

The following publication Man, Y., Hu, Y., & Ren, J. (2019). Forecasting COD load in municipal sewage based on ARMA and VAR algorithms. Resources, Conservation and Recycling, 144, 56–64 is available at <https://doi.org/10.1016/j.resconrec.2019.01.030>.

# **COD load forecasting model of municipal sewage for wastewater treatment plants based on ARMA and VAR algorithms**

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1 **Abstract**

2       Due to different sources and the water using habits, the influent COD of municipal  
3 sewage fluctuate sharply over time. To ensure the treatment quality of sewage, the  
4 wastewater treatment plants (WWTP) often over-aerate the air and over-add the  
5 chemicals. This results in a waste of energy consumption and increases the operation  
6 cost for WWTP. With the rapid expansion of industrialization and urbanization,  
7 municipal sewage has increased by years. Energy conservation and sustainable water  
8 management for municipal WWTP are becoming an urgent issue that needs to be solved.  
9 This paper proposes a COD load forecasting model for municipal WWTP using hybrid  
10 artificial intelligence algorithms. The auto-regressive moving average (ARMA)  
11 algorithm is used for sewage inflow forecasting, and a vector auto-regression (VAR)  
12 algorithm is used for COD forecasting. The real-time data from a municipal WWTP is  
13 used for model verification. Besides the proposed ARMA+VAR model, the BPNN,  
14 LSSVM, GA-BPNN based COD load forecasting models are also studied as the  
15 contrasting cases. The verification results reveal that the ARMA+VAR model is  
16 superior to the other forecasting models for future application in the wastewater  
17 treatment plants. The accuracy of the proposed model is as high as 99%.

18

19 **Keywords:** municipal sewage; wastewater treatment plants; COD load; forecasting  
20 model; sustainable water management

## 21 **1. Introduction**

22 With the rapid expansion of industrialization and urbanization, the quantity of  
23 municipal wastewater effluent has been growing at a rate of 5% per year over the past  
24 decade (Yang, et al., 2017). The energy consumption for municipal sewage treatment is  
25 constantly rising. How to reduce energy consumption is an issue that must be solved in  
26 municipal wastewater treatment plants (WWTP) (López-Morales & Rodríguez-Tapia,  
27 2019).

28 The activated sludge process that contains secondary bio-treatment process has  
29 been applied in most of municipal WWTP in China (Man et al., 2017). The energy  
30 consumption is mainly concentrated in the influent pump station for improving sewage  
31 and the aeration system for the secondary bio-treatment process. The energy  
32 consumption of these two operation units accounts for about 70% of total energy  
33 consumption (Man et al., 2018). The power consumption of the aeration system  
34 generally accounts for 40% to 50% of the whole plant (Li et al., 2017). It is the largest  
35 power consumption operation unit in the municipal WWTP.

36 Chemical oxygen demand (COD) is one of the most commonly measured items in  
37 water quality monitoring and analysis. It directly reflects the extent of contamination  
38 of the water which is polluted by reducing substances (Wang at al., 2018). COD is one  
39 of the most important indicators to demonstrate whether the effluent fits the discharge  
40 standard after treatment. Municipal sewage mainly comes from the urban human living  
41 area, precipitation, and some industrial wastewater. Unlike the industrial wastewater,  
42 the influent COD load of municipal sewage plants has changed greatly due to the

43 difference in climate change and living habits of residents. To ensure that the treated  
44 effluent can meet the discharge standard, the COD content of the discharged effluent  
45 should be monitored in WWTP. The aeration rate and chemicals dosage should be  
46 controlled according to the COD content of the discharged effluent (Babu & Reddy,  
47 2014). However, the COD detection needs a quite long time and it is an off-line  
48 operation process, which will cause problems such as time lag and inaccurate feedback  
49 during the process control. Meanwhile, due to the wide range of municipal sewage  
50 sources and the large fluctuation of influent mass flow, a large design margin is often  
51 reserved for aeration process in WWTP. In the treatment process, the air flow is often  
52 over-aerated and chemicals are over-added in order to ensure treatment quality of  
53 sewage when the sewage inlet mass flow or the COD content fluctuates sharply.  
54 However, in spite of well effluent quality control, this operation not only sacrifices a  
55 large amount of unnecessary energy input, but also causes problems such as secondary  
56 contamination of chemicals (Sen et al., 2016). Moreover, the excessive dissolved  
57 oxygen will cause the destruction of the flocculating agent and result in poor settling of  
58 suspended solids, thus reducing the quality of effluent. If the aeration rate and the  
59 chemicals dosage in the treatment process can be accurately controlled by establishing  
60 a "feed-forward and feedback" control system, the energy consumption and cost of the  
61 treatment process can be both reduced on the premise of ensuring the effluent quality.  
62 However, the influent COD load of the sewage must be forecasted for establishing such  
63 "feed-forward and feedback" control system.

64 Some research achievements have been made on the forecasting of municipal

65 sewage quality based on different mathematics or mechanism models, such as  
66 regression model (Park & Engel, 2015; Suchetana et al., 2019), grey forecasting model  
67 (Chen et al., 2010), neural network (Vrečko et al., 2011; Gebler et al., 2018), and auto-  
68 regressive moving average (ARMA) (Yuan et al., 2016; Barak & Sadegh, 2016). Due  
69 to the simple mathematical structure, the mechanism model has the advantages of fast  
70 convergence speed and high forecasting accuracy for stable data sequence. However,  
71 the accuracy will largely decrease when the raw data fluctuates sharply because such  
72 models usually pay much attention to data fitting for the search of data sequence rule.  
73 The heuristic algorithms such as neural network, particle swarm optimization (PSO),  
74 etc. with strong adaptability and learning ability are usually used for dealing with  
75 nonlinear and uncertain problems. However, they are easy to appear the shortcoming  
76 such as long learning time and local optimization, which results in non-convergence  
77 and reduces the industrial application scope (Son & Kim, 2017; Ye et al., 2018). The  
78 time series based forecasting method is a kind of intelligent algorithms based on the  
79 essential law of data reflected by time series. Compared with other intelligent  
80 algorithms, the most outstanding advantage of the time series based algorithms is that  
81 they can rapidly capture the trends of the data sequence. The rapid calculation process  
82 opens up possibilities for its industrial applications. In recent years, although there are  
83 many applications in of forecasting models based on time series algorithms (Deng &  
84 Wang, 2017; Deng et al., 2015), it is still in the initial stage for the application for  
85 sewage treatment.

86 In order to increase the control accuracy of the aeration process, this paper

87 proposes a COD load forecasting model for municipal sewage based on ARMA and  
88 vector auto-regression (VAR) algorithms. The industrial real-time data is used for  
89 modeling and model verification. The proposed COD load forecasting model will  
90 provide a scientific basis for precise control of the aeration rate, which will reduce  
91 energy consumption and operation cost.

