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Highlights

Multi-objective portfolio selection considering expected and total utility

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- Proposing a novel multi-objective portfolio selection model that takes into account of expected and total utility.
- Quantitatively calculating the total utility through prospect theory and disappointment theory to avoid investment losses caused by irrational behavior of investors.
- The effectiveness of the proposed portfolio selection model in balancing investment return with risk is validated by real market data.

Multi-objective portfolio selection considering expected and total utility

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ABSTRACT

To address the challenge of minimizing the risk of irrational investment decisions made by investors in the face of wealth fluctuations, this article incorporates prospect theory and disappointment theory into the framework of multi-objective portfolio selection. The proposed portfolio selection model aims to balance return and risk by simultaneously maximizing expected and total utility. The expected utility objective drives the model to chase investment returns, while the total utility drives the model to avoid investment risk. The effectiveness of the proposed multi-objective portfolio selection model is tested by comparing it to three control portfolio selection models.

1. Introduction

Portfolio selection began with the work of Markowitz (1952). The core issue of portfolio selection is to make rational asset allocation strategy to achieve a balance of risk and return. It's a common knowledge that investment risk grows with return. Inspired by Markowitz's mean-variance framework, many works refine portfolio selection by adopting novel return and risk metrics, including distributionally robust mean-variance (Blanchet et al., 2022), independent component analysis risk-control method (Lassance et al., 2022), semiparametric method (Han and Wang, 2022), combination of semi-variance and conditional value-at-risk (CVaR) (Kaucic et al., 2019), etc. All these works push portfolio selection forward but share with a potential pitfall still.

These works always assume the investment risk result from the volatility capital market and the investor is an apathetic person who obeys the Expected Utility Theory (EUT) without any violation of reason. But this assumption sometimes contradicts reality. Chen observes the return chasing behavior of retail investors by analysing data on 18 million individual equity accounts (Chen et al., 2022). This irrational behavior can lead to risk concentration and retail investor returns significantly below the market average. Tversky and Kanehman found people's decisions are not fully consistent with EUT's predictions when under risk and random. And these findings form the Prospect Theory (PT) (Kahneman and Tversky, 1979). PT uses distorted value function and weight function to quantify phenomena of risk-seeking and loss-aversion.

There is a plethora of evidence showing PT is reference dependent, both in the lab and in the field (Baillon et al., 2020). PT does not know much about how the reference point is formed and then assigns arbitrary value, as long as seems reasonable, which is a fundamental problem. PT could not explain some existing experimental results, thus Nagarajan and Shechter (2014) pose the question "why PT, a well-known and widely accepted framework for decision making under uncertainty, may not apply to a fundamental operations management problem". In order to revise PT, Baillon et al. (2020) conduct a series of experiments and proves the best rule is the reference point rule named as the Status Quo. Therefore this rule is adopted by us to build the portfolio selection model.

While PT has contributed significantly to our understanding of decision-making under risk, it is not without drawbacks and limitations that should be taken into account when applying the theory to real-world situations. Therefore, in order to develop a more comprehensive understanding of human decision-making, Bell (1985) proposes

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Disappointment Theory (DT). DT is an extension of PT that focuses on how individuals react to outcomes that are worse than their expectations. While PT provides a general framework for understanding decision-making under uncertainty, the disappointment model offers complementary insights into the role of expectations in shaping people's emotions and behavior.

Therefore, considering that the risk of investing can arise from both the volatile market and emotional impulses of investors, this study proposes a novel portfolio selection model grounded on EUT, PT and DT. The concept of reference point dependency in PT emphasizes the importance of understanding how investors evaluate outcomes and make decisions under risk, and highlights the need for decision-makers to take into account the various reference points that may be relevant for a given decision. By concurrently utilizing PT and DT, this framework takes emotional factors into consideration then provides a complete picture of how individuals make decisions and can help explain some of the deviations from traditional economic models that have been observed in empirical studies.

In the subsequent section, we provide an overview of the theoretical foundation, followed by a discussion of the preliminary concepts employed in this research in Section 3. Section 4 delves into the comprehensive description of the innovative multi-objective portfolio selection model. Section 5 outlines the methodology adopted for this investigation. Section 6 presents one case study to evaluate the performance of the multi-period portfolio selection model. Finally, Section 7 offers concluding remarks and implications of the research.

2. Literature Review

Portfolio selection theory has been trying to provide an feasible answer to the fundamental problem: How should an investor allocate his/her wealth among possible choices (Anadu et al., 2020). The literature has enhanced and refined Markowitz's mean-variance framework by incorporating the features of actual capital markets and investment limitations (Qarni and Gulzar, 2021; Masoud et al., 2020; Bertsimas et al., 2022). As uncertainty can be aroused from market and investor which poses a challenge to be characterized through probability theory, the literature resorts to employing fuzzy set theory in order to address randomness and uncertainty when it comes to portfolio selection (Wang and Zhu, 2002).

