A two-stage optimization approach for aircraft hangar maintenance planning and staffing assignment problems under MRO outsourcing mode

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Abstract

Aircraft Maintenance Repair and Overhaul (MRO) is essential to ensure aviation safety and air transport operations. Aircraft has to temporally suspend its service and receive extensive maintenance in the hangar upon meeting the prescribed flying hours and take-off/landing times. Traditionally, each airline company carries the hangar maintenances for its own fleets. A transition of MRO operations has emerged with the rapid development to air transport demands. Outsourcing hangar maintenance to maintenance service company has been increasing among airlines, enabling airlines to reduce the cost of MRO while meeting the aircraft's safety requirements. After receiving the maintenance demands with their specifications, the maintenance service company needs to determine: the maintenance schedules, parking stand allocation, aircraft movement path as well as staff assignment through the planning period. A mixed-integer linear programming (MILP) mathematical model, integrating the abovementioned factors is developed. In the model, the geometric factors are considered, which is integrated with the multi-skill manpower assignment afterwards. We consider staff with multiple types of maintenance skills, aligning the practice of sophisticated hangar maintenance. Secondly, given the complexity of the integrated problem, a two-stage optimization approach is developed by decomposing the original model, which is coordinated by the linkage constrains between geometric and numeric decision-making scattering in the decomposed subproblems. The results and analysis of computational experiments are reported, which shows: (i) the adaptability and effectiveness of two-stage optimization approach and (ii) the scalability of the two-stage optimization approach that is able to provide good feasible solutions for medium- to large-size instances covering various planning period. The impacts of maintenance demand intensity and manpower supply variation are analysed afterward to provide managerial insights.

Keywords: Aircraft hangar maintenance planning, MRO outsourcing, Multi-skill staffing, Mixed-integer linear programming, heuristic decomposition approach

1. Introduction

Aircraft Maintenance, Repair and Overhaul (MRO) are significant supporting activities in aviation industry. Aircraft maintenances are strictly regulated by aviation authorities to keep aircraft's safety and airworthiness [1, 2]. The aircraft have to temporally suspend its service and receive maintenance upon meeting the prescribed flying hours and/or take-off/landing circles. In MRO industry, aircraft hangar maintenance involves a high operating cost, including the aircraft hangar establishment, facility asset as well as manpower. From the perspective of airlines, MRO are not value adding but mandatory activities in term of airline's fleet operations [3, 4]. The past decade has witnessed a rapid growth of civil aviation industry, which stimulates significant global economic growth as well as demand for commercial air transport [2, 5, 6]. Given that the MRO activities is the third highest cost behind fuel and labour cost [7], the sever competition in aviation industry has brought stimulus to airlines for reconsidering the MRO practice so as to reduce the MRO costs while maintaining the safety level of fleets [8-10]. A transition of MRO practice has emerged, and some airline company switch to outsource the MRO operations for their fleets to an independent service company so as to focus on their own high value adding commercial flying business. It is estimated that the percentage of outsourcing has risen from about 25 per cent to around 70 per cent of maintenance activities between the mid-1990s and 2012 [11].

The aircraft hangar maintenance operations under MRO outsourcing mode is studied in this paper. Optimization approach for hangar maintenance planning problem from the perspective of aircraft hangar maintenance company is developed. In MRO outsourcing mode, aircraft hangar maintenance demands are initiated by multiple airlines with specified requirement of maintenance service, according to each company's internal maintenance plan of its operating fleet. Faced with increasing hangar maintenance demands, maintenance service provider has to carry out an integrated maintenance plan align with multiple resources constraints. The integrated maintenance plan includes the determining the service time of each incoming aircraft, the parking position of each aircraft in the hangar as well as proper maintenance technician assignment to maintenance tasks. Specifically, the service time, rolling operations of aircraft should align with the parking plans over the planning horizon. In addition, the assignment of maintenance staff shall be based on the licenses (also known as skill) of each technician [12], as each technician can only perform the particular qualified maintenance task. The licences that the technician holds also relate to the maintenance manpower cost as the senior technician holding advanced license usually involve higher wages. Moreover, other consideration, such as team size and rest time, shall be included while assigning proper technicians to respective maintenance tasks. The development of such a plan is challenging as there exist interdependent relations among the aforementioned three core elements. The number of aircraft that maintenance hangar can accommodate changes along the planning period as the maintenance company receives different size of aircraft from different airlines, and the parking stand is not predetermined as in the conventional maintenance hangar operated by single airlines. In addition, due to the different arrival time, departure time and service time of incoming aircraft, the roll in and out time of each aircraft differ, then the blocking may occur when there are many incoming maintenance requests arriving at similar times, or the improper parking stand allocation is made. Moreover, the assignment of technicians may also influence the service time of maintenance task, which results in the changes of service time windows and fulfilling time of maintenance demands.

To address these issues and provide a systematic approach to solve the problem, we propose an optimization methodology to develop maintenance plans from the perspective of the independent aircraft maintenance service company. The work described in this paper is developed based on the journal and conference paper we published earlier [13, 14]. Additions of technician assignment problem render a challenging optimization model to tackle than the previous work. The hangar parking capacity, flexible parking assignment, and multi-skill technician assignment are three core difficulties in solving the problem. We focus on the modelling the correlations among maintenance service time scheduling, hangar layout planning and staffing. A Mixed Integer Linear Programming (MILP) model is firstly developed to take in the aforementioned practical factors in hangar maintenance operations under MRO outsourcing mode. Afterwards, a two-stage optimization approach is proposed to provider good quality solution for large-scale instances. The contributions of the studied problem can be summarized as follows: 1) an integrated planning model incorporating the aircraft maintenance scheduling, hangar layout planning and multi-skill technician assignment problem is developed, which is tailored for the hangar maintenance service company under the MRO outsourcing mode. 2) The proposed problem bridges the research gaps in literature regarding the aircraft maintenance problem and multi-skill technician assignment problem with the consideration of MRO outsourcing, which involves geometric factors and practical consideration in staffing. The problem studied in this paper is an extension of hangar planning model in literature, which fulfilling the lack of understanding in the overall maintenance operations planning problem.

The remainder of this paper is organized as follows. Section 2 provide an overview of related literature, then identify the research gaps. The problem description, objective and a set of constraints constituting the optimization problem is presented in Section 3. Section 4 introduce the two-stage optimization approach after analysing the problem structure. The results of computational experiment are reported in Section 5. Finally, the conclusions and future work are discussed in Section 6.

2. Literature review

2.1 Hangar maintenance scheduling and staffing problem

Aircraft maintenance tasks are conducted in a set of checks periodically to ensure the aviation safety, and the frequency of various maintenance checks is prescribed by the combinations of flying hours as well as the number of take-off and landing cycles [15]. There are four major types of checks (Type A, B, C and D checks) that are regulated by the Federal Aviation Administration according to the maintenance scope, duration as well as the frequency [16]. Aircraft maintenance is high cost activity regarding the equipment, inventory and manpower. Samaranayake, Lewis [17] studied the complexity of conducting aircraft maintenance checks involving extensive equipment, tools and materials, then developed an engineering structure to efficiently manage the scheduling of aircraft maintenance. While classifying the maintenance checks according to their work places, the maintenance checks can be categorized into line maintenance and hangar maintenance [18]. Line maintenance refers to "on line" maintenance that is conducted within the turnaround time between two flights, as the aircraft is parked at the gate or the apron, to guarantee a reliable aircraft dispatch [19], and Type A check is usually classified into line maintenance. For the other check types (B, C and D checks), they usually refer to "hangar" maintenance, as they require intensive maintenance inputs and long maintenance lead-times compared with the line maintenance. It is identified that in the hangar maintenance under MRO outsourcing mode is facing with the bottleneck of limited hangar space, and the Recently, Qin, Chan [20] proposed an aircraft parking stand allocation model for a maintenance company serving different size aircraft in batches, considering the variation of hangar capacity.

Traditionally, the staffing problem in aircraft maintenance are frequently considered together with the aircraft maintenance routing problem [21], as aircraft maintenance activities are conducted by airlines. For example, Chen, He [12] considered a technicians assignment optimization problem in the context of an aircraft maintenance hangar operated within single airline company, assuming constant hangar capacity. With the development of MRO outsourcing, some studies covered workforce scheduling problems from the aircraft maintenance company's perspective [21, 22]. De Bruecker, Van den Bergh [23] considered an aircraft maintenance personnel rosters problem from an independent aircraft line maintenance company serving several airline companies. Liang, Feng [24] considered an aircraft maintenance routing problem incorporating propagated delays in optimization, and Gavranis and Kozanidis [25] proposed an exact algorithm to solve a maintenance scheduling problem that maximized the fleet availability of a military aircraft unit. From the perspective of maintenance service provider in MRO outsourcing mode, the intensity of workload cannot be changed by revising the maintenance routing decision of fleet in each airline, so as to alleviate or balance the workload during a period of time [21]. As the maintenance outsourcing decisions are pre-determined by multiple airlines, the maintenance service provider aims to fulfill the maintenance demands within their permissible time windows by utilizing the available maintenance resource. Given the complexity of aircraft maintenance tasks, consideration of multiple skill type and skill levels are commonly adopted and indispensable in the aircraft maintenance staffing optimization. Yan, Yang [26] considered a technician assignment problem in short-term airline maintenance manpower planning, which incorporates multiple types of maintenance skill licenses with flexible management strategies in the mathematical model. Chen, He [12] considered a multiple-skill technicians' assignment and problem in an aircraft hangar maintenance operated by a single airline company, with a bi-objective optimization approach to minimize the total labor cost and achieve workload allocation fairness. In literature, most of the staff assignment and rostering problem in aircraft maintenance are correlated to the line maintenance of maintenance company, or hangar maintenance operated by single airline company. The research proposed in this paper aims to bridge the gaps between the multi-skill technician assignment problem in MRO industry and hangar maintenance operation under the MRO outsourcing mode.

