Optimal Pricing Decisions of Competing Air-Cargo-Carrier Systems – Impacts of Risk Aversion, Demand and Cost Uncertainties[1]

Xin Wen[2], Xiaoyan Xu[3], Tsan-Ming Choi[4], Sai-Ho Chung[2]

Prepared for ***IEEE Transactions on Systems, Man, and Cybernetics: Systems***

Abstract: Air cargo transportation has become increasingly important for global logistics systems nowadays. However, due to the intensive market competition and diverse uncertainties arising from demand and operating costs, the pricing decisions for air cargo carriers are extremely challenging but under-explored. Besides, many airlines are holding risk-averse attitudes in decision making to survive in the highly volatile and competitive market. Therefore, in this paper, we apply the mean-variance theory to characterize the risk-averse behaviors of decision makers, and analytically derive the equilibrium prices for two competing risk-averse air cargo carriers under demand and cost uncertainties. We then uncover how the crucial factors like risk sensitivity coefficients, market competition, market share, demand uncertainty and cost uncertainty affect the carriers' optimal prices. Besides, important cost thresholds and relative risk-averse attitude thresholds are identified for the impacts of these factors on the equilibrium prices. Our analytical results demonstrate the symmetry in the optimal prices and critical thresholds for the two carriers. Besides, we reveal the importance to consider both carrier's own and the competitor's risk attitudes and operating characteristics in decision making when market competition exists. Moreover, we find the direct and indirect impacts of risk attitudes on the optimal prices, thus highlighting the importance to integrate risk considerations into the optimal pricing decision framework. Finally, we show that market situations play a critical role in characterizing the effects of diverse parameters on the equilibrium prices, which should be carefully evaluated by decision makers.

Keywords: Optimization; risk analysis; stochastic supply chain systems; mean-variance theory; air cargo carrier systems.

---

[1] We sincerely thank the editors and three anonymous reviewers for their critical and constructive comments, which help us improve the quality of this research paper.

[2] Department of Industrial and Systems Engineering, The Hong Kong Polytechnic University.

[3] Glorious Sun School of Business and Management, Donghua University.

[4] Corresponding author: Tsan-Ming Choi, Professor of Fashion Business, Business Division, Institute of Textiles and Clothing, The Hong Kong Polytechnic University, email: jason.choi@polyu.edu.hk, phone: 852-27666450.

Optimal Pricing Decisions of Competing Air-Cargo-Carrier Systems – Impacts of Risk Aversion, Demand and Cost Uncertainties

I. INTRODUCTION

A. Background and Motivation

Air-cargo carrier systems are an important part of transportation logistics systems. Due to the increased global trades, higher demand for fast shipment, and companies' efforts in keeping low inventory level through quick and frequent replenishments [1, 2], the air freight transportation industry is growing rapidly in recent years, with the industry-wide revenue reaching 95.9 billion US dollars in 2017 [3, 4]. As reported by IATA [5], 35% of the global trade value of goods are transported by air in 2015. Besides, Airbus [6] and Boeing [7] have forecasted that the international air freight volume will double in the next two decades. Currently, air cargo has become a crucial component of revenue not only for dedicated cargo air carriers (e.g., Cargolux), but also for combinatorial air carriers (e.g., Cathay Pacific) [8]. Moreover, it is reported that cargo transportation produces more than twice revenue than the first-class cabin passenger transport, and the throughput of air cargo grows 50% faster than that of air passenger [3, 9].

Despite the fast growth and increasing importance, the air cargo transportation industry is facing with diverse challenges. First of all, the industry is characterized by fierce and intensive market competition. As predicted, approximate 350 new-built air cargo carriers in North America and 200 in Asia-Pacific will appear in the next twenty years [6, 7]. Second, the market is highly volatile and uncertain, with remarkable variations in consumer demand [4, 10]. For example, IATA [5] reports that the monthly industry-wide freight traffic kept varying throughout the year of 2017, and the difference between the highest (in November) with the lowest (in February) volumes reaches around 5 billion FTKs. Third, as the fuel consumption comprises the largest part of an airline's operating costs, the fluctuation in crude oil price creates significant challenges for the profitability and development of air freight companies [11, 12]. As reported by Airbus [6], the international jet-fuel price kept fluctuating since 2000, which climbed by more than 200% from 2000 to 2008, followed by a sharp reduction by 50% in 2009. After that, the oil price grew rapidly to the 2008-level in 2010. Airbus [6] has also predicted a great fluctuation in fuel price in the next two decades. Although some airlines adopt financial instruments like fuel hedging to alleviate the impact of oil price fluctuation, cost uncertainty still exists. For instance, Cathay Pacific is reported to lose 6.45 billion HK dollars in fuel hedging in 2017[5], causing great financial burden for the corporate. Therefore, the significant uncertainties in operating costs should be carefully considered during decision making for cargo airlines.

Consequently, it is seen that air freight carriers are challenged by uncertainties from both demand and cost perspectives, together with intensive market competition. Therefore, it is reasonable that some freight airlines hold a risk-averse attitude against profit losses to maintain profitability in the highly volatile and competitive environment. As a result, enhancing the strategic decision making, especially with risk considerations, becomes crucial for air cargo carriers. As pointed out by Azadian and Murat

[5] https://hongkongbusiness.hk/aviation/news/cathay-pacific-hit-massive-645b-fuel-hedging-loss-in-2017.

[12], among the air freight operations management issues, the pricing problem is the most important but challenging one. It is reported that modern companies are keen to identify the optimal pricing decisions that enable them to adapt to the increasingly competitive market [13, 14]. Although it has been identified that the objectives and equilibrium decisions[6] of loss-averse entities are totally different from those of risk-neutral ones [15], the optimal pricing decisions for competing risk-averse cargo airlines in the presence of demand and cost uncertainties are under-explored.

In this paper, we focus on examining the performance of two competing carriers. It is standard practice to consider two players in the operations management science literature, as the two-player case is fundamental when examining multi-player games and could show the impact of competition. This is similar to the exploration of duopoly scenarios in supply chain systems because having two players under duopoly can uncover the impacts of competition [51]. In academia, there are a substantial number of references to support this assumption (e.g. [49-51], etc.). In real practices, it is also practical to consider two-player situations in terms of the air cargo carriers. For instance, Shenzhen Bao'an International Airport, which focuses on expanding the domestic and international cargo transportation, is now cooperating with 4 air freight carriers (SF Express, UPS, DHL and YTO Express). However, in most cases, only two cargo carriers compete for the majority of the market in the same route (for example, only UPS and DHL operate the cargo flights between SZX and ANC).[7].

Therefore, it is important and meaningful to explore such a problem and derive insightful managerial implications for both practitioners and academics on how to enhance the competitiveness of freight airlines through investigating the impacts of risk aversion and market uncertainties on the equilibrium pricing decisions.

B. Research Questions and Major Findings

Motivated by the importance of the air freight transportation industry and the various challenges faced by freight airlines, in this work, we analytically study the pricing decisions of cargo airlines with the consideration of risk-averse behaviors. Specifically, we consider a system consisting of two competing carriers who are risk-averse to profit losses. First in the basic model, we explore the optimal prices for the carriers under market demand uncertainty, and investigate the impacts of diverse parameters on the equilibrium prices to generate respective managerial insights. Then, we extend the analyses to integrate cost uncertainty into consideration, and highlight the importance of considering this crucial factor for decision making.

To be specific, we aim to address the following research questions: 1. What are the optimal pricing decisions for the two risk-averse air cargo carriers when they compete under stochastic demands? 2.How do the crucial factors (e.g., market competition, risk sensitivity coefficients, demand uncertainty, market share) affect the optimal prices? 3. What are the optimal prices if the two carriers face uncertain costs (e.g., related to the volatile oil prices)? How does cost uncertainty influence the decision making of the two carriers?

As we will show later on in this paper, by addressing the above research questions, we find a number of major important insights. First, the equilibrium prices for the two competing risk-averse cargo airlines are perfectly symmetric, determined by various critical parameters. Second, we show

---

[6] In this paper, the terms “optimal” decision and “equilibrium” decision are used interchangeably.
7 The specific details can be found on the official website of Shenzhen Bao'an International Airport (http://www.szairport.com/szairport/hyhzhb/tthkhy.shtml).

that carriers should consider not only its own risk attitudes and costs, but also the competitor's risk preferences and operating characteristics during decision making when market competition exists. Third, we find that the impacts of risk attitudes of decision makers on the optimal prices are twofold as follows: (i) A carrier's risk attitude could directly increase the optimal prices for both carriers if its operating cost is sufficiently large in a duopoly market with competition; and (ii) risk behaviors could affect the optimal prices indirectly by characterizing the effects of other crucial parameters (e.g., demand and cost uncertainties, market competition). For instance, it is identified that a carrier is prone to charge a higher price when the market demand is becoming more volatile if its operating cost is sufficiently high and it is very risk-averse relative to its competitor in a duopoly market with competition. On the other hand, if the operating cost becomes increasingly stochastic, a carrier will not increase its price unless the fixed part of its cost is sufficiently low and its relative risk-averse attitude is very high compared to its competitor. Besides, we find that carriers are inclined to raise their prices when the market competition becomes intensified due to the aversion to profits losses. Fourth, it is shown that market situations affect the impacts of diverse critical factors on the optimal prices significantly. For example, results indicate if a carrier dominates the market, the risk attitude of the other carrier then becomes nonsignificant. Moreover, market share is demonstrated to influence the optimal prices differently when demand is deterministic or uncertain and when market competition does or does not exist.

C. Contribution Statements and Paper's Structure

To the best of our knowledge, this paper is the first analytical study that comprehensively explores how risk-aversion, market competition, demand uncertainty and cost uncertainty affect the optimal pricing decisions for air-cargo carrier operations. The incorporation of risk sensitivity in decision making derives novel insights and implications regarding the impact of risk considerations on the pricing mechanisms for air cargo carriers. The mean-variance theory is applied to model the risk-averse behaviors of decision makers. Besides, cost uncertainty is considered, which provides useful information for practitioners to deal with the volatility arising from the crude oil market. All results are derived in closed-form and proven mathematically. Considering the importance of optimal price decisions for cargo airlines and the increasing attention from both academia and industry, our study provides crucial managerial implications to advance the understanding on the optimal pricing decisions for the air freight transportation industry, and helps enhance the competitiveness of air cargo carriers in the highly uncertain market.

This paper is organized as below. First, Section II reviews the related literature. Then, we build a basic model and construct the mean-variance objectives for two competing risk-averse air cargo carriers under demand uncertainty in Section III. Next, Section IV derives the equilibrium solutions and managerial insights based on the basic model. In Section V, we extend the analyses to integrate the factor of cost uncertainties. Finally, Section VI concludes for this work and proposes several future research directions.

II. RELATED LITERATURE

First, this study belongs to the stream of operations management in the air cargo transportation industry. Second, as we consider the risk behaviors of decision makers, it also relates to the stream of decisions with risk considerations. Last, we review the application of the mean-variance theory which we will utilize to model the risk-averse attitudes of the air cargo carriers in this work.

A. Operations in Air Cargo Logistics Systems

In the literature, the majority of airline-related analytical operations research concentrates on passenger transport [16-20], while much less explores air cargo transportation [21]. Most of the existing literature on air freight operations investigates the topics like revenue management, capacity management, entry decisions, and booking control. For instance, Barz and Gartner [22] construct heuristics for network air freight revenue management based on linear programming, decomposition and approximate dynamic programming, while Wada et al. [23] investigate the capacity allocation problem for risk-averse cargo airlines. From the perspective of market entry decisions, Wang et al. [21] study an air freight service supply chain with promised delivery time competition. The authors identify the win-win and lose-lose situations are identified for both mainline carriers and regional carriers if the mainline carriers enter the upstream regional market. They also find that the multi-dimensional competition could reduce the negative impact of the upstream entry on the incumbent regional carriers. On the other hand, Hellermann et al. [24] propose an option contract to derive the optimal booking policy for a system consisting of a freight forwarder and a cargo airline. They analyze the impact of overbooking on cargo airline's profitability, and demonstrate the advantageous performance of the proposed contractual agreement over the existing one by applying industrial real data. Other analytical research topics related to air cargo management include shipment integration and consolidation [25], network planning [26], and loading planning [27].

Regarding the pricing problem, although the significance and challenges of this crucial decision for air freight carriers have been realized, only a few pieces of studies have explored this critical issue. First, Azadian and Murat [12] study a group pricing problem for an air cargo company. The authors state that it is a common practice for the transportation industry to group several locations and price these services on a group basis. Therefore, they formulate an integrated model to simultaneously decide the optimal group service locations and the corresponding prices. Besides, the authors construct a mixed-integer nonlinear programming model for the integrated problem which is solved by algorithms based on decomposition approaches. Second, considering a service supply chain consisting of an air freight airline and freight forwarders who compete for uncertain demand, Tao et al. [4] explore the option contracts between the agents, and derive the optimal prices for the airline and the optimal reservation strategies for freight forwarders to maximize their expected profits. A Stackelberg game is established to model the behaviors of the supply chain members, while numerical experiments and sensitivity analyses are conducted to generate managerial insights in Tao et al. [4]. Similar to the above two studies, we also explore the pricing problems for air cargo carriers. However, different from them, our study simultaneously incorporates the uncertainties from both demand and operating costs, and market competition into the decision framework. Accordingly, we are able to investigate the impacts of these crucial factors on the equilibrium prices, thus generating useful insights and implications. More importantly, our work considers the carriers' risk attitudes towards profit losses, which is novel in the air cargo pricing literature.

B. Decisions with Risk Considerations

Risk analysis is one of the most crucial topics in systems engineering over the past decade [28-30]. For example, Shen et al. [31] study the performance of markdown money policy in a fashion supply chain composed of a loss-averse manufacturer and a risk-neutral retailer. Besides, Choi [32] examines the supply chain coordination issues with risk-sensitive retail buyers under both symmetric and asymmetric information settings. The author illustrates that the risk attitudes of decision makers significantly influence the achievability of prefer coordination for a supply chain. Similarly, Xie et al. [33] explore the conditions to achieve supply chain coordination with the consideration of retailers' risk behaviours. In the model setting of Xie et al. [33], retailers could be risk-neutral, risk-averse, or risk-take in a unified framework. Therefore, the significant impact of risk attitudes on decision making is demonstrated through comparing the various settings [33]. Furthermore, Zhang et al. [34] investigate the effects of loss-averse behaviour and capital constraint on the optimal price and ordering quantity decisions for a newsvendor supply chain. In addition, risk analysis has been widely applied in the areas like personnel assignment [35], cybersecurity protection [36], Bayesian network modelling [37], and contamination of food production facilities [38].