The motivation of this article is to incorporate investors' emotional factors in the portfolio selection framework. As fuzzy set theory is an effective tool to characterize an uncertain environment with vagueness in many aspects of capital market, such as the predictable behavior of investors (Tiryaki and Ahlatcioglu, 2009), thus this article builds a multi-objective portfolio selection model based on fuzzy set theroy.

The literature have endeavored to depict and anticipate human behavior in many cases within the paradigm of PT (Ferro et al., 2021; Gao et al., 2021; Grant et al., 2021). And portfolio selection research has been improved by adopting PT, in random environment (Fulga, 2016) and fuzzy environment (Tian et al., 2018). Based on the premise that the investor holds securities prior to the portfolio optimization, Wang et al. (2022, 2023) propose portfolio selection models based on PT.

Lu et al. (2015) incorporate the financial reference point and the social reference point as the double reference points which forms the total utility framework in the comparison theory. Koop and Johnson (2012); Wang and Johnson (2012) propose triple financial reference points, the goal, the status quo and the minimum requirement. These three points create four disjointed regions, failure, loss, gain and success and provides a reasonable explanation of decision-makers' behaviours.

Efforts to model subjective emotions with underlying cognitive dissonance date back to Akerlof and Dickens (1982). DT is built upon the findings that the felicity associated with a given uncertain outcome increases as the difference between the expectation and realization grows (Gul, 1991; Loomes and Sugden, 1986). Later Brunnermeier and Parker (2005); Brunnermeier et al. (2007) propose the optimal belief approach in which the investor holds the endogenous belief that expectations of future pleasures could lead to ex ante felicity. Gollier and Muermann (2010) build a more precise structural model on the optimal beliefs approach, in which the subject has to balance between ex ante savoring and ex post disappointment.

3. Preliminaries

This section contains preliminary knowledge which will be used in this study.

3.1. Fuzzy Set Theory

Suppose $\mu(x)$ is the membership function of fuzzy variable ξ. ξ is normal when existing $\mu(l) = 1$, $l \in \mathbf{R}$. Fuzzy variable in this article are always normal. And the following credibility measurement method is used to define the likelihood of event $\xi \leq l$ happening:

$$\mathbf{Cr}\{\xi \leq l\} = \frac{1}{2}[\sup_{\xi \leq l} \mu(x) + 1 - \sup_{\xi > l} \mu(x)]. \tag{1}$$

It is easy to prove the credibility measurement method is self-dual that satisfies $\mathbf{Cr}\{\xi \leq l\} = 1 - \mathbf{Cr}\{\xi > l\}$. The expected value of ξ could be calculated by:

$$\mathbf{E}[\xi] = \int_0^{+\infty} \mathbf{Cr}\{\xi \geq l\} dl - \int_{-\infty}^{0} \mathbf{Cr}\{\xi \leq l\} dl. \tag{2}$$

3.2. Prospect Theory

In the realm of finance, a prospect can be defined as a probability distribution that pertains to different monetary values. Prospects allocate a probability of **1** to a finite set of potential outcomes, denoted by A. Prospects are denoted as $(p_1, m_1; \cdots; p_n, m_n)$ which means the investor pays m_i with probability $p_i, i = 1, \cdots, n$. Outcomes are characterized as either losses and gains relative to a specified reference point r. The outcome m is a gain if $m > r$ and a loss when $m < r$.

In PT, a probability weight function $\mathbf{w}$ and an utility distortion function $\mathbf{U}$ such that prospects are represented by:

$$A \to \mathbf{PT}_r[A] = \int_{m \leq r} \mathbf{U}(m - r) d\mathbf{w}(A) + \int_{m \geq r} \mathbf{U}(m - r) d\mathbf{w}(1 - A). \tag{3}$$

When the function $\mathbf{w}$ is linear, PT reduces to EUT with reference-free utility:

$$A \to EU(A) = \int m dA. \tag{4}$$

This article adopts the utility function $\mathbf{U}$ and the weight function $\mathbf{w}$ modeled by Baillon et al. (2020), using the following examples they provided.

$$\mathbf{U}(m) = \begin{cases} (m - r)^{\alpha}, & \text{if} \quad m \geq r, \\ \lambda * (r - m)^{\alpha}, & \text{if} \quad m < r, \end{cases} \tag{5}$$

$$\mathbf{w} = e^{-(-\ln p)^{\gamma}}, \tag{6}$$

$\alpha = 0.48$, $\lambda = -2.34$, $\gamma = 0.43$. Fig. 1 (a) shows the gain and loss utility functions in PT, represented by the red and the green lines, respectively. These two functions intersect at the reference point. Fig. 1 (b) presents the probability weight function in PT, in which the dotted line is the expected weight in EUT, while the solid line shows the probability weight in PT.

