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Demand Signal Transmission in a Certified Refurbishing Supply Chain: Rules and Incentive Analysis

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Abstract: Retailers, who sell certified refurbished products, usually have accumulated big data on demand properties, and hence, hold demand signal advantages over the other supply chain parties. In practice, we observe that this signal might be voluntarily shared to a rival who sells regular products. We are therefore interested in the incentives of demand signal transmission of the retailer selling certified refurbished products, and the value of an accurate signal for the other supply chain parties, especially in a one-to-two supply chain comprising a manufacturer (producing both regular and certified refurbished products) and two retailers (selling regular and certified refurbished products, respectively). We formulate the two retailers' competition and demand signal properties, and find that it is of the best interest for the manufacturer to produce two products, regardless of the possible downstream competition. We derive interesting demand signal transmission rules that the retailer selling certified refurbished products would *voluntarily* transmit the signal to the retailer (the rival) selling regular products, while it will not transmit the signal to the upstream manufacturer (the business partner). Even if the retailer selling regular products obtains the signal, it will not transmit the signal to the manufacturer either. We discuss the resulting insights regarding the production cost reduction, the government subsidy, and the product quality improvement. We find that the signal transmission rule is robust, and the retailers' profits may be *reduced* by the quality improvement of the certified refurbished product.

Keywords: Manufacturing Systems; Certified Refurbishing; Demand Signal Update; Supply Chain Collaboration

1. Introduction

Certified refurbishing recovers value from used products by replacing components or reprocessing used parts to bring the product to like-new condition (Atasu et al., 2008, Naeem et al., 2013), and extends the product life cycle (Yang et al. 2016). In practice, certified refurbishing of end of use (EOU) products can reduce the quantity of the EOU products polluting the environment and in turn improves supply chain sustainability. In economic sense, certified refurbishing provides an additional source of income for a manufacturer by extracting value from the EOU products (Galbreth et al., 2013), which induces some manufacturers to actively carry out certified refurbishing operations (Zou et al., 2016). For example, in 2007, Caterpillar established a certified refurbishing division which generated a business volume of \$ 2 billion (Ferguson and Souza, 2010). Xerox increased around 50% profits through its green manufacturing program (Savaskan et al., 2004).

While manufacturers carry out certified refurbishing, many retailers begin to sell certified refurbished products to the consumers. In recent years, there have been more and more retailers who exclusively sell certified refurbished products. For example, TonerGreen, an environmentally-conscious online store specializes in eco-friendly toner cartridges, offers US-made, certified refurbished toner cartridges for most of the popular printer brands such as HP, Xerox, Canon, and so on. In China, Beijing Ouruifu is a retailer specializing in selling various certified refurbished engines supplied by Sinotruck Jinan Fuqiang Power Co., LTD.

However, demand uncertainty for the certified refurbished products can be very high because of varying customers' acceptance cognition of social responsibility (Niu and Zou, 2017). Usually, the retailer selling the certified refurbished product knows the demand signal well, when processing and analyzing the big data stored in point-of-sales (POS) databases (Shang et al. 2016, Choi et al. 2017). This helps the retailer to reduce the potential profit loss caused by the huge demand uncertainty of certified refurbished products. In this paper, we assume demand uncertainty mainly comes from the certified refurbished products while the demand of the regular products is stable (we relax this assumption in the extensions.), because, in general, regular products are mature and consumers have clear cognition for it. In contrast, for most products, certified refurbishing is an emerging technology and consumers do not have clear cognition. The variance of the certified refurbished products' demand can also be viewed as the difference of the demand uncertainty degrees between these two products. Without loss of generality, we assume two retailers sell regular and certified refurbished products, respectively, and the retailer selling the certified refurbished products knows the demand signal better. He might share the signal with the supply chain parties (Lan et al. 2017, Niu and Zou 2017), but the incentives can be complicated.

From our recent interviews with several CEOs and CIOs in certified refurbishing industry, we found managers had different perspectives on investment in demand information gathering and processing. Some fully agreed with the application of data mining technology, but a few managers said that too much information was not beneficial for them. In fact, Taylor and Xiao (2010) and Chen and Xiao (2012) revealed that over-investment in information forecast technology, rather than being beneficial to a firm, even hurt the firm under certain conditions.

We aim to study the signal transmission among supply chain parties, and identify the implementations for the government to improve channel sustainability. Our main findings are summarized as follows: (1) The retailers have no incentives to share demand signal with the manufacturer, but the retailers selling certified refurbished products will share the signal with his rival. This changes their profits from product selling and demand learning, and hence, achieves a co-opetition relationship with each other (competing for customers and cooperating via demand signal sharing); (2) Government's encouragement of production cost reduction and certified refurbishing subsidy will increase channel sustainability, but will not change the demand signal transmission rules. Interestingly, the retailer selling certified refurbished products might be hurt by quality improvement, which induces the government to design a rewarding mechanism to keep improving the quality of certified refurbished products.

The rest of the paper is organized as follows. We review the related literature in Section 2. In Section 3, the base model is introduced, and the equilibrium analysis is conducted. In Section 4, we explore some extension of the base model and demonstrate the robustness of our findings. Section 5 concludes this paper.

2. Related Literature

Our paper is related to three streams of literature, namely: (1) remanufacturing closed-loop supply chain, (2) sustainability operations management, and (3) supply chain information management.

Firstly, literature on remanufacturing closed-loop supply chain mainly focuses on EOU collection, production and channel conflict problems. Savaskan et al. (2004) found that outsourcing the EOU product collection operation to a retailer was more effective than doing it by the manufacturer itself or a third party. However, Liu et al. (2017) found that, under certain conditions, the manufacturer and retailer dual collecting model was better than the retailer collecting model. As for production decisions, most existing studies on remanufacturing closed-loop supply chain (e.g., Atasu et al. 2008, Guide and Van Wassenhove 2009 and Souza 2013) showed that the competition from third-party remanufacturers was detrimental for manufacturers. In contrast, Agrawal et al. (2015) found that the presence of remanufactured products (sold by the manufacturer) could reduce the consumers' perceived value of new products by up to 8%, while the presence of third-party-remanufactured products could increase the perceived value of new products by up to 7%. Regarding channel conflict in remanufacturing closed-loop supply chain, there are three typical papers. Wang et al. (2014) considered a remanufacturer sold the remanufactured products either to the manufacturer or to the customers. They found, regardless of the remanufacturer's channel choice, a government subsidy could incentivize remanufacturing activity. Then, Yan et al. (2015) found that, compared to running its own e-channel, the manufacturer preferred to subcontract the marketing operation to a third party. Recently, Wang et al. (2016) studied whether it was necessary to develop an e-channel for the manufacturer. They found that the manufacturer preferred to operate a direct online channel. Which type of products (regular or remanufactured) the manufacturer should sell through the online channel depended on multiple factors. Aydin et al. (2016) formulated the coordination problem in a closed-loop supply chain simultaneously considering production line design into a multi-objective optimization model. They proposed an algorithm to find the Pareto optimal solutions. In contrast to their papers, we contribute by investigating signal transmission rules, which can be a new driver for channel cooperation, especially when the demand of remanufactured products is more uncertain than that of regular products.

Our paper is also closely related to the literature on sustainable operations management (SOM). SOM refers to these operational decisions for improving firm's ecological efficiency (Drake and Spinler, 2013). De et al. (2017) formulated a maritime inventory routing problem using mixed integer non-linear programming, and integrate sustainability factors by a non-linear relationship between vessel speed and fuel consumption. Choi (2015) considered environmental sustainability as a constraint, and proposed an optimization model under this constraint. He found the risk aversion of the firm played an important role in choosing of pollutant reduction technology. Xu et al. (2013) provided an effective method to evaluate the pressure to manage green supply chain management, and helped policy makers to induce companies to carry out sustainable green manufacturing. In recent years, carbon emission or carbon regulations on operation strategies have been a hot topic in SOM. Hua et al. (2011) investigated how firms manage carbon footprints in inventory management under the carbon emission trading mechanism. Song et al. (2017) compared the effects of carbon emission trading and carbon tax mechanisms on capacity expansion. Dong et al. (2016) studied order quantity and sustainability investment decision problems under carbon emission regulation. Xu et al. (2017) analyzed production and pricing problems in a make-to-order supply chain, incorporating the cap-and-trade regulation constraint, and found carbon trade price has an opposite effect on the total emissions and production quantity. In contrast, remanufacturing of the EOU products would reduce the quantity of the EOU products. Then, reduction and reuse of the EOU products indicate less EOU products polluting the environment which improves sustainability. Choi and Shen (2019) developed a framework to improve fashion supply chain's sustainability using information technologies. Shen et al. (2019) studied the problem of green and non-green products' selling sequence. They found that selling green product first always hurt environment more when the service level of green product was lower than the non-green one.

Literature on supply chain information management is also closely related. Li (2002) focused on supply chain consisting of an upstream manufacturer and n downstream retailers, and found vertical information sharing had two opposite effects. Wang et al. (2009) found that, in most cases, the upstream manufacturer had no motivation to share its producing cost information with the downstream retailer. Taylor and Xiao (2010) found the improvement of downstream retailer's forecasting accuracy was either beneficial for the upstream manufacturer, or harmful to the upstream manufacturer. In fact, information sharing might happen not only among the partners in a supply chain, but also between the firms and the consumers. Lan et al. (2017) found competition reduced the retailers' incentives to share product quality information with the consumers, and the firms preferred sequential information sharing case to simultaneous case. Jha et al. (2017) studied the demand information sharing between a product and a technology development company which collaboratively developed a new product. Recently, Shen and Chan (2017) comprehensively reviewed the literature on forecast information sharing, pointing out the value and obstacles of forecast information sharing. These papers focus on information sharing in a forward supply chain, but we discuss information sharing in a closed-loop supply chain. The products (regular vs. remanufactured) sold result in different customer utilities, and the demand variance of the remanufactured product is higher. Shen et al. (2018) reviewed the literature about supply chain contracting and information, and they discussed the interaction effect of contracting and information. Some literature studied information leak, which focused on player's incentives of leaking its partner's information to other players (Anand and Goya 2009, Kong et al. 2013). We study the supply chain parties' incentives of signal transmission in such an imbalanced supply chain. Note that, our results can be insightful in a general setting where a manufacturer sells two substitutable products with quality differentiation. Therefore, our work is comparable with the literature on information sharing, which usually assumes retailers sell the same products.

Table 1: *Summary of main features of related papers and the contributions*

	Supply Chain Collaboration	**Sustainability**	**Information Sharing**	**Closed-loop Supply Chain**
Our study	√	√	√	√
Agrawal et al. (2015)		√		√
Atasu et al. (2008)		√		√
Aydin et al. (2016)	√	√		√
Chen and Xiao (2012)			√	
Choi (2015)		√		
De et al. (2017)		√		
Dong et al. (2016)		√		
Dukes et al. (2011)			√	
Ferguson and Souza (2010)		√		√
Galbreth et al. (2013)		√		√
Gal-Or et al. (2008)			√	
Ha and Tong (2008)			√	
Hahm and Lee (2011)			√	
Huang et al. (2018)			√	
Hua et al. (2011)		√		
Jha et al. (2017)	√		√	
Li (2002)			√	
Liu et al. (2017)		√		√
Mishra (2007)			√	
Naeem et al. (2013)		√		√
Wang et al. (2014)	√	√		√
Wang et al. (2016)		√		√
Yang et al. (2015)		√		√
Zou et al. (2016)		√		√

3. Model Framework

Consider a supply chain that consists of one manufacturer producing both regular and certified refurbished products, and two retailers specializing in selling regular and certified refurbished products, respectively. For convenience, we denote the manufacturer as m, the retailer specializing in selling regular products as retailer n and the retailer specializing in selling certified refurbished products as retailer r. It's worth noting that, we assume certified refurbished products have a lower degree of product quality than the regular one. Thus, customers who purchase refurbished products are driven by a lower price, instead of consumer sustainability consciousness. However, as mentioned above, the demand uncertainty of the certified refurbished products can be very high. The assumption is reasonable for durable products such as printer, turbine, diesel engines, and brakes (Yang et al. 2015, Niu and Zou 2017). In general, Retailer r can forecast the demand uncertainty by keeping a close interaction with the end consumers, collecting massive historical data and using big data technology to analyze the data. Following Ha and Tong (2008), we employ the "signal" to denote the *processed demand information based on retailer r's POS data*. Clearly, there is a signal difference regarding r's demand when the two retailers make order quantity decisions where retailer n only has a vague signal of retailer r's demand. Transmission of the signal will change the parties' profits. Therefore, retailer r should decide whether to share the signal with other supply chain parties or not. If retailer r shares its demand signal to manufacturer m (or retailer n), then, the latter will decide whether to accept the signal or not. If manufacturer m (or retailer n) decides to accept the signal, m (or retailer n) then should decide whether to transmit the signal to retailer n (or manufacturer m).

We do not study signaling game in the main context, but use "signal" as an item that stands for more accurate demand information. In section 4.5, we make efforts to study the "signaling effect" based on the literature on information sharing; however, we still do not study signaling game, which is beyond the scope of this paper.

We first consider basic models to answer the following two questions: **Question 1**: Is it beneficial for manufacturer m to produce certified refurbished products? **Question 2:** How would the retailer r's signal be transmitted? Then we study production cost and government subsidy (either of the upstream or of the downstream), and the certified refurbished product's quality to provide some insights for the government.

3.1 Model

Notations

m: Manufacturer;

n: Retailer who sales regular products;

r: Retailer who sales certified refurbished products;

δ: Quality of the certified refurbished product;
q_n: Retailer n's order quantity;
q_r: Retailer r's order quantity;
w_n: Regular product's wholesale price;
w_r: Certified refurbished product's wholesale price;
p_n: Regular product's market price;
p_r: Certified refurbished product's market price;
v: Customer's location;
s_n: Surplus utility of the customer who purchase regular product;
s_r: Surplus utility of the customer who purchase certified refurbished product;
Γ: The demand signal of certified refurbished product;
π_m: Manufacturer's ex post profit;
π_n: Retailer n's ex post profit;
π_r: Retailer r's ex post profit;
$\widehat{\Pi}_m$: Manufacturer's ex ante profit;
$\widehat{\Pi}_n$: Retailer n's ex ante profit;
$\widehat{\Pi}_r$: Retailer r's ex ante profit;

We first consider the case when manufacturer m carries out certified refurbishing. Manufacturer m provides products respectively to retailer n and retailer r, as showed in Figure 1. This structure is common in practice. For example, Pantip Plaza and Overcart are respective the largest marketplaces for refurbished products in Thailand and India.[1,2] Manufacturer can contract with a boutique to sell regular and also contract with a retailer in refurbished products market to sell certified refurbished products. Being able to guide game players to make decisions in production, signal is of no value if obtained after the production decisions. Therefore, we assume the transmission of signal happens before all the game players make their decisions on ordering and production quantities. The whole problem involves two stages, i.e., signal sharing stage and production stage.

We begin with the signal sharing stage. The informed player may choose to transmit the signal to the uninformed player who has no signal. The uninformed player may then decide to accept it or not. For example, at the beginning, retailer r may transmit the signal to either manufacturer m or retailer n, or to both of them. Manufacturer m (retailer n) can choose to accept the signal or not. If yes, m (or n) may continue to transmit the signal to n (or M), who then chooses to accept it or not, as shown in Figure 2. We assume that only the demand signal is uncertain and can be updated, and all other parameters are common knowledge to all parties.

1 http://www.bangkok.com/shopping-mall/pantip-plaza.htm
2 https://www.crunchbase.com/organization/overcart#section-overview

Therefore, the signal sharing agreement reached in the first stage is known to all supply chain members. In order to avoid the moral hazard issues, the signal sharing agreement among the supply chain members is reached before the obtain of the updated demand signal.