In the air cargo industry, if uncertainties (like uncertain demand and cost) exist, the performance of airlines will be affected and their profitability thus becomes volatile [39]. Therefore, how to improve decision making under an uncertain environment to alleviate profit risks becomes a critical problem for freight airlines. For instance, Wada et al. [23] enhance the capacity allocation strategy for risk-averse freight airlines, and the results obtained from risk-averse and risk-neutral models are compared to demonstrate the great impact of risk attitudes on the optimal solutions. Besides, Sample Average Approximation approach is applied to test the models using real data in Wada et al. [23].

Some research has integrated risk considerations into the pricing decision framework. For example, Zheng et al. [40] study the optimal pricing decisions for liner shipping companies who compete for uncertain demand, and the risk-averse behaviour of one participant is modelled by the conditional value at risk approach. Conditions when the equilibrium prices will increase or decrease along with competition level are analysed in Zheng et al. [40]. Besides, Agrawal and Seshadri [41] explore the pricing and ordering decisions for a risk-averse newsvendor facing with uncertain demand. They show that the loss-averse attitude could either raise and lower the retail price in different model settings. Besides, Li et al. [42] study the impact of risk preference of a retailer on the optimal pricing decisions for a dual-channel supply chain. They show that when the retailer becomes more risk-averse, the equilibrium retail price will decline if the uncertain demand follows a uniform distribution. A similar study could be found in Liu et al. [43], which concentrates on the pricing and coordination decisions under a dual-channel supply chain. They employ the standard deviation of payoff as a measure of risk and find that the optimal prices under a risk-averse case are lower than those in a risk-neutral case. Furthermore, Li et al. [44] analytically explore how the risk attitudes of a retailer could affect the optimal price and promised delivery time decisions.

From the above discussion, it is clear that relatively limited research has studied the pricing decisions for air cargo carriers. Besides, none of the current studies has investigated the integrated impact of market competition, demand uncertainty and cost uncertainty on the optimal prices. More importantly, to be best of our knowledge, no previous research has explored how risk behaviours of decision makers affect the equilibrium pricing decisions for the air freight industry. This work hence aims to bridge these significant literature gaps. **Table 1** indicates the positioning of this work in the literature. As could be seen from the table, this work differs from other studies and becomes the first research that use the mean-variance theory to explores the pricing problem for competing risk-averse air cargo carriers facing uncertain demand and costs.

C. Mean-Variance Theory

Regarding risk analysis, one of the most commonly applied analytical approaches is the mean-variance (MV) theory. Although there exist different types of measures capturing risk behaviour, such like the von Neumann–Morgen-stern utility (VNMU) approach, and Black-Scholes Formula, their real world applications are limited [48]. The superiority of the MV approach is that it can provide a practical and implementable solution when considering risk hedging, which means that it is more understandable and extensively applied by industrialists.

The MV theory was firstly introduced for portfolio optimization in financial engineering, and then is widely used in supply chain and logistics operations problems [45-48]. For instance, Chiu et al. [47] solve the supply chain coordination problem with several loss-averse retailers and a risk-neutral manufacturer based on the framework of the MV theory. The authors find that the risk parameters play a crucial role in determining the efficiency of coordination contracts. They also show that the manufacturer could manage the retailer profit variance through adjusting the risk indicators. Similarly, building an analytical MV optimization model, Li et al. [48] examine a fast fashion supply chain with returns policy. They find that a simple returns policy could coordinate a fast fashion supply chain even when multiple retailers exist. In our work, we follow this research stream to apply the MV theory to measure the loss aversion behaviours of air cargo carriers.

Table 1. Literature positioning of this paper.

Literature	Air cargo	Pricing decisions	Market competition	Demand uncertainty	Cost uncertainty	Risk considerations
Barz and Gartner [22]	√			√		
Wada et al. [23]	√			√		√
Wang et al. [21]	√		√			
Hellermann et al. [24]	√			√		√
Leung et al. [25]	√					
Derigs et al. [26]	√					
Li et al. [27]	√					
Azadian and Murat [12]	√	√				
Tao et al. [4]	√	√	√	√		
Shen et al. [31]				√		√
Choi [32]				√		√
Xie et al. [33]				√		√
Zhang et al. [34]				√		√
Lazzerini and Pistolesi [35]						√
Qin et al. [36]						√
Yang et al. [37]						√
Chang et al. [38]						√
Zheng et al. [40]		√	√	√		√
Agrawal and Seshadri [41]		√		√		√
Li et al. [42]		√		√		√
Liu et al. [43]		√	√	√		√

Li et al. [44]		√	√			√
This paper	√	√	√	√	√	√

III. BASIC MODEL

In the basic model, we consider an air transport system consisting of two competing risk-averse freight carriers who need to determine their optimal pricing decisions with volatile market demand. The two carriers are denoted by r=1 or 2. Here, the unit operating cost for each carrier is fixed as c_r , while the competition level between the two players is denoted by λ. The uncertain market demand $\tilde{a}$ ($\tilde{a} = a_0 + \varepsilon$) consists of a fixed part a_0 and an uncertain part ε which follows a normal distribution[8] with the mean of zero and the standard deviation of σ (i.e., $\varepsilon \sim N(0, \sigma^2)$). We use θ to represent the market share of Carrier 2 which is determined by various factors like reputation, service quality and company size. Accordingly, $1-\theta$ stands for the market share of Carrier 1. Besides, note that θ could also be treated as consumer preference or loyalty. Therefore, it is sensible that θ is usually not affected by prices. The unit price for each carrier is represented by P_r $(r = 1$ or $2)$. Following the literature in supply chain and logistics management [21, 40, 43], we model the demand functions for the two carriers (D_1, D_2) as in Eq. (1) and Eq. (2), where the demand uncertainty and the product competition in air cargo carriers can be reflected.

Eq. (1) $$D_1 = (1-\theta)\tilde{a} - P_1 + \lambda P_2,$$

Eq. (2) $$D_2 = \theta\tilde{a} - P_2 + \lambda P_1.$$

Then, the profits for the two carriers could be expressed in Eq. (3) and Eq. (4). To be specific, the demand for one carrier is dependent on both its own and competitor's prices.

Eq. (3) $$\pi_1 = (P_1 - c_1)[(1-\theta)\tilde{a} - P_1 + \lambda P_2],$$

Eq. (4) $$\pi_2 = (P_2 - c_2)(\theta\tilde{a} - P_2 + \lambda P_1).$$

The competition parameter λ actually indicates the impact of the price adjustment of one carrier on its competitor's demand. For example, when the price of Carrier 2 (P_2) increases by one unit, the demand for its competitor (D_1) would increase by λ. Note that we only consider the situation when the unit price is no smaller than the unit cost, and the demand for each carrier is non-negative (i.e., $P_r \geq c_r$ and $D_r \geq 0$) to assure no lose for the carriers. With Eq. (3) and Eq. (4), the expected profit functions for the two carriers could be obtained in Eq. (5) and Eq. (6).

Eq. (5) $$E(\pi_1) = (P_1 - c_1)[(1-\theta)\ a_0 - P_1 + \lambda P_2],$$

Eq. (6) $$E(\pi_2) = (P_2 - c_2)(\theta a_0 - P_2 + \lambda P_1).$$

Considering that both carriers are risk-averse to profit losses, we adopt the mean-variance (MV) theory to model the risk-averse preference of the decision makers. The objective function for the MV theory is shown in Eq. (7), which equals the expected profit minus the variance of profit multiplying the risk sensitivity coefficient (k).

Eq. (7) Maximize: $$O = E(\pi) - kV(\pi).$$

Therefore, the respective MV objectives for the two carriers are formulated in Eq. (8) and Eq. (9), where the demand uncertainty and risk attitudes are taken into consideration. We can observe that, the first parts in Eq. (8) and Eq. (9) are the expected profits for Carrier r and the second parts are the

[8] Actually, our results will hold for the case when the randomness follows any bounded symmetric distribution with a zero mean.

variance of profit multiplying the risk sensitivity coefficient (k_r). Specifically, the risk sensitivity coefficient for Carrier r (k_r) is a risk aversion indicator for Carrier r. An increase in k_r represents the increasing risk aversion against profit volatility for the decision maker. When $k_r = 0$, the freight airline is risk-neutral. In addition, recall that σ is the standard deviation of the market demand, which refers to the demand uncertainty.

Eq. (8) $\quad Max\colon\ O_1 = (P_1 - c_1)[(1-\theta)\ a_0 - P_1 + \lambda P_2] - k_1[(P_1 - c_1)^2(1-\theta)^2\sigma^2],$

Eq. (9) $\quad Max\colon\ O_2 = (P_2 - c_2)(\theta a_0 - P_2 + \lambda P_1) - k_2(P_2 - c_2)^2\theta^2\sigma^2.$

Besides, it is pointed out that in our problem setting, consumers could place an order to the carriers long before the event date, which enables the carriers to suitably manage the utilization of aircrafts. Therefore, the capacity limitation is not considered in this work. This is commonly observed in the practice. For instance, the Switzerland-based freight forwarder Panalpina is reported to encourage shippers to book their air cargo shipment orders as early as possible before the start of peak seasons to avoid capacity shortages[9]. Besides, many air cargo carriers allow customers to make ordering one month or even months in advance (like UPS and Emirates SkyCargo). On the other hand, in the literature, Wada et al. [23] consider long-term agreement orders where capacity is allocated in advance. Therefore, our model setting is reasonable in both practice and academics.

IV. OPTIMAL DECISIONS: AN EQUILIBRIUM ANALYSIS

To focus on exploring the impact of the risk-averse behaviours of carriers, in the following analyses, we limit our attention to the cases when both carriers are risk averse (i.e., $k_r > 0$)[10]. The optimal pricing decisions for the two competitors in the basic model (P_1^*, P_2^*) could be obtained by solving Eq. (8) and Eq. (9), which are summarized in Lemma 1. Note that the list of notation used in the analyses is summarized in **Table 3** (Appendix A1). Besides, in the analyses, some important relative risk-averse attitude thresholds and cost thresholds are identified, which are listed in **Table 4** (Appendix A1) and **Table 5** (Appendix A1), respectively. Furthermore, all mathematical proofs are relegated to Appendix (A2).

Lemma 1. *In the basic model with uncertain demand and fixed costs, the MV objective functions for the two competing risk-averse carriers are strictly concave, and the respective optimal prices are given as follows*:

$$P_1^* = \frac{2(1+S_2k_2)[(1-\theta)a_0 + c_1(1+2S_1k_1)] + \lambda[\theta a_0 + c_2(1+2S_2k_2)]}{4(1+S_1k_1)(1+S_2k_2) - \lambda^2},$$

$$P_2^* = \frac{2(1+S_1k_1)[\theta a_0 + c_2(1+2S_2k_2)] + \lambda[(1-\theta)a_0 + c_1(1+2S_1k_1)]}{4(1+S_1k_1)(1+S_2k_2) - \lambda^2}.$$

Using the notation summarized in **Table 3**, P_1^* and P_2^* can be represented as $P_1^* = \frac{A_2A_3 + \lambda A_4}{A_1A_2 - \lambda^2}$ and $P_2^* = \frac{A_1A_4 + \lambda A_3}{A_1A_2 - \lambda^2}$, respectively. Lemma 1 shows that when a risk-averse freight airline facing market competition tries to maximize its own MV objective under demand uncertainty, an optimal pricing

---

9 https://www.aircargonews.net/news/freight-forwarder/single-view/news/shippers-warned-to-book-now-or-face-rate-spikes-and-delays-in-busy-peak-for-air-cargo.html

10 The case when the carriers are risk neutral can be explored by setting the risk coefficient k_r to be 0.

decision exists. Besides, the equilibrium prices for the two carriers are perfectly symmetric. The major parameters, like market share (θ), competitional level (λ) and demand uncertainty (σ), all impose great effects on the optimal solutions for the two players involved. More importantly, it is interesting to note that the risk attitudes of both carriers impose critical influences on the equilibrium prices for each individual participant (that is, P_r^* is determined by both k_1 and k_2). Therefore, the importance of considering not only the carrier's own risk attitude, but also the risk behavior of its competitor in the decision process is highlighted. Next, we will investigate the impacts of the diverse crucial factors on the equilibrium prices sequentially. Since the competition level (λ) and operating cost (c_r) influence the optimal prices most directly, we first evaluate the implications of them in Proposition 1 as follows.

Proposition 1. *In the basic model where two risk-averse carriers compete for uncertain demand with fixed costs, we have:*

(i) The optimal prices for the two carriers increase with market competition (i.e., $\frac{\partial P_1^*}{\partial \lambda} \geq 0, \frac{\partial P_2^*}{\partial \lambda} \geq 0$*);*

(ii) With market competition ($\lambda > 0$), a carrier's optimal price increases with both its own and competitor's operating costs, while the increase is faster with its' own cost than with the competitor's;

(iii) Without market competition ($\lambda = 0$), a carrier's optimal price increases with its own operating cost, but is unrelated to its competitor's operating cost.

Proposition 1 summarizes the important insights regarding the impacts of market competition and operating costs on the optimal pricing decisions for the risk-averse carriers. From Proposition 1(i), we could see that under demand uncertainty, both two participants intend to increase their prices when market competition becomes more furious. The intuition is explained as follows. Considering that the two carriers are risk-averse to profit losses, when the two companies compete against each other more fiercely, the threats of demand shrinkage drive them to raise their prices, with the aim of maintaining profitability in the uncertain market. Therefore, it is implied that the risk attitudes of decision makers are crucial in characterizing the impact of market competition on the equilibrium prices.

From Proposition 1(ii), it is reasonable that the carrier will charge a higher price if its own operating cost increases. On the other hand, due to the competition between the two participants, it is interesting to note that the rise in competitor's cost could also drive a carrier to raise its price. This is mainly because the competitor is prone to increase its price according to the growth of its cost to hedge against profit risks, which leaves a room for the carrier to charge a higher price. Consequently, it is identified that the operating costs of both entities are important determinants for loss-averse carriers when they engage in a competition. However, the influencing power of competitor's cost growth on a carrier's price is smaller than that of the company's own cost growth. Naturally, the driving force of competitor's cost growth vanishes if the two players terminate competition, as shown in Proposition 1(iii).

Based on the above analysis, we can easily derive that both the competition level and operating cost will influence the pricing decisions significantly. To further explore the joint effect of the competition level and operating cost on the optimal pricing decisions, we conduct the numerical study and depict the Figure 1. Notice that, we denote $\Delta c = c_1 - c_2$ as the operating cost difference, thus a higher Δc means a higher operating cost for the Carrier 1 relative to its rival.

