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# Stochastic Link Flow Model for Signalized Traffic Networks with Uncertainty in Demand

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Abstract: In order to investigate the stochastic features in urban traffic dynamics, we propose a Stochastic Link Flow Model (SLFM) for signalized traffic networks with demand uncertainties. In the proposed model, the link traffic state is described using four different link state modes, and the probability for each link state mode is determined based on the stochastic link states. The SLFM model is expressed as a finite mixture approximation of the link state probabilities and the dynamic link flow models for all the four link state modes. Using data from microscopic traffic simulator SUMO, we illustrate that the proposed model can provide a reliable estimation of the link traffic states, and as well as good estimations on the link state uncertainties propagating within a signalized traffic network.



*Keywords:* Stochastic traffic model; Traffic signals; Urban traffic network.

## 1. INTRODUCTION

In real-life traffic networks, uncertainties always exist in traffic flow dynamics. However, these uncertainties are seldom explicitly expressed in traffic models, especially for urban traffic networks with traffic signals. In order to better understand the stochastic features of urban traffic networks, this research investigates the randomness in a link flow model under the traffic demand uncertainties.

There are many reasons for demand uncertainties in a traffic network, e.g. weather conditions, public holidays, special events, etc. These demand uncertainties may either vary from time to time within a day as short-term variations, or vary in a week, a month, or a year as long-term variations (Lam et al., 2008; Chen et al., 2011). The demand uncertainty coming from outside of a network is usually considered as an exogenous source of uncertainty (Shao et al., 2006).

There are some research works on dynamic stochastic traffic models, which describe the traffic dynamics as well as the stochastic characteristics in the traffic state evolution (e.g. traffic flow, density). According to the evolution method, the dynamic models can be generally classified into two categories, i.e. state transition model and spatio-temporal dynamic model. For the state transition model, the randomness in the traffic dynamics is modeled directly with certain probability distributions, and the traffic state evolution is described by a Markov Process in which the traffic states are updated with pre-obtained transition probabilities. For instance, in (Yu and Recker, 2006), the arriving traffic flow in a link is described by a Poisson distribution, and the dynamics of traffic evolution is modeled by a Markov decision process, in which the next discrete traffic state depends on the previous traffic state, the control action, and its transition probability. A Markov jump traffic model is also proposed considering the receiving ability of the downstream links in (Tordeux et al., 2014). This kind of stochastic traffic model has discrete traffic states and constant transition probabilities derived from historical real traffic data or simulations. It is heuristic in describing the traffic evolution, for they do not rely on existing traffic dynamic models. For the spatio-temporal dynamic models, the uncertainty modeling is integrated into existing dynamic traffic models, e.g. the LWR model (Lighthill and Whitham, 1955), the CTM model (Daganzo, 1994), and other dynamic traffic models (Papageorgiou, 1995; Yperman, 2007; Lin et al., 2012). There are several research works about stochastic traffic models based on the CTM model. Boel and Mihaylova (2006) proposed a cell-based stochastic model by introducing randomness into the flow speed of each cell. Sumalee et al. (2011) proposed a stochastic CTM model (i.e. SCTM) for freeways by integrating random link Fundamental Diagrams (FDs) into the CTM dynamics to generate a stochastic traffic state evolution, and also extended for signalized traffic networks by Zhong et al. (2013). After that, the SCTM model was applied for journey time estimation and short-term traffic state prediction in (Sumalee et al., 2013; Pan et al., 2013). Jabari and Liu (2012) proposed a stochastic traffic flow model with random vehicle headways depending on the traffic states of successive cells, and the model was shown to be consistent


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with the CTM model. The model was validated in real traffic field by Jabari and Liu (2013). Flötteröd and Osorio (2017) proposed a Stochastic Link Transmission Model (SLTM) to describe the stochastic queuing in LTM. These stochastic traffic models usually have traffic states defined discrete in time, and the transition probabilities for transferring from one time step to the next time step are state dependent, and thus can vary with time. This kind of model integrates the stochastic modeling into the mechanism of the existing dynamic traffic models, and is more focus on the inherent stochastic features in the network supply and the traffic flow propagation.