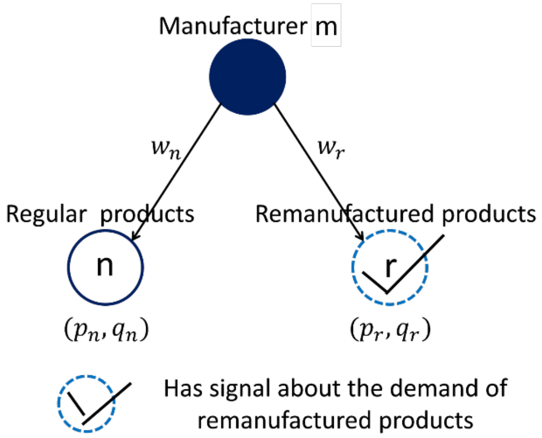

Figure 1: Supply chain structure before signal transmission

The three supply chain parties and their signal transmission are taken as a system, which forms four different states depending on whether parties get the signal or not:

State R: Only retailer r (selling certified refurbished products) has the signal;

State RM: Manufacturer m and retailer r have the signal;

State RN: Retailer n (selling regular products) and retailer r have the signal;

State RNM: All supply chain parties have the signal.

The four states fully cover the equilibrium outcomes of the signal transmission in this stage, as shown in Figure 2.

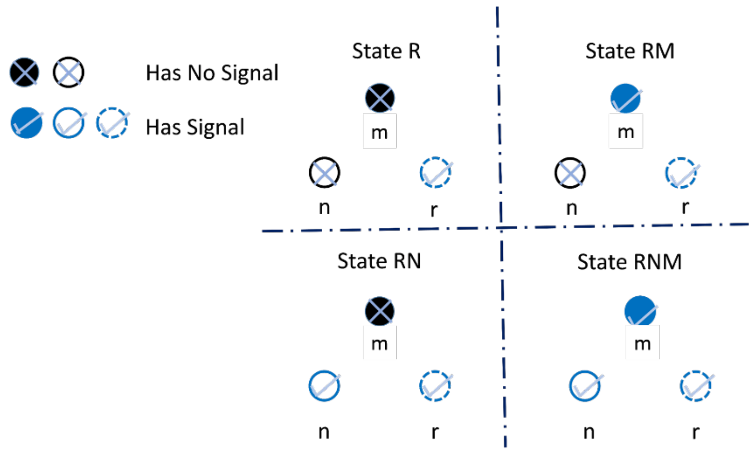

Figure 2 Four demand signal states of the system

Next we deal with the production stage. There are two sequential events in this stage. First, manufacturer m sets the wholesale prices w_n, w_r for the regular product and the certified refurbished product, respectively. Second, retailer n and retailer r decide the order quantity q_n, q_r simultaneously. The sequence of events is illustrated in Figure 3:

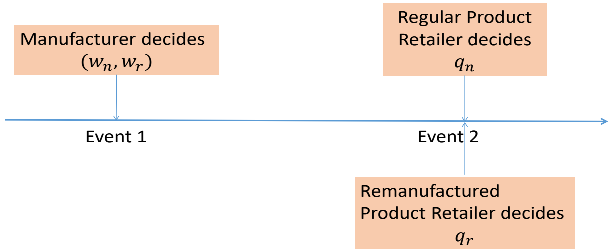

Figure 3 Sequence of events in the production stage

Regardless of the uncertainty of the certified refurbished product, we mark the quality of the regular product as 1 and the certified refurbished product as $\delta, (\delta \in (0,1))$ since the durability of the latter is generally worse. Denote the consumer's willing-to-pay for the regular product by *v*, which is assumed to be uniformly distributed in the interval [0,1]. Therefore, the willing-to-pay for the certified refurbished product purchased by the same consumer is δv. Denote the retail price of the regular product by p_n, and the certified refurbished product by p_r. Therefore, the surplus value of the regular product and the certified refurbished product by the same consumer are s_n, s_r, which can be written as:

$$s_n = v - p_n$$

$$s_r = \delta v - p_r$$

Obviously, the consumer whose surplus $s_n \geq s_r$ chooses to purchase the regular product, i.e.

$$v \geq \frac{p_n - p_r}{1 - \delta}$$

While the consumer whose surplus $0 \leq s_r < s_n$ would prefer the certified refurbished product, i.e.

$$\frac{p_r}{\delta} \leq \mathrm{v} < \frac{p_n - p_r}{1 - \delta}$$

Thus, the market demands for the two types of the products are:

$$q_n = 1 - \frac{p_n - p_r}{1 - \delta}$$

$$q_r = \frac{p_n - p_r}{1 - \delta} - \frac{p_r}{\delta}$$

We derive the inverse demand functions as follows:

$$p_n = 1 - q_n - \delta q_r$$

$$p_r = \delta(1 - q_n - q_r)$$

To capture demand uncertainty of the certified refurbished products, the inverse demand function is rewritten as follows:

$$p_r = \delta(1 - q_n - q_r) + \epsilon$$

where ϵ is normally distributed with mean zero and variance σ^2.

In summary, the functions for the actual profit of the three players are：

$$\Pi_m = w_n q_n + w_r q_r \quad (\text{for } m)$$

$$\Pi_n = (1 - q_n - \delta q_r - w_n) q_n \quad (\text{for retailer } n)$$

$$\Pi_r = [\delta(1 - q_n - q_r) + \epsilon - w_r] q_r \ (\text{for retailer } r)$$

Retailer r has collected a good amount of related data and has predicted ϵ by means of big data technology. Denote the predicted value (signal) by Γ, and the deviation by ϵ_1, then, $\Gamma = \epsilon + \epsilon_1$. In addition, we assume $\epsilon_1 \sim N(0, \sigma_1^2)$. According to Vives (1984), Raju and Roy (2000), we assume $E[\epsilon|\Gamma] = \frac{\sigma^2 \Gamma}{\sigma^2 + \sigma_1^2}$. Suppose manufacturer m and retailer n is able to obtain the signal only by signal transmission, rather than any other ways. Since the production stage occurs after the signal sharing stage, by backward induction, we need to first capture the equilibrium profits of the three parties before addressing the equilibrium solutions in the signal sharing stage.

3.2 Results in Each State

3.2.1 State R

In State R，the equilibrium outcomes in State R are:

State R

$w_n^R = \frac{1}{2}$	$w_r^R = \frac{\delta}{2}$
$q_n^R = \frac{2-\delta}{2(4-\delta)}$	$q_r^R = \frac{1}{2(4-\delta)} + \frac{\sigma^2 \Gamma}{2\delta(\sigma^2 + \sigma_1^2)}$
$\widehat{\Pi}_n^R = \frac{(2-\delta)^2}{4(4-\delta)^2}$	$\widehat{\Pi}_r^R = \frac{\delta}{4(4-\delta)^2} + \frac{\sigma^4}{4\delta(\sigma^2 + \sigma_1^2)}$
$\widehat{\Pi}_m^R = \frac{1}{2(4-\delta)}$	

In which，$\widehat{\Pi}_n^R = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \pi_n^R \, d\epsilon d\epsilon_1, \widehat{\Pi}_r^R = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \pi_r^R \, d\epsilon d\epsilon_1, \widehat{\Pi}_m^R = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \pi_m^R \, d\epsilon d\epsilon_1$。

3.2.2 State RM

In State RM,

the equilibrium outcomes:

State RM

$w_n^{RM} = \frac{1}{2} + \frac{\sigma^2 \Gamma}{4(\sigma^2 + \sigma_1^2)}$	$w_r^{RM} = \frac{\delta}{2} + \frac{\sigma^2 \Gamma}{2(\sigma^2 + \sigma_1^2)}$
$q_n^{RM} = \frac{2-\delta}{2(4-\delta)}$	$q_r^{RM} = \frac{1}{2(4-\delta)} + \frac{\sigma^2 \Gamma}{4\delta(\sigma^2 + \sigma_1^2)}$
$\widehat{\Pi}_n^{RM} = \frac{(2-\delta)^2}{4(4-\delta)^2}$	$\widehat{\Pi}_r^{RM} = \frac{\delta}{4(4-\delta)^2} + \frac{\sigma^4}{16\delta(\sigma^2 + \sigma_1^2)}$
$\widehat{\Pi}_m^{RM} = \frac{1}{2(4-\delta)} + \frac{\sigma^4}{8\delta(\sigma^2 + \sigma_1^2)}$	

In which， $\widehat{\Pi}_n^{M} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \pi_n^M \, d\epsilon d\epsilon_1, \widehat{\Pi}_r^{M} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \pi_r^M \, d\epsilon d\epsilon_1, \widehat{\Pi}_m^{M} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \pi_m^M \, d\epsilon d\epsilon_1$

3.2.3 State RN

In State RN, the equilibrium outcomes are given as follows:

State RN

$w_n^{RN} = \frac{1}{2}$	$w_r^{RN} = \frac{\delta}{2}$
$q_n^{RN} = \frac{2-\delta}{2(4-\delta)} - \frac{\sigma^2 \Gamma}{(4-\delta)(\sigma^2 + \sigma_1^2)}$	$q_r^{RN} = \frac{1}{2(4-\delta)} + \frac{2\sigma^2 \Gamma}{\delta(4-\delta)(\sigma^2 + \sigma_1^2)}$
$\widehat{\Pi}_n^{RN} = \frac{(2-\delta)^2}{4(4-\delta)^2} + \frac{\sigma^4}{(4-\delta)^2(\sigma^2 + \sigma_1^2)}$	$\widehat{\Pi}_r^{RN} = \frac{\delta}{4(4-\delta)^2} + \frac{4\sigma^4}{\delta(4-\delta)^2(\sigma^2 + \sigma_1^2)}$
$\widehat{\Pi}_m^{RN} = \frac{1}{2(4-\delta)}$	

In which, $\widehat{\Pi}_n^{RN} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \pi_n^{RN} \, d\epsilon d\epsilon_1, \widehat{\Pi}_r^{RN} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \pi_r^{RN} \, d\epsilon d\epsilon_1, \widehat{\Pi}_m^{RN} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \pi_m^{RN} \, d\epsilon d\epsilon_1.$

3.2.4 State RNM

In State RNM, the equilibrium outcomes as follows:

State RNM

$w_n^{RNM} = \frac{1}{2}$	$w_r^{RNM} = \frac{\delta}{2} + \frac{\sigma^2 \Gamma}{2(\sigma^2 + \sigma_1^2)}$
$q_n^{RNM} = \frac{2-\delta}{2(4-\delta)} - \frac{\sigma^2 \Gamma}{2(4-\delta)(\sigma^2 + \sigma_1^2)}$	$q_r^{RNM} = \frac{1}{2(4-\delta)} + \frac{\sigma^2 \Gamma}{\delta(4-\delta)(\sigma^2 + \sigma_1^2)}$
$\widehat{\Pi}_n^{RNM} = \frac{(2-\delta)^2}{4(4-\delta)^2} + \frac{\sigma^4}{4(4-\delta)^2(\sigma^2 + \sigma_1^2)}$	$\widehat{\Pi}_r^{RNM} = \frac{\delta}{4(4-\delta)^2} + \frac{\sigma^4}{(4-\delta)^2\delta(\sigma^2 + \sigma_1^2)}$
$\widehat{\Pi}_m^{RNM} = \frac{1}{2(4-\delta)} + \frac{\sigma^4}{2\delta(4-\delta)(\sigma^2 + \sigma_1^2)}$	

Where $\widehat{\Pi}_n^{RNM} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \pi_n^{RNM} \, d\epsilon d\epsilon_1$, $\widehat{\Pi}_r^{RNM} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \pi_r^{RNM} \, d\epsilon d\epsilon_1$, $\widehat{\Pi}_m^{RNM} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \pi_m^{RNM} \, d\epsilon d\epsilon_1$.

3.3 Comparison

Based on the equilibrium profits of the three supply chain parties under the four states, we derive the following results:

Lemma 1 $\widehat{\Pi}_{\mathrm{n}}^{RN} > \widehat{\Pi}_{\mathrm{n}}^{RNM} > \widehat{\Pi}_{\mathrm{n}}^{R} = \widehat{\Pi}_{\mathrm{n}}^{RM}$; $\widehat{\Pi}_{\mathrm{r}}^{RN} > \widehat{\Pi}_{\mathrm{r}}^{R} > \widehat{\Pi}_{\mathrm{r}}^{RNM} > \widehat{\Pi}_{\mathrm{r}}^{RM}$; $\widehat{\Pi}_{\mathrm{m}}^{RNM} > \widehat{\Pi}_{\mathrm{m}}^{RM} > \widehat{\Pi}_{\mathrm{m}}^{R} = \widehat{\Pi}_{\mathrm{m}}^{RN}$.

We discuss and answer the two questions stated in the introduction. If a supply chain party with the signal transmits it to another party who has no signal, and it is accepted, we deem that a successful signal transmission occurs. Obviously, every successful signal transmission incurs change in the state of the system. Our focus is on the change in the expected profits of the three players resulting from every successful signal transmission. For ease of exposition, we denote the system transformation from State i to State j (where $i, j \in \{\mathrm{R}, \mathrm{RM}, \mathrm{RN}, \mathrm{RNM}\}$) as Event ij. Thus, there are four possible events in the system, i.e. Event RRM, Event RRN, Event RMRNM, and Event RNRNM, as illustrated in Figure 4

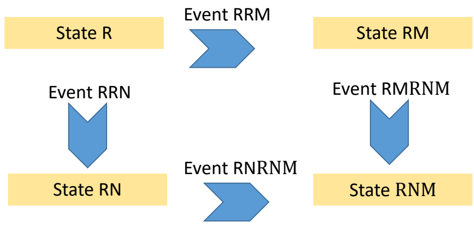

Figure 4: Four events

Note that signal transmission between retailers is referred to horizontal transmission, while signal transmission between supply chain layers is referred to vertical transmission in this paper. By Lemma 1 the following propositions are derived:

Proposition 1

(i) When the signal is obtained, the expected profit of an uninformed party will increase;

(ii) The horizontal transmission of the signal will enable increase in the expected profit of the informed party;

(iii) Signal transmission from upstream to downstream will increase the expected profit of an informed party;

(iv) Signal transmission from downstream to upstream will reduce the expected profit of an informed party;

(v) The expected profit of an uninformed party is independent of the signal transmission all the time.

Proposition 1 shows several important rules about signal transmission. Firstly, according to Proposition 1 (iv), manufacturer m never obtains the demand information (Neither retailer n or retailer r voluntarily transmits the signal to manufacturer M). Secondly, Proposition 1 (ii) implies that, retailer r will voluntarily share the information with retailer n. According to Proposition 1 (i), retailer n receives the information from retailer r exclusively. In other words, Event RRN will always happen, as illustrated in Figure 5.

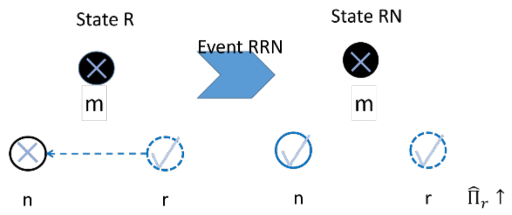

Figure 5 Horizontal signal transmission

Therefore, we focus on the case where retailer r shares the demand signal with retailer n. When the market price of the certified refurbished product is higher than the expectation, retailer n should have cut down the order quantity, which is, however, hindered by the absence of signal. As a result, retailer r is compelled to sell the product at a price lower than the market price, which is detrimental to r. Now retailer n has the signal which enables the order quantity to be revised timely, thus retailer r's profit increases. By contrast, when the market price of certified refurbished product is lower than the expectation, retailer n should have increased the order quantity, which is, however, hindered by the absence of signal. As a result, retailer r sells product at a price higher than the market price, which is beneficial to r. Now retailer n has the signal which enables the order quantity to be revised timely, thus retailer r's profit decreases. In equilibrium, the impact from retailer n obtaining a positive signal with $\epsilon > 0$ is more significant, because the profit margin of retailer r (selling certified refurbished products) is higher. That is, $p_r - w_r = \delta + \epsilon - \delta(q_n + q_r) - w_r$ is more sensitive to a positive ϵ. This incentivizes retailer r' to share the demand signal with retailer n.