2.2 Layout planning problem and non-overlapping constraints for irregular polygons

The problem studied in this paper involves a dynamic layout planning problem. In the literature, some optimization problems share some similarities in the problem nature and assumptions. The extension of the traditional Vehicle Routing Problem (VRP) incorporating simultaneous picks-up and deliveries, and two-dimensional loading constraints (2L-SPD) belongs to the class of the composite routing-packing optimization problem [27]. In Vehicle Routing Problem with Two-dimensional Loading and picks-up/deliveries constraints, one has to determine the route of a vehicle that satisfies customers at different demand and delivery points and consider a two-dimensional packing problem for the placing the goods in the vehicle for different customers [28], requiring that the routing of the vehicle must satisfy the Last-In-First-Out (LIFO) loading and unloading constraint. In addition, in the literature, the items to be arranged in the vehicle are all rectangle [27-31]. Moreover, the Facility Layout Problem (FLP) is another classic layout planning problem, which aims to determine the locations of rectangular facilities at different sites, minimizing the material handling costs between the facilities [32-35]. Dynamic Facility Layout Problems consider arranging the facilities over a planning period instead of one-time planning [36, 37]. Though layout planning problems have been extensively studied in the literature from various perspectives, such as the manufacturing industry [38-42], the relevant approaches cannot be directly applied in our problem due to the following considerations: (i) the shape of an aircraft is irregular. (ii) The Last-In-First-Out constraint can be relaxed as a soft constraint in the maintenance scheduling problem. (iii) Blocking during the aircraft roll in/out operations significantly affects the efficiency and needs to be characterized.

The aircraft parking stand allocation problem embedded in the maintenance scheduling problem can be modelled as a cutting and packing problem in a two-dimensional fixed dimension container. The most widely used tool for checking whether two irregular polygons overlap in the cutting and packing problem is the No-Fit Polygon (NFP). Bennell and Oliveira [43] and Bennell and Oliveira [44] provided a detailed tutorial on how to generate NFP between two non-convex irregular polygons. Alvarez-Valdes, Martinez [45] introduced a horizontal slices formulation approach to enhance the formulation of Fischetti and Luzzi [46]. Martinez-Sykora, Alvarez-Valdes [47] adopted horizontal slices in their MIP formulation to solve the irregular pieces packing problem with guillotine cuts. Cherri, Mundim [48] proposed two robust mixed-integer formulations for the irregular polygon packing problem that decompose the non-convex polygons into several convex pieces to generate NFP.

3. Problem statement and mathematical formulation

3.1 Problem statement

Aircraft hangar maintenance has to be conducted in the aircraft hangar after meeting the flying hours prescribed by the aviation authorities[18]. The aircraft is taken out of service and sent to a maintenance service company for hangar maintenance. The maintenance service company receives the maintenance requests from multiple airlines according to the pre-determine aircraft maintenance routing plan of their fleets. To fulfill these maintenance requests from multiple airlines, a hangar maintenance plan has to determine:

- a maintenance schedule specifying the service period of each aircraft, consisting of the timing of movement operations for each aircraft;
- hangar parking layouts covering the planning period, which aligns with the maintenance schedule. The hangar parking layouts specify the movement operations of all aircraft that induce the changes of the hangar layouts.
- Staff assignment to each maintenance tasks associated with the incoming aircraft for maintenance service.

The main goal is to minimize the penalty costs induced in fulfilling the maintenance requests and the manpower costs. The transitions of hangar layout plans are illustrated in Figure 1, which specifies the parking positions of the aircraft along the planning period. The continuity of the hangar layout is ensured by examining the positions of aircraft in the present shift, last shift and coming shift.

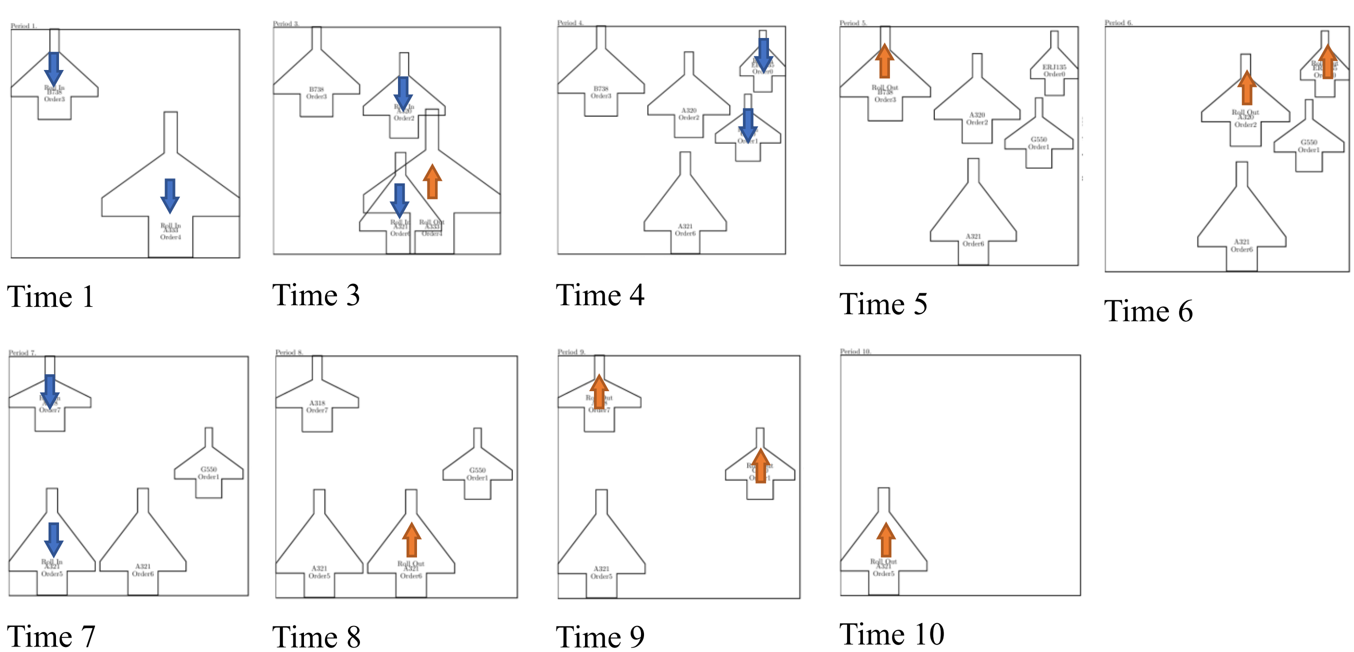


Figure 1 Hangar maintenance problem

3.2 Aircraft non-overlapping approach and three-dimensional parking

As we consider the physical shape of an aircraft in undertaking the parking planning, appropriate modelling of aircraft is fundamental to fully utilize the hangar space. The non-overlapping approach discussed in this section is incorporated in the mathematical model. Given the geometric shape of an aircraft, it can be characterized as a non-convex polygon (Figure 2). We denote the reference point of each aircraft to be the middle point at the bottom of the aircraft, and the coordinates of the reference point of aircraft p_i in two-dimensional space are denoted as (x_i, y_i). For a pair of aircraft p_i and p_j, the No-fit polygon NFP_{ij} is the region in which the reference point of aircraft p_j cannot be placed if aircraft p_i remains stationary since it would overlap aircraft p_i. A feasible zone for placing aircraft p_j without overlap with p_i is the region outside NFP_{ij}. Given these two polygons, the NFP_{ij} is generated by tracing the path of the reference point on p_j as p_j slides around the boundary of p_i, such that two polygons always touch but never overlap (Figure 3). Therefore, if the reference point of *j* moves into the NFP_{ij} then the two polygons overlap, and the interior of the NFP_{ij} represents all overlapping positions.

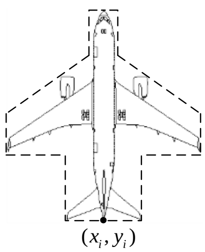


Figure 2 Geometric Representation and reference point of aircraft

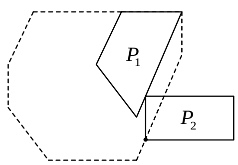
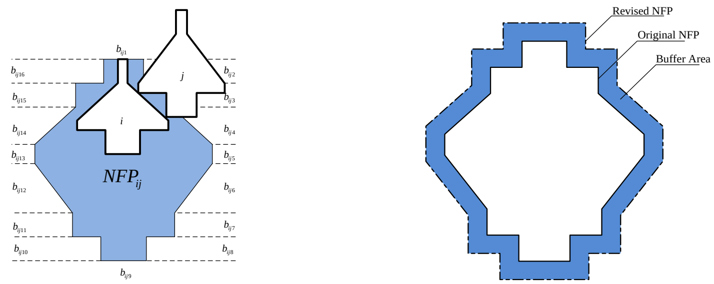


Figure 3 No Fit Polygon of P_1 and P_2



(a) **(b)**

Figure 4 Horizontal slices outside NFP

According to Alvarez-Valdes, Martinez [45], each horizontal slice is defined by drawing one or two horizontal line(s) outwards from each vertex of the NFP, and they are then characterized by one or two horizontal edge(s) as well as the part of boundary of the NFP (Figure 4 (a)). A set of variables b_{ijk} is associated with each horizontal slice and the reference point of p_j is placed in the slice *k* if $b_{ijk} = 1$. Therefore, a general form of the constraint preventing overlap is

$$\alpha_{ij}^{kf}(x_j - x_i) + \beta_{ij}^{kf}(y_j - y_i) \le q_{ijk} + M \cdot (1 - b_{ijk}),\ \forall i, j \in P, i \ne j, k = 1, 2, ..., m_{ij}$$

where $\alpha_{ij}^{kf}(x_j - x_i) + \beta_{ij}^{kf}(y_j - y_i) = q_{ijk}$ is the equation of the line of the *f*th edge of the *k*th slice in NFP_{ij} and m_{ij} is the number of slices outside the NFP_{ij}. In a real situation, we cannot allow two aircraft to touch each other during the movement operation. Therefore, a safety margin between aircraft needs to be imposed in NFPs. Imposing a safety margin for an aircraft is equivalent to adding a buffer area outside each aircraft. Moving the edges of NFP for a pair of aircraft outward is equivalent to enlarging the boundary of the non-allowable area for the reference point of the relative movable aircraft in that pair. Each edge of the original NFPs is moved outwards by distance *n* (Figure 4 (b)), and the minimum safety distance between two aircraft is prescribed as one meter.