3.3. Disappointment Theory

The DT is a decision-making theory that seeks to explain how people respond to disappointment or loss, and how these responses can affect their behavior. According to this theory, people experience disappointment when the outcome of a decision or event falls short of their expectations or reference point. Disappointment can cause individuals to experience negative emotions, such as regret, frustration, or anger, and can also influence their subsequent decisions and actions.

In particular, when a decision-maker receives a certain outcome m_i, he/she experience disappointment with respect to outcomes better than m_i. Suppose all the possible outcomes are arranged as $(m_1, \cdots, m_n)$, and their corresponding possibilities are $(p_i, \cdots, p_n)$. To quantify disappointment, Bell proposed the use of a Disappointment Index, which is a measure of the difference between an individual's expected and actual outcome. The Disappointment Index can be calculated by the following formula:

$$\mathbf{D}(A) = \sum_{i=1}^{n} \sum_{m_j \geq m_i} p_i p_j \mathbf{H}(m_i - m_j). \tag{7}$$

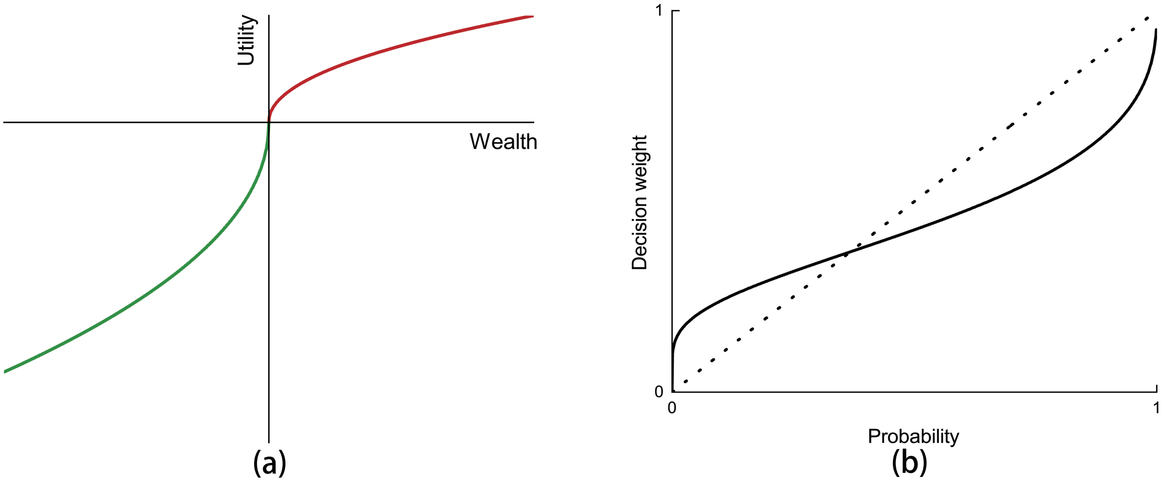

Figure 1: The gain-loss utility function and the probability weight function.

$\mathbf{H}(\cdot)$ is a non-negative function defined over an interval, which captures the decision maker's preference for dispersion. Cillo and Delquié (2014) demonstrate that setting $\mathbf{H}(z) = z$ in portfolio selection is reasonable, and this approach is adopted in this article.

If the outcomes are continuous and $\mathbf{F}$ is the cumulative distribution of A, then Disappointment Index could be calculated as:

$$\begin{aligned}\mathbf{D}(A) &= \int_{-\infty}^{+\infty} -\mathbf{E}[\int_{-\infty}^{A} \mathbf{H}(A-x)d\mathbf{F}(x)] \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{A} \mathbf{H}(y-x)d\mathbf{F}(x)d\mathbf{F}(y).\end{aligned} \tag{8}$$

4. Multi-objective Portfolio Selection Model

In this section, we describe the mathematical model for our multi-objective portfolio selection problem.

4.1. Expected and Total Utility

Suppose the investor allocates his funds among a set of possible securities, denoted by $(a_1, \ldots, a_n)$. During the given time period the investor makes investment decisions based on forecasts for these possible securities. For the security a_i, its forecasted price in the given time period is represented by a fuzzy variable p_i and its current price is $p_i^{'}$. During the given time period, security a_i pays a dividend of d_i. Therefore the return rate of security a_i during the given time period is represented by the fuzzy variable ξ_i, which can be calculated as follow:

$$\xi_i = \frac{p_i - p_i^{'} + r_i}{p_i^{'}} \tag{9}$$

The expected return rate of security a_i could be presented by the expected value of fuzzy variable ξ_i as calculated by Equ. (2):

$$\mathbf{E}[\xi_i] = \int_0^{+\infty} \mathbf{Cr}\{\xi_i \geq l\}dl - \int_{-\infty}^{0} \mathbf{Cr}\{\xi_i \leq l\}dl. \tag{10}$$