92

## 93 **2. Materials and methodology**

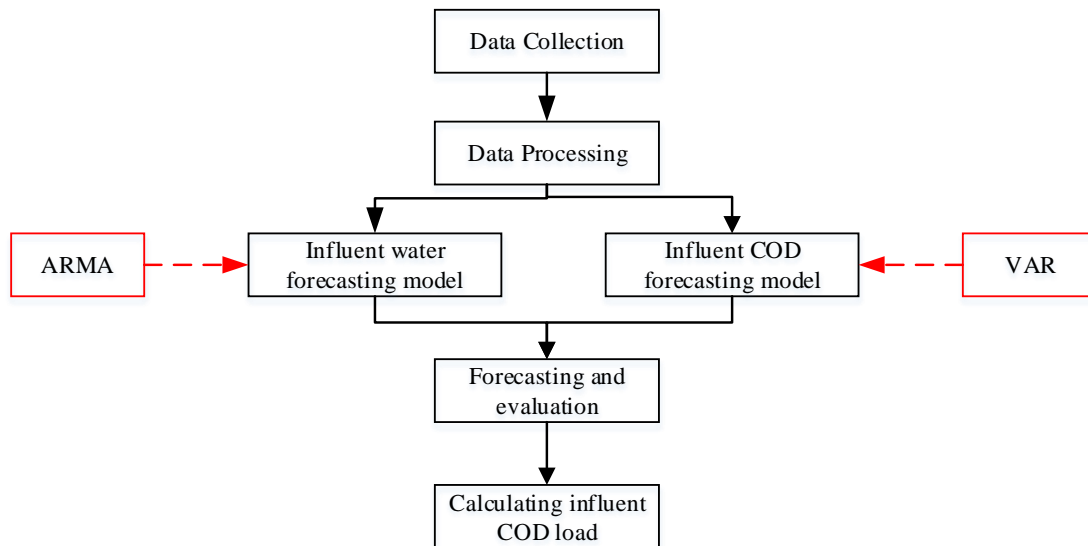
94 In the WWTP, the COD load is usually used as an indicator of aeration and  
95 chemical dosage. The COD load is the product of the sewage mass inflow and the  
96 absolute value of influent COD. Since the amount of sewage mass inflow and COD are  
97 two independent variables, they can be modeled separately and thereby obtaining the  
98 forecasting model of influent COD load.

99 Affected by residents living habits and precipitation, the mass flow of municipal  
100 sewage influent presents the characteristics of strong timeliness and seasonality.  
101 Therefore, the ARMA algorithm is used to model municipal sewage inflow in this paper.  
102 The influent COD is related to many internal correlation factors or variables. It is  
103 necessary to analyze the influence of the variables on the influent COD in time series.  
104 Therefore, the VAR algorithm is used to forecast the COD of municipal sewage inflow  
105 in this paper.

106 This research consists of 4 steps, as shown in Fig. 1. (1) Data collection: The data  
107 in this paper come from the real-time data from a municipal WWTP. (2) Data pre-  
108 processing: The real-time data usually have problems such as data missing and error, it

109 is necessary to filter the error data and fill up the missing data. The data preprocessing  
110 will help to improve the accuracy of the model. (3) Modeling: The sewage influent mass  
111 flow forecasting model is established based on ARMA algorithm, and the influent COD  
112 forecasting model is established based on VAR algorithm. (4) Forecasting and  
113 verification: The influent COD load is forecasted and the industrial real-time data are  
114 used to verify the accuracy of the forecasting model.

115



116

117

Figure 1. Roadmap of the research

118

## 119 2.1. Data preparation

120 The original data used in this paper are collected from a municipal WWTP in  
121 Qingyuan, Guangdong Province. The annual treatment capacity of this WWTP is 5  
122 million tons. The temperature of sewage varies from around 8 °C to 30 °C. This research  
123 is carried out based on the A2O wastewater treatment technology. The collected real-  
124 time data is obtained from the historical database of the WWTP. The sampling

125 frequency of sewage inflow is every 1 hour and influent COD is every 1 minute.

126 Since the object of this paper is to obtain the phase forecasting model of sewage  
127 inflow and influent COD based on time series analysis, the relevant factors that affect  
128 the influent water inflow and influent COD are analyzed and selected in this section.

129 Unlike the industrial WWTP, the sewage inflow of municipal WWTP is mainly  
130 related to human water using habits and natural precipitation. The former enables the  
131 water inflow to present a strong cyclical change, and the latter results in an abrupt  
132 change of water inflow in the time series. Therefore, it is necessary to introduce  
133 precipitation data during the modeling process to forecast the sewage inflow of  
134 municipal WWTP.

135 The influent COD is affected by the pH, the concentration of ammonia and nitride  
136 ( $\text{NH}_3\text{-N}$ ), and the influent sewage temperature ( $T$ ). The time sequence  $\{Z_{t1}\}$  of the  
137 influent COD ( $\text{mg}\cdot\text{L}^{-1}$ ), the time sequence  $\{Z_{t2}\}$  of influent pH, the time sequence  $\{Z_{t3}\}$   
138 of  $\text{NH}_3\text{-N}$  ( $\text{mg}\cdot\text{L}^{-1}$ ), and the time sequence  $\{Z_{t4}\}$  of influent sewage temperature ( $T$ ) are  
139 selected as the model input variable.

140

## 141 **2.2. ARMA algorithm based forecasting model**

142 ARMA algorithm is an effective method to forecast time series based data  
143 sequence, which can be explained by the time-delay term and random error term of  
144 variable  $\mu$ . ARMA algorithm can find a suitable forecasting model on the premise of  
145 the given data pattern. The algorithm of the auto-regressive moving-average model ( $p$ ,  
146  $q$ ) is as shown in Eq. (1) (Wang et al, 2018):



147  $\mu_t = c + \varphi_1 \times \mu_{t-1} + \dots + \varphi_p \times \mu_{t-p} + \varepsilon_t + \theta_1 \times \varepsilon_{t-1} + \dots + \theta_q \times \varepsilon_{t-q}, \quad t = 1, 2, \dots, T \quad (1)$

148 Where,  $c$  is a constant,  $\varphi_1, \varphi_2, \dots, \varphi_p$  are the autoregressive model coefficient,  $P$   
149 is autoregressive model order;  $\varepsilon_t$  is white noise series that the mean value is 0 with the  
150 variance  $\delta^2$ ,  $\mu$  is a constant parameter,  $\theta_1, \theta_2, \dots, \theta_q$  are coefficients of the  $q$ -order  
151 moving average model.

152 The ARMA based sewage inflow forecasting model has two main procedures:  
153 Firstly, based on the preprocessed data, the autocorrelation coefficient (ACF) and  
154 partial autocorrelation coefficient (PACF) are calculated to identify the model and the  
155 estimate the parameters; secondly, the preliminary model and the estimated model  
156 parameters shall be certified. The Akaike Information Criterion (AIC) method is used  
157 to certify and determine the appropriate order of the model. The flow diagram of the  
158 ARMA modeling flowchart is shown in Figure 2.

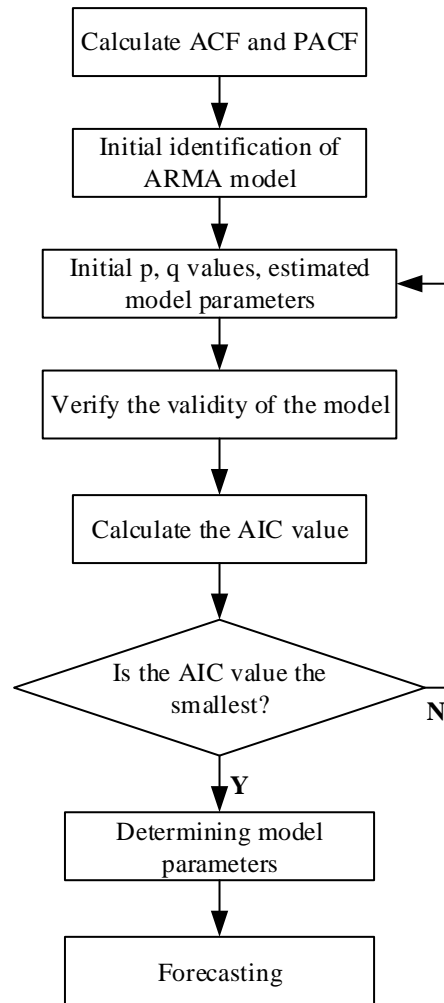


Figure 2. The programming chart of ARMA

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161

162 The specific steps for the modeling process are as follows:

163 (1) Model identification: Since only the time series data are available to be  
 164 obtained, the ARMA ( $p, q$ ) model should be identified based on the two statistics  
 165 parameters, autocorrelation coefficient (ACF) and partial autocorrelation coefficient  
 166 (PACF).