In this paper, we propose a Stochastic Link Flow Model (SLFM) for signalized traffic networks. The model is a stochastic spatio-temporal dynamic model, and exogenous sources of uncertainty (i.e. demand uncertainty) are considered in the model. The dynamic evolution of the model is realized by iteratively updating the link densities with input and output averaged link flows for the four link state modes. According to the stochastic features of link states, the probability for each link state mode is determined based on the boundary states of the two ends of the link. The link density is expressed as a finite mixture approximation of probabilistic dynamic traffic models for the four link state modes. The model has a comparatively low computational complexity, for its dimensions both in time and space are all reduced. The SLFM can describe the variations of the link density and flow under the influence of traffic signal splits. It provides the possibility of investigating the propagation of the traffic demand and its uncertainty through urban traffic networks.

## 2. LINK FUNDAMENTAL DIAGRAM WITH TRAFFIC SIGNALS

Fundamental Diagram (FD) provides the relation between flow $q$ and density $\rho$ on a link. In the model, we suppose that the Fundamental Diagram (FD) on each link has a theoretical triangular shape. However, in signalized traffic networks, the capacity of the FD of a link will be restricted by the signal timings due to the stop-and-go of the flow. Thus, the triangular link FD is chopped at the capacity because of the traffic signal restrictions on the intersection, and turns into a trapezoid shape with a restricted capacity (Wu et al., 2011; Lo, 1999, 2001), as shown in Fig. 1. In this paper, we define a one-way link with signals at the downstream end as a subsystem, and we assume that the link in the subsystem is long enough such that it has the normal triangular FD at the upstream boundary and the constrained trapezoidal FD at the downstream boundary.

The capacity is restricted by the fraction of the green time over the cycle time on the link:

$$Q_{\mathrm{M}}^{'} = \frac{Q_{\mathrm{M}} \cdot g}{c}, \tag{1}$$

where $Q_{\mathrm{M}}$ and $Q_{\mathrm{M}}^{'}$ are the capacity and the restricted capacity of the FD, $g$ is the effective green time on the link, and $c$ is the cycle time of the downstream intersection connecting to the link.

As Fig. 1 shows, $\rho_{\mathrm{c}}$ is the critical density, $\rho_{\mathrm{cd}}$ and $\rho_{\mathrm{cu}}$ are the lower bound and the upper bound critical densities for the restricted capacity; $\rho_{\mathrm{J}}$ is the junction density, $v_{\mathrm{f}}$ is the free-flow speed, and $w_{\mathrm{c}}$ is the spillback speed.