3.3 Mathematical formulation

3.3.1 Assumptions in maintenance scheduling

The basic assumptions that describe the proposed problem are as follow:

- the estimated time of arrival, estimated time of departure, and required maintenance time are assumed to be deterministic, and the time spent on movement is incorporated in the required maintenance time;
- once the aircraft is rolled into the hangar, its parking position cannot be adjusted until the maintenance task is finished and the aircraft leaves the hangar;
- once the aircraft is rolled into the hangar, the maintenance task must be finished before leaving the hangar. If the planning period ends before finishing the maintenance task (due to the delays of rolling in), such maintenance request is deemed as failed to deliver
- if the arriving aircraft (or the departing aircraft) is blocked by any parked aircraft in the hangar, its movement operations cannot be conducted until its pathway is cleared;
- the moving path of an aircraft is a straight line and turning is not allowed due to safety consideration.
- the aircraft cannot revisit the maintenance hangar after leaving, i.e. the rolling in and rolling out operations can be conducted only once.
- the time spent on roll in and roll out operations are incorporated in the required maintenance time.
- the model applies to planning for regular maintenance. Unexpected events or demands are not considered

3.3.2 Assumptions in manpower planning

- For each incoming aircraft, the complete maintenance request is breakdown into a series of maintenance tasks with precedence relations.
- Each maintenance task requires one or more types of maintenance skills, which correlates to the licenses held by technicians as shown in Figure 5.

- At each shift, exact number of required technicians shall be assigned if the maintenance task is scheduled to conducted at the shift.
- For the senior maintenance technicians, it is allowed to conduct the maintenance tasks required junior-level skills.
- If the precedence relations between two maintenance tasks on one aircraft is imposed, then the later task cannot be conducted before finishing the previous one.
- The assignments of maintenance technician shall conform with the resting time requirement, i.e. no consecutive two shifts are allowed.

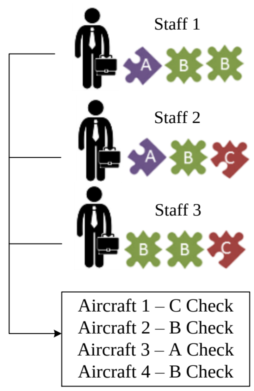


Figure 5 Multi-skill maintenance technician assignment

3.3.3 Parameters and decision variables

The given information (parameters) of the hangar maintenance planning problem consists of:

- **Maintenance demand details:** The information of incoming aircraft for hangar maintenance, including aircraft type, the breakdowns of maintenance checks with the specifications of maintenance skills and size of maintenance team. Each maintenance request has its own desired service window, including the estimated time of arrival (ETA) to the hangar, and desired estimated time of departure (ETD). The delivery after ETA induces the tardiness cost. The weightiness of each maintenance request is also predetermined.
- **Geometric information:** The necessary geometric information related to the dimensions of aircraft, and the No-Fit Polygons for generating non-overlapping constraints. The dimensions of the maintenance hangar.

- **Manpower information:** The set of multi-skill maintenance technicians, with the licenses held by respective person. The available working time of particular subsection and the manpower cost.

The list of notations for parameters mentioned above are as follows:

Notations

a_t	Set of scheduled arrival maintenance request at shift *t*
d_t	Set of schedule departure aircraft in hangar at shift *t*
A_t	Set of cumulative scheduled arrival aircraft in hangar from beginning to shift *t*. $A_t \in \bigcup_{i=0}^{t} a_t$
D_t	Set of cumulative scheduled departure aircraft in hangar from beginning to shift *t*. $D_t \in \bigcup_{i=0}^{t} d_t$
I	Set of maintenance requests received during planning horizon, $i \in I$
t	Index of shift, where *T* is the length of planning horizon
ETA_i	Estimated time of arrival of maintenance request associated with aircraft *i*
ETD_i	Estimated time of departure of maintenance request associated with aircraft *i*
$MTime_i$	Required maintenance time of maintenance request associated with aircraft *i*
w'_{ij}	Adjusted aircraft width *i* when aircraft *j* placed next to it
TD_i	Tail distance of aircraft *i*
$penalty1$	Penalty of not serving aircraft *i* during planning period (per request)
$penalty2$	penalty of late delivery of aircraft *i* during planning period (per minute)
$penalty3$	Penalty of failure to deliver aircraft *i* during planning period (per request)
$Weightness_i$	Weightiness of maintenance request *i*
W	width of hangar
H	length of hangar
w_i	width of aircraft *i*
h_i	length of aircraft *i*
NFP_{ij}	*NFP* of aircraft *i* and *j* with minimal safety distance
s_{ij}^k	*k*th slice of the region outside the NFP_{ij}
$\alpha_{ij}^{kf}, \beta_{ij}^{kf}, q_{ij}^{kf}$	parameters used to define the *f*th linear equation of the slice s_{ij}^k outside the NFP_{ij}
m_{ij}	number of slices outside NFP_{ij}
t_{ij}^k	number of linear equations used to define the slice s_{ij}^k
MPW	Set of technicians. $m \in MPW$
MPW^{is}	Set of technicians compatible for the maintenance task s associated with aircraft i
θ_m	Manpower cost of maintenance technician m undertaking compatible tasks for one shift
Div	Set of manpower planning division, $d \in Div$
r_{is}	Required working hours by skill s to finish aircraft task s (on aircraft i)
R_{mt}	Availability (hours) of technician m during planning division d, $d \in Div$
a_{mt}	1, if worker m is available at shift t

S_i	Set of maintenance tasks associated with aircraft i
S_m	Set of maintenance tasks compatible with technician m
S_m^t	Set of maintenance task compatible with technician m at shift t
h_t	The duration of shift t
τ_{is}	Required number of qualified technicians to perform maintenance task s for aircraft i
PD_{is}	Set of predecessors before conducting task s associated with aircraft i
M	A large number

To determine a maintenance schedule to fulfill the maintenance requests as well as hangar layouts at different times, the following decision variables are introduced, and the uses of auxiliary decision variables in developing specific constraints are discussed in Section 3.3.3.

Decision Variables

(x_i, y_i)	position of reference point of aircraft *i* in the hangar
out_{it}	binary decision variable that takes the value 1 if aircraft *i* is rolled out at shift *t*, and 0 otherwise
in_{it}	binary decision variable that takes the value 1 if aircraft *i* is rolled in at shift *t*, and 0 otherwise
out_{iT^*}	binary decision variable that takes the value 1 if fail to deliver aircraft *i* at the end of planning horizon, and 0 otherwise
p_{it}	binary decision variable that takes the value 1 if aircraft *i* is parked in hangar at shift *t*, and 0 otherwise
h_{ijt}	binary decision variable that takes the value 1 if aircraft *j* blocks aircraft *i* from rolling in or out at shift *t*, and 0 otherwise
L_{ij}	binary decision variable that takes the value 1 if aircraft *i* is on the left side of aircraft *j* without overlap, and 0 otherwise
R_{ij}	binary decision variable that takes the value 1 if aircraft *i* is on the right side of aircraft *j* without overlap, and 0 otherwise
U_{ij}	binary decision variable that takes the value 1 if aircraft *i* is above aircraft *j* without overlap, and 0 otherwise
b_{ijkt}	binary decision variable that takes the value 1 if the reference point of aircraft *j* is placed into the slice s_{ij}^k of the region outside NFP_{ij} at shift *t*, and 0 otherwise
$z_{mt,is}$	1, if technician m is assigned to task s (belonging to aircraft i) on shift t
y_{ist}	1, maintenance task s is conducted at shift t if minimum number of worker is met to conducted maintenance task s (on aircraft I) on shift d & precedence requirement is met
f_{ist}	1, if working hours of maintenance task s (on aircraft i) is completed by the end of shift t
zl_{mD}	1, if technician m's working hours in D division has met the limit

3.3.4 Objective and constraints

$$\min \sum_{\forall i \in A_T} Weightness_i \cdot \left[(1 - \sum_{t \geq ETA_i} in_{it}) \cdot penalty_1 + \sum_{t \geq ETD_i} out_{it}(t - ETD_i) \cdot penalty2_i + out_{iT^*} \cdot penalty3_i \right]$$
$$+ \sum_{m \in M} \sum_{t \in T} \sum_{s \in S_m} z_{ms,it} \cdot \theta_m$$

The objective function minimizes sum of overall penalty costs and manpower cost in servicing the incoming aircraft for hangar maintenance from multiple airlines. The objective function includes the penalty costs of 1) lateness in fulfilling the maintenance tasks; 2) failure to deliver the maintenance requests by the end of the planning period; 3) the profit lose cost in failing to accept the maintenance request and 4) the cost of utilizing manpower

s.t.

In the hangar maintenance operations planning are indexed by the shift along the entire planning period (Figure 6). Each point on the timeline represents each shift *t*. The integrated decision at shift *t* involves the movement operations decision, the parking position and the maintenance tasks' status (whether the task is conducted or finished). The position decision variables are not indexed by shift as its position keep unchanged once rolls in, while the auxiliary The other auxiliary decision variables, determine the position relation and movement operations, i.e. out_{iT^*}, p_{it}, h_{ijt}, L_{ij}, R_{ij}, U_{ij} and b_{ijkt}, are indexed by shift *t* to establish the continuity through multiple shifts.