Proposition 2 The signal transmission rules finally reduce to one equilibrium state, i.e. State RN. Namely, there is a horizontal transmission but no vertical transmission.

We find that retailer r will voluntarily transmit the demand signal to retailer n. The underlying reason is as follows. In state NR, the quantity of regular products, q_n^{RN}, is negatively related to information Γ, but the quantity of certified refurbished products, q_r^{RN}, is positively related to information Γ. Similar to Dukes et al. (2011), we measure the congruence using the covariance of the quantities q_n and q_r, i.e., $\operatorname{cov}(q_n, q_r)$. Negative value represents congruence, but positive value represents confrontation. We find $\operatorname{cov}\left(q_n^{RN}, q_r^{RN}\right) = -4\sigma^4 \Big/ \left(\delta(4-\delta)^2\left(\sigma^2+\sigma_1^2\right)\right) < \operatorname{cov}\left(q_n^R, q_r^R\right) = 0$. This indicates that retailer n and retailer r decide quantities more congruently with each other in state NR which leads to a weaker quantity competition, and even coordination between these two competing retailers, because one retailer's quantity is increased while the other retailer's is reduced. Similar coordination induced by demand information sharing has been observed in Hahm and Lee (2011), Wu and Zhang (2014), Huang et al. (2018), and Tian and Jiang (2018), although they do not study a co-opetitive supply chain structure like ours.

In addition, we find $\partial \operatorname{cov}\left(q_n^{RN}, q_r^{RN}\right) \Big/ \partial\delta > 0$, $\partial \operatorname{cov}\left(q_n^{RN}, q_r^{RN}\right) \Big/ \partial\sigma^2 < 0$ and $\partial \operatorname{cov}\left(q_n^{RN}, q_r^{RN}\right) \Big/ \partial\sigma_1^2 > 0$, which illustrate that retailer r is less willing to share information with retailer n when the substitutability of the two products is high, or retailer r's forecasting accuracy is low. However, when the demand uncertainty is high, retailer r is more willing to share information with retailer n. It's worth noting that, the retailers will not voluntarily share the demand signal with the manufacturer. The reason is that, if manufacturer m has the demand signal, m will occupy most of the information value by determining the wholesale price.

So far, signal transmission issues for Question 2 are resolved. Regarding Question 1, we need to investigate State RN to see whether certified refurbishing benefits the manufacturer. We derive the following result:

Proposition 3 Compared to the case without certified refurbishing, the expected profit of the manufacturer increases when it produces and sells both regular and certified refurbished products.

Clearly, the presence of the certified refurbished product which attracts purchase from low-end consumers, enlarges manufacturer m's market share, and in turn increases m's expected profit.

4. Discussions

In this section, we extend our base models to five different settings to demonstrate the robustness of our main findings. In the first setting, we focus on the effect of production cost difference. In the second setting, we consider the effect of government subsidy. In the third setting, we consider the effect of quality improvement of the certified refurbished products. In the fourth setting, we assume that regular product has demand uncertainty and retailer n has the signal. In the fifth setting, we consider the uncertainty of the consumer value discount for certified refurbished products. Finally, we consider the constraint condition on the quantity of the certified refurbished products.

4.1 Insights about Production Cost

Now we consider the production cost by assuming cost per unit regular product (and per unit certified refurbished product) as c_n(and c_r), then the profit functions for the three players are:

$$\pi_n = (1 - q_n - \delta q_r - \mathrm{w_n})q_n$$

$$\pi_r = [\delta(1 - q_n - q_r) + \epsilon - w_r]q_r$$

$$\pi_m = (w_n - c_n)q_n + (w_r - c_r)q_r$$

The equilibrium is derived in the appendix: Cost-Equilibrium Analysis.

Proposition 4 With positive production costs, the equilibrium signal transmission rule is that, retailer r will share the signal with its rival. The downstream retailers will not share signal with the manufacturer.

Proposition 4 indicates that, although production cost affects the expected profit of the supply chain parties, it has no effect on the signal transmission rules. Namely, the qualitative results from the base models continue to hold in this more general setting. In fact, the quantity of the certified refurbished product decreases in certified refurbishing cost. We prove that government's encouragement in production cost reduction will increase channel sustainability, because certified refurbishing will reduce the quantity of the EOU products.

We also find that, cost has no impact on manufacturer m's decision on producing certified refurbished product. The profit of the manufacturer decreases in the production cost $c_n(c_r)$. This illustrates that manufacturer m has incentives to invest in production cost reduction, even if government's encouragement is absent.

4.2 Insights about Government Subsidy

We consider the upstream and downstream subsidy for certified refurbished products provided by the government. Denote the subsidy per unit certified refurbished product as η_u (upstream subsidy) and η_d (downstream subsidy).

4.2.1 Upstream Subsidy

The profit functions for the supply chain parties are:

$$\pi_{\mathrm{n}} = (1 - q_n - \delta q_r - \mathrm{w_n})q_n$$

$$\pi_{\mathrm{r}} = [\delta(1 - q_n - q_r) + \epsilon - w_r]q_r$$

$$\pi_{\mathrm{m}} = w_n q_n + (w_r + \eta_{\mathrm{u}})q_r$$

The equilibrium results are given in the appendix: Upstream Subsidy -Equilibrium Analysis.

4.2.2 Downstream Subsidy

The profit functions for the supply chain parties are:

$$\pi_{\mathrm{n}} = (1 - q_n - \delta q_r - \mathrm{w_n})q_n$$

$$\pi_{\mathrm{r}} = [\delta(1 - q_n - q_r) + \eta_{\mathrm{d}} + \epsilon - w_r]q_r$$

$$\pi_{\mathrm{m}} = w_n q_n + w_r q_r$$

The equilibrium results are given in the appendix: Downstream Subsidy-Equilibrium Analysis.

Proposition 5 Regardless of which party the government subsidy is given to the retailer or the manufacturer, the signal transmission rules will remain the same.

Regardless of which party the government subsidy is given to, they reallocate the benefit (government subsidy) via transfer pricing (wholesale price). From the proof for proposition 5 in appendix, we find the manufacturer and retailer r share the government subsidy equally, regardless of who receives the subsidy. Furthermore, upstream subsidy and downstream subsidy has the same impact on other decision variables except wholesale prices. Therefore, we conclude that the government subsidy does not change the signal transmission rules.

Proposition 6

(i) $q_r^{i-U}\left(q_r^{i-D}\right), \widehat{\Pi}_r^{i-U}\left(\widehat{\Pi}_r^{i-D}\right)$ *and* $\widehat{\Pi}_m^{i-U}\left(\widehat{\Pi}_m^{i-D}\right)$ *increase in* $\eta_u(\eta_d)$;

(ii) $q_n^{i-U}\left(q_n^{i-D}\right)$ *and* $\widehat{\Pi}_n^{i-U}\left(\widehat{\Pi}_n^{i-D}\right)$ *decrease in* $\eta_u(\eta_d)$.

Where $i \in \{R, RN, RM, RNM\}$.

Proposition 6 indicates that, government subsidy can lead to a change of market share of the two products, where the market share of the regular product decreases, while the market share of the certified refurbished product increases.

4.3 Insights about Quality Improvement of Certified Refurbished Products

Note that the expected profits of retailer n and retailer r are as follows

$$\widehat{\Pi}_N^{RN} = \underbrace{\frac{(2-\delta)^2}{4(4-\delta)^2}}_{I} + \underbrace{\frac{\sigma_{R0}^4}{(4-\delta)^2(\sigma_{R0}^2+\sigma_{R1}^2)}}_{II}$$

$$\widehat{\Pi}_R^{RN} = \underbrace{\frac{\delta}{4(4-\delta)^2}}_{III} + \underbrace{\frac{4\sigma_{R0}^4}{\delta(4-\delta)^2(\sigma_{R0}^2+\sigma_{R1}^2)}}_{IV}$$

If neither of them has the signal, i.e. $\sigma_{R1}^2 = +\infty$, then

$$\widehat{\Pi}_N^{RN} = I$$

$$\widehat{\Pi}_R^{RN} = III$$

Note: II and IV are the additional profit when the demand signal is transmitted to the retailer selling the regular products and the retailer selling the certified refurbished units, we denote them as the value of the signal. In addition, we denote I and III as basic profits and let $A = \frac{\sigma_{R0}^4}{\sigma_{R0}^2+\sigma_{R1}^2}$. Obviously, A represents the magnitude of the signal value. By analyzing how the quality of the certified refurbished product affect the expected profits of the two retailers, we obtain two propositions as follows.

Proposition 7

(i) *If* $A \in \left(0, \frac{1}{2}\right]$, $\widehat{\Pi}_N^{RN}$ *decreases in* δ;

(ii) *If* $A \in \left(\frac{1}{2}, 1\right)$, $\widehat{\Pi}_N^{RN}$ *decreases in* $\delta \in (0, 2-2A]$, *but increases in* $\delta \in (2-2A, 1)$;

(iii) *If* $A \in [1, +\infty)$, $\widehat{\Pi}_N^{RN}$ *increases in* δ.

Proposition 8

(i) *If* $A \in \left(0, \frac{5}{16}\right)$, $\widehat{\Pi}_R^{RN}$ *decreases in* $\delta \in (0, \hat{\delta}]$, *but increases in* $\delta \in (\hat{\delta}, 1)$;

(ii) *If* $A \in \left[\frac{5}{16}, +\infty\right)$, $\widehat{\Pi}_R^{RN}$ *decreases in* δ, *where* $\hat{\delta}$ *is a root of* $\delta^3 + 4\delta^2 + 48A\delta - 64A = 0$.

Proposition 7 is quite counterintuitive. Intuitively, high quality of the certified refurbished products hurts the rival retailer n's. However, in Proposition 7, we find that retailer n might benefit from the quality improvement of the certified refurbished products.. To illustrate, recall that in Proposition 1, we find retailer r always transmits the demand signal to retailer n. Therefore, retailer n always obtains the signal information value at the expense of market cannibalized by the certified refurbished product sold by retailer r. In fact, the expected profit $\widehat{\Pi}_N^{RN}$ consists of two parts, part I (basic profit) and part II (signal information value). Interestingly, the basic profit, part I, has a negative correlation with the quality of the certified refurbished products δ. In contrast, the signal information value, part II, has a positive correlation with the quality of the certified refurbished products δ. Thus, when the demand uncertainty is sufficiently high, i.e., the signal information value is sufficiently high, the positive correlation effect exceeds the negative effect.

In contrast, in Proposition 8, we find that, retailer r's profit might be hurt when its quality is improved, especially when the quality is lower than a threshold, or the value of the signal is high. Essentially, that is also due to the profit structure: quality improvement of the certified refurbished products might benefit the rival via the signal value externality. With a small δ, the profit loss from the tense competition dominates; with a large A, the value of the signal spills over to retailer n, which benefits n buy hurts r. The government should design a rewarding policy to help retailer r who is subject to low quality products and high demand variance.

4.4 Insights about Revenue Sharing Contract

Now we consider the contract between retailer r and manufacture m is revenue sharing. The revenue sharing rate is r. The profit functions for the three players are:

$$\pi_n = (1 - q_n - \delta q_r - \mathrm{w_n}) q_n$$

$$\pi_r = (1 - r)[\delta(1 - q_n - q_r) + \epsilon - w_r] q_r$$

$$\pi_m = (w_n - c_n) q_n + (w_r - c_r) q_r + r[\delta(1 - q_n - q_r) + \epsilon - w_r] q_r$$

The equilibriums are derived in the appendix: Revenue Sharing-Equilibrium Analysis. The demand signal transmission rule is identified as

Proposition 9 *Retailer r will transmit signal to manufacturer m if and only if* $r > \frac{4-\delta}{4}$

Clearly, when revenue sharing rate r is high, retailer r and manufacturer m have a high cooperation degree, so retailer r will transmit signal to manufacturer m. This breaks the horizontal information sharing alliance between retailer n and r, when revenue sharing contract is not considered.

4.5 Signaling effect in state RM

In state RM, manufacturer m has the demand information shared by retailer r (the retailer selling certified refurbished products), however, the manufacturer doesn't share information with retailer n (the retailer selling regular products). In this subsection, we assume retailer n can infer it from the wholesale price w_n. Similar to Gal-Or et al. (2008), we assume that in state RM, manufacturer m sets wholesale prices as follows: $w_n = \alpha_0 + \alpha_1\Gamma$ and $w_r = \beta_0 + \beta_1\Gamma$. Retailer n can infer Γ from the wholesale prices because it is a one-to-one match from Γ to w_n (w_r). However, in state RM, the wholesale prices are not only a cost but also a source of demand information for retailer n. Retailer r has signal, so he sets his quantity according to the wholesale prices w_n, w_r and signal Γ, i. e., $q_r = B_1 + B_2 w_r + B_3 w_n + B_4\Gamma$. The expected profit of retailer n conditionally depends on wholesale price w_n

$$\pi_n = E[[1 - q_n - \delta q_r - w_n]q_n | w_n]$$

and retailer r's expected profit conditionally depends on signal Γ

$$\pi_r = E[[\delta(1 - q_n - q_r) + \varepsilon - w_r]q_r | \Gamma]$$

The two first-order conditions yield

$$q_n = \frac{1 - \delta E[q_r | w_n] - w_n}{2}$$

$$q_r = \frac{\delta\left(1 - E[q_n | \Gamma]\right) + E[\varepsilon | \Gamma] - w_r}{2\delta}$$

Substituting $q_n = A_1 + A_2 w_n + A_3 w_r$ and $q_r = B_1 + B_2 w_r + B_3 w_n + B_4\Gamma$, we obtain

$$\begin{aligned}
q_n &= \frac{1 - \delta E[q_r | w_n] - w_n}{2} \\
&= \frac{1 - \delta\left(B_1 + B_2 w_r + B_3 w_n + B_4 E[\Gamma | w_n]\right) - w_n}{2} \\
&= \frac{1 - \delta B_1 - \delta B_2 w_r - \delta B_3 w_n - \delta B_4 \dfrac{w_n - \alpha_0}{\alpha_1} - w_n}{2} \\
&= A_1 + A_2 w_n + A_3 w_r
\end{aligned}$$

$$
\begin{aligned}
q_r &= \frac{\delta\left(1-E[q_n|\Gamma]\right)+E[\varepsilon|x]-w_r}{2\delta} \\
&= \frac{\delta\left(1-A_1-A_2w_n-A_3w_r\right)+\dfrac{\sigma\Gamma}{\sigma+s}-w_r}{2\delta} \\
&= B_1+B_2w_r+B_3w_n+B_4\Gamma
\end{aligned}
$$

Therefore, we have

$$
\begin{cases}
A_1 = \dfrac{1-\delta B_1}{2}+\delta B_4\dfrac{\alpha_0}{2\alpha_1} \\
A_2 = -\dfrac{1+\delta B_3}{2}-\dfrac{\delta B_4}{2\alpha_1} \\
A_3 = -\dfrac{\delta B_2}{2} \\
B_1 = \dfrac{1-A_1}{2} \\
B_2 = -\dfrac{1+\delta A_3}{2\delta} \\
B_3 = -\dfrac{A_2}{2} \\
B_4 = \dfrac{\sigma}{2\delta\left(\sigma+s\right)}
\end{cases}
$$

Solving the above equations leads to

$$
\begin{cases}
A_1 = \dfrac{2-\delta}{4-\delta}+\dfrac{\alpha_0\sigma}{\alpha_1\left(4-\delta\right)\left(\sigma+s\right)} \\
A_2 = -\dfrac{2}{4-\delta}-\dfrac{\sigma}{\alpha_1\left(4-\delta\right)\left(\sigma+s\right)} \\
A_3 = \dfrac{1}{4-\delta} \\
B_1 = \dfrac{1}{4-\delta}-\dfrac{\alpha_0\sigma}{2\alpha_1\left(4-\delta\right)\left(\sigma+s\right)} \\
B_2 = -\dfrac{2}{\delta\left(4-\delta\right)} \\
B_3 = \dfrac{1}{4-\delta}+\dfrac{\sigma}{2\alpha_1\left(4-\delta\right)\left(\sigma+s\right)} \\
B_4 = \dfrac{\sigma}{2\delta\left(\sigma+s\right)}
\end{cases}
$$