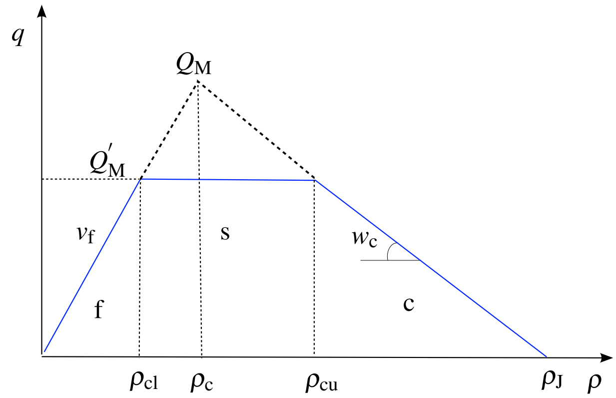


Figure 1. Illustration for the traffic light constrained link FD

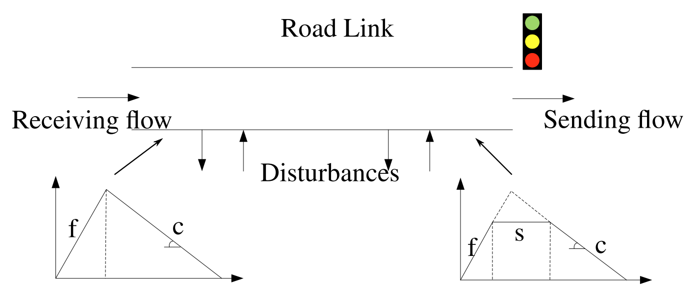


Figure 2. Signalized urban road link with upstream and downstream FDs

## 3. STOCHASTIC LINK FLOW MODEL (SLFM) WITH TRAFFIC SIGNALS

### 3.1 Link state modes and probabilities

A link and the FDs on its two boundaries are shown in Fig. 2. The normal triangular FD at the entry of the link is separated into two linear states, i.e. free flow (f) on left-hand side and congestion (c) on the right-hand side; the constrained trapezoidal FD in front of the traffic signals is divided into three linear states, i.e. free flow (f), congestion (c), and saturation (s) at the restricted capacity.

For each subsystem, we can define 4 link state modes according to the two boundary states at the ends of the link (i.e. at the entry of the link and at the stop line). As shown in Fig. 3, the link state mode of a subsystem can be classified into

* *Free-flow mode* (F): free-flow state at the upstream boundary of a link and free-flow state at the downstream boundary of the link;
* *Accumulating mode* (A): free-flow state at the upstream boundary of a link and saturation state at the downstream boundary of the link;
* *Congestion mode* (C): congestion state at the upstream boundary of a link and congestion state at the downstream boundary of the link;
* *Dissipating mode* (D): congestion state at the upstream boundary of a link and saturation state at the downstream boundary of the link.

Suppose a link is in Free-flow mode at the beginning. When the traffic demand increases, the traffic will gradually accumulate and the traffic state of the link will switch from Free-flow mode to Accumulating mode, in which the entering flow is still free flow but the leaving flow is saturated. If the demand

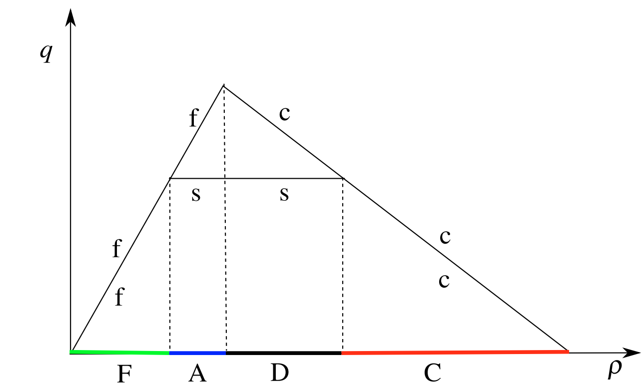

Figure 3. Illustration for the link state modes

keeps on increasing, the traffic state of the link will switch from Accumulating mode to Congestion mode, in which the traffic congestion is built up and propagates to the upstream link. When the traffic demand reduces, the traffic state of the link will switch from Congestion mode to Dissipating mode, in which the congested vehicles will discharge gradually from the link with saturation flow. The traffic state of the link switches back to Free-flow mode when the discharging flow reduces to a value below the restricted capacity (i.e. saturation flow). Similar state modes was defined for CTM freeway model in (Muñoz et al., 2006).