167 The parameter selection principle: The values of  $p, q$  are determined by the  
 168 truncation and tailing characteristics of ACF and PACF. With the increase of lag order,  
 169 if AC or PAC shows sinusoidal attenuation or exponential attenuation approaching zero,

170 they have trailing property. If AC or PAC quickly approaches 0 from a certain lag period,  
171 it has truncation. By this method, only preliminary order determination can usually be  
172 carried out. For further precise order determination, it shall be tested from bottom to  
173 top. In this paper, the most widely used AIC method is used to determine the order  
174 determination of the model.

175 (2) Parameter estimation. The most commonly used methods, nonlinear least  
176 squares method (NLLS), is used to estimate the parameters in this paper.

177 (3) Model verification. Check whether the residual sequence of the fitted model is  
178 a white noise sequence. If the residual error meets the requirement of white noise  
179 sequence, the model selection is reasonable; otherwise, repeat the steps (1) ~ (2) until  
180 the appropriate model is determined.

181 (4) The model order determination. The AIC values of the verified model with  
182 different orders are then calculated based on the AIC method. The model order is  
183 determined when the smallest AIC value appears.

184

### 185 **2.3. VAR algorithm based forecasting model**

186 The VAR algorithm structures the model by using each endogenous variable as a  
187 function of the hysteresis value of all endogenous variables. The VAR algorithm is  
188 similar to the multivariate linear regression model that is widely used in multivariate  
189 statistical analysis. Therefore, many methods involved in multivariate linear regression  
190 with multiple dependent variables can be applied to the VAR model.

191 Proceed from the data; VAR algorithm does not contain exogenous variables. The

192 mathematical form of the model with  $p$ -order is shown in Eq. (2) (Chan & Eisenstat,  
193 2018):

$$194 \quad Z_t = \phi_0 + \sum_{i=1}^p \phi_i \times Z_{t-i} + a_t \quad (2)$$

195 Where:  $Z_t$  is a multivariate time series with one-dimensional endogenous variables,  $\phi$   
196 is the one-dimensional constant vector. When  $\phi$  is not equal to 0,  $Z_t$  is a random vector  
197 sequence with independent and identical distribution. The mean value is 0.

198 This equation of model is convenient to analyze the dynamic relationship between  
199 endogenous variables. The dynamic relationship is the relation between the variable to  
200 be studied as well as the  $p$ -phase lag of itself and other variables. In view of the  
201 backward shift operator, the model is converted into Eq. (3):

$$202 \quad \Phi(B) \times Z_t = \Phi_0 + a_t \quad (3)$$

203 Where,  $\Phi(B) = I_t - \sum_{i=1}^p \phi_i \times B^i$ , it is a matrix polynomial with  $p$ -order.

204 After the preliminary model is determined, it needs to be tested. The residuals,  
205 which plays an important role in the modeling process, need to be tested. After the  
206 establishment of the model, it is more important to test the stability of the model. If the  
207 VAR based model is stable, it will not produce spurious regression and is trusted to be  
208 effective for practical forecasting. The content of the model test mainly includes two  
209 parts: (1) Ensure the stability of the model; (2) Give the direction of further  
210 improvement if necessary. In this paper, the residuals of the model are tested by the  
211 multivariate portmanteau test method. The null hypothesis of the test method is:  $H_0$ :  
212  $R_1 = \dots = R_{m-0}$ , the alternative hypothesis is:  $H_1$ :  $R_j \neq 0, \exists j \in [1, m]$ , where  $m$  is a  
213 predetermined positive integer. The sequence of residuals can be calculated by Eq. (4)

214 (Patilea & Raïssi, 2015):

$$215 \quad Q_k(m) = T^2 \times \sum_{p=1}^m \frac{1}{T-p} \times tr(\widehat{C}_p' \times \widehat{C}_0^{-1} \times \widehat{C}_p \times C_0^{-1}) \sim \chi^2((m-p) \times k^2) \quad (4)$$

216 where,  $Q_k(m)$  is a chi-square distribution with a progressive order of freedom  $(m-p) \times$   
217  $k^2$ .

218 Since the VAR based model is established based on time-series data, the VAR  
219 based model is a non-theoretical model in practical applications and the influence of  
220 the variables of the model is determined by Granger causality method. The details of  
221 this method are shown in the Appendix. In the meanwhile, another method, Impulse  
222 Response Function (IRF), is used to explore the relationship between variables. When  
223 calculating the impulse response, the model must be guaranteed to be stable. If the  
224 model is unstable, the impulse response of a changing model has not only the effects of  
225 disturbances, but also the effects of changes in the system itself in the calculation  
226 process. The impulse response can be used to describe the dynamic response of  
227 disturbances generated by one endogenous variable to other variables in the VAR based  
228 model. The variance decomposition of the error is also used to further evaluate the  
229 importance of different impacts by analyzing the contribution of endogenous variables.  
230 The specific modeling process is shown in Figure 3:

231 (1) Augmented Dickey-Fuller (ADF) test: Judge whether the sequence is stable by  
232 the ADF test. If it fails to pass the ADF test, the difference of the sequence is carried  
233 out until the sequence passes the ADF test.

234 (2) Initial model order selection. The information criterion values in different  
235 orders are calculated and ranked. The model order  $p$  referred to the smallest information

236 criterion values will be selected. This paper calculates the information criterion values  
237 at different model orders by using the AIC method, Bayesian Information Criterion  
238 (BIC) method and Hannan-Quinn Criterion (HQC) method. The smallest  $p$  value is  
239 selected for model initialization.

240 (3) Granger causality test. The input variable of the VAR based model is selected  
241 in order to analyze the causality between influent COD and other influent variables by  
242 using Granger causality test method.

243 (4) Model order determination. The model order is determined by the IRF method.  
244 The preliminary several different orders are selected, and the VAR model is established  
245 to determine whether the effect of different single variables on other variables is  
246 consistent with the established VAR mode. The corresponding order is selected as the  
247 order of the final model to establish the VAR forecasting model.

248 (5) Model test and modification. The cross-correlation of the residual error for the  
249 preliminary model is tested by the multivariate portmanteau test method. When the  
250 residual error has no strong correlation or cross-correlation, the validity of the model is  
251 determined. Otherwise, repeat the steps (2) ~ (4).