Thus, given the wholesale prices w_n and w_r, retailers' quantities are

$$q_n=\left(\frac{2-\delta}{4-\delta}+\frac{\alpha_0\sigma}{\alpha_1(4-\delta)(\sigma+s)}\right)-\left(\frac{2}{4-\delta}+\frac{\sigma}{\alpha_1(4-\delta)(\sigma+s)}\right)w_n+\frac{1}{4-\delta}w_r$$

$$q_r=\left(\frac{1}{4-\delta}-\frac{\alpha_0\sigma}{2\alpha_1(4-\delta)(\sigma+s)}\right)-\frac{2}{\delta(4-\delta)}w_r+\left(\frac{1}{4-\delta}+\frac{\sigma}{2\alpha_1(4-\delta)(\sigma+s)}\right)w_n+\frac{\sigma}{2\delta(\sigma+s)}\Gamma$$

Next, we analyze the wholesale price decision of manufacturer m in the first stage. In state RM, manufacturer can accurately forecast the order quantities of retailer n and retailer r in the second stage. Its objective function is

$$\begin{aligned}\pi_m=\max_{w_n,w_r}E[w_n&\left(\left(\frac{2-\delta}{4-\delta}+\frac{\alpha_0\sigma}{\alpha_1(4-\delta)(\sigma+s)}\right)-\left(\frac{2}{4-\delta}+\frac{\sigma}{\alpha_1(4-\delta)(\sigma+s)}\right)w_n+\frac{1}{4-\delta}w_r\right)\\&+w_r\left(\left(\frac{1}{4-\delta}-\frac{\alpha_0\sigma}{2\alpha_1(4-\delta)(\sigma+s)}\right)-\frac{2}{\delta(4-\delta)}w_r+\left(\frac{1}{4-\delta}+\frac{\sigma}{2\alpha_1(4-\delta)(\sigma+s)}\right)w_n+\frac{\sigma}{2\delta(\sigma+s)}\Gamma\right)|\Gamma]\end{aligned}$$

From the first-order conditions, we have

$$\begin{cases}w_n=\dfrac{4\left(2-\delta+\dfrac{\alpha_0\sigma}{\alpha_1(\sigma+s)}\right)+\left(2+\dfrac{\sigma}{2\alpha_1(\sigma+s)}\right)\left(\delta-\dfrac{\delta\alpha_0\sigma}{2\alpha_1(\sigma+s)}\right)+\left(2+\dfrac{\sigma}{2\alpha_1(\sigma+s)}\right)\dfrac{(4-\delta)\sigma}{2(\sigma+s)}\Gamma}{8\left(2+\dfrac{\sigma}{\alpha_1(\sigma+s)}\right)-\delta\left(2+\dfrac{\sigma}{2\alpha_1(\sigma+s)}\right)^2}\\[2ex] w_r=\dfrac{2\left(2+\dfrac{\sigma}{\alpha_1(\sigma+s)}\right)\left(\delta-\dfrac{\delta\alpha_0\sigma}{2\alpha_1(\sigma+s)}\right)+\delta\left(2+\dfrac{\sigma}{2\alpha_1(\sigma+s)}\right)\left(2-\delta+\dfrac{\alpha_0\sigma}{\alpha_1(\sigma+s)}\right)+\left(2+\dfrac{\sigma}{\alpha_1(\sigma+s)}\right)\dfrac{(4-\delta)\sigma}{(\sigma+s)}\Gamma}{8\left(2+\dfrac{\sigma}{\alpha_1(\sigma+s)}\right)-\delta\left(2+\dfrac{\sigma}{2\alpha_1(\sigma+s)}\right)^2}\end{cases}$$

Substituting $w_n=\alpha_0+\alpha_1\Gamma$ and $w_r=\beta_0+\beta_1\Gamma$ into above equations leads to

$$\begin{cases}\alpha_0=\dfrac{(6-\delta)(4-\delta)-(2-\delta)\sqrt{(4-\delta)^2+16(4-\delta)}}{8(4-\delta)}\\[2ex]\alpha_1=\dfrac{\sqrt{(4-\delta)^2+16(4-\delta)}-(4-\delta)}{8(4-\delta)(\sigma+s)}\sigma\end{cases}$$

$$\begin{cases}\beta_0=\dfrac{(10-\delta)(4-\delta)-(2-\delta)\sqrt{(4-\delta)^2+16(4-\delta)}}{16(4-\delta)}\delta\\[2ex]\beta_1=\dfrac{(8-\delta)(4-\delta)+\delta\sqrt{(4-\delta)^2+16(4-\delta)}}{16(4-\delta)(\sigma+s)}\sigma\end{cases}$$

Therefore, the equilibrium outcomes are:

$$\begin{cases} w_n^{RM} = \dfrac{(6-\delta)(4-\delta)-(2-\delta)\sqrt{(4-\delta)^2+16(4-\delta)}}{8(4-\delta)} + \dfrac{\sqrt{(4-\delta)^2+16(4-\delta)}-(4-\delta)}{8(4-\delta)\left(\sigma^2+\sigma_1^2\right)}\sigma^2\Gamma \\ w_r^{RM} = \dfrac{(10-\delta)(4-\delta)-(2-\delta)\sqrt{(4-\delta)^2+16(4-\delta)}}{16(4-\delta)}\delta + \dfrac{(8-\delta)(4-\delta)+\delta\sqrt{(4-\delta)^2+16(4-\delta)}}{16(4-\delta)\left(\sigma^2+\sigma_1^2\right)}\sigma^2\Gamma \end{cases}$$

$$\begin{cases} q_n^{RM} = \dfrac{(2-\delta)\left((4+\delta)+\sqrt{(4-\delta)^2+16(4-\delta)}\right)}{16(4-\delta)} - \dfrac{(4+\delta)+\sqrt{(4-\delta)^2+16(4-\delta)}}{16(4-\delta)\left(\sigma^2+\sigma_1^2\right)}\sigma^2\Gamma \\ q_r^{RM} = \dfrac{1}{2(4-\delta)} + \dfrac{\sigma^2\Gamma}{\delta(4-\delta)\left(\sigma^2+\sigma_1^2\right)} \end{cases}$$

$$\begin{cases} \hat{\Pi}_n^{RM} = \dfrac{(2-\delta)^2\left((4+\delta)+\sqrt{(4-\delta)^2+16(4-\delta)}\right)^2}{256(4-\delta)^2} + \dfrac{\left((4+\delta)+\sqrt{(4-\delta)^2+16(4-\delta)}\right)^2\sigma^4}{256(4-\delta)^2\left(\sigma^2+\sigma_1^2\right)} \\ \hat{\Pi}_r^{RM} = \dfrac{\delta}{4(4-\delta)^2} + \dfrac{\sigma^4}{\delta(4-\delta)^2\left(\sigma^2+\sigma_1^2\right)} \\ \hat{\Pi}_m^{RM} = \dfrac{(2-\delta)^2\sqrt{(4-\delta)^2+16(4-\delta)}-(4-\delta)\left(4-12\delta+\delta^2\right)}{64(4-\delta)} + \dfrac{(8-\delta)(4-\delta)+\delta\sqrt{(4-\delta)^2+16(4-\delta)}}{64\delta(4-\delta)\left(\sigma^2+\sigma_1^2\right)}\sigma^4 \end{cases}$$

Based on the equilibrium profits, we derive Proposition 10.

Proposition 10. When retailer n has information inference ability, retailer r will not transmit signal to manufacturer m.

The reason is that, when retailer n has the information inference ability, manufacturer m has the incentive to set a high wholesale price to both retailers, so as to induce retailer n to infer a high demand signal. Consequently, retailer n will order and sell more, which hurts retailer r in the downstream market. For retailer r, the procurement cost w_n is increased, which also hurts retailer r's profit. Then Proposition 10 becomes immediate.

5. Conclusion

Motivated by industrial observation that more and more retailers are selling certified refurbished products, who might share demand information with the retailers selling regular products, we characterized the competition and the cooperation via demand transmission in a one-to-two supply chain. The manufacturer was beneficial to produce both regular and certified refurbished products. We found that there existed signal transmission rules in this system, and the equilibrium was that, the horizontal transmission held while the vertical transmission would not hold. We showed that this finding was robust with positive production costs, government subsidy, and quality improvement of the certified refurbished products.

Besides, both production cost and government subsidy were found to influence the market share of the regular product and the certified refurbished product, which in turn affected the environment. So the government should encourage the development of new technology to reduce the production cost of the certified refurbished product, and increase the subsidy for the certified refurbished product.

We analyzed the effect of the quality of the certified refurbished products on the profits of two retailers and obtained interesting findings as follows. We found that, when the demand uncertainty of the certified refurbished products was high, quality improvement of the certified refurbished products would increase the expected profit of the retailer (the rival) who sold the regular products. It indicated that the government should encourage the certified refurbished product's quality improvement, when the quality level was low, or the demand variance was high, because retailer r had profit loss with the quality improvement of certified refurbished product.

We discuss four research directions to conclude this paper. First, it could be interesting to study alterative supply chain contract forms among the retailers and the manufacturer. In practice, the manufacturer may use consignment contract with retailer r, to reduce r's concern of low demand. Second, we have assumed that the sharing decisions are made ex ante (before getting the demand signal). If the sharing decision is made after getting the demand signal, the other supply chain members can infer the demand signal from retailer r's sharing decision. That is, there exists the signaling issues that require signaling game formulation. Third, signal transmission decisions (and the reject/acceptance decisions of the player to whom the information is transferred) are known to every player. It can be interesting to study the information asymmetry issues about the signal transmission decisions. That might result in interesting results, however, is complicated and beyond the scope of this research.

Acknowledgements

The authors are grateful to the managing editor Prof. Li Guo, the guest editor, and the reviewers for their helpful comments. The first author's work was supported by NSFC Excellent Young Scientists Fund (No. 71822202), NSFC (No. 71571194), Chang Jiang Scholars Program (Niu Baozhuang 2017), GDUPS (Niu Baozhuang 2017). The corresponding author is Zongbao Zou. The *co-first author Lei Chen's work was supported by the Joint Supervision Scheme with the Chinese Mainland, Taiwan and Macao Universities from HKPolyU and RGC of Hong Kong (No. G-SB0T).*

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Appendix

Process of Solving Game in Each State

1 State R

In State R，only retailer r has signal. We first consider the retailers' decisions in the second stage given the wholesale prices w_n and w_r for the two types of products. The uninformed retailer n's decision is made according to the manufacturer's wholesale prices. Similar to Gal-Or (2008) and Wu and Zhang (2014), we assume that retailer n uses the decision rule $q_n = A_1 + A_2 w_n + A_3 w_r$ to set the quantity. However, the informed retailer r sets quantity according to the wholesale prices w_n and w_r and signal Γ, i. e., $q_R = B_1 + B_2 w_r + B_3 \Gamma + B_4 w_n$. The expected profit of retailer n is

$$\pi_n = E[(1 - q_n - \delta q_r - \mathrm{w_n})q_n],$$

and retailer r's expected profit conditional on signal Γ is

$$\pi_r = E[[\delta(1 - q_n - q_r) + \varepsilon - w_r]q_r|\Gamma]$$

The two first-order conditions yield

$$q_n = \frac{1}{2}\left(1 - \delta E[q_r] - w_n\right) \tag{1}$$

$$q_r = \frac{1}{2\delta}\left(\delta\left(1 - E[q_n|\Gamma]\right) + E[\varepsilon|\Gamma] - w_r\right) \tag{2}$$

Substituting $q_n = A_1 + A_2 w_n + A_3 w_r$ and $q_R = B_1 + B_2 w_r + B_3 \Gamma + B_4 w_n$ into the right side of (1) and (2), we can obtain

$$\begin{cases} A_1 = \dfrac{1-\delta B_1}{2} \\ A_2 = -\dfrac{1+\delta B_4}{2} \\ A_3 = -\dfrac{\delta B_2}{2} \\ B_1 = \dfrac{1-A_1}{2} \\ B_2 = -\dfrac{1+\delta A_3}{2\delta} \\ B_3 = \dfrac{\sigma^2}{2\delta\left(\sigma^2 + \sigma_1^2\right)} \\ B_4 = -\dfrac{A_2}{2} \end{cases} \tag{3}$$

Solving the equations in (3) leads to

$$\begin{cases} A_1 = \dfrac{2-\delta}{4-\delta} \\ A_2 = -\dfrac{2}{4-\delta} \\ A_3 = \dfrac{1}{4-\delta} \\ B_1 = \dfrac{1}{4-\delta} \\ B_2 = -\dfrac{2}{\delta(4-\delta)} \\ B_3 = \dfrac{\sigma^2}{2\delta\left(\sigma^2+\sigma_1^2\right)} \\ B_4 = \dfrac{1}{4-\delta} \end{cases}$$

Thus, given the wholesale prices w_n and w_r, retailers' quantities are

$$q_n = \frac{2-\delta}{4-\delta} - \frac{2}{4-\delta} w_n + \frac{1}{4-\delta} w_r$$

$$q_r = \frac{1}{4-\delta} - \frac{2}{\delta(4-\delta)} w_r + \frac{\sigma^2}{2\delta\left(\sigma^2+\sigma_1^2\right)} \Gamma + \frac{1}{4-\delta} w_n$$

Then we proceed to analyze the wholesale price decision of manufacturer m in the first stage. Without signal, the objective function of the manufacturer m maximizing the expected profit is

$$\pi_m = E[w_n \left(\frac{2-\delta-2w_n+w_r}{4-\delta} \right) + w_r \left(\frac{\delta-2w_r+\delta w_n}{\delta(4-\delta)} + \frac{\sigma^2}{2\delta\left(\sigma^2+\sigma_1^2\right)} \Gamma \right)]$$

The wholesale prices set by manufacturer m satisfy

$$\begin{cases} \dfrac{\partial \pi_m}{\partial w_n} = \dfrac{2-\delta-4w_n+2w_r}{4-\delta} = 0 \\ \dfrac{\partial \pi_m}{d w_r} = \dfrac{\delta+2\delta w_n-4w_r}{\delta(4-\delta)} + \dfrac{\sigma^2}{2\delta\left(\sigma^2+\sigma_1^2\right)} E[\Gamma] = 0 \end{cases}$$

which yields equilibrium wholesale prices

$$\mathrm{w}_\mathrm{n}^R = \frac{1}{2}, w_r^R = \frac{\delta}{2}$$

2 State RM

In State RM, manufacturer m and retailer r have signal. Similarly, we first consider the retailers' decisions in the second stage for given the wholesale prices w_n and w_r of the two products. We assume that the manufacturer doesn't share information with retailer n, though the former has demand information. We assume retailer n doesn't have inference ability. Because retailer n only observes wholesale price w_n and w_r, we assume that retailer n uses the decision rule $q_n = A_1 + A_2 w_n + A_3 w_r$ to set the quantity. However, the informed retailer r sets quantity according to the wholesale prices w_n and w_r and signal Γ, i.e., $q_R = B_1 + B_2 w_r + B_3 \Gamma + B_4 w_n$.

