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Real Options Model of Toll-Adjustment Mechanism in Concession Contracts of Toll Road Projects

Qing Chen¹; Geoffrey Shen²; Fan Xue³; and Bo Xia⁴

Abstract

The toll-adjustment mechanism (TAM) is a hybrid of a price cap regulation mechanism and a revenue sharing mechanism. It is one solution to saving private investors from severe traffic demand risk and the government from heavy fiscal burden, while ensuring the private investor a reasonable but not excessive rate of return in a public-private partnership (PPP) concession contract. This research models TAM as a real option to assess the value of flexibility of the right (but not obligation) to toll adjustments. A hypothetical case study derived from a real-life project (the Western Harbour Crossing in Hong Kong) is illustrated in detail to demonstrate the application of the framework developed and to validate the effectiveness and robustness of the framework. Outcomes of the research can help the government to design reasonable concession contracts and help the private investors to make sound investment decisions through effective management of the traffic demand risk. Therefore, a win-win prospect can be achieved in PPP concession contracts for both parties.

Introduction

Facing increasing difficulties in funding public facilities and utilities, governments around the world resort to the private sector for a large amount of funds, resources, and expertise. One way of achieving this objective is through a public-private partnership (PPP), which facilitates private finance, especially in public infrastructure development, for improved quality, efficiency, and costeffectiveness (Zheng and Tiong 2010). In the transportation sector, PPP toll roads, normally in the build-operate-transfer (BOT) scheme (a form of project financing in which the concessionaire receives a concession from the concessioner to finance, design, construct, and operate a facility stated in the concession contract, after which the facility will be handed back to the

concessioner), are gaining popularity throughout the world (Chung et al. 2010). This is in response to the increasing demand for transport infrastructure.

Meanwhile, PPP projects are characterized by a huge sunk cost, a high level of uncertainties and risks of various sorts, long-term financing agreements, and a nonrecourse or limited-recourse project financing scheme (Debande 2002; Zhang 2005b). Demand risk, defined as the inability to determine the behavior of real traffic

movement compared with forecasted traffic, plays a particularly significant role in PPP toll road projects (Estache et al. 2000). To attract private investors into the highly uncertain, capital-intensive toll road projects, the host government initiated multiple policies and schemes to alleviate the demand risks, usually through extensive guarantees (e.g., minimum traffic guarantees and minimum revenue guarantees) against a variety of risks, politically and economically, in assorted forms. Nevertheless, poorly designed guarantee/support types of governmental schemes (in essence contingent liabilities) could induce substantial fiscal burdens to the host government and taxpayers, which can counterintuitively diminish and even eliminate the advantage of applying PPP to infrastructure projects. According to Irwin (2003), to put in place sound policies that generally reduce risks and increase expected returns is better than issuing guarantees to attract private investors.

The toll-adjustment mechanism (TAM) is such a solution to the demand risk problem yet it induces no fiscal burden to the host government. In PPP toll road contracts, the concessionaires' main sources of revenue are the tolls that they charge users for the entire length of the concession. Given the high level of uncertainties involved in the long-term contract (usually in decades), errors of forecasted traffic are inevitable, and they sometimes can be significant. To address this demand risk, TAM gives concessionaires the right, not obligation, to adjust tolls (when meeting certain

conditions agreed between parties in concession contracts) to achieve reasonable profits. Nevertheless, the stakes involved in TAM are also huge because any change in the toll will change end-users' behaviors (demand) and lead to the change of cash waterfalls, such as revenue, operating income, and profit (Ye and Tiong 2003). Given the unique and distinct characteristics of the TAM, it is of great importance and urgency to quantitatively assess the value of flexibility of the right (but not obligation) of toll adjustments. However, to the best of the authors' knowledge, it is seldom studied in the academic field.

To bridge the research gap, this study modeled TAM as real options (the right to charge any toll below the price cap) to assess the value of toll adjustments, which will provide practitioners an effective tool to analyze and manage such arrangements in the contract. The basic features of TAM arrangement and the rationale of applying real options for the TAM value assessment will be introduced first. After that, a theoretical real options model will be developed, followed by a hypothetical case study derived from a real-life project (Western Harbour Crossing in Hong Kong) to demonstrate the application of the real options model developed and to validate the effectiveness and robustness of the model.

TAM

Research on TAM is relatively scarce. Of the few papers published, most just introduced it as one means of incentive among others, such as guarantees and subsidies, with plaintive descriptions quoted from the concession contracts of how the mechanism operates without going in depth and in detail (Kumaraswamy and Zhang 2001; Tam 1999; Zhang 2005a, c). The remaining publications went further to make tentative quantitative analyses (L. Athias and S. Saussier,

“Contractual design of toll adjustment processes in infrastructure concession contracts,” working paper, ATOM-U. of Paris, Paris; [Loo 2003](#); [Ye and Tiong 2003](#)).

In a BOT toll road project, the concessionaire is granted the right to finance, build, operate, and maintain the toll road; the revenues collected will be used to recoup the loans and investment costs. The concession period is N years. At the end of the concession period, the project will be transferred back to the government at no cost. The contract contains a TAM, which is characterized by typical features as follows.

Minimum Revenues

There are minimum levels of revenues specified explicitly for each year in the concession contract. If in any year the actual revenue falls short of the minimum level, the project company will be entitled to the right to adjust the toll according to the maximum toll increment level stipulated in the contract.

Revenue Caps and Sharing

There are maximum levels of revenues specified explicitly for each year in the concession contract. If in any year the actual revenue exceeds the maximum level, the revenue in excess of the maximum level will go to the government; for the project company, the revenue will be capped at the level of the maximum revenue.

Price Caps

In each year there is a price cap, specifying the maximum level of toll the project company may charge. If in any year a toll adjustment is possible, the price cap will be updated accordingly.

Toll Adjustment

Toll adjustment can only be made when the actual revenue is less than the minimum level stipulated, and the extent of adjustment cannot exceed the maximum level specified in the contract.

Methodology

Real Options Theory

The development of a general and flexible method for valuing the TAM is underpinned by real options theory. Amram and Kulatilaka (1999) defined a real option as the right, not the obligation, to make some strategic decisions depending on the uncertainty level of the conditions. Real options theory is derived from the traditional option pricing theory, which tries to value financial options (Black and Scholes 1973; Cox et al. 1979; Bookstaber and Clarke 1983). It helps to determine the alternative actions for the uncertain future, when to apply these actions, and the prices of choosing these actions. Unlike making all strategic decisions at the early project stage, using flexible strategies and delaying decisions considered as real options reduce project risks and increase project value.