Suppose the investment proportions of security $(a_1, \ldots, a_n)$ are presented by $(x_1, \ldots, x_n)$. So the expected return rate of this portfolio selection scheme could be presented by the expected value of fuzzy variable $\mathbb{A}$.

$$\mathbb{A} = x_1 * \xi_1 + \ldots + x_n * \xi_n. \tag{11}$$

In EUT, the investor will always maximize the expected utility, i.e., the expected return rate of the portfolio selection scheme which is calculated as below,

$$\max \mathbf{E}[\mathbb{A}] = \max(\int_{0}^{+\infty} \mathbf{Cr}\{\mathbb{A} \geq l\}dl - \int_{-\infty}^{0} \mathbf{Cr}\{\mathbb{A} \leq l\}dl). \tag{12}$$

According to Equ. (3), the prospect value of the portfolio selection scheme can be calculated as:

$$\mathbf{PT}_r[\mathbb{A}] = \int_{\mathbb{A}\leq r} \mathbf{U}(\mathbb{A})d\mathbf{w}(\mathbb{A}) + \int_{\mathbb{A}\geq r} \mathbf{U}(\mathbb{A})d\mathbf{w}(1-\mathbb{A}), \tag{13}$$

in which the reference point is chosen as the status quo.

According to Equ. (8), the Disappointment Index of the portfolio selection scheme could be calculated as:

$$\mathbf{D}(\mathbb{A}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{\mathbb{A}} \mathbf{H}(y-x)d\mathbf{F}(x)d\mathbf{F}(y). \tag{14}$$

According to Wang and He (2022), the total utility is the sum of the prospect value and the Disappointment Index, which can be calculated as follows:

$$\mathbf{TU}(\mathbb{A}) = \mathbf{PT}_r(\mathbb{A}) - \mathbf{D}(\mathbb{A}) \tag{15}$$

4.2. Conditional Value-at-Risk

The mean-variance framework introduced by Markowitz (1952) has been widely adopted as the basis for portfolio selection. Under this framework, the expected return is considered the investment return, while the variance serves as a measure of investment risk. However, there has been criticism regarding the use of variance as a risk indicator, as it does not provide a precise indication of the potential losses an investor may experience.

To address this issue, Value-at-Risk (VaR) (Jorion, 1997) has emerged as a widely used tool for measuring downside risk. VaR represents the maximum loss that an investor may incur with a probability no greater than a predefined confidence level γ. This is expressed mathematically as:

$$\mathbf{VaR}_{1-\gamma}(\mathbb{L}) = \sup\{\lambda \mid \Pr(\mathbb{L} \geq \lambda) \geq \gamma\}, \tag{16}$$

where $\mathbb{L}$ represents the possible loss variable may suffered by the investor. Although the VaR method has gained widespread acceptance as a risk measurement tool, it has been criticized for its non-smoothness, non-convexity, and tendency to have numerous local minima. As an alternative, Conditional Value-at-Risk (CVaR) (Artzner et al., 1999), which is a coherent risk measurement approach, has emerged as an appealing optimization method to the VaR method. CVaR is represented by the expected conditional loss above VaR in the given confidence level, as below:

$$\mathbf{CVaR}_{1-\gamma}(\mathbb{L}) = \mathbf{E}[x \mid x > \mathbf{VaR}_{1-\gamma}(\mathbb{L})]. \tag{17}$$

Another calculation method of CVaR can be provided without relying on the definition of VaR:

$$\mathbf{CVaR}_{1-\gamma}(\mathbb{L}) = \max_{z}\{z + \frac{1}{\gamma}\mathbf{E}[(x-z)^+]\}, \tag{18}$$

where $(x-z)^+ = \max\{x-z, 0\}$.

4.3. Mathematical Model

This multi-objective portfolio selection model incorporates EUT, PT and DT, and seeks to simultaneously maximize the investor's expected utility and total utility. And CVaR is adopted as the risk metric. The mathematical model of the proposed multi-objective portfolio selection model (**E&TU**) is shown as below:

$$
\begin{cases}
\max \mathbf{E}[\mathbb{A}] = \max(\int_0^{+\infty} \mathbf{Cr}\{\mathbb{A} \geq l\}dl - \int_{-\infty}^{0} \mathbf{Cr}\{\mathbb{A} \leq l\}dl), \\
\max \mathbf{TU}(\mathbb{A}) = \max[\int_{\mathbb{A}\leq r} \mathbf{U}(\mathbb{A})d\mathbf{w}(\mathbb{A}) + \int_{\mathbb{A}\geq r} \mathbf{U}(\mathbb{A})d\mathbf{w}(1-\mathbb{A}) \\
+ \int_{-\infty}^{+\infty} \int_{-\infty}^{\mathbb{A}} \mathbf{H}(y-x)d\mathbf{F}(x)d\mathbf{F}(y)], \\
\text{s.t.} \\
\mathbf{CVaR}_{1-\gamma}(\mathbb{L}) < S, \\
x_i \geq 0,\ i = 1, 2, \dots,\ n, \\
\sum_{i=1}^{n} x_i = 1.
\end{cases}
\tag{19}
$$