The expected profit of retailer n is

$$\pi_n = E[(1 - q_n - \delta q_r - \mathrm{w_n}) q_n],$$

and retailer r's expected profit conditional on signal Γ is

$$\pi_r = E[[\delta(1 - q_n - q_r) + \varepsilon - w_r] q_r | \Gamma]$$

The two first-order conditions yield

$$q_n = \frac{1}{2}(1 - \delta E[q_r] - E[w_n]) \tag{4}$$

$$q_r = \frac{1}{2\delta}\left(\delta\left(1 - E[q_n|\Gamma]\right) + E[\varepsilon|\Gamma] - w_r\right) \tag{5}$$

Substitute $q_n = A_1 + A_2 w_n + A_3 w_r$ and $q_r = B_1 + B_2 w_r + B_3 \Gamma + B_4 w_n$ into (4) and (5), and we have

$$\begin{cases} A_1 = \dfrac{1 - \delta B_1}{2} \\ A_2 = -\dfrac{1 + \delta B_4}{2} \\ A_3 = -\dfrac{\delta B_2}{2} \\ B_1 = \dfrac{1 - A_1}{2} \\ B_2 = -\dfrac{1 + \delta A_3}{2\delta} \\ B_3 = \dfrac{\sigma^2}{2\delta\left(\sigma^2 + \sigma_1^2\right)} \\ B_4 = -\dfrac{A_2}{2} \end{cases}$$

Solving the equations leads to

$$\begin{cases} A_1 = \dfrac{2-\delta}{4-\delta} \\ A_2 = -\dfrac{2}{4-\delta} \\ A_3 = \dfrac{1}{4-\delta} \\ B_1 = \dfrac{1}{4-\delta} \\ B_2 = -\dfrac{2}{\delta\left(4-\delta\right)} \\ B_3 = \dfrac{\sigma^2}{2\delta\left(\sigma^2+\sigma_1^2\right)} \\ B_4 = \dfrac{1}{4-\delta} \end{cases}$$

Therefore, the order quantity decisions of retailers are

$$q_n = \frac{2-\delta}{4-\delta} - \frac{2}{4-\delta} w_n + \frac{1}{4-\delta} w_r$$

$$q_r = \frac{1}{4-\delta} - \frac{2}{\delta\left(4-\delta\right)} w_r + \frac{\sigma^2}{2\delta\left(\sigma^2+\sigma_1^2\right)} \Gamma + \frac{1}{4-\delta} w_n$$

Next, the wholesale price decision by the informed manufacturer m is in the first stage, who can accurately forecast the order quantity decision by retailer n and retailer r in the second stage, to maximize the profit. The objective function is

$$\pi_m = E[w_n\left(\frac{2-\delta-2w_n+w_r}{4-\delta}\right) + w_r\left(\frac{\delta-2w_r+\delta w_n}{\delta\left(4-\delta\right)} + \frac{\sigma^2}{2\delta\left(\sigma^2+\sigma_1^2\right)}\Gamma\right)|\Gamma]$$

Compared to state R, in state RM the manufacturer has the signal Γ. Thus, the wholesale prices set by manufacturer m satisfy

$$\begin{cases} \dfrac{\partial \pi_m}{\partial w_n} = \dfrac{2-\delta}{4-\delta} - \dfrac{4}{4-\delta} w_n + \dfrac{2}{4-\delta} w_r = 0 \\ \dfrac{\partial \pi_m}{\partial w_r} = \dfrac{1}{4-\delta} - \dfrac{4}{\delta\left(4-\delta\right)} w_r + \dfrac{\sigma^2}{2\delta\left(\sigma^2+\sigma_1^2\right)} \Gamma + \dfrac{2}{4-\delta} w_n = 0 \end{cases}$$

Therefore, we derive the equilibrium wholesale prices

$$w_n^{RM} = \frac{1}{2} + \frac{\sigma^2 \Gamma}{4(\sigma^2+\sigma_1^2)}, w_r^{RM} = \frac{\delta}{2} + \frac{\sigma^2 \Gamma}{2(\sigma^2+\sigma_1^2)}$$

3 State RN

In State RN, only retailer n and retailer r have signal. First we consider the retailers' decisions in the second stage when manufacturer m has given the wholesale price w_n, w_r for the two products. Since both retailers have the same signal, the expected profits of the retailers are

$$\pi_n = E[\left(1-q_n-\delta q_r-w_n\right)q_n \left|\Gamma\right]$$

$$\pi_r = E[\left(\delta\left(1-q_n-q_r\right)+\varepsilon-w_r\right)q_r \left|\Gamma\right]$$

The two first-order conditions yield

$$q_n = \frac{1-\delta E[q_r \left|\Gamma\right]-w_n}{2} \tag{6}$$

$$q_r = \frac{1}{2\delta}\left(\delta\left(1-E[q_n \left|\Gamma\right]\right)+E[\varepsilon \left|\Gamma\right]-w_r\right) \tag{7}$$

In this state, having signal, the retailers set their quantities according to the wholesale prices w_n and w_r and signal Γ, i. e., $q_n = A_1 + A_2 w_n + A_3 w_r + A_4 \Gamma$ and $q_R = B_1 + B_2 w_r + B_3 \Gamma + B_4 w_n$. Substituting $q_n = A_1 + A_2 w_n + A_3 w_r + A_4 \Gamma$ and $q_R = B_1 + B_2 w_r + B_3 \Gamma + B_4 w_n$ into the right sides of (6) and (7), we can obtain

$$q_n = \frac{1-\delta B_1}{2} - \frac{1+\delta B_3}{2} w_n - \frac{\delta B_2}{2} w_r - \frac{\delta B_4 \Gamma}{2}$$

$$q_r = \frac{1-A_1}{2} - \frac{1+\delta A_3}{2\delta} w_r - \frac{A_2}{2} w_n + \frac{\left(\sigma^2 - \delta\left(\sigma^2+\sigma_1^2\right)A_4\right)}{2\delta\left(\sigma^2+\sigma_1^2\right)}\Gamma$$

Therefore, we have

$$
\begin{cases}
A_1 = \dfrac{1-\delta B_1}{2} \\
B_1 = \dfrac{1-A_1}{2} \\
A_2 = -\dfrac{1+\delta B_3}{2} \\
B_2 = -\dfrac{1+\delta A_3}{2\delta} \\
A_3 = -\dfrac{\delta B_2}{2} \\
B_3 = -\dfrac{A_2}{2} \\
A_4 = -\dfrac{\delta B_4}{2} \\
B_4 = \dfrac{\left(\sigma^2 - \delta\left(\sigma^2 + \sigma_1^2\right) A_4\right)}{2\delta\left(\sigma^2 + \sigma_1^2\right)}
\end{cases}
\tag{8}
$$

Solving the system of equations in (8) leads to

$$
\begin{cases}
A_1 = \dfrac{2-\delta}{4-\delta} \\
B_1 = \dfrac{1}{4-\delta} \\
A_2 = -\dfrac{2}{4-\delta} \\
B_2 = -\dfrac{2}{\delta\left(4-\delta\right)} \\
A_3 = \dfrac{1}{4-\delta} \\
B_3 = \dfrac{1}{4-\delta} \\
A_4 = -\dfrac{\sigma^2}{\left(4-\delta\right)\left(\sigma^2 + \sigma_1^2\right)} \\
B_4 = \dfrac{2\sigma^2}{\delta\left(4-\delta\right)\left(\sigma^2 + \sigma_1^2\right)}
\end{cases}
$$

Thus, given the wholesale prices w_n and w_r, retailers' quantities are

$$
q_n = \frac{2-\delta}{4-\delta} - \frac{2}{4-\delta} w_n + \frac{1}{4-\delta} w_r - \frac{\sigma^2}{\left(4-\delta\right)\left(\sigma^2 + \sigma_1^2\right)} \Gamma
$$

$$
q_r = \frac{1}{4-\delta} - \frac{2}{\delta\left(4-\delta\right)} w_r + \frac{1}{4-\delta} w_n + \frac{2\sigma^2}{\delta\left(4-\delta\right)\left(\sigma^2 + \sigma_1^2\right)} \Gamma
$$

Then we analyze the wholesale price decision by manufacturer m in the first stage. Without signal, manufacturer m maximizes the expected profit and the objective function is

$$\pi_m = E[w_n\left(\frac{2-\delta-2w_n+w_r}{4-\delta}-\frac{\sigma^2}{(4-\delta)(\sigma^2+\sigma_1^2)}\Gamma\right)+w_R\left(\frac{\delta-2w_r+\delta w_n}{\delta(4-\delta)}+\frac{2\sigma^2}{2\delta(4-\delta)(\sigma^2+\sigma_1^2)}\Gamma\right)]$$

$$=w_n\left(\frac{2-\delta-2w_n+w_r}{4-\delta}\right)+w_R\left(\frac{\delta-2w_r+\delta w_n}{\delta(4-\delta)}\right)$$

The two first-order conditions lead to the following two equations

$$\begin{cases}\dfrac{\partial\pi_m}{\partial w_n}=\dfrac{2-\delta-4w_n+w_r}{4-\delta}=0\\[2ex] \dfrac{\partial\pi_m}{\partial w_r}=\dfrac{\delta-4w_r+\delta w_n}{\delta(4-\delta)}=0\end{cases}$$

Thus, we have

$$w_n^{RN}=\frac{1}{2},w_r^{RN}=\frac{\delta}{2}$$

4 State RNM

In State RNM, all players have signal. First we consider the retailers' decisions in the second stage, for given the wholesale prices w_n and w_r for the two products. In state RNM, both retailers have the same signal Γ. The retailers set their quantities according to the wholesale prices w_n and w_r and signal Γ, i. e., $q_n=A_1+A_2w_n+A_3w_r+A_4\Gamma$ and $q_R=B_1+B_2w_r+B_3\Gamma+B_4w_n$. The expected profits of the retailers are

$$\pi_n=E[(1-q_n-\delta q_r-w_n)q_n|\Gamma]$$

$$\pi_r=E[(\delta(1-q_n-q_r)+\varepsilon-w_r)q_r|\Gamma]$$

The two first-order conditions yield

$$q_n=\frac{1-\delta E[q_r|\Gamma]-w_n}{2} \tag{9}$$

$$q_r=\frac{1}{2\delta}\left(\delta(1-E[q_n|\Gamma])+E[\varepsilon|\Gamma]-w_r\right) \tag{10}$$

Substituting $q_n=A_1+A_2w_n+A_3w_r+A_4\Gamma$ and $q_R=B_1+B_2w_r+B_3\Gamma+B_4w_n$ into the right side of (9) and (10), we can obtain

$$q_n=\frac{1-\delta B_1}{2}-\frac{1+\delta B_3}{2}w_n-\frac{\delta B_2}{2}w_r-\frac{\delta B_4\Gamma}{2}$$

$$q_r = \frac{1-A_1}{2} - \frac{1+\delta A_3}{2\delta} w_r - \frac{A_2}{2} w_n + \frac{\left(\sigma^2 - \delta\left(\sigma^2 + \sigma_1^2\right) A_4\right)}{2\delta\left(\sigma^2 + \sigma_1^2\right)} \Gamma$$

Therefore, we have

$$\begin{cases} A_1 = \dfrac{1-\delta B_1}{2} \\ B_1 = \dfrac{1-A_1}{2} \\ A_2 = -\dfrac{1+\delta B_3}{2} \\ B_2 = -\dfrac{1+\delta A_3}{2\delta} \\ A_3 = -\dfrac{\delta B_2}{2} \\ B_3 = -\dfrac{A_2}{2} \\ A_4 = -\dfrac{\delta B_4}{2} \\ B_4 = \dfrac{\left(\sigma^2 - \delta\left(\sigma^2 + \sigma_1^2\right) A_4\right)}{2\delta\left(\sigma^2 + \sigma_1^2\right)} \end{cases}$$

Solving the equations leads to

$$\begin{cases} A_1 = \dfrac{2-\delta}{4-\delta} \\ B_1 = \dfrac{1}{4-\delta} \\ A_2 = -\dfrac{2}{4-\delta} \\ B_2 = -\dfrac{2}{\delta\left(4-\delta\right)} \\ A_3 = \dfrac{1}{4-\delta} \\ B_3 = \dfrac{1}{4-\delta} \\ A_4 = -\dfrac{\sigma^2}{\left(4-\delta\right)\left(\sigma^2 + \sigma_1^2\right)} \\ B_4 = \dfrac{2\sigma^2}{\delta\left(4-\delta\right)\left(\sigma^2 + \sigma_1^2\right)} \end{cases}$$

So, the retailers' quantity decisions are as follows

$$q_n = \frac{2-\delta}{4-\delta} - \frac{2}{4-\delta} w_n + \frac{1}{4-\delta} w_r - \frac{\sigma^2}{(4-\delta)(\sigma^2+\sigma_1^2)}\Gamma$$

$$q_r = \frac{1}{4-\delta} - \frac{2}{\delta(4-\delta)} w_r + \frac{1}{4-\delta} w_n + \frac{2\sigma^2}{\delta(4-\delta)(\sigma^2+\sigma_1^2)}\Gamma$$

Next we consider the wholesale price decision by manufacturer M, with the signal, m can accurately forecast the order quantity decision by retailer n and retailer r so as to maximize the profit. Therefore manufacturer m's objective function is

$$\pi_m = E[w_n\left(\frac{2-\delta-2w_n+w_r}{4-\delta} - \frac{\sigma^2}{(4-\delta)(\sigma^2+\sigma_1^2)}\Gamma\right) + w_R\left(\frac{\delta-2w_r+\delta w_n}{\delta(4-\delta)} + \frac{2\sigma^2}{2\delta(4-\delta)(\sigma^2+\sigma_1^2)}\Gamma\right)|\Gamma]$$

The wholesale prices given by manufacturer m satisfy

$$\begin{cases} \frac{\partial \pi_m}{\partial w_n} = \frac{2-\delta-4w_n+2w_r}{4-\delta} - \frac{\sigma^2}{(4-\delta)(\sigma^2+\sigma_1^2)}\Gamma = 0 \\ \frac{\partial \pi_m}{d w_r} = \frac{\delta+2\delta w_n-4w_r}{\delta(4-\delta)} + \frac{2\sigma^2}{2\delta(4-\delta)(\sigma^2+\sigma_1^2)}\Gamma = 0 \end{cases}$$

by the first-order conditions, we derive the equilibrium wholesale prices

$$w_n^{RNM} = \frac{1}{2}, w_r^{RNM} = \frac{\delta}{2} + \frac{\sigma^2 \Gamma}{2(\sigma^2+\sigma_1^2)}$$

PROOF FOR PROPOSITION 4

Cost-Equilibrium Analysis

We first summarize the equilibrium outcomes as follows.