Since the traffic demands and the traffic disturbances are stochastic, the traffic state of a link is also stochastic. The 4 link state modes are defined as a mode set $M = \{\mathrm{F}, \mathrm{A}, \mathrm{C}, \mathrm{D}\}$, and the probability of the link state in each mode is defined as $P_m$ $(m \in M)$. So we have

$$\sum_{m \in M} P_m = 1. \tag{2}$$

Assume that the density on a link follows the Normal distribution as $\rho(k) \sim N(\mu(k), \sigma(k))$, then the probabilities of all the link state modes can be written as

$$P_{\mathrm{F}} = Pr\{0 \le \rho(k) < \rho_{\mathrm{cl}}\} \tag{3}$$
$$P_{\mathrm{A}} = Pr\{\rho_{\mathrm{cl}} \le \rho(k) \le \rho_{\mathrm{c}}\} \tag{4}$$
$$P_{\mathrm{C}} = Pr\{\rho_{\mathrm{cu}} < \rho(k) \le \rho_{\mathrm{J}}\} \tag{5}$$
$$P_{\mathrm{D}} = Pr\{\rho_{\mathrm{c}} < \rho(k) \le \rho_{\mathrm{cu}}\}. \tag{6}$$

Then, the link density can be expressed as a linear combination of the densities in the 4 link state modes:

$$f(\rho(k)) = \sum_{m \in M} P_m(k) f_m(\rho(k)), \tag{7}$$

where $f(\cdot)$ is the probability density function (PDF) of the link traffic density, $f_m(\cdot)$ is the PDF of the link traffic density in mode $m$, $P_m(k)$ is the probability of link state mode $m \in M$ at time step $k$, and $\rho(k)$ is the density of a link at time step $k$. Consequently, the expectation of the link density is

$$\mathbb{E}(\rho(k)) = \sum_{m \in M} P_m(k) \mathbb{E}(\rho_m(k)). \tag{8}$$

Define $\mu_m(k) = \mathbb{E}(\rho_m(k))$ and $\mu(k) = \mathbb{E}(\rho(k))$. Then the covariance can be calculated as

$$\mathrm{Var}(\rho(k)) = \sum_{m \in M} P_m(k) \Omega_m(k) - (\mu(k))^2, \tag{9}$$

where $\Omega_m(k) = \mathbb{E}((\rho_m(k))^2)$ is the autocorrelation function of $\rho_m(k)$. Thus, the standard deviation of the joint link density is $\sigma(k) = \sqrt{\mathrm{Var}(\rho(k))}$. For more details, one can refer to Sumalee et al. (2011).

### 3.2 Link models for different link state modes

For each of the link state mode in $M$, a time-variant model is formulated to update the traffic state on the link based on the averaged entering and leaving flows. Moreover, we also take into account of the influence of the traffic signals. Averaged CTM model is also proposed by Grandinetti et al. (2015). Therefore, the dynamic evolution of the traffic density at time step $k$ on a link is

$$\rho(k+1) = A\rho(k) + B_0\rho(k)U(k) + B_1U(k) + Dd(k) + C, \tag{10}$$

$$U(k) = \beta(k)r(k), \tag{11}$$

where $A$, $B_0$, $B_1$, $D$, and $C$ are constant parameters, $U(k)$ is the scalar control input at time step $k$ which is assumed to be a deterministic value, $\beta(k)$ is the vector of turning ratios in front of the traffic signals on a link at time step $k$, $r(k)$ is the vector of green time splits on a link at time step $k$, $d(k)$ contains the stochastic disturbances from outside of the link (i.e. demands from upstream links, supply from the downstream links, and input/output disturbances on the link).

The disturbance vector $d(k)$ is the vector containing the disturbances from the input flow $q_{\mathrm{E}}(k)$ (i.e. entering flow) of the link, which is a sum of the leaving flows from the upstream links, the output flow $q_{\mathrm{A}}(k)$ (i.e. accepted flow) of the link, which is decided by the available receiving flows of the downstream links, and the disturbance flows that getting in and out along the link (i.e. $d_i(k)$ and $d_o(k)$). Thus, we have $d(k) = [q_{\mathrm{E}}(k)\ d_{\mathrm{i}}(k)\ d_{\mathrm{o}}(k)\ q_{\mathrm{A}}(k)]^{\mathrm{T}}$.