Fuzzy variable $\mathbb{A}$ and $\mathbb{L}$ represent the return rate and the possible loss of the portfolio selection scheme. It is easy to prove $\mathbb{A} = -\mathbb{L}$. γ and S represent the predefined confidence level and risk threshold. Constraint ($x_i \geq 0,\ i = 1, 2, \dots,\ n$) represents no short-selling.

5. Solution Algorithm

In this section, we present the fuzzy simulation approach and an improved cooperative particle swarm optimization algorithm (ICPSO) as the proposed solution algorithm for the proposed multi-objective portfolio selection model.

5.1. Fuzzy Simulation

In Section 4.1 and Section 4.2, we provide the definitions of the expected value of a fuzzy variable and CVaR, as well as a method to calculate these values for a single fuzzy variable. However, due to the interdependence of future returns of securities in the alternative pool, a complete analysis to the aforementioned $\mathbf{E}(x_1 * \xi_1 + \dots + x_n * \xi_n)$, $\mathbf{PT}_r(x_1 * \xi_1 + \dots + x_n * \xi_n)$, $\mathbf{D}(x_1 * \xi_1 + \dots + x_n * \xi_n)$ and $\mathbf{CVaR}_{1-\gamma}(-x_1 * \xi_1 - \dots - x_n * \xi_n)$ is challenging. To overcome this challenge, we employ fuzzy simulation (Liu and Iwamura, 1998; Liu and Liu, 2002) in this study to obtain an approximate value of the expected utility, total utility and CVaR.

5.2. Improved Cooperative Particle Swarm Optimization

The proposed models in this study involve complex nonlinear optimization that cannot be resolved through conventional means or current software. Hakli and Kiran (2020) demonstrated the effectiveness of heuristic algorithms in solving sophisticated optimization problems and obtaining sufficiently accurate solutions. Particle Swarm Optimization (PSO), which can explores the searching space and achieves the best solution by modifying particle speed and position of the particles based on population intelligence, is one such heuristic algorithm. Due to its fast convergence and ease of implementation, PSO has been widely used to solve various optimization problems (Wang et al., 2021; Nayak and Misra, 2020; Tharwat and Schenck, 2021).

Multi-objective optimization problems have become increasingly important in a variety of real-world applications and have garnered significant attention in recent decades. In response to the success of evolutionary computation algorithms in single-objective optimization, numerous researchers have attempted to extend these algorithms to solve multi-objective optimization problems (Hua et al., 2021). However, when applying multi-objective evolutionary algorithms to solve these problems, selecting appropriate individuals for the iteration presents a challenge (Zhan et al., 2013).

To address the issue of fitness assignment in multi-objective evolutionary algorithms, we adopt the objective aggregation approach in this study. This approach involves assigning weights to the multiple objectives to form a single objective, which is then optimized (Parsopoulos and Vrahatis, 2002). The main challenge with this approach is determining the appropriate weights for each objective. As the two objectives of the proposed multi-objective portfolio selection model are inherently correlated, the formed single objective of the model is the summary of the expected utility and total utility.

To address the issue of local convergence in PSO, Wang et al. (2023) proposed an improved cooperative PSO (ICPSO) algorithm that incorporates swarm decomposition and novel evolving strategy. Given the demonstrated feasibility and effectiveness of this algorithm in solving portfolio selection problems, we utilize it in this study to obtain solutions for the proposed models. The solution process of the algorithm can be summarized as follows:

[Step 1]. Initializing particles: Initializing the locations of a swarm containing N particles, and the location $\mathcal{P}_i$ of the ith particle in the virtual K-dimensional space is represented by the $K \times 1$ real-valued matrix:

$$\mathcal{P}_i = [x_{i,1}, x_{i,2}, ..., x_{i,K}]^{\mathrm{T}}, \tag{20}$$

where $\forall k$ $(1 \leq k \leq K)$, $x_{i,k}$ is the kth dimensional coordinates of the ith particle in the K-dimensional search space.

[Step 2]. Adjusting particles: The randomly generated particles may not satisfy the constraints of the specific optimization problem, if there are constraints at all. Therefore, each non-conforming particle is corrected to meet the constraints by the following steps.

[*Step 2.1*]. $i = 1$.

