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### Fast Dynamic Hysteresis Modeling Using a Regularized On-Line Sequential

### **Extreme Learning Machine with Forgetting Property**

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Abstract—Piezoelectric ceramics(PZT)actuator has been widely used in flexure-guided micro/nano positioning stage because of their high resolution. However, it is quite hard to achieve high-rate precision positioning control because of the complex hysteresis nonlinearity effect of PZT actuator. Thus, an online **RELM** algorithm forgetting with property (FReOS-ELM) is proposed to handle this issue. Firstly, we adopt regularized extreme learning machine (RELM)to build an intelligent hysteresis model. The training of the algorithm is completed only in one step, which avoids the shortcomings of the traditional hysteresis model based on artificial neural network (ANN) that slow training speed and easy to fall into the local minimum. Then, based on the regularized extreme learning on-line sequential (ReOS-ELM), an on-line RELM algorithm with forgetting property (FReOS-ELM) is designed, which can avoid the computational load of ReOS-ELM in the process of adding new data for learning on-line. In the experiment, a real-time voltage signal with varying frequencies and amplitudes is adopted, and the output displacement data of the micro/nano positioning stage is also acquired and analyzed. The experimental results show that the RELM-based hysteresis modeling algorithm has higher efficiency and more stable learning ability and generalization ability than the traditional neural network; In the aspect of online modeling, FReOS-ELM hysteresis modeling can achieve a better result than ReOS-ELM.

**Keywords:** Piezoelectric ceramics · micro/nano positioning stage · flexure · hysteresis nonlinearity · extreme learning machine.

#### 1 Introduction

PZT actuator has the advantages of small size, light weight, high stiffness, large output force, high resolution, and fast dynamic response, etc. Hence, it has been widely applied in various flexure-guided micro nano positioning system[1], such as the computer components[2-3], adaptive optics[4] and scanning probe microscopes(SPMs)[5]. However, PZT actuators made of piezoelectric materials have inherent hysteresis nonlinearity effect, it is a big challenge to compensate the control error that caused by the hysteresis nonlinearity in the micro/nano positioning system, also it is the key problem for improving the control precision[6-7].

A few of groups have focused on designing advanced methods to model and compensate the hysteresis nonlinearity of piezoelectric actuators. The general hysteresis models can be divided into three types: 1) physical type, such as Maxwell model, Duhe model and JA model; 2) semiphysical type, such as Preisach model, PI model, and the modified PI models[8-9], etc.; 3) intelligent learning algorithms, such as ANN model[10-11], support vector machine (SVM)[12-13] and adaptive fuzzy[14]. In the aspect of intelligent hysteresis modeling, ANN has achieved fruitful results[15], it has also attracted the widely attention of the scholars. For instance, Weichuan Liu et al.[16] presents a hysteresis model based on multilayer feedforward neural network(MFNN) which can be used to compensate the control precision without employing inverted hysteresis model. X. Zhang et al.[17] presents a neural network-based robust adaptive output-feedback motion controller to solve

hysteresis nonlinearity problem in the micro/nano positioning system. However, most of the ANN algorithm require iterative learning in the training process[18-19]. Besides, ANN training based on gradient iterative learning is slow and easy to fall into local minimum point[20-21]. Thus, this paper adopt RELM[22] proposed by W. Y. Deng et al. to fulfill hysteresis nonlinearity modeling task. It is based on the ELM algorithm proposed by Huang et al.[23], which has the advantages including fast learning and stable calculation speed. However, it cannot realize on-line parameter updating. Therefore, Hieu Trung Huynh et al. proposed ReOS-ELM[24], which can update the parameters of the model on-line, but still will appear computational load.

In order to carter for this requirement, an on-line **RELM** algorithm with forgetting property (FReOS-ELM) is proposed in this work. A real-time complex voltage signal with varying frequencies and amplitudes is used to study of the complex hysteresis nonlinearity. A series of experimental tests and performance analysis are carried out for validating the performance of the proposed method in the presence of complex hysteresis effect. The experimental results show that the training time of the RELM-based hysteresis model is shorter, the fitting precision is higher and the generalized ability of complex hysteresis is stronger than that of neural network hysteresis model. As for on-line parameter tuning, although the ReOS-ELM is better than the RELM in terms of long-term predictions, the FReOS-ELM can achieve better predictability than ReOS-ELM at the appropriate forgetting factor.