If we define the simulation time step as the cycle time, i.e. $T_s = c$, then the control input can be written as

$$U(k) = \beta_{\mathrm{th}}(k)r_{\mathrm{th}}(k) + \beta_{\mathrm{l}}(k)r_{\mathrm{l}}(k), \tag{12}$$

in which $\beta_{\mathrm{th}}(k)$ and $\beta_{\mathrm{l}}(k)$ are the through and left turning ratios of the flow in the link (see Fig. 4 and 5), $r_{\mathrm{th}}(k)$ and $r_{\mathrm{l}}(k)$ are the green time splits for the flows going straight and turning left, e.g. the throughput green time split is defined as $r_{\mathrm{th}}(k) = g_{\mathrm{th}}(k)/c$. If the set of phases in one cycle time is $P$, then the green time splits of the cycle time satisfy

$$\sum_{p \in P} r_p(k) = 1. \tag{13}$$

In this context, the dynamic model can be further written explicitly for all 4 modes under the mode probabilities.

For the link state mode F, the upstream and downstream of the link are all having free flows, thus the dynamic model for updating the density becomes

$$\rho(k+1) = A\rho(k) + B_0\rho(k)U(k) + Dd(k), \tag{14}$$

where $A = 1 + \beta_{\mathrm{r}}B_0$, $B_0 = -\frac{T_{\mathrm{s}}}{l}v_{\mathrm{f}}$, $D = [\frac{T_{\mathrm{s}}}{l}\ 0\ 0\ 0]$, and $l$ is the length of the link.

For the link state mode A, the upstream link has free flow, but the downstream link is saturated, thus the dynamic model for updating the density becomes

$$\rho(k+1) = A\rho(k) + B_1U(k) + Dd(k) + C, \tag{15}$$

where $A = 1$, $B_1 = -\frac{T_{\mathrm{s}}}{l}Q_{\mathrm{M}}$, $D = [\frac{T_{\mathrm{s}}}{l}\ 0\ 0\ 0]$, and $C = \beta_{\mathrm{r}}B_1$.

For the link state mode C, the upstream and the downstream link are all congested, thus the dynamic model for updating the density becomes

$$\rho(k+1) = A\rho(k) + Dd(k) + C, \tag{16}$$

where $A = 1 + \frac{T_{\mathrm{s}}}{l}w_{\mathrm{c}}$, $D = [0\ 0\ 0\ -\frac{T_{\mathrm{s}}}{l}]$, and $C = -\frac{T_{\mathrm{s}}}{l}w_{\mathrm{c}}\rho_{\mathrm{J}}$.

For the link state mode D, the upstream link is congested, but the downstream link becomes saturated, thus the dynamic model for updating the density becomes

$$\rho(k+1) = A\rho(k) + B_1 U(k) + Dd(k) + C, \tag{17}$$

where $A = 1 + \frac{T_s}{l} w_c$, $B_1 = -\frac{T_s}{l} Q_M$, and $C = -\frac{T_s}{l} w_c \rho_J + \beta_r B_1$.

### 3.3 Link mean density and auto-correlation

Let $\mu(k) = \mathbb{E}(\rho(k))$. Then according to the evolution formula for link density in eq. (10), the mean of the link density can be iteratively calculated as

$$\mu(k+1) = (A + B_0 U(k))\mu(k) + B_1 U(k) + DE(d(k)) + C, \tag{18}$$

where how much the previous link mean density will affect the current link mean density is influenced by the previous control signal input.