Recently, the real option concept is also gaining recognition in the field of construction engineering and management. This is primarily attributed to the multiple flexibilities that are often embedded or intentionally structured within the various stages during the lifecycle of a complex infrastructure project (Huang and Chou 2005; Iyer and Sagheer 2011; Doan and Menyah 2012). For example, Yiu and Tam (2006) proposed a real options model and analyzed a real-life construction project tender to examine how underpricing in tendering provides real options value (ROV). Liu and Cheah (2009) applied the real options approach in a PPP/private finance initiative (PFI) project negotiation. Of particular note is that since the 1990s, real options analyses have been

frequently used in PPP projects for the evaluation of government guarantees (Alleman and Rappoport 2002; Cheah and Liu 2006; Chiara et al. 2007; Ashuri et al. 2010, 2012; Kim et al. 2012). In a guarantee arrangement, if a project underperforms in a particular year, the investor has the option to demand that the government reimburse the shortfall, up to a preestablished level of guarantee. Because of these characteristics, the valuation of these guarantees requires the use of real options analysis (Dixit and Pindyck 1994; Trigeorgis 2005; Jun 2010). Cui et al. (2004) developed a real options model to value warranties in highway projects, describing the mechanics of the warranty option and its advantages over the conventional warranty.

A toll-adjustment provision and an option are similar in that they can provide a downside protection to their holders. One of the major problems with TAM is that it is difficult to value. The real options method has demonstrated to be an effective approach for value assessing, and represents an important step toward improving risk mitigation and facilitating contractual and financial negotiations (Chiara et al. 2007).

However, due to the unique and distinct characteristics of the TAM, which are different from guarantees and financial options, simple application, or naive adaptation of real options analysis, such as in the case of real options analysis of guarantees in concession contracts, it is no longer satisfactory or adequate within this setting. Thus, a significant portion of the research was devoted to developing a new theoretical real options model for the TAM.

Once the model is set up, the TAM will be analyzed and the ROV calculated. The technical analysis was performed by developing a decision tree model and determining the optimal path by the branch-and-bound method.

A hypothetical case study based on a real-life project (the Western Harbour Crossing in Hong Kong) is performed (1) to demonstrate the application of the model developed and (2) to validate

and verify the model. Reasonable and appropriate simplifications were made to modify the real-life project into the hypothetical case study, to reduce the computational complexity, and to emphasize the essence of the problem.

Real Options Model

A population of Q commuters traveling from City A to City B faces two options: one route is a toll road that charges each user a toll of P [unit: dollars per trip (dpt)] for traveling on it and the other is a free road (Fig. 1). The Q_T^{\max} and Q_F^{\max} are, respectively, the maximum traffic capacity of the toll road and that of the free road [unit:

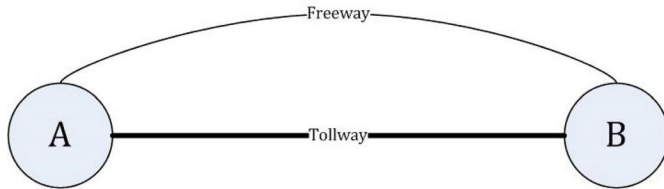


Fig.1. Model setup

vehicles per day (vpd)]; Q_T and Q_F are, respectively, the traffic volume commuters on the toll road and that on the free road (unit: vehicles per day); t_T^0 and t_F^0 are, respectively, the free-flow (no congestion) travel time per vehicle on the toll road and that on the free road (unit: minutes); t_T and t_F are, respectively, the actual (with congestion) travel time per vehicle on the toll road and that on the free road (unit: minutes); t_T is an increasing and nonconcave function (i.e., convex or linear function, in which the line segment between any two points on the graph of the function lies above or on the graph; equivalently, if the function is twice differentiable, and the second derivative is always greater than or equal to zero for its entire domain) of Q_T , where both t_T and t_T^0 are continuous (which means sufficiently small changes in the input result in arbitrarily small changes

in the output); similarly, t_F is an increasing and nonconcave function of Q_F , where both t_F and t_F^0 are continuous; and t_{CT} and t_{CF} are, respectively, the travel cost per vehicle usage of the toll road and that of the free road (dollars).

Each commuter has a unique value of time (VOT) [unit: dollars per minute (dpm)]. Let $f(x)$ and $F(x)$ denote, respectively, the probability distribution function (PDF) and cumulative distribution function (CDF) of VOT for the entire population of commuters, and $x_0 = VOT$ of the commuter who is indifferent when choosing the toll road or the free road, that is, travel cost for the commuter is the same for the two routes. For the commuter whose VOT is x_0

$$t_{CT} \leq t_{CF}; \text{ that is; } P \leq t_{TX_0} \leq t_{FX_0} \quad (1)$$

The following is assumed:

1. All commuters are completely rational decision makers, seeking to minimize their traveling costs.
2. Each commuter knows his or her own unique VOT as well as the distribution of VOT for the entire population of commuters.
3. The same toll is charged for all the vehicles regardless of the vehicle types.

For any commuter whose VOT is less than x_0 , he or she will definitely travel on the free road; for any commuter whose VOT is larger than x_0 , he or she will definitely travel on the toll road.

Therefore

$$\int_0^{x_0} Q_F \cdot f(x) \cdot dx + \int_{x_0}^{\infty} Q_T \cdot f(x) \cdot dx = Q \quad (2)$$

that is

$$Q_F - \frac{1}{4} F x \delta 0 P - F \delta P 0 \frac{1}{4} F x \delta 0 P \quad (3) Q$$

in which

P

$$x 0 \frac{1}{4} \quad \text{---} > 0 \quad (4) t_F \quad t_T$$

From the equation

$$Q_F \quad P$$

$$\text{---} \frac{1}{4} F x \delta 0 P \frac{1}{4} F \text{---} \delta \quad P \quad \delta \quad P \quad (5)$$

$$Q \quad t_F Q_F \quad t_T Q \quad Q_F$$

the number of commuters who use the free road given the distribution of VOT and toll P can be determined. Thus, the number of commuters who use the toll road can be determined.

Here the authors adopt the link (arc) congestion (or volume delay, or link performance) function developed by the U.S. Bureau of Public Roads (BPR), which will be termed as $S_a \delta v_a P$

$$S_a \delta v_a P \quad " \quad \frac{1}{4} t_a 1 p 0:15 a \# \quad (6) v \quad 4 c_a$$

in which t_a = free-flow travel time on link a; v_a = volume of traffic on link a per unit of time (somewhat more accurately: flow attempting to use link a); c_a = capacity of link a per unit of time; and $S_a \delta v_a P$ = average travel time for a vehicle on link a.

In the BOT contract, the concessionaire is entitled to operate the project for N years, that is, the contract duration is N (excluding the construction duration), and at the end of the operational period the project will be transferred back to the government at no cost. At $t = 0$, that is, the beginning of the first operational period, the government set the price cap for this period as P_{OTAM} ,

that is, the price charged by the concessionaire cannot exceed this level. The discount factor is $\beta_{TAM} = 1/(1+r_{TAM})$, in which r_{TAM} is the discount rate of the concessionaire for the project. R_t^{min} and R_t^{max} are, respectively, the minimum revenue and maximum revenue (revenue cap) stipulated in the TAM: if in any year i the actual revenue of the concessionaire is below R_t^{min} , the price cap for the next year (and thus the following years) will be allowed to be raised by DP_i , and let P_{iTAM}^c denote the price cap for period i . In each period, the demand function is modeled as $Q_t = Q_t^0 - \delta P_{iTAM}$, in which P_{iTAM} is the price actually charged by the concessionaire, and the demand function is decreasing and nonconcave and price cannot be negative. For the convenience of illustration, let R_t^{opt} denote the maximum revenue, given $Q_t = Q_t^0 - \delta P_{iTAM}$, which can be attained at the price level P_t^{opt} . Attention should be paid so that P_t^{opt} may exceed P_{iTAM}^c ; therefore, neither R_t^{opt} nor P_t^{opt} is attainable for the concessionaire. In this case the project can only seek to maximize its revenue under the according price cap in the period. Q_{max} is the maximum supply capacity of the project, and in the case of a toll road, it is the maximum daily volume of vehicles it can handle. During the entire contract duration period, the maximum toll that the concessionaire may charge cannot exceed P_{max} .