Equilibrium Outcomes in State R

$$w_n^{R-O} = \frac{1}{2}(1+c_n)$$

$$w_r^{R-O} = \frac{1}{2}(\delta+c_r)$$

$$q_n^{R-O} = \frac{2-\delta-2c_n+c_r}{8-2\delta}$$

$$q_r^{R-O} = \frac{\delta+\delta c_n-2c_r}{2\delta(4-\delta)} + \frac{\sigma^2\Gamma}{2\delta(\sigma^2+\sigma_1^2)}$$

$$\widehat{\Pi}_n^{R-O} = \frac{(2-\delta-2c_n+c_r)^2}{4(4-\delta)^2}$$

$$\widehat{\Pi}_r^{R-O} = \frac{(\delta+\delta c_n-2c_r)^2}{4\delta(4-\delta)^2} + \frac{\sigma^4}{4\delta(\sigma^2+\sigma_1^2)}$$

$$\widehat{\Pi}_m^{R-O} = \frac{\delta-2\delta c_n+\delta^2 c_n+\delta c_n^2-\delta c_r-\delta c_n c_r+c_r^2}{2\delta(4-\delta)}$$

Equilibrium Outcomes in State RM

$$w_n^{RM-O} = \frac{1}{2}(1 + c_n) + \frac{\sigma^2\Gamma}{4(\sigma^2 + \sigma_1^2)}$$

$$w_r^{RM-O} = \frac{1}{2}(\delta + c_r) + \frac{\sigma^2\Gamma}{2(\sigma^2 + \sigma_1^2)}$$

$$q_n^{RM-O} = \frac{2 - \delta - 2c_n + c_r}{2(4 - \delta)}$$

$$q_r^{RM-O} = \frac{\delta + \delta c_n - 2c_r}{2\delta(4 - \delta)} + \frac{\sigma^2\Gamma}{4\delta(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_n^{RM-O} = \frac{(2 - \delta - 2c_n + c_r)^2}{4(4 - \delta)^2}$$

$$\widehat{\Pi}_r^{RM-O} = \frac{(\delta + \delta c_n - 2c_r)^2}{4\delta(4 - \delta)^2} + \frac{\sigma^4}{16\delta(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_m^{RM-O} = \frac{\delta - 2\delta c_n + \delta^2 c_n + \delta c_n^2 - \delta c_r - \delta c_n c_r + c_r^2}{2\delta(4 - \delta)} + \frac{\sigma^4}{8\delta(\sigma^2 + \sigma_1^2)}$$

Equilibrium Outcomes in State RN

$$w_n^{RN-O} = \frac{1}{2}(1 + c_n)$$

$$w_r^{RN-O} = \frac{1}{2}(\delta + c_r)$$

$$q_n^{RN-O} = \frac{2 - \delta - 2c_n + c_r}{2(4 - \delta)} - \frac{\sigma^2\Gamma}{(4 - \delta)(\sigma^2 + \sigma_1^2)}$$

$$q_r^{RN-O} = \frac{\delta + \delta c_n - 2c_r}{2\delta(4 - \delta)} + \frac{2\sigma^2\Gamma}{\delta(4 - \delta)(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_n^{RN-O} = \frac{(2 - \delta - 2c_n + c_r)^2}{4(4 - \delta)^2} + \frac{\sigma^4}{(4 - \delta)^2(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_r^{RN-O} = \frac{(\delta + \delta c_n - 2c_r)^2}{4\delta(4 - \delta)^2} + \frac{4\sigma^4}{\delta(4 - \delta)^2(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_m^{RN-O} = \frac{\delta - 2\delta c_n + \delta^2 c_n + \delta c_n^2 - \delta c_r - \delta c_n c_r + c_r^2}{2\delta(4 - \delta)}$$

Equilibrium Outcomes in State RNM

$$w_n^{RNM-O} = \frac{1}{2}(1 + c_n)$$

$$w_r^{RNM-O} = \frac{1}{2}(\delta + c_r) + \frac{\sigma^2 \Gamma}{2(\sigma^2 + \sigma_1^2)}$$

$$q_n^{RNM-O} = \frac{2 - \delta - 2c_n + c_r}{2(4-\delta)} - \frac{\sigma^2 \Gamma}{2(4-\delta)(\sigma^2 + \sigma_1^2)}$$

$$q_r^{RNM-O} = \frac{\delta + \delta c_n - 2c_r}{2\delta(4-\delta)} + \frac{\sigma^2 \Gamma}{\delta(4-\delta)(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_n^{RNM-O} = \frac{(2 - \delta - 2c_n + c_r)^2}{4(4-\delta)^2} + \frac{\sigma^4}{4(4-\delta)^2(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_r^{RNM-O} = \frac{(\delta + \delta c_n - 2c_r)^2}{4\delta(4-\delta)^2} + \frac{\sigma^4}{\delta(4-\delta)^2(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_m^{RNM-O} = \frac{\delta - 2\delta c_n + \delta^2 c_n + \delta c_n^2 - \delta c_r - \delta c_n c_r + c_r^2}{2\delta(4-\delta)} + \frac{\sigma^4}{2\delta(4-\delta)(\sigma^2 + \sigma_1^2)}$$

Then, we conduct the comparison and have the results as follows:

Comparison: State R VS. State RM

$$\widehat{\Pi}_n^{R-O} - \widehat{\Pi}_n^{RM-O} = 0$$

$$\widehat{\Pi}_r^{R-O} - \widehat{\Pi}_r^{RM-O} = \frac{3\sigma^4}{16\delta(\sigma^2 + \sigma_1^2)} > 0$$

$$\widehat{\Pi}_m^{R-O} - \widehat{\Pi}_m^{RM-O} = -\frac{\sigma^4}{8\delta(\sigma^2 + \sigma_1^2)} < 0$$

$$\widehat{\Pi}_n^{R-O} + \widehat{\Pi}_r^{R-O} + \widehat{\Pi}_m^{R-O} - \left(\widehat{\Pi}_n^{RM-O} + \widehat{\Pi}_r^{RM-O} + \widehat{\Pi}_m^{RM-O}\right) = \frac{\sigma^4}{16\delta(\sigma^2 + \sigma_1^2)} > 0$$

Comparison: State R VS. State RN

$$\widehat{\Pi}_n^{R-O} - \widehat{\Pi}_n^{RN-O} = -\frac{\sigma^4}{(4-\delta)^2(\sigma^2 + \sigma_1^2)} < 0$$

$$\widehat{\Pi}_r^{R-O} - \widehat{\Pi}_r^{RN-O} = -\frac{(8-\delta)\sigma^4}{4(4-\delta)^2(\sigma^2 + \sigma_1^2)} < 0$$

$$\widehat{\Pi}_m^{R-O} - \widehat{\Pi}_m^{RN-O} = 0$$

$$\widehat{\Pi}_n^{R-O} + \widehat{\Pi}_r^{R-O} + \widehat{\Pi}_m^{R-O} - \left(\widehat{\Pi}_n^{RN-O} + \widehat{\Pi}_r^{RN-O} + \widehat{\Pi}_m^{RN-O}\right) = -\frac{(12-\delta)\sigma^4}{4(4-\delta)^2(\sigma^2 + \sigma_1^2)} < 0$$

Comparison: State R VS. State RNM

$$\widehat{\Pi}_n^{R-O} - \widehat{\Pi}_n^{RNM-O} = -\frac{\sigma^4}{4(4-\delta)^2(\sigma^2 + \sigma_1^2)} < 0$$

$$\widehat{\Pi}_r^{R-O} - \widehat{\Pi}_r^{RNM-O} = \frac{(12-8\delta+\delta^2)\sigma^4}{4\delta(4-\delta)^2(\sigma^2+\sigma_1^2)} > 0$$

$$\widehat{\Pi}_m^{R-O} - \widehat{\Pi}_m^{RNM-O} = -\frac{\sigma^4}{2\delta(4-\delta)(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_n^{R-O} + \widehat{\Pi}_r^{R-O} + \widehat{\Pi}_m^{R-O} - \left(\widehat{\Pi}_n^{RNM-O} + \widehat{\Pi}_r^{RNM-O} + \widehat{\Pi}_m^{RNM-O}\right)$$

$$= \frac{(4-7\delta+\delta^2)\sigma^4}{4\delta(4-\delta)^2(\sigma^2+\sigma_1^2)}\begin{cases} \geq 0, if\ \delta \in \left[0, \dfrac{7-\sqrt{33}}{2}\right] \\ < 0, if\ \delta \in \left(\dfrac{7-\sqrt{33}}{2}, 1\right) \end{cases}$$

Comparison: State RM VS. State RN

$$\widehat{\Pi}_n^{RM-O} - \widehat{\Pi}_n^{RN-O} = -\frac{\sigma^4}{(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_r^{RM-O} - \widehat{\Pi}_r^{RN-O} = -\frac{(48+8\delta-\delta^2)\sigma^4}{16\delta(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_m^{RM-O} - \widehat{\Pi}_m^{RN-O} = \frac{\sigma^4}{8\delta(\sigma^2+\sigma_1^2)} > 0$$

$$\widehat{\Pi}_n^{RM-O} + \widehat{\Pi}_r^{RM-O} + \widehat{\Pi}_m^{RM-O} - \left(\widehat{\Pi}_n^{RN-O} + \widehat{\Pi}_r^{RN-O} + \widehat{\Pi}_m^{RN-O}\right) = -\frac{(16+40\delta-3\delta^2)\sigma^4}{16\delta(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

Comparison: State RM VS. State RNM

$$\widehat{\Pi}_n^{RM-O} - \widehat{\Pi}_n^{RNM-O} = -\frac{\sigma^4}{4(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_r^{RM-O} - \widehat{\Pi}_r^{RNM-O} = -\frac{(8-\delta)\sigma^4}{16(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_m^{RM-O} - \widehat{\Pi}_m^{RNM-O} = -\frac{\sigma^4}{8(4-\delta)(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_n^{RM-O} + \widehat{\Pi}_r^{RM-O} + \widehat{\Pi}_m^{RM-O} - \left(\widehat{\Pi}_n^{RNM-O} + \widehat{\Pi}_r^{RNM-O} + \widehat{\Pi}_m^{RNM-O}\right) = -\frac{(20-3\delta)\sigma^4}{16(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

Comparison: State RN VS. State RNM

$$\widehat{\Pi}_n^{RN-O} - \widehat{\Pi}_n^{RNM-O} = \frac{3\sigma^4}{4(4-\delta)^2(\sigma^2+\sigma_1^2)} > 0$$

$$\widehat{\Pi}_r^{RN-O} - \widehat{\Pi}_r^{RNM-O} = \frac{3\sigma^4}{\delta(4-\delta)^2(\sigma^2+\sigma_1^2)} > 0$$

$$\widehat{\Pi}_m^{RN-O} - \widehat{\Pi}_m^{RNM-O} = -\frac{\sigma^4}{2\delta(4-\delta)(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_n^{RN-O} + \widehat{\Pi}_r^{RN-O} + \widehat{\Pi}_m^{RN-O} - \left(\widehat{\Pi}_n^{RNM-O} + \widehat{\Pi}_r^{RNM-O} + \widehat{\Pi}_m^{RNM-O}\right) = \frac{(4+5\delta)\sigma^4}{4\delta(4-\delta)^2(\sigma^2+\sigma_1^2)} > 0$$

Based on the foregoing equilibria and the comparison results, we have

$$\widehat{\Pi}_n^{RN-O} > \widehat{\Pi}_n^{RNM-O} > \widehat{\Pi}_n^{R-O} = \widehat{\Pi}_n^{RM-O}$$
$$\widehat{\Pi}_r^{RN-O} > \widehat{\Pi}_r^{R-O} > \widehat{\Pi}_r^{RNM-O} > \widehat{\Pi}_r^{RM-O}$$

$$\widehat{\Pi}_m^{RNM-O} > \widehat{\Pi}_m^{RM-O} > \widehat{\Pi}_m^{R-O} = \widehat{\Pi}_m^{RN-O}$$

$$\widehat{\Pi}_n^{RN-O} + \widehat{\Pi}_r^{RN-O} + \widehat{\Pi}_m^{RN-O} > \widehat{\Pi}_n^{RNM-O} + \widehat{\Pi}_r^{RNM-O} + \widehat{\Pi}_m^{RNM-O}(\widehat{\Pi}_n^{R-O} + \widehat{\Pi}_r^{R-O} + \widehat{\Pi}_m^{R-O}) > \widehat{\Pi}_n^{RM-O} + \widehat{\Pi}_r^{RM-O} + \widehat{\Pi}_m^{RM-O}$$

It is clearly, the equilibrium state is state RN, i.e., the retailer r will transmit demand signal to the retailer n. neither retailer r or the retailer n voluntarily transmits the signal to manufacturer.

PROOF FOR PROPOSITION 5

Upstream Subsidy-Equilibrium Analysis

Similar to that for Proposition 1(in Section 3.2), we derive the outcomes in state R, RM, RN, and RNM as follows:

Equilibrium Outcomes in State R

$$w_n^{R-U} = \frac{1}{2}$$

$$w_r^{R-U} = \frac{1}{2}(\delta + \eta_u)$$

$$q_n^{R-U} = \frac{2 - \delta - \eta_u}{2(4 - \delta)}$$

$$q_r^{R-U} = \frac{\delta + 2\eta_u}{2\delta(4 - \delta)} + \frac{\sigma^2 \Gamma}{2\delta(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_r^{R-U} = \frac{(\delta + 2\eta_u)^2}{4\delta(4 - \delta)^2} + \frac{\sigma^4}{4\delta(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_m^{R-U} = \frac{\delta + \delta\eta_u + \eta_u^2}{2\delta(4 - \delta)}$$

Equilibrium Outcomes in State RM

$$w_n^{RM-U} = \frac{1}{2} + \frac{\sigma^2 \Gamma}{4(\sigma^2 + \sigma_1^2)}$$

$$w_r^{RM-U} = \frac{1}{2}(\delta + \eta_u) + \frac{\sigma^2 \Gamma}{2(\sigma^2 + \sigma_1^2)}$$

$$q_n^{RM-U} = \frac{2 - \delta - \eta_u}{2(4 - \delta)}$$

$$q_r^{MU} = \frac{\delta + 2\eta_u}{2\delta(4 - \delta)} + \frac{\sigma^2 \Gamma}{4\delta(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_n^{RM-U} = \frac{(2 - \delta - \eta_u)^2}{4(4 - \delta)^2}$$

$$\widehat{\Pi}_r^{RM-U} = \frac{(\delta + 2\eta_u)^2}{4\delta(4 - \delta)^2} + \frac{\sigma^4}{16\delta(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_m^{RM-U} = \frac{\delta + \eta_u\delta + \eta_u^2}{2\delta(4 - \delta)} + \frac{\sigma^4}{8\delta(\sigma^2 + \sigma_1^2)}$$

Equilibrium Outcomes in State RN

$$w_n^{RN-U} = \frac{1}{2}$$

$$w_r^{RN-U} = \frac{1}{2}(\delta + \eta_u)$$

$$q_n^{RN-U} = \frac{2-\delta-\eta_u}{2(4-\delta)} - \frac{\sigma^2\Gamma}{(4-\delta)(\sigma^2+\sigma_1^2)}$$

$$q_r^{RN-U} = \frac{\delta+2\eta_u}{2\delta(4-\delta)} + \frac{2\sigma^2\Gamma}{\delta(4-\delta)(\sigma^2+\sigma_1^2)}$$

$$\widehat{\Pi}_n^{RN-U} = \frac{(2-\delta-\eta_u)^2}{4(4-\delta)^2} + \frac{\sigma^4}{(4-\delta)^2(\sigma^2+\sigma_1^2)}$$

$$\widehat{\Pi}_r^{RN-U} = \frac{(\delta+2\eta_u)^2}{4\delta(4-\delta)^2} + \frac{4\sigma^4}{\delta(4-\delta)^2(\sigma^2+\sigma_1^2)}$$

$$\widehat{\Pi}_m^{RN-U} = \frac{\delta+\delta\eta_u+\eta_u^2}{2\delta(4-\delta)}$$

Equilibrium Outcomes in State RNM

$$w_n^{RNM-U} = \frac{1}{2}$$

$$w_r^{RNM-U} = \frac{1}{2}(\delta + \eta_u) + \frac{\sigma^2\Gamma}{2(\sigma^2+\sigma_1^2)}$$

$$q_n^{RNM-U} = \frac{2-\delta-\eta_u}{2(4-\delta)} - \frac{\sigma^2\Gamma}{2(4-\delta)(\sigma^2+\sigma_1^2)}$$

$$q_r^{RNM-U} = \frac{\delta+2\eta_u}{2\delta(4-\delta)} + \frac{\sigma^2\Gamma}{\delta(4-\delta)(\sigma^2+\sigma_1^2)}$$

$$\widehat{\Pi}_n^{RNM-U} = \frac{(2-\delta-\eta_u)^2}{4(4-\delta)^2} + \frac{\sigma^4}{4(4-\delta)^2(\sigma^2+\sigma_1^2)}$$

$$\widehat{\Pi}_r^{RNM-U} = \frac{(\delta+2\eta_u)^2}{4\delta(4-\delta)^2} + \frac{\sigma^4}{\delta(4-\delta)^2(\sigma^2+\sigma_1^2)}$$

We then conduct the comparison and derive the results as follows:

Comparison: State R VS. State RM

$$\widehat{\Pi}_n^{R-U} - \widehat{\Pi}_n^{RM-U} = 0$$

$$\widehat{\Pi}_r^{R-U} - \widehat{\Pi}_r^{RM-U} = \frac{3\sigma^4}{16\delta(\sigma^2+\sigma_1^2)} > 0$$

$$\widehat{\Pi}_m^{R-U} - \widehat{\Pi}_m^{RM-U} = -\frac{\sigma^4}{8\delta(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_n^{R-U} + \widehat{\Pi}_r^{R-U} + \widehat{\Pi}_m^{R-U} - \left(\widehat{\Pi}_n^{RM-U} + \widehat{\Pi}_r^{RM-U} + \widehat{\Pi}_m^{RM-U}\right) = \frac{\sigma^4}{16\delta(\sigma^2 + \sigma_1^2)} > 0$$

Comparison: State R VS. State RN

$$\widehat{\Pi}_n^{R-U} - \widehat{\Pi}_n^{RN-U} = -\frac{\sigma^4}{(4-\delta)^2(\sigma^2 + \sigma_1^2)} < 0$$

$$\widehat{\Pi}_r^{R-U} - \widehat{\Pi}_r^{RN-U} = -\frac{(8-\delta)\sigma^4}{4(4-\delta)^2(\sigma^2 + \sigma_1^2)} < 0$$

$$\widehat{\Pi}_m^{R-U} - \widehat{\Pi}_m^{RN-U} = 0$$

$$\widehat{\Pi}_n^{R-U} + \widehat{\Pi}_r^{R-U} + \widehat{\Pi}_m^{R-U} - \left(\widehat{\Pi}_n^{RN-U} + \widehat{\Pi}_r^{RN-U} + \widehat{\Pi}_m^{RN-U}\right) = -\frac{(12-\delta)\sigma^4}{4(4-\delta)^2(\sigma^2 + \sigma_1^2)} < 0$$

Comparison: State R VS. State RNM

$$\widehat{\Pi}_n^{R-U} - \widehat{\Pi}_n^{RNM-U} = -\frac{\sigma^4}{4(4-\delta)^2(\sigma^2 + \sigma_1^2)} < 0$$

$$\widehat{\Pi}_r^{R-U} - \widehat{\Pi}_r^{RNM-U} = \frac{(12 - 8\delta + \delta^2)\sigma^4}{4\delta(4-\delta)^2(\sigma^2 + \sigma_1^2)} > 0$$

$$\widehat{\Pi}_m^{R-U} - \widehat{\Pi}_m^{RNM-U} = -\frac{\sigma^4}{2\delta(4-\delta)(\sigma^2 + \sigma_1^2)} < 0$$

$$\widehat{\Pi}_n^{R-U} + \widehat{\Pi}_r^{R-U} + \widehat{\Pi}_m^{R-U} - \left(\widehat{\Pi}_n^{RNM-U} + \widehat{\Pi}_r^{RNM-U} + \widehat{\Pi}_m^{RNM-U}\right)$$

$$= \frac{(4 - 7\delta + \delta^2)\sigma^4}{4\delta(4-\delta)^2(\sigma^2 + \sigma_1^2)} \begin{cases} \geq 0, if\ \delta \in \left[0, \dfrac{7-\sqrt{33}}{2}\right] \\ < 0, if\ \delta \in \left(\dfrac{7-\sqrt{33}}{2}, 1\right) \end{cases}$$

Comparison: State RM VS. State RN

$$\widehat{\Pi}_n^{RM-U} - \widehat{\Pi}_n^{RN-U} = -\frac{\sigma^4}{(4-\delta)^2(\sigma^2 + \sigma_1^2)} < 0$$

$$\widehat{\Pi}_r^{RM-U} - \widehat{\Pi}_r^{RN-U} = -\frac{(48 + 8\delta - \delta^2)\sigma^4}{16\delta(4-\delta)^2(\sigma^2 + \sigma_1^2)} < 0$$

$$\widehat{\Pi}_m^{RM-U} - \widehat{\Pi}_m^{RN-U} = \frac{\sigma^4}{8\delta(\sigma^2 + \sigma_1^2)} > 0$$

$$\widehat{\Pi}_n^{RM-U} + \widehat{\Pi}_r^{RM-U} + \widehat{\Pi}_m^{RM-U} - \left(\widehat{\Pi}_n^{RN-U} + \widehat{\Pi}_r^{RN-U} + \widehat{\Pi}_m^{RN-U}\right) = -\frac{(16 + 40\delta - 3\delta^2)\sigma^4}{16\delta(4-\delta)^2(\sigma^2 + \sigma_1^2)} < 0$$

Comparison: State RM VS. State RNM

$$\widehat{\Pi}_n^{RM-U} - \widehat{\Pi}_n^{RNM-U} = -\frac{\sigma^4}{4(4-\delta)^2(\sigma^2 + \sigma_1^2)} < 0$$

$$\widehat{\Pi}_r^{RM-U} - \widehat{\Pi}_r^{RNM-U} = -\frac{(8-\delta)\sigma^4}{16(4-\delta)^2(\sigma^2 + \sigma_1^2)} < 0$$

$$\widehat{\Pi}_m^{RM-U} - \widehat{\Pi}_m^{RNM-U} = -\frac{\sigma^4}{8(4-\delta)(\sigma^2 + \sigma_1^2)} < 0$$

$$\widehat{\Pi}_n^{RM-U} + \widehat{\Pi}_r^{RM-U} + \widehat{\Pi}_m^{RM-U} - \left(\widehat{\Pi}_n^{RNM-U} + \widehat{\Pi}_r^{RNM-U} + \widehat{\Pi}_m^{RNM-U}\right) = -\frac{(20-3\delta)\sigma^4}{16(4-\delta)^2(\sigma^2 + \sigma_1^2)} < 0$$

Comparison: State RM VS. State RNM

$$\hat{\Pi}_n^{RN-U} - \hat{\Pi}_n^{RNM-U} = \frac{3\sigma^4}{4(4-\delta)^2(\sigma^2+\sigma_1^2)} > 0$$

$$\hat{\Pi}_r^{RN-U} - \hat{\Pi}_r^{RNM-U} = \frac{3\sigma^4}{\delta(4-\delta)^2(\sigma^2+\sigma_1^2)} > 0$$

$$\hat{\Pi}_m^{RN-U} - \hat{\Pi}_m^{RNM-U} = -\frac{\sigma^4}{2\delta(4-\delta)(\sigma^2+\sigma_1^2)} < 0$$

$$\hat{\Pi}_n^{RN-U} + \hat{\Pi}_r^{RN-U} + \hat{\Pi}_m^{RN-U} - \left(\hat{\Pi}_n^{RNM-U} + \hat{\Pi}_r^{RNM-U} + \hat{\Pi}_m^{RNM-U}\right) = \frac{(4+5\delta)\sigma^4}{4\delta(4-\delta)^2(\sigma^2+\sigma_1^2)} > 0$$

Based on the foregoing equilibria and the comparison results, we have:

$$\hat{\Pi}_n^{RN-U} > \hat{\Pi}_n^{RNM-U} > \hat{\Pi}_n^{R-U} = \hat{\Pi}_n^{RM-U}$$
$$\hat{\Pi}_r^{RN-U} > \hat{\Pi}_r^{R-U} > \hat{\Pi}_r^{RNM-U} > \hat{\Pi}_r^{RM-U}$$
$$\hat{\Pi}_m^{RNM-U} > \hat{\Pi}_m^{RM-U} > \hat{\Pi}_m^{R-U} = \hat{\Pi}_m^{RN-U}$$
$$\hat{\Pi}_n^{RN-U} + \hat{\Pi}_r^{RN-U} + \hat{\Pi}_m^{RN-U} > \hat{\Pi}_n^{RNM-U} + \hat{\Pi}_r^{RNM-U} + \hat{\Pi}_m^{RNM-U}(\hat{\Pi}_n^{R-U} + \hat{\Pi}_r^{R-U} + \hat{\Pi}_m^{R-U})$$
$$> \hat{\Pi}_n^{RM-U} + \hat{\Pi}_r^{RM-U} + \hat{\Pi}_m^{RM-U}$$

We find that, when the government subsidy is given to the retailer, the five rules in Proposition 1 still hold.

Downstream Subsidy -Equilibrium Analysis

We derive the outcomes in state R, RM, RN, and RNM as follows:

Equilibrium Outcomes in State R

$$w_n^{R-D} = \frac{1}{2}$$

$$w_r^{R-D} = \frac{1}{2}(\delta - \eta_d)$$

$$q_n^{R-D} = \frac{2-\delta-\eta_d}{2(4-\delta)}$$

$$q_r^{R-D} = \frac{\delta+2\eta_d}{2\delta(4-\delta)} + \frac{\sigma^2\Gamma}{2\delta(\sigma^2+\sigma_1^2)}$$

$$\hat{\Pi}_n^{R-D} = \frac{(2-\delta-\eta_d)^2}{4(4-\delta)^2}$$

$$\hat{\Pi}_r^{R-D} = \frac{(\delta+2\eta_d)^2}{4\delta(4-\delta)^2} + \frac{\sigma^4}{4\delta(\sigma^2+\sigma_1^2)}$$

$$\hat{\Pi}_m^{R-D} = \frac{\delta+\delta\eta_d+\eta_d^2}{2\delta(4-\delta)}$$

Equilibrium Outcomes in State RM

$$w_n^{RM-D} = \frac{1}{2} + \frac{\sigma^2\Gamma}{4(\sigma^2+\sigma_1^2)}$$

$$w_r^{RM-D} = \frac{1}{2}(\delta - \eta_d) + \frac{\sigma^2\Gamma}{2(\sigma^2+\sigma_1^2)}$$

$$q_n^{RM-D} = \frac{2-\delta-\eta_d}{2(4-\delta)}$$

$$q_r^{RM-D} = \frac{\delta + 2\eta_d}{2\delta(4-\delta)} + \frac{\sigma^2 \Gamma}{4\delta(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_n^{RM-D} = \frac{(2-\delta-\eta_d)^2}{4(4-\delta)^2}$$

$$\widehat{\Pi}_r^{RM-D} = \frac{(\delta + 2\eta_d)^2}{4\delta(4-\delta)^2} + \frac{\sigma^4}{16\delta(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_m^{RM-D} = \frac{\delta + \eta_d\delta + \eta_d^2}{2\delta(4-\delta)} + \frac{\sigma^4}{8\delta(\sigma^2 + \sigma_1^2)}$$

Equilibrium Outcomes in State RN

$$w_n^{RN-D} = \frac{1}{2}$$

$$w_r^{RN-D} = \frac{1}{2}(\delta - \eta_d)$$

$$q_n^{RN-D} = \frac{2-\delta-\eta_d}{2(4-\delta)} - \frac{\sigma^2 \Gamma}{(4-\delta)(\sigma^2 + \sigma_1^2)}$$

$$q_r^{RN-D} = \frac{\delta + 2\eta_d}{2\delta(4-\delta)} + \frac{2\sigma^2 \Gamma}{\delta(4-\delta)(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_n^{RN-D} = \frac{(2-\delta-\eta_d)^2}{4(4-\delta)^2} + \frac{\sigma^4}{(4-\delta)^2(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_r^{RN-D} = \frac{(\delta + 2\eta_d)^2}{4\delta(4-\delta)^2} + \frac{4\sigma^4}{\delta(4-\delta)^2(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_m^{RN-D} = \frac{\delta + \delta\eta_d + \eta_d^2}{2\delta(4-\delta)}$$

Equilibrium Outcomes in State RNM

$$w_n^{RNM-D} = \frac{1}{2}$$

$$w_r^{RNM-D} = \frac{1}{2}(\delta - \eta_d) + \frac{\sigma^2 \Gamma}{2(\sigma^2 + \sigma_1^2)}$$

$$q_n^{RNM-D} = \frac{2-\delta-\eta_d}{2(4-\delta)} - \frac{\sigma^2 \Gamma}{2(4-\delta)(\sigma^2 + \sigma_1^2)}$$

$$q_r^{RNM-D} = \frac{\delta + 2\eta_d}{2\delta(4-\delta)} + \frac{\sigma^2 \Gamma}{\delta(4-\delta)(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_n^{RNM-D} = \frac{(2-\delta-\eta_d)^2}{4(4-\delta)^2} + \frac{\sigma^4}{4(4-\delta)^2(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_r^{RNM-D} = \frac{(\delta + 2\eta_d)^2}{4\delta(4-\delta)^2} + \frac{\sigma^4}{\delta(4-\delta)^2(\sigma^2 + \sigma_1^2)}$$

$$\widehat{\Pi}_m^{RNM-D} = \frac{\delta + \delta\eta_d + \eta_d^2}{2\delta(4-\delta)} + \frac{\sigma^4}{2\delta(4-\delta)(\sigma^2 + \sigma_1^2)}$$

We then conduct the comparison and derive the results as follows:

Comparison: State R VS. State RM

$$\widehat{\Pi}_n^{R-D} - \widehat{\Pi}_n^{RM-D} = 0$$

$$\widehat{\Pi}_r^{R-D} - \widehat{\Pi}_r^{RM-D} = \frac{3\sigma^4}{16\delta(\sigma^2+\sigma_1^2)} > 0$$

$$\widehat{\Pi}_m^{R-D} - \widehat{\Pi}_m^{RM-D} = -\frac{\sigma^4}{8\delta(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_n^{R-D} + \widehat{\Pi}_r^{R-D} + \widehat{\Pi}_m^{R-D} - \left(\widehat{\Pi}_n^{RM-D} + \widehat{\Pi}_r^{RM-D} + \widehat{\Pi}_m^{RM-D}\right) = \frac{\sigma^4}{16\delta(\sigma^2+\sigma_1^2)} > 0$$

Comparison: State R VS. State RN

$$\widehat{\Pi}_n^{R-D} - \widehat{\Pi}_n^{RN-D} = -\frac{\sigma^4}{(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_r^{R-D} - \widehat{\Pi}_r^{RN-D} = -\frac{(8-\delta)\sigma^4}{4(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_m^{R-D} - \widehat{\Pi}_m^{RN-D} = 0$$

$$\widehat{\Pi}_n^{R-D} + \widehat{\Pi}_r^{R-D} + \widehat{\Pi}_m^{R-D} - \left(\widehat{\Pi}_n^{RN-D} + \widehat{\Pi}_r^{RN-D} + \widehat{\Pi}_m^{RN-D}\right) = -\frac{(12-\delta)\sigma^4}{4(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

Comparison: State R VS. State RNM

$$\widehat{\Pi}_n^{R-D} - \widehat{\Pi}_n^{RNM-D} = -\frac{\sigma^4}{4(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_r^{R-D} - \widehat{\Pi}_r^{RNM-D} = \frac{(12-8\delta+\delta^2)\sigma^4}{4\delta(4-\delta)^2(\sigma^2+\sigma_1^2)} > 0$$

$$\widehat{\Pi}_m^{R-D} - \widehat{\Pi}_m^{RNM-D} = -\frac{\sigma^4}{2\delta(4-\delta)(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_n^{R-D} + \widehat{\Pi}_r^{R-D} + \widehat{\Pi}_m^{R-D} - \left(\widehat{\Pi}_n^{RNM-D} + \widehat{\Pi}_r^{RNM-D} + \widehat{\Pi}_m^{RNM-D}\right)$$
$$= \frac{(4-7\delta+\delta^2)\sigma^4}{4\delta(4-\delta)^2(\sigma^2+\sigma_1^2)} \begin{cases} \geq 0, if\ \delta \in \left[0, \frac{7-\sqrt{33}}{2}\right] \\ < 0, if\ \delta \in \left(\frac{7-\sqrt{33}}{2}, 1\right) \end{cases}$$