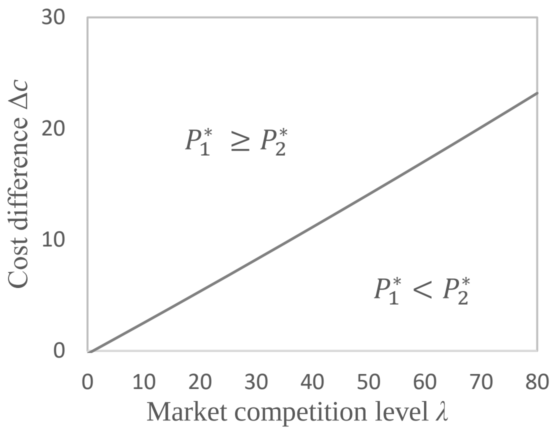


Figure 1. The comparison between optimal pricing decisions with respect to the competition level λ and cost difference Δc (We set $a_0 = 100,\ \theta = 0.4,\ \sigma = 5,\ k_1 = 80,\ k_2 = 100$ and varied λ from 0 to 80[11].)

From Figure 1 we can observe that, the competition level and operating cost can jointly influence the pricing decisions. Specifically, when the operating cost is relatively low, setting a lower price than rival's is more advisable when market competition is more keen. This result is reasonable and intuitive, because when confronted with a fierce business competition, a lower price can grab more market share and then boost profit. While when the operating cost is relatively high, the decision maker always tends to set a higher price than its rival, irrespective of the competition level. Thus, it is suggested that when the decision makers want to make pricing decisions, they should pay attention to the operating cost first and then market competition level.

Next, in order to figure out the influences of the risk attitude (k_r), which is the indispensable factor in terms of the risk-averse carriers, we proceed to analyze the relationships between the risk attitudes and optimal prices in Proposition 2.

Proposition 2. *In the basic model where two risk-averse carriers compete for uncertain demand with fixed costs, the impacts of risk sensitivity coefficients of carriers (i.e.,* k_1, k_2*) on the optimal prices are diverse as follows:*

(i) With deterministic demand ($\sigma = 0$*), the risk attitudes of the two carriers impose no impact on* P_1^* *and* P_2^**.*

(ii) With uncertain demand ($\sigma \neq 0$*),*

a) Under a duopoly[12] *market with carrier competition (*$0 < \theta < 1,\ \lambda > 0$*):*
P_1^* *and* P_2^* *are increasing with the risk sensitivity coefficient of Carrier r (*k_r*), if* c_r *is sufficiently large (i.e.,* $c_r > CT_r$*), or decreasing with* k_r *if* c_r *is sufficiently small (i.e.,* $c_r < CT_r$*). The threshold* CT_r *is increasing in* c_{3-r}*.*

b) When there is no competition in the market ($\lambda = 0$*):*
Carrier r's risk attitude (k_r*) would not affect the optimal price of Carrier (3-r). Besides,* CT_r

[11] All the data we set in the numerical studies (Figures 1-3) follow the model assumptions (e.g. $0 < \theta < 1,\ \lambda \geq 0,\ \sigma \geq 0$, etc.) and can help show the effects (which have been analytically proven) intuitively.

[12] In the analyses, the market is named as "duopoly" market if $0 < \theta < 1$, or "monopoly" market if $\theta = 0$ or 1. When $\theta = 0$, Carrier 1 is called "dominator", while Carrier 2 becomes "dominator" if $\theta = 1$.

is unrelated with c_{3-r} .

c) *Under a monopoly market* ($\theta = 0$ *or* 1)*:*
If Carrier r occupies all the market, the other carrier's risk attitude (k_{3-r}) *would not affect the optimal prices of both carriers.*

Proposition 2 indicates that the impacts of risk attitudes of decision makers on the equilibrium prices depend on market situations. To be specific, whether the market demand is fixed or uncertain, whether market competition exists, and whether the market is monopoly or duopoly, are crucial in determining the role of risk behaviors in decision making.

First of all, as all risks are derived from uncertainties, Proposition 2(i) shows that the risk-averse attitudes of the carriers are irrelevant to the optimal pricing decisions when the market demand is deterministic without any uncertainty.

On the other hand, with volatile market demand, risk attitudes impact decision making significantly, which is further affected by market segmentation and market competition. Firstly, under a duopoly market ($0 < \theta < 1$), if the two carriers compete for uncertain market ($\lambda > 0$), there exists a critical cost threshold (CT_r) to determine the influence of risk attitudes, which is illustrated in Proposition 2(ii)a). Specifically, if the operating cost of a carrier is very high ($c_r > CT_r$), when it becomes more loss-averse, both two carriers will raise their equilibrium prices. However, due to the competition in the market and uncertainties in demand, a carrier would not increase its price along with risk aversion unless its operating cost is sufficiently high, which creates great challenges for the company to maintain profitability. Observing the growth in the competitor's price, the other carrier in the market will thus follow. On the opposite, if a carrier's cost is very low ($c_r < CT_r$), the two participants will increase their prices if it becomes less risk-averse. The latent reason is because when the operating cost is low enough, the difficulty in achieving a target profit is low. Therefore, a carrier could be more ambitious to make higher profits by increasing its price, especially when it is less loss-averse. After that, the other carrier in the market will react to follow. Regarding the cost threshold CT_r for a carrier, an increase in the competitor's cost will lead to a higher CT_r . That is, with competition, the increase in the competitor's operating cost would make it more difficult for the two carriers to raise their optimal prices when a carrier becomes more risk-averse. It implies that the competitor's cost imposes a moderating effect on the impact of the risk behavior of a carrier on the optimal prices for the two carriers under the influence of market competition.

While without the driving force of competition ($\lambda = 0$), it is natural that the other carrier's operating cost has no influence on a carrier's own cost threshold, while the optimal price of a carrier is independent from the other carrier's risk behaviors (see Proposition 2(ii)b)). In addition, when one carrier dominates the whole market ($\theta = 0$ *or* 1), it is reasonable to observe that the impact of the other carrier's risk attitude becomes nonsignificant on the equilibrium decisions for both two participants (as shown in Proposition 2(ii)c)).

In conclusion, Proposition 2 underlines the importance of considering the risk attitudes of decision makers in the optimal pricing decisions when market demand is volatile. Besides, the significant impacts of market competition in determining the role of competitor's risk behaviors and operating costs in the optimal prices of a carrier are highlighted.

Since we have proved that the risk behavior plays a vital role in pricing decisions, we would like to further explore the relationships between the optimal pricing decisions of two carriers with respect to their risk attitudes in Figure 2.

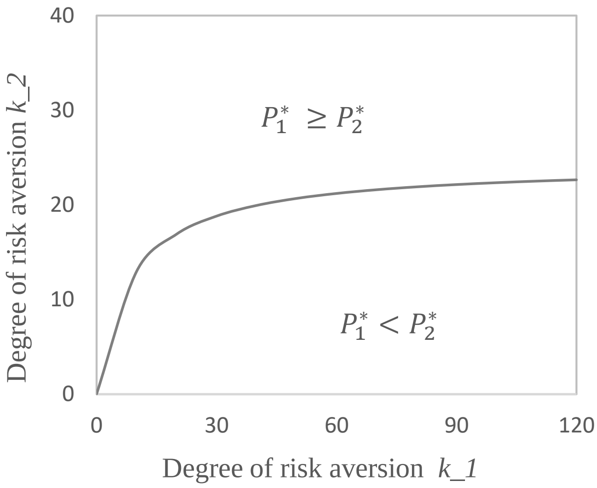


Figure 2. The comparison between optimal pricing decisions with respect to the degree of risk aversion k_1 and k_2 (We set $a_0 = 100$, $\theta = 0.4$, $\sigma = 5$, $c_1 = 500$, $c_2 = 400$ and $\lambda = 40$.)

As shown in Figure 2, the degree of risk aversion of the competing carriers will jointly influence their pricing decisions. For a carrier, when it competes with a low risk-averse rival, a lower price (compared with its competitor) would be charged if its own risk-averse level is relatively high. Whereas if the competitor is very risk-averse, its optimal price would always be higher than the rival's. Particularly, if a carrier is totally risk-neutral ($k_r = 0$), a definitely higher price will be set relative to its rival, while when its competitor is totally risk-neutral, it is more advisable for the carrier to charge a lower price. By following these results, the risk-averse air cargo carriers can make more scientific and reasonable decisions as well as know their opponents better under demand uncertainty.

Furthermore, market segmentation is shown to be crucial in the impacts of risk attitudes on decision making. Thus, we conduct the analysis about the impact of market share on carriers' optimal pricing decisions in Proposition 3.

Proposition 3. *In the basic model where two risk-averse carriers compete for uncertain demand with fixed costs, the impacts of market share on the optimal prices are shown as follows:*

(i) With deterministic demand ($\sigma = 0$): Carrier r increases its optimal price according to the expansion of its own market share.

(ii) With uncertain demand ($\sigma \neq 0$), for Carrier r:

a) With market competition ($\lambda > 0$), its optimal price increases (or decreases) along with its own market share if $c_r > DT_r$ (or $c_r < DT_r$). Besides, DT_r *is positively related to* c_{3-r} .

b) With market competition ($\lambda > 0$), its optimal price increases (or decreases) along with its competitor's market share if $c_{3-r} > ET_{3-r}$ (or $c_{3-r} < ET_{3-r}$). Besides, ET_{3-r} is positively related to c_r .

c) Without market competition ($\lambda = 0$), its optimal price increases along with its own market share.

Proposition 3 demonstrates the various influences of market share on carriers' optimal pricing decisions. First, as shown in Proposition 3(i), when there is no uncertainty in the market threatening decision makers, carriers will increase their prices to improve profitability according to the expansion of its own market share.

However, Proposition 3(ii) indicates that the impacts of market share under demand uncertainty are much different, which is further affected by market competition. Specifically, under competition, a carrier is prone to charge a higher (or lower) price along with the increase of its own market share if its own cost is high (or low) enough (see Proposition 3(ii)a)). The principle behind is that, the risk of profit losses (due to the competition and demand uncertainty in the market) would prevent a carrier from increasing its price when it occupies a larger market unless its operating cost is too high to achieve a targeted profitability level. Since the threshold DT_r is positively related to the competitor's cost, we could expect that when a carrier's own cost is becoming increasingly high while the competitor's cost is becoming increasingly low, the carrier will be easier to raise its price along with the expansion of its market size. On the other hand, when the operating cost is sufficiently low, in order to compete with its competitor for uncertain demand, it is optimal for a carrier to decline its price to attract more consumers when its market share is expanded. In this case, the carrier could maintain a certain profitability level owing to the low cost and expanded market share. Interestingly, this strategy is easier to operate when the competitor's cost becomes higher (which means DT_r becomes higher, and $c_r < DT_r$ becomes easier).

Regarding the impact of competitor's market share, critical thresholds for competitor's operating cost (i.e., ET_{3-r}) also exist, as indicated in Proposition 3(ii)b). To be specific, Carrier *r* intends to raise (or reduce) its price when its competitor's market share increases, with the condition that the competitor's operating cost is higher (or lower) than ET_{3-r}. As threshold ET_{3-r} is positively related to the carrier's own cost (c_r), we could expect that when c_r is sufficiently low (that is, $c_{3-r} > ET_{3-r}$ is easier to be satisfied), the risk-averse carrier has to rise its price to keep profitability if the competitor seizes more and more market share. On the other hand, if there is no competition, carriers could always promote their profits in the volatile market by increasing prices along with the expansion of market share (as shown in Proposition 3(ii)c)).

As a remark, Proposition 3 highlights the significant effects of demand uncertainty and risk attitudes of decision makers on the impacts of market share on the optimal prices. Besides, the critical role of market competition in loss-averse decision making is further demonstrated.

Next, we explore how demand uncertainty affects loss-averse decision making. Denote τ_r as the *relative risk-averse attitude of Carrier r over its competitor*, which is equal to $\frac{k_r}{k_{3-r}}$. Then, we have Proposition 4 as follows.

Proposition 4. *In the basic model where two risk-averse carriers compete for uncertain demand with fixed costs, the impacts of demand uncertainty on the optimal prices are derived as follows:*

(i) Under a duopoly market ($0 < \theta < 1$) with competition ($\lambda > 0$), for Carrier r:

a) If $\tau_r > \Lambda_r$, its optimal price increases with demand uncertainty if $c_r > YT_r$.

b) If $\tau_{3-r} > \Omega_{3-r}$, its optimal price increases with demand uncertainty if $c_{3-r} > PT_{3-r}$.

(ii) Under a duopoly market ($0 < \theta < 1$) without competition ($\lambda = 0$), for Carrier 1, its optimal price increases with demand uncertainty if $c_1 > a_0(1-\theta)$, while for Carrier 2, its optimal price increases with demand uncertainty if $c_2 > a_0\theta$.

(iii) Under a monopoly market ($\theta = 0$ or 1):

a) With market competition ($\lambda > 0$), when the market is dominated Carrier r, $\frac{\partial P_1^*}{\partial \sigma}$ and $\frac{\partial P_2^*}{\partial \sigma}$ are positive if $c_r > OT_r$.

b) Without market competition ($\lambda = 0$), when the market is dominated by Carrier r, we have $\frac{\partial P_{3-r}^*}{\partial \sigma} = 0$. Besides, we have $\frac{\partial P_r^*}{\partial \sigma} > 0$ if $c_r > a_0$.

Proposition 4 shows that the relative risk-averse attitudes and market situations (i.e., market segmentation and competition) play critical roles in determining the impacts of demand uncertainty on the optimal prices. Proposition 4(i) considers a duopoly market shared by two competing carriers ($0 < \theta < 1,\ \lambda > 0$). Specifically, Proposition 4(i)a) indicates that if the operating cost is very large, a carrier with sufficiently higher relative risk-averse attitude would charge a higher price when the market becomes more volatile. A higher relative risk-averse attitude actually implies that a carrier is becoming more risk-averse to profit losses relative to its competitor. Therefore, when a carrier with high relative risk-averse attitude is facing with increasing demand uncertainty, due to the fear of demand losses, it will not raise its price unless it is challenged by a high operating cost. Interestingly, a carrier will also increase its price according to demand uncertainty if both the relative risk-averse attitude and the operating cost of its competitor are sufficiently large (see Proposition 4(i)b)). This is essentially motivated by the competitor's intension to raise price to deal with profit losses in a more volatile environment.

On the other hand, if there is no competition in the duopoly market, each carrier will increase its price to hedge against the increasing demand volatility if its own operating cost is high enough, while the other carrier's risk attitudes and costs are irrelevant, as shown in Proposition 4(ii).

Moreover, Proposition 4(iii) considers a monopoly market where the market is dominated by one player ($\theta = 0$ or 1). First, if competition exists, both two carriers will choose a higher price with the increase of demand uncertainty if the dominator's operating cost is sufficiently large (as shown in Proposition 4(iii)a)). It is intuitive that the dominator will increase its price to withstand the profit risks brought by the increased demand volatility when its operating cost is very high. Second, as shown in Proposition 4(iii)b), if there is no market competition, the rise in demand uncertainty will lead the dominator to increase its price if its cost is sufficiently high, which is similar to the situation with market competition. However, the other carrier in the market will keep its price no matter when demand becomes increasingly or decreasingly uncertain, which is different from the situation with market competition.