The ultimate objective of the company is to maximize the net present value (NPV) of the project, so the objective function is

N

$$\max NPV_{TAM} = \max \sum_{t=1}^N \beta_{TAM}^t (R_t - P_t) \quad (7)$$

$$\beta_{TAM} = 1/(1+r_{TAM})$$

which indicates that to maximize the NPV one must seek an optimal series of pricing strategies for each period, $f_{tTAM}^{N_1} = f_{P_{1TAM}; P_{2TAM}; \dots; P_{NTAM}} g$, given b_{TAM} , P_{0TAM} , P_{max} , Q_{max} , $D_{P_t}; t = 1; \dots; N-1$, $R_{t; t=1; \dots; N-1}^{min}$, and $R_{t; t=1; \dots; N}^{max}$.

Subject to

$$R_{t; t=1; \dots; N} \leq \min\{P_{tTAM} Q_{tTAM}; Q_{max}; R_{t; t=1; \dots; N}^{max}\} \quad (8)$$

which indicates first, that the supply of the project cannot exceed the maximum capacity Q_{max} . If the traffic demand exceeds the maximum capacity, the concessionaire can only satisfy part of the demand in this case. Second, the revenue collected by the concessionaire is capped at the level $R_{t; t=1; \dots; N}^{max}$, which is stipulated in the contract

$$Q_{tTAM} \leq Q_{t; t=1; \dots; N} \quad (9)$$

which indicates that the demand is solely determined by the price (toll). So as the revenue

$$\frac{\partial Q_t}{\partial P_t} < 0; t = 1; \dots; N \text{ and } \frac{\partial Q_t}{\partial P_t} > 0; t = 1; \dots; N \quad (10)$$

which means that the demand function is a general decreasing and nonconcave function

$$0 < P_{1TAM} \leq P_{1TAM}^c \leq P_0 \leq P_{max} \quad (11)$$

which indicates that, in the first operational period, the price charged by the concessionaire cannot exceed the price cap of that period P_{1TAM}^c , that is, P_0 stipulated by the government in the contract

t1

$$0 < P_{tTAM} \leq P_{tTAM} \leq P_0 \leq P_{max}; c = \min$$

1/41

$$t = 1/2; \dots; N \quad (12)$$

in which

$$\text{sgn}(\delta P_x) = \begin{cases} 1; & \text{if } x > 0 \\ 0; & \text{if } x = 0 \end{cases} \quad (13)$$

which means that, in each period, there exists a price cap for that period, P^c_t , and P^c_t is determined by different combinations of price escalations DP_t s and, ultimately, determined by the revenues in each prior period. If revenue of one period is less than the minimum level stipulated in the TAM, then $\text{sgn}(\delta P_x) = 1$, that is, a toll adjustment is allowed; otherwise, $\text{sgn}(\delta P_x) = 0$. In any period, the price cap cannot exceed the maximum level of price for the entire operational period P_{\max} .

Thus, the ROV of the TAM is

$$\text{ROV} = \max \text{NPV}_{\text{TAM}} - \max \text{NPV}_{\text{PCM}} \quad (14)$$

which indicates that the value of flexibility in the contract with the automatic toll adjustment is equal to the difference between (1) the optimal NPV of the project under the optimal pricing strategy in the contract with the automatic TAM and (2) the optimal NPV of the project under the optimal pricing strategy in the contract with the pure price cap regulation mechanism.

Numerical Analysis

To solve the optimization problem, a tree structure model (Fig. 2) is built to facilitate the solving process. For the simplicity of notation,

$\pi_{t|t_0}^{t_1}$ means in the 0 or 1 $g_{t|t_0}^{t_1}$ denote the pricing strategy for period t period, $\pi_{t|t_0}^{t_1}$ $\pi_{t|t_0}^{t_1}$ means in t , i.e.,

the penultimate period, $N-1$, that is, $\pi_{N-1|N-1}^{N-1}$ $\pi_{N-1|N-1}^{N-1}$. Let $k_{N-1|N-1}^{N-1}$ P . For the last period, $t=N-1$ $t=N-1$. Thus, $k_{N-1|N-1}^{N-1}$

$k_{N-1|N-1}^{N-1}$ corresponds to the pricing strategy $f_0;0;\dots;0;0;1$ N , $k_{N-1|N-1}^{N-1}$ corresponds to the pricing strategy $f_0;0;\dots;0;1;1$ g_N , ..., and $k_{N-1|N-1}^{N-1}$ 2^{N-1} corresponds to the pricing strategy $1f;1;\dots;1;1;1$ g_N .

For instance, $k_{N-1|N-1}^{N-1} \in 2^{N-1}$, so the pricing strategy is therefore $f_0;1;\dots;1;1;1$ g_N , which means that the concessionaire will only seek one opportunity of toll adjustment, which is from the second period, thus the revenue for the first period is

