units of goods, respectively.

3.2 Physarum-Inspired Model for Designing Sustainable Supply Chain Network Design

In the sustainable supply chain design problem, it is required that the flow is less that its actual capacity. In our view, in the optimal solution to such problem, its capacity u_a is equal to its actual flow f_a. Suppose the capacity u_a on link a is more than its flow f_a, this will generate extra cost. On the contrary, if the capacity u_a on link a is equal to its flow f_a, it not only can satisfy the requirement of the passing flow, but also can decrease the total cost. From this point of view, in the optimal solution, the capacity on each link should be equal to its actual flow. In other words, in the optimal solution we have $f_a = u_a$. Note, that in the *Physarum* model, the flow on link a is Q_{ij}.

The sustainable supply chain network design problem is a system optimum (SO) problem from the view point of flow theory in the transportation systems. It aims at minimizing the total cost in the supply chain network. To date, *Physarum* has been successfully employed to handle the user equilibrium (UE) problem in the transportation network, e.g. see (Bi and Zhang, 2001; Huang and Lam, 2002; Liu et al, 2009; Ziyou and Yifan, 2002). In order to solve the UE problem, we just need to use the cost functions to replace the length of each link. Algorithm 1 illustrates the detailed procedures for solving UE in the sustainable supply chain network.

In the *Physarum* model, all the flow tends to pass through the path with shortest length while the cost function on each link is in association with the passing flow. By taking advantage of the *Physarum* model, we can formulate the optimal pattern for the flow corresponding to the UE state. When the algorithm converges to the UE solution, no flow can reduce its cost by changing the path. In other words, all the flow has the same cost.

However, different from UE solution, in the sustainable supply chain network, we aim at minimizing the total cost. For the purpose of using *Physarum* to solve this issue, we transform the SO state into the corresponding UE state using the following Eq. (18) (Bell and Iida, 1997; Bingfeng and Ziyou, 2013).

$$\widetilde{t_a}\left(x_a\right) = t_a\left(x_a\right) + x_a \frac{dt_a\left(x_a\right)}{dx_a},\ \forall a \in L \tag{18}$$

where x_a represents the flow on link a, $t_a(x_a)$ is the cost function per unit of flow on link a while $\widetilde{t_a}\left(x_a\right)$ denotes the transformed cost function per unit of flow.

In the sustainable supply chain network, L_{ij} is the cost when the flow is Q_{ij}. Hence, the following Eq. (19) is derived to express the cost per unit of flow:

$$LF_{ij} = \frac{\hat{c}_a\left(Q_{ij}\right) + \hat{\pi}_a\left(Q_{ij}\right) + \omega\left(e_a\left(Q_{ij}\right) + \hat{f}_a\left(Q_{ij}\right)\right)}{Q_{ij}} \tag{19}$$

Algorithm 1 *Physarum*-Inspired Model for the User Equilibrium Solution in the Sustainable Supply Chain Network Design (L,1,N,R)

// N is the size of the network;
// L_{ij} is the link connecting node i with node j;
// 1 is the starting node while R is the set of retail outlets;
$D_{ij} \leftarrow (0,1]\ (\forall i,j = 1,2,\ldots,N)$;
$Q_{ij} \leftarrow 0\ (\forall i,j = 1,2,\ldots,N)$;
$p_i \leftarrow 0\ (\forall i = 1,2,\ldots,N)$;
$L_{ij} \leftarrow 0.001\ (\forall i,j = 1,2,\ldots,N)$;
$count \leftarrow 1$;
repeat
  Calculate the pressure associated with each node according to the following Eq. (17)

$$\sum_i \frac{D_{ij}}{L_{ij}}(p_i - p_j) = \begin{cases} +\sum_{i=1}^{n_R} d_{R_i} & for\ j = 1, \\ -d_{R_j} & for\ j = R_1, R_2, \cdots R_{n_R}, \\ 0 & otherwise \end{cases}$$