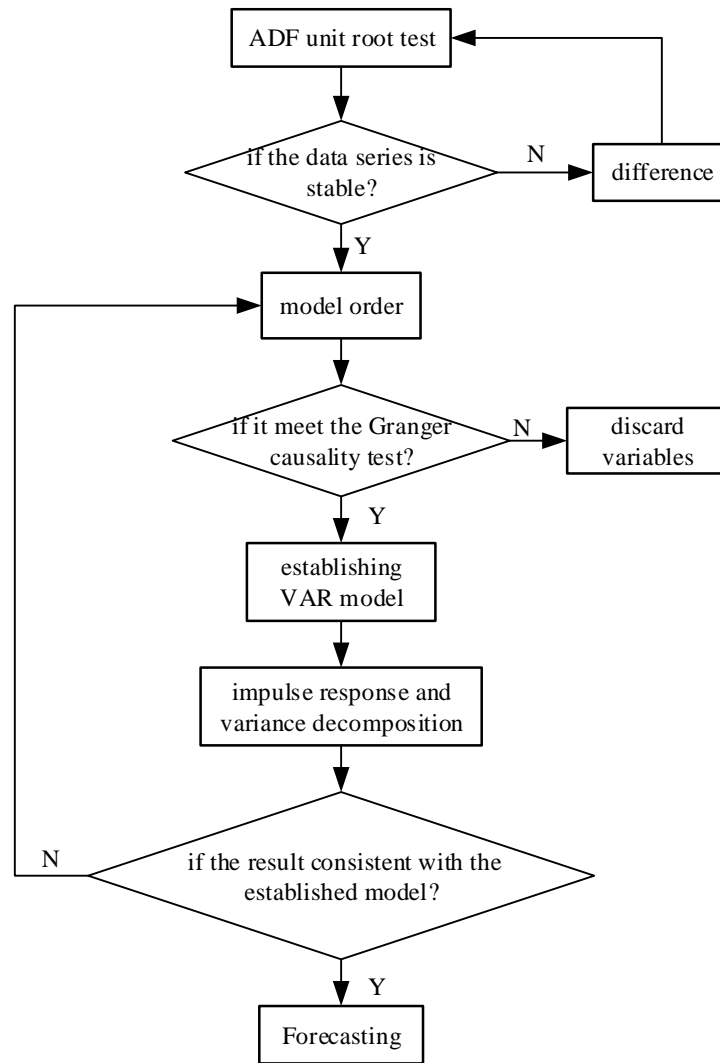


Figure 3. The programming picture of VAR

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253

254

### 255 3. Results and discussion

#### 256 3.1. The sewage inflow forecasting model

257 The original data of the sewage inflow from June 3, 2018 to June 23, 2018 with a

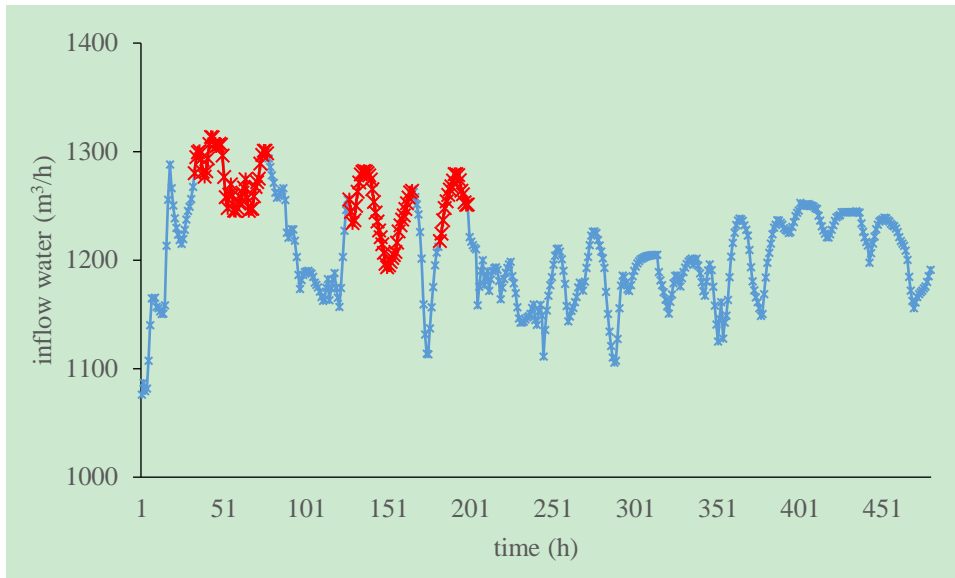
258 sampling time of every 1 hour are collected from a municipal WWTP in Qingyuan.

259 After preprocessing, a total of 481 sampling points are obtained for the cumulative

260 sewage mass inflow. And the mass inflow of the sewage for every 1 hour is obtained

261 by doing the first-order difference for the cumulative inflow data. Figure 4 shows the

262 preprocessed data for sewage mass inflow, where the red part is the rainfall period  
263 released by the local meteorological department.



264

265 Figure 4. Preprocessed data for sewage mass inflow

266

267 The preprocessed data are divided into two parts. One part including the data of  
268 the first 19 days is used to train the model parameters. The other part including the data  
269 of the 20th day is used for model testing. Since the original data sequence fails to meet  
270 the stability requirement of the ARMA algorithm, the data sequence needs to be  
271 differentiated. Here, the first-order difference of the data sequence can be carried out to  
272 meet the sequence stationarity, and then the relevant ACF and PACF are solved to judge  
273 the tailing and truncation of the model to select the appropriate model order. The NLLS  
274 method is used to estimate the model parameters, and the model lag of the determined  
275 coefficients is obtained to test the model. The rationality of the model is judged by  
276 whether the residual error is the white noise sequence. Finally, the AIC method is used  
277 to determine the order of the model, as shown in Table 1. According to Table 1, it can



278 be found that the AIC value is the smallest when  $p=5$ , and  $q=3$ . Therefore, the  
 279 forecasting model of sewage inflow per unit time in the sewage treatment plant is  
 280 ARMA (5, 3).

281

282 Table 1. AIC value table for different  $p$  and  $q$  orders

$q$ value $p$ value	1	2	3	4	5	6
1	4.6678	4.6207	4.6252	4.6291	4.6318	4.6266
2	4.6237	4.6195	4.6190	4.6232	4.6261	4.5754
3	4.5520	4.6244	4.6288	4.5721	4.5755	4.5614
4	4.5579	4.6074	4.5669	4.5641	4.5977	4.6043
5	4.5617	4.5673	4.5135	4.5273	4.5951	4.5188
6	4.6152	4.5991	4.5202	4.5745	4.6036	4.5535

283

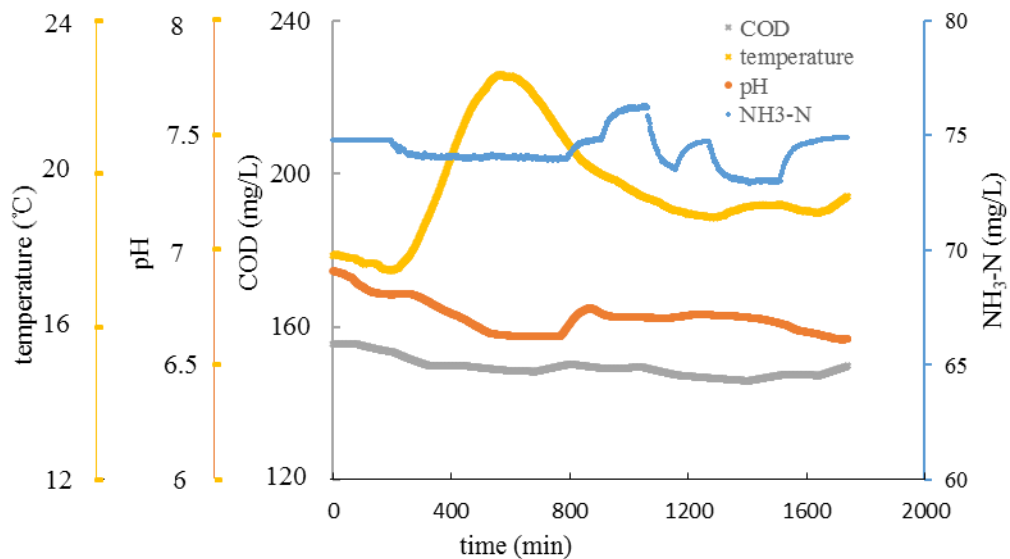
284 At the same time, the adaptive mechanism is used in the model for rolling  
 285 forecasting. The collected real-time sewage inflow data during the forecasted time  
 286 period will be added to the historical sewage database for re-calculating the parameters  
 287 of the ARMA (5, 3). The updated forecasting model with new parameters is then used  
 288 to forecast the next time sewage inflow data. In this way, the dynamic forecasting model  
 289 is established by modifying the parameters in real time.