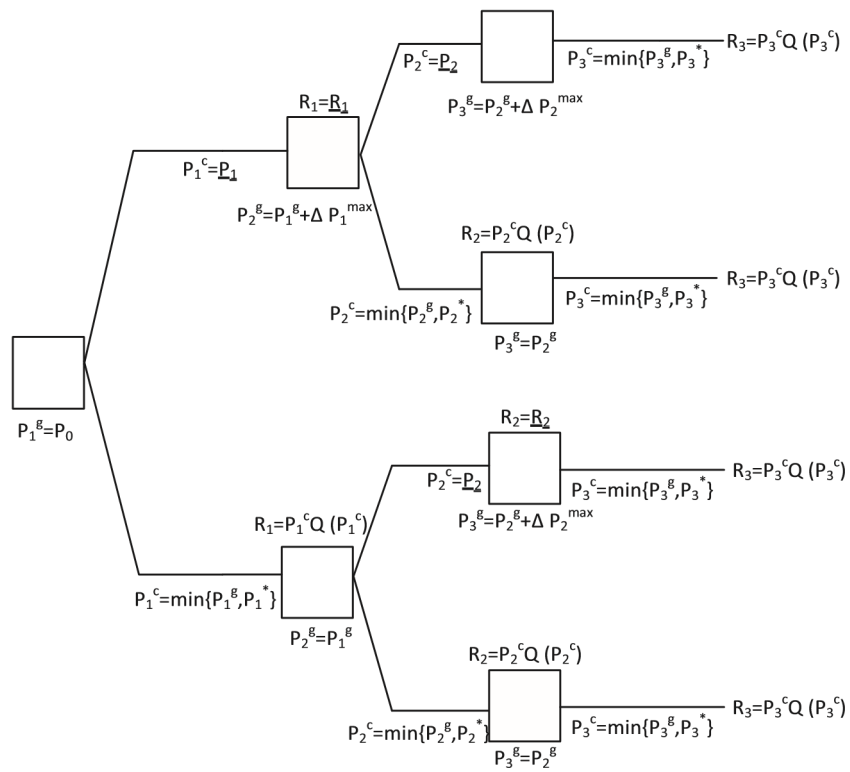


Fig.2. Tree structure model for real options analysis of TAM

$$R_2 = \underline{R_2}$$

the minimum level stipulated in the contract. This is beneficial because price caps for the following periods are increased by DP_1 , that is, the price caps for each period are $P_{TAM}^c - N_1 \frac{1}{4} f_{P0TAM}; P_{TAM} - DP_1; \dots; P_{TAM} - DP_1; P_{TAM} - DP_1; \dots; P_{TAM} - DP_1$, therefore, the revenues of the individual periods are $TAM - \frac{1}{4} TAM - R_{TAM} - N_1 \frac{1}{4} R^{\min}_1; \min R_2 - P_2; R_{\max 2}; \dots; \min R_N - P_N; R_{\max N} - N$, thus, the

NPV of the project under this pricing strategy is

$$NPV_{TAM}^k = \frac{1}{4} \sum_{t=1}^N (P_t - R_t) \frac{1}{(1+k)^t} \quad (15)$$

$$P_t = \begin{cases} P_{TAM} - R_{\max t} & \text{if } P_{TAM} - R_{\max t} \geq R^{\min}_1 \\ R^{\min}_1 & \text{otherwise} \end{cases} \quad (15)$$

in which $P_t = P_{TAM} - R_{\max t} \frac{1}{4}$

$$P_t = \begin{cases} P_{TAM} - R_{\max t} & \text{if } P_{TAM} - R_{\max t} \geq R^{\min}_1 \\ R^{\min}_1 & \text{otherwise} \end{cases} \quad (16)$$

Thus, for all the k s, the NPV of the project under this strategy is

$$NPV_{TAM}^k = \frac{1}{4} \sum_{t=1}^N (P_t - R_t) \frac{1}{(1+k)^t} \quad (17)$$

for each k

which indicates, for each pricing strategy k , there is a correspondent set of revenues of N periods, therefore, the NPV for each pricing strategy k can be found. Exhausting k from 1 to 2^{N_1} , the optimal

pricing strategy for the concessionaire is the one that can maximize the NPV of the project among the k combinations.

At period t , there are 2^{t-1} decision nodes in the tree model, each with a unique price cap, corresponding to its route of previous pricing strategies. For the j th decision node, the price cap is

t

$$P_{c,t} = P_{0,t} + \sum_{i=1}^{t-1} \delta^i P_i \quad (18)$$

$i=1$

The lower bound (LB) for this decision node is

N

$$LB_t = \sum_{i=1}^N X_{i,t} R_i P_{c,t} \quad (19)$$

$i=1$

which means, from period t and on, the concessionaire will charge the toll at the price cap of period t , $P_{c,t} = P_{0,t} + \sum_{i=1}^{t-1} \delta^i P_i$. The NPV of this decision node under such a pricing strategy is called the LB for the decision node j at period t . It is straightforward to see that for any pricing strategy actually chosen by the concessionaire, $f_t = 0$ or $1g^N$, any of the correspondent net present value $NPV_{i,t}$ is no less than the lower bound LB_t :

Similarly, the upper bound (UB) for this decision node can be defined as

N

$$UB_t = \sum_{i=1}^N X_{i,t} R_i P_{c,t}$$

$i=1$

(N

$$R_i = \min(P_{optt} ; P_{ctTAM} \delta P_j \beta XDP_j) ; R_{maxt}) \quad (20)$$

$i \geq t$

which indicates that from period t and on, the concessionaire will seek to maximize the revenue in each period under the condition that all price escalations are permissible regardless of the relationship between actual revenues and the minimum levels stipulated in the contract. The revenue may or may not be higher than the optimal

Table 1. Maximum and Minimum Revenues

t	min R	max R
1	365	1,095
2	402	1,205
3	442	1,325
4	486	1,457
5	534	1,603
6	588	1,764
7	647	1,940
8	711	2,134
9	782	2,347
10	861	2,582
11	947	2,840

12	1,041	3,124
13	1,146	3,437
14	1,260	3,780
15	1,386	4,158
16	1,525	4,574
17	1,677	5,031
18	1,845	5,535
19	2,029	6,088
20	2,232	6,697
21	2,456	7,367
22	2,701	8,103
23	2,971	8,914
24	3,268	9,805
25	3,595	10,785
26	3,955	11,864
27	4,350	13,050
28	4,785	14,355
29	5,264	15,791
30	5,790	17,370

Note: t = period, which is usually year in toll road projects; R = millions of HKD.

revenue without any price caps because it is still possible that with all the price escalations, the price cap still can be lower than the optimal level. The revenues must be lower than the revenue

caps, that is, revenue caps still work. The NPV of this decision node under such a pricing strategy is called the UB for the decision node j at period t . It is straightforward to see that for any pricing strategy actually chosen by the concessionaire, $f_t \in [0, 1]^{N_t}$, any of the correspondent net present value $NPV_{t,TAM}(\delta_j)$ is less than the upper bound $UB_t(\delta_j)$:

At any period, for any decision nodes, both LB and UB can be attained, in that the pricing strategies assumed in this fashion are comprised of independent prices, unlike in the case of active pricing strategy, in which the prices the concessionaire may charge for each period are interrelated.

At any period t , exhausting $j \in \{1, 2, \dots, 2^{t-1}\}$, the authors will

generate 2^{t-1} pairs of $LB_t(\delta_j); UB_t(\delta_j)$, let $\delta_j^k \in \mathcal{P}_j^k$ denote one route leading to node j . For any two decision nodes

m and l the one to node n , if

$$NPV_{t,TAM}(\delta_k) \leq UB_t(\delta_m) < NPV_{t,TAM}(\delta_l) \leq LB_t(\delta_n) \quad (21)$$

which means that the pricing strategy corresponding to route k is strictly inferior to the pricing strategy corresponding to route l ; thus, route k can be safely discarded.

Applying this process for $t = 1, \dots, N-1$ and in each period for decision node $j \in \{1, 2, \dots, 2^{t-1}\}$, will effectively reduce the search space, and, therefore, computation complexity of the problem.

Application of the branch and bound algorithm will be illustrated in the case study.

Case Study

A free crossing harbor tunnel connects Kowloon and Hong Kong Island. The current crossing harbor traffic is 150,000 vpd, which exceeds the maximum designed capacity of 100,000 vpd. This created severe traffic jams during peak hours of the day, which caused a huge waste of time for

the commuters and a huge social and environmental cost for society. The free-flow travel time without congestion in the free tunnel is 6 min per vehicle (mpv); however, the actual travel time can be as long as 10 min or 20 min due to congestion. In light of this situation, the government of Hong Kong intends to build another crossing harbor tunnel to relieve the traffic on the existing one. The maximum designed capacity of the new tunnel is set to be 300,000 vpd, and the cost of construction is estimated to be 7 billion Hong Kong dollars (HKD). This tunnel has a better location, which means a shorter travel distance, and the more advanced design of the tunnel allows for a higher speed. Free travel time in the new tunnel is only half that of the current at just 3 mpv.