  $Q_{ij} \leftarrow D_{ij} \times (p_i - p_j)/L_{ij}$ // Using Eq. (10);
  $D_{ij} \leftarrow Q_{ij} + D_{ij}$ // Using Eq. (16)
  Update the cost on each link;
  for $i = 1 : N$ **do**
    for $j = 1 : N$ **do**
      if $Q_{ij} \neq 0$ **then**
        $L_{ij} = L_{ij} + \widehat{c}_a(Q_{ij}) + \widehat{\pi}_a(Q_{ij}) + \omega\left(e_a(Q_{ij}) + \widehat{e}_a(Q_{ij})\right)$;
      end if
    end for
  end for
  $L = L/2$;
  $count \leftarrow count + 1$
until a termination criterion is met

According to the above method, we can construct the procedures for constructing the optimal sustainable supply chain network, which is shown in Algorithm 2.

There are several possible solutions to decide when to stop execution of Algorithm 2, e.g. the maximum number of iterations is achieved or a flux

Algorithm 2 *Physarum*-Inspired Model for Constructing the Optimal Sustainable Supply Chain Network Design (L,1,N,R)

// N is the size of the network;
// L_{ij} is the link connecting node i with node j;
// 1 is the starting node while R is the set of retail outlets;
$D_{ij} \leftarrow (0, 1]$ $(\forall i, j = 1, 2, \ldots, N)$;
$Q_{ij} \leftarrow 0$ $(\forall i, j = 1, 2, \ldots, N)$;
$p_i \leftarrow 0$ $(\forall i = 1, 2, \ldots, N)$;
$L_{ij} \leftarrow 0.001$ $(\forall i, j = 1, 2, \ldots, N)$;
$count \leftarrow 1$;
repeat
  Calculate the pressure associated with each node according to the following Eq. (17)

$$\sum_i \frac{D_{ij}}{L_{ij}} (p_i - p_j) = \begin{cases} +\sum\limits_{i=1}^{n_R} d_{R_i} & for\ j = 1, \\ -d_{R_j} & for\ j = R_1, R_2, \cdots R_{n_R}, \\ 0 & otherwise \end{cases}$$

  $Q_{ij} \leftarrow D_{ij} \times (p_i - p_j)/L_{ij}$ // Using Eq. (10);
  $D_{ij} \leftarrow Q_{ij} + D_{ij}$ // Using Eq. (16)
  Update the cost on each link;
  for $i = 1 : N$ **do**
    for $j = 1 : N$ **do**
      if $Q_{ij} \neq 0$ **then**
        $L_{ij} = L_{ij} + LF_{ij} + Q_{ij} * \left. \frac{dLF_{ij}}{dQ_{ij}} \right|_{Q_{ij}=Q_{ij}}$;
      end if
    end for
  end for
  $L = L/2$;
  $count \leftarrow count + 1$
until a termination criterion is met

through each tube remains unchanged for some period of time. In the examples illustrated in the next section we use the following termination criterion. In numerical experiments discussed in next section the algorithm halts when the $\sum\limits_{i,j=1,2,\ldots,n} \left(\left| D_{ij}^{count+1} - D_{ij}^{count} \right| \right) \leq 0.001$, where $D_{ij}^{count+1}, D_{ij}^{count}$ is the conductivity associated with the edge L_{ij} during the $n+1_{th}$ andn_{th} iterations respectively, and n is the number of nodes in the network.

4 Numerical Examples

We demonstrate efficiency of our algorithm in several numerical examples. The algorithm is implemented using Matlab on an Intel Pentium Dual-Core E4600 processor (2.40GHz) with 2GB of RAM under Windows Seven. The baseline for all the examples are shown in Fig. 2. In this figure, the numbers along these links represent the sequence. It can be noticed that there are three alternative manufacturing plants and each of them has two possible technologies. Each manufacturer is in association with two possible distribution centers. Similarly, each distribution center is associated with two possible storage centers. The firm has to satisfy the demand from three possible retail outlets. The basic data for the following examples is shown in Table 1.