290

### 291 3.2. The influent COD forecasting model

292 The original data of influent COD is also collected from this municipal WWTP.  
 293 The sampling time for influent COD is every 1 minute. The data preprocessing method  
 294 of the influent COD is similar to the method for the sewage inflow. The preprocessed  
 295 data of the influent COD is shown in Figure 5. The variation trend of four groups of

296 correlation parameters (influent COD, pH, NH<sub>3</sub>-N, and temperature) in 28 hours is also  
 297 shown in Figure 5. The variation range of influent COD is between [146, 156]. The  
 298 general trend is not evidently related to the human water using period. The influent  
 299 NH<sub>3</sub>-N fluctuates within the range of [72, 78]. Combined with the variation range of  
 300 influent pH and temperature, it can be found that the correlation between the influent  
 301 COD and other variables of the influent has a mutual influence. However, the  
 302 appropriate influencing variables shall be selected in combination with the quantitative  
 303 mathematical analysis.



304  
 305 Figure 5. The variation range of COD and the related variables

306  
 307 **3.2.1. ADF unit root test**

308 In the ADF unit root test, the availability of intercept and time trend items has a  
 309 significant impact on the results of the test. From Figure 5, it can be found that the four  
 310 variables do not show the consistent trend, so the ADF test with an intercept but without  
 311 trend is used. If the ADF test value is less than the critical values of 1%, 5%, and 10%,

312 it indicates that the data sequence is stable. The test results are shown in Table 2, where:  
 313  $Z_{t1}$  is the influent COD in mg/L.  $Z_{t2}$  is influent pH, and  $Z_{t3}$  is influent  $\text{NH}_3\text{-N}$  in mg/L.  
 314  $\Delta Z_{t3}$  is the influent  $\text{NH}_3\text{-N}$  with the first order difference.  $\Delta Z_{t4}$  is the influent sewage  
 315 temperature with the first order difference.

316 According to the ADF test results it can be found that the COD and pH are stable  
 317 at different significance levels on time series, while the  $\text{NH}_3\text{-N}$  and temperature are  
 318 unstable. However, these two variables are stable with a first-order difference.  
 319 Therefore,  $\Delta Z_{t3}$  and  $\Delta Z_{t4}$  are selected as the input parameters together with  $Z_{t1}$  and  $Z_{t2}$ .

320

321

Table 2. ADF unit root test results

Variable	ADF test value	1% threshold	5% threshold	10% threshold	<i>p</i> value	Conclusion
$Z_{t1}$	-10.6451	-2.5691	-1.9416	-1.6168	0	Stable
$Z_{t2}$	-11.1794	-2.5691	-1.9416	-1.6168	0	Stable
$Z_{t3}$	0.0518	-2.5691	-1.9416	-1.6168	0.349	unstable
$\Delta Z_{t3}$	-58.0250	-2.5691	-1.9416	-1.6168	0	Stable
$Z_{t4}$	3.9861	-2.5691	-1.9416	-1.6168	0.281	Unstable
$\Delta Z_{t4}$	-7.8256	-2.5691	-1.9416	-1.6168	0	Stable

322

### 323 3.2.2. Model order selection

324 Table 3 shows the test results of different information criterions with a maximum  
 325 lag order of 13. When the lag order is 9, the BIC shows the minimal information content.  
 326 When the lag order is 10, the HQC shows the minimal information content. The  
 327 information content of AIC is decreased with the increasing lag orders. The results  
 328 indicate that different information criterions have different emphases due to the  
 329 different penalty factors of them. In the comparison of the results of AIC, BIC, and

330 HQC, it can be found that BIC and HQC are consistent to some extent: With the  
 331 increasing of model order  $p$ , the trend of BIC and HQC is almost the same, they both  
 332 show the trend of decreasing first and then increasing. From Table 3, the model order  
 333 selection by AIC needs to be beyond 13th order, while 9th order by BIC, and 10th order  
 334 by HQC. Therefore,  $p=9$ , namely VAR(9), is firstly selected. Since all the results of  
 335 AIC, BIC, and HQC show a slow decreasing trend after 3rd order,  $p=3$ , namely VAR(3),  
 336 is therefore selected.

337

338 Table 3. Statistical results of different information criteria for different lagged orders

$P$	AIC	BIC	HQ	$p$ -value
0	22.8746	22.8746	22.8746	0
1	10.2092	10.2374	10.2196	0
2	3.7387	3.7593	3.7596	0
3	2.0151	2.0999	2.0464	0
4	1.9391	2.0521	1.9809	0
5	1.6998	1.8411	1.7521	0
6	1.5770	1.7673	1.6604	0
7	1.5164	1.7143	1.5896	0
8	1.4483	1.6744	1.5319	0
9	1.1692	1.4236	1.2633	0
10	1.1500	1.4326	1.2545	0
11	1.1495	1.4603	1.2644	0.0313
12	1.1399	1.4791	1.2654	0.0001
13	1.0989	1.4663	1.2348	0

339

### 340 3.2.3. Granger causality test

341 Granger causality test is used to analyze the causality between influent COD and  
 342 other influent variables. The results of the Granger causality test is shown in Table 4.  
 343 The sig value is the indicator of the credibility. It is the error probability of the results.  
 344 The higher the sig value means less credibility. A sig value of 0.05 is generally

345 considered to be acceptable at the wrong boundary level. That means if the sig is lower  
 346 than 0.05, the original hypothesis needs to be rejected. Otherwise, it is acceptable.

347 The test results show that influent pH is not the Granger cause of influent COD;  
 348 in the meanwhile, the influent COD is not Grange cause of the influent pH, and there  
 349 is no statistical causality between the two parameters. However, the NH<sub>3</sub>-N with first  
 350 order difference and the temperature first order difference are the Granger cause of  
 351 influent COD. The three correlation parameters are interacted.

352

353

Table 4. Granger causality test results

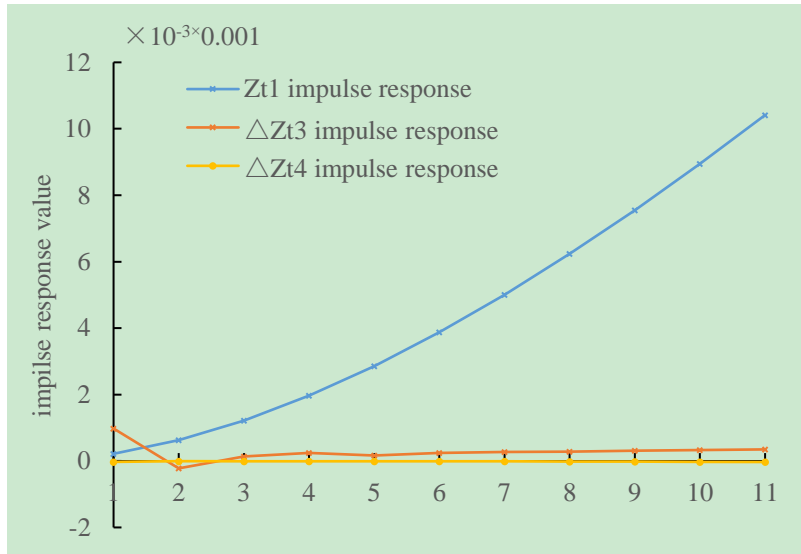
Null hypothesis	<i>F</i> -statistics	Sig value
$Z_{t2}$ is not a reason for $Z_{t1}$	10.736	0
$\Delta Z_{t3}$ is not a reason for $Z_{t1}$	1.6099	0.2002
$\Delta Z_{t4}$ is not a reason for $Z_{t1}$	1.6815	0.1864
$Z_{t1}$ is not a reason for $Z_{t2}$	10.736	0
$Z_{t1}$ is not a reason for $\Delta Z_{t3}$	0.0124	0.9877
$Z_{t1}$ is not a reason for $\Delta Z_{t4}$	5.1059	0.0062