The government decided to procure this capital-intensive project through a BOT scheme, signing a concession contract with a private investor. The private investor, often in the form of a project company, will be responsible for the financing and construction of the tunnel according to the technical specifications set by the government and then will be entitled to the exclusive franchise to operate the tunnel for 30 years on completion of construction. The revenues generated through toll collection will be used to recoup the huge initial investment of the project, comprised mainly of construction and financial costs. When the concession contract terminates, the tunnel will be transferred back to the government at no cost.

To prevent the project company from exploiting the commuters arbitrarily for excessive profits, which is quite politically sensitive for a public utility, the government sets (1) revenue caps for each year in the concession period, and the excessive revenues beyond the caps will automatically go to the government, and (2) price caps for each year. During the first operational year the maximum toll the project company may charge is 20 HKD per trip; during the entire concession period of 30 years the toll cannot exceed 100 HKD per trip. Because the project is massively capital intensive and involves a variety of risks and uncertainties, e.g., the traffic demand, to make it

financially feasible and properly profitable to attract investors, the government also devises a TAM. This is written in the contract, in which a set of minimum revenue levels are defined clearly ex ante. During any year, if the actual revenue is less than the minimum level, the project company is entitled to raise the price cap by a certain degree for the remaining period. The extents of toll adjustment are 5 HKD each year for the first decade, 10 HKD for the middle decade, and 15 HKD for the last decade to offset inflation. Levels of maximum and minimum of revenues can be found in Table 1 (in millions of HKD).

There are a total of nine scenarios in the future (Table 2). For the low VOT scenario, the expected VOT of the first year is 3 dpm; for the medium VOT scenario it is 5 dpm; and for the high VOT scenario it is 7 dpm. The expected VOTs from year 2 to 30 increase by 2% annually. In all three cases the standard variance of VOT is one-fifth of the expected value. For the low traffic scenario, the total crossing harbor traffic will grow at an annual rate of 2% with first-year traffic to be 100,000 vpd, for the medium traffic scenario it will be 2% with 200,000 vpd, and for the high traffic scenario it will be 2% with 300,000 vpd.

A contract with a pure price cap regulation mechanism (with no TAM) will be used as a benchmark for the real options analysis. The price caps for the first decade in the concession contract are 20 dpt; the middle decade caps are 25 dpt; and the last decade caps are 30 dpt. The project company's discount rate in calculating the NPV is 12%. The risk-free rate of return in the market is 5%.

The actual tolls charged (P), price caps (P_c), and actual revenues (R) under TAM pricing strategy in each period are shown in Tables 3–5.

It can be seen from Table 3 that because the project company seeks to maximize the overall NPV, there are several tolls in certain years that are notably low and several notably high. In other

words, the volatility of tolls between consecutive periods is high. For example, the toll can be increased from 6.6 to 29.9 dpt, and then be reduced to 7.8 dpt, and then jumped to a much higher level of 35 dpt. These increases especially occur when the traffic is at a medium or high level. Because in some years revenues higher than the minimum revenues are achievable through certain patterns of pricing, the concessionaire may seek higher revenue in one period by artificially reducing the revenue in the prior period, resulting in a series

Table 2. Scenarios of the Future

VOT	Traffic demand		
	Low	Medium	High
Low	S1	S4	S7
Medium	S2	S5	S8
High	S3	S6	S9

Table 3. Actual Tolls Charged under the TAM Pricing Strategy (dpt)

t	Scenario								
	1	2	3	4	5	6	7	8	9
1	7.0	11.7	16.4	19.9	11.8	16.6	20.0	20.0	20.0
2	7.2	12.0	16.8	5.5	12.1	16.9	20.0	19.9	20.0
3	7.3	12.2	17.1	25.0	5.8	17.3	3.9	19.9	20.0
4	7.5	12.5	17.4	8.4	25.0	17.7	4.2	19.9	20.0
5	7.6	12.7	17.8	13.0	6.6	18.1	20.8	20.0	20.0

6	7.8	13.0	18.1	34.9	29.9	18.6	30.0	4.9	20.0
7	7.9	13.2	18.5	17.0	7.8	19.1	29.9	24.3	20.0
8	8.1	13.5	18.9	39.8	35.0	19.7	29.8	25.0	20.0
9	8.3	13.8	19.3	39.9	35.0	20.0	29.9	25.0	20.0
10	8.4	14.0	19.6	40.0	34.9	20.0	29.9	25.0	20.0
11	8.6	14.3	20.0	25.6	11.0	10.2	9.0	7.2	6.6
12	8.8	14.6	20.4	27.9	12.2	11.4	9.9	7.8	7.6
13	8.9	14.9	20.9	30.4	14.5	12.3	49.4	39.2	37.8
14	9.1	15.2	21.3	33.0	19.4	13.4	49.9	44.8	40.0
15	9.3	15.5	21.7	36.0	22.3	14.6	13.1	10.7	9.8
16	9.5	15.8	22.1	38.9	85.0	69.9	59.6	53.3	48.6
17	9.7	16.1	22.6	99.7	84.7	69.8	15.6	54.7	49.6
18	9.9	16.4	23.0	99.8	84.8	69.9	69.5	14.7	13.2
19	10.1	16.8	23.5	99.8	84.7	70.0	69.5	64.7	60.0
20	10.3	17.1	23.9	99.7	84.9	69.9	69.9	64.8	59.8
21	10.5	17.5	24.4	99.9	40.5	32.4	21.5	19.4	17.9
22	10.7	17.8	24.9	100.0	99.9	36.1	23.4	80.0	74.9
23	10.9	18.2	25.4	99.9	100.0	99.9	99.8	23.0	21.6
24	11.1	18.5	25.9	99.6	99.9	99.8	99.6	94.5	89.3
25	11.3	18.9	26.5	99.7	99.6	100.0	99.9	94.4	89.8

26	11.6	19.3	27.0	99.6	99.7	99.5	99.2	94.8	89.3
27	11.8	19.7	27.6	99.9	99.6	100.0	99.1	94.2	89.3
28	12.1	20.1	28.1	99.4	100.0	99.7	99.7	94.2	89.1
29	12.3	20.5	28.7	100.0	99.6	99.8	99.1	94.9	89.6
30	12.6	20.9	29.3	99.8	99.6	99.8	99.2	94.4	89.8

Note: t = period, which is usually year in toll road projects.