In short, Proposition 4 indicates that in addition to market competition and market segmentation, the relative risk-averse attitude of the two carriers also plays a pivotal role in characterizing the impacts of demand uncertainty on the optimal prices. Therefore, it is suggested for an air cargo carrier that it is essential to consider not only its own risk behaviors and operating characteristics, but also its competitor's decisions and features for strategic decision making in the uncertain and competitive environment.

Finally, we summarize some important relationships between the optimal price (P_r^*) with major coefficients and corresponding conditions identified in the basic model in **Table 2**. With this table, we aim to provide a quick look about the impacts of the diverse crucial parameters on the optimal prices for risk-averse air cargo carriers. Specifically, the first three lines in Table 2 provide the summaries for Proposition 1, which show the impacts of market competition level and operating costs on the optimal pricing decisions. Then, the results in the fourth and fifth lines summarize the important insights of Proposition 2. The results not only underline the importance of considering the risk attitudes of air cargo carriers in the optimal pricing decisions, but also highlight the impacts of market competition in determining the role of competitor's risk behaviors and operating costs in the optimal prices. In the sixth line, we give a quick view for Proposition 3, in which the relationships between the optimal prices and market share are evaluated. Finally, Proposition 4 is summarized in the last line, where we can observe that the market competition level, market segmentation, relative risk-averse attitude and operating cost all play the vital role in characterizing the impacts of demand uncertainty on the optimal prices. Notice that, all the specific details can be found in the previous propositions.

Table 2. Important relationships between P_r^* with major parameters.

	A	Conditions	B	Conditions	C	Conditions	D	Conditions	Remarks
λ	↑								
c_r	↑								
c_{3-r}	↑	$\lambda > 0$							
k_r	→	$\sigma = 0$	↑	$\sigma \neq 0$,$0 < \theta < 1$, $\lambda > 0$, $c_r > CT_r$	→	$\sigma \neq 0$, the market is dominated by Carrier (3-*r*)			
k_{3-r}	→	$\sigma = 0$	↑	$\sigma \neq 0$,$0 < \theta < 1$, $\lambda > 0$,$c_{3-r} > CT_{3-r}$	→	$\sigma \neq 0$, the market is dominated by Carrier *r*	→	$\sigma \neq 0$, $\lambda = 0$	
θ (1-θ)	↑	$\sigma = 0$	↑	$\sigma \neq 0$, $\lambda > 0$, $c_r > DT_r$	↑	$\sigma \neq 0$, $\lambda > 0$, $c_{3-r} < ET_{3-r}$	↑	$\sigma \neq 0$, $\lambda = 0$	For Carrier 2 (Carrier 1)
σ	↑	$0 < \theta < 1,\ \lambda > 0,$ $\tau_r > \Lambda_r,\ c_r > YT_r$	↑	$0 < \theta < 1,\ \lambda > 0,$ $\tau_{3-r} > \Omega_{3-r}$, $c_{3-r} > PT_{3-r}$	↑	$\lambda > 0$, $c_r > OT_r$, the market is dominated by Carrier *r*			

V. EXTENDED ANALYSES – UNCERTAIN COSTS

As discussed in the introduction, cost uncertainty is a crucial and challenging problem which significantly affects the profitability and development of air cargo carriers. Therefore, after evaluating the pricing decisions under uncertain demand, in this section, we extend the basic model to the case when the operating cost is stochastic, to study the impact of cost uncertainty on the optimal pricing decisions. Here, the air freight carriers are facing with uncertain unit costs, $\widetilde{c_r}$ $(\widetilde{c_r} = c_{r0} + \varphi)$. $\widetilde{c_r}$ consists of a fixed part c_{r0} and an uncertain part φ which follows a normal distribution with the mean of zero and standard deviation of δ $(\varphi \sim N(0, \delta^2))$. Besides, we denote the unit price for the carriers in the extended model as P_r^e. Therefore, we could obtain the updated profit functions for the two players as in Eq. (10) and Eq. (11). The updated expected profits are then shown in Eq. (12) and Eq. (13), while the corresponding variances of profits are illustrated in Eq. (14) and Eq. (15).

Eq. (10) $\quad \pi_1^e = (P_1^e - \widetilde{c_1})[(1-\theta)\tilde{a} - P_1^e + \lambda P_2^e],$

Eq. (11) $\quad \pi_2^e = (P_2^e - \widetilde{c_2})(\theta\tilde{a} - P_2^e + \lambda P_1^e).$

Eq. (12) $\quad E(\pi_1^e) = (P_1^e - c_{10})[(1-\theta)\ \ a_0 - P_1^e + \lambda P_2^e],$

Eq. (13) $E(\pi_2^e) = (P_2^e - c_{20})(\theta a_0 - P_2^e + \lambda P_1^e)$.

Eq. (14) $V(\pi_1^e) = (1-\theta)^2\sigma^2[\delta^2 + (P_1^e - c_{10})^2] + \delta^2[(1-\theta)\ a_0 - P_1^e + \lambda P_2^e]^2$,

Eq. (15) $V(\pi_2^e) = \theta^2\sigma^2[\delta^2 + (P_2^e - c_{20})^2] + \delta^2(\theta a_0 - P_2^e + \lambda P_1^e)^2$.

Similarly, the MV theory is applied to measure the impact of loss-averse attitudes on pricing decisions with the influence of cost uncertainty. The MV objective function for the extended model is constructed in Eq. (16).

Eq. (16) Maximize: $O_r^e = E(\pi_r^e) - k_r V(\pi_r^e)$. ($r$=1,2)

Solving Eq. (16), we could obtain the optimal pricing decisions for the two risk-averse competing carriers under demand and cost uncertainties (i.e., P_1^{e*} and P_2^{e*}), which are summarized in Lemma 2.

Lemma 2. *In the extended model with uncertain demand and uncertain costs, the MV objective functions for the two competing risk-averse carriers are strictly concave, and the respective optimal prices are given as follows*:

$$P_1^{e*} = \frac{2(1+S_2k_2+k_2\delta^2)[(1-\theta)a_0(1+2k_1\delta^2)+c_{10}(1+2S_1k_1)]+\lambda(1+2k_1\delta^2)[\theta a_0(1+2k_2\delta^2)+c_{20}(1+2S_2k_2)]}{4(1+S_1k_1+k_1\delta^2)(1+S_2k_2+k_2\delta^2)-\lambda^2(1+2k_1\delta^2)(1+2k_2\delta^2)}$$

$$, P_2^{e*} = \frac{2(1+S_1k_1+k_1\delta^2)[\theta a_0(1+2k_2\delta^2)+c_{20}(1+2S_2k_2)]+\lambda(1+2k_2\delta^2)[(1-\theta)a_0(1+2k_1\delta^2)+c_{10}(1+2S_1k_1)]}{4(1+S_1k_1+k_1\delta^2)(1+S_2k_2+k_2\delta^2)-\lambda^2(1+2k_1\delta^2)(1+2k_2\delta^2)}.$$

Similar to the basic model, Lemma 2 shows that optimal pricing decisions exist for air cargo carriers to maximize the MV objectives when they face stochastic costs. Besides, the equilibrium prices for the two risk-averse carriers are perfectly symmetric in the extended model. In addition to the major parameters like market share (θ), market competition (λ), market uncertainty (σ), and risk attitudes of the two players (k_r), it is important to note that cost uncertainty (δ) also plays an important role in determining the optimal prices. In the expressions, $\eta_r = 1+2k_r\delta^2$ is defined as the *cost uncertainty risk coefficient*, which reflects the integrated impact of cost uncertainty and loss aversion on the optimal decision making. Applying the notation listed in **Table 3** (Appendix A1), P_1^{e*} and P_2^{e*} could be represented as $P_1^{e*} = \frac{B_2B_3+\lambda\eta_1B_4}{B_1B_2-\lambda^2\eta_1\eta_2}$ and $P_2^{e*} = \frac{B_1B_4+\lambda\eta_2B_3}{B_1B_2-\lambda^2\eta_1\eta_2}$, respectively. Obviously, P_r^{e*} and P_r^* have similar forms expect that P_r^{e*} is featured with the cost uncertainty risk coefficient η_r. Next, Proposition 5 demonstrates the specific impacts of cost uncertainty on the equilibrium prices under different scenarios.

Proposition 5. *In the extended model where two risk-averse carriers facing stochastic costs compete for uncertain demand, the impacts of cost uncertainty on the optimal prices are diverse as follows:*

(i) With market competition ($\lambda > 0$), for Carrier r:

a) If $\tau_r > \vartheta_r$, its optimal price increases with cost uncertainty if $c_{r0} < WT_r$.

b) If $\tau_{3-r} > \varsigma_{3-r}$, its optimal price increases with cost uncertainty if $c_{(3\text{-}r)0} < UT_{3-r}$.

(ii) Without market competition ($\lambda = 0$):

a) Under a duopoly market ($0 < \theta < 1$), for Carrier 1, its optimal price increases with cost uncertainty if $c_{10} < (1-\theta)a_0$, while for Carrier 2, its optimal price increases with cost *uncertainty if* $c_{20} < \theta a_0$.

b) *Under a monopoly market (*$\theta = 0$ *or* 1*), when the market is dominated by Carrier r, we have* $\frac{\partial P_{3-r}^{e\ *}}{\partial \delta} \leq 0$. *Besides, we have* $\frac{\partial P_r^{e*}}{\partial \delta} > 0$ *if* $c_{r0} < a_0$.

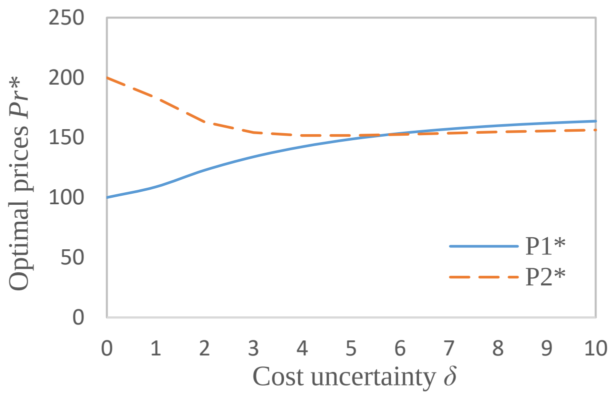
From Proposition 5, we could see that the role of cost uncertainty not only depends on market conditions such as market competition and segmentation, but also hinges on the relative risk-averse attitude (τ_r), which is similar to the effect of demand uncertainty as discussed in Proposition 4.

First of all, with market competition, Proposition 5(i)a) shows that an increase in cost uncertainty would lead to a growth in the optimal price if the fixed part of a carrier's cost (c_{r0}) is low enough ($c_{r0} < WT_r$) and its relative risk-averse attitude over its competitor is high enough. Intuitively, a carrier will charge a higher price to hedge against the increased uncertainty in its operating cost. However, due to demand uncertainty and market competition, a rise in price may result in a reduction in consumer demand. Thus, a carrier with sufficiently higher relative risk-averse attitude will not rise its price when its cost is becoming increasingly stochastic unless the fixed part of its cost could be controlled in a low level (i.e., $c_{r0} < WT_r$). Besides, a carrier would also increase its price along with cost uncertainty if its competitor's relative risk-averse attitude is very high and the competitor's cost (fixed part) is very low (see Proposition 5(i)b)). The motivation is the competitor's proneness to deal with the profit risks caused by the increased cost uncertainty through raising price.

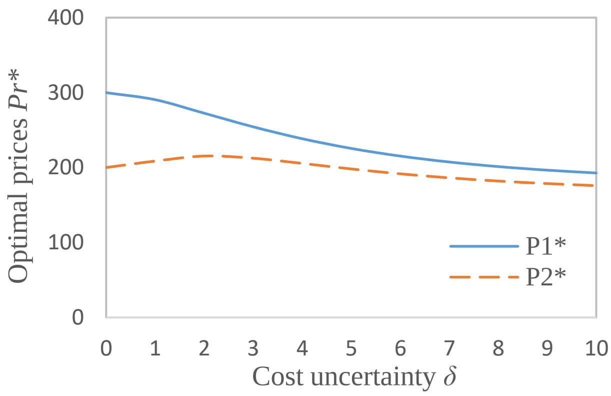
On the other hand, if the two participants do not compete ($\lambda = 0$), the influence of cost uncertainty on a carrier's optimal prices further depends on market segmentation, while the operating characteristics of the other carrier in the market becomes irrelevant. Specifically, in a duopoly market (see Proposition 5(ii)a)), the optimal price of a carrier would be positively related to cost uncertainty if the fixed part of its cost is low enough. The intuition is that only when the fixed component of cost is sufficiently low, it will be possible for the carrier to maintain profitability after increasing price (demand decreases accordingly) to deal with the increased cost uncertainty. Similarly, Carrier *r* will raise its price along with cost uncertainty if $c_{r0} < a_0$ when the market is dominated by Carrier *r* (in Proposition 5(ii)b)). However, in the monopoly market, the other carrier has to decrease its price when the operating cost becomes more stochastic to keep profitability.

In short, Proposition 5 derives insights regarding the impacts of cost uncertainty on the optimal pricing decisions for risk-averse air cargo carriers under a competitive and uncertain market environment.

Then, aiming at comparing the optimal pricing decisions between two participants with volatile operating cost, we proceed to illustrate the difference of optimal prices with respect to the cost uncertainty in Figure 3.



(a) When $c_{10} < WT_1$



(b) When $c_{10} \geq WT_1$

Figure 3. The comparison between optimal pricing decisions with respect to the cost uncertainty δ (We set $a_0 = 100,\ \theta = 0.4,\ \sigma = 5,\ \lambda = 0.7,\ c_{20} = 200,\ k_1 = 80,\ k_2 = 100$ and $c_{10} = 100$ in (a), $c_{10} = 300$ in (b))

Figure 3 uncovers the relationships between the optimal prices of two participants. To be specific, Figure 3 (a) reveals that, with the growth of cost uncertainty, the Carrier 1's optimal price may surpass its rival's when the fixed part of its cost is relatively low ($c_{10} < WT_1$). While from Figure 3 (b) we can observe that, when the fixed part of Carrier 1's cost is controlled in a relatively high level ($c_{10} \geq WT_1$), the carrier's optimal price is always higher than its competitor's, regardless of the cost uncertainty. It is understandable that there is no need for the carrier to charge a higher price if the cost uncertainty is relatively low. However, if the fixed part of the operating cost is extremely high, it is unadvisable for the carrier to charge a lower price, which may hurt its profit. Moreover, the numerical analysis in Figure 3 could attest and verify the analytical results in Proposition 5(i) in a visualized way.