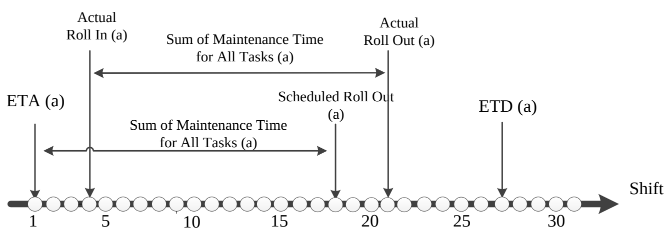


Figure 6 Planning Horizon indexed by shifts

The constraints in the mathematical model can be divided into several functions:

1) Non-overlapping constraint

The aircraft received by the maintenance service company should be served within the boundary of hangar, and the aircraft should be separated with the minimum safety margin while parked in the hangar, using the No-Fit Polygons given in Section 3.2.

$$x_i + w_i / 2 \le W, \ \forall i \in I \tag{1}$$

$$x_i \ge w_i / 2, \ \forall i \in I \tag{2}$$

$$y_i + h_i \le H, \ \forall i \in I \tag{3}$$

$$\alpha_{ij}^{kf}(x_j - x_i) + \beta_{ij}^{kf}(x_j - x_i) \le q_{ij}^{kf} + M \cdot (1 - b_{ijkt}), \ \forall i, j \in A_t, \ \forall k = 1, 2, ..., m_{ij}, \ \forall f = 1, 2, ..., t_{ij}^{k},, \forall t \ge 0 \tag{4}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \le p_{it}, \ \forall i, j \in A_t, \forall t \ge 0 \tag{5}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \le p_{jt}, \ \forall i, j \in A_t, \forall t \ge 0 \tag{6}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \le 1 - out_{it}, \ \forall i \in D_t, \forall t \ge 0 \tag{7}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \le 1 - out_{jt}, \ \forall j \in D_t, \forall t \ge 0 \tag{8}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \ge p_{it} + p_{jt} - 1, \ \forall i, j \in A_t \backslash D_t, \forall t \ge 0 \tag{9}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \ge p_{it} + p_{jt} - (out_{it} + out_{jt}) - 1, \ \forall i, j \in D_t, \forall t \ge 0 \tag{10}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \ge p_{it} + p_{jt} - out_{it} - 1, \ \forall i \in D_t, \forall j \in A_t \setminus D_t, \forall t \ge 0 \tag{11}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \ge p_{it} + p_{jt} - out_{jt} - 1, \ \forall i \in A_t \setminus D_t, \forall j \in D_t, \forall t \ge 0 \tag{12}$$

Constraints (1) – (3) ensure that the aircraft are placed within the boundary of the maintenance hangar. No-Fit Polygons between two aircraft are expressed in Constraint (4). Constraints (4) – (12) are entire non-overlapping constraints set for a pair of aircraft parking at shift *t*. In particular, the non-overlapping constraint is activated when two aircraft are parked in the hangar simultaneously at shift *t* (constraints (9) – (12)), and the non-overlapping is deactivated if any one of them is not arranged to be parked at shift *t* or one of them is rolled out altogether at that shift (constraints (5) – (8)). The auxiliary decision variable p_{it} indicates if aircraft *i* is placed in the hangar at shift *t*, activating the non-overlapping constraints. The set of binary variables b_{ijkt} associated with the horizontal slice k outside the NFP between aircraft i and j in constraint (4).

2) Movement blocking constraints

During the movement operations of aircraft, there shall not have any obstacles blocking its path of movement. If an aircraft is about to leave or enter the hangar, the other aircraft parking in the hangar should not become the obstacle, blocking the moving aircraft. In this regard, the position between two aircraft need to be determined by the auxiliary decision variables h_{ijt}, L_{ij}, R_{ij}, U_{ij}. If the aircraft about to move at shift t is blocked by any other aircraft, its movement operation has to be cancelled at this shift.

$$(x_i + w'_{ij}/2) - (x_j - w'_{ji}/2) \le M \cdot (1 - L_{ij}) \quad \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \ge 0 \tag{13}$$

$$(x_i - w'_{ij}/2) - (x_j + w'_{ji}/2) \ge -M \cdot (1 - R_{ij}) \quad \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \ge 0 \tag{14}$$

$$(y_i + TD_i) - (y_j + TD_j) \ge -M \cdot (1 - U_{ij}) \quad \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \ge 0 \tag{15}$$

$$(1 - h_{ijt}) \ge \frac{1}{6} \cdot \left[L_{ij} + R_{ij} + U_{ij} + in_{jt} + out_{jt} + (1 - p_{jt}) \right] \quad \forall i \in A_t, \forall j \in D_t, \forall t \ge 0 \tag{16}$$

$$(1 - h_{ijt}) \le L_{ij} + R_{ij} + U_{ij} + in_{jt} + out_{jt} + (1 - p_{jt}) \quad \forall i \in A_t, \forall j \in D_t, \forall t \ge 0 \tag{17}$$

$$(1 - h_{ijt}) \ge \frac{1}{5} \cdot \left[L_{ij} + R_{ij} + U_{ij} + in_{jt} + (1 - p_{jt}) \right] \quad \forall i \in A_t, \forall j \in A_t \setminus D_t, \forall t \ge 0 \tag{18}$$

$$(1 - h_{ijt}) \le L_{ij} + R_{ij} + U_{ij} + in_{jt} + (1 - p_{jt}) \quad \forall i \in A_t, \forall j \in A_t \setminus D_t, \forall t \ge 0 \tag{19}$$

Constraints (13) – (19) indicate and prescribe the correlation between the parking position of the aircraft and the blocking in aircraft movement operations. In particular, binary variables L_{ij}, R_{ij} and U_{ij} prescribe that if they take value 1, then aircraft i is placed on the left-hand side, right-hand side and upper position of aircraft j, respectively, so that aircraft j does not block the movement operations of aircraft i.

The binary variable h_{ijt} reflecting whether aircraft i is blocked by aircraft j is controlled by constraints (16) – (19). Specifically, aircraft j does not block the movement of aircraft i under the following conditions: 1) aircraft j undertakes the movement operations at the same shift as aircraft i; 2) aircraft j is not placed in the hangar at shift t; 3) aircraft i is on the the left-hand side, right-hand side or the upper position of aircraft j, as indicated by binary variables L_{ij}, R_{ij} and U_{ij}, respectively.

3) Movement Operations and aircraft blocking:

The constraints in this section prescribe that if the movement path of the aircraft rolling in and rolling out is blocked by other aircraft parked in the hangar, the movement actions cannot be conducted. In particular, for an aircraft pending leaving the hangar, the rolling out operation has to wait until the aircraft blocking the path leaves first (or concurrently). For the arrival aircraft, its parking position can be adjusted so that the aircraft can be timely moved in, or the movement operation has to be postponed until the aircraft blocking the pathway leaves the hangar.

$$out_{it} \le 1 - \frac{1}{|A_t \setminus i|} \cdot \sum_{\forall j \in A_t \setminus i} h_{ijt}, \forall i \in D_t, \forall t \ge 0 \quad (20)$$

$$in_{it} \le 1 - \frac{1}{|A_t \setminus i|} \cdot \sum_{\forall j \in A_t \setminus i} h_{ijt}, \forall i \in A_t, \forall t \ge 0 \quad (21)$$

Constraints (20) and (21) state that the rolling out and rolling in operations of aircraft i cannot be conducted if it is blocked by any parked aircraft in the hangar at shift t. The auxiliary decision variable h_{ijt} indicates the relations between each pair of aircraft at shift t acting as the mediator between the movement operations decision variable (out_{it} , in_{it}) and the movement blocking constraints (Constraints (13)-(19)).

4) Staying time requirements:

The duration that each aircraft stays in the hangar should sufficient for conducting the maintenance task. The constrains set in this section ensure the staying time of an aircraft served by the company equals or is longer than its required maintenance. Moreover, the rolling in and rolling out operations for each aircraft can be conducted only once, as the aircraft cannot revisit the hangar during the planning period. The auxiliary decision variable p_{it} acts as a mediator, establishing the relation between the non-overlapping constraint in constraint set 1) and the staying time requirement in this section.

$$(\sum_{t \ge ETD_i} out_{it} \cdot t - \sum_{t \ge ETA_i} in_{it} \cdot t) + M \cdot (1 - \sum_{t \ge ETA_i} in_{it}) + M \cdot (1 - \sum_{t \ge ETD_i} out_{it}) \ge MTime_i, \forall i \in I \quad (22)$$

$$p_{it} = \sum_{ETA_i \le m \le t} in_{im}, \forall i \in A_T, \forall ETA_i \le t \le ETD_i \quad (23)$$

$$p_{it} = \sum_{ETA_i \le m \le t} in_{im} - \sum_{ETD_i \le m \le t-1} out_{im}, \forall i \in A_T, \forall t \ge ETD_i + 1 \tag{24}$$

$$\sum_{t \ge ETA_i} in_{it} \le 1, \forall i \in I \tag{25}$$

$$\sum_{t \ge ETD_i} out_{it} \le 1, \forall i \in I \tag{26}$$

$$out_{it} \le \sum_{ETA_i \le m < t} in_{im}, \forall i \in I, \forall t \ge ETD_i \tag{27}$$

$$(1 - out_{iT*}) \le \sum_{t \ge ETD_i} out_{it} + M \cdot (1 - \sum_{t \ge ETA_i} in_{it}), \forall i \in I \tag{28}$$

$$(1 - out_{iT*}) \le \sum_{t \ge ETD_i} out_{it} + M \cdot (1 - \sum_{t \ge ETA_i} in_{it}), \forall i \in A_T \tag{29}$$

Constraint (22) determines the duration of stay for each aircraft, prescribing that if such aircraft is accepted by the service company then its parking time must equal or be longer than its required maintenance time, which equals to the sum of the maintenance time of each maintenance task associated with the aircraft. Constraint (22) acts as a bounding constraint for the aircraft staying time in the hangar.

Constraints (23) and (24) prescribe that p_{it} indicates whether the aircraft is parked in the hangar takes value 1 by the time it rolls into hangar until it rolls out. If the value of p_{it} equals to one, the respective non-overlapping constraints are activated accordingly.

Constraints (25) – (27) ensure that the rolling in operations happens after the arrival time of the maintenance request (ETA), and rolling out operations are conducted only after the aircraft has been rolled in. Constraints (28) – (29) imposes that out_{iT^*} equals to one if the aircraft is still parked in the hangar at the end of the planning horizon.