354

### 355 3.2.4. Model order determination

356 The impulse response results of different associated variables to the influent COD  
 357 is shown in Figure 6. The impulse response of COD and influent temperature with the  
 358 first-order difference to themselves is raising with the increasing of model forecasting  
 359 period, as shown in Figure 6 (a) and (c). However, the impulse responses of the three  
 360 variables to the other variables are quickly attenuated to zero, which indicates the  
 361 variables have influence relationship.

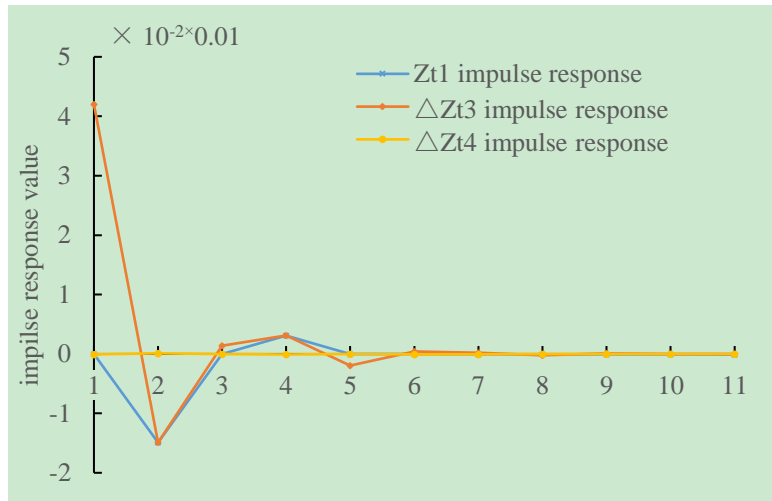
362



363

364

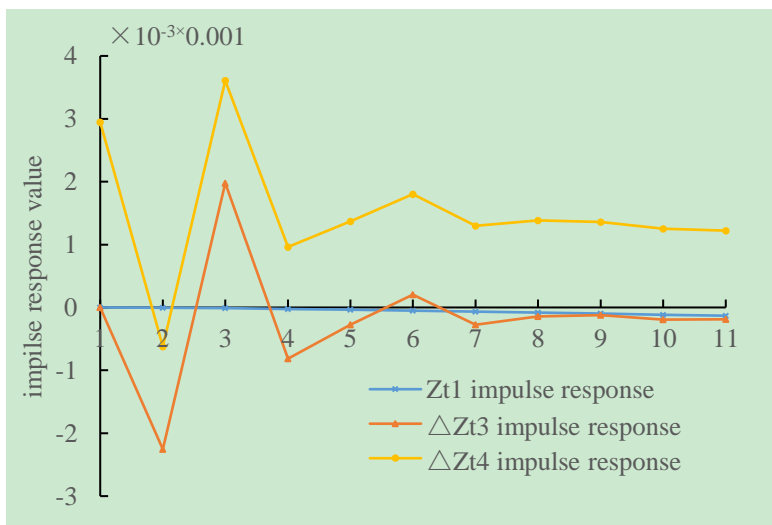
(a) The impulse response function caused by  $Z_{t1}$



365

366

(b) The impulse response function caused by  $\Delta Z_{t3}$



367

368 (c) The impulse response function caused by  $\Delta Z_{t4}$

369 Figure 6. Impulse response for different variables with different model forecasting  
370 period

371  
372 In order to understand the contribution of each variable to the influent COD, the  
373 variance decomposition shall be carried out. The results obtained by the decomposition  
374 are shown in Table 5. It can be found that the main contribution of COD is relatively  
375 large in the first forecasting period. With the increasing of forecasting period, the  
376 influence of  $\Delta Z_{t3}$  and the  $\Delta Z_{t4}$  on COD increase gradually. The impulse response  
377 begins to decrease after the third forecasting period. It means the first three forecasting  
378 period has the highest influence on COD. Therefore, VAR(3) is finally selected as the  
379 influent COD forecasting model.

380  
381 Table 5. Variance decomposition of model forecast period

Period	Forecast variance decomposition		
	$Z_{t1}$ variance decomposition	$\Delta Z_{t3}$ variance decomposition	$\Delta Z_{t4}$ variance decomposition
1	1.0000	0	0
2	0.9999	0.9969	0.9986
3	0.99986	0.9949	0.9987
4	0.9997	0.9946	0.9987
5	0.9997	0.9945	0.9987

382  
383 **3.2.5. Model test and modification**

384 Once the initial model has been obtained, the cross-correlation of residual error  
385 needs to be tested by the multivariate portmanteau test method. When  $m > m_0$  ( $m_0$ :  
386 determined model order),  $p < 0.05$ , there is no strong correlation or cross-correlation

387 between the residual errors to determine the validity of the model. Otherwise, the model  
 388 order determination shall be carried out again.

389 According to the obtained statistics results of the multivariate portmanteau test  
 390 method, there are 9 parameters of VAR(3) model. As a result, the order of freedom of  
 391 chi-square distribution of the test statistics  $Q_k(m)$  is set as  $9m-9$ . For the VAR(9) model,  
 392 there are 27 parameters. Thus the order of freedom of chi-square distribution of the test  
 393 statistics  $Q_k(m)$  for VAR(9) is  $9m-27$ . The p values of the two model test statistics  $Q_k$   
 394 ( $m$ ) are given in Table 6. For the VAR(3) model, when  $m>3$ ,  $p<0.05$ . That means the  
 395 residual errors of the established the VAR(3) model have no strong correlation or cross-  
 396 correlation at a significant level of 5%. However, for the VAR (9) model when  $m=5$   
 397 ( $m<m_0$ ),  $p>0.05$ . That means the VAR(9) model have a strong correlation or cross-  
 398 correlation and the forecasting result is not reliable. Therefore, the VAR(3) model is  
 399 finally selected as the influent COD forecasting model. Here, the core equation of the  
 400 influent COD forecasting model is shown in Eq. (5):

$$\begin{aligned}
 401 \quad Z_{1,t} &= 2.9028 \times Z_{1,t-1} - 0.001 \times \Delta Z_{2,t-1} - 0.00147 \times \Delta Z_{3,t-1} - 2.8073 \times \\
 402 \quad Z_{1,t-2} &+ 0.000818 \times \Delta Z_{3,t-2} + 0.9045 \times Z_{1,t-3} + 0.000818 \times Z_{3,t-3} \quad (5)
 \end{aligned}$$

403

404 Table 6. The  $Q$ - statistic test value of different VAR models

$m$	The $p$ -value of $Q$ - statistics of VAR(3)	The $p$ -value $Q$ - statistics of VAR(9)
1	1	1
2	1	1
3	1	1
4	0	0.02
5	0	0.16
6	0	0