Comparison: State RM VS. State RN

$$\widehat{\Pi}_n^{RM-D} - \widehat{\Pi}_n^{RN-D} = -\frac{\sigma^4}{(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_r^{RM-D} - \widehat{\Pi}_r^{RN-D} = -\frac{(48+8\delta-\delta^2)\sigma^4}{16\delta(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_m^{RM-D} - \widehat{\Pi}_m^{RN-D} = \frac{\sigma^4}{8\delta(\sigma^2+\sigma_1^2)} > 0$$

$$\widehat{\Pi}_n^{RM-D} + \widehat{\Pi}_r^{RM-D} + \widehat{\Pi}_m^{RM-D} - \left(\widehat{\Pi}_n^{RN-D} + \widehat{\Pi}_r^{RN-D} + \widehat{\Pi}_m^{RN-D}\right) = -\frac{(16+40\delta-3\delta^2)\sigma^4}{16\delta(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

Comparison: State RM VS. State RNM

$$\widehat{\Pi}_n^{RM-D} - \widehat{\Pi}_n^{RNM-D} = -\frac{\sigma^4}{4(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_r^{RM-D} - \widehat{\Pi}_r^{RNM-D} = -\frac{(8-\delta)\sigma^4}{16(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

$$\widehat{\Pi}_m^{RM-D} - \widehat{\Pi}_m^{RNM-D} = -\frac{\sigma^4}{8(4-\delta)(\sigma^2+\sigma_1^2)} < 0$$
$$\widehat{\Pi}_n^{RM-D} + \widehat{\Pi}_r^{RM-D} + \widehat{\Pi}_m^{RM-D} - \left(\widehat{\Pi}_n^{RNM-D} + \widehat{\Pi}_r^{RNM-D} + \widehat{\Pi}_m^{RNM-D}\right) = -\frac{(20-3\delta)\sigma^4}{16(4-\delta)^2(\sigma^2+\sigma_1^2)} < 0$$

Comparison between State RN vs. State RNM

$$\widehat{\Pi}_n^{RN-D} - \widehat{\Pi}_n^{RNM-D} = \frac{3\sigma^4}{4(4-\delta)^2(\sigma^2+\sigma_1^2)} > 0$$
$$\widehat{\Pi}_r^{RN-D} - \widehat{\Pi}_r^{RNM-D} = \frac{3\sigma^4}{\delta(4-\delta)^2(\sigma^2+\sigma_1^2)} > 0$$
$$\widehat{\Pi}_m^{RN-D} - \widehat{\Pi}_m^{RNM-D} = -\frac{\sigma^4}{2\delta(4-\delta)(\sigma^2+\sigma_1^2)} < 0$$
$$\widehat{\Pi}_n^{RN-D} + \widehat{\Pi}_r^{RN-D} + \widehat{\Pi}_m^{RN-D} - \left(\widehat{\Pi}_n^{RNM-D} + \widehat{\Pi}_r^{RNM-D} + \widehat{\Pi}_m^{RNM-D}\right) = \frac{(4+5\delta)\sigma^4}{4\delta(4-\delta)^2(\sigma^2+\sigma_1^2)} > 0$$

Based on the foregoing equilibria and the comparison results, we have:

$$\widehat{\Pi}_n^{RN-D} > \widehat{\Pi}_n^{RNM-D} > \widehat{\Pi}_n^{R-D} = \widehat{\Pi}_n^{RM-D}$$
$$\widehat{\Pi}_r^{RN-D} > \widehat{\Pi}_r^{R-D} > \widehat{\Pi}_r^{RNM-D} > \widehat{\Pi}_r^{RM-D}$$
$$\widehat{\Pi}_m^{RNM-D} > \widehat{\Pi}_m^{RM-D} > \widehat{\Pi}_m^{R-D} = \widehat{\Pi}_m^{RN-D}$$
$$\widehat{\Pi}_n^{RN-D} + \widehat{\Pi}_r^{RN-D} + \widehat{\Pi}_m^{RN-D} > \widehat{\Pi}_n^{RNM-D} + \widehat{\Pi}_r^{RNM-D} + \widehat{\Pi}_m^{RNM-D}(\widehat{\Pi}_n^{R-D} + \widehat{\Pi}_r^{R-D} + \widehat{\Pi}_m^{R-D})$$
$$> \widehat{\Pi}_n^{RM-D} + \widehat{\Pi}_r^{RM-D} + \widehat{\Pi}_m^{RM-D}$$

Thus, similar to upstream subsidy model, we find the three rules in Proposition 1 still hold.

PROOF FOR PROPOSITION 6

We take the derivate of the equilibrium outcomes with respect to η_u and η_d respectively, and derive:

In upstream subsidy model ($\mathrm{i} \in \{\mathrm{R}, \mathrm{RM}, \mathrm{RN}, \mathrm{RNM}\}$)

$$\frac{dw_n^{i-U}}{d\eta_u} = 0$$
$$\frac{dw_r^{i-U}}{d\eta_u} = \frac{1}{2} > 0$$
$$\frac{dq_n^{i-U}}{d\eta_u} = \frac{-1}{2(4-\delta)} < 0$$
$$\frac{dq_r^{i-U}}{d\eta_u} = \frac{1}{\delta(4-\delta)} > 0$$
$$\frac{d\widehat{\Pi}_n^{i-U}}{d\eta_u} = -\frac{2-\delta-\eta_u}{2(4-\delta)^2} < 0$$
$$\frac{d\widehat{\Pi}_r^{i-U}}{d\eta_u} = \frac{\delta+2\eta_u}{\delta(4-\delta)^2} > 0$$
$$\frac{d\widehat{\Pi}_m^{i-U}}{d\eta_u} = \frac{\delta+2\eta_u}{2\delta(4-\delta)} > 0$$

In downstream subsidy model ($\mathrm{i} \in \{\mathrm{R}, \mathrm{RM}, \mathrm{RN}, \mathrm{RNM}\}$)

$$\frac{dw_n^{i-D}}{d\eta_d} = 0$$

$$\frac{dw_r^{i-D}}{d\eta_d} = -\frac{1}{2} > 0$$
$$\frac{dq_n^{i-D}}{d\eta_d} = \frac{-1}{2(4-\delta)} < 0$$
$$\frac{dq_r^{i-D}}{d\eta_d} = \frac{1}{\delta(4-\delta)} > 0$$
$$\frac{d\widehat{\Pi}_n^{i-D}}{d\eta_d} = -\frac{2-\delta-\eta_d}{2(4-\delta)^2} < 0$$
$$\frac{d\widehat{\Pi}_r^{i-D}}{d\eta_d} = \frac{\delta+2\eta_d}{\delta(4-\delta)^2} > 0$$
$$\frac{d\widehat{\Pi}_m^{i-D}}{d\eta_d} = \frac{\delta+2\eta_d}{2\delta(4-\delta)} > 0$$
According to the sensitivity analysis, it is easy to obtain the results in Proposition 6.

PROOF FOR PROPOSITION 7

The first-order derivation of $\widehat{\Pi}_n^{RN}$ with respect to δ is as follows:
$$\frac{\partial\widehat{\Pi}_n^{RN}}{\partial\delta} = \frac{2A-2+\delta}{(4-\delta)^3}$$
Obviously, If $A \in \left(0,\frac{1}{2}\right]$, $\frac{\partial\widehat{\Pi}_n^{RN}}{\partial\delta} < 0$; If $A \in \left(\frac{1}{2},1\right)$, $\frac{\partial\widehat{\Pi}_n^{RN}}{\partial\delta} \le 0$when $\delta \in (0,2-2A]$, but $\frac{\partial\widehat{\Pi}_n^{RN}}{\partial\delta} > 0$ when $\delta \in (2-2A,1)$; If $A \in [1,+\infty)$, $\frac{\partial\widehat{\Pi}_n^{RN}}{\partial\delta} > 0$.

Thus, we obtain the result of Proposition 7.

PROOF FOR PROPOSITION 8

The first-order derivation of $\widehat{\Pi}_r^{RN}$ with respect to δ is as follows:
$$\frac{\partial\widehat{\Pi}_r^{RN}}{\partial\delta} = \frac{\delta^3+4\delta^2+48A\delta-64A}{4\delta^2(4-\delta)^3}$$
We define a function $f(\delta)=\delta^3+4\delta^2+48A\delta-64A$. It is clear that, function $f(\delta)$ is increasing in δ, and $f(\delta)\big|_{\delta\to0^+}<0$, $f(\delta)\big|_{\delta\to1^-}\approx 5-16A$. Therefore, we find that, if $A\ge\frac{5}{16}$, then $f(\delta)<0$ for $\delta\in(0,1)$; if $A<\frac{5}{16}$, there is a solution $\hat{\delta}\in(0,1)$ to let $f(\hat{\delta})=0$. According to the monotonically increasing property, we have
$$f(\hat{\delta})=\begin{cases}<0, & 0<\delta<\hat{\delta}\\ =0, & \delta=\hat{\delta}\\ >0, & \hat{\delta}<\delta<1\end{cases}.$$

PROOF FOR PROPOSITION 9

Revenue Sharing Contract-Equilibrium Analysis

Similar to that for Proposition 1(in Section 3.2), we derive the outcomes in state R, RM, RN, and RNM as follows:

Equilibrium Outcomes in State R

$$w_n^{R-R} = \frac{1}{2}$$

$$w_r^{R-R} = \frac{\delta(-4+3r+\delta)}{2(-4+2r+\delta)}$$

$$q_n^{R-R} = \frac{-2+r+\delta}{2(-4+2r+\delta)}$$

$$q_r^{R-R} = \frac{1}{2}\left(\frac{1}{4-2r-\delta} + \frac{\sigma^2\Gamma}{\delta(\sigma^2+\sigma_1^2)}\right)$$

$$\widehat{\Pi}_n^{R-R} = \frac{(-2+r+\delta)^2}{4(-4+2r+\delta)^2}$$

$$\widehat{\Pi}_r^{R-R} = \frac{(-1+r)\left(-\frac{\delta^2}{(-4+2r+\delta)^2} - \frac{\sigma^4}{\sigma^2+\sigma_1^2}\right)}{4\delta}$$

$$\widehat{\Pi}_m^{R-R} = \frac{1}{4}\left(\frac{-2+r}{-4+2r+\delta} - \frac{r\sigma^4}{\delta(\sigma^2+\sigma_1^2)}\right)$$

Equilibrium Outcomes in State RM

$$w_n^{RM-R} = \frac{1}{2} + \frac{\sigma^2\Gamma}{4(\sigma^2+\sigma_1^2)}$$

$$w_r^{RM-R} = \frac{2\delta(-4+3r+\delta)}{4(-4+2r+\delta)} + \frac{\sigma^2\Gamma(-8+8r+2\delta-r\delta)}{4(-4+2r+\delta)(\sigma^2+\sigma_1^2)}$$

$$q_n^{RM-R} = \frac{2(-2+\delta+r)}{4(-4+2r+\delta)} + \frac{r\sigma^2\Gamma}{4(-4+2r+\delta)(\sigma^2+\sigma_1^2)}$$

$$q_r^{RM-R} = \frac{-2\delta}{4\delta(-4+2r+\delta)} + \frac{\Gamma\sigma^2(-4+\delta)}{4\delta(-4+2r+\delta)(\sigma^2+\sigma_1^2)}$$

$$\widehat{\Pi}_n^{RM-R} = \frac{4(-2+r+\delta)^2 + \frac{r(8-3r-2\delta)\sigma^4}{\sigma^2+\sigma_1^2}}{16(-4+2r+\delta)^2}$$

$$\widehat{\Pi}_r^{RM-R} = (1-r)\frac{4\delta^2 + \frac{(-4+\delta)^2\sigma^4}{\sigma^2+\sigma_1^2}}{16\delta(-4+2r+\delta)^2}$$

$$\widehat{\Pi}_m^{RM-R} = \frac{4(-2+r)^2 - 4\delta + \frac{(4r(-4+\delta)+(-4+\delta)^2+r^2\delta)\sigma^4}{\delta(\sigma^2+\sigma_1^2)}}{8(-4+2r+\delta)^2}$$

Equilibrium Outcomes in State RN

$$w_n^{RN-R} = \frac{1}{2}$$

$$w_r^{RN-R}=\frac{\delta(-4+3r+\delta)}{2(-4+2r+\delta)}$$

$$q_n^{RN-R}=\frac{-2+r+\delta}{2(-4+2r+\delta)}+\frac{\sigma^2\Gamma}{(-4+\delta)(\sigma^2+\sigma_1^2)}$$

$$q_r^{RN-R}=\frac{1}{8-4r-2\delta}-\frac{2\sigma^2\Gamma}{(-4+\delta)\delta(\sigma^2+\sigma_1^2)}$$

$$\widehat{\Pi}_n^{RN-R}=\frac{(-2+r+\delta)^2}{4(-4+2r+\delta)^2}+\frac{\sigma^4}{(-4+\delta)^2(\sigma^2+\sigma_1^2)}$$

$$\widehat{\Pi}_r^{RN-R}=\frac{(1-r)\delta}{4(-4+2r+\delta)^2}+\frac{4(1-r)\sigma^4}{(-4+\delta)^2\delta(\sigma^2+\sigma_1^2)}$$

$$\widehat{\Pi}_m^{RN-R}=\frac{-2+r}{4(-4+2r+\delta)}+\frac{4r\sigma^4}{(-4+\delta)^2\delta(\sigma^2+\sigma_1^2)}$$

Equilibrium Outcomes in State RNM

$$w_n^{RNM-R}=\frac{1}{2}$$

$$w_r^{RNM-R}=\frac{\delta(-4+3r+\delta)}{2(-4+2r+\delta)}+\frac{\sigma^2\Gamma(-4+4r+\delta)}{2(-4+2r+\delta)(\sigma^2+\sigma_1^2)}$$

$$q_n^{RNM-R}=\frac{-2+r+\delta}{2(-4+2r+\delta)}+\frac{\sigma^2\Gamma}{2(-4+2r+\delta)(\sigma^2+\sigma_1^2)}$$

$$q_r^{RNM-R}=\frac{\delta}{8\delta-4r\delta-2\delta^2}+\frac{\sigma^2\Gamma}{(4\delta-2r\delta-\delta^2)(\sigma^2+\sigma_1^2)}$$

$$\widehat{\Pi}_n^{RNM-R}=\frac{(-2+r+\delta)^2+\frac{\sigma^4}{\sigma^2+\sigma_1^2}}{4(-4+2r+\delta)^2}$$

$$\widehat{\Pi}_r^{RNM-R}=(1-r)\frac{\delta^2+\frac{4\sigma^4}{\sigma^2+\sigma_1^2}}{4\delta(-4+2r+\delta)^2}$$

$$\widehat{\Pi}_m^{RNM-R}=\frac{-2+r-\frac{2\sigma^4}{\delta(\sigma^2+\sigma_1^2)}}{4(-4+2r+\delta)}$$

We then conduct the comparison and derive the results as follows:

$$\widehat{\Pi}_r^{R-R}-\widehat{\Pi}_r^{RM-R}=\frac{(1-r)(4r+3(-4+\delta))(-4+4r+\delta)\sigma^4}{16\delta(-4+2r+\delta)^2(\sigma^2+\sigma_1^2)}\begin{cases}>0, r<\frac{4-\delta}{4}\\ \leq 0, r\geq\frac{4-\delta}{4}\end{cases}$$

$$\widehat{\Pi}_r^{RN-R}-\widehat{\Pi}_r^{RNM-R}=\frac{(1-r)(4r+3(-4+\delta))(-4+4r+\delta)\sigma^4}{(-4+\delta)^2\delta(-4+2r+\delta)^2(\sigma^2+\sigma_1^2)}\begin{cases}>0, r<\frac{4-\delta}{4}\\ \leq 0, r\geq\frac{4-\delta}{4}\end{cases}$$

PROOF FOR PROPOSITION 10

Based on equilibrium outcomes, we conduct the comparison and derive the results as follows:

$$\widehat{\Pi}_r^R - \widehat{\Pi}_r^{RM} = \frac{(-6+\delta)(-2+\delta)\sigma^4}{4(-4+\delta)^2\delta(\sigma^2+\sigma_1^2)} > 0$$

According to the comparison result, it is easy to obtain the results in Proposition 10.