Example 1 In this example, the demands for each retail outlet is

$$d_{R_1} = 45, d_{R_3} = 35, d_{R_3} = 5$$

The cost functions and emission functions are shown in Table 1. In this example, we assume that the firm does not care about the emission generated in its supply chain. Therefore, $\omega = 0$. Fig. 3 shows us the flux changing trend during the iterative process. It can be seen that the Physarum converges to the optimal solution after 25 iterations. Table 2 indicated the specific flow on each link. As expected, the flow associated with each link is equal to its capacity. According to Eq. (9), we can obtain that the total cost is 10716.33 and the result is consistent with that in Ref. (Nagurney and Nagurney, 2010a). From Table 2 we can see that link 14 has zero capacity and zero flow. Thus, in the final optimal sustainable supply chain network, link 14 will be removed.

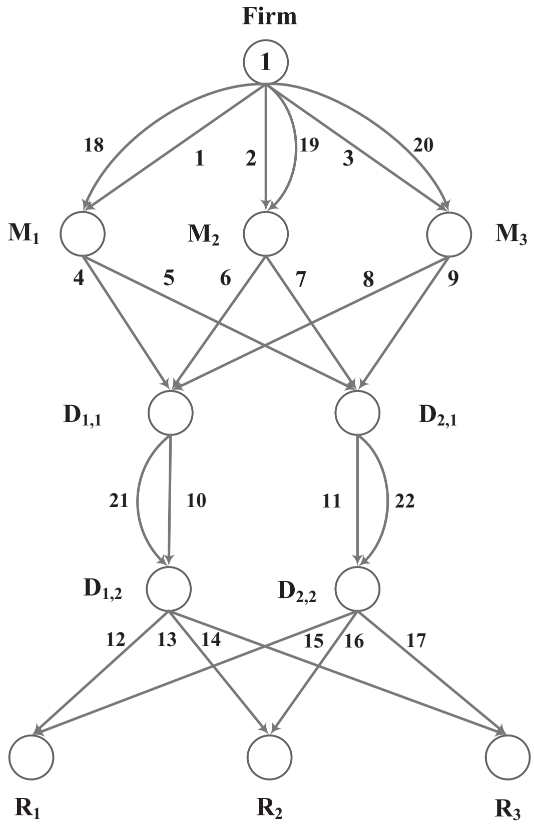

Fig. 2 The baseline supply chain network topology for all the examples. Adopted from Ref. (Nagurney and Nagurney, 2010a)

Example 2 Here we modify data from Example 1 by adopting parameter $\omega = 5$, which shows a degree of the firm's concern about the environment. The optimal solution to this example is given in Table 3. The total cost as shown in Eq. (7) for this example is 11288.27. The total emission cost is 7735.71. The result is different from that in Ref. (Nagurney and Nagurney, 2010a). To be specific, for the node M_1, its inflow is equal to the sum of the flow on link 18 and link 1, which is equal to 33.22 (19.32+13.90) in Nagurney's solution. As for the outflows associated with node M_1, it is composed of two separate flows on link 4 and link 5, which is equal to 33.23 (19.43+13.80). Obviously,

Table 1 Total Cost and Emission Functions for the Numerical Examples. Adopted from Ref. (Nagurney and Nagurney, 2010a)

