The effect of connected automated vehicles on macroscopic fundamental diagram of freeway traffic

Tianlu Pan[a], William H.K. Lam[a], Agachai Sumalee[a], Renxin Zhong[b*]
[a]*Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University, Hong Kong SAR, China.*
[b]*Guangdong Provincial Key Laboratory of Intelligent Transportation Systems, School of Engineering, Sun Yat-Sen University, Guangzhou, China.*

Abstract

This paper quantifies the effect of penetrated connected automated vehicles (CAVs) on macroscopic fundamental diagrams of mixed freeway traffic, while the impact of the variable speed limit (VSL) issued by freeway management system is also studied. The effect is introduced by penetrated CAVs which could tolerate smaller headway compared with regular human piloted vehicles (RHVs), as CAVs spend smaller response time to unexpected events. Finally, the findings are demonstrated in numerical examples for freeway traffic with different penetration rate.

Keywords: Connected automated vehicles; Penetration rate; Minimum space headway; Macroscopic fundamental diagram

1 Introduction

The number of connected automated vehicles (CAVs) equipped with vehicle automated VACS will be rapidly increasing in the coming decade (Roncoli et al., 2015a, b). Meanwhile, the regular human-piloted vehicles (RHVs) still play the major role on the market in short term (Levin and Boyles, 2016a,b). Therefore, it will be very common that the road is shared by CAVs and RHVs in the near future. The penetration of CAVs may lead to improvements in freeway network performance and traffic flow efficiency. However, the improvement is not exactly quantified in literature.

2. Minimum space headway and fundamental diagram

Headway, defined as the time/space between the same positions of two consecutive vehicles, is an important measure of traffic flow characteristics, and thus it is essential for studying traffic flow. Because CAVs have significantly different operating characteristics compared to RHVs, it is deemed that CAVs can significantly reduce the headway between vehicles and hence increase the roadway capacity (Levin and Boyles, 2016a). A recent experiment at the California Partners for Advanced Transportation Technology (PATH) showed that CAVs in platoons can maintain a time headway as small as 0.6 s, compared to 1.5 s for RHVs (Chen et al., 2017). In view of this, it is necessary to define a headway distribution law to model the potential capacity enhancement with the introduction of CAVs and to enable minimum headway control for mixed traffic.

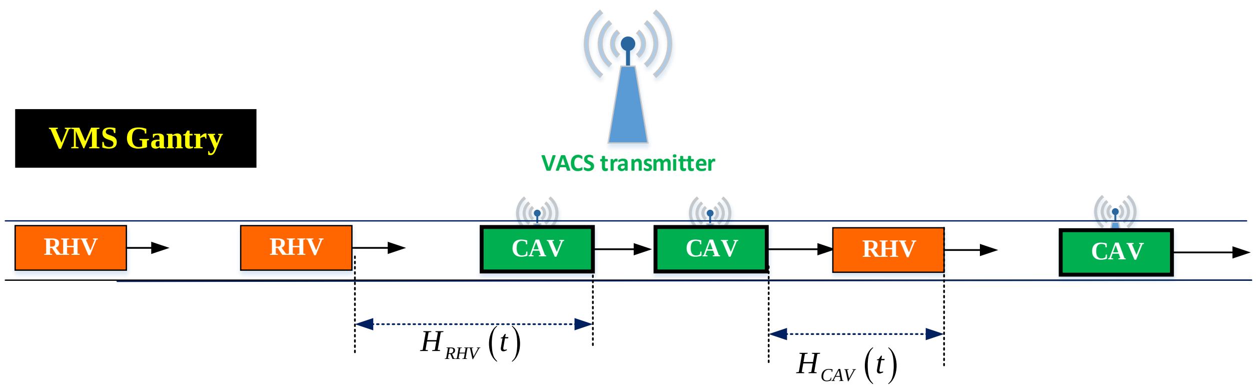

Figure 1. The illustration of space headway of mixed freeway traffic

In line with Levin and Boyles (2016a) and Chen et al. (2017), the effect of CAVs on roadway capacity was investigated by considering a single-lane freeway segment, as depicted in Figure . Suppose a platoon of vehicles is traveling along a freeway section in the same lane and that the traffic flow is stable without interruption from on-ramp/off-ramp or traffic incidents. Based on the rear-end collision avoidance principle (Jepsen, 1998; Levin and Boyles, 2016a, b), for vehicles traveling at speed $v(t)$ (miles/h) the space headway (from the head of the leading vehicle to the head of the following vehicle) criterion $H_D(t)$ (mile) ahead of a specific vehicle that is driving in mode D is defined as below:

$$H_D(t) \geq v(t)\Delta T_D + l + C, \quad for\ D = CAV, RHV \tag{1}$$

where ΔT_{CAV} and ΔT_{RHV} (h) denote the response times of CAVs and RHVs, respectively, l (mile) denotes the vehicle length of the leading vehicle, and C is the safety gap (Jepsen, 1998) or minimum safe constant gap (Hidas, 2005) when all the vehicles are at a standstill. Compared with RHVs, CAVs can tolerate a much smaller space headway because of the smaller response time. Supposing that the traffic on this unit length freeway segment (otherwise, multiply both sides of equation (2) by the segment length) comprises a proportion, $P(t)$, of CAVs and RHVs, $1-P(t)$, at time t, the relationship between space headway and traffic density is described as

$$\rho(t)\big(P(t)H_{CAV}(t) + (1-P(t))H_{RHV}(t)\big) = 1 \tag{2}$$

where $H_{CAV}(t)$ and $H_{RHV}(t)$ denote the space headway of CAVs and RHVs, respectively, as illustrated in Figure 1.

Based on equation (1), we have the following equations:

$$\rho(t)P(t)H_{CAV}(t) \geq \rho(t)P(t)\big(v(t)\Delta T_{CAV} + l + C\big)$$

$$\rho(t)(1-P(t))H_{RHV}(t) \geq \rho(t)(1-P(t))\big(v(t)\Delta T_{RHV} + l + C\big)$$

Summing the left and right sides, respectively,

$$\rho(t)\big(P(t)H_{CAV}(t) + (1-P(t))H_{RHV}(t)\big)$$
$$\geq \rho(t)P(t)\big(v(t)\Delta T_{CAV} + l + C\big) + \rho(t)(1-P(t))\big(v(t)\Delta T_{RHV} + l + C\big)$$

This in conjunction with equation (2) provides

$$1 \geq \rho(t)P(t)\left(v(t)\Delta T_{CAV} + l + C\right) + \rho(t)\left(1-P(t)\right)\left(v(t)\Delta T_{RHV} + l + C\right)$$

Therefore, the maximum speed $\tilde{v}_s(t)$ that can be tolerated by specific traffic density and penetration rate is evaluated as follows:

$$\tilde{v}_s(t) = \frac{\left(1 - l \cdot \rho(t) - C \cdot \rho(t)\right)}{\rho(t)} \cdot \frac{1}{\left(P(t)\ \Delta T_{CAV} + \left(1-P(t)\right)\ \Delta T_{RHV}\right)} \tag{3}$$

As reported in the literature, the average speed of RHVs is roughly equal to the average speed of CAVs in a traffic stream (Fountoulakis et al., 2017). To this end, it is assumed that this maximum speed will be **spontaneously** followed by both CAVs and RHVs as it guarantees the minimum space gap for avoiding collisions in accordance with different levels of congestion specified by traffic density. Therefore, in this section, the variable $\tilde{v}_s(t)$ is named the **spontaneous speed limit** (SSL). However, when the traffic density tends to zero (i.e., no vehicle is traveling on the freeway), the SSL approaches an infinitely large value. To remedy this, one can simply set an upper bound to this SSL, called a vehicle's **maximum mechanical speed**. For example, 170 mile/h is a common upper bound of the speedometer of private vehicles. Generally speaking, the speed is actually restricted by the posted **permanent compulsory** (upper bound) **speed limit** for freeway traffic management purpose. In the United States under normal conditions, the posted permanent compulsory freeway speed limit ranges from 90 miles/h in rural areas to 40 miles/h in urban areas (Highway Capacity Manual, 2010). Additionally, a temporary VSL can be issued as a control strategy for traffic incident management or congestion resolution if needed. As it can be expected, **maximum mechanical speed > permanent compulsory speed limit > temporary VSL**; therefore, the **implemented speed limit (ISL)** is generally given as either the permanent compulsory speed limit or the temporary VSL.

