Robust Airline Crew Scheduling with Flight Flying Time Variability

Abstract: The crew pairing problem is one of the most important but challenging tasks for commercial airlines. However, the operation environment of the aviation industry is highly volatile with diverse uncertainties. Flight flying time variability is an important disruption that usually causes deviations of flight departure/arrival times from the schedule. Traditional crew pairing frameworks without considering flight flying time variability can generate pairings that are fragile to flight delays. However, the impact of flight flying time variability on crew pairings is under-explored. In this paper, we propose two robustness enhancement strategies based on the consideration of flight flying time variability (i.e., encouraging deviation-affected-free flights and discouraging deviation-affected flights). Besides, two robustness measurements are developed to construct two novel robust crew pairing models. One is time based while the other is number based. A customized column generation based solution algorithm is proposed. Computational experiments based on real flight schedules show that our new models can greatly enhance solution robustness (e.g., 49.1% more deviation-buffer time) at a price of an acceptable increase in operating costs (e.g., 9.7%) compared with the traditional model. Besides, extreme-delay flights can be completely avoided in the proposed models. Moreover, the solutions obtained from the time-based model show higher resistance against the disruption of flight flying time variability with a lower operating cost than the number-based model.

Keywords: Airline crew pairing; Robust scheduling; Flying time variability; Column generation

1. Introduction

The aviation industry is crucial for the global economy by facilitating passenger and cargo transportation all over the world (Button et al., 2019; Choi et al., 2019; Ng et al., 2017; Qin et al., 2019; Wen et al., 2019). According to International Air Transport Association (IATA), 4.3 billion passengers travelled through air in 2018, and the air passenger traffic is predicted to double in the next two decades[1]. Meanwhile, the aviation industry is characterized with intensive market competition and high operating costs. Therefore, improving the quality of operational decisions is of great importance for modern airlines to survive in the furiously competitive market (Bock et al., 2020; Sheng et al., 2019; Sun et al., 2020a; Wang & Wang, 2019). Due to the large problem scale and high problem complexity, the airline operational scheduling problem is usually divided into four sub-problems (Şafak et al., 2018). They are flight scheduling, fleet assignment, aircraft maintenance routing, and crew scheduling. Among them, crew scheduling has attracted increasing attention from both the industry and the academia as crew costs have become the second highest expenditure for

[1] https://www.iata.org/contentassets/c81222d96c9a4e0bb4ff6ced0126f0bb/iata-annual-review-2019.pdf. Retrieved on 10 Sept.

airlines, just after fuel costs. It is believed that even a small improvement in crew schedules can bring a remarkable cost saving (Chung et al., 2017; Ng et al., 2020; Wen et al., 2020). Crew scheduling is conducted by sequentially solving a crew pairing problem (CPP) and a crew assignment problem (CAP). The CPP is to construct sufficient anonymous pairings to cover all the scheduled flights with a minimum cost, while the CAP is to connect the pairings generated in the CPP to form monthly schedules to be assigned to specific crew members. Authorities, labor unions, and airlines usually impose diverse strict working rules and regulations on airline crew members, like the maximum time away from base and minimum flight connection times.

Although the air transportation industry is becoming increasingly essential for the world economy, the industry is challenged by various uncertainties in the operating environment, such as bad weather, airport congestion, mechanical failures, and crew absence. Accordingly, flights always deviate from the scheduled departure and arrival times. Flight delays and cancellations are commonly seen nowadays. According to Bureau of Transportation Statistics of United States Department of Transportation, the flight on-time rate was below 85% since the year of 2011, while the flight delay rate even grew to 24.53% in 2014[2]. It is reported that the cost related to disruptions for airlines reaches 25 to 35 billion USD each year. If we take the costs for other entities like passengers into account, this number goes up to 60 billion USD[3]. Regarding airline crew scheduling, as crew schedules are constructed based on pre-determined flight schedules with the aim of cost minimization, the constructed schedule plans are fragile and sensitive to disruptions. The gap between the planned schedules and the real operations for crew members can lead to huge negative impacts to airlines (e.g., catering services and lodging) and passengers (e.g., inconvenience) (Chung et al., 2015). Therefore, it is of great importance to build airline crew schedules with disruption considerations.

According to the implementation stage, the disruption management operations for airline scheduling are divided into two categories: recovery planning and robust planning (Clausen et al., 2010). Recovery planning refers to the re-scheduling of the original plan if a disruption occurs. The result of the recovery planning process can be the re-timing or cancellation of flights, reassigning standby crew members, or re-accommodating passengers. On the other hand, robust planning is to integrate the possible occurrence of disruptions into the decision framework in the planning phase, with the aim of producing schedules that are capable to remain feasible and less vulnerable in disrupted environments. Buffer time is a common and useful approach to help scheduled activities cover unexpected events. However, increasing buffer time will inevitably lead to a reduced utilization of airline resources (like aircraft and crew members). Accordingly, it is essential to balance the robustness and operating costs of airline schedules.

Among various uncertainties, variations in flight flying time (which is mainly caused by different

[2] https://www.transtats.bts.gov/HomeDrillChart.asp. Retrieved on 11 Sept 2020.
[3] https://digitaltravelapac.wbresearch.com/blog/4-travel-disruptions-affecting-otas-airlines. Retrieved on 12 Sept 2020.

cruise speeds) are commonly seen and have become increasingly important for the aviation industry. In the domain of Air Traffic Management, the adjustment of flight cruise speed serves several purposes such as conflicts reduction and noise abatement (Delgado & Prats, 2009). For example, data analytics on historical flight data demonstrates that an aircraft may speed up if it encounters a delayed departure, in order to alleviate the delay of the flight arrival (Sun et al., 2020b). Previous research also shows that speed fluctuation is a crucial consideration in aircraft fuel management (Khan et al., 2019a; Khan et al., 2019b; Wang et al., 2020). Moreover, various uncertainties related to flights, weather conditions (e.g., unexpectedly strong headwinds), traffic control and navigational error, airport congestions, etc., make flight flying time variation inevitable (Delgado & Prats, 2013). Accordingly, the actual flying time of scheduled flights keeps varying from time to time, which further leads to deviations in flight arrivals and flight departures.

In our study, we consider flight flying time as the source of disruption, and our objective is to alleviate the impact of flight flying time variability on the generated crew schedules. To be specific, we propose two novel robustness enhancement strategies based on the flying time variability and flight departure/arrival interdependency constructed by Sun et al. (2020b). The first strategy is to encourage flights that will not be affected by the expected arrival delay of the preceding flight, while the second strategy is to discourage flights that will be influenced by the expected arrival delay of the preceding flight. Regarding each robustness strategy, we also propose two different measures: number-based measure and time-based measure. A customized column generation based solution algorithm is proposed to solve the developed two novel robust crew pairing models (namely the *Robust Crew Pairing Model with Number-based Measure* (RCPN) and the *Robust Crew Pairing Model with Time-based Measure* (RCPT)). Computational experiments based on real flight schedules show that our new models can greatly enhance the robustness of pairing solutions at a price of an acceptable increase in basic operating costs compared with the traditional model without considering flight flying time variability in pairing generation. Moreover, both RCPN and RCPT can completely avoid extreme-delay cases, while the traditional model generates many extreme-delay flights. Regarding the comparison between RCPN and RCPT, it is found that RCPT can generate an average of 60.3% more deviation-buffer time, while consumes 3.8% less operating costs than RCPN. Besides, we reveal that although RCPT applies a time-based measure, it achieves satisfactory and more stable performances in terms of the number-based measure than RCPN. Therefore, the solutions obtained from RCPT show higher resistance against the disruption of flight flying time variability with a lower operating cost than RCPN.

Contributions

To the best of our knowledge, this research is the first study proposing crew pairing robustness enhancement strategies based on the consideration of flight flying time variability in the airline crew scheduling literature. The newly proposed time-based and number-based robustness measures are novel.

The constructed new robust crew pairing models are proved to achieve great robustness enhancement with an acceptable increase in basic operating costs through computational experiments based on real-world collected flight schedules. Our study thus demonstrates the importance of considering the flight flying time variability led by the fluctuating cruise speed, which has become a common disruption source in the air transportation industry, during the planning stage of airlines. In order to solve the proposed robust crew pairing models, a column generation based solution methodology which constructs customized sub-problems by the novel robustness-related costs is established.

The rest of the paper is structured as below. First, Section 2 reviews the existing literature. The problem definition is introduced in Section 3. Next, Section 4 demonstrates the proposed robust crew pairing model. The solution approach is illustrated in Section 5, while Section 6 carries out computational experiments. At last, Section 7 makes conclusion for this work.

2. Literature Review

In this section, we review the related literature from two aspects: Airline crew pairing problems and robust airline crew pairing problems.

2.1 Airline crew pairing problems

Due to the high operating costs, the airline crew pairing problem has been studied extensively in recent years (Parmentier & Meunier, 2019; Sun et al., 2020b). The CPP is generally formulated as a set covering problem or a set partitioning problem. Recently, a polynomial-sized nonlinear model is also applied to model the daily crew pairing problem which can be linearized by the reformulation-linearization technique (Haouari et al., 2019). Generally, there are millions or even billions of possible pairings for a weekly flight schedule with thousands of flights, which is common for commercial airlines nowadays. Therefore, column generation is widely applied in solving the large-scale airline crew pairing problem both in the academia and in the industry since 1980s (Desaulniers et al., 1997; Lavoie et al., 1988).

Recently, Quesnel et al. (2019) integrate the language considerations into the CPP to better satisfy the requirements of the CAP, and construct a new CPP variant named as the CPP with language constraints (CPPLC). In Quesnel et al. (2019), an efficient partial pricing technique is proposed to deal with the large number of sub-problems. Computational experiments based on real flight schedules show that the pairings generated from the proposed CPPLC can reduce the language requirement violation in the CAP by 61%-96% compared with those derived from the traditional CPP (Quesnel et al., 2019). Similarly, Wen et al. (2020) consider the manpower availability restriction for each class of cabin crews in the stage of CPP to alleviate the shortcoming of the rigid separated airline crew scheduling approach (i.e., the sequential CPP and CAP). In Saddoune et al. (2012), the airline pilot scheduling problem is solved in a single step.

Saddoune et al. (2012) show that the single step crew scheduling can result in a 3.37% reduction in operating costs compared with the sequential approach. However, the problem scale and complexity increase intensively.

On the other hand, some research tries to enhance the optimization efficiency for airlines by integrating various stages of airline scheduling into a single decision framework (Papadakos, 2009; Ruther et al., 2017). For example, Cacchiani and Salazar-González (2017) solve a fleet assignment, aircraft routing, and crew pairing integrated problem for a regional airline, with the objective of minimizing the weighted sum of the number of crew pairings, the waiting times of crew members between consecutive flights, and the number of aircraft routes. In a similar study, Shao et al. (2017) construct a Benders decomposition approach together with several acceleration techniques to handle an integrated airline scheduling problem, to derive fleet assignments, aircraft routing plans, and crew pairings at the same time. Besides, Özener et al. (2017) study an integrated scheduling problem for an European Airline by simultaneously considering the fleet assignment and crew pairing problem with the aim of maximizing the total profit of the airline.