Link a	$\widehat{c}_a(f)$	$\widehat{\pi}_a(u_a)$	$e_a(f_a)$	$\widehat{e}_a(f_a)$
1	$f_1^2+2f_1$	$0.5u_1^2+u_1$	$0.05f_1^2+f_1$	$1.5u_1^2+2u_1$
2	$0.5f_2^2+f_2$	$2.5u_2^2+u_2$	$0.1f_2^2+f_2$	$2u_2^2+2u_2$
3	$0.5f_3^2+f_3$	$u_3^2+2u_3$	$0.15f_3^2+2f_3$	$2.5u_3^2+u_3$
4	$1.5f_4^2+2f_4$	$u_4^2+u_4$	$0.05f_4^2+0.1f_4$	$0.1u_4^2+0.2u_4$
5	$f_5^2+3f_5$	$2.5u_5^2+2u_5$	$0.05f_5^2+0.1f_5$	$0.05u_5^2+0.1u_5$
6	$f_6^2+2f_6$	$0.5u_6^2+u_6$	$0.1f_6^2+0.1f_6$	$0.05u_6^2+0.1u_6$
7	$0.5f_7^2+2f_7$	$0.5u_7^2+u_7$	$0.05f_7^2+0.2f_7$	$0.1u_7^2+0.2u_7$
8	$0.5f_8^2+2f_8$	$1.5u_8^2+u_8$	$0.05f_8^2+0.1f_8$	$0.1u_8^2+0.3u_8$
9	$f_9^2+5f_9$	$2u_9^2+3u_9$	$0.05f_9^2+0.1f_9$	$0.1u_9^2+0.2u_9$
10	$0.5f_{10}^2+2f_{10}$	$u_{10}^2+5u_{10}$	$0.2f_{10}^2+f_{10}$	$1.5u_{10}^2+3u_{10}$
11	$f_{11}^2+f_{11}$	$0.5u_{11}^2+3u_{11}$	$0.25f_{11}^2+3f_{11}$	$2u_{11}^2+3u_{11}$
12	$0.5f_{12}^2+2f_{12}$	$0.5u_{12}^2+u_{12}$	$0.05f_{12}^2+0.1f_{12}$	$0.1u_{12}^2+0.2u_{12}$
13	$0.5f_{13}^2+5f_{13}$	$0.5u_{13}^2+u_{13}$	$0.1f_{13}^2+0.1f_{13}$	$0.05u_{13}^2+0.1u_{13}$
14	$f_{14}^2+7f_{14}$	$2u_{14}^2+5u_{14}$	$0.15f_{14}^2+0.2f_{14}$	$0.1u_{14}^2+0.1u_{14}$
15	$f_{15}^2+2f_{15}$	$0.5u_{15}^2+u_{15}$	$0.05f_{15}^2+0.3f_{15}$	$0.1u_{15}^2+0.2u_{15}$
16	$0.5f_{16}^2+3f_{16}$	$u_{16}^2+u_{16}$	$0.05f_{16}^2+0.1f_{16}$	$0.1u_{16}^2+0.1u_{16}$
17	$0.5f_{17}^2+2f_{17}$	$0.5u_{17}^2+u_{17}$	$0.15f_{17}^2+0.3f_{17}$	$0.05u_{17}^2+0.1u_{17}$
18	$0.5f_{18}^2+1f_{18}$	$u_{18}^2+2u_{18}$	$0.2f_{18}^2+2f_{18}$	$2u_{18}^2+3u_{18}$
19	$0.5f_{19}^2+2f_{19}$	$u_{19}^2+u_{19}$	$0.25f_{19}^2+3f_{19}$	$3u_{19}^2+4u_{19}$
20	$1.5f_{20}^2+1f_{20}$	$u_{20}^2+u_{20}$	$0.3f_{20}^2+3f_{20}$	$2.5u_{20}^2+5u_{20}$
21	$0.5f_{21}^2+2f_{21}$	$u_{21}^2+3u_{21}$	$0.1f_{21}^2+3f_{21}$	$1.5u_{21}^2+4u_{21}$
22	$f_{22}^2+3f_{22}$	$0.5u_{22}^2+2u_{22}$	$0.2f_{22}^2+4f_{22}$	$2.5u_{22}^2+4u_{22}$

the inflows are not equal to the outflows. This contradicts the law of flow conservation. Similarly, it is also observed that such kind of phenomenon can be found in the node $D_{2,1}$.

The Physarum model converges to the optimal solution after 22 iterations. In the optimal solution all links have positive capacity and flows. In addition, the flows are equal to the capacity on all the links. In Example 1, links 1 and 18 have the same flow. However, in Example 2, the flow on link 1 is increased