Considering the highly volatile crude oil market and the significant fuel price fluctuation in the modern world, the analytical challenging analysis with cost uncertainty is important. Thus, we believe that the extended analyses in this section is definitely imperative and critical. It could provide useful implications and guidelines for both practitioners and academics on the strategies to deal with the challenges arising from air freight operating costs.

VI. CONCLUSION

A. Concluding Remarks, Insights and Implications

Nowadays, air freight transportation is becoming increasingly important for global logistics systems to facilitate quick and reliable logistics services. However, despite the fast growth, the industry is challenged by intensive market competition and highly volatile consumer demand. Besides, airlines are also facing with significant operating cost uncertainty caused by jet-fuel price fluctuation. As a result, in order to enhance their survivability and profitability in the highly competitive and stochastic market environment, many air cargo carriers become conservative and behave as risk-averse in decision making. Among the strategic decisions of cargo airlines, the optimal pricing problem is the most crucial but challenging one, which significantly impacts the development of air cargo carriers. However, although the importance of pricing decisions with risk considerations has been realized, the optimal pricing decisions for loss-averse air cargo carriers in the presence of cost and demand uncertainties are still under-investigated. This paper thus aims to examine the impacts of risk attitudes of decision makers, market competition, demand uncertainty and cost uncertainty on the optimal pricing decisions for air cargo carriers by applying the MV theory.

Through analytically exploring a basic model where two risk-averse air cargo carriers with deterministic operating costs compete for uncertain demand and an extended model where both demand and cost are uncertain, we have studied the equilibrium prices for the two carriers and explored the impacts of diverse crucial parameters on the optimal decision making. Our analytical and numerical results have generated the following major findings and insights. First, we have identified that the optimal prices and important thresholds for the two carriers are perfectly symmetric either in the basic or the extended model. Second, we have found that the optimal price of a carrier is affected not only by its own risk attitudes and costs, but also by the competitor's characteristics (e.g., costs, risk preferences) if they engage in a competition. Otherwise, without competition, the decisions of the two carriers are irrelevant. Third, we have demonstrated that the risk-averse behaviors of carrier managers could affect the optimal prices either directly or indirectly. For instance, if a carrier's cost is high enough, its increasing risk-averse attitude could directly lead to a growth in the optimal prices for the two participants in a duopoly market with competition. On the other hand, the loss-averse behaviors could impose indirect impacts on the optimal prices through affecting the effects of other important parameters. More importantly, the relative risk-averse attitudes of the two carriers are demonstrated to be crucial in evaluating the impacts of both demand and cost uncertainties on the optimal decision making. Fourth, we have revealed that market situations are critical determinants in the risk-averse pricing decisions. For example, when the two carriers compete more fiercely, both two players will increase their prices. Besides, market segmentation (i.e., whether the market is duopoly or monopoly) imposes great impacts on the effects of risk sensitivity coefficients, demand uncertainty and cost uncertainty. For instance, if the market is dominated by a carrier without competition, then the risk attitude of the other carrier becomes nonsignificant. Besides, the other carrier's optimal price is irrelevant to demand uncertainty (or negatively related to cost uncertainty), while the dominator's optimal price is positively related to demand uncertainty (or cost uncertainty) if the dominator's cost is sufficiently high (or low).

To conclude, this work contributes to the existing literature of systems engineering and science by integrating risk considerations, market competition and market uncertainties (demand and cost) into the optimal pricing decisions for air cargo carriers. We have analytically explored the equilibrium solutions and investigated how the crucial factors impact the optimal prices. Through comprehensive investigation, we have highlighted the importance to enhance pricing decisions for cargo airlines by considering these critical factors in the current highly volatile and competitive market.

B. Future Studies

This paper generates useful insights regarding the optimal pricing decisions for competing air cargo carriers who are risk-averse to profit losses caused by uncertain demand and stochastic operating costs. For future research, it will be interesting to explore the situation where the decision makers have different risk attitudes (e.g., one is risk-neutral and the other is risk-averse). Besides, more risk measurements like mean-downside risk (MDR) approach [15] could be applied to characterize the risk behaviors of air cargo carriers.

Acknowledgments

This research was partially supported by The Hong Kong Polytechnic University (under the account

code RUE7).

REFERENCES

[1] N. Li, F. T. Chan, S. H. Chung, and A. H. Tai, "A stochastic production-inventory model in a two-state production system with inventory deterioration, rework process, and backordering," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 6, pp. 916-926, 2017.

[2] M. Li, Z. Wang, and F. T. Chan, "An inventory routing policy under replenishment lead time," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 12, pp. 3150-3164, 2017.

[3] W. H. Wong, A. Zhang, Y. Van Hui, and L. C. Leung, "Optimal baggage-limit policy: Airline passenger and cargo allocation," *Transportation Science*, vol. 43, no. 3, pp. 355-369, 2009.

[4] Y. Tao, E. P. Chew, L. H. Lee, and L. Wang, "A capacity pricing and reservation problem under option contract in the air cargo freight industry," *Computers & Industrial Engineering*, vol. 110, pp. 560-572, 2017.

[5] IATA, "Air Freight Market Analysis," 2018, Available: https://www.iata.org/publications/economics/Reports/freight-monthly-analysis/freight-analysis-apr-2018.pdf.

[6] Airbus, "Global Market Forcast 2017-2036," 2017, Available: https://www.airbus.com/search.page.html?tags=corporate-topics%3Astrategy/global-market-forecast.

[7] Boeing, "Current Market Outlook 2017-2036," 2017, Available: http://www.boeing.com/resources/boeingdotcom/commercial/market/current-market-outlook-2017/assets/downloads/2017-cmo-6-19.pdf.

[8] B. Feng, Y. Li, and Z.-J. M. Shen, "Air cargo operations: Literature review and comparison with practices," *Transportation Research Part C: Emerging Technologies*, vol. 56, pp. 263-280, 2015.

[9] IATA, "IATA Cargo Chartbook December 2017," 2017, Available: https://www.iata.org/publications/economics/Reports/freight-monthly-analysis/freight-analysis-dec-2017.pdf.

[10] W. Powell and C. Winston, "A numerical investigation of the impact of uncertain demand and varying risk preferences on the pricing and capacity decisions of transportation firms: The case of airlines," *Transportation Research Part B: Methodological*, vol. 17, no. 6, pp. 471-490, 1983.

[11] C.-C. Chao and C.-W. Hsu, "Cost analysis of air cargo transport and effects of fluctuations in fuel price," *Journal of Air Transport Management*, vol. 35, pp. 51-56, 2014.

[12] F. Azadian and A. Murat, "Service location grouping and pricing in transportation: Application in air cargo," *European Journal of Operational Research*, vol. 267, no. 3, pp. 933-943, 2018.

[13] J. Zhang, R. Dai, Q. Zhang, and W. Tang, "An optimal energy efficiency investment and product pricing strategy in a two-market framework," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 4, pp. 608-621, 2018.

[14] Z. He, T. Cheng, J. Dong, and S. Wang, "Evolutionary location and pricing strategies in competitive hierarchical distribution systems: A spatial agent-based model," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 44, no. 7, pp. 822-833, 2014.

[15] H.-L. Chan, T.-M. Choi, Y.-J. Cai, and B. Shen, "Environmental taxes in newsvendor supply chains: A mean-downside-risk analysis," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2018 (in press).

[16] C.-H. Chen and J.-H. Chou, "Multiobjective optimization of airline crew roster recovery

problems under disruption conditions," *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* vol. 47, no. 1, pp. 133-144, 2017.
[17] S. H. Chung, H. L. Ma, and H. K. Chan, "Cascading delay risk of airline workforce deployments with crew pairing and schedule optimization," *Risk Analysis,* vol. 37, no. 8, pp. 1443-1458, 2017.
[18] T. Doi, T. Nishi, and S. Voß, "Two-level decomposition-based matheuristic for airline crew rostering problems with fair working time," *European Journal of Operational Research,* vol. 267, no. 2, pp. 428-438, 2018.
[19] Z. Liang, F. Xiao, X. Qian, L. Zhou, X. Jin, X. Lu, and S. Karichery, "A column generation-based heuristic for aircraft recovery problem with airport capacity constraints and maintenance flexibility," *Transportation Research Part B: Methodological,* vol. 113, pp. 70-90, 2018.
[20] K. Wei and V. Vaze, "Modeling crew itineraries and delays in the national air transportation system," *Transportation Science,* 2018 (in press).
[21] F. Wang, X. Zhuo, and B. Niu, "Strategic entry to regional air cargo market under joint competition of demand and promised delivery time," *Transportation Research Part B: Methodological,* vol. 104, pp. 317-336, 2017.
[22] C. Barz and D. Gartner, "Air cargo network revenue management," *Transportation Science,* vol. 50, no. 4, pp. 1206-1222, 2016.
[23] M. Wada, F. Delgado, and B. K. Pagnoncelli, "A risk averse approach to the capacity allocation problem in the airline cargo industry," *Journal of the Operational Research Society,* vol. 68, no. 6, pp. 643-651, 2017.
[24] R. Hellermann, A. Huchzermeier, and S. Spinler, "Options contracts with overbooking in the air cargo industry," *Decision Sciences,* vol. 44, no. 2, pp. 297-327, 2013.
[25] L. C. Leung, Y. Van Hui, Y. Wang, and G. Chen, "A 0–1 LP model for the integration and consolidation of air cargo shipments," *Operations Research,* vol. 57, no. 2, pp. 402-412, 2009.
[26] U. Derigs, S. Friederichs, and S. Schäfer, "A new approach for air cargo network planning," *Transportation Science,* vol. 43, no. 3, pp. 370-380, 2009.
[27] Y. Li, Y. Tao, and F. Wang, "A compromised large-scale neighborhood search heuristic for capacitated air cargo loading planning," *European Journal of Operational Research,* vol. 199, no. 2, pp. 553-560, 2009.
[28] X. Gan, S. P. Sethi, and H. Yan, "Coordination of supply chains with risk‐averse agents," *Production and Operations Management,* vol. 13, no. 2, pp. 135-149, 2004.
[29] Y. Shang, D. Dunson, and J.-S. Song, "Exploiting big data in logistics risk assessment via Bayesian nonparametrics," *Operations Research,* vol. 65, no. 6, pp. 1574-1588, 2017.
[30] S. Asian and X. Nie, "Coordination in supply chains with uncertain demand and disruption risks: Existence, analysis, and insights," *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* vol. 44, no. 9, pp. 1139-1154, 2014.
[31] B. Shen, T.-M. Choi, Y. Wang, and C. K. Lo, "The coordination of fashion supply chains with a risk-averse supplier under the markdown money policy," *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* vol. 43, no. 2, pp. 266-276, 2013.
[32] T.-M. Choi, "Supply chain systems coordination with multiple risk sensitive retail buyers," *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* vol. 46, no. 5, pp. 636-645, 2016.
[33] Y. Xie, H. Wang, and H. Lu, "Coordination of supply chains with a retailer under the mean-CVaR criterion," *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* vol. 48, no. 7, pp. 1039-1053, 2018.
[34] B. Zhang, D. Wu, L. Liang, and D. L. Olson, "Supply chain loss averse newsboy model with capital constraint," *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* vol. 46, no. 5, pp. 646-658, 2016.
[35] B. Lazzerini and F. Pistolesi, "Multiobjective personnel assignment exploiting workers'

sensitivity to risk," *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* vol. 48, no. 8, pp. 1267-1282, 2018.
[36] Y. Qin, Q. Zhang, C. Zhou, and N. Xiong, "A risk-based dynamic decision-making approach for cybersecurity protection in industrial control systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* no. 99, pp. 1-8, 2018.
[37] Z.-X. Yang, G.-G. Wu, L. Song, and L.-P. Zhang, "Control change cause analysis-based Bayesian network modeling for system risk assessment," *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* 2018 (in press).
[38] Y. Chang, A. L. Erera, and C. C. White, "Risk assessment of deliberate contamination of food production facilities," *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* vol. 47, no. 3, pp. 381-393, 2017.
[39] C.-H. Chiu and T.-M. Choi, "Supply chain risk analysis with mean-variance models: A technical review," *Annals of Operations Research,* vol. 240, no. 2, pp. 489-507, 2016.
[40] W. Zheng, B. Li, and D.-P. Song, "Effects of risk-aversion on competing shipping lines' pricing strategies with uncertain demands," *Transportation Research Part B: Methodological*, vol. 104, pp. 337-356, 2017.
[41] V. Agrawal and S. Seshadri, "Impact of uncertainty and risk aversion on price and order quantity in the newsvendor problem," *Manufacturing & Service Operations Management*, vol. 2, no. 4, pp. 410-423, 2000.
[42] B. Li, P. Chen, Q. Li, and W. Wang, "Dual-channel supply chain pricing decisions with a risk-averse retailer," *International Journal of Production Research,* vol. 52, no. 23, pp. 7132-7147, 2014.
[43] M. Liu, E. Cao, and C. K. Salifou, "Pricing strategies of a dual-channel supply chain with risk aversion," *Transportation Research Part E: Logistics and Transportation Review,* vol. 90, pp. 108-120, 2016.
[44] Y. Li, Q. Lin, and F. Ye, "Pricing and promised delivery lead time decisions with a risk-averse agent," *International Journal of Production Research,* vol. 52, no. 12, pp. 3518-3537, 2014.
[45] C. H. Chiu, T. M. Choi, X. Dai, B. Shen, and J. H. Zheng, "Optimal advertising budget allocation in luxury fashion markets with social influences: A mean - variance analysis," *Production and Operations Management*, vol. 27, no. 8, pp. 1611-1629, 2018.
[46] T. M. Choi, J. Zhang, and T. E. Cheng, "Quick response in supply chains with stochastically risk sensitive retailers," *Decision Sciences*, vol. 49, no. 5, pp. 932-957, 2018.
[47] C.-H. Chiu, T.-M. Choi, G. Hao, and X. Li, "Innovative menu of contracts for coordinating a supply chain with multiple mean-variance retailers," *European Journal of Operational Research,* vol. 246, no. 3, pp. 815-826, 2015.
[48] J. Li, T.-M. Choi, and T. E. Cheng, "Mean variance analysis of fast fashion supply chains with returns policy," *IEEE Transactions on Systems, Man, and Cybernetics: Systems,* vol. 44, no. 4, pp. 422-434, 2014.
[49] J. Shao, Jing , H. Krishnan , and S. T. Mccormick . "Price incentives and coordination in a two-product decentralized supply chain," *Decision Sciences,* vol. 45, no. 3, pp. 507-533, 2014.
[50] Guthrie, B., P. Parikh, and N. Kong. "Evaluating warehouse strategies for two-product class distribution planning," *International Journal of Production Research*, vol. 55, no. 21, pp. 6470–6484, 2017.
[51] B. Shen, X. Xu, and T. M. Choi. "Simplicity is beauty: pricing coordination in two-product supply chains with simplest contracts under voluntary compliance." *International Journal of Production Research*, vol. 57, no. 9, pp. 2769-2787, 2019.
[52] P Kouvelis, Z Pang, Q Ding. " Integrated commodity inventory management and financial hedging: A dynamic mean-variance analysis," *Production and Operations Management*, vol. 27, no. 6, pp.1052-1073, 2018.