Let the auto-correlation of link density be $\Omega(k) = \mathbb{E}(\rho^2(k))$, we can further calculate the iterative equation for the auto-correlation as follows:

$$\begin{aligned}\Omega(k+1) =& F_1(k)\Omega(k) + F_0(k)\mu(k) \\ &+ G(k)E(d(k)) + E(d^{\mathrm{T}}(k)D^{\mathrm{T}}(k)Dd(k)) \\ &+ H(k),\end{aligned} \tag{19}$$

where

$$F_1(k) = A^2 + B_0^2 U^2(k) + 2AB_0 U(k), \tag{20}$$

$$\begin{aligned}F_0(k) =& 2B_0 B_1 U^2(k) + 2(B_0 DE(d(k)) + AB_1 + B_0 C)U(k) \\ &+ 2ADE(d(k)) + 2AC,\end{aligned} \tag{21}$$

$$G(k) = 2B_1 DU(k) + 2CD, \tag{22}$$

$$H(k) = B_1^2 U^2(k) + 2B_1 CU(k) + C^2. \tag{23}$$

Therefore, the mean and the auto-correlation of the link density evolve iteratively with time based on the previous traffic states both inside the link and outside the link. Consequently, the demand uncertainties can propagate along the travel routes and also with time, but the propagating speed may vary according to the green time splits.

### 3.4 Link leaving and receiving flows

The leaving flow of a link is the estimated linear mixture of the leaving flows in different link modes, and the receiving flow of a link is the estimated linear mixture of the receiving flows that can be accepted by the link in all the link modes. The leaving and receiving flows of the link are parts of the input and output flows of its neighbor links. Therefore, the receiving flow and the leaving flow of a link need to be calculated.

The leaving flow of a link can be written as a linear combination of the leaving flows in all the link modes as

$$q(k) = P_F \rho(k) v_f U(k) + (P_A + P_D) Q_M U(k) + P_C A(k), \tag{24}$$

which means that the leaving flow of a link depends on the free flow, the saturation flow (the restricted capacity), and the accepting flow of the downstream links.

The receiving flow of a link is a linear combination of the receiving flows in all link modes as

$$q_R(k) = (P_A + P_F) Q_M + (P_C + P_D) w_c (\rho(k) - \rho_J), \tag{25}$$

which means that the receiving flow of a link depends on the capacity and the congestion flow of the link.

### 3.5 Entering and accepted link flows

In some of the link model modes, we need to know the entering flow and the accepted flow of the link. In F and A modes, the entering flow is provided by the upstream links, because the link has enough space to receive the flow due to the free-flow state in upstream of the link; in C mode, the accepted flow depends on the available space of the downstream links, because the congestion propagates upwards along the links.

Let the output link set of link $i$ be $O_i$ with downstream links as the elements, the input link set of link $i$ be $I_i$ with upstream links as the elements.

Then, the entering flow of link $i$ can be written as the sum of all the flows from upstream links as

$$q_{E,i}(k) = \sum_{u \in I_i} \beta_{u,i} q_u(k), \tag{26}$$

where $q_{E,i}(k)$ is the entering flow of link $i$ at time step $k$, $\beta_{u,i}$ is the turning ratio that the flow turning from link $u$ to link $i$, and $q_u(k)$ is the leaving flow of link $u$ at time step $k$.

In the C mode, due to the traffic congestion in downstream link, the accepted flow depends on the available space of the downstream link. However, this may not be the case for all the downstream links, some of the downstream links of link $i$ maybe not congested. In such case, for each downstream link $d$, the accepted traffic flow can be separated into two situations with probabilities as

$$P_{1,d}(k) = Pr\{q_{R,d}(k) \le Q_{M,i} \beta_{i,d} r_{i,d}(k)\}, \tag{27}$$

$$P_{2,d}(k) = Pr\{q_{R,d}(k) > Q_{M,i} \beta_{i,d} r_{i,d}(k)\}, \tag{28}$$

where $q_{R,d}(k)$ is the receiving flow of link $d$ at time step $k$, $Q_{M,i}$ is the capacity flow of link $i$, and $r_{i,d}(k)$ is the green time split for the flow turning from link $i$ to link $d$ at time step $k$. $P_{1,d}(k)$ represents the probability that the receiving flow of link $d$ is less equal than the separated and restricted capacity flow of link $i$ at time step $k$; $P_{2,d}(k)$ represents the probability that the receiving flow of link $d$ is larger than the separated and restricted capacity flow of link $i$ at time step $k$, i.e. downstream link $d$ does not contribute to the congestion in link $i$.