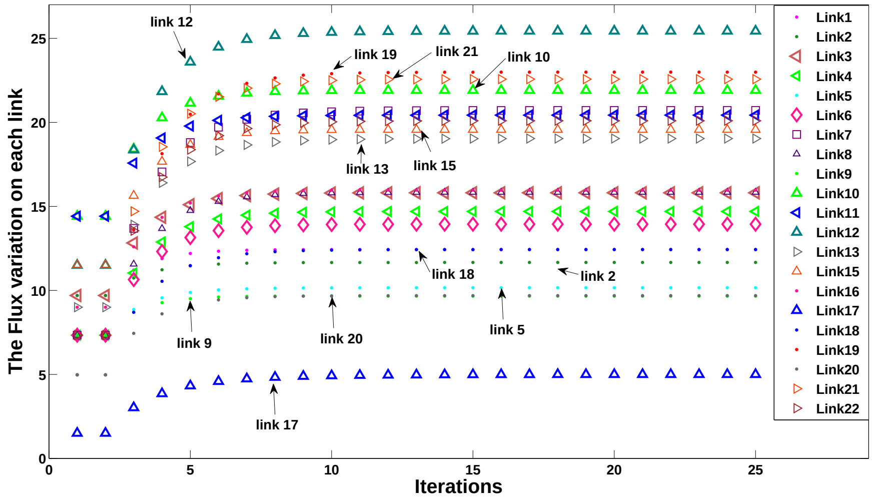

Fig. 3 The flux variation during the iterative process in the Example 1

Table 2 The optimal solution of Example 1

Link a	f_a^*	u_a^*	Link a	f_a^*	u_a^*
1	12.43	12.43	12	25.44	25.44
2	11.67	11.67	13	19.03	19.03
3	15.81	15.81	14	0.00	0.00
4	14.69	14.69	15	19.56	19.56
5	10.16	10.16	16	15.97	15.97
6	13.94	13.94	17	5.00	5.00
7	20.70	20.70	18	12.43	12.43
8	15.83	15.83	19	22.98	22.98
9	9.66	9.66	20	9.69	9.69
10	21.90	21.90	21	22.57	22.57
11	20.43	20.43	22	20.10	20.10

by 50% while the flow on link 18 only increases by about 10%. This is because the emission cost on link 1 is less than that on link 18. Such kind of behavior, also can be found on links 2 and 19.

Table 3 The optimal solution to Example 2

Link a	f_a^*	u_a^*	Link a	f_a^*	u_a^*
1	19.33	19.33	12	26.65	26.65
2	15.68	15.68	13	20.65	20.65
3	13.45	13.45	14	1.69	1.69
4	19.45	19.45	15	18.35	18.35
5	13.78	13.78	16	14.35	14.35
6	13.78	13.78	17	3.31	3.31
7	13.24	13.24	18	13.90	13.90
8	15.76	15.76	19	11.34	11.34
9	8.99	8.99	20	11.30	11.30
10	24.20	24.20	21	24.79	24.79
11	19.66	19.66	22	16.35	16.35