APPENDIX (A1)

Table 3 summarizes the important notation used in the analyses (both in the main context and in the online mathematical proofs).

Table 3. Important notation used in the analyses.

Notation	Remarks	Notation	Remarks
$S_1 = (1-\theta)^2\sigma^2$	$S_1 \geq 0$	$T_3 = 1 + S_1k_1$	$T_3 \geq 1$
$S_2 = \theta^2\sigma^2$	$S_2 \geq 0$	$T_4 = 1 + S_2k_2$	$T_4 \geq 1$
$A_1 = 2(1+S_1k_1)$	$A_1 \geq 2$	$\eta_1 = 1 + 2k_1\delta^2$	$\eta_1 \geq 1$
$A_2 = 2(1+S_2k_2)$	$A_2 \geq 2$	$\eta_2 = 1 + 2k_2\delta^2$	$\eta_2 \geq 1$
$A_3 = (1-\theta)a_0 + c_1(1+2S_1k_1)$	$A_3 \geq 0$	$B_1 = 2(1+S_1k_1+k_1\delta^2)$	$B_1 \geq 2$
$A_4 = \theta a_0 + c_2(1+2S_2k_2)$	$A_4 \geq 0$	$B_2 = 2(1+S_2k_2+k_2\delta^2)$	$B_2 \geq 2$
$T_1 = 1 + 2S_1k_1$	$T_1 \geq 1$	$B_3 = (1-\theta)a_0(1+2k_1\delta^2) + c_{10}(1+2S_1k_1)$	$B_3 \geq 0$
$T_2 = 1 + 2S_2k_2$	$T_2 \geq 1$	$B_4 = \theta a_0(1+2k_2\delta^2) + c_{20}(1+2S_2k_2)$	$B_4 \geq 0$

The crucial relative risk-averse attitude thresholds identified in the analyses are listed in **Table 4**. We could see that each pair of relative risk-averse attitude thresholds for the two carriers are perfectly symmetric (e.g., $\Lambda_1$andΛ_2).

Table 4. Crucial relative risk-averse attitude thresholds identified in the analyses.

Λ_1	$\frac{\lambda^2\theta^2T_1}{(1-\theta)^2A_2(A_2-\lambda^2}$	ϑ_1	$\frac{\lambda^2\eta_1(B_2-\eta_2)}{B_2(B_2-\lambda^2\eta_2)}$
Λ_2	$\frac{\lambda^2(1-\theta)^2T_2}{\theta^2A_1(A_1-\lambda^2)}$	ϑ_2	$\frac{\lambda^2\eta_2(B_1-\eta_1)}{B_1(B_1-\lambda^2\eta_1)}$
Ω_2	$\frac{(1-\theta)^2A_2T_2}{\theta^2(A_1-\lambda^2)}$	ς_1	$\frac{B_1(B_2-\eta_2)}{\eta_2(B_2-\lambda^2\eta_2)}$
Ω_1	$\frac{\theta^2A_1T_1}{(1-\theta)^2(A_2-\lambda^2)}$	ς_2	$\frac{B_2(B_1-\eta_1)}{\eta_1(B_1-\lambda^2\eta_1)}$

Besides, **Table 5** concludes the important cost thresholds for the two carriers in determining the impacts of diverse parameters on the optimal prices. It is seen that all pairs of cost thresholds are perfectly symmetric (e.g., CT_1 and CT_2).

Table 5. Crucial cost thresholds identified in the analyses.

CT_1	$\frac{2(1+S_2k_2)(1-\theta)a_0 + \lambda\left[\theta a_0 + c_2(1+2S_2k_2)\right]}{2(1+S_2k_2)-\lambda^2}$
CT_2	$\frac{2(1+S_1k_1)\theta a_0 + \lambda\left[(1-\theta)a_0 + c_1(1+2S_1k_1)\right]}{2(1+S_1k_1)-\lambda^2}$

DT_1

$$\frac{a_0(\lambda - A_2)(A_1A_2 - \lambda^2) + 4\sigma^2\left\{\begin{array}{l} k_1(1-\theta)A_2a_0\left[A_2(1-\theta)+\lambda\theta\right] - \lambda k_2\theta a_0\left[\theta A_1 + \lambda(1-\theta)\right] \\ +[k_1(1-\theta)A_2(1+2S_2k_2) + k_2\theta(A_1-\lambda^2)]\lambda c_2 \end{array}\right\}}{4\sigma^2\left[k_1(1-\theta)A_2(A_2-\lambda^2)+\lambda^2k_2\theta T_1\right]}$$

DT_2

$$\frac{a_0(\lambda - A_1)(A_1A_2 - \lambda^2) + 4\sigma^2\left\{\begin{array}{l} k_2\theta A_1a_0[A_1\theta+\lambda(1-\theta)] - \lambda k_1(1-\theta)a_0\left[(1-\theta)A_2+\lambda\theta\right] \\ +[k_2\theta A_1(1+2S_1k_1) + k_1(1-\theta)(A_2-\lambda^2)]\lambda c_1 \end{array}\right\}}{4\sigma^2\left[k_2\theta A_1(A_1-\lambda^2)+\lambda^2k_1(1-\theta)T_2\right]}$$

ET_1

$$\frac{a_0(A_1 - \lambda)(A_1A_2 - \lambda^2) + 4\sigma^2\left\{\begin{array}{l} -k_2\theta A_1a_0\left[A_1\theta+(1-\theta)\lambda\right] + \lambda k_1(1-\theta)a_0\left[(1-\theta)A_2+\lambda\theta\right] \\ +[k_2\theta A_1(A_1-\lambda^2) + \lambda^2k_1(1-\theta)(1+2S_2k_2)]c_2 \end{array}\right\}}{4\sigma^2\left[k_2\theta A_1\lambda T_1+\lambda k_1(1-\theta)(A_2-\lambda^2)\right]}$$

ET_2

$$\frac{a_0(A_2 - \lambda)(A_1A_2 - \lambda^2) + 4\sigma^2\left\{\begin{array}{l} -k_1(1-\theta)A_2a_0\left[A_2(1-\theta)+\theta\lambda\right] + \lambda k_2\theta a_0[\theta A_1+\lambda(1-\theta)] \\ +[k_1(1-\theta)A_2(A_2-\lambda^2) + \lambda^2k_2\theta(1+2S_1k_1)]c_1 \end{array}\right\}}{4\sigma^2\left[k_1(1-\theta)A_2\lambda T_2+\lambda k_2\theta(A_1-\lambda^2)\right]}$$

YT_1

$$\frac{a_0[k_1(1-\theta)^2A_2[(1-\theta)A_2+\lambda\theta] + \lambda k_2\theta^2[\theta A_1+\lambda(1-\theta)]] - c_2[\lambda k_2\theta^2(A_1-\lambda^2) - k_1(}{k_1(1-\theta)^2A_2(A_2-\lambda^2) - \lambda^2k_2\theta^2T_1}$$

YT_2

$$\frac{a_0[k_2\theta^2A_1[\theta A_1+\lambda(1-\theta)] + \lambda k_1(1-\theta)^2[(1-\theta)A_2+\lambda\theta]] - c_1[\lambda k_1(1-\theta)^2(A_2-\lambda^2)}{k_2\theta^2A_1(A_1-\lambda^2) - \lambda^2k_1(1-\theta)^2T_2}$$

PT_1

$$\frac{a_0[k_2\theta^2A_1[\theta A_1+\lambda(1-\theta)] + \lambda k_1(1-\theta)^2[(1-\theta)A_2+\lambda\theta]] - c_2[k_2\theta^2A_1(A_1-\lambda^2) - \lambda^2}{\lambda k_1(1-\theta)^2(A_2-\lambda^2) - k_2\theta^2A_1\lambda T_1}$$

PT_2

$$\frac{a_0[k_1(1-\theta)^2A_2[(1-\theta)A_2+\lambda\theta] + \lambda k_2\theta^2[\theta A_1+\lambda(1-\theta)]] - c_1[k_1(1-\theta)^2A_2(A_2-\lambda^2}{\lambda k_2\theta^2(A_1-\lambda^2) - k_1(1-\theta)^2A_2\lambda T_2}$$

WT_1

$$\frac{\begin{array}{l}[k_2\lambda\eta_1(B_1-\lambda^2\eta_1) - k_1B_2\lambda(B_1-\eta_1)]B_4 + [k_1B_2(B_2-\lambda^2\eta_2) - k_2\lambda^2\eta_1(B_2-\eta_2)](1-\theta)a_0\eta_1 \\ -[(1-\theta)a_0k_1B_2 + \lambda k_2\eta_1\theta a_0](B_1B_2-\lambda^2\eta_1\eta_2)\end{array}}{[k_2\lambda^2\eta_1(B_2-\eta_2) + k_1B_2(\lambda^2\eta_2 - B_2)](1+2S_1k_1)}$$

WT_2

$$\frac{\begin{array}{l}[k_1\lambda\eta_2(B_2-\lambda^2\eta_2) - k_2B_1\lambda(B_2-\eta_2)]B_3 + [k_2B_1(B_1-\lambda^2\eta_1) - k_1\lambda^2\eta_2(B_1-\eta_1)]\theta a_0\eta_2 \\ -[\theta a_0k_2B_1 + \lambda k_1\eta_2(1-\theta)a_0](B_1B_2-\lambda^2\eta_1\eta_2)\end{array}}{[k_1\lambda^2\eta_2(B_1-\eta_1) + k_2B_1(\lambda^2\eta_1 - B_1)](1+2S_2k_2)}$$

UT_1

$$\frac{\begin{array}{l}[k_2B_1(B_1-\lambda^2\eta_1) - k_1\lambda^2\eta_2(B_1-\eta_1)]B_4 + [k_1\lambda\eta_2(B_2-\lambda^2\eta_2) - k_2B_1\lambda(B_2-\eta_2)](1-\theta)a \\ -[\theta a_0k_2B_1 + \lambda k_1\eta_2(1-\theta)a_0](B_1B_2-\lambda^2\eta_1\eta_2)\end{array}}{[k_2B_1\lambda(B_2-\eta_2) + k_1\lambda\eta_2(\lambda^2\eta_2 - B_2)](1+2S_1k_1)}$$

UT_2

$$\frac{\begin{array}{l}[k_1B_2(B_2-\lambda^2\eta_2) - k_2\lambda^2\eta_1(B_2-\eta_2)]B_3 + [k_2\lambda\eta_1(B_1-\lambda^2\eta_1) - k_1B_2\lambda(B_1-\eta_1)]\theta a_0\eta_2 \\ -[(1-\theta)a_0k_1B_2 + \lambda k_2\eta_1\theta a_0](B_1B_2-\lambda^2\eta_1\eta_2)\end{array}}{[k_1B_2\lambda(B_1-\eta_1) + k_2\lambda\eta_1(\lambda^2\eta_1 - B_1)](1+2S_2k_2)}$$

APPENDIX (A2): ALL PROOFS

Basic model

Proof of Lemma 1. Checking the second-order derivatives of Eq. (8) and Eq. (9), it is found that $\frac{\partial^2 O_1}{\partial (P_1)^2} = -2 - 2S_1k_1 < 0$ and $\frac{\partial^2 O_2}{\partial (P_2)^2} = -2 - 2S_2k_2 < 0$, which shows that both objective functions are concave in the respective unit price. Consequently, we could identify the reactive functions for the two players through solving the first-order conditions as follows:

$$P_1 = \arg_{P_1}\left\{\frac{\partial O_1}{\partial P_1} = 0\right\} \rightarrow P_1 / P_2 = \frac{(1-\theta)a_0 + \lambda P_2 + c_1(1+2S_1k_1)}{2(1+S_1k_1)}$$

$$P_2 = \arg_{P_2}\left\{\frac{\partial O_2}{\partial P_2} = 0\right\} \rightarrow P_2 / P_1 = \frac{\theta a_0 + \lambda P_1 + c_2(1+2S_2k_2)}{2(1+S_2k_2)}$$

Solving the reactive functions, the optimal prices (P_1^* and P_2^*) for the two carriers could be identified. Besides, $A_1A_2 > \lambda^2$ always holds. (Q.E.D.)

Proposition 1

(i) Checking the first-order derivatives of P_1^* and P_2^* with respect to λ, we get

$$\frac{\partial P_1^*}{\partial \lambda} = \frac{1}{\left[A_1A_2 - \lambda^2\right]^2}\left\{A_4\left(A_1A_2 - \lambda^2\right) + \left(A_2A_3 + \lambda A_4\right)2\lambda\right\}$$ and

$$\frac{\partial P_2^*}{\partial \lambda} = \frac{1}{\left[A_1A_2 - \lambda^2\right]^2}\left\{A_3\left(A_1A_2 - \lambda^2\right) + \left(A_1A_4 + \lambda A_3\right)2\lambda\right\}.$$

It is easily seen that $\frac{\partial P_1^*}{\partial \lambda} \geq 0, \frac{\partial P_2^*}{\partial \lambda} \geq 0$. (Q.E.D.)

(ii) When $\lambda > 0$, checking the first-order derivatives of P_1^* and P_2^* with respect to c_1 and c_2, we get $\frac{\partial P_1^*}{\partial c_1} = \frac{2(1+S_2k_2)(1+2S_1k_1)}{4(1+S_1k_1)(1+S_2k_2)-\lambda^2} > 0$, $\frac{\partial P_2^*}{\partial c_2} = \frac{2(1+S_1k_1)(1+2S_2k_2)}{4(1+S_1k_1)(1+S_2k_2)-\lambda^2} > 0$, $\frac{\partial P_1^*}{\partial c_2} = \frac{\lambda(1+2S_2k_2)}{4(1+S_1k_1)(1+S_2k_2)-\lambda^2} > 0$, and $\frac{\partial P_2^*}{\partial c_1} = \frac{\lambda(1+2S_1k_1)}{4(1+S_1k_1)(1+S_2k_2)-\lambda^2} > 0$.

Besides, we have $\frac{\partial P_1^*}{\partial c_1} > \frac{\partial P_1^*}{\partial c_2}$ and $\frac{\partial P_2^*}{\partial c_2} > \frac{\partial P_2^*}{\partial c_1}$. (Q.E.D.)