The main contribution of this work is to propose a novel intelligent learning algorithm for fast dynamic hysteresis modeling for the PZT actuators, which is significant for improving the positioning performance of the PZT-actuated flexure micro/nano positioning stage. In this paper, the section II introduces the experimental system and research contents; the section III introduces the RELM theory and its hysteresis modeling method; In section IV, we compare the RELM based hysteresis model with the gradient iteration based hysteresis model; In section V, the

principle of ReOS-ELM and FReOS-ELM are introduced, respectively. Then, the long-term online prediction performance of the hysteresis model based on the mentioned algorithms are studied and compared; In section VI, a series of practical control experiments are carried out to validate the performance of the proposed algorithm.

# 2 The experimental setup and problem statement

### 2.1 The experimental setup

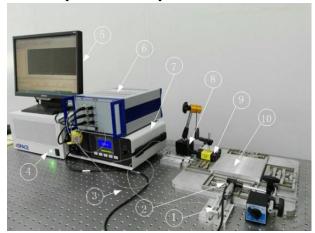


Fig. 1 The experimental setup of this study. 1) PZT actuator, 2) Fiber-optic displacement sensor probe 3) Vibration isolation stage, 4) dSPACE control system, 5) Host controlling computer, 6) Voltage amplifier, 7) Fiber-optic displacement sensor, 8) Magnetic gauge stand, 9) Laser displacement sensor, 10) Micro/nano positioning stage.

The whole experimental system is displayed in Fig.1. A developed flexure-guided micro/nano positioning stage with two degrees-of-freedom(DOF), motion range 1.036mm, and first resonance frequency 119.6 HZ is adopted in this experiment. Two piezoelectric ceramic actuators(P-840.60, PI) with a 3-channel voltage amplifier(E-503, PI) are used to actuate the micro/nano positioning stage. The laser displacement sensor(LTS-025-04, MTI) and fiber-optic displacement sensor (MTI-2100, MTI) are utilized to measure the precision displacement of the stage. The open-loop control of this stage is achieved by using the dSPACE prototyping simulating system(DS-1007, rapid dSPACE). In order to avoid the external vibration from all kinds of interference, all instruments (except the PC) are placed on the vibration isolation stage(WN01AL, Winner optics). Finally, the input voltage signal and

the output displacement of the micro/nano positioning stage are acquired and recorded to investigate the hysteresis nonlinearity.

#### 2.2 Problem statement

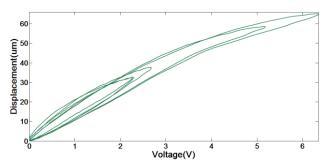


Fig. 2 The curve of complex hysteresis loop.

The hysteresis nonlinearity of PZT actuator is affected by various factors. For example, the load, the frequency of the input voltage and the history value of the working state. However, as for the hysteresis problem of piezoelectric system, we always choose the simple driving voltage signals to carry out the studies, such as periodic signal[25], frequency-varying signal[26], amplitudeand varying signal[27]. In practice, the input voltage signal is very complex and variable in the practical process. As shown in Fig.2, a complex analog voltage signal with varying amplitudes and frequencies is used as the input voltage to drive the experimental system.

Moreover, in the research of intelligent hysteresis modeling, most of the training of ANN is carried out by gradient iteration, which can be expressed as follows:

$$\omega_i^{k+1} = \omega_i^k - \eta \frac{\partial E(\omega_1, \omega_2 ... \omega_n)}{\partial \omega_i}$$
 (1)

where  $\omega_i^k$  is the *i*th parameters after the *k*th iteration,

 $E(\omega_1, \omega_2, ...\omega_n)$  is the training loss function. Once E is

small enough, the model training is over. Although the ANN has achieved satisfactory results in the hysteresis modeling, this training method is time-consuming, and the accuracy is often difficult to achieve one-time training. Therefore, it will take long time, and it is not suitable for the use of large sample size and complicated rules. In this paper, a kind of intelligent

algorithm named FReOS-ELM is proposed and implemented, which can learn quickly and has high precision in the presence of complex hysteresis effect.

# 3 RELM theory and hysteresis nonlinearity modeling

### 3.1 ELM and RELM theory

As we know, the principle of extreme learning machine (ELM) is:

$$y_i = \sum_{j=1}^{s} \beta_j f(\alpha_j, \theta_j, x_i)$$
  $i = 1, 2...N$  (2)

where  $x_i$  is *i*th input,  $y_i$  is *i*th output,  $x_i$  is the connection weights between *j*th neurons in the hidden layer and the output layer,  $\alpha_j$  is the connection weights between *j*th neurons in the hidden layer and the input layer,  $\theta_j$  is the threshold of *j*th neurons in the hidden layer.