[*Step 2.2*]. Checking whether the ith particle meets the constraints. If the constraints are not fully met, then go to Step 2.3, else go to Step 2.4.

[*Step 2.3*]. The ith particle is non-conforming, the location of this particle is re-initialized and then back to Step 2.2.

[*Step 2.4*]. If $i = N$, these N particles are feasible and go to **Step 3** for subsequent optimization, else $i = i + 1$ and go to Step 2.2.

[Step 3]. Decomposing swarm: The entire swarm is randomly divided into M sub-swarms, where $1 \leq M \leq N$, $M \in \mathbb{Z}$. Each sub-swarm contains N/M particles. Note that if $N\%M \neq 0$, the population size of the first to the $(M-1)$th sub-swarm is $(N - (N\%M))/(M-1)$, the last sub-swarm contains $N\%M$ particles, where % is the modulo calculator.

[Step 4]. Calculating fitness value: Calculating the value of the fitness function for each particle. Then we initialize personal best fitness value $\mathcal{P}value$, personal best location $\mathcal{P}best$, sub-swarm best fitness value $\mathcal{S}value$, sub-swarm best location $\mathcal{S}best$, global best fitness value $\mathcal{G}value$, global best location $\mathcal{G}best$, where $\mathcal{S}value$ and $\mathcal{S}best$ represent the best fitness value and its corresponding location in each sub-swarm.

[Step 5]. Updating particles: Based on the $\mathcal{P}best$, $\mathcal{S}best$ and $\mathcal{G}best$ in **Step 4**, each particle could be updated as following procedures.

If the ith particle is not the one corresponding to the $\mathcal{S}best$, its velocity and location can be updated as:

$$\begin{aligned} & v_{i,k} \leftarrow \tau * v_{i,k} + C_1 * rand(0,1) * (\mathcal{P}best_{i,k} - x_{i,k}) \\ & + C_2 * rand(0,1) * (\mathcal{S}best_{i,k} - x_{i,k}), \\ & x_{i,k} \leftarrow x_{i,k} + v_{i,k}, \end{aligned} \tag{21}$$

where τ is the inertia weight, C_1 and C_2 are the learning weights and $\mathcal{S}best_{i,k}$ is the kth-dimensional coordinate of the best sub-swarm location $\mathcal{S}best$ of which sub-swarm the ith particle is in. Otherwise if the ith particle is the one corresponding to the $\mathcal{S}best$, its velocity and location can be updated as:

$$\begin{aligned} & v_{i,k} \leftarrow \tau * + v_{i,k} + \mathcal{G}best_k, \\ & x_{i,k} \leftarrow v_{i,k} + x_{i,k}. \end{aligned} \tag{22}$$

[Step 6]. Checking feasibility: Each newly generated particle is checked whether the constraints in the optimization problem have been met and regenerate invalid ones using the above procedures in **Step 5**.

[Step 7]. Iterating swarm: Iterating **Steps 4 ∼ 6** for a predefined number of times. $\mathcal{G}value$ obtained after the last iteration is taken as the optimal value to the optimization problem, and its corresponding location $\mathcal{G}best$ is the final answer of the problem.

6. Numerical Example

In this section, one case is given to test the effectiveness of the proposed multi-objective portfolio selection model.

6.1. Control Model

In this section, we present three portfolio selection models that serve as control models to evaluate the potential effectiveness of the proposed multi-objective portfolio selection model.

A) Buy&Hold Portfolio Selection (**B&H**)

B&H is frequently utilized as a benchmark model in portfolio selection (Wang et al., 2022). In this model, the investor does not modify the investment proportion of their held securities. Assuming equal importance of all securities, we can assume that the investment proportion of each security is the same without sacrificing generality. Due to its straightforwardness, **B&H** is commonly employed to compare performance with other models. The mathematical representation of **B&H** is:

$$\begin{cases} \max \mathbf{E}[x_1 * \xi_1 + \ldots + x_n * \xi_n], \\ \text{s.t.} \\ x_1 = \ldots = x_n \geq 0,\ i = 1, \ldots, n, \\ x_1 + \ldots + x_n = 1, \end{cases} \tag{23}$$

in which $(x_i = \ldots = x_n,\ i = 1, \ldots, n)$ ensures that the asset is equally distributed on the securities.