5) Variable domination constraints

$$x_i, y_i \ge 0 \quad \forall i \in I \tag{30}$$

$$b_{ijkt} \in \{0,1\} \ \forall i, j \in A_t, k = 1, 2, ..., m_{ij}, \forall t \ge 0 \tag{31}$$

$$p_{it} \in \{0,1\} \quad \forall i \in A_t, \forall t \ge 0 \tag{32}$$

$$in_{it} \in \{0,1\}, \forall i \in A_t, \forall t \ge 0 \tag{33}$$

$$out_{it} \in \{0,1\}, \forall i \in D_t, \forall t \geq 0 \tag{34}$$

$$h_{ijt}, L_{ij}, R_{ij}, U_{ij} \in \{0,1\} \quad \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \geq 0 \tag{35}$$

Constraint (30) ensures that the coordinates of the aircraft are positive, and constraints (31) – (35) indicate the binary variables in the mathematical model. The following constraints are used to tighten the geometric relations among aircraft parking in the hangar:

$$L_{ij} + L_{ji} \leq 1, \forall i, j \in I, j \neq i \tag{36}$$

$$R_{ij} + R_{ji} \leq 1, \forall i, j \in I, j \neq i \tag{37}$$

$$L_{ij} \leq R_{ji}, \forall i, j \in I, j \neq i \tag{38}$$

$$R_{ij} \leq L_{ji}, \forall i, j \in I, j \neq i \tag{39}$$

The feasibility of the tentative solution is examined by firstly determining a feasible maintenance schedule, then fixing the position-related binary variables. After branching on all the position-related variables, the geometry constraints are imposed to examine if such a parking plan is feasible. In this regard, the LP relaxation of the model is not tight, and the updates of the lower bound do not progress well to tighten the optimality gap. Constraints (36-39) impose a side-by-side relation between a pair of aircraft

6) Staff Assignment Components:

$$y_{ist} \leq p_{it}, \forall i \in I_t, \forall s \in S_i, t \geq ETA_i \tag{40}$$

$$y_{ist} \leq \frac{1}{|PD_s|} \cdot \sum_{s' \in PD_{is}} f_{s'd}, \forall s \in S, t \geq ETA_i \tag{41}$$

$$y_{ist} \leq \frac{1}{\tau_{is}} \cdot \sum_{m \in MPW^{is}} z_{mt,is}, \forall i \in I, s \in S_i, t \geq ETA_i \tag{42}$$

$$y_{ist} \geq \frac{1}{\tau_{is}} \cdot \sum_{m \in MPW^{is}} z_{mt,is}, \forall i \in I, s \in S_i, t \geq ETA_i \tag{43}$$

$$\sum_{m \in MPW^{is}} z_{mt,is} \leq \tau_{is}, \forall i \in I, s \in S_i, t \geq ETA_i \tag{44}$$

$$\sum_{m \in MPW^{is}} z_{mt,is} \leq \tau_{is} \cdot (1 - f_{ist}), \forall i \in I, s \in S_i, t \geq ETA_i \tag{45}$$

$$z_{mt,is} \le p_{it}, \forall m \in MPW^{is}, \forall i \in I_t, \forall s \in S_i, t \ge ETA_i \tag{46}$$

$$M \cdot f_{ist} \ge \sum_{t > t' \ge ETA_i} y_{ist'} \cdot h_{t'} - r_{is}, \forall s \in S_i, \forall i \in I, t \ge ETA_i \tag{47}$$

$$-M \cdot (1 - f_{ist}) \le \sum_{t > t' \ge ETA_i} y_{ist'} \cdot h_{t'} - r_{is}, \forall s \in S_i, \forall i \in I, t \ge ETA_i \tag{48}$$

$$\sum_{t \ge ETA_i} y_{ist} \cdot h_t \ge r_{is} - M \cdot (1 - \sum_{t \ge ETA_i} in_{it}), i \in I, s \in S_i \tag{49}$$

$$\sum_{s \in S_m^t, i \in I} z_{mt,is} \le a_{mt}, \forall m \in MPW, \forall t \in T \tag{50}$$

$$\sum_{t' \in Div_d, t > t'} \sum_{i \in I} \sum_{s \in S_m^t} z_{mt',is} \cdot h_{t'} \le R_{md}, \forall m \in MPW, \forall d \in Div \tag{51}$$

$$\sum_{s \in S_t^m} z_{mt,is} + \sum_{s \in S_{t+1}^m} z_{m(t+1),is} \le 1, \forall m \in MPW, \forall t \in T \tag{52}$$

$$out_{it} \le \frac{1}{|S_i|} \cdot \sum_{s \in S_i} f_{ist}, \forall i \in I, t \ge ETD_i \tag{53}$$

$$z_{mt,is} \in \{0,1\}, \forall m \in MPW, \forall t \in T, \forall i \in I, \forall s \in S_i \tag{54}$$

The constraints (40) – (55) are relevant to staff assignment's decision-making, which characterize the assumptions and requirements while forming the maintenance team, assigning maintenance technicians and arranging each individual's maintenance roster. Constraint (40) ensures that any maintenance task associates with aircraft *i* can be conducted as long as the aircraft is parking in the hangar. The situation that the aircraft is leaving but the maintenance tasks of aircraft is scheduled to conduct at shift t, i.e. $y_{isd} = 1, p_{it} = 1, out_{it} = 1$, is not a possible scenario, since Constraint (54) has prescribed that the roll out operation cannot be triggered before completing all maintenance tasks for the aircraft and therefore Constraint (40) do not involve the aircraft rolling out decision variable. Constraint (41) imposes the precedence relations between the maintenance tasks associate with aircraft *i*, implying that the subsequent maintenance tasks cannot be conducted before the preceding tasks have been finished. Constraints (42) – (44) ensures that each maintenance task shall have enough qualified maintenance technician to conduct, and the number of capable technicians shall not exceed the required number by particular task to avoid wastage on manpower input. Constraint (45) prescribes that no more maintenance technicians are assigned to the finished maintenance tasks. Constraint (46) ensures that the maintenance technicians cannot be allocated to the aircraft not parking in the hangar. Constraints (47) – (48) determine if the maintenance task s associates with aircraft i have finished by the shift t. Constraint (49) prescribes that the maintenance time of each maintenance tasks shall be equal or larger than the required maintenance time. Constraints (50) – (54) are the regulations in assigning multi-skill maintenance technicians. Constraints (50) and (51) impose that the technician can be assigned at shift t if the individual is available, and the working time of individual cannot exceed the prescribed working time limit of that division d. Constraint (52) prescribes that the individual maintenance technician cannot undertake maintenance tasks in two consecutive shifts to ensure the technician has enough resting time. Constraint (53) regulates that the aircraft cannot leave the hangar before completing all maintenance tasks associated with that aircraft. Constraint (54) prescribes that the manpower assignment decision variables as binary type.

3.4 Branching strategy

Considering the large number of binary variables involves in the mathematical model, the difficulties in updating the incumbent solutions and lower bounds are expected during the optimization process as eliminating the unpromising solution and its duplicate solution is time-consuming. For example, improper branching strategy may create the branching tree indicating the manpower allocation variables on the top of branching trees, which determines the manpower allocation before fixing the service period of aircraft. Alternatively, improper branching sequence may indicate the position-related variables at the top of branching trees, which determines the position of aircraft without confirming their respective service period. Such unwise or default branching ways may result in the adjustment of relevant decision variables or pruning unpromising subtrees from the top of the branching tree in an inefficient way. Therefore, given the hierarchal structure of the binary decision variables, a branching strategy that cater to the features of mathematical model can be developed to avoid the inefficient default branching strategy.

The hierarchal structure of the binary variables can be listed in a descending order as follows:

1) determine the service period (parking period) of each aircraft with the associated in_{it}, out_{it}, p_{it} and out_{iT^*}. Normally the value of out_{iT^*} for all incoming maintenance demands equals to zero, implying that the maintenance demands can be delivered by the end of planning period;

2) After determining the service period of each incoming aircraft, the non-overlapping constraints and movement path blocking constraints are imposed to validate the tentative service period. Therefore, the value of binary variables h_{ijt}, L_{ij}, R_{ij}, U_{ij} and b_{ijkt} are branched to determine the coordinate (x_i, y_i) of each aircraft, which examines the hangar capacity and clearance of movement path in the tentative parking positions;

3) Upon determining the service period and geometric positions of each aircraft, the allocation of multi-skill maintenance technicians to each maintenance tasks associated with aircraft is conducted. In the problem instances with overwhelming maintenance demands or with peak maintenance period, it is the case that negative impact of incoordination among the service time decisions, parking positions and manpower allocations is amplified, which manifests the interdependent relationships among these three core decision makings in hangar maintenance planning problem.

For these instances with high demands, branching on the binary variables and updating bounds can be trapped for a long time, and therefore the branching strategy tailored for this problem is proposed to assist the branch-and-bound algorithm in searching for the incumbent solutions. The branching priorities assigned to the binary variables follows the hierarchal structure of the mathematical model, i.e. the priorities are assigned to service period-related, position-related and manpower-related binary variables in a decreasing order. In this section, we conduct the computational experiment to find the performance of the MILP model and the proposed branching strategy in solving the small-sized instances.

4. A two-stage optimization approach

The MILP model presented in Section 3 involves great numbers of geometric constraints, resource constraints and respective decision variables, which makes the medium- and large-scale instance intractable by the default solver. In this section, a two-stage optimization based on model decomposition is presented to enhance the efficiency in solving the problem.

4.1 Decomposition of original model

In the original MILP model, the timing constraint, geometric constraints and resources constraints are integrated to reflect the interdependent relations among the three-core decision-making elements in this problem, namely the parking period of each aircraft in the hangar, the parking stand position and the assignment of technicians. As the mathematical model involves lager number of binary decision variables, the branching progress takes up a large amount of time. Updating bounds or finding new incumbent solutions becomes difficult for the medium- to large-size instances, since the default branching strategies provided by the solver CPLEX are incapable to analyze the complex relations and practical meaning behind the set of binary variables. Specifically, a hierarchal structure exists in the MILP model, and branching progress may be trapped into investigating the unpromising pending solution for a relatively long time. The hierarchal structure of the mathematical model can be presented as follows: the service period (parking period) of each aircraft is determined according to their ETA, and ETD first. Afterwards, the non-overlapping constraints and movement path blocking constraints are imposed to validate the tentative service period, so as to find if the hangar has enough capacity for aircraft parking and clear movement paths to their dedicated positions. If the infeasible solution return with the tentative determined service period, then one or more aircraft's service period or parking positions need to be adjusted in order to align with the geometric constraints. Our previous research and computational analysis have revealed that finding feasible parking plan for single time or multi-period is challenging while dealing with large number of incoming aircraft [14, 49]. In this extended model, we incorporate the multi-skill technician assignment problem, which is another bottleneck in fulfilling hangar maintenance demand under MRO mode. The hierarchal structure incorporating multi-skill technician assignment in the extended model makes the branching strategy incapable in tackling the medium-size integrated instance, especially dealing with the instance with overwhelming maintenance demands and limited maintenance resource (hangar space and manpower). In this regard, we propose a two-stage optimization approach inspired by the decomposition method to reduce the difficulty in solving the original model as a whole.