2.2 Robust airline crew pairing problems

Although optimization models aim to improve airline profitability by generating more efficient and cost-effective schedules, these schedules may be susceptible to unforeseen disruptions or irregular operations, such as extreme weather, airport congestion or propagated delays. To recover from these unexpected situations, recovery actions may be adopted, such as flight delay or cancellation, crew swapping, and crew deadheads. However, these activities can be expensive and time-consuming, and even hurt the brand image of the airline. Therefore, it has become increasingly popular to establish robust schedules with the aim of reducing the sensitivity of the planned schedules to minor or major disruptions. Generally, the robust airline crew pairing problem is categorized into non-integrated robust planning and integrated robust planning problems (Chung et al., 2015).

For the first category, researchers focus on improving the robustness of crew pairings. For example, Sun et al. (2020b) propose a data-driven bicriteria robust crew pairing model, in which the flying time variability and flight departure/arrival interdependency are formulated. With this interdependency, the difference between the scheduled departure/arrival times and the expected departure/arrival times can be obtained. Accordingly, the robustness of the crew pairing solutions can be enhanced by minimizing the obtained differences. Computational experiments based on real flight schedules show that the data-driven bicriteria robust crew pairing model can greatly improve the reliability of solutions without sacrificing much basic operating costs (Sun et al., 2020b). Tekiner et al. (2009) consider a special disruption which comes from the fact that airlines sometimes add flights during operations. To deal with this uncommon disruption, Tekiner et al. (2009) propose a model that can accommodate additional flights without affecting others, while controlling the increased crew cost at a user-specified level. A similar study can be found in Muter et al. (2013). Most recently, Antunes et al. (2019) present a robust crew pairing model without the requirement of knowing the details of the underlying delay disruptions. The linearity of the constraints and objective function are retained so that the robust crew pairing model can be solved by commercial solvers.

Regarding the second category, the aircraft routing problem is usually considered with the crew pairing problem simultaneously to enhance the solution robustness. Dunbar et al. (2012) point out that the sequential scheduling approach fails to account for the dependencies between different scheduling stages (e.g., aircraft routing, crew scheduling). Therefore, Dunbar et al. (2012) propose a new robust aircraft and crew integrated scheduling model which can accurately calculate and minimize the overall propagated delay. Later, Dunbar et al. (2014) extend the robust model of Dunbar et al. (2012) by proposing two new delay propagation reduction algorithms via the incorporation of stochastic delay information. Similarly, Weide et al. (2010) aim to improve the airline schedule robustness against typical stochastic variability in airline operations by considering the aircraft routing problem with two types of airline crew pairing problems (including the cabin crew pairing problem and the cockpit crew pairing problem).

2.3 Research gaps

From the review presented above, it is seen that the impact of flight flying time variability is studied rather less in the crew pairing literature. However, Sun et al. (2020b) have shown that flight flying time is a random variable which is directly affected by the departure time of the flight, while the flying time variation further affects flight arrival and the departure of the next flight, causing propagated delay effects. Therefore, it is valuable and important to integrate these considerations (i.e., flight flying time variability and the related flight departure/arrival interdependency) into the crew pairing framework to enhance the solution robustness. Our study is mostly related to Sun et al. (2020b). However, in Sun et al. (2020b), the crew pairing robustness is represented by the minimization of the expected differences between the actual flight schedule and the scheduled plan for air crews. Although the total expected arrival/departure time deviation can be minimized, the number of departure-delayed flights or the level of departure delay is not considered in Sun et al. (2020b). Through our discussion with the managers from a major Hong Kong airline, it is known that airlines are eager to minimize the number of departure-delayed flights or the level of departure delays of affected flights (i.e., delay time length). For example, one flight with a two-hour delay is definitely different from four flights with a half-hour delay each. Therefore, in the current study, we propose two novel robustness strategies, with one encouraging the occurrence of flights that will not be affected by the expected arrival delay of the preceding flight, while the other discouraging the occurrence of flights that will be affected. More importantly, the specific levels of the delays and non-delays (i.e., number of flights or the time length) are considered to provide more insightful guidelines for airlines, which is new to the literature.

3. Problem Description

This section defines the problem to be studied, constructs the duty-based flight network used for pairing generation, and introduces flying time variability and flight departure-arrival interdependency.

3.1 Definitions and regulations

The robust airline crew pairing problem aims to determine sufficient legal pairings to cover all flights' requirements that are robust to disruptions with the minimum cost, while satisfying all the regulations imposed by authorities, labor unions, and airlines. A *duty* consists of a sequence of flights connected by sits (also named as transits or connections), while the *duty period* refers to the elapsed time of a duty. A feasible *pairing* is a sequence of duties (separated by rests) operated by the same crew member, starting from and ending at the home base, and satisfies diverse rules and regulations. The elapsed time of a pairing is the *time away from base* (TAFB).

Authorities, labor unions, and airlines have imposed various rigorous rules and regulations on air crews, in order to ensure the safety level of air transportation. In the pairing construction of this study, we consider the following rules according to the practical operations of a major Hong Kong airline and CAD 371[4]. First of all, two flights can be connected (i.e., one flight is the immediate following flight for another flight) only if there is a legal sit time between the two flights, while the arrival airport of the former flight is the departure airport of the latter flight. The maximum sit time (*Maxsit*) is 240 minutes, while the minimum (*Minsit*) is 30 minutes. Second, a briefing before a duty and a debriefing after a duty are compulsory. Third, the legal duration of a duty varies based on the local starting time of the first flight and the number of flights in the duty, as summarized in **Table 1**. Maximum three flights are allowed for a duty. Fourth, a rest lasting from 720 minutes to 2160 minutes is required between two duties if the period of the former duty is no longer than 12 hours. Otherwise, an 840-minute rest is the minimum requirement. Last, maximum five duties, twelve flights, and 7200 minutes of TAFB are allowed for a pairing.

Table 1. Maximum duty periods (in hours).

Local starting time of the first flight of the duty	Number of flight legs in the duty		
	1	2	3
07:00-07:59	13	12.25	11.5
08:00-12:59	14	13.25	12.5
13:00-17:59	13	12.25	11.5
18:00-21:59	12	11.25	10.5
22:00-06:59	11	10.25	9.5

[4] CAD 371: The Avoidance of Fatigue in Aircrews published by the Civil Aviation Department of the Government of the HKSAR.

3.2 Duty-based flight network

We use F to represent the set of flights to be scheduled which is indexed by j, and P to stand for the set of potential pairings in the duty-based flight network which is indexed by p. In this work, we apply the duty-based flight network for pairing generation to enhance optimization efficiency as some working regulations have already been satisfied in the network construction process (Vance et al., 1997). An example of the duty-based flight network is shown in **Figure 1**. The acyclic duty-based network is denoted as $G = (N, A)$. N represents the set of nodes (including a source node, a sink node, and duty nodes), while A is the set of arcs in the network. A source node (s) and a sink node (k) are used to stand for the home base. An example duty is illustrated in the upper right corner of **Figure 1**. As we can see, a duty (d) is composed of a sequence of flights connected by sit arcs. Accordingly, we denote the set of sit arcs in duty d by SA_d, while the set of flights contained in duty d by F_d. All possible duties are constructed according to the rules and regulations. Duties are connected by rests in the network. All duties starting from the home base link with the source node by a starting arc, while all duties ending at the home base connect with the sink node by an ending arc. The sets of starting arcs and ending arcs are denoted by A_s and A_e, respectively. For the duties that do not start from the home base, a deadhead starting arc is used to link the home base (s) with the duty. Similarly, for the duties that do not end at the home base, a deadhead ending arc is used to link the duty with the home base (k). We use A_{ds} and A_{de} to denote the sets of deadhead starting arcs and deadhead ending arcs, respectively. A legal pairing refers to a resource-feasible s-t path in the constructed duty-based network. The set of all arcs contained in Pairing p is represented by A_p.

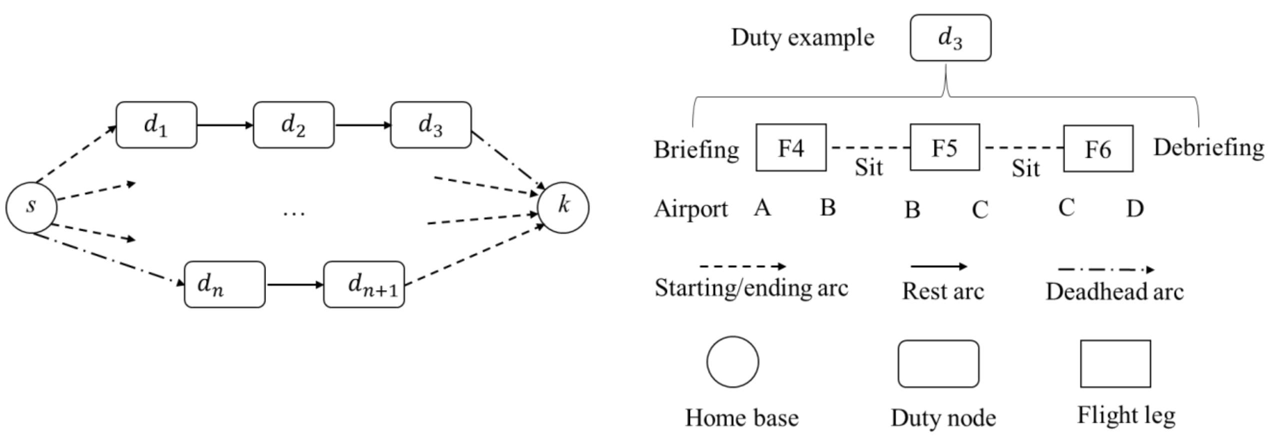

Figure 1. An example of the duty-based flight network.

3.3 Flying time variability & flight departure-arrival interdependency

Although each flight in the flight schedule is assigned a departure time, an arrival time, together with the flying time, the real flight flying times vary significantly. As discussed, airlines and pilots may control the flight cruise speed to adjust the flying time. For example, if a flight departs late from an airport at a peak time, the aircraft can fly at a higher speed to reduce the impact of the late departure on its arrival. If a flight arrives at the destination airport in peak times, the total flying time will increase greatly. There are many other important factors that affect the real flying time, like weather conditions, traffic control, and navigational error. For example, if a flight encounters a strong headwind in the first half of the journey, the pilot may speed up in the second half of the trip. Considering the random nature of flight schedules, academia has been devoted to discovering the relationship among flight departure times, flying times, and arrival times, with the aim of improving the decision making for the important but volatile air transportation industry. A recent study by Chung et al. (2017) has revealed that flight arrival times are significantly affected by flight departure times. A related work by Sun et al. (2020b) further uncovers that flight flying times are actually dependent on the real departure time of the flight through analytics on 2-year real historical flight data provided by a major Hong Kong airline. A heteroscedastic regression model successfully predicts the structure of flight flying times by characterizing the influence of flight departure time on the mean and variance of the actual flying time. Based on the identified flight flying time structure, the expected arrival and departure times of consecutive flights can be modelled mathematically in Sun et al. (2020b). In our work, we build a novel robust crew pairing model which is resistant against potential flight delays based on the flight departure-arrival interdependency as well as the departure-dependent flight flying time variability developed by Sun et al. (2020b) (see Section 4). Note that to make the problem tractable, regarding the flight (departure-dependent) flying time variability, we only consider its impact on the expected arrival time of the immediate subsequent flight, without the propagated impact on the other latter flights in the pairing. Next, the flight expected arrival time based on the departure-dependent flying time is introduced.