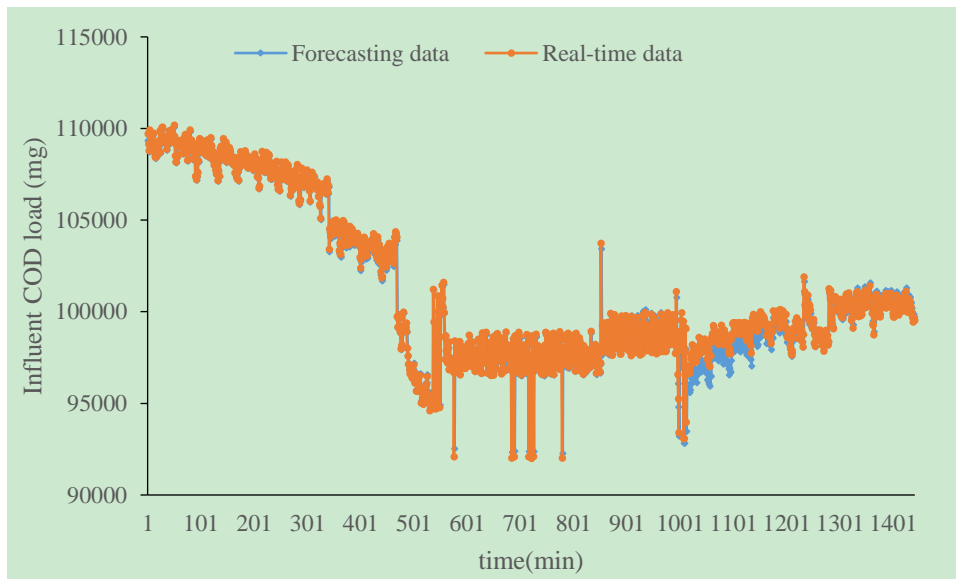
7	0	0
8	0	0
9	0	0

405

### 406 3.3. Verification of the influent COD load forecasting model

407 As mentioned before, the influent COD load is equal to the product of sewage mass  
 408 inflow and influent COD. Therefore, the forecasting model of influent COD load can  
 409 be obtained by forecasting the sewage inflow and influent COD. In order to test the  
 410 forecasting performance, the real-time data of the influent COD load for 24 hours in  
 411 another period of this municipal WWTP is used for verification. The comparison  
 412 between the forecasting results and real-time measured data is shown in Figure 7.

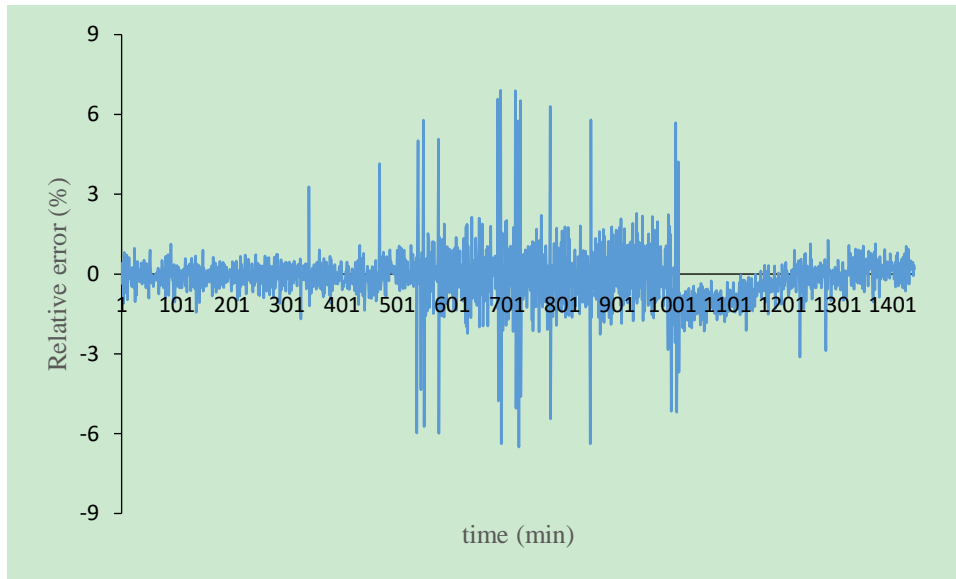
413



414

415

(a) Forecasting results for influent COD load



(b) Relative error

Figure 7. Forecasting result

416

417

418

419

420 From Figure 7, no matter the influent COD load is in the stable period or in the

421 large fluctuation period, the relative errors of forecasting results are within  $[-7\%, 7\%]$ .

422 For more than 95% of the relative errors of the forecasting results are within  $[-5\%, 5\%]$ ,

423 which is much less than industrial acceptable standard  $[-5\%, 5\%]$  for process control.

424 The proposed influent COD load forecasting model has good reliability.

425 In order to objectively verify the feasibility of the model, the evaluation indicators

426 are calculated as shown in Table 7. For the evaluation indicators:  $R^2$  of the forecasting

427 results is as high as 0.94, which shows high fitness between the forecasting results and

428 the measured data. The mean absolute percentage error (MAPE) is 1.08%, which is far

429 less than the judgment standard (the accuracy of the model is high if  $MAPE < 10$ ). The

430 value of Theil inequality coefficient (TIC) is also close to zero. These evaluation

431 indicators reveal that the influent COD load forecasting model for municipal WWTP

432 proposed in this paper is reliable and has high accuracy.

433

434

Table 7. Evaluation indicator

Evaluation index	R <sup>2</sup>	MAPE (%)	TIC
COD load	0.94	0.68	0.00003

435

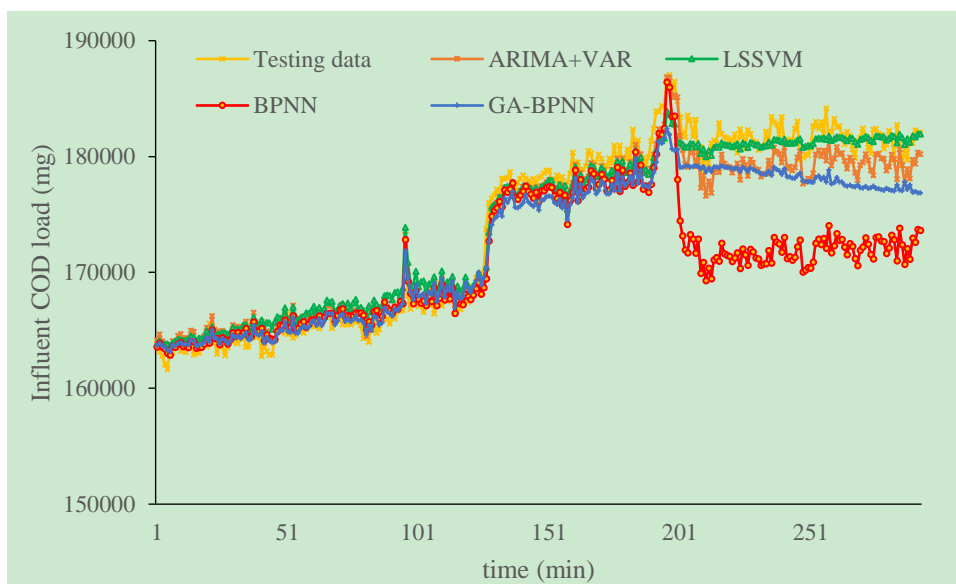
### 436 3.4 Comparison and discussion for the forecasting performance of different models

437 The comparative analysis of the forecasting performances of the ARMA+VAR  
438 algorithms based model, BPNN algorithm based model, LSSVM algorithm based  
439 model, and hybrid GA-BPNN algorithm based model are presented in this section. All  
440 the four forecasting models are developed under the same study case. In order to show  
441 the forecasting results of four models more clearly, this paper selected another 7.5 hours  
442 of data in the WWTP. The forecasting results as shown in Figure 8..