The original problem is decomposed into two subproblem. The first-stage problem consists of all geometric constraints to determine parking stands, movement path blocking and service time of each aircraft in the hangar. In the second-stage problem, the resource constraints related to multi-skill technician assignment problem is included. To ensure the connectivity between two subproblems, an iterative process with linkage constraints are proposed to develop integrate solution between geometric- and manpower-related decision makings. The detailed descriptions of the two-stage optimization approach are discussed in Sections 4.2. The characteristic of proposed approach is that the service time solution given from the first stage is flexible and can be adjusted during the optimization process of second stage problem. The overview of optimization procedures is presented in Algorithm 1

4.2 First stage problem

The decision variables and constraints related to decision-making in service period and geometric aspects are included, i.e. determine parking period of aircraft, parking position and movement path clearance. Instead of determining the working schedule of each maintenance task associated with the aircraft, the first stage problem only determine the time period that aircraft is staying in the hangar. The first stage problem's optimization process consists of two scenarios:

1) the first scenario is the problem initialization. It is assumed that the manpower supply is sufficient to meet all maintenance task at any time, which means that the maintenance tasks for each aircraft can be conducted consecutively one by one during the whole planning period. After determinizing the parking periods, parking stands and movement paths for all aircraft, these decisions are passed to the second stage problem for assignments of maintenance technicians, as well as the feasibility checking;

2) the second scenario is the iteration process after the problem initialization. After inputting the initial solution (or the solution in the previous iterations), the mismatch between two problems might occur. In the problem initialization stage, it is assumed that the manpower supply is sufficient to meet all maintenance tasks at any time along the planning period. However, usually the initial solution given by the first stage problem do not comply with the staff assignment problem in the second stage at the problem initialization step, which means that the manpower supply cannot meet all maintenance tasks with the predetermined desired time windows given by the first stage problem. Given that the available manpower is not able to meet the desired service period, the service time decision has to be adjusted to align with the manpower supply. A possible way is to extend the parking time of aircraft so as to allow enough time to finish the maintenance tasks correlated to the aircraft. The alignment between the first stage and second stage problem can be found after the adjustment of parking time of aircraft.

4.3 Second stage problem

The decision variables and constraints related to service time scheduling and multi-skill technician assignment form the second stage problem, which determines the technicians that serve the maintenance tasks within the tentative service period decision passed from the first stage problem. In the second stage problem, the fulfillment of maintenance tasks is specified, based on service period determined by the first stage problem. The optimization process of the second stage problem consists of two scenarios:

1) the second stage problem is able to identify a feasible solution with the service period passed from the first stage problem, which means that the manpower is able to meet fulfill all maintenance tasks within the desired service period given by the first stage problem. No more service time adjustment is needed, and the combined solution can be finalized.

2) infeasibility returns in the second stage problem with the input desired service period from the first stage problem. Under such circumstance, the service periods have to be adjusted in order to allow longer timeframe for fulfillment of maintenance task.

Decision variable

out_break_i	1, if the roll out time of aircraft *i* is amended in the stage two problem, and 0 otherwise
$out^*\_break_i$	1, if fail to deliver aircraft *i* in the stage two problem, and 0 otherwise

The auxiliary decision variables are introduced in the second stage problem.

out_break_i	1, if the roll out time of aircraft *i* is amended in the stage two problem, and 0 otherwise
$out^*\_break_i$	1, if fail to deliver aircraft *i* in the stage two problem, and 0 otherwise

The service time decisions for all incoming aircraft made in the first stage problem serve as the initial solution for the second stage problem. The initial solution consists of $\{\overline{in_{it}}, \overline{out_{it}}, \overline{out_{iT^*}}\}$ for all aircraft, and the initial solution imposes as “soft constraints” in the second stage problem, which means that the initial service time decision can be adjusted deemed necessary when the manpower is insufficient to serve them within the desired period. The relevant constraints are presented as follows:

$$in\_sp_{it} \geq \overline{in_{it}}, \forall i \in A_T, \forall t \geq 0 \qquad (55)$$

$$out\_sp_{it} \geq \overline{out_{it}} - out\_break_i, \forall i \in D_t, \forall t \geq 0 \tag{56}$$

$$out_{T^*}\_sp_{it} \geq \overline{out_{iT^*}}, \forall i \in D_t, \forall t \geq 0 \tag{57}$$

Constraints (55)-(57) impose the initial service time determined by the first stage solution. It is noted that the tentative roll in time from first stage is not permitted to revise. The ETA plus MTime for all tasks of incoming aircraft are quite close to the ETD, and we do not allow postpone of roll in time in the second stage problem. In this regard, the changeable service time decision is the roll out time of aircraft, which allows to extend the aircraft staying time in the hangar to finish the maintenance tasks.

4.2 Linkage constraints for two-stage problem

When misalignment between two problems occurs, the service time adjustment shall be imposed. Intuitively, the service time extended constraint can be the revised version of maintenance time constraint (58):

$$\left(\sum_{t \geq ETD_i} out_{it} \cdot t - \sum_{t \geq ETA_i} in_{it} \cdot t\right) + M \cdot \left(1 - \sum_{t \geq ETA_i} in_{it}\right) + M \cdot \left(1 - \sum_{t \geq ETD_i} out_{it}\right) \geq MTime_i^{REVISED}, \forall i \in A_T \tag{58}$$

The linkage constraint (58) is added into the first stage problem and resolve again in the next iteration. The revised maintenance time $MTime_i^{REVISED}$ can be derived by the revised roll out time of aircraft *i* out_sp_{it} from the second stage problem and the original roll in time $\overline{in_{it}}$ from the first stage problem. $MTime_i^{REVISED} = out\_sp_{it} - \overline{in_{it}}$ for the aircraft needs the extension of staying time in the hangar. After adding the revised maintenance time constraint back to the first stage problem, two possible outcomes of service time decisions are expected: 1) a revised service time have been found, and the aircraft can be serviced during the planning period; 2) after imposing the revised staying time constraint, the aircraft is rejected for service, as the delay cost induced due to the extended staying time or the blocking with other aircraft.

Algorithm 1

Two-stage optimization approach for the hangar maintenance planning problem

Notations	Meanings
$in_{it}^{1}, out_{it}^{1}$	Roll in and roll out time decision for aircraft in first stage problem
$in_{it}^{2}, out_{it}^{2}$	Roll in and roll out time decision for aircraft in second stage problem
A_T	All maintenance aircraft during the planning period
out_break_i	Indicator of adjusting roll out time of aircraft *i* in the second stage problem
$RollOutT*_i^2$	Indicator of failure to deliver the aircraft *i* at the end of planning period in the second stage problem
$MTime_i^{REVISED}$	The revised staying time requirement of aircraft *i* after solving the second stage problem

1: Solve the first stage problem and derive the first stage service time decision, including the service time and movement operations decisions.
2: Input the first stage decision into the second stage problem. Solve the second stage problem to determine the staff rosters along the planning period.
3: If the second stage solution is infeasible. The misalignment between service time and manpower supply exists.
4: For *i* in A_T
5: If $out\_break_i = 1$
(The roll out time of aircraft *i* is adjusted)
6: Calculate the revised staying time required for the aircraft *i*.
$MTime_i^{REVISED} = \sum_{t \geq ETD_i} out\_sp_{it} \cdot t - \sum_{t \geq ETA_i} \overline{in_{it}} \cdot t)$
7: If $MTime_i^{REVISED} > \sum_{t \geq ETD_i} \overline{out_{it}} \cdot t - \sum_{t \geq ETA_i} \overline{in_{it}} \cdot t$
8: Generate the revised staying time constraint for the aircraft *i*. Add the constraint to the first stage problem
9: If $RollOutT*_i^2 = 1$
10: Generate the $RollOut_i^* \geq 1 - M \cdot (1 - \sum_{t \geq ETA_i} in_{it})$ constraint. Add the constraint to the first stage problem
11: Go to Step 1.
12: Else if the second stage solution is feasible
13: The current service time solution can find feasible technician assignment plan. Go to Step 14.
14: End

5. Computational experiments

In this section, we describe the way of generating problem instances based on the real data collected from an aircraft maintenance company, and the analysis of the numerical experiment results. All the procedures described in the previous sections are coded in C# in Visual Studio 2010 and run on a computer with an Intel Core i7 processor, at 3.6 GHz with 32 Gb of RAM. The Mixed-Integer Linear Programming is solved by the CPLEX 12.7 serial model.

5.1 Description of test instances

The problem instances are generated from the data of maintenance demands derived from an aircraft hangar maintenance service provider located in Hong Kong, which is serving over 50 clients, including airlines, business jet companies and utility aircraft companies. The necessary information of maintenance demands, including the estimated arrival time (*ETA*), estimated time of departure also known as desirable service completion time(*ETD*), aircraft type and maintenance checks type of each maintenance demands, is collected from multiple clients over 157 days from January to May in 2015 to create instances. We have utilized these set of data to generate problem instances in our previous study [14, 20]. The aircraft to be maintained are classified into three categories according to their physical size as presented in Table 1.