We use $arr(j)$ and $dep(j)$ to represent the actual arrival and departure times of Flight j, respectively, while $arr^s(j)$ and $dep^s(j)$ to denote the scheduled arrival and departure times of Flight j, respectively. $FT_j(dep(j))$ stands for the real flying time of Flight j, which is dependent on the actual departure time of $dep(j)$. $E[(\cdot)]$ denotes the expect value. $Minsit/Maxsit$ is the minimum/maximum legal sit time between two consecutive flights. According to Sun et al. (2020b), most of the flight regular uncertainties can be approximated by Normal distribution. Therefore, the flight flying time is modelled as a Normal distribution with the mean of $\mu_j\big(dep(j)\big)$ and standard deviation of $\sigma_j\big(dep(j)\big)$ which depends on the actual departure time of the flight, $dep(j)$. Therefore, $arr(j)$ and $E[arr(j)]$ can be formulated as in Eq. (1) and Eq. (2) in the following.

$$arr(j) = dep(j) + FT_j(dep(j)) \tag{1}$$

$$E[arr(j)] = E[dep(j)] + \mu_j(E[dep(j)]) \tag{2}$$

Using Eq. (2), we can obtain the expected arrival time of Flight j by considering flight flying time variability. The actual departure time of the next flight (i.e., Flight $j+1$) is dependent on the arrival of the immediate preceding flight (i.e., Flight j)[5], as shown in Eq. (3). The first part in the max {} of Eq. (3) represents the scheduled departure time of Flight $j+1$, while the second part represents the earliest legal departure time for Flight $j+1$ based on the real arrival of its previous flight (i.e., Flight j).

$$dep(j+1) = Max\{dep^s(j+1), arr(j) + Minsit\} \quad (3)$$

Therefore, with $E[arr(j)]$, we are able to judge whether the following flight (i.e., Flight $j+1$) is expected to depart on time or not. That is, the status of a flight is affected by its previous flight(s).

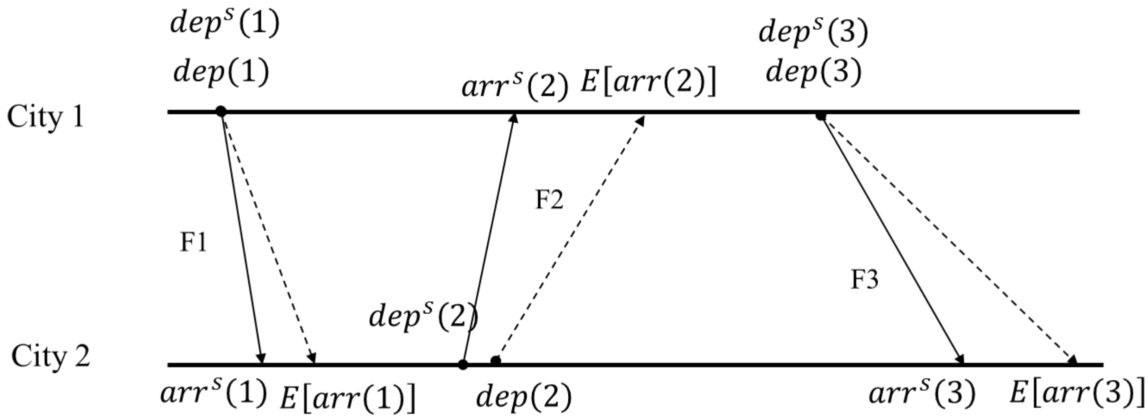

Figure 2. An example of the impact of flight flying time variability on crew pairing.

Following Sun et al. (2020b), we assume that the aircraft is always available for each flight to exclude the impact of aircraft disruptions. Besides, every first flight leg of a duty (Flight j_s^d) departs on time. That is, $dep(j_s^d) = dep^s(j_s^d)$. Besides, the arrivals before the scheduled arrival time point are regarded as arrival on time. **Figure 2** shows an example of the impact of flight flying time variability on crew pairing. The duty in **Figure 2** involves three consecutive flights. They are F1, F2, and F3. The actual departure time for F1 is its scheduled departure time (i.e., $dep(1) = dep^s(1)$). Due to the variation in flight flying time, the expected arrival time of F1 is later than the scheduled time (i.e., $E[arr(1)] = dep^s(1) + \mu_1(dep^s(1)) > arr^s(1)$). Base on $E[arr(1)]$, according to Eq. (3), it is identified that the departure of F2 is delayed (i.e., $E[arr(1)] + Minsit > dep^s(2)$, $dep(2) > dep^s(2)$). Then, applying Eq. (2), it is found that the expected arrival of F2 is also later than scheduled. For F3, although its immediate preceding flight (F2) encounters both departure and arrival delays, it can depart on time as suggested by Eq. (3) (i.e., $E[arr(2)] + Minsit \leq dep^s(3)$, $dep(3) = dep^s(3)$). However, due to flight flying time variability, F3 is also expected to arrive late at its destination. As we can see from **Figure 2**, if we only apply the scheduled departure/arrival times

[5] Note that Flight j^- denotes the immediate preceding flight of Flight j.

for the flights on the schedule without considering flight flying time variability in the pairing generation process, crews can easily encounter flight delays.

4. A Robust Crew Pairing Model

This section proposes two novel robust crew pairing models with the consideration of flight flying time variability and flight departure-arrival interdependency as discussed in Section 3.3. As departure delays cause stress, anxiety and anger among passengers and increase costs for airlines due to extra food and lodging[6], in this study, our focus is to enhance the robustness of flights in the constructed pairings against departure delays. Accordingly, two novel robustness enhancement strategies are developed. In Section 4.1, we introduce the robustness concept proposed in this work, based on which we will then construct novel robust crew pairing models in Section 4.2.

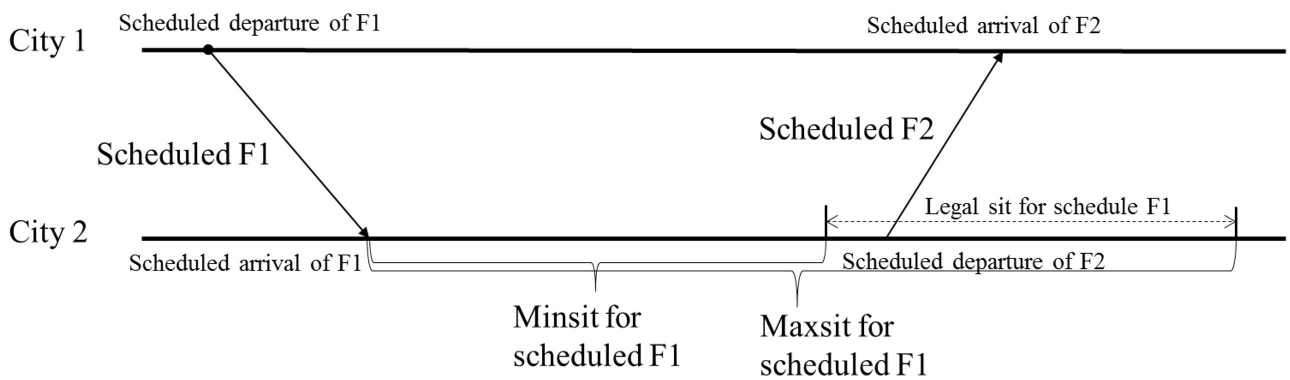

Figure 3. A typical duty.

4.1 Crew pairing robustness

First of all, we illustrate our strategies of robustness enhancement. **Figure 3** shows a typical duty which consists of two flights between City 1 and City 2. The scheduled departure time of F2 falls in the legal sit time range regarding its immediate preceding flight (F1) (i.e., [*Minsit*, *Maxsit*]). According to the schedule, a crew member will operate F2 after the work on F1. If no disruption occurs, the two flights will be operated sequentially as scheduled.

However, as discussed, the real flying time of a flight varies according to the actual departure time together with many other factors. The actual flight arrival thus may deviate from the scheduled time point, which will further affect the departure of the next flight in the duty. Let's take a look at **Figure 4**. F1 arrives late at City 2. The scheduled departure time of the next flight (i.e., F2) is not in the legal sit time range for

[6] https://www.trefis.com/stock/dal/articles/375013/what-is-the-impact-of-flight-delays/2016-08-31 retrieved on 02 Sept 2020.

the actual F1. Accordingly, F2 will encounter a departure delay, and the earliest departure time of F2 is at the beginning of the legal sit time range for the actual F1. As illustrated by **Figure 4**, if the flight flying time variability is not considered, the constructed crew pairing is fragile where the deviations of previous flights can easily lead to departure delays for later flights.

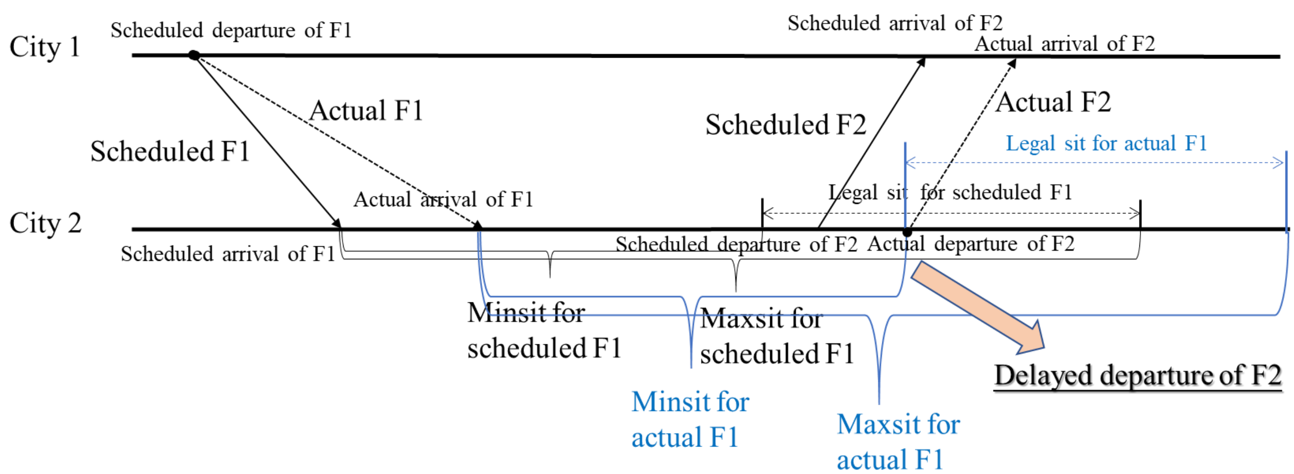

Figure 4. Delayed F1 and disrupted F2.

Due to the diverse uncertainties and volatilities in the aviation industry, it is difficult to predict the real departure time, flying time, and arrival time for each individual flight in the future. However, through data analytics, the expected arrival time of a flight can be formulated mathematically by considering flight flying time variability (see Section 3.3), based on which we can enhance the robustness of crew pairings to some extent. The main strategies are to (i) encourage flights that will not be affected by the expected arrival delay of the preceding flight, and (ii) discourage flights that will be influenced by the expected arrival delay of the preceding flight.