Scenario									
t	1	2	3	4	5	6	7	8	9
1	20	20	20	20	20	20	20	20	20
2	25	20	20	20	20	20	20	20	20
3	30	25	20	25	20	20	20	20	20
4	35	30	20	25	25	20	25	20	20
5	40	35	20	30	25	20	30	20	20
6	45	40	20	35	30	20	30	20	20
7	50	45	20	35	30	20	30	25	20
8	55	50	20	40	35	20	30	25	20
9	60	55	25	40	35	20	30	25	20
10	65	60	30	40	35	20	30	25	20
11	70	65	35	40	35	20	30	25	20
12	80	75	45	50	45	30	40	35	30

13	90	85	55	60	55	40	50	45	40
14	100	95	65	70	65	50	50	45	40
15	100	100	75	80	75	60	50	45	40
16	100	100	85	90	85	70	60	55	50
17	100	100	95	100	85	70	60	55	50
18	100	100	100	100	85	70	70	55	50
19	100	100	100	100	85	70	70	65	60
20	100	100	100	100	85	70	70	65	60
21	100	100	100	100	85	70	70	65	60
22	100	100	100	100	100	85	85	80	75
23	100	100	100	100	100	100	100	80	75
24	100	100	100	100	100	100	100	95	90
25	100	100	100	100	100	100	100	95	90
26	100	100	100	100	100	100	100	95	90
27	100	100	100	100	100	100	100	95	90
28	100	100	100	100	100	100	100	95	90
29	100	100	100	100	100	100	100	95	90
30	100	100	100	100	100	100	100	95	90

Table 4. Price Caps under the TAM Pricing Strategy (dpt)

Note: t = period, which is usually year in toll road projects.

Table 5. Actual Revenues under the TAM Pricing Strategy (Millions of

HKD)

t	Scenario								
	1	2	3	4	5	6	7	8	9
1	221	369	516	466	728	1,020	1,145	1,380	1,598
2	230	384	537	400	757	1,060	1,194	1,429	1,653
3	239	399	559	556	437	1,101	437	1,481	1,705
4	249	415	581	486	881	1,145	480	1,533	1,761
5	259	432	605	534	521	1,190	1,388	1,588	1,816
6	270	449	629	737	1,039	1,236	1,854	588	1,869
7	280	467	654	646	639	1,285	1,934	1,943	1,925
8	292	486	681	889	1,229	1,335	2,012	2,049	1,978
9	304	506	708	965	1,299	1,388	2,098	2,119	2,033
10	316	526	737	1,043	1,368	1,441	2,182	2,188	2,089
11	328	547	766	946	944	902	946	946	874
12	342	569	797	1,040	1,041	1,031	1,039	1,037	1,031
13	355	592	829	1,143	1,145	1,128	3,552	3,394	3,577
14	370	616	863	1,258	1,260	1,250	3,730	3,897	3,851
15	385	641	897	1,386	1,385	1,384	1,381	1,383	1,377
16	400	667	933	1,523	2,880	3,106	4,622	4,785	4,780
17	416	693	971	2,287	3,064	3,260	1,674	5,054	5,014

18	433	721	1,010	2,510	3,257	3,424	5,625	1,842	1,841
19	450	750	1,050	2,737	3,451	3,588	5,858	6,196	6,219
20	468	780	1,092	2,968	3,656	3,752	6,123	6,415	6,403
21	487	811	1,136	3,208	2,454	2,453	2,449	2,452	2,450
22	506	844	1,182	3,451	4,467	2,697	2,700	8,150	8,200
23	527	878	1,229	3,696	4,717	5,379	9,071	2,966	2,961
24	548	913	1,278	3,943	4,967	5,627	9,420	10,004	10,116
25	570	949	1,329	4,201	5,215	5,894	9,820	10,337	10,489
26	592	987	1,382	4,460	5,480	6,135	10,141	10,735	10,759
27	616	1,027	1,437	4,734	5,744	6,424	10,515	11,030	11,095
28	640	1,067	1,494	4,992	6,031	6,683	10,955	11,392	11,406
29	666	1,110	1,554	5,287	6,289	6,967	11,297	11,836	11,799
30	692	1,154	1,616	5,561	6,570	7,247	11,708	12,154	12,171

Note: t = period, which is usually year in toll road projects.

of toll escalations. Therefore, the actual tolls when the traffic is high are much higher than those when the traffic is low.

Table 5 also shows volatile revenues, especially with high traffic levels: in some years they are as low as the minimum levels, whereas in other years they are as high as twice the minimum levels. With TAM pricing strategies, when the traffic is high, and in the last several periods, therevenues are usually far higher, which can offset the prior lower revenues, resulting in a maximum overall NPV throughout the entire concession period.

The previously mentioned results in the tables are illustrated in Figs. 3–5. The X in these figures is the ratio of the actual toll in 1 year to that in the previous year. A high X means that the concessionaire can charge any level of toll as long as it is below the price cap. Differentiating the traffic demand is necessary to see the impact of different VOTs on the concessionaire’s pricing strategies, price caps, and revenues. When traffic demand is medium (Scenarios 4–6), the impact of different VOTs to the concessionaire’s pricing strategies, price caps, and revenues are shown in Figs. 3–5. Fig. 3 shows that a dynamic pricing strategy is adopted by the concessionaire, and the tolls in the final stage reach the maximum level stipulated in the contract. Fig. 4 shows that the higher the VOT is, the slower the price cap reaches the maximum level. Fig. 5 shows that through dynamic pricing revenues are arranged to maximize the NPV in a zigzag fashion. The higher the VOT is, the higher are the revenues.

To see the impact of different traffic demands (Q_s) on the concessionaire’s pricing strategies, price caps, and revenues, differentiating VOTs is necessary. When the traffic VOT is medium (Scenarios 2, 5, and 8), the impact of different Q_s to the concessionaire’s pricing strategies, price caps, and revenues are shown in Figs. 6–8. Fig. 6 shows that the actual final toll can reach the maximum level only with medium traffic demand; the final actual toll for high traffic demand is slightly smaller, whereas those for the low traffic demand are much less than the price caps. Fig. 7 shows that in each period the price caps for the low traffic demand scenario are higher than that for the medium or high traffic demand scenario. Fig. 8 shows that with a higher level of VOT the dynamic feature of the revenues magnifies and the adjustments tend to be in a periodic fashion. Attention should be paid in the final stages when the revenues from high traffic demand are as high as 2 and even 10 times the revenues from medium and low traffic demand, respectively.

Table 6 shows the NPVs under the TAM and pure price cap regulation mechanism (with no TAM used as a benchmark), and the ROV of TAM accordingly.