Table 1 Classification of aircraft size

Classification	Total Number	Aircraft models
Small-sized	10	G200 CL600 CL605 F900LX F2000EX F2000LX ERJ135 F7X G450 GIV
Medium-sized	11	GL5T G550 G5000 G6000 G650 A318 ERJ190 A319 A320 B738 A321
Large-sized	2	A332 A333

We refer to Ertogral and Öztürk [3]'s principle while determining the cost of maintenance technicians, which prescribes the cost of worker on hourly rate basis. As we consider the multi-skill maintenance technician's setup in this problem, it is reasonable to prescribe that the individual technician equipped with more maintenance skill is associated with a higher wage rate, regardless of the maintenance tasks assigned to the technician in actual implementation. Such cost setting encourages assigning senior maintenance technicians, i.e. the individuals equipped with more maintenance skills and senior maintenance licenses, to the maintenance tasks requiring senior technicians to avoid improper utilization of manpower and wastage of manpower. The number of maintenance technicians required for the different tasks for aircraft under different categories are listed in Table 2. To examine the performance of proposed two-stage optimization approach, the following parameters are adjusted across the problem instances: 1) the skill levels of maintenance technicians; 2) number of available staff; 3) required maintenance service and associated maintenance tasks and 4) shift settings.

Table 2 Generic number of require maintenance technicians

	Small Aircraft	Medium Aircraft	Large Aircraft

Minor Maintenance Task	2	3	5
Medium Maintenance Task	3	4	6
Major Maintenance Task	4	6	9

For the setting chance lose cost, tardiness cost and failure to deliver cost upon the end of planning period, each maintenance demand has its own set of costs according to the maintenance types, aircraft types and the required maintenance skill level. In details, the chance lose cost refers to the situation that the maintenance company do not have enough maintenance capacity to receive the maintenance demand. Originally, the profit of completing the maintenance service for an aircraft is regarded as two times more than the required input of manpower and each aircraft requires different input of manpower and maintenance time. Therefore, the expected profit is recorded as the chance lose cost of particular aircraft. The penalty cost is determined on minute basis, which is induced maintenance service cannot be accomplished by the end of desirable delivery time (ETD). For the cost of failure to finish the maintenance task by the end of planning period, its cost shall be larger than the tardiness cost from ETD of each maintenance demand to the end of planning period, and we impose the cost of failure to deliver as a portion of chance lose in the computational experiments.

5.2 Computational results

Two sets of computational experiments are conducted in this section. Section 5.2.1 reports the computational results of basic MILP model in small- size instances. Afterwards, the medium- and large-size instances were solved by the two-stage optimization approach, whose performances are reported in Section 5.2.2.

5.2.1 Model and branching strategy’s evaluation

In this section, we examine the performance of the proposed MILP formulation and branching strategy presented in Section 3.4.

It can be inferred that extending the length of planning period also influence the complexity of solving a single instance as it determines the scale of time-related decision variables.

Table 3 reports the results for 4 sets of instances solved by the original model and the model with branching strategy. The number of shifts for single day is prescribed as three, and the length of each shift is 8 hours, which aligns with the normal staff rostering setting. The first column in the table denotes the name of instance. The maintenance demand data collected from the maintenance company are organized to create different groups of problem instances, which are divided into subsections to create problem instances in different sizes or with different parameters. The maintenance demands data are sorted on monthly basis, and further divided into multiple set of problem instances within the month. The name of the instance is presented in "number of requests_maintenance tasks's demand level (number of planning days)" form, e.g. 5_1(14) stands for the instance covering the instance with 5 incoming aircraft to be serviced maintenance with standard maintenance task demand, which covers 14 days. To investigate the performance of solution approach, instances covering half month to two months are created to examiner the impact of planning period and number of maintenance demands. Each instance was solved by the original model and the model incorporating branching strategy mentioned in the beginning of this section. The second column reports the number of binary variables involved in the mathematical model, and the third column denotes the preprocessing time before implementing the branch-and-bound algorithm embedded in CPLEX to solves the instance respectively. To examine the impact of increasing manpower requirement (the required number of qualified maintenance technicians for particular maintenance task) on the objective value as well as the computational performance, the adjustment on number of requiring capable maintenance technicians is conducted to create a variation of manpower requirement. Specifically, Instance x_1(planning days) denotes the original problem instance with the maintenance team size align with Table 2, and Instance x_2(planning days) and x_3(planning days) refers to the problem instances with more manpower demands associated with each maintenance tasks along the entire instance in a progression manner, i.e. instance x_3(planning days) prescribes larger maintenance team size requirements than x_2(planning days). Therefore, the same group of instances with manpower requirement variation have the same number of binary variables. The preprocessing time includes the initialization of the mathematical model, i.e. defining the decision variables and initializing the constraints. The best-known solution (upper bound), lower bound, optimality gap and the CPU time elapsed when the termination criterion was met are recorded from the fourth to eleventh columns for two models for comparison, respectively. The time limit for each instance was 3,600 seconds for both models.

The overall results in It can be inferred that extending the length of planning period also influence the complexity of solving a single instance as it determines the scale of time-related decision variables.

Table 3 show that within one hour, both models cannot solve the problem instances to optimal. Nevertheless, a minor advantage of branching strategy is manifested compared with the original model in most instances, in terms of the best-known solution and the optimality gap for some instances. In particular, the branching strategy finds better incumbent solution or tighten optimality gap over the original model in the problem instance with moderate number of maintenances demands or higher manpower requirement, e.g. 8_3(22) and 10_2(32), while such advantage do not reflect in challenging instances with large number of maintenance demands and high manpower requirements, e.g. 15_3(63) . It is noted that the mathematical model involves a significant number of binary variables in each instance, which grows significantly along the lengthen of the planning period and the increase of number of maintenance tasks. The number of maintenance requests in each instance is one of determinants of the model scale, while the difficulties of tackling instance also lie on the distribution of arrival, maintenance tasks and precedent relations associated with the aircraft as well as the manpower requirement of maintenance tasks. The analysis on the same group of problem instances with a variation of manpower requirements on maintenance task reveals an increasing trend of objective value, which is associated with the enlargement of maintenance manpower cost and the rejections of maintenance requests due to the insufficient manpower supply. The rejections of maintenance request also reflect the shrinkages of optimality gap as the branching efforts are saved and eliminated for examining the service period, parking stand allocation, geometric relations with other aircraft and the manpower assignment of the rejected maintenance requests. The minor advantages of branching strategy demonstrate the hierarchical structure of the mathematical model impose the computational difficulties in tackling the problem instance. However, the results in solving instances also reflect the inefficiency of tackling the problem solely with the branching strategy. It can be inferred that extending the length of planning period also influence the complexity of solving a single instance as it determines the scale of time-related decision variables.

Table 3 Comparison between original model and model with branching strategy

Instance	Binary Variables	MILP Model without Branching Strategy	MILP Model with Branching Strategy

		Preprocessing Time (s)	Upper Bound	Lower Bound	Gap (%)	CPU (s)	Upper Bound	Lower Bound	Gap	CPU (s)
5_1(14)	11734	0.48	43719.00	38198.50	12.63	3600	41583.00	35235.32	15.22	3600
5_2(14)		0.40	58266.00	45532.41	21.85	3600	57930.00	43953.50	24.13	3600
5_3(14)		0.42	54234.00	53982.92	0.46	3600	54234.00	50490.13	6.90	3600
8_1(22)	67927	2.52	83595.00	12357.97	85.21	3600	83595.00	12343.76	85.23	3600
8_2(22)		2.76	95115.00	40431.77	57.49	3600	95091.00	40421.21	57.49	3600
8_3(22)		2.69	113091.00	71447.20	36.82	3600	112827.00	111968.31	0.76	3600
10_1(32)	116022	4.44	129351.75	11640.78	91	3600	82311.75	11631.62	85.87	3600
10_2(32)		4.41	176055.75	35495.72	79.84	3600	38577.75	35503.16	7.97	3600
10_3(32)		4.60	194877.75	57955.29	70.26	3600	166002.75	57979.18	65.07	3600
15_1(63)	162657	8.29	195156.00	59608.39	69.46	3600	216510.00	59600.51	72.47	3600
15_2(63)		8.61	413802.00	87353.60	78.89	3600	226206.00	87679.71	61.24	3600
15_3(63)		7.96	490815.00	132279.25	73.05	3600	490815.00	132279.25	73.05	3600

5.2.2 Two-stage optimization approach evaluation

The computational results in Section 5.2.1 have demonstrated minor advantages of branching strategy in tackling the instances over the original model. However, it is intractable to deal with the instances solely by the MILP model with branching strategy as the instance cannot be solved optimally after meeting the stopping criterion. In this section, we further implement the computational experiment to examine the performance of two-stage optimization approach presented in Section 4. In the report of computational results, the performance of MILP model incorporating branching strategy is compared with the two-stage optimization approach. We examine the performance of the two-stage optimization approach with different setting of shift one day, i.e. 3 shifts per day and 4 shifts per day. The MILP model tailored for the two-stage optimization approach is embedded in solving the first stage problem and second stage problem with branch-and-bound algorithm. The time limit for solving each first and second stage problem is 1,800 seconds.

Table 4 reports the computational results of two-stage optimization approach. The seventh to eighth column of Table 4 report the binary variables involved in the problems in two stages and the number of iterations, respectively. The computational time and objective function values are two indicators in comparing the performance of model with branching strategy and the proposed heuristic approach. The two-stage optimization approach is able to obtain good quality solution within around an hour time, compared with the MILP model with branching strategy. The advantages of two-stage approach were manifested regarding the objective value while solving the instances with high maintenance demands, i.e. the problem instances with more than 5 aircraft maintenance requests. It is found that setting 4 shifts daily operations do not reflect the benefit from controlling the cost in view of objective value. The required computational time and model scale for solving the same instances increases in the 4-shift per day setting. The strengths of the two-stage optimization approach over the MILP model for large-scale or high demand instances implies the incapability of the MILP model in connecting the three independent core elements of decision-making, as the branch-and-bound algorithm is likely to probe the subtree associated with infeasible solution repeatedly before updating bounds and default branch-and-bound algorithm cannot infer the pattern of infeasible solution in the previous pruned subtree.