A simple example for strategy (i) is demonstrated in **Figure 5**. In this example, F1 is expected to encounter an arrival delay. After F1, the next flight that crews will operate is F3, whose scheduled departure time is exactly located in the overlap of the legal sit time range for the scheduled F1 and the expected arrival-late F1. Therefore, F3 can depart on time as scheduled even when its preceding flight (i.e., F1) is expected to arrive late. By constructing more flights like the example F3 as shown in **Figure 5**, the constructed crew pairing plan can be more robust against disruptions. In the following, flights like F3 in this example are called as *deviation-affected-free flights*. The length of the legal sit time range overlap for the scheduled and expected arrival of Flight j (i.e., $Overlap(j)$) is calculated by Eq. (4).

$$Overlap(j) = arr^s(j) + Maxsit - E[arr(j)] - Minsit \tag{4}$$

Strategy (ii) is explained by using **Figure 6**. In this example, F1 is expected to arrive late at its destination which is followed by F4. The scheduled departure time of F4 is not in the legal sit time range for the expected arrival-late F1. Therefore, F4 is not allowed to depart until the minimum legal sit time for the expected arrival-late F1 is reached. That is, F4 encounters a departure delay due to the arrival delay of its preceding flight. In the following, flights like F4 are called *deviation-affected flights*.

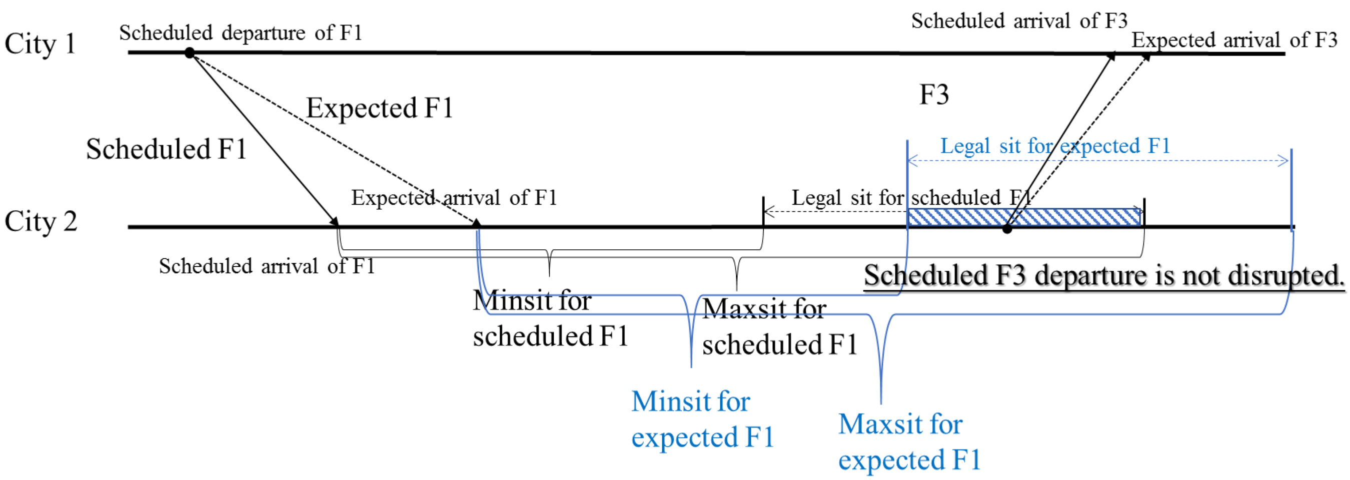

Figure 5. Strategy (i) for robustness.

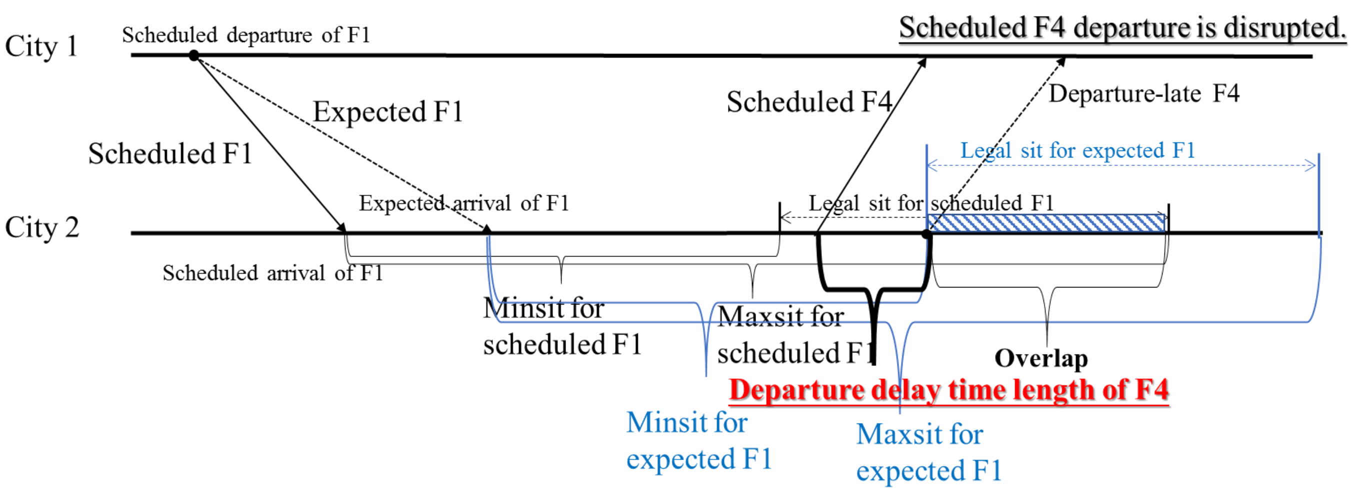

Figure 6. Strategy (ii) for robustness.

For the remaining flights in the duty which are not shown in **Figure 5** and **Figure 6**, the same rule will be applied to judge whether the flight is a deviation-affected-free or deviation-affected flight. We utilize **Figure 7** to show a full duty with the consideration of flight flying time variability that involves two categories of flights (i.e., deviation-affected-free flights and deviation-affected flights).

Note that if the legal sit time range for the expected arrival of a flight and that for the scheduled arrival of this flight do not have an overlap. That is, the flight is expected to arrive at its destination with a delay more than 210 minutes (as *Maxsit* - *Minsit*=210), we consider this flight is in an extreme-delay case. **Figure**

8 shows an example. In this example, the minimum legal sit time for the expected arrival-late F1 is beyond the legal sit time range for the scheduled F1. Therefore, the expected earliest departure time for F5 is at the beginning of the legal sit time range for the expected arrival-late F1. If this extreme case happens, a very big penalty cost (ϑ) will be generated. Actually, this extreme case will occur with a very small probability. This is because a delay longer than three hours is regarded as a severe disruption which cannot be handled by proactive planning (Ionescu et al., 2016).

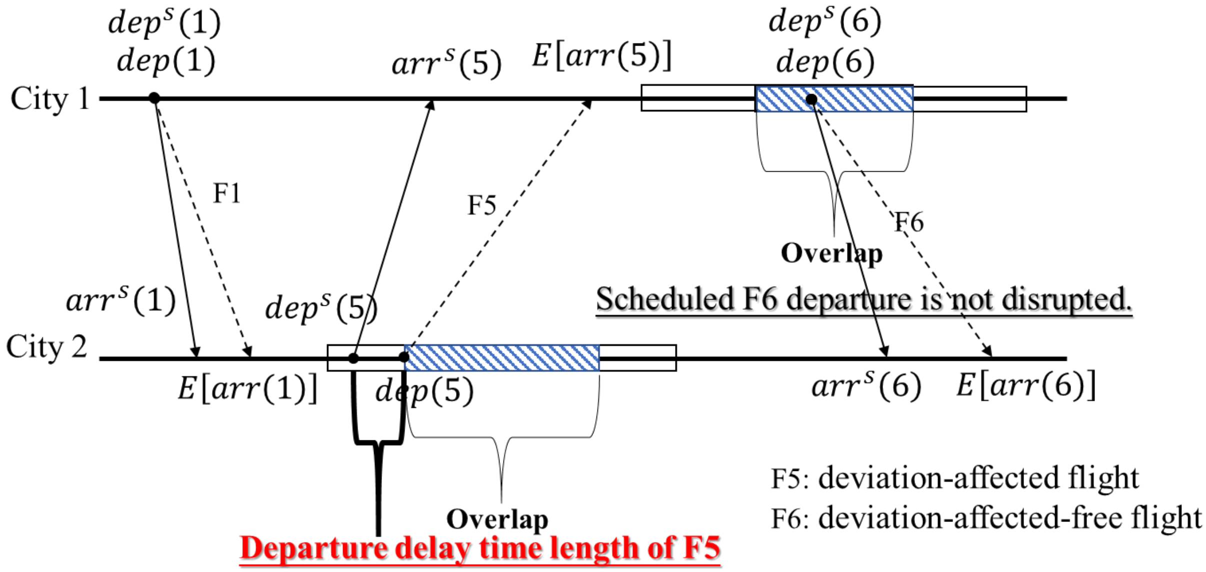

Figure 7. An example of a full duty with flying time variability.

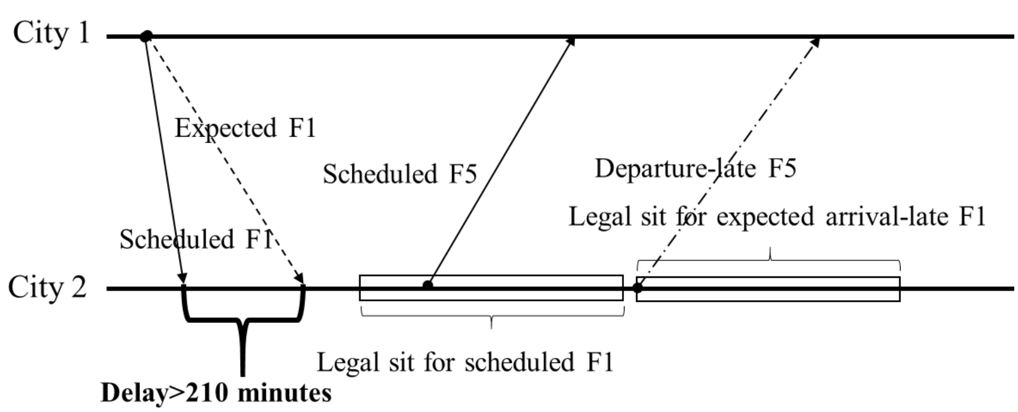

Figure 8. An example with an extreme-delay case.

As the status of a flight depends on its preceding flight (i.e., the departure time, the departure-dependent flight flying time), the expected arrival time of the flight may vary in different potential pairings, which shall be calculated along with the pairing construction process. One flight may be a deviation-affected-free flight in one pairing, while be a deviation-affected flight if it is covered by another pairing. To enhance the robustness of crew pairing solutions, it is important to discourage the occurrence of deviation-affected flights, while construct more deviation-affected-free flights. The objective of this study is to construct a set of pairings with the minimum basic operating cost and the maximum robustness.

4.2 Mathematical model

Based on the robustness enhancement strategies proposed in Section 4.1, we then construct the mathematical model for the robust crew pairing problem. The crew pairing problem is generally modelled as a set-covering or set-partitioning problem. The objective is to determine a set of legal pairings ($p \in P$) with the minimum total costs, while the constraint is to ensure that each flight is covered by (at least) one pairing, as formulated in Eq. (5) to Eq. (7).