(iii) When $\lambda = 0$, checking the first-order derivatives of P_1^* and P_2^* with respect to c_1 and c_2, we get $\frac{\partial P_1^*}{\partial c_2} = 0$, $\frac{\partial P_2^*}{\partial c_1} = 0$, $\frac{\partial P_1^*}{\partial c_1} = \frac{2(1+S_2k_2)(1+2S_1k_1)}{4(1+S_1k_1)(1+S_2k_2)-\lambda^2} > 0$ and $\frac{\partial P_2^*}{\partial c_2} = \frac{2(1+S_1k_1)(1+2S_2k_2)}{4(1+S_1k_1)(1+S_2k_2)-\lambda^2} > 0$.

(Q.E.D.)

Proof of Proposition 2

Regarding k_1, the first-order derivatives of P_1^* and P_2^* are as follows:

$$\frac{\partial P_1^*}{\partial k_1} = \frac{2A_2(1-\theta)^2\sigma^2}{\left[A_1A_2 - \lambda^2\right]^2}\left[-A_2(1-\theta)a_0 - \lambda A_4 + c_1(A_2 - \lambda^2)\right]$$, and

$\frac{\partial P_2^*}{\partial k_1}=\frac{2\lambda(1-\theta)^2\sigma^2}{\left[A_1A_2-\lambda^2\right]^2}\left[-A_2(1-\theta)a_0-\lambda A_4+c_1(A_2-\lambda^2)\right]$. For k_2, we could obtain the following:

$\frac{\partial P_1^*}{\partial k_2}=\frac{2\lambda\theta^2\sigma^2}{\left[A_1A_2-\lambda^2\right]^2}\left[-A_1\theta a_0-\lambda A_3+c_2\left(A_1-\lambda^2\right)\right]$, and $\frac{\partial P_2^*}{\partial k_2}=\frac{2A_1\theta^2\sigma^2}{\left[A_1A_2-\lambda^2\right]^2}\left[-A_1\theta a_0-\lambda A_3+c_2\left(A_1-\lambda^2\right)\right]$.

(i) When σ =0, we have $\frac{\partial P_1^*}{\partial k_1}=\frac{\partial P_2^*}{\partial k_1}=\frac{\partial P_1^*}{\partial k_2}=\frac{\partial P_2^*}{\partial k_2}=0$. (Q.E.D.)

(ii) When $\sigma\neq 0$,

a) When $0<\theta<1$ and $\lambda>0$, if $c_1>(<)\frac{2(1+S_2k_2)(1-\theta)a_0+\lambda\left[\theta a_0+c_2(1+2S_2k_2)\right]}{2(1+S_2k_2)-\lambda^2}=CT_1$,

then, we get $\frac{\partial P_1^*}{\partial k_1},\frac{\partial P_2^*}{\partial k_1}>(<)0$. Besides, if

$c_2>(<)\frac{2(1+S_1k_1)\theta a_0+\lambda\left[(1-\theta)a_0+c_1(1+2S_1k_1)\right]}{2(1+S_1k_1)-\lambda^2}=CT_2$, then, we get $\frac{\partial P_1^*}{\partial k_2},\frac{\partial P_2^*}{\partial k_2}>(<)0$. Besides, it could be identified that $\frac{\partial CT_r}{\partial c_{3-r}}>0$. (Q.E.D.)

b) When $\lambda=0$, we have $\frac{\partial CT_r}{\partial c_{3-r}}=0$, and $\frac{\partial P_2^*}{\partial k_1}=\frac{\partial P_1^*}{\partial k_2}=0$. (Q.E.D.)

c) When $\theta=0$, we have $\frac{\partial P_1^*}{\partial k_2}=\frac{\partial P_2^*}{\partial k_2}=0$; When $\theta=1$, we have $\frac{\partial P_1^*}{\partial k_1}=\frac{\partial P_2^*}{\partial k_1}=0$. (Q.E.D.)

Proof of Proposition 3

Checking the first order derivatives of P_1^* and P_2^* with respect to θ, we get:

$$\frac{\partial P_1^*}{\partial\theta}=\frac{1}{\left[A_1A_2-\lambda^2\right]^2}\left\langle\begin{array}{l}a_0\left(\lambda-A_2\right)\left(A_1A_2-\lambda^2\right)+\\4\sigma^2\left\{k_1(1-\theta)A_2\left[A_2(1-\theta)a_0+\lambda A_4+c_1\left(\lambda^2-A_2\right)\right]+\lambda k_2\theta\left[c_2\left(A_1-\lambda^2\right)-\theta a_0A_1-\lambda A_3\right]\right\}\end{array}\right\rangle$$ and

$$\frac{\partial P_2^*}{\partial\theta}=\frac{1}{\left[A_1A_2-\lambda^2\right]^2}\left\langle\begin{array}{l}a_0\left(A_1-\lambda\right)\left(A_1A_2-\lambda^2\right)+\\4\sigma^2\left\{k_2\theta A_1\left[-A_1\theta a_0-\lambda A_3+c_2\left(A_1-\lambda^2\right)\right]+\lambda k_1(1-\theta)\left[c_1\left(\lambda^2-A_2\right)+(1-\theta)a_0A_2+\lambda A_4\right]\right\}\end{array}\right\rangle.$$

(i) If $\sigma=0$, we get $\frac{\partial P_1^*}{\partial\theta}\leq 0$ (which equals $\frac{\partial P_1^*}{\partial(1\text{-}\theta)}\geq 0$) and $\frac{\partial P_2^*}{\partial\theta}\geq 0$. (Q.E.D.)

(ii) When $\sigma\neq 0$,

a) With competition ($\lambda>0$), for carrier 1, we have $\frac{\partial P_1^{BD*}}{\partial\theta}<(>)0$ (which equals $\frac{\partial P_1^*}{\partial(1\text{-}\theta)}>(<)0$) when $c_1>(<)\frac{a_0(\lambda-A_2)(A_1A_2-\lambda^2)+4\sigma^2\{k_1(1-\theta)A_2a_0[A_2(1-\theta)+\lambda\theta]-\lambda k_2\theta a_0[\theta A_1+\lambda(1-\theta)]+[k_1(1-\theta)A_2(1+2S_2k_2)+k_2\theta(A_1-\lambda^2)]\lambda c_2\}}{4\sigma^2[k_1(1-\theta)A_2(A_2-\lambda^2)+\lambda^2k_2\theta T_1]}=DT_1$ is satisfied. For carrier 2, we have $\frac{\partial P_2^{BD*}}{\partial\theta}>(<)0$ when $c_2>(<)\frac{a_0(\lambda-A_1)(A_1A_2-\lambda^2)+4\sigma^2\{k_2\theta A_1a_0[A_1\theta+\lambda(1-\theta)]-\lambda k_1(1-\theta)a_0[(1-\theta)A_2+\lambda\theta]+[k_2\theta A_1(1+2S_1k_1)+k_1(1-\theta)(A_2-\lambda^2)]\lambda c_1\}}{4\sigma^2[k_2\theta A_1(A_1-\lambda^2)+\lambda^2k_1(1-\theta)T_2]}=$

DT_2 is satisfied. Besides, we have $\frac{\partial DT_r}{\partial c_{3-r}} > 0$ (when $\lambda > 0$). (Q.E.D.)

b) With competition ($\lambda > 0$), for carrier 2, we have $\frac{\partial P_2^{BD*}}{\partial \theta} < (>) 0$ (which equals $\frac{\partial P_1^{*}}{\partial (1\text{-}\theta)} > (<) 0$) when $c_1 > (<) \frac{a_0(A_1-\lambda)(A_1A_2-\lambda^2)+4\sigma^2\{-k_2\theta A_1a_0[A_1\theta+(1-\theta)\lambda]+\lambda k_1(1-\theta)a_0[(1-\theta)A_2+\lambda\theta]+[k_2\theta A_1(A_1-\lambda^2)+\lambda^2k_1(1-\theta)(1+2S_2k_2)]c_2\}}{4\sigma^2[k_2\theta A_1\lambda T_1+\lambda k_1(1-\theta)(A_2-\lambda^2)]} = ET_1$ is satisfied. For carrier 1, we have $\frac{\partial P_1^{BD*}}{\partial \theta} > (<) 0$ when $c_2 > (<) \frac{a_0(A_2-\lambda)(A_1A_2-\lambda^2)+4\sigma^2\{-k_1(1-\theta)A_2a_0[A_2(1-\theta)+\theta\lambda]+\lambda k_2\theta a_0[\theta A_1+\lambda(1-\theta)]+[k_1(1-\theta)A_2(A_2-\lambda^2)+\lambda^2k_2\theta(1+2S_1k_1)]c_1\}}{4\sigma^2[k_1(1-\theta)A_2\lambda T_2+\lambda k_2\theta(A_1-\lambda^2)]} = ET_2$ is satisfied. Besides, we have $\frac{\partial ET_{3-r}}{\partial c_r} > 0$. (Q.E.D.)

c) Without competition ($\lambda = 0$), we have $\frac{\partial P_1^*}{\partial \theta} = \frac{1}{A_1^2}[2a_0(k_1S_1-1) - 4\sigma^2 k_1(1-\theta)c_1] < 0$ and $\frac{\partial P_2^*}{\partial \theta} = \frac{1}{A_2^2}[2a_0(1-k_2S_2) + 4\sigma^2 k_2\theta c_2] > 0$. (Q.E.D.)

Proof of Proposition 4

Checking the first order derivatives of P_1^* and P_2^* with respect to σ, we get:

$$\frac{\partial P_1^*}{\partial \sigma} = \frac{4\sigma}{\left[A_1A_2-\lambda^2\right]^2}\left\{\begin{array}{l} c_1\left[k_1(1-\theta)^2A_2\left(A_2-\lambda^2\right)-\lambda^2k_2\theta^2T_1\right]+c_2\left[\lambda k_2\theta^2\left(A_1-\lambda^2\right)-k_1(1-\theta)^2A_2\lambda T_2\right] \\ -a_0\left[k_1(1-\theta)^2A_2\left[(1-\theta)A_2+\lambda\theta\right]+\lambda k_2\theta^2\left[\theta A_1+\lambda(1-\theta)\right]\right] \end{array}\right\}$$ and

$$\frac{\partial P_2^*}{\partial \sigma} = \frac{4\sigma}{\left[A_1A_2-\lambda^2\right]^2}\left\{\begin{array}{l} c_2\left[k_2\theta^2A_1\left(A_1-\lambda^2\right)-\lambda^2k_1(1-\theta)^2T_2\right]+c_1\left[\lambda k_1(1-\theta)^2\left(A_2-\lambda^2\right)-k_2\theta^2A_1\lambda T_1\right] \\ -a_0\left[k_2\theta^2A_1\left[\theta A_1+\lambda(1-\theta)\right]+\lambda k_1(1-\theta)^2\left[(1-\theta)A_2+\lambda\theta\right]\right] \end{array}\right\}.$$

(i) When $0 < \theta < 1$ and $\lambda > 0$,

Let $\Lambda_1 = \frac{\lambda^2\theta^2T_1}{(1-\theta)^2A_2(A_2-\lambda^2)}$, $\Omega_2 = \frac{(1-\theta)^2A_2T_2}{\theta^2(A_1-\lambda^2)}$, and $\Lambda_2 = \frac{\lambda^2(1-\theta)^2T_2}{\theta^2A_1(A_1-\lambda^2)}$, and $\Omega_1 = \frac{\theta^2A_1T_1}{(1-\theta)^2(A_2-\lambda^2)}$.

a) For carrier 1, when $\tau_1 > \Lambda_1$, when c_1 is sufficiently large, that is,

$$c_1 > \frac{a_0\left[k_1(1-\theta)^2A_2\left[(1-\theta)A_2+\lambda\theta\right]+\lambda k_2\theta^2\left[\theta A_1+\lambda(1-\theta)\right]\right]-c_2\left[\lambda k_2\theta^2\left(A_1-\lambda^2\right)-k_1(1-\theta)^2A_2\lambda T_2\right]}{k_1(1-\theta)^2A_2\left(A_2-\lambda^2\right)-\lambda^2k_2\theta^2T_1} = YT_1$$

, then we have $\frac{\partial P_1^*}{\partial \sigma} > 0$. For carrier 2, if $\tau_2 > \Lambda_2$, if c_2 is sufficiently large, that is, $c_2 > \frac{a_0\left[k_2\theta^2A_1[\theta A_1+\lambda(1-\theta)]+\lambda k_1(1-\theta)^2[(1-\theta)A_2+\lambda\theta]\right]-c_1[\lambda k_1(1-\theta)^2(A_2-\lambda^2)-k_2\theta^2A_1\lambda T_1]}{k_2\theta^2A_1(A_1-\lambda^2)-\lambda^2k_1(1-\theta)^2T_2} = YT_2$, then we have $\frac{\partial P_2^*}{\partial \sigma} > 0$. (Q.E.D.)

b) For carrier 2, when $\tau_1 > \Omega_1$, when c_1 is sufficiently large, that is, $c_1 > \frac{a_0\left[k_2\theta^2A_1[\theta A_1+\lambda(1-\theta)]+\lambda k_1(1-\theta)^2[(1-\theta)A_2+\lambda\theta]\right]-c_2[k_2\theta^2A_1(A_1-\lambda^2)-\lambda^2k_1(1-\theta)^2T_2]}{\lambda k_1(1-\theta)^2(A_2-\lambda^2)-k_2\theta^2A_1\lambda T_1} = PT_1$, then we have $\frac{\partial P_2^*}{\partial \sigma}>0$. For carrier 1, when $\tau_2>\Omega_2$, if c_2 is sufficiently large, that is,

$$c_2 > \frac{a_0\left[k_1(1-\theta)^2 A_2\left[(1-\theta)A_2+\lambda\theta\right]+\lambda k_2\theta^2\left[\theta A_1+\lambda(1-\theta)\right]\right]-c_1\left[k_1(1-\theta)^2 A_2\left(A_2-\lambda^2\right)-\lambda^2 k_2\theta^2 T_1\right]}{\lambda k_2\theta^2\left(A_1-\lambda^2\right)-k_1(1-\theta)^2 A_2\lambda T_2}=PT_2$$

, then we have $\frac{\partial P_1^*}{\partial \sigma}>0$. (Q.E.D.)

(ii) When $0<\theta<1$ and $\lambda=0$, we get $\frac{\partial P_1^*}{\partial \sigma}=\frac{4\sigma k_1(1-\theta)^2 A_2^2}{(A_1A_2)^2}[c_1-a_0(1-\theta)]$ and $\frac{\partial P_2^*}{\partial \sigma}=\frac{4\sigma k_2\theta^2 A_1^2}{(A_1A_2)^2}(c_2-a_0\theta)$. Therefore, we have $\frac{\partial P_1^*}{\partial \sigma}>0$ if $c_1>a_0(1-\theta)$ and $\frac{\partial P_2^*}{\partial \sigma}>0$ if $c_2>a_0\theta$. (Q.E.D.)