In ELM, the weights and thresholds between the hidden layer and the input layer are assigned randomly, and (3) is transformed into a linear equation,

$$Y = F\beta \tag{3}$$

where: 
$$Y = \begin{bmatrix} y_1 & \cdots & y_N \end{bmatrix}^T$$
,  $\beta = \begin{bmatrix} \beta_1 & \cdots & \beta_N \end{bmatrix}^T$ , and  $F$  is a

 $N \times S$  dimensional matrix.

The connection weights between the hidden layer and the output layer can be obtained by generalized inverse operation,

$$\beta = (F^T F)^{-1} F^T Y = F^+ Y \tag{4}$$

In addition, Huang et al.[23] has proved that when the number of neurons in the hidden layer S is less than or equal to the number of training samples, the model can get the best learning effect with zero errors.

However, because ELM gives the weights and thresholds between the hidden layer and the input layer, the output weight is obtained via calculating the generalized inverse, which leads to the model easy to generate generalization ability and the stability is not ideal[22]. By adding parameter  $\lambda$  to adjust the proportion of the two kinds of risks, the RELM can achieve a better compromise between empirical risk and structural risk. Firstly, the objective function and constraint conditions are constructed:

$$\min \frac{1}{2} \beta^T \beta + \frac{\lambda}{2} \varepsilon^T \varepsilon$$

$$st \quad O = F\beta - \varepsilon$$
(5)

where  $\varepsilon = [\varepsilon_1 \quad \varepsilon_2 \quad \cdots \quad \varepsilon_N]$  is error, then the objective function and constraint conditions are represented by Lagrange function. Finally, let's take the partial derivative for  $\beta$  and  $\varepsilon$ , respectively, and let it equal to 0, the result is derived as,

$$\beta = (F^T F + \frac{I}{\lambda})^{-1} F^T O \tag{6}$$

### 3.2 Hysteresis modeling based on RELM

The hysteresis nonlinearity model based on RELM is shown in Fig.3.

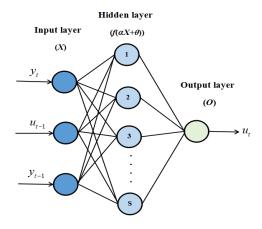


Fig. 3 The RELM-based hysteresis model.

In order to solve the problem of multi valued mapping of hysteresis nonlinearity. Here the input layer is set to three neurons, respectively, the output displacement value of t moment y(t), the last moment of the output displacement y(t-1) and the corresponding input voltage value u(t-1), that is, the input of each training x=(y(t), u(t-1), y(t-1)); There are S neurons in the hidden layer, and the output layer is the input voltage value corresponding to the output displacement of t moment u(t), namely O = u(t-1).

Additionally, the output displacement value and the input voltage value can be taken as the input of the hysteresis model. The performance of the model is also different for the different input samples of the dimension and the amount of information. However, in some cases, the greater the input dimension, the worse the model fitting and prediction performance(see Fig.4). In order to reduce the cost of time and information, as long as the accuracy of the model can meet the requirements, it cannot need too much input.

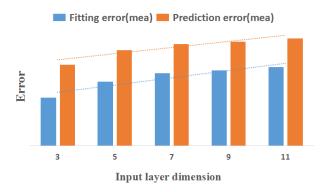


Fig. 4 The hysteresis modeling error under various input layer dimensions.

### 4 Performance analysis of hysteresis model based on RELM

In this section, we first analyze the effects of different parameters  $\lambda$  and the number S of neurons in the hidden layer on the performance of the model, then the hysteresis model based on RELM and hysteresis model of traditional gradient descent method based on the error back return is compared to further analysis the characteristics of the model.

#### 4.1 Experimental items

1) Activation function of hidden layer: The Sigmoid function is used as the excitation function of the hidden layer:

$$f(x) = \frac{1}{1 + e^{-x}} \tag{7}$$

The Sigmoid function is often used as the threshold function of the ANN since the Sigmoid function has a single increase property in terms of function and its inverse function.

2) Experimental error evaluation method: MAE and RMSE are used to calculate the error value, they are described as follows,

$$MAE = \frac{1}{N} \sum_{i=1}^{N} |e_i|, \quad RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} e_i^2}$$
 (8)

In order to avoid accidental results in RELM arising from randomly determined  $\alpha_j$  and  $\theta_j$ , the final error value is equal to the average of every experimental error.