Crew pairing problem (CPP)

$$min \sum_{p \in P} c_p x_p, \tag{5}$$

$$s.t. \quad \sum_{p \in P} a_{pj} \cdot x_p \geq 1, \forall j \in F, \tag{6}$$

$$x_p \in \{0,1\}, \forall p \in P. \tag{7}$$

c_p is the cost of Pairing p. a_{pj} represents whether Pairing p contains Flight j. When $a_{pj} = 1$, Flight j is covered by Pairing p; Otherwise, $a_{pj} = 0$. The decision variable, x_p, is equal to 1 if Pairing p is selected in the final solution, and 0 otherwise. In the following Section 4.2.1, we will first introduce the traditional basic operating cost for crew pairings (c_p^o), followed by our proposed robustness-related costs (c_p^{rb}). Then, the robust model is shown in Section 4.2.2.

4.2.1 Costs

We consider a bi-criteria model which integrates the minimization of basic operating costs (c_p^o) and the minimization of robustness-related costs (c_p^{rb}).

4.2.1.1 Basic operating costs

Traditionally, only the basic operating costs for crews are considered in the crew pairing framework. For a pairing, its basic operating cost consists of a fixed component (K) and arc-related costs[7]. Arcs in a pairing can be further divided into duty arcs (d^-, d), (deadhead) starting arcs (s, d), and (deadhead) ending arcs (d, k). For duty arcs, the arc-related cost ($t_{d^-,d}$, Eq. (8)) is composed of waiting cost during flight connections ($c_{d^-,d}^w$, Eq. (11)), rest cost ($c_{d^-,d}^r$, Eq. (13)), and pairing minimum duty guaranteed cost (PMDG) ($c_{d^-,d}^D$, Eq. (14)). Specifically, the waiting cost during flight connections is a function ($g_{j^-}(\cdot)$) dependent on the flight connection time ($\delta_{j^-,j}$). Similarly, the rest cost is a function ($l_{d^-}(\cdot)$) dependent on the rest

[7] In this work, we model the basic cost for pairings by arcs.

period between two duties ($\delta_{d^-,d}$). For the detailed functions for $g_{j^-}(\cdot)$ and $l_{d^-}(\cdot)$, readers are referred to Saddoune et al. (2013) and Chung et al. (2017). The PMDG cost is applied to force the algorithm to form longer duties. That is, if the duty credit flying time (v_d) is shorter than a threshold (V_{min}), a unit PMDG cost (v) will be incurred. For (deadhead) starting arcs (s, d), the arc-related cost ($t_{s,d}$, Eq. (9)) consists of waiting cost during flight connections ($c^w_{s,d}$, Eq. (12)), pairing minimum duty guaranteed cost (PMDG) ($c^D_{s,d}$, Eq. (15)), and deadhead cost ($c^h_{s,d}$, Eq. (16) if this arc is deadhead and Eq. (17) otherwise). For (deadhead) ending arcs (d, k), the arc-related cost ($t_{d,k}$, Eq. (10)) only involves the deadhead cost ($c^h_{d,k}$, Eq. (18) if this arc is deadhead and Eq. (19) otherwise). Note that the deadhead cost is fixed as β for each deadhead (starting or ending) arc.

Arc-related basic operating cost	$t_{d^-,d} = c^w_{d^-,d} + c^r_{d^-,d} + c^D_{d^-,d}, \forall (d^-, d) \in \{A - A_e - A_{de} - A_s - A_{ds}\},$	(8)
	$t_{s,d} = c^w_{s,d} + c^D_{s,d} + c^h_{s,d}, \forall (s, d) \in \{A_s + A_{ds}\},$	(9)
	$t_{d,k} = c^h_{d,k}, \forall (d, k) \in \{A_e + A_{de}\},$	(10)
Waiting cost	$c^w_{d^-,d} = \sum_{(j^-,j) \in SA_d} g_{j^-}(\delta_{j^-,j}), \ \forall (d^-, d) \in \{A - A_e - A_{de} - A_s - A_{ds}\},$	(11)
	$c^w_{s,d} = \sum_{(j^-,j) \in SA_d} g_{j^-}(\delta_{j^-,j}), \ \forall (s, d) \in \{A_s + A_{ds}\},$	(12)
Rest cost	$c^r_{d^-,d} = l_{d^-}(\delta_{d^-,d}), \forall (d^-, d) \in \{A - A_e - A_{de} - A_s - A_{ds}\},$	(13)
PMDG cost	$c^D_{d^-,d} = v * \max\{0, (V_{min} - v_d)\}, \forall (d^-, d) \in \{A - A_e - A_{de} - A_s - A_{ds}\},$	(14)
	$c^D_{s,d} = v * \max\{0, (V_{min} - v_d)\}, \forall (s, d) \in \{A_s + A_{ds}\},$	(15)
Deadhead cost	$c^h_{s,d} = \beta, \forall (s, d) \in A_{ds},$	(16)
	$c^h_{s,d} = 0, \forall (s, d) \in A_s,$	(17)
	$c^h_{d,k} = \beta, \forall (d, k) \in A_{de},$	(18)
	$c^h_{d,k} = 0, \forall (d, k) \in A_e.$	(19)

In summary, if we denote the arcs in Pairing p by $(n_p^-, n_p) \in A_p$ which includes a (deadhead) starting arc, duty arcs, and a (deadhead) ending arc, the total basic operating cost for this pairing c^o_p can be formulated as in Eq. (20).

$$c^o_p = K + \sum_{(n_p^-, n_p) \in A_p} t_{n_p^-, n_p} \tag{20}$$

4.2.1.2 Robustness-related costs

As we discussed, if flight flying time variability and flight departure/arrival interdependency are not considered, the crew pairings generated according to the scheduled departure/arrival times may encounter propagated disruptions along the itinerary. Therefore, in this section, we propose novel robustness objectives according to the two strategies proposed in Section 4.1. Specifically, along pairing construction, the expected flight arrival times are calculated based on flying time variability, and the departure time of the next flight is then obtained (as presented in Section 3.3). Accordingly, the undelay-bonus (bn_p^{rb}) and delay-cost (dc_p^{rb}) can be formulated, and the robustness-related cost (c_p^{rb}) is equal to delay-cost (dc_p^{rb}) minus undelay-bonus (bn_p^{rb}). The cost formulation for strategy (i) and strategy (ii) are introduced sequentially in the following.

Strategy (i): The purpose here is to encourage the occurrence of deviation-affected-free flights in the pairings generated. Therefore, a undelay-bonus (bn_p^{rb}) is given to Pairing p if it contains deviation-affected-free flights. Let F^p represent the set of flights contained in Pairing p, and F_{daf}^p denote the set of deviation-affected-free flights in Pairing p. Flight j ($j \in F^p$) belong to F_{daf}^p if $E[arr(j^-)] + Minsit \leq dep^s(j) \leq arr^s(j^-) + Maxsit$. Here, we have two ways to enhance the occurrence of deviation-affected-free flights. They are: (a) Maximize the number of deviation-affected-free flights (Number-based measure); (b) Maximize the total *deviation-buffer time* length of deviation-affected-free flights (Time-based measure). The total number of deviation-affected-free flights in Pairing p equals to $|F_{daf}^p|$. Note that the symbol "| |" here stands for the size of a set. The deviation-buffer time ($Buffer(j)$) refers to the time difference between the scheduled departure time of a flight with the sum of the expected arrival time of its preceding flight with *Minsit*, which is equivalent to Eq. (21). Accordingly, the total deviation-buffer time for Pairing p is $\sum_{j \in F_{daf}^p} Buffer(j)$.

$$Buffer(j) = dep^s(j) - E[arr(j^-)] - Minsit \tag{21}$$

In a word, bn_p^{rb} is in the form of Eq. (22) if the Number-based measure is used, and Eq. (23) if the Time-based measure is applied.

$$bn_p^{rb} = |F_{daf}^p| \tag{22}$$

$$bn_p^{rb} = \sum_{j \in F_{daf}^p} Buffer(j) \tag{23}$$

Strategy (ii): The aim here is to discourage the occurrence of deviation-affected flights in the pairings generated. Therefore, a delay-cost (bn_p^{rb}) is given to Pairing p if it contains deviation-affected flights. Let F_{da}^p denote the set of deviation-affected flights in Pairing p. Flight j ($j \in F^p$) belongs to F_{da}^p if $arr^s(j^-) + Minsit \leq dep^s(j) < E[arr(j^-)] + Minsit$. Here, similar to strategy (i), we also have two methods to discourage the occurrence of deviation-affected flights. They are: (a) Minimize the number of deviation-affected flights (Number-based measure); (b) Minimize the total *deviation-delay time* length of deviation-affected flights (Time-based measure). The total number of deviation-affected flights in Pairing p equals to $|F_{da}^{p}|$. For deviation-affected Flight j, the deviation-delay time ($Devdelay(j)$) refers to the time difference between the sum of the expected arrival time of its preceding flight with *Minsit* minus the scheduled departure time of Flight j, which is equivalent to Eq. (24). Accordingly, the total deviation-delay time for Pairing p is $\sum_{j\in F_{da}^{p}} Devdelay(j)$.

$$Devdelay(j) = E[arr(j^{-})] + Minsit - dep^{s}(j) \tag{24}$$

In a word, dc_p^{rb} is in the form of Eq. (25) if the Number-based measure is used, and Eq. (26) if the Time-based measure is applied.

$$dc_p^{rb} = |F_{da}^{p}| \tag{25}$$

$$dc_p^{rb} = \sum_{j\in F_{da}^{p}} Devdelay(j) \tag{26}$$

Consequently, the total robustness-related costs (c_p^{rb}) for Pairing p is then equal to delay-cost (dc_p^{rb}) minus undelay-bonus (bn_p^{rb}), as modelled in Eq. (27).

$$c_p^{rb} = dc_p^{rb} - bn_p^{rb} \tag{27}$$

4.2.2 Model

With the constructed cost functions, we can derive the mathematical model for the robust crew pairing problem. Note that the units for the basic operating cost and robustness-related cost are not the same. Therefore, it is necessary to normalize the two costs. We use the symbol "$\widehat{\ }$" to represent the normalized value. The normalization function is $\hat{z} = \frac{z-\underline{z}}{\overline{z}-\underline{z}}$, where $\underline{z}$ is the minimum value of z, while $\overline{z}$ is the maximum value of z. Note that for any constructed Pairing p, if $E[arr(j)] + Minsit > arr^{s}(j) + Maxsit$ ($\forall j \in F^{p}$), an extreme-delay case happens. A very big penalty cost (ϑ) will be assigned to this pairing. Let $\aleph_p = 1$ represents that Pairing p involves extreme-delay cases, and $\aleph_p = 0$ otherwise. Therefore, the extreme-delay penalty cost is $\aleph_p \cdot \vartheta$. The total cost for Pairing p (c_p) is then equal to the normalized basic operating cost ($\widehat{c_p^{o}}$) plus the normalized robustness-related cost ($\widehat{c_p^{rb}}$) plus the extreme-delay penalty cost ($\aleph_p \cdot \vartheta$), as shown in Eq. (28). We use w_1 and w_2 ($w_1 + w_2 = 1$) to represent the weights for the basic operating cost and robustness-related cost, respectively.

$$c_p = w_1\widehat{c_p^{o}} + w_2\widehat{c_p^{rb}} + \aleph_p \cdot \vartheta \tag{28}$$

Therefore, if the Number-based robustness measure is applied, the Robust Crew Pairing Model with Number-based Measure (named as RCPN) can be formulated in Eq. (29) to Eq. (31).