Therefore, the accepted flow of link $i$ by the downstream links can be calculated as

$$q_{A,i}(k) = \sum_{d \in O_i} [P_{1,d}(k) q_{R,d}(k) + P_{2,d}(k) Q_{M,i} \beta_{i,d} r_{i,d}(k)]. \tag{29}$$

The entering flow and accepted flow are used in the link models of different modes as the input and output flows provided and accepted by the upstream and downstream links, and then the link states are updated with these flows at the next time step for all the link state modes.

## 4. SIMULATIONS

To test the SLFM, we use the microscopic traffic simulator SUMO to simulate a real traffic environment, which provides the stochastic traffic demands, as well as the stochastic traffic densities and the traffic flows. We give the SLFM the traffic demands generated by SUMO, and compare the estimated link densities and the link leaving flows from the SLFM with the real values generated by SUMO.

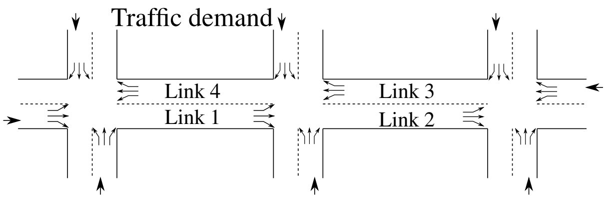


Figure 4. The urban network used for the case study

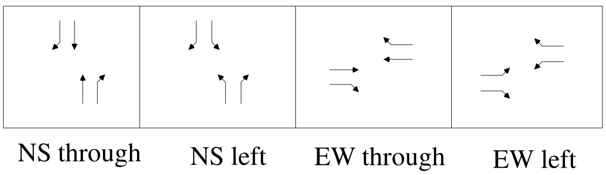


Figure 5. The phases for traffic signals

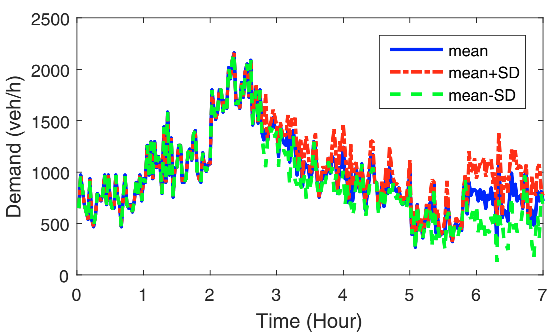


Figure 6. The stochastic traffic demand

### 4.1 Network and traffic signal setup

We consider an urban network with 3 signalized intersections, 4 links inside the network (see Fig. 4). The traffic demands are provided to the network in all the 8 entry links. There are 4 traffic signal phases, North-South through, North-South left, East-West through, and East-West left, as in Fig. 5.

Stochastic traffic demands are generated for all the entry links of the network for 7 hours, and a traffic flow peak is simulated during the time. The mean and standard deviation of the traffic demand is shown in Fig. 6.

### 4.2 Results

The comparison is given for the densities between the SLFM and the real values from SUMO for link 1, 2, 3 and 4 in Fig. 7. The results show that the estimated mean link densities from SLFM match well with the link densities from SUMO, and link density peaks emerge on all the links due to the existence of the peaks in traffic demands. The estimated standard deviations of the link density from SLFM increase when the links are approaching the congested state (i.e. when the link densities grow high), which verifies that the higher the mean link density is, the higher the uncertainty of the link density will be. In addition, the estimated standard deviations of links 1 and 3 are higher than those of links 2 and 4. By comparing the variations of probabilities for all link state modes in Fig. 8, we can see that links 1 and 3 are more congested than links 2 and 4 during the peak, where links 1 and 3 are more in D mode, but links 2 and 4 are more in A mode. If a link is more congested, based on the previous derivation of the SLFM, the standard deviation