3) Driving voltage signal: In order to obtain the nonlinear hysteresis value in terms of output displacement and input voltage u(t), as shown in (9),

here select the voltage signal u(t) as the input signal, thus the amplitude and frequency of the signal is changed continuously with no delay.

$$u(t) = 1.5(1 - \cos\frac{2}{3}\pi t)e^{\frac{(\sin\frac{\pi}{2}t)(\sin0.05t^{1.5})}{2}} \qquad t > 0$$
 (9)

Here u(t) and the according acquired filtered output displacement are shown in fig.5(a-c).

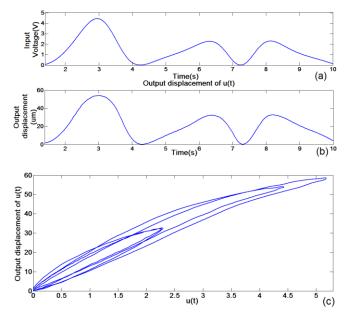
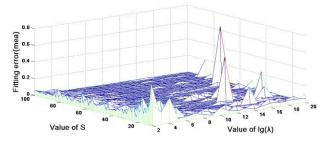


Fig. 5 The measured input and output signals of the positioning stage.

# 4.2 Experimental analysis of the influence of parameters $\lambda$ and S of neurons in hidden layers

The learning effect of the hysteresis nonlinearity model determines the generalization performance of the model, so it is very important to investigate the influence mechanism of parameters  $\lambda$  and the number S of neurons in the hidden layer.



**Fig. 6** The fitting error in terms of the parameter  $\lambda$  and S.

As shown in Fig. 6:

- 1) when S is too small, such as S < 30, the learning error of the model is relatively large, for different  $\lambda$  values, the fitting effect is poor;
- 2) when  $\lambda$  is too small, such as  $\lg(\lambda)$  <6, the learning error of the model is relatively large, for different S values, the fitting effect is poor;
- 3) when  $\lambda$  is too large, such as  $\lg(\lambda) > 12$ , for different S values, the model learning error may be volatile, the fitting effect is not stable. This is because when the parameter  $\lambda$  is too large, the connection weights between the hidden layer and the output layer are:

$$\beta = (F^{T}F + \frac{I}{\lambda})^{-1}F^{T}O \approx (F^{T}F)^{-1}F^{T}O$$
 (10)

It can be seen from (10) that the  $\beta$  value obtained by RELM is approximately equal to the  $\beta$  value obtained by ELM, so it is very likely get the disadvantage of ELM that the matrix is ill, which leads to poor learning performance.

4) when  $\lambda$  and S value is appropriate, such as 6<lg ( $\lambda$ ) <12, S>30, the model has a good learning effect. Especially when 30<S<60, with the increase of S, the effect is better and better, but when 60<S, with the increase of S, the effect is not obvious.

Based on the above analysis we can know that the parameter  $\lambda$  values and the number of neurons in the hidden layer S model will affect the learning effect. Generally, when the right value is determined, the S value is greater, the better effect of learning model. But when the learning accuracy is satisfied, the value S don't need too great, which can be used to avoid computing load.

#### 4.3 Comparing experiments of different algorithms

The experiment was carried out using u(t) as the driving signal. The results of fitting and prediction are displayed as follows.

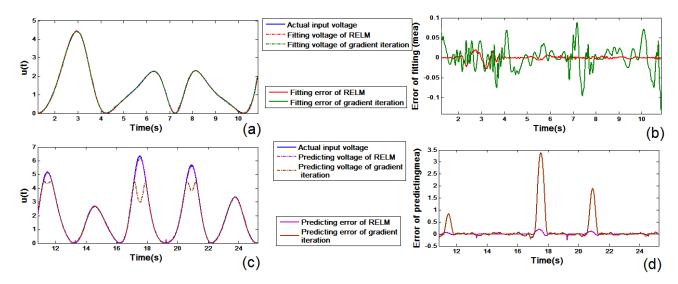


Fig. 7 Comparing experiments between different models. a) The comparisons of the model fitting results. b) The comparisons of the fitting error. c)The comparisons of the model predicting results. d) The comparisons of the predicting error.

Table I The modeling results of the RELM method and gradient iteration method

	MEA1	RMSE1	MEA2	RMSE2	Training time(s)
	(Fitting error)	(Fitting error)	(predition error)	(predition error)	
RELM	0.0034	0.5831	0.0193	4.8855	3.7988
Iteration	0.0218	2.9113	0.2078	73.1725	30.9719

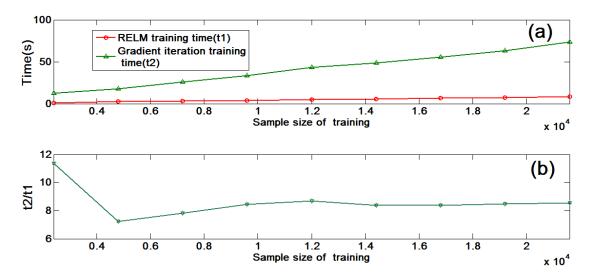


Fig. 8 The comparisons of the training time in terms of different sample size.