(iii) When $\theta=0$ or 1,

a) When $\theta=1$ and $\lambda>0$, we get $\frac{\partial P_1^*}{\partial \sigma}=\frac{4\sigma}{\left[A_1A_2-\lambda^2\right]^2}\left\{-\lambda^2 k_2T_1c_1+\lambda k_2\left(A_1-\lambda^2\right)c_2-a_0\lambda k_2A_1\right\}$,

$\frac{\partial P_2^*}{\partial \sigma}=\frac{4\sigma}{\left[A_1A_2-\lambda^2\right]^2}\left\{-k_2A_1\lambda T_1c_1+k_2A_1\left(A_1-\lambda^2\right)c_2-a_0k_2A^2{}_1\right\}$; Therefore, we have $\frac{\partial P_1^*}{\partial \sigma}>0$ and $\frac{\partial P_2^*}{\partial \sigma}>0$ if $c_2>\frac{(\lambda T_1c_1+a_0A_1)}{\left(A_1-\lambda^2\right)}=OT_2$.

When $\theta=0$ and $\lambda>0$, we get $\frac{\partial P_1^*}{\partial \sigma}=\frac{4\sigma}{\left[A_1A_2-\lambda^2\right]^2}\left\{k_1A_2\left(A_2-\lambda^2\right)c_1-k_1A_2\lambda T_2c_2-a_0k_1A^2{}_2\right\}$,

$\frac{\partial P_2^*}{\partial \sigma}=\frac{4\sigma}{\left[A_1A_2-\lambda^2\right]^2}\left\{\lambda k_1\left(A_2-\lambda^2\right)c_1-\lambda^2k_1T_2c_2-a_0\lambda k_1A_2\right\}$. Therefore, we have $\frac{\partial P_1^*}{\partial \sigma}>0$ and $\frac{\partial P_2^*}{\partial \sigma}>0$ if $c_1>\frac{(\lambda T_2c_2+a_0A_2)}{\left(A_2-\lambda^2\right)}=OT_1$. (Q.E.D.)

b) When $\theta=1$ and $\lambda=0$, we have $\frac{\partial P_1^*}{\partial \sigma}=0$, and $\frac{\partial P_2^*}{\partial \sigma}>0$ if $c_2>a_0$; When $\theta=0$ and $\lambda=0$, we have $\frac{\partial P_2^*}{\partial \sigma}=0$, and $\frac{\partial P_1^*}{\partial \sigma}>0$ if $c_1>a_0$. (Q.E.D.)

Extended analyses

Proof of Lemma 2. Checking the second-order derivatives of Eq. (16), it is found that $\frac{\partial^2 O_1^e}{\partial (P_1^e)^2}=-2-2S_1k_1-2\delta^2k_1<0$ and $\frac{\partial^2 O_2^e}{\partial (P_2^e)^2}=-2-2S_2k_2-2\delta^2k_2<0$, which proves that both objective functions are concave in the respective unit price. Consequently, we could identify the reactive functions for the two players through solving the first-order conditions as follows:

$$P_1^e = \arg_{P_1}\left\{\frac{\partial O_1^e}{\partial P_1^e}=0\right\} \to P_1^e \big/ P_2^e = \frac{(1-\theta)a_0[1+2k_1\delta^2]+\lambda P_2^e[1+2k_1\delta^2]+c_{10}(1+2S_1k_1)}{2(1+S_1k_1+k_1\delta^2)}$$

$$P_2^e = \arg_{P_2}\left\{\frac{\partial O_2^e}{\partial P_2^e}=0\right\} \to P_2^e \big/ P_1^e = \frac{\theta a_0(1+2k_2\delta^2)+\lambda P_1^e(1+2k_2\delta^2)+c_{20}(1+2S_2k_2)}{2(1+S_2k_2+k_2\delta^2)}$$

Solving the reactive functions, the optimal prices for the two carriers could be obtained for the extended model. Besides, $B_1B_2 > \lambda^2\eta_1\eta_2$ always holds. (Q.E.D.)

Proof of Proposition 5

Checking the first order derivatives of P_1^{e*} and P_2^{e*} with respect to δ, we get:

$$\frac{\partial P_1^{e*}}{\partial \delta} = \frac{4\delta}{\left[B_1B_2-\lambda^2\eta_1\eta_2\right]^2}\left\{\begin{array}{l}[k_2\lambda^2\eta_1(B_2-\eta_2)-k_1B_2(B_2-\lambda^2\eta_2)](1+2S_1k_1)c_{10}+\lambda[k_1B_2(B_1-\eta_1)-k_2\eta_1(B_1-\lambda^2\eta_1)](1+2S_2k_2)c_{20} \\ +[k_2\lambda^2\eta_1(B_2-\eta_2)-k_1B_2(B_2-\lambda^2\eta_2)](1-\theta)a_0\eta_1+\lambda[k_1B_2(B_1-\eta_1)-k_2\eta_1(B_1-\lambda^2\eta_1)]\theta a_0\eta_2 \\ +[(1-\theta)a_0k_1B_2+\lambda k_2\eta_1\theta a_0](B_1B_2-\lambda^2\eta_1\eta_2)\end{array}\right\}$$

, and

$$\frac{\partial P_2^{e*}}{\partial \delta} = \frac{4\delta}{\left[B_1B_2-\lambda^2\eta_1\eta_2\right]^2}\left\{\begin{array}{l}[k_1\lambda^2\eta_2(B_1-\eta_1)-k_2B_1(B_1-\lambda^2\eta_1)](1+2S_2k_2)c_{20}+\lambda[k_2B_1(B_2-\eta_2)-k_1\eta_2(B_2-\lambda^2\eta_2)](1+2S_1k_1)c_{10} \\ +[k_1\lambda^2\eta_2(B_1-\eta_1)-k_2B_1(B_1-\lambda^2\eta_1)]\theta a_0\eta_2+\lambda[k_2B_1(B_2-\eta_2)-k_1\eta_2(B_2-\lambda^2\eta_2)](1-\theta)a_0\eta_1 \\ +[\theta a_0k_2B_1+\lambda k_1\eta_2(1-\theta)a_0](B_1B_2-\lambda^2\eta_1\eta_2)\end{array}\right\}$$

. Besides, we have $B_r-\eta_r = 1+2S_rk_r \ge 1$ and $\lambda^2\eta_r - B_r = (\lambda^2-2)+(\lambda^2-1)2k_r\delta^2-2S_rk_r < 0$.

(i) When $\lambda > 0$, let $\vartheta_1 = \dfrac{\lambda^2\eta_1(B_2-\eta_2)}{B_2(B_2-\lambda^2\eta_2)}$, $\varsigma_2 = \dfrac{B_2(B_1-\eta_1)}{\eta_1(B_1-\lambda^2\eta_1)}$, $\vartheta_2 = \dfrac{\lambda^2\eta_2(B_1-\eta_1)}{B_1(B_1-\lambda^2\eta_1)}$, $\varsigma_1 = \dfrac{B_1(B_2-\eta_2)}{\eta_2(B_2-\lambda^2\eta_2)}$

a) For carrier 1, when $\tau_1 > \vartheta_1$, when c_{10} is sufficiently small, that is,

$$c_{10} < \frac{\begin{array}{l}[k_2\lambda\eta_1(B_1-\lambda^2\eta_1)-k_1B_2\lambda(B_1-\eta_1)]B_4+[k_1B_2(B_2-\lambda^2\eta_2)-k_2\lambda^2\eta_1(B_2-\eta_2)](1-\theta)a_0\eta_1 \\ -[(1-\theta)a_0k_1B_2+\lambda k_2\eta_1\theta a_0](B_1B_2-\lambda^2\eta_1\eta_2)\end{array}}{[k_2\lambda^2\eta_1(B_2-\eta_2)+k_1B_2(\lambda^2\eta_2-B_2)](1+2S_1k_1)} = WT_1,$$ then

we have $\dfrac{\partial P_1^{e*}}{\partial \delta} > 0$.

For carrier 2, if $\tau_2 > \vartheta_2$, if c_{20} is sufficiently small, that is,

$$c_{20} < \frac{\begin{array}{l}[k_1\lambda\eta_2(B_2-\lambda^2\eta_2)-k_2B_1\lambda(B_2-\eta_2)]B_3+[k_2B_1(B_1-\lambda^2\eta_1)-k_1\lambda^2\eta_2(B_1-\eta_1)]\theta a_0\eta_2 \\ -[\theta a_0k_2B_1+\lambda k_1\eta_2(1-\theta)a_0](B_1B_2-\lambda^2\eta_1\eta_2)\end{array}}{[k_1\lambda^2\eta_2(B_1-\eta_1)+k_2B_1(\lambda^2\eta_1-B_1)](1+2S_2k_2)} = WT_2,$$ then we

have $\dfrac{\partial P_2^{e*}}{\partial \delta} > 0$. (Q.E.D.)

b) For carrier 2, when $\tau_1 > \varsigma_1$, when c_{10} is sufficiently small, that is,

$$c_{10} < \frac{\begin{array}{l}[k_2B_1(B_1-\lambda^2\eta_1)-k_1\lambda^2\eta_2(B_1-\eta_1)]B_4+[k_1\lambda\eta_2(B_2-\lambda^2\eta_2)-k_2B_1\lambda(B_2-\eta_2)](1-\theta)a_0\eta_1 \\ -[\theta a_0k_2B_1+\lambda k_1\eta_2(1-\theta)a_0](B_1B_2-\lambda^2\eta_1\eta_2)\end{array}}{[k_2B_1\lambda(B_2-\eta_2)+k_1\lambda\eta_2(\lambda^2\eta_2-B_2)](1+2S_1k_1)} = UT_1,$$ then we

have $\dfrac{\partial P_2^{e*}}{\partial \delta} > 0$.

For carrier 1, when $\tau_2 > \varsigma_2$, if c_{20} is sufficiently small, that is,

$$c_{20} < \frac{[k_1B_2(B_2-\lambda^2\eta_2)-k_2\lambda^2\eta_1(B_2-\eta_2)]B_3+[k_2\lambda\eta_1(B_1-\lambda^2\eta_1)-k_1B_2\lambda(B_1-\eta_1)]\theta a_0\eta_2 - [(1-\theta)a_0k_1B_2+\lambda k_2\eta_1\theta a_0](B_1B_2-\lambda^2\eta_1\eta_2)}{[k_1B_2\lambda(B_1-\eta_1)+k_2\lambda\eta_1(\lambda^2\eta_1-B_1)](1+2S_2k_2)} = UT_2$$

, then we have $\frac{\partial P_1^{e*}}{\partial \delta} > 0$. (Q.E.D.)

(ii) When $\lambda = 0$,

a) When $0 < \theta < 1$, we get $\frac{\partial P_1^{e*}}{\partial \delta} = \frac{4\delta k_1 B_2^2}{(B_1B_2)^2}\{[(1-\theta)a_0 - c_{10}](1+2S_1k_1)\}$. Therefore, we have $\frac{\partial P_1^{e*}}{\partial \delta} > 0$ if $c_{10} < (1-\theta)a_0$. Besides, we get $\frac{\partial P_2^{e*}}{\partial \delta} = \frac{4\delta k_2 B_1^2}{\left[B_1B_2 - \lambda^2\eta_1\eta_2\right]^2}\{(\theta a_0 - c_{20})(1+2S_2k_2)\}$.

Therefore, we have $\frac{\partial P_2^{e*}}{\partial \delta} > 0$ if $c_{20} < \theta a_0$. (Q.E.D.)

b) When $\theta = 0$ or 1,

When $\theta = 1$, we could obtain $\frac{\partial P_1^{e*}}{\partial \delta} = \frac{4\delta}{(B_1B_2)^2}[-k_1B_2^2(1+2S_1k_1)c_{10}] \le 0$. Besides, we get $\frac{\partial P_2^{e*}}{\partial \delta} = \frac{4\delta k_2 B_1^2}{\left[B_1B_2\right]^2}(a_0 - c_{20})(1+2S_2k_2)$. Therefore, we have $\frac{\partial P_2^{e*}}{\partial \delta} > 0$ if $c_{20} < a_0$.

When $\theta = 0$, we could obtain $\frac{\partial P_2^{e*}}{\partial \delta} = \frac{4\delta}{(B_1B_2)^2}[-k_2B_1^2(1+2S_2k_2)c_{20}] \le 0$. Besides, we get $\frac{\partial P_1^{e*}}{\partial \delta} = \frac{4\delta k_1 B_2^2}{(B_1B_2)^2}(a_0 - c_{10})(1+2S_1k_1)$. Therefore, we have $\frac{\partial P_1^{e*}}{\partial \delta} > 0$ if $c_{10} < a_0$. (Q.E.D.)

Biographical note

Xin Wen is currently a PhD student in Department of Industrial and Systems Engineering at The Hong Kong Polytechnic University. She has published in journals such as International Journal of Production Economics, and Transportation Research – Part E. Her current research interest is on air logistics operations.

Xiaoyan Xu is currently a postgraduate student in Glorious Sun School of Business and Management at Donghua University. She has published in International Journal of Production Research. She is interested in research topics on logistics operations and services.

Tsan-Ming Choi is currently a full professor at The Hong Kong Polytechnic University. His expertise is on supply chain systems engineering and applied optimization. He has authored/edited sixteen research handbooks and published over 180 papers in high impact leading citation journals such as Automatica, European Journal of Operational Research, Decision Sciences, IEEE Transactions on Automatic Control, IEEE Transactions on Automation Science and Engineering, IEEE Transactions on Cybernetics, IEEE Transactions on Engineering Management, IEEE Transactions on Industrial Informatics, IEEE Transactions on Intelligent Transportation Systems, IEEE Transactions on Systems, Man and Cybernetics - Systems, INFORMS Service Science, Naval Research Logistics, Production and Operations Management, Transportation Research – Part E, etc. He is currently the Co-Editor-in-Chief of Transportation Research – Part E, a senior editor of Production and Operations Management, and Decision Support Systems, and an associate editor of IEEE Transactions on Systems, Man, and Cybernetics - Systems, and Information Sciences.

Sai-Ho Chung, PhD, is an Associate Professor in Industrial and Systems Engineering at the Hong Kong Polytechnic University. He obtained his PhD degree from the University of Hong Kong. His research interests include logistics and supply chain management, supply chain collaboration, production scheduling, distributed scheduling, distribution network, etc. Dr Chung has published over 70 SCI journal papers. He has guest edited a number of special issues for various journals, e.g., Transportation Research Part E: Logistics and Transportation Review, Industrial Management & Data Systems.