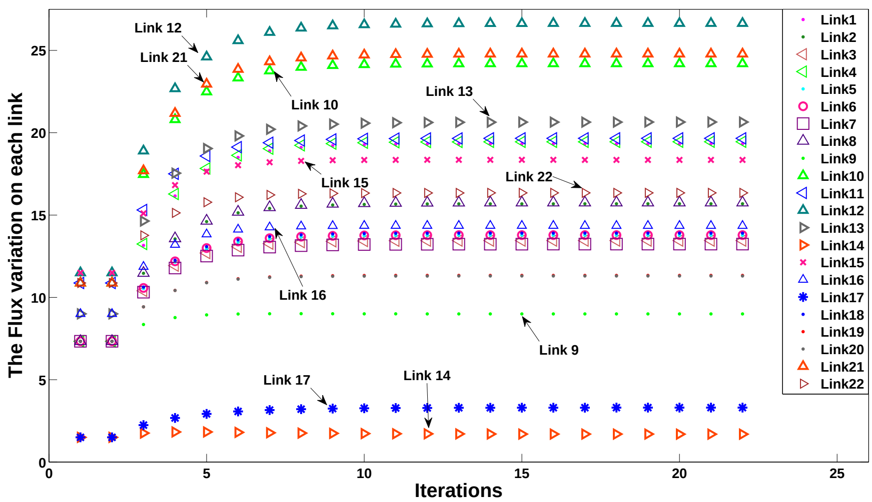

Fig. 4 The flux variation during the iterative process in the Example 2

Example 3 Example 3 has the same data as Example 2 except that the firm cares more about the environment: $\omega = 10$. The optimal solution is given in Table 4. Fig. 5 shows the changing trend of each link during the iterative process. In this example, the total cost is 11418.44. The results are also different from that in Ref. (Nagurney and Nagurney, 2010a). The problem appearing in Example 2 is also found in the solution of Ref. (Nagurney and Nagurney, 2010a). Specifically speaking, for node M_1, its inflow is equal to 26.35 (15.80+10.55) while its outflow is 26.36 (14.37+11.99). As for node M_3, its inflow is 24.32 (13.10+11.22) while its outflow is 24.33 (15.45+8.88). In a similar way, for node $D_{1,1}$, its inflow is 49.48 (19.66+14.37+15.45) while its outflow is 49.47 (24.30+25.17). This is contradictory with the law of the flow conservation. From this point of view, such kind of solution is invalid.

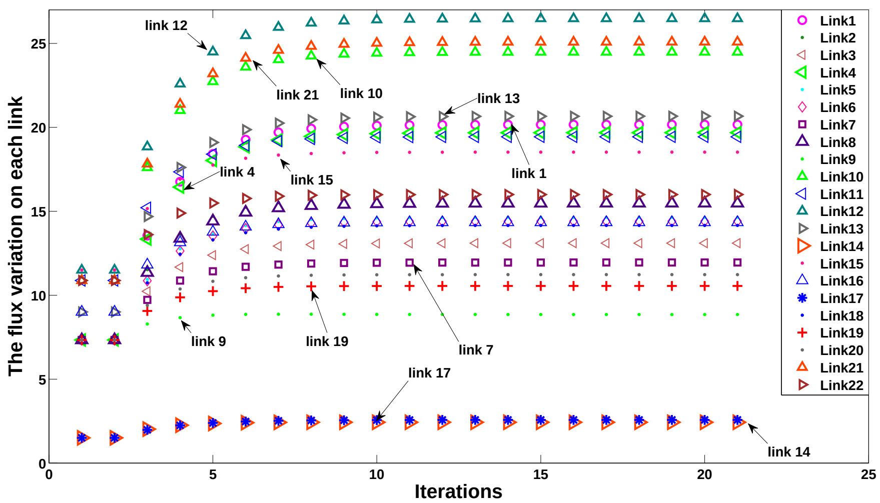

Fig. 5 The flux variation during the iterative process in the Example 3

In the proposed Physarum model, it converges to the optimal solution after 21 iterations. As in Example 2, all the links have positive flows and capacity. This has demonstrated the efficiency and flexibility of the proposed approach.

Table 4 The optimal solution to Example 3

Link a	f_a^*	u_a^*	Link a	f_a^*	u_a^*
1	20.16	20.16	12	26.48	26.48
2	15.80	15.80	13	20.66	20.66
3	13.10	13.10	14	2.43	2.43
4	19.68	19.68	15	18.52	18.52
5	14.64	14.64	16	14.34	14.34
6	14.41	14.41	17	2.57	2.57
7	11.95	11.95	18	14.16	14.16
8	15.45	15.45	19	10.55	10.55
9	8.85	8.85	20	11.22	11.22
10	24.48	24.48	21	25.08	25.08
11	19.44	19.44	22	16.00	16.00

5 Conclusions

In this paper, based on *Physarum* solver, we propose a biologically inspired solution to the sustainable supply chain network design problem, in which both the product flow and the links' capacity are design variables. In addition, we have considered the emission cost generated by activities of the supply chain. By employing the continuity of the flux during the iterative process and the protoplasmic network adaptivity, we successfully solve the sustainable supply chain network. By comparing with the results in Ref. (Nagurney and Nagurney, 2010a), we have demonstrated the practicality and flexibility of the proposed approach.

Future research can be carried out in the following directions. On one hand, we will research the supply chain network design problem under more complicated environment, such as the supply chain network design under profit maximization and oligopolistic completion. On the other hand, we will try to apply *Physarum* into other fields, such as the design of wireless networks.

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