Based on the above results, we can know:

1) On the whole, the hysteresis model based REInM is better than the traditional neural network algorithm(gradient iteration). Especially in the extrapolation prediction, the MEA error of RELM is only about 1/10 under the gradient iteration, and the RMSE value of RELM is far less than the RMSE value under the gradient iteration, and the RELM has

stronger generalization ability.

- 2) As shown in Fig. 5.b, the fitting error value of the hysteresis model based on the gradient iteration has obvious oscillation, but hysteresis model based on RELM value is relatively stable, which shows that the memory effect of RELM algorithm is more stable and accurate.
  - 3) Comparing the common point of Fig.5.b and 5.d,

it can be found that although RELM-based hysteresis model has local sensitivity in fitting training and extrapolation process, that is, when u(t) achieve to peak value at the point of 3s, 6s, 11s, 17.5s and 21s, the performance of fitting, training, and extrapolation will become worse, but the error of RELM modeling is still smaller than the one based on gradient iteration. The results show that RELM can still achieve good stability even the hysteresis behavior at some of the extreme points is difficult to identify.

4) As shown in Fig. 6, it can be seen that RELM has higher learning efficiency since the consuming time of gradient iteration method is 8-9 times than the RELM-based method in different sample sizes.

# 5 On-line hysteresis nonlinearity modeling based on REOS-RELM/FReOS-ELM

#### 5.1 Principle of ReOS-RELM and FReOS-ELM

When a new sample is added to the actual control process, it is necessary to consider the usage of on-line modeling method to update the model. Thus, HT Huynh et al. proposed an on-line RELM (ReOS-ELM)[24] based on recursive least square(RLS).

In the case of N samples, the solution for the model weight is:

$$\beta_{N} = (F_{N}^{T} F_{N} + \frac{I}{\lambda})^{-1} F_{N}^{T} O_{N} = P_{N} F_{N}^{T} O_{N}$$
 (11)

where  $P_N = (F_N^T F_N + \frac{I}{\lambda})^{-1}$ , when a new sample

 $(x_{N+1}, o_{N+1})$  is added to the model:

$$\beta_{N+1} = P_{N+1} F_{N+1}^T O_{N+1} \tag{12}$$

where 
$$F_{N+1} = \begin{bmatrix} F_N \\ f(\alpha x_{N+1} + \theta) \end{bmatrix} = \begin{bmatrix} F_N \\ f_{N+1} \end{bmatrix}$$
,  $O_{N+1} = \begin{bmatrix} O_N \\ o_{N+1} \end{bmatrix}$ .

Then,

$$\begin{split} P_{N+1} &= (F_{N+1}^T F_{N+1} + \frac{I}{\lambda})^{-1} = (F_N^T F_N + f_{N+1}^T f_{N+1} + \lambda^{-1} I)^{-1} \\ &= (P_N^{-1} + f_{N+1}^T f_{N+1})^{-1} = (I - P_N \frac{f_{N+1}^T f_{N+1}}{1 + f_{N+1} P_N f_{N+1}^T}) P_N \end{split}$$

(13)

The denominator  $1+f_{N+1}P_Nf_{N+1}^T$  is a scalar, also,

$$\beta_{N+1} = P_{N+1} F_{N+1}^T O_{N+1} = P_{N+1} (F_N^T O_N + f_{N+1}^T O_{N+1})$$

$$= P_{N+1} (P_N^{-1} P_N F_N^T O_N + f_{N+1}^T O_{N+1})$$

$$= P_{N+1} ((P_{N+1}^{-1} - f_{/N+1}^T f_{N+1}) \beta_N + f_{N+1}^T O_{N+1})$$

$$= \beta_N + P_{N+1} f_{N+1}^T (O_{N+1} - f_{N+1} \beta_N)$$

$$= \beta_N + P_{N+1} f_{N+1}^T \mathcal{E}_{N+1}$$

$$= \beta_N + P_{N+1} f_{N+1}^T \mathcal{E}_{N+1}$$
(14)

Then

$$\begin{cases}
P_{N+1} = \left(I - P_N \frac{f_{N+1}^T f_{N+1}}{1 + f_{N+1} P_N f_{N+1}^T}\right) P_N \\
\beta_{N+1} = \beta_N + P_{N-1} f_{N+1}^T (o_{N+1} - f_{N+1} \beta_N)
\end{cases}$$
(15)

According to (15), we can see that the second term on the right side of the equation is the

term of weight 
$$\beta$$
, where  $\varepsilon_{N+} = (o_{N+} - f_{N+}^{\text{calibration}} \beta)$  is

prediction error,  $P_{N} \int_{1}^{T} f_{N}^{T} f_{N}^{T} dx$  is the calibration parameters.