RCPN

$$min\ \sum_{p\in P}\{w_1(K+\widehat{\sum_{(n_p^-,n_p)\in A_p}} t_{n_p^-,n_p}) + w_2(\widehat{|F_{da}^p| - |F_{daf}^p|}) + \aleph_p \cdot \vartheta\}\cdot x_p, \tag{29}$$

$$s.t.\quad \sum_{p\in P} a_{pj}\cdot x_p \geq 1, \forall j\in F, \tag{30}$$

$$x_p \in \{0,1\}, \forall p\in P. \tag{31}$$

On the other hand, if the Time-based robustness measure is utilized, the Robust Crew Pairing Model with Time-based Measure (named as RCPT) is presented in Eq. (32) to Eq. (34).

RCPT

$$min\ \sum_{p\in P}\{w_1(K+\widehat{\sum_{(n_p^-,n_p)\in A_p}} t_{n_p^-,n_p}) + w_1(\widehat{\sum_{J\in F_{da}^p} Devdelay(j) - \sum_{J\in F_{daf}^p}} Buffer(j)) + \aleph_p \cdot \vartheta\}\cdot x_p, \tag{32}$$

$$s.t.\quad \sum_{p\in P} a_{pj}\cdot x_p \geq 1, \forall j\in F, \tag{33}$$

$$x_p \in \{0,1\}, \forall p\in P. \tag{34}$$

To be specific, $t_{n_p^-,n_p}$ is determined by Eq. (8) to Eq. (19), $Buffer(j)$ is defined by Eq. (21), while $Devdelay(j)$ is decided by Eq. (24).

5. Solution Approach

In this work, we build a column-generation based solution algorithm to solve the proposed robust crew pairing models. Column generation is a continuous optimization technique to deal with large-scale linear programming problems, without explicitly considering the whole solution pool. Using column generation, the CPP is divided into a restricted master problem and a sub-problem, which are demonstrated in Sections 5.1 and 5.2, respectively.

5.1 Restricted master problem

The restricted master problem (RMP) is the linear relaxation of the CPP with a limited number of feasible pairings. We initialize the solution pool by forming a pairing for each duty (deadhead arcs are used if the duty does not start from or end at the home base). The RMP is solved iteratively to optimality, and the dual prices derived in each iteration are transferred to the sub-problem, in order to identify better pairings which can be used to update the pairing pool of the RMP. The whole column generation terminates if no better pairings could be found. Mixed integer programming technique is then used to obtain integer solutions.

5.2 Sub-problem

The sub-problem is usually formulated as a resource-constrained shortest path problem to identify promising pairings from the whole solution pool. Promising pairings refer to those with negative reduced costs that can further lower down the objective value of the robust crew pairing model. Therefore, the purpose of the sub-problem is to identify the pairing with the most negative cost (i.e., reduced cost). That is, to solve a shortest path problem in the duty-based flight network with resource constraints. Resources in the crew pairing problem are usually related to rules and regulations, like the maximum time away from base and the maximum number of flights in a pairing. A pairing is feasible only when all rules and regulations are satisfied. The identified better pairing is then added to the solution pool of the RMP, and the next iteration starts. If no better pairings (i.e., no paths are with negative costs) can be identified, the algorithm ends. Let π_j represent the dual price for the j_{th} row (Flight f) of the flight coverage constraint (i.e., Eq. (30) in RCPN or Eq. (33) in RCPT). The reduced cost of decision variable x_p (Pairing or Path p) is formulated in Eq. (35). Note that $\bar{c_p}$ stands for the reduced cost for x_p, and c_p is defined in Eq. (28). In RCPN, $c_p = w_1(K + \sum_{\widehat{(n_{\bar{p}},n_p)\in A_p}} t_{n_{\bar{p}},n_p}) + w_2(\widehat{|F_{da}^p| - |}F_{daf}^p|) + \aleph_p \cdot \vartheta$, while in RCPT, $c_p = w_1(K + \sum_{\widehat{(n_{\bar{p}},n_p)\in A_p}} t_{n_{\bar{p}},n_p}) + w_1(\sum_{j\in F_{da}^p} \widehat{Devdelay(j) -} \sum_{j\in F_{daf}^p} Buffer(j)) + \aleph_p \cdot \vartheta$.

$$\bar{c_p} = c_p - \sum_{j\in F} a_{pj} \cdot \pi_j \tag{35}$$

6. Computational Experiments

This section carries out computational experiments to examine the performances of the proposed two novel robust crew pairing models (i.e., RCPN and RCPT). Experiments were conducted on a PC with Windows 7 operation system and Intel (R) Core (TM) i7-4790 @ 3.60 GHz (32 GB RAM). The implementations are coded in Java programming language. The RMP is solved using CPLEX Concert Technology in IBM ILOG CPLEX Optimization Studio (Version 12.6.3). In the following, Section 6.1 introduces the flight data sets used for experiments, while Section 6.2 illustrates the robustness strategies constructed in this work through two typical duties generated from an instance. Last, section 6.3 demonstrates the superior performances of the proposed models in robustness enhancement.

Table 2. Instance characteristics.

Instance	No. of flights	Cities involved	Days involved
1	36	7	1
2	39	4	3
3	56	4	2
4	67	7	2
5	96	7	3
6	98	7	2

6.1 Data sets

We test the proposed models using a set of instances derived from real flight schedules (**Table 2**). To be specific, six instances with an increasing size are involved, with 36, 39, 56, 67, 96, and 98 flights, respectively. Instances 1, 4, 5, and 6 contain routes among 7 cities (i.e., airports) (GUM, HKG, ICN, KIX, OKA, SIN, and TPE). Instances 2 and 3 involve 4 cities (i.e., airports) (HKG, ICN, KIX, and TPE). Besides, the spans of the instance schedules are from 1 day to 3 days.

Table 3. Sample duties.

Duty 1 (Pairing 65)					
			F9: deviation-affected-free flight		
F7 Scheduled departure_HKG	**F7 Scheduled arrival_TPE**	**MinSit (30)**	**MaxSit (240)**	**F9 Scheduled departure_TPE**	**F9 Scheduled arrival_HKG**
5:10	6:45	7:15	10:45	7:19	9:10
Expected departure	**Expected arrival**	**MinSit (30)**	**MaxSit (240)**		
5:10	6:23 (early arrival)	7:15	10:45		
Duty 2 (Pairing 65)					
			F38: deviation-affected flight		
F37 Scheduled departure_HKG	**F37 Scheduled arrival_TPE**	**MinSit (30)**	**MaxSit (240)**	**F38 Scheduled departure_TPE**	**F38 Scheduled arrival_HKG**
0:01	3:45	4:15	7:45	5:05	8:50
Expected departure	**Expected arrival**	**MinSit (30)**	**MaxSit (240)**		
0:01	5:53	6:23	9:53		

6.2 Demonstration of the constructed robustness strategies

We demonstrate the robustness strategies proposed in this work through two duties identified from Instance 3. The details of the duties are shown in **Error! Reference source not found.**. To be specific, Duty 1 (as illustrated in **Figure 9**) from Pairing 65 consists of Flight 7 (HKG-TPE) and Flight 9 (TPE-HKG). According to Eq. (2), it is found that Flight 7 is expected to arrive at its destination earlier than the scheduled time. As discussed, early arrival is regarded as on-time arrival. Therefore, the legal sit time ranges for the scheduled Flight 7 and the expected Flight 7 are the same. Accordingly, the scheduled departure time of Flight 9 falls in the overlap of the legal sit time range for the scheduled Flight 7 and the expected Flight 7. Therefore, Flight 9 is a deviation-affected-free flight, while the deviation-buffer time for Flight 9 is 4 minutes. Duty 2 (as illustrated in **Figure 10**) from Pairing 65 is composed of Flight 37 (HKG-TPE) and Flight 38 (TPE-HKG). As calculated, Flight 37 is expected to arrive late at its destination by 128 minutes (2h 8m). Unfortunately, the scheduled departure time of Flight 38 is not in the legal sit time range for the expected arrival-late Flight 37. Therefore, Flight 38 is a deviation-affected flight which is expected to be late for 78 minutes (1h 18m, deviation-delay time).

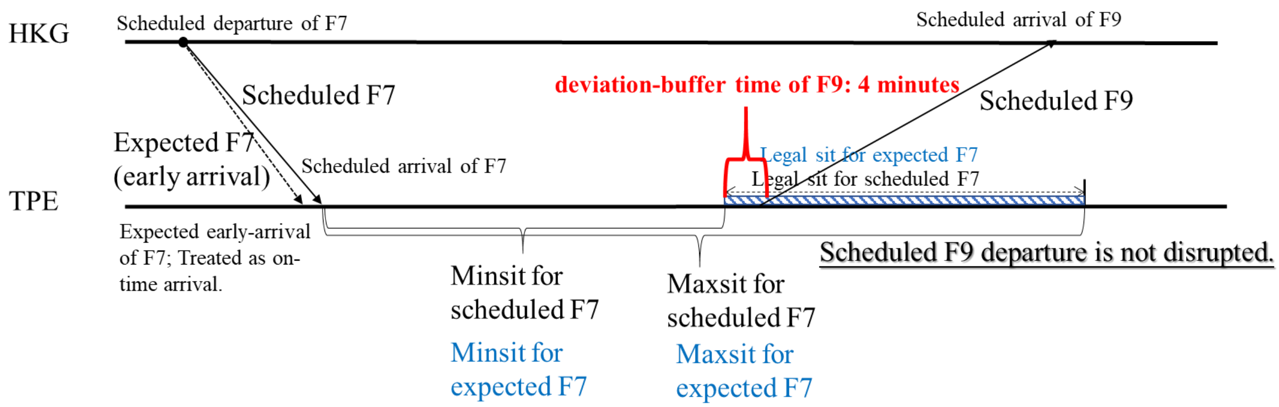

Figure 9. Sample Duty 1 from Pairing 65.

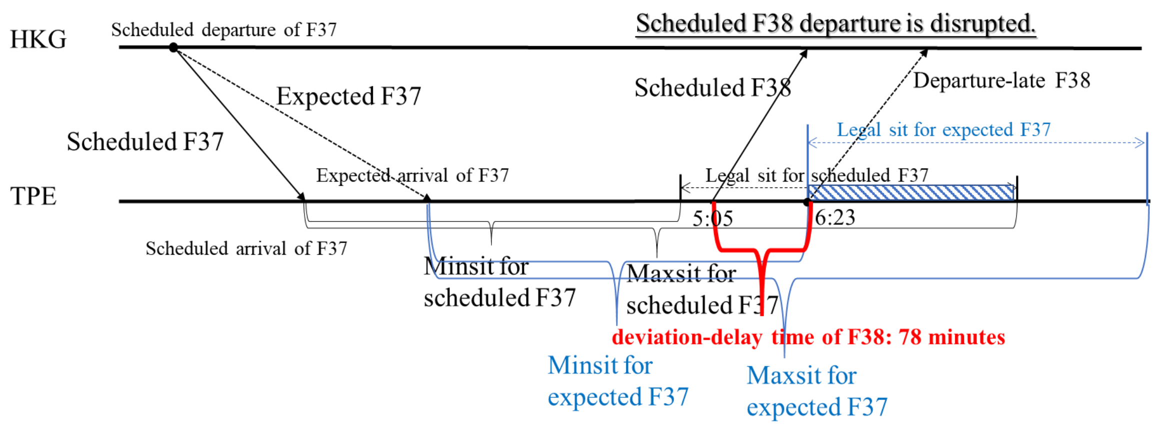

Figure 10. Sample Duty 2 from Pairing 65.

6.3 Advantages of the proposed models in robustness enhancement

After demonstrating our proposed idea of robustness enhancement strategies facilitated by considering flight flying time variability, in this part, we compare our constructed novel robust crew pairing models (i.e., RCPT and RCPN) with the existing model which is widely used in the literature, to illustrate the advantages of our proposed models. Note that the traditional crew pairing model without the consideration of solution robustness is named as "Base model" hereafter.

The solution details of RCPT, RCPN, and Base model are summarized in **Table 4** and **Table 5**. From the third column of **Table 4**, it is reasonable to observe that the number of pairings required to cover all flights in the schedule becomes larger when the instance size grows. The second column in **Table 5** gives the number of flights that belong to single-flight duties for each instance under each model. As these duties only involve one flight, these flights are free from the deviations of its previous flights (actually there is no previous flight in the duty). Therefore, we list this number separately. Besides, it should be pointed out that the construction of single-flight duty greatly depends on the structure of the considered flight schedule. Moreover, we distinguish the normal deviation-affected flights (as shown in the fourth column of **Table 5**) and the extreme-delay flights (as shown in the fifth column of **Table 5**). In our proposed models, a very large penalty cost will be incurred if an extreme-delay case happens. Therefore, in the experiments, the proposed RCPT and RCPN avoid the happening of any extreme-delay flight, while the Base model generates many. As a result, it is concluded that our proposed models are superior in avoiding extreme-delay cases, which can significantly benefit commercial airlines. Note that in the last column of **Table 4**, the average delay time is calculated by using the total deviation-affected flights (including both normal and extreme cases). Therefore, we can even observe an average deviation-delay time of 543 minutes for Instance 2 if the Base model is applied.

Table 4. Solution details (Part I).

Instance	Model	No. of pairings in the solution	w1	w2	Total deviation-buffer time	Average buffer time for deviation-affected-free flights	Total deviation-delay time	Average delay time for total deviation-affected flights
1	RCPT	21	0.5	0.5	2627	202	0	0
	RCPN	24	0.5	0.5	1411	128	0	0
	Base	24	1	0	1379	138	31	31
2	RCPT	23	0.5	0.5	877	175	62	62
	RCPN	24	0.5	0.5	567	189	0	0
	Base	22	1	0	718	120	1629	543
3	RCPT	35	0.5	0.5	1115	223	63	63
	RCPN	37	0.5	0.5	479	120	0	0
	Base	34	1	0	829	118	414	138
4	RCPT	38	0.5	0.5	697	116	78	78
	RCPN	45	0.5	0.5	656	109	78	78
	Base	43	1	0	471	94	670	134
5	RCPT	54	0.5	0.5	339	85	39	39
	RCPN	68	0.5	0.5	236	59	39	39
	Base	63	1	0	244	61	825	275
6	RCPT	65	0.5	0.5	1876	171	78	78
	RCPN	56	0.5	0.5	1356	136	78	78
	Base	58	1	0	1169	130	1785	255

For the weights used in the models, in this section, we set w_1 and w_2 are 0.5, to place equal emphasis on the normalized basic operating cost ($\widehat{c_p^o}$) and the normalized robustness-related cost ($\widehat{c_p^{rb}}$). On the other hand, as robustness measures are not considered in the Base model, the weight for the basic operating cost (w_1) is equal to 1 for the Base model. The performance comparisons of the three models are summarized in **Table 6**. Note that in the categories of "Basic operating cost", "Total deviation-buffer time", and "No. of deviation-affected-free flights", the first column represents the performance comparison of RCPT over RCPN, while for the categories of "Total deviation-delay time" and "No. of total deviation-affected flights", the first column represents the performance comparison of RCPN over RCPT. Next, we analyze the performances of the proposed RCPT & RCPN, compared with the Base model from different perspectives.

Table 5. Solution details (Part II).

Instance	No. of flights in single-flight duties	No. of deviation-affected-free flights	No. of deviation-affected flights (not extreme delay)	No. of extreme-delay flights	No. of total deviation-affected flights	Basic Operating Cost
1	23	13	0	0	0	3.75E+06
	25	11	0	0	0	3.95E+06
	25	10	1	0	1	3.95E+06
2	33	5	1	0	1	3.46E+06
	36	3	0	0	0	3.91E+06
	30	6	2	1	3	3.00E+06
3	50	5	1	0	1	6.25E+06
	52	4	0	0	0	6.60E+06
	46	7	1	2	3	5.40E+06
4	60	6	1	0	1	9.76E+06
	60	6	1	0	1	9.76E+06
	57	5	3	2	5	8.98E+06
5	91	4	1	0	1	1.34E+07
	91	4	1	0	1	1.34E+07
	89	4	1	2	3	1.26E+07
6	86	11	1	0	1	1.12E+07
	87	10	1	0	1	1.14E+07
	82	9	1	6	7	9.56E+06

Basic operating cost. The basic operating cost performances of the three models are summarized in the second to the fourth columns of **Table 6**. It is reasonable to observe an increase in the monetary expenditure for the proposed RCPT and RCPN compared with the Base model, as solution robustness is considered in the proposed models, while not for the Base model. The proposed models improve the robustness of the pairings generated at a price of an average of 9.7% increase in cost for RCPT, and an average of 14.3% increase in cost for RCPN. It is also worthwhile to note that RCPT achieves cost savings over RCPN with an average of 3.8%. Besides, it is interesting to see that for Instance 1, RCPT and RCPN can even save costs compared with the Base model with an improvement in robustness (which is facilitated by an increase in deviation-buffer time & number of deviation-affected-free flights, and a reduction in deviation-delay time & number of deviation-affected flights).

Total deviation-buffer time & Total deviation-delay time. As RCPT aims to improve the pairing resistance against the deviations of previous flights through assigning buffer time, we can observe a remarkable increase in the deviation-buffer time of the solutions generated by RCPT compared with the Base model (with an average of 49.1%). However, RCPN performs worse than the Base model by generating shorter deviation-buffer time (1.5% less on average). On the other hand, when compared with the Base model, RCPN shows slightly better performances than RCPT regarding the reduction of deviation-delay time. However, the degree of increase in deviation-buffer time achieved by RCPT is far more significant than the degree of reduction in deviation-delay time obtained by RCPN when the Base model is regarded as a benchmark.

Table 6. Performance comparisons.

Instance	Basic operating cost			Total deviation-buffer time			Total deviation-delay time			No. of deviation-affected-free flights			No. of total deviation-affected flights		
	RCPT/RCPN	**RCPT/Base**	**RCPN/Base**	**RCPT/RCPN**	**RCPT/Base**	**RCPN/Base**	**RCPN/RCPT**	**RCPT/Base**	**RCPN/Base**	**RCPT/RCPN**	**RCPT/Base**	**RCPN/Base**	**RCPN/RCPT**	**RCPT/Base**	**RCPN/Base**
1	-5.0%	-5.0%	0.0%	86.2%	90.5%	2.3%	0.0%	-100.0%	-100.0%	18.2%	30.0%	10.0%	0.0%	-100.0%	-100.0%
2	-11.4%	15.2%	30.1%	54.7%	22.1%	-21.0%	-100.0%	-96.2%	-100.0%	66.7%	-16.7%	-50.0%	-100.0%	-66.7%	-100.0%
3	-5.3%	15.8%	22.2%	132.8%	34.5%	-42.2%	-100.0%	-84.8%	-100.0%	25.0%	-28.6%	-42.9%	-100.0%	-66.7%	-100.0%
4	0.0%	8.7%	8.7%	6.3%	48.0%	39.3%	0.0%	-88.4%	-88.4%	0.0%	20.0%	20.0%	0.0%	-80.0%	-80.0%
5	0.0%	5.8%	5.8%	43.6%	38.9%	-3.3%	0.0%	-95.3%	-95.3%	0.0%	0.0%	0.0%	0.0%	-66.7%	-66.7%
6	-1.0%	17.6%	18.8%	38.3%	60.5%	16.0%	0.0%	-95.6%	-95.6%	10.0%	22.2%	11.1%	0.0%	-85.7%	-85.7%
Mean	-3.8%	9.7%	14.3%	60.3%	49.1%	-1.5%	-33.3%	-93.4%	-96.5%	20.0%	4.5%	-8.6%	-33.3%	-77.6%	-88.7%

No. of deviation-affected-free flights & No. of total deviation-affected flights. As RCPN aims to improve the pairing robustness through a number-based measure, it is intuitive to witness a great reduction

(88.7% on average) in the number of deviation-affected flights of the solutions obtained from RCPN compared with the Base model. However, it is also noted that RCPN performs worse than the Base model by generating less deviation-affected-free flights (8.6% less on average). On the contrary, RCPT can greatly reduce the existence of deviation-affected flights than the Base model by an average of 77.6%, while also increase the number of deviation-affected-free flights with a mean of 4.5%. Therefore, it is implied that even when judging the solutions by applying the number-based measure, RCPT performs more stable than RCPN.

After comparing the proposed models with the Base model, we then concentrate on the performance comparisons of RCPT and RCPN, which is discussed in the following.

RCPT vs RCPN. As discussed, RCPT improves solution robustness through a time-based measure (i.e., maximize deviation-buffer time while minimize deviation-delay time), while RCPN enhances solution robustness by a number-based measure (i.e., maximize the number of deviation-affected-free flights while minimize the number of deviation-affected flights). Then, the performances of RCPT and RCPN are analyzed regarding these two measures.

First, focusing on the time-base measure, it is reasonable to find that RCPT can generate 60.3% more deviation-buffer time than RCPN averagely. Recall that RCPT can also achieve a cost reduction than RCPN with an average of 3.8%. Therefore, the solutions obtained from RCPT show higher resistance against disruptions with a lower operating cost than RCPN. However, RCPT also incurs longer deviation-delay time for Instances 2 & 3 than RCPN. This is because RCPN is able to avoid the occurrence of deviation-affected flights for these two instances, while RCPT fails. But compared with the much higher deviation-buffer time produced by RCPT, the slight increase in deviation-delay time becomes acceptable.

Second, if we focus on the number-based measure, it is surprising to identify that RCPT creates more deviation-affected-free flights than RCPN (with an average of 20%), although the emphasis of RCPT is the time measure. This is because the maximization of deviation-buffer time leads to the growing occurrence of deviation-affected-free flights. In terms of the production of deviation-affected flights, RCPT is also able to show the same ability in avoiding deviation-affected flights as RCPN for four instances among the total six instances (i.e., Instances 1, 4, 5, and 6). Therefore, it is concluded that although RCPT applies a time-based measure, it also achieves satisfactory performances in terms of the number-based measure.

7. Concluding Remarks

The crew pairing problem is crucial for modern airlines. Challenged by diverse uncertainties, it is important for airlines to enhance the robustness of their constructed crew pairings. Flight flying time variability is commonly seen nowadays, and has become one major source of disruption for airline operational planning. However, the impact of flight flying time variability on airline crew pairings is under-investigated. In this paper, we propose two robustness enhancement strategies based on the consideration of flight flying time variability. The first strategy is to encourage flights that will not be affected by the expected arrival delay of the preceding flight, while the second strategy is to discourage flights that will be influenced by the expected arrival delay of the preceding flight. Regarding each robustness strategy, we propose two different measures: number-based measure and time-based measure. Accordingly, two novel robust crew pairing models, namely the Robust Crew Pairing Model with Number-based Measure (RCPN) and the Robust Crew Pairing Model with Time-based Measure (RCPT), are constructed. A customized column generation based solution algorithm is proposed to solve the models. Computational experiments based on data sets derived from real flight schedules show that our new models can greatly enhance the robustness of pairing solutions (e.g., 49.1% more deviation-buffer time) at a price of an acceptable increase in basic operating costs (e.g., 9.7%) compared with the traditional model without considering flight flying time variability in pairing generation. Moreover, both RCPN and RCPT can completely avoid extreme-delay cases, while the traditional model generates many extreme-delay flights. Extreme-delay flights cause significant damages for the image of airlines. Therefore, our proposed RCPN and RCPT can benefit airlines a lot. In terms of the performance comparison between RCPN and RCPT, we find that RCPT can generate an average of 60.3% more deviation-buffer time, while consumes 3.8% less operating costs than RCPN. Additionally, it is revealed that although RCPT applies a time-based measure, when compared with RCPN regarding the number-based measure, RCPT achieves satisfactory and more stable performances. Therefore, the solutions obtained from RCPT show higher resistance against the disruption of flight flying time variability with a lower operating cost than RCPN.

Managerial implications

From this study, it is seen that airline crew scheduling departments should carefully consider the impact of flight flying time variability on the generated crew pairings, in addition to the basic operating costs. As flight flying time variability led by the fluctuating cruise speed has become a common disruption for the daily operations of the air transportation industry, failing to integrating the consideration of flight flying time variability may produce fragile pairings that are easily disrupted in real practice. Therefore, airlines will be benefited from this study by applying the proposed robust crew pairing models with the novel robustness enhancement strategies based on the consideration of flight flying time variability (i.e., encouraging deviation-affected-free flights and discouraging deviation-affected flights through a number-based measure and a time-based measure). By utilizing our proposed models, airlines can greatly enhance their resistance against flight flying time disruptions in daily operations.

As for future research, one interesting and valuable direction is to consider the impact of global pandemic (e.g., the recent outbreak of COVID-19) on the crew scheduling decisions in the aviation industry (Cai & Choi, 2020; Choi, 2020; Govindan et al., 2020; Ivanov, 2020). As the epidemic leads to large-scale flight cancellation and re-scheduling, how to manage air crew resources becomes even more challenging.

References

Antunes, D., Vaze, V., & Antunes, A. P. (2019). A robust pairing model for airline crew scheduling. *Transportation Science, 53*(6), 1751-1771.

Bock, S., Mantin, B., Niemeier, H.-M., & Forsyth, P. J. (2020). Bankruptcy in international vs domestic markets: Evidence from the airline industry. *Transportation Research Part A: Policy and Practice, 132*, 728-743.

Button, K., Martini, G., Scotti, D., & Volta, N. (2019). Airline regulation and common markets in Sub-Saharan Africa. *Transportation Research Part E: Logistics and Transportation Review, 129*, 81-91.

Cacchiani, V., & Salazar-González, J.-J. (2017). Optimal solutions to a real-world integrated airline scheduling problem. *Transportation Science, 51*(1), 250-268.

Cai, Y.-J., & Choi, T.-M. (2020). A United Nations' sustainable development goals perspective for sustainable textile and apparel supply chain management. *Transportation Research Part E: Logistics and Transportation Review, 141*, 102010.

Choi, T.-M. (2020). Innovative "bring-service-near-your-home" operations under Corona-virus (COVID-19/SARS-CoV-2) outbreak: Can logistics become the messiah? *Transportation Research Part E: Logistics and Transportation Review, 140*, 101961.

Choi, T.-M., Wen, X., Sun, X., & Chung, S.-H. (2019). The mean-variance approach for global supply chain risk analysis with air logistics in the blockchain technology era. *Transportation Research Part E: Logistics and Transportation Review, 127*, 178-191.

Chung, S. H., Ma, H. L., & Chan, H. K. (2017). Cascading delay risk of airline workforce deployments with crew pairing and schedule optimization. *Risk Analysis, 37*(8), 1443-1458.

Chung, S. H., Tse, Y. K., & Choi, T.-M. (2015). Managing disruption risk in express logistics via proactive planning. *Industrial Management & Data Systems*.

Clausen, J., Larsen, A., Larsen, J., & Rezanova, N. J. (2010). Disruption management in the airline industry—Concepts, models and methods. *Computers & Operations Research, 37*(5), 809-821.

Delgado, L., & Prats, X. (2009). *Fuel consumption assessment for speed variation concepts during the cruise phase.* Paper presented at the Conference on Air Traffic Management Economics.

Delgado, L., & Prats, X. (2013). Effect of wind on operating-cost-based cruise speed reduction for delay absorption. *IEEE Transactions on Intelligent Transportation Systems, 14*(2), 918-927.

Desaulniers, G., Desrosiers, J., Dumas, Y., Marc, S., Rioux, B., Solomon, M. M., & Soumis, F. (1997). Crew pairing at air france. *European Journal of Operational Research, 97*(2), 245-259.

Dunbar, M., Froyland, G., & Wu, C.-L. (2012). Robust airline schedule planning: Minimizing propagated delay in an integrated routing and crewing framework. *Transportation Science, 46*(2), 204-216.

Dunbar, M., Froyland, G., & Wu, C.-L. (2014). An integrated scenario-based approach for robust aircraft routing, crew pairing and re-timing. *Computers & Operations Research, 45*, 68-86.

Govindan, K., Mina, H., & Alavi, B. (2020). A decision support system for demand management in healthcare supply chains considering the epidemic outbreaks: A case study of coronavirus disease 2019 (COVID-19). *Transportation Research Part E: Logistics and Transportation Review, 138*, 101967.

Haouari, M., Mansour, Z. F., & Sherali, H. D. (2019). A new compact formulation for the daily crew pairing problem. *Transportation Science, 53*(3), 811-828.

Ionescu, L., Gwiggner, C., & Kliewer, N. (2016). Data analysis of delays in airline networks. *Business & Information Systems Engineering, 58*(2), 119-133.

Ivanov, D. (2020). Predicting the impacts of epidemic outbreaks on global supply chains: A simulation-based analysis on the coronavirus outbreak (COVID-19/SARS-CoV-2) case. *Transportation Research Part E: Logistics and Transportation Review, 136*, 101922.

Khan, W. A., Chung, S.-H., Ma, H.-L., Liu, S. Q., & Chan, C. Y. (2019a). A novel self-organizing constructive neural network for estimating aircraft trip fuel consumption. *Transportation Research Part E: Logistics and Transportation Review, 132*, 72-96.

Khan, W. A., Chung, S. H., Awan, M. U., & Wen, X. (2019b). Machine learning facilitated business intelligence (Part I): Neural networks learning algorithms and applications. *Industrial Management & Data Systems*.

Lavoie, S., Minoux, M., & Odier, E. (1988). A new approach for crew pairing problems by column generation with an application to air transportation. *European Journal of Operational Research, 35*(1), 45-58.

Muter, İ., İlker Birbil, Ş., Bülbül, K., Şahin, G., Yenigün, H., Taş, D., & Tüzün, D. (2013). Solving a robust airline crew pairing problem with column generation. *Computers & Operations Research, 40*(3), 815-830.

Ng, K., Lee, C., Chan, F. T., & Qin, Y. (2017). Robust aircraft sequencing and scheduling problem with arrival/departure delay using the min-max regret approach. *Transportation Research Part E: Logistics and Transportation Review, 106*, 115-136.

Ng, K., Lee, C. K., Chan, F. T., Chen, C.-H., & Qin, Y. (2020). A two-stage robust optimisation for terminal traffic flow problem. *Applied Soft Computing, 89*, 106048.

Özener, O. Ö., Matoğlu, M. Ö., Erdoğan, G., Haouari, M., & Sözer, H. (2017). Solving a large-scale integrated fleet assignment and crew pairing problem. *Annals of Operations Research, 253*(1), 477-500.

Papadakos, N. (2009). Integrated airline scheduling. *Computers & Operations Research, 36*(1), 176-195.

Parmentier, A., & Meunier, F. (2019). Aircraft routing and crew pairing: Updated algorithms at Air France. *Omega*, 102073.

Qin, Y., Wang, Z., Chan, F. T., Chung, S., & Qu, T. (2019). A mathematical model and algorithms for the aircraft hangar maintenance scheduling problem. *Applied Mathematical Modelling, 67*, 491-509.

Quesnel, F., Desaulniers, G., & Soumis, F. (2019). A branch-and-price heuristic for the crew pairing problem with language constraints. *European Journal of Operational Research*.

Ruther, S., Boland, N., Engineer, F. G., & Evans, I. (2017). Integrated aircraft routing, crew pairing, and tail assignment: Branch-and-price with many pricing problems. *Transportation Science, 51*(1), 177-195.

Saddoune, M., Desaulniers, G., Elhallaoui, I., & Soumis, F. (2012). Integrated airline crew pairing and crew assignment by dynamic constraint aggregation. *Transportation Science, 46*(1), 39-55.

Saddoune, M., Desaulniers, G., & Soumis, F. (2013). Aircrew pairings with possible repetitions of the same flight number. *Computers & Operations Research, 40*(3), 805-814.

Şafak, Ö., Çavuş, Ö., & Aktürk, M. S. (2018). Multi-stage airline scheduling problem with stochastic passenger demand and non-cruise times. *Transportation Research Part B: Methodological, 114*, 39-67.

Shao, S., Sherali, H. D., & Haouari, M. (2017). A novel model and decomposition approach for the integrated airline fleet assignment, aircraft routing, and crew pairing problem. *Transportation Science, 51*(1), 233-249.

Sheng, D., Li, Z.-C., & Fu, X. (2019). Modeling the effects of airline slot hoarding behavior under the grandfather rights with use-it-or-lose-it rule. *Transportation Research Part E: Logistics and Transportation Review, 122*, 48-61.

Sun, X., Chung, S.-H., Choi, T.-M., Sheu, J.-B., & Ma, H. L. (2020a). Combating lead-time uncertainty in global supply chain's shipment-assignment: Is it wise to be risk-averse? *Transportation Research Part B: Methodological, 138*, 406-434.

Sun, X., Chung, S. H., & Ma, H. L. (2020b). Operational risk in airline crew scheduling: Do features of flight delays matter? *Decision Sciences*.

Tekiner, H., Birbil, Ş. İ., & Bülbül, K. (2009). Robust crew pairing for managing extra flights. *Computers & Operations Research, 36*(6), 2031-2048.

Vance, P. H., et al. (1997). A heuristic branch-and-price approach for the airline crew pairing problem. *preprint*.

Wang, C., & Wang, X. (2019). Airport congestion delays and airline networks. *Transportation Research Part E: Logistics and Transportation Review, 122*, 328-349.

Wang, Z., Khan, W. A., Ma, H.-L., & Wen, X. (2020). Cascade neural network algorithm with analytical connection weights determination for modelling operations and energy applications. *International Journal of Production Research*, 1-18.

Weide, O., Ryan, D., & Ehrgott, M. (2010). An iterative approach to robust and integrated aircraft routing and crew scheduling. *Computers & Operations Research, 37*(5), 833-844.

Wen, X., Chung, S.-H., Ji, P., Shue, J.-B., & Choi, T.-M. (2020). Multi-class cabin crew pairing problems with controlled crew substitution in airline operations. *working paper*.

Wen, X., Xu, X., Choi, T.-M., & Chung, S.-H. (2019). Optimal pricing decisions of competing air-cargo-carrier systems--Impacts of risk aversion, demand, and cost uncertainties. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, in press.