Table 6 shows that in Scenarios 1–3, when the traffic levels are low, the two contract strategies produce the same NPV. However, the TAM mechanism is better in the other six scenarios. The TAM pricing strategy does not perform better than the benchmark strategy in the first three cases not because the strategy itself is defective, but because the traffic level in the future is too low to generate enough profits to make it financially acceptable for the project, although it still can be economically feasible to the government taking into account the externalities it brought to the society. In this

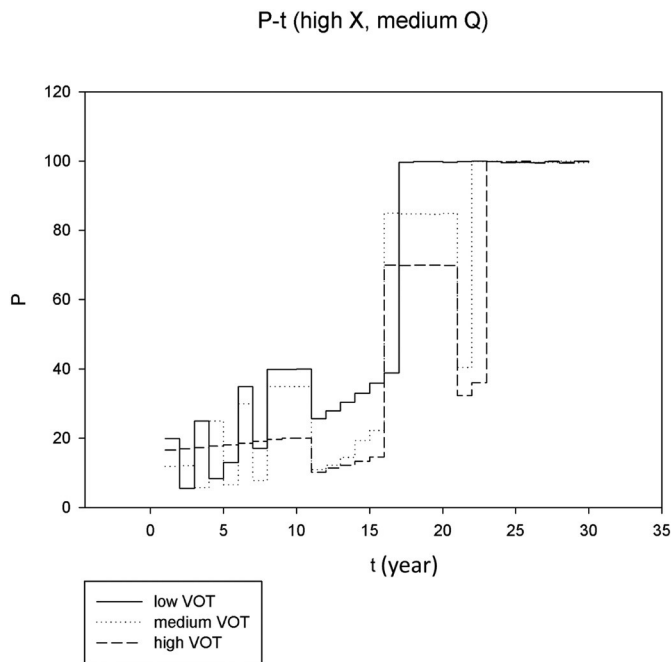


Fig.3. TAM pricing with medium traffic

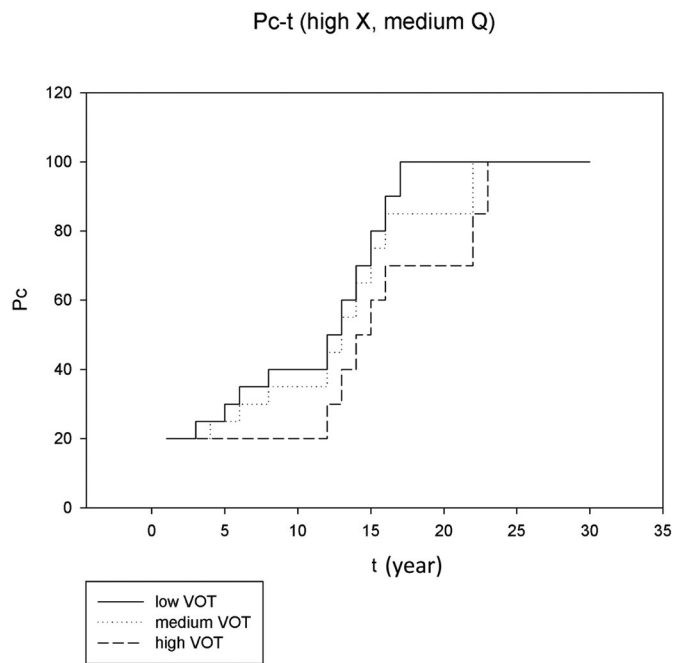


Fig. 4. Pricecaps under TAM pricing with medium traffic

case, the government should devise more incentives other than the TAM in the contract, such as subsidies or guarantees, to make the project attractive for private investors; otherwise, no tenders will be proposed at all. This is the very situation facing the Western Harbour Crossing Company. Because of a variety of reasons, the actual traffic in the tunnel is far less than the projected level, and it is only about one-quarter of the designed capacity after nearly 20 years of operation in a 30-year concession contract. Even the TAM cannot protect the concessionaire from the far less than expected or even negative NPV, which may not be sufficient to recoup the huge initial capital investment (2,310, 3,849, and 5,389 million HKD are all less than the 7 billion HKD cost).

When the traffic level is medium to high, the TAM pricing strategy is strictly superior to the pure price cap mechanism. Table 6 implies that the TAM is of significance to the concessionaire if the traffic is medium or not very high. On the one hand, if the traffic is so low that even the

maximum revenues are less than the minimum levels stipulated in the contract, the TAM is pointless. In other words, it will work and toll escalations will be allowed, but the revenues are still low. In contrast, if the traffic is so high that even a pure price cap regulation mechanism can guarantee that the revenues will be beyond the maximum levels stipulated in the contract, there is no point in seeking multiple toll escalations through the TAM, because higher price caps will not help to increase revenues. This is exactly the same for low traffic.

Conclusion

A real options model of the TAM was constructed in this study. It was found that the TAM was an ideal alternative for a guarantee arrangement in concession contracts of infrastructure projects, such as toll roads. In each period of the TAM, pricing strategies to maximize the current revenue and to reach the revenue at the minimum guaranteed level through micro toll adjustments within a single period need to be considered. Therefore, the optimal decision comprised of a series of tolls charged in each period can be determined, which was the one maximizing the NPV of the project. The ROV of managerial flexibility embedded in the TAM can then be determined.

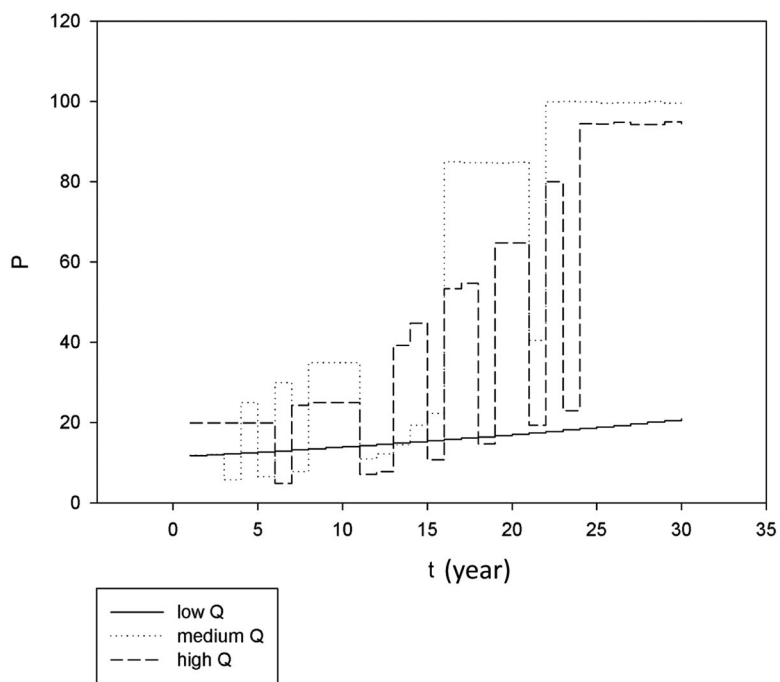
The model developed to assess the ROV of such a mechanism can benefit both the public and private sectors. For the public sector this means ensuring a reasonable but not excessive profit margin to attract private investors and, at the same time, not compromising the benefit to the public, and preventing possible opportunistic behaviors through clauses amended to the TAM. For the private sector this means making investment decisions and mitigating the demand risk in a better way as well as beating competitors who failed to recognize the value of flexibility and therefore undervalued the project. A win-win prospect was then achieved for both parties.