Each weight in the real-time online update process is the use of the latest information, the theory is conducive to the long-term stable and effective work. However, because of the continuous accumulation of data information, the algorithm will lead to more computational load[28] and new information is flooded with old information, so it is necessary to abandon the old information.

In this paper, we use a new method named RELM, which combines the gradually weakened memory method(GWMM), that is, the forgetting parameter is introduced into the original online RELM algorithm, so that the model based on the proposed algorithm has the characteristics of forgetting. Its working principle is:

$$E = \mu \sum_{i=1}^{N} \varepsilon_i^2 + \varepsilon_{N+1}^2$$
 (16)

That is, when each new sample is added, the sum of the residuals obtained from the training of the sample is forgotten at rate  $\mu$ . RLS can be used to obtain the parameters of the following equation:

$$\begin{cases}
P_{N+1} = \frac{1}{\mu} \left( I - P_N \frac{f_{N+1}^T f_{N+1}}{\mu + f_{N+1} P_N f_{N+1}^T} \right) P_N \\
\beta_{N+1} = \beta_N + P_{N+1} f_{N+1}^T \left( o_{N+1} - f_{N+1} \beta_N \right)
\end{cases} (17)$$

Here  $\mu$  is a forgotten parameter, the value of the general range (0.95,1), when the  $\mu$ =1, FReOS-ELM

is the traditional ReOS-ELM.

The FReOS-ELM algorithm is described as follows:

Step1:Initializes the sample set (X,O), parameter  $\lambda$ , randomized weight  $\alpha$  and threshold  $\theta$ ;

Step2:Select the appropriate hidden layer function f, calculate the hidden layer function matrix F;

Step3: Calculate  $\beta$  and P:

Step4:Prediction of o with  $\hat{o} = f(\alpha x + \theta)\beta$  for predefined input x;

Step 5: When there is a calibration sample o , record prediction error  $\varepsilon = o - f$  ;

Step6:Update online 
$$P \leftarrow \frac{1}{\mu} (I - \frac{f^T f}{\mu + f P f^T}) P$$
, then

update 
$$\beta Pf + \beta^T \varepsilon$$
;

Step7: Repeat Step 4-6 until the end of forecast.

# 5.2 Hysteresis modeling experiment based on ReOS-ELM and FReOS-ELM

In order to evaluate the long-term predictive performance of RELM, ReOS-ELM, and FReOS-ELM, the first 20% of the total number of samples are used for training, and the remaining 80% samples are used for predicting. The experimental results are displayed as follows,

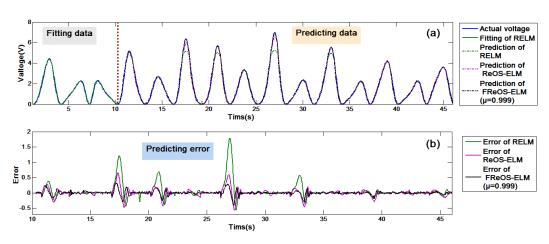


Fig. 9 The modeling results with different methods. a) The modeling results with RELM, ReOS-ELM, and FReOS-ELM algorithms. b) The modeling errors with RELM, ReOS-ELM, and FReOS-ELM algorithms.

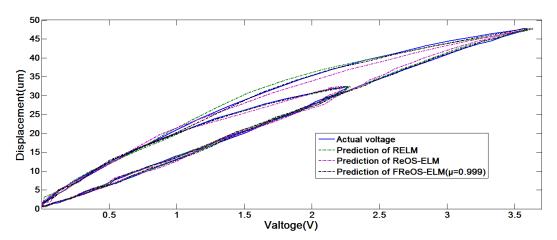


Fig. 10 The curves of the predicted hysteresis loops with RELM, ReOS-ELM, and FReOS-ELM algorithms.