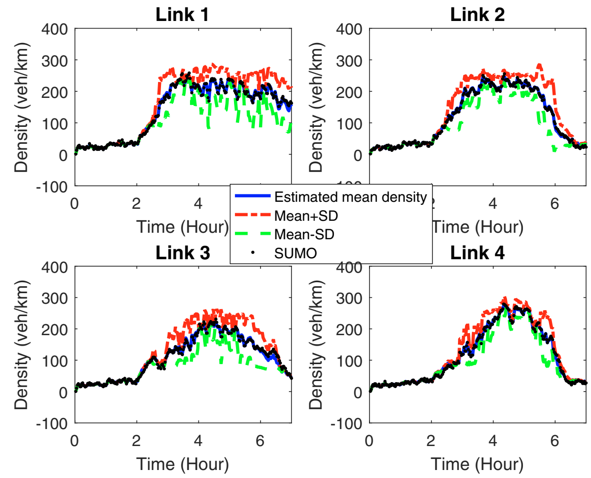


Figure 7. The estimated link densities vs. the SUMO real link densities

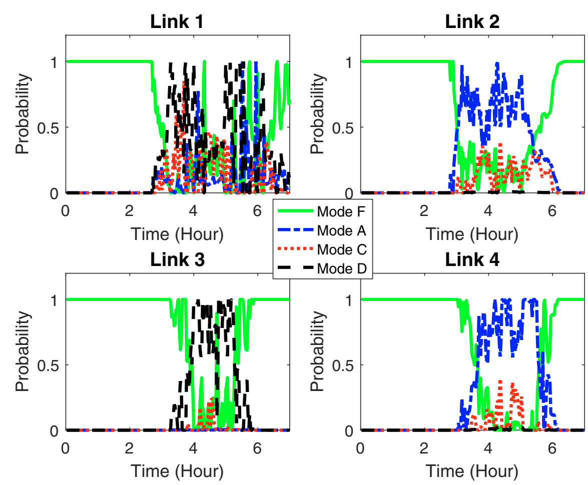


Figure 8. The variations of probabilities for all link state modes

of the link density is influenced more by the high mean and auto-correlation of the link in previous time step. This means that congestion is created in links 1 and 3 due to the increase of the entering flows, and congestion will increase the uncertainty in the link density estimation. Uncertainties in traffic demands can propagate through links in the traffic network; however congestion is the main cause of the stochastic disturbances in link traffic flows, and thus causes more uncertainties.

As Fig. 8 shows, the probabilities for the 4 link state modes oscillate a lot due to the peak of the traffic demands. This means that traffic congestion under high traffic demands will make the SLFM model have strong randomness in determining the link state modes.

The simulation results show that the SLFM model could be used to investigate the propagation of urban traffic demand and its uncertainties, or be used in traffic signal optimizations for urban traffic networks with demand variations.

## 5. CONCLUSIONS

In order to investigate the stochastic features in urban traffic dynamics, we have proposed a Stochastic Link Flow Model

(SLFM) for signalized traffic networks. In the proposed model, the link traffic state is defined into four different link state modes, and the probability for each mode was derived on the basis of the stochastic link states. The SLFM model was presented as a finite mixture approximation of the link state probabilities and the dynamic link flow models for all the four link state modes.

In the numerical example, we compared the estimated link traffic states obtained using the SLFM against the link traffic states generated from SUMO. It was found that the estimated link states were close to the simulation results from SUMO. It was also observed that the mean link states simulated the peaks similar to the one of the real traffic demands. In addition, the link state uncertainties could also be estimated taking account the uncertainty in traffic demands propagating into the urban network along the links. Note that the highest uncertainty was due to the congestion state when the link traffic density was comparatively high. It was shown that the SLFM model could be used to investigate the propagation of urban traffic demand and its uncertainties, or used in traffic signal optimizations for urban traffic networks with demand variations.

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