Limitation and Future Study

The following limitations of the research are mainly in the two-route choice model for commuters with heterogeneous VOTs:

1. Only two routes are considered. In real life there can be more than two connections between two cities; for example, there are three tunnels linking Kowloon and Hong Kong Island.
2. Only one transport mode, that is, traveling by driving private car, is considered. There are other choices, such as buses, taxis, and so forth. Also, for those vehicles traveling on the roads, competing routes such as a subway (in Hong Kong, Massive Transport Railway), can also be chosen by commuters.
3. The same toll is charged for all types of vehicles. In practice, different tolls are charged for different types of vehicles based on their size, potential damage made to the road, and other factors.
4. Traffic peak hours are neglected. In this research, congestion is considered by observing the actual daily traffic divided by the daily road capacity. However, in real life, traffic flow on a road is not even; during peak workday hours the traffic can be several times that during the rest of the day.

P-t (high X, medium VOT)



R-t (high X, medium Q)

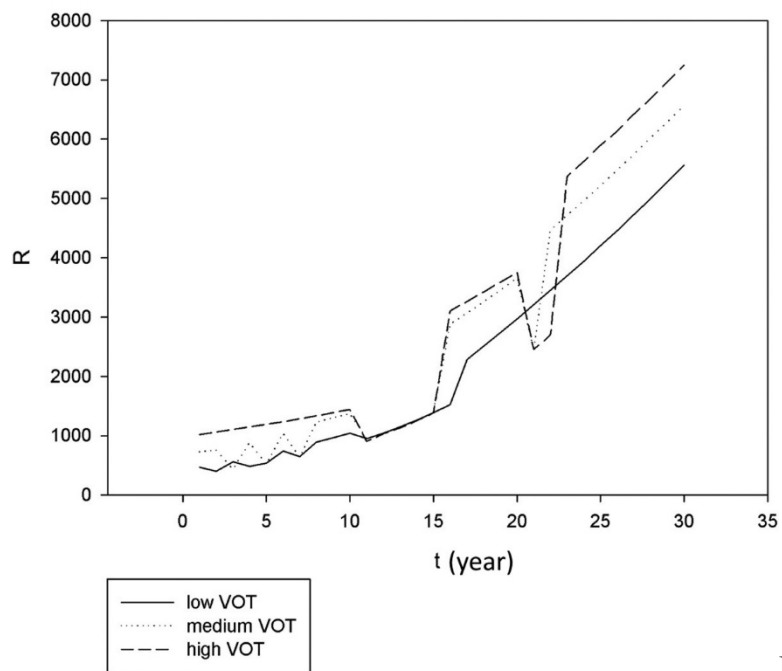


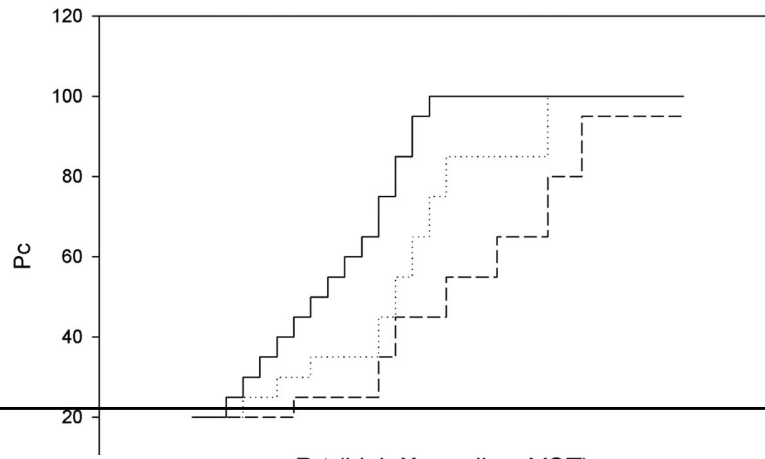
Fig.

Revenues under TAM pricing with medium traffic

Fig.6. TAM pricing with medium VOT

Four main research threads are recommended for future research. First, the traffic assignment model can be expanded. In the real world, the route choice can be much more complicated than the neat two-route choice model depicted in the research. Also, there are various route choices and transport mode choices available. Public transport can be considered for passengers with VOTs different from drivers.

Pc-t (high X, medium VOT)



R-t (high X, medium VOT)

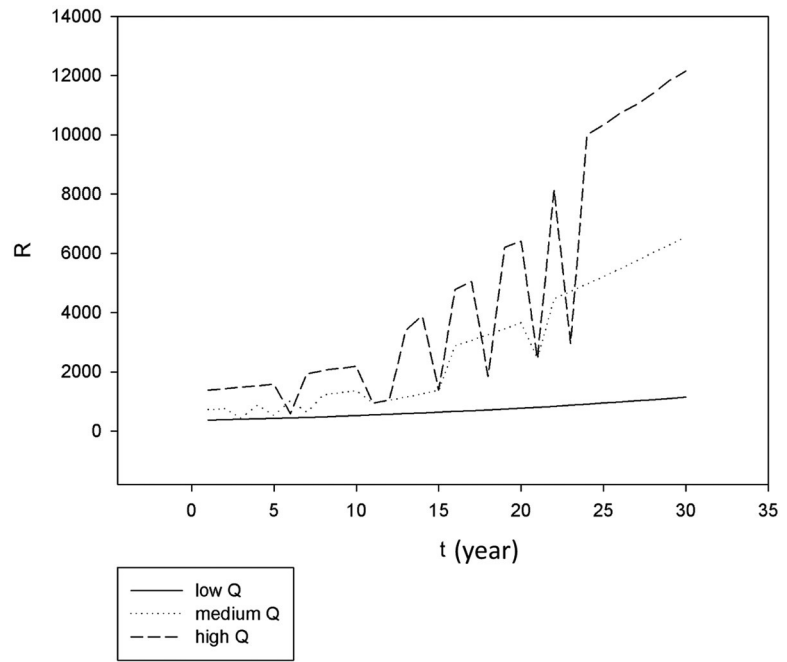


Fig. 8. Revenues under TAM pricing with medium VOT

Second, to be more realistic, differentiations of tolls among different vehicle types as well as operational costs should be taken into consideration. In this case the objective of the private investor is to optimize the NPV of profit rather than that of the revenue; therefore, pricing strategies may change accordingly with different tolls for different vehicle types. A combination of prices rather than a single price can be charged to achieve a certain revenue/profit goal, which gives the concessionaire even more managerial flexibilities.

Third, regulations on opportunistic pricing behavior, which is exactly the TAM pricing strategy described in the research, should be introduced, because in practice commuters would dislike the constant fluctuations of tolls, especially when consecutive tolls can be two to three times as high or even higher. However, such regulations should also ensure the private investor with a reasonable rate of return.

Table 6. NPVs and ROVs (Millions of HKD)

Parameter	Scenario								
	1	2	3	4	5	6	7	8	9
NPV(TAM)	2,310	3,849	5,389	7,931	9,970	12,114	15,813	16,816	17,298
NPV(pure PCs)	2,310	3,849	5,389	6,086	8,223	10,661	12,761	14,646	16,330
ROV	0	0	0	1,845	1,747	1,452	3,052	2,170	969

Note: PC = price cap.

Last, mechanisms to mitigate demand risk in a more fundamental way should be investigated. For the private investors, the TAM described in the research is not as effective as minimum revenue guarantees when the traffic demand is so low that optimal revenues in each period are still less than the minimum levels, respectively, which is exactly the case with Western Harbour Crossing in Hong Kong.

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