Table II The modeling results of the RELM, ReOS-ELM, and FReOS-ELM

Methods	MEA	RMSE
RELM	0.1014	0.0669
ReOS-ELM	0.0733	0.0174
FReOS-ELM	0.0429	0.0057

As indicated in Fig.9(a-b) and Table II, the long-term predicting performance of ReOS-ELM and FReOS-ELM are better than RELM. Moreover, when the forgetting factor is specified to 0.999, the FReOS-ELM's long-term prediction performance is better than ReOS-ELM, while its sensitivity is lower and the effect is more stable at the peak voltage.

# 5.3 Experimental analysis of the forgetting characteristic of FReOS-ELM hysteresis model

Not all of the forgetting factor can improve the performance of the online parameter update. Especially, when the parameters  $\mu$  are too small, or the number of training samples is small, it is possible to cause the deterioration of model performance. This

is because every time the online correction parameters, will be in accordance with a certain rate to forget all the old data and information, the smaller the  $\mu$ , the faster the forgotten, it is likely to reduce the learning effect of the model.

In order to be more intuitive to see the effect of different parameters and different number of online correction on the model. In this experiment, the original sample is specified to 270, the prediction (test) sample is specified to 630, the new samples are added one by one according to the constant interval. Finally, the on-line update in terms of total number of 158, 126, and 105, respectively, are completed. The experimental results are plotted as follows:

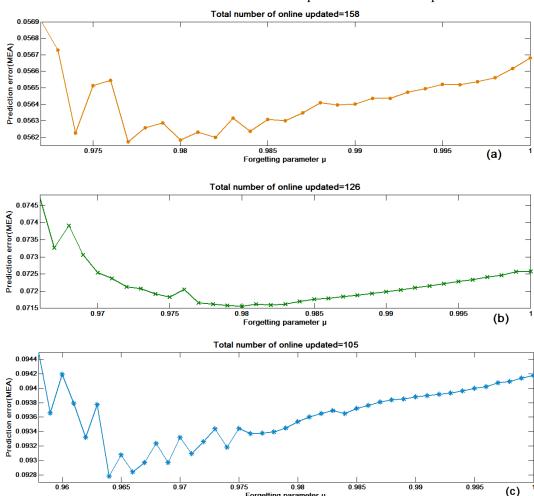


Fig. 11 The comparison of FReOS-ELM prediction error under different online correction times and different forgetting parameters.

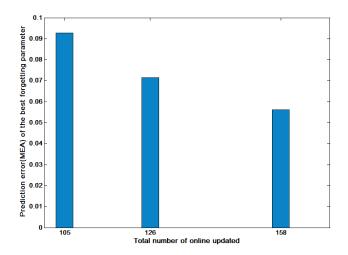


Fig.12 The error histogram of the best prediction results under different online correction times.

It can be found via the experimental results:

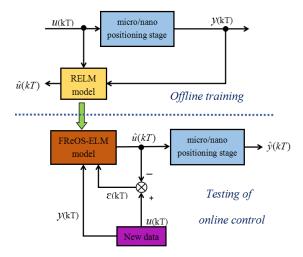
1)As shown in Fig.11(a-c), in a certain range  $\mu$ , the smaller the  $\mu$ , the smaller the online prediction error based on the FReOS-ELM hysteresis model, and better than the hysteresis model based on ReOS-ELM ( $\mu$ =1). But when the parameters  $\mu$  are too small, the accuracy of the model will be lower and more unstable.

- 2) From Fig. 11(a-c) also can be found, the more the total number of online correction, the greater the value of the forgetting parameter when the model deteriorates. This is just to verify that "the number of online correction will cause the deterioration of the model performance".
- 3) From Fig.12, the greater the number of online correction, the smaller the prediction error of the model. This is because with the increase of the number of online correction, the parameters of the model are gradually approaching the optimal value. 6

#### **Control experiment**

The experimental control system is displayed in Fig. 1. A series of control experiments in terms of gradient iteration, RELM hysteresis model, ReOS-ELM model, and FReOS-ELM hysteresis model are carried out to validate the superior performance of the proposed algorithm.

#### 6.1 Experiment design



**Fig.13** The schematic diagram of the control experiment with FReOS-ELM.

The schematic diagram of the control experiment with FReOS-ELM is shown in Fig. 13, the u(t) is used as the driving signal, and the actual output displacement data of the platform are acquired and recorded, and then the first 4500 samples are selected for training. Then the next 4500 samples are selected as online test data set, that is, the displacement data v(kT) (T is the sampling period, k=1, 2, ...) is acted as the reference displacement input, which can be used to predict the value of the driving voltage  $\hat{u}(kT)$ , then, it will be employed as the driving voltage to drive the positioning platform. Finally, 150 error values of the true driving voltage value u(kT) of expected output displacement and the predicted voltage value  $\hat{u}(kT)$ are selected as the input value to update the FReOS-ELM-based hysteresis model online. Besides, the control experiments with the traditional gradient

iterative training method, RELM method, and ReOS-ELM method are also implemented(150 times).