The effectiveness of two-stage optimization approach reflects that the staying time constraint, i.e. MTime requirement imposing on each incoming aircraft, successfully acts as an efficient connecting bridge between two stages problem, as the main effect of insufficient manpower supply directly result in the tardiness in fulfilling the maintenance tasks and extension of service time window. For the maintenance service company, facing with hard instances for decision-making within short period of time is common for real-world operations. To provide solution for practical use, two-stage optimization approach can be considered as a reliable heuristic when the service providers are in need of better-quality solutions in less time than the exact method provided by the commercial solver when the allowable computational time is limited.

Table 4 Computational results in solving instances by two-stage optimization approach

Instance	Shift Setting	MILP Model with Branching Strategy				Two-stage Optimization Approach					
		Upper Bound	Lower Bound	Gap	CPU	Binary Variables in Stage 1 Problem	Binary Variables in Stage 2 Problem	Iteration	Generated Constraints	Objective Value	CPU
5_1(14)	3	41583.00	35235.32	15.22	3600	5487	6721	1	4	44223.00	1348.66
5_2(14)		57930.00	43953.50	24.13	3600			2	10	59055.00	1884.95
5_3(14)		54234.00	50490.13	6.90	3600			1	7	56442.00	185.29
8_1(22)		83595.00	12343.76	85.23	3600	34100	35347	1	1	21429.00	2268.05
8_2(22)		95091.00	40421.21	57.49	3600			1	4	50733.00	3631.54
8_3(22)		112827.00	111968.31	0.76	3600			2	9	117483.00	2100.73
10_1(32)		82311.75	11631.62	85.87	3600	68320	49953	1	1	29025.75	192.59
10_2(32)		38577.75	35503.16	7.97	3600			1	1	53313.75	2175.83
10_3(32)		166002.75	57979.18	65.07	3600			2	11	127695.75	2090.25
15_1(63)		216510.00	59600.51	72.47	3600	112153	55069	1	13	114303.00	4140.59
15_2(63)		226206.00	87679.71	61.24	3600			2	36	225720.00	2367.86
15_3(63)		490815.00	132279.25	73.05	3600			4	48	260271.00	3896.76
5_1(14)	4	51669.00	39097.39	24.33	3600	7372	9045	1	6	49275.00	1894.04
5_2(14)		69900.00	47597.70	31.91	3600			2	12	73626.00	1824.27
5_3(14)		58671.00	51300.52	12.56	3600			1	7	59625.00	847.89
8_1(22)		121539.00	38119.44	68.64	3600	45505	35310	2	6	87267.00	4408.53
8_2(22)		184776.00	57023.04	69.14	3600			3	8	134208.00	1901.62
8_3(22)		207024.00	62777.81	69.68	3600			1	4	146700.00	1850.29
10_1(32)		118938.75	34805.94	70.74	3600	91700	43629	2	18	94057.50	7567.64
10_2(32)		204966.75	55514.02	72.92	3600			1	16	142711.50	1904.96
10_3(32)		335211.00	65667.83	80.41	3600			1	14	166316.25	1880.97
15_1(63)		341013.00	59779.58	82.47	3600	149634	73565	3	31	156357.00	3840.18
15_2(63)		448527.00	85264.98	80.99	3600			3	38	227919.00	3840.54
15_3(63)		510036.00	130365.20	74.44	3600			1	26	318888.00	2295.45

5.2.3 Enhancement of maintenance technician's skill

The computational experiment in Sections 5.2.1 and 5.2.2 focus on the computational efficiency in dealing with the problem instance without considering the variation of manpower supply. In this section, an analysis is conducted to discover the impact of the manpower supply variation on the solution, which provides better understanding for the service provide to better exploit the maintenance capability and realize the profitable portfolio.

Two settings of manpower supply enhancement are prescribed for each instance in this section. The column "Manpower supply enhancement 1" refers to the setting of "minor" manpower supply enhancement, and the "Manpower supply enhancement 2" refers to the "major" manpower supply enhancement, respectively. The specification of maintenance demands, including the estimated arrival time, desirable deliver time, requirement of maintenance tasks (required skill and team size), remains unchanged through the manpower supply enhancement. Table 5 reports the computational results of enhancing the manpower supply for the instances. After enhancing the manpower supply in a progression pattern, it is found that the objective values of all instances maintain a decreasing trend along the increase of available manpower supply across all skill level. The iterations and generated additional constraints during the optimization progress also maintain a similar trend, which reflects that the difficulties in finding solution have reduced. The computational times for the larger size instances reduce significantly compared with the original manpower supply. After further investigation on the outcome of solution, it is found that the major elements contributing to the objective value transits from the tardiness in fulfilling maintenance demands/chance lose cost of rejecting maintenance demands to the manpower utilization cost. The above findings along the variation of manpower supply reveals that the manpower supply becomes a significant resource bottleneck over the maintenance hangar space in fulfilling the maintenance demands in peak hours, as the penalty costs result in the incapability of manpower contribute significantly to the overall cost. However, these findings of manpower supply do not necessarily imply that having more manpower available at all times benefit most to the maintenance service provider. The difference of objective values between Manpower supply enhancement 1 & 2 has narrowed down, comparing with the setting of original instance, In the mathematical model, the manpower cost is calculated based on an hourly rate as adopted in relevant literature, while the hiring cost is not incorporated. In this regard, a strategic planning of manpower shall be another important issue before carrying out the maintenance planning optimization mainly studied in this paper.

Table 5 Enhancement of maintenance manpower supply

Instance	Before Manpower Supply Enhancements				Manpower Supply Enhancement 1				Manpower Supply Enhancement 2			
	Iteration	Generated Constraints	Objective Value	CPU	Iteration	Generated Constraints	Objective Value	CPU	Iteration	Generated Constraints	Objective Value	CPU
5_1(14)	1	4	44223.00	1348.66	1	1	17520.00	58.12	1	1	15240.00	46.04
5_2(14)	1	10	59055.00	1884.95	1	4	37716.00	919.30	1	2	25476.00	96.38
5_3(14)	1	7	56442.00	185.29	1	7	53010.00	1842.99	1	2	35340.00	142.22
8_1(22)	1	1	21429.00	2268.05	0	0	13821.00	94.69	1	1	12285.00	53.31
8_2(22)	1	4	50733.00	3631.54	0	0	35901.00	540.01	0	0	34605.00	29.28
8_3(22)	2	9	117483.00	2100.73	2	3	53877.00	5519.37	0	0	52509.00	1832.27
10_1(32)	1	1	29025.75	192.59	1	1	13209.75	104.25	0	0	13137.75	38.24
10_2(32)	1	1	53313.75	2175.83	0	0	38193.75	302.58	0	0	36465.75	297.68
10_3(32)	2	11	127695.75	2090.25	1	1	59145.75	3684.18	0	0	58045.75	1843.52
15_1(63)	1	13	114303.00	4140.59	3	9	64605.00	9012.99	2	3	63717.00	5900.67
15_2(63)	2	36	225720.00	2367.86	1	6	12145.00	4183.36	2	7	100246.00	6855.17
15_3(63)	4	48	260271.00	3896.76	5	39	221961.00	4582.51	4	12	167325.00	9722.56

6. Conclusions

Efficient service implementation in fulfilling the increasing aircraft hangar maintenance demands has been emerging as a crucial factor from the perspective of MRO service company. An aircraft hangar maintenance planning problem is studied from the context of MRO outsourcing mode as a new problem of aircraft maintenance planning in literature. To fulfill the incoming maintenance demands from multiple airlines and other clients, the maintenance company has to determine the service period, multi-period hangar parking plan and the manpower assignment roster altogether. The research work presented in this paper makes a novel contribution closing the research gap in maintenance optimization problem in aviation industry. In particular, we propose a mixed integer linear programming (MILP) model to formulate the geometric constraints, manpower assignment constraints so as to integrate and characterize the interdependent relations of decision-making. For the maintenance company, developing the hangar parking plan and multi-skill maintenance technicians' roster are bottleneck in fulfilling maintenance demands and implementing maintenance tasks. The limited hangar capacity, flexible parking arrangement, movement blocking together with the multi-skill maintenance technician rostering make the problem intractable by the MILP model with default branching and bound method given the significant growth of model size. To tackle the medium- to large-size problem, a two-stage optimization approach is developed to decompose the original model into two subproblem linked by constraints, and its effectiveness is examined afterwards. The developed approach is tested on a large set of instances, based on the maintenance demands data from an aircraft hangar maintenance company located in Hong Kong. We assessed the effectiveness of the proposed approach in solving medium- to large-size instances in providing good quality solutions, then carry out an analysis to study the impacts of the variation of parameters in maintenance demands and manpower supply. Given the interrelations among three core elements of decision-making, the service capacity fluctuates with the incoming maintenance demands. The computational results on solving problem instances have manifested the challenges in coordinating the maintenance resources, and congestion of arrival maintenance requests may result in requirement of maintenance resource within short period of time, tardiness as well as high chance lose cost upon rejecting some aircraft maintenance request. From the perspective of management, the negative impact of incoordination of maintenance resources and should not be underestimated with the rising maintenance demands, which induces clients' dissatisfaction and adverse profit lost in the company. Moreover, the arrival pattern of maintenance demands shall be carefully studied well in advance to prepare the arrangement of resource supply before carrying out the specific hangar maintenance planning in real-world practice.

Further avenues of this research topic include: (1) the consideration of advanced joint decision making among independent service company and multiple airlines ahead of time to arrange the maintenance plan is suggested, which aims to avoid overwhelming maintenance requests arrive at similar time in a proactive manner and understand the arrival pattern. The joint maintenance planning is expected to enhance the service level of maintenance service provider in serving the increasing demands and fulfill the maintenance requirement of different airlines' fleets, allowing MRO service provider to have enough time to review and adjust their service capacity especially manpower supply. When congestions of maintenance demands, the joint decision making allow maintenance company and multiple airlines to negotiate and adjust the respective plans from multiple parties in a flexible manner; (2) stochastic modelling incorporating the uncertainties due to unscheduled maintenance and limited information regarding the maintenance demand arrival pattern can be considered, which makes the optimization approach close to the real operations; (3) the development of exact algorithms, heuristic algorithms and improvement of the decomposition approaches in solving the more challenging instances, given the effectiveness of the linkage effect in using aircraft staying time constraints in the two-stage optimization approach in this paper.

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