443 The forecasting performance of ARMA+VAR, BPNN, LSSVM, and GA-BPNN  
444 of the study case is shown in Fig. 8 (a) and the relative error is shown in Fig. 8 (b).  
445 Setting a benchmark of [-2%, 2%] in forecasting error makes it easy to find out the best  
446 consistent performer among the employed forecasting models. The discretized time  
447 points where the forecasting error lies within this benchmark are specifically shown in  
448 Fig. 8 (b). Considering the mentioned benchmark in forecasting error, the proposed  
449 ARMA+VAR model provides permissible forecasting error with 385 time points. On  
450 the contrary, for the BPNN model, the number of time points which lie within the  
451 forecasting error [-2%, 2%] is 221 for the BPNN model, 293 for the LSSVM model,  
452 and 248 for the hybrid GA-BPNN model. This reveals that the proposed ARMA+VAR

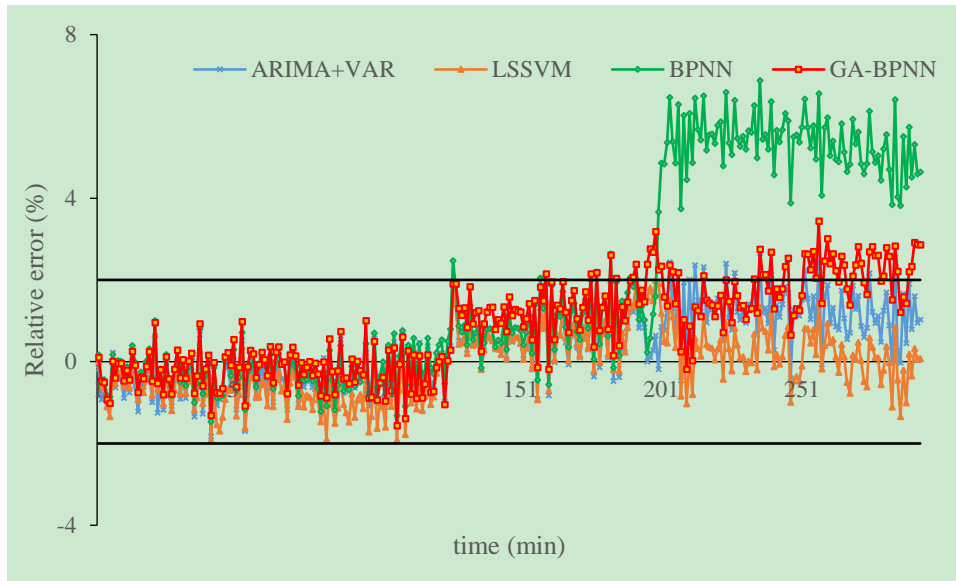
453 model has the best consistent performance among all the employed models.

454 The mean absolute percent error (MAPE) and root mean square error (RMSE) of  
455 the forecasting performance for the BPNN, the LSSVM, the hybrid GA-BPNN, and  
456 proposed ARMA+VAR models are shown in Table 8. The MAPE of the ARMA+VAR  
457 model is 2 times less than that of BPNN. The MAPE of the ARMA+VAR model is  
458 reduced by 89.8% when compared with the hybrid GA-BPNN model and by 42.6%  
459 when compared with the LSSVM model. The verification results using industrial data  
460 show that the proposed ARMA+VAR model achieves the highest accuracy than the  
461 compared three models.



462  
463

(a) Forecasting result



(b) Relative error

Fig.8. Forecasting results comparison of the four models

Table 8. The forecasting performance analysis

ARMA+VAR		BPNN		LSSVM		GA-BPNN	
MAPE	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE	RMSE
1.08	113.56	2.65	303.51	1.54	182.6	2.05	232.6

#### 4. Conclusion

This paper proposed an influent COD load forecasting model based on hybrid artificial intelligence algorithms for municipal wastewater treatment plants. The real-time data are used for modeling and model verification. The influent COD load forecasting model consists of two parts, the sewage inflow forecasting model based on ARMA algorithm, and the influent COD forecasting model based on VAR algorithm. The forecasting model is established based on the historical data of sewage inflow and

477 the correlation analysis of some key variables (include pH, NH<sub>3</sub>-N, and temperature).  
478 The forecasting model is the basis of the feedforward-feedback control system for the  
479 aeration process in WWTP.

480 The proposed influent COD load forecasting model shows good reliability and  
481 high accuracy. The relative errors of the forecasting results are within [-7%, 7%], which  
482 meets the industrial acceptable standard for process control. Compared with three  
483 employed contrast forecasting models (BPNN, LSSVM, and hybrid GA-BPNN), the  
484 forecasting performance shows that the proposed ARMA+VAR model has the highest  
485 accuracy. It reveals that the ARMA+VAR model is superior to the other three  
486 forecasting models for future application in the papermaking process since its MAPE  
487 is only 1.08%. The forecasting model supplies the basis for the feedforward-feedback  
488 control system of the aeration process in WWTP and makes it possible for precise  
489 control for energy conservation for municipal WWTP.

490

#### 491 **Acknowledgments**

492 This work is supported by the Fund of State Key Laboratory of Pulp and Paper  
493 Engineering (No. 201830), the Nature Science Funds of Guangdong Province  
494 (No.2017A030310562), and the Fundamental Research Funds for the Central  
495 Universities (No. 2017BQ023).

496

497 **Appendix**

498 **Granger causality test**

499 Granger (1969) put forward the concept of causality, which is easy to deal with  
500 VAR algorithm based forecasting model. Granger causality test can be used to analyze  
501 the relationship between two time series variables. In general, for the variables Y and  
502 X, Granger causality requires the estimation (Farokhzadi et al, 2018):

503 
$$Y_t = \sum_{i=1}^m a_i X_{t-i} + \sum_{i=1}^m \beta_i Y_{t-i} + u_{1t} \quad (\text{A.1})$$

504 
$$X_t = \sum_{i=1}^m \lambda_i Y_{t-i} + \sum_{i=1}^m \delta_i X_{t-i} + u_{2t} \quad (\text{A.2})$$

505 Where:  $m$  is the number of time-lag term X, namely the number of parameters to be  
506 estimated in the constrained regression equation;  $t$  is the time in min;  $a_i$  and  $\lambda_i$  are  
507 parameter coefficients;  $u_{1t}$  and  $u_{2t}$  are irrelevant white noises.

508 The Granger causality test is completed by a constrained F test, as shown in Eq.  
509 (A.3) (Farokhzadi et al, 2018):

510 
$$F = \frac{(RSS_R - RSS_U)/m}{RSS_U/(n - k)} \quad (\text{A.3})$$

511 Where,  $RSS_R$  is the sum of residual errors obtained by a constrained regression that does  
512 not contain X time-lag terms;  $RSS_U$  is the sum of residual errors of unconstrained  
513 regression that contains X time-lag terms, and  $n$  is the sample size;  $k$  is the number of  
514 parameters to be estimated in the unconstrained regression.

515 If  $F > F_\alpha(m, n-k)$ , the null hypothesis is rejected, and X is considered to be the  
516 Granger cause of Y (Meng & Han, 2018).

517

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