### 6.2 Experimental results and analysis

After that, the trained models are applied in the open-loop control experiment. The output

of displacement data the positioning by the fiber-optical displacement is measured sensor and collected by dSPACE, and then the displacement data and the reference displacement data are compared and analyzed. The experimental results are plotted and shown in Fig. 14-15.

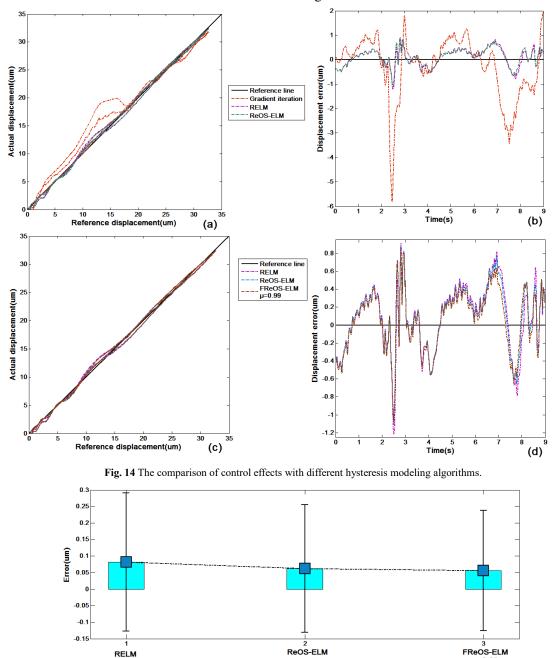


Fig. 15 The diagram of control displacement variance with RELM, ReOS-ELM and FReOS-ELM.

As shown in Fig.14(a-b), It can be found that the performance of the hysteresis model based on gradient iteration is obviously worse than RELM, and the

maximum displacement error is about 6μm. Also, as shown in Fig.15, It can be found that the effect of ReOS-ELM is better than that of RELM. It means that

the online RELM hysteresis model is better than the off-line RELM hysteresis model. The maximum displacement error of RELM, ReOS-ELM and FReOS-ELM is less than 1.2um. As shown in Fig.15, it can be found that the mean error and variance of FReOS-ELM are less than RELM and ReOS-ELM.

In addition, the data of Table III can be obtained. Comparing with the gradient iteration, the *MAE* values of RELM, ReOS-ELM, and FReOS-ELM are reduced

by 59.72%, 66.83%, and 67.92%, respectively, and the *RMSE* value is decreased by 66.70%, 72.74%, and 73.83%, respectively. It can also be found that the control effect of the RELM hysteresis model is more stable than that of the gradient iteration hysteresis model, and the control effect of the FReOS-ELM hysteresis model is more stable than that of ReOS-ELM.

Table III Comparison of control results of Iteration, RELM, ReOS-ELM and FReOS-ELM

	Iteration	RELM	ReOS-ELM	FReOS-ELM
MAE (um)	0.8881	0.3577	0.2945	0.2849
RMSE (um)	1.2918	0.4302	0.3521	0.3380
Error range (um)	(-5.8180, 1.9917)	(-1.2167, 0.9154)	(-1.1307, 0.8609)	(-1.0911, 0.8447)

#### 7 Conclusions

In order to cater for the requirement of fast and accurate modeling of complex hysteresis nonlinearity for the PZT-actuated micro/nano positioning system, a novel FReOS-ELM algorithm is proposed in this paper. In the aspect of off-line modeling, a real-time voltage signal with variable frequency is adopted to drive the nanopositioning stage, and the predicting, modeling, and experimental analysis are carried out in detail. Comparing with the traditional gradient iterative based ANN hysteresis modeling method, the off-line training time of RELM is shorter, the fitting precision is higher, and the complex hysteresis generalization ability is much stronger. In the aspect of on-line modeling, although ReOS-ELM is better than RELM in long-term prediction, FReOS-ELM can get more satisfactory prediction performance than ReOS-ELM once the parameters are appropriately assigned.

A series of open-loop control experiments with the mentioned algorithms have been carried out to make a comparison. All the results uniformly verify that the proposed FReOS-ELM algorithm can be employed to perform the high dynamic hysteresis modeling and on-line micro/nano positioning control.

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