B) Expected Utility-CVaR portfolio Selection (**EU-C**)

Traditional fuzzy portfolio selection models aim to maximize investment returns at a specific level of risk. We propose a fuzzy single-objective portfolio selection model with CVaR to show the potential effectiveness of the mutli-objective framework. The mathematical representation of **EU-C** is:

$$\begin{cases} \max \mathbf{E}[x_1 * \xi_1 + \ldots + x_n * \xi_n], \\ \text{s.t.} \\ \mathbf{CVaR}_{1-\gamma}(-x_1 * \xi_1 - \ldots - x_n * \xi_n) < S, \\ x_i \geq 0,\ i = 1, \ldots, n, \\ x_1 + \ldots + x_n = 1, \end{cases} \tag{24}$$

C) Multi-objective portfolio selection with VaR (**MO-V**)

Wang et al. (2018) proposed a multi-objective portfolio selection model that aims to maximize both Sharpe ratio and VaR ratio simultaneously. We adopt this model to evaluate the potential effectiveness of the proposed multi-objective portfolio selection model. The mathematical representation of **MO-V** is:

$$\begin{cases} \max \mathbf{SR}[x_1 * \xi_1 + \ldots + x_n * \xi_n], \\ \max \mathbf{VR}[x_1 * \xi_1 + \ldots + x_n * \xi_n], \\ \text{s.t.} \\ x_i \geq 0,\ i = 1, \ldots, n, \\ x_1 + \ldots + x_n = 1, \end{cases} \tag{25}$$

in which **SR** and **VR** represent Sharpe ratio and VaR ratio respectively. It should be noted that the VaR ratio is not the same with VaR value, VaR ratio could be calculated by Equ. (26).

$$\mathbf{VR}[x_1 * \xi_1 + \ldots + x_n * \xi_n] = \frac{\mathbf{E}[x_1 * \xi_1 + \ldots + x_n * \xi_n] - x_0}{\mathbf{VaR}_{1-\beta}[x_1 * \xi_1 + \ldots + x_n * \xi_n]}, \tag{26}$$

in which x_0 is the reference return rate.

Table 1

Randomly selected stocks in case study.

Stock No.	Code	Company Name
1	600010	Inner Mongolia Baotou Steel Union Co.,Ltd.
2	600028	China Petroleum & Chemical Corporation
3	600196	Shanghai Fosun Pharmaceutical(Group) Co.,Ltd.
4	600276	Jiangsu Hengrui Pharmaceuticals Co.,Ltd.
5	600406	NARI Technology Co.,Ltd.
6	600436	Zhangzhou Pientzezhuang Pharmaceutical Co.,Ltd.
7	603260	Hoshine Silicon Industry Co.,Ltd.
8	603288	Foshan Haitian Flavouring and Food Company Ltd.
9	603501	Will Semiconductor Co.,Ltd. Shanghai
10	603986	GigaDevice Semiconductor Inc.

Table 2

Return forecast for stocks.

Stock No.	Return	Stock No.	Return
1	(0.95, 0.51)	2	(1.02, 0.77)
3	(0.88, 1.01)	4	(1.08, 1.07)
5	(0.83, 1.05, 1.22)	6	(0.88, 0.93, 1.10)
7	(0.87, 1.02, 1.42)	8	(0.83, 0,94, 1.03, 1.2)
9	(0.88, 1.01, 1.11, 1.41)	10	(0.83, 0.99, 1.03, 1.33)

Table 3

Stock position.

Model \ Stock	1	2	3	4	5	6	7	8	9	10
E&TU	0	0	0.301	0.203	0	0	0	0	0.224	0.272
B&H	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
EU-C	0	0	0	0.654	0.346	0	0	0	0	0
MO-V	0	0	0.178	0.174	0.124	0	0	0.251	0.166	0.107

6.2. Case Study

In this section, one case based on real market data is adopted to compare investment performance with control models.

In this case, 10 stocks were randomly selected from Shanghai Securities 50 Index, as listed in Table 1.

This portfolio selection case spans from January 1st, 2023 to March 31st, 2023, with a total of 90 days, 59 of which are trading days. Based on expert knowledge and observation of public financial reports, we present return forecasts for each candidate stock in the form of fuzzy variables, which are listed in the Table 2. In Table 2, (a, b) represents a Gaussian fuzzy variable, (a, b, c) represents a triangular fuzzy variable, and (a, b, c, d) represents a trapezoidal fuzzy variable.

There are some parameters needed to be assigned first, including the risk tolerance level S, confidence level $1 - \gamma$ and reference point r in Equ. (19). Following the suggestions in (Wang et al., 2018), we set $S = 0.1$ and $1 - \gamma = 0.9$. The reference point r, designated as 1.003, is the status quo.

The performance of each portfolio selection model is determined by its stock position, which is presented in Table 3. While **B&H** is characterized by over-diversification, **EU-C** concentrates its assets on only two stocks, which may increase the risk but also have the potential for higher returns. Compared to the diversification of **B&H** and the concentration of **EU-C**, the transitional state achieved by **MO-V**, which involves investing in six stocks, is the result of maximizing returns (maximizing Sharpe ratio) while minimizing risk (minimizing VaR). And the proposed multi-objective portfolio selection model **E&TU** invests in four stocks, No.3, No.4, No.9 and No.10.