In line with the VSL control literature, e.g. Hegyi, et al. (2005), the turning point of ISL and SSL is located at the critical density $\rho_c(t)$ (noting that the critical density is also affected by the implemented speed limited control itself), as shown in Figure 2. From the above analysis, the traffic flow speed $v(t)$ is finally defined as a function of ISL (the lower of the posted compulsory speed limit and the VSL), traffic density, and the penetration rate of CAVs.

$$v(t) = \min\left(\tilde{v}_l(t), \tilde{v}_s(t)\right)$$

$$= \begin{cases} \tilde{v}_l(t) & \text{if } \rho(t) \leq \rho_c(t) \\ \tilde{v}_s(t) = \dfrac{\left(1 - l \cdot \rho(t) - C \cdot \rho(t)\right)}{\rho(t)} \cdot \dfrac{1}{\left(P(t)\ \Delta T_{CAV} + \left(1-P(t)\right)\ \Delta T_{RHV}\right)} & \text{if } \rho(t) > \rho_c(t) \end{cases} \tag{4}$$

where $\tilde{v}_l(t)$ denotes the ISL and $\rho_c(t)$ is a function of the penetration rate of CAVs and ISL as follows:

$$\rho_c(t) = \frac{1}{\tilde{v}_l(t)\left(P(t)\ \Delta T_{CAV} + \left(1-P(t)\right)\ \Delta T_{RHV}\right) + l + C} \tag{5}$$

In traffic flow theory, the flow (rate) is the product of speed $v(t)$, and density $\rho(t)$, whereas the capacity is the maximum traffic flow (rate) observed at critical density $\rho_c(t)$. By $Q_m(t)=\rho_c(t)\tilde{v}_l(t)$, we have the roadway capacity as a function of the penetration rate of CAVs and ISL:

$$Q_m(t)=\frac{\tilde{v}_l(t)}{\tilde{v}_l(t)\left(P(t)\ \Delta T_{CAV}\ +\left(1-P(t)\right)\ \Delta T_{RHV}\right)+l+C} \tag{6}$$

The back-wave speed is evaluated by:

$$w_c(t)=\frac{l+C}{\left(P(t)\ \Delta T_{CAV}\ +\left(1-P(t)\right)\ \Delta T_{RHV}\right)} \tag{7}$$

On other hand, the jam density is determined by the average vehicle length and the minimum safe constant gap:

$$\rho_J(t)=\frac{1}{l+C} \tag{8}$$

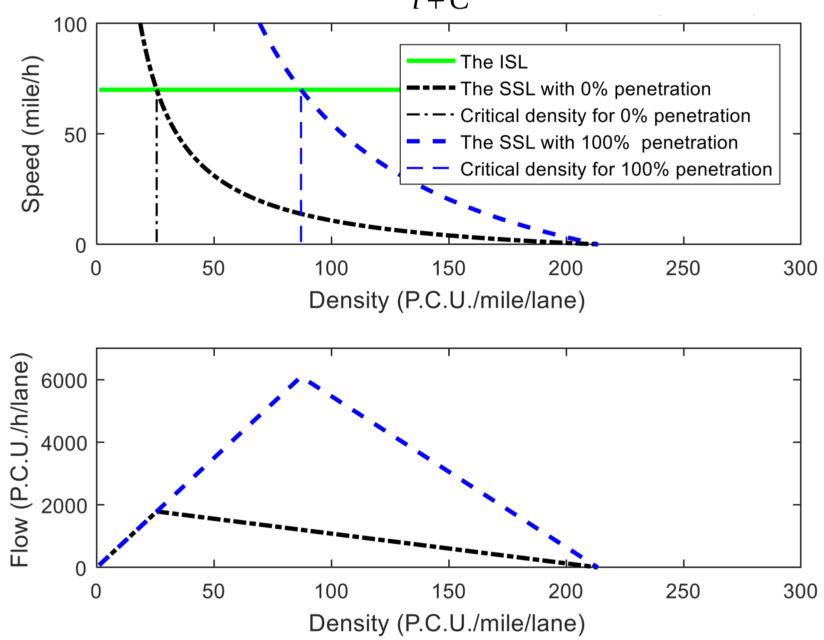

Figure 2. Lane-specific fundamental diagram as function of penetration rate of CAVs.

To ensure safety, the minimum space headway criteria $\tilde{H}_D(t)$ for D = *CAV and RHV* are evaluated respectively as

$$\begin{aligned}\tilde{H}_{CAV}(t)&=v(t)\Delta T_{CAV}+l+C\\ \tilde{H}_{RHV}(t)&=v(t)\Delta T_{RHV}+l+C\end{aligned} \tag{9}$$

The total space $\tilde{O}(t)$ reserved for the minimum-space headway of all vehicles traveling on the freeway segment is calculated according to the proportions of the two vehicle classes.

$$\