Table 4

Brief summary of selected criteria.

Type	Criterion	Abbr.	Descriptions
Return	Cumulative Wealth	**CW**	The most elementary criterion.
	Annual Yield	**AY**	The average return evaluation.
Risk	Maximum Drawdown	**MD**	Downside risk evaluation.
	Volatility	**Vol**	Evaluating risk comprehensively.
Risk-adjusted return	Sharpe Ratio	**SR**	Evaluates return and risk.
	Calmar Ratio	**CR**	Widely adopted criterion.

Table 5

Criterion values of portfolio selection models.

Model \ Criterion	**CW**	**AY**	**MD**	**Vol**	**SR**	**CR**
E&TU	1.197	0.801	0.127	0.313	15.016	6.307
B&H	1.037	0.147	0.072	0.087	9.118	2.042
EU-C	1.238	0.973	0.167	0.479	14.517	5.826
MO-V	1.151	0.613	0.134	0.351	12.214	4.575

All the aforementioned stock positions will be testified in the real market. The investment performances of these four portfolio selection models in real market will be evaluated under six criteria which listed in Table 4. These six criteria to the four portfolio selection models could be calculated by Equ. (27)- (32) at the end of the investment period.

$$\mathbf{CW} = \sum_{i=1}^{n} x_i^{'} * p_i^{'}, \tag{27}$$

$x_i^{'}$ and $p_i^{'}$ represent the investment propotion and the price of security i at the end of the investment period.

$$\mathbf{AY} = \mathbf{CW}^{n/252} - 1, \tag{28}$$

the average trading years consists of 252 days.

$$\mathbf{MD} = \max_{1\leq i\leq n} \frac{\mathrm{M}_i - \mathbf{CW}_i}{\mathrm{M}_i}, \tag{29}$$

the notations M_i and $\mathbf{CW}_i$ represent the running maximum and cumulative wealth, respectively, up until the ith trading day.

$$\mathbf{Vol} = \sqrt{252} * \sigma, \tag{30}$$

σ is the standard deviation of the portfolio selection model.

$$\mathbf{SR} = \frac{\mathbf{AY} - x_0}{\sigma}, \tag{31}$$

x_0 is the reference return rate.

$$\mathbf{CR} = \frac{\mathbf{AY}}{\mathbf{MD}}. \tag{32}$$

The criterion values of these four portfolio selection models are listed in Table 5.

From the Table 5, it can be seen that the cumulative wealth at the end of the investment period for all models is greater than at the beginning. This is because most of the 10 stocks experienced an increase during the investment period. Due to **B&H**'s over-diversified investment strategy, both its returns and risk levels are minimized. Meanwhile, due to **EU-C**'s over-concentrated investment strategy, both its returns and risk levels are maximized. Compared to these two models, **MO-V** and **E&TU** achieve a transitional state in both return and risk levels, demonstrating the effectiveness of the multi-objective selection framework in balancing returns and risk.

The goal of **E&TU** is to maximize expected and total utility simultaneously. The goal of maximizing expected utility represents the tendency of **E&TU** to chase returns. Due to the presence of loss aversion in PT and disappointment avoiding in DT, the goal of maximizing total utility will drive the portfolio selection model to make some investment decisions that reduce risk. From the perspectives of return, the return chasing ability of **E&TU** is better than **B&H** and **MO-V**. From the perspective of risk, the risk avoiding ability of **E&TU** is better than **EU-C** and **MO-V**. In terms of returns, **E&TU** demonstrates superior return chasing ability compared to **B&H** and **MO-V**. In terms of risk, **E&TU** shows better risk avoidance ability compared to **EU-C** and **MO-V**. Moreover, in terms of risk-adjusted return, **E&TU** shows the best ability of balancing returns and risk. In conclusion, it can be inferred that **E&TU** is capable of generating more consistent profits compared to other control portfolio selection models.

7. Conclusions and Discussions

A novel multi-objective portfolio selection model based on expected utility theory, prospect theory and disappointment theory is proposed in this study. The objective of maximizing expected utility drives the model to chase returns while the objective of maximizing total utility which consists of prospect value and Disappointment Index drives the model to avoid risk.

Three control portfolio selection models are compared with the proposed model. We conduct an real market data experiment to test the effectiveness of the proposed portfolio selection model. The experiment proves the proposed portfolio selection model is able to gain stable profits than other three models under evaluation of return, risk and risk-adjusted return.

As the future trends of security prices embody a multitude of factors, treating the risk/expected-return level as a static value may not align with certain investors' behavior. Consequently, it is necessary to revise the forecasts of security return rates based on the investment outcomes of the previous period and adjust the portfolio accordingly. Thus, incorporating the suggested methodology into multi-period portfolio selection represents a compelling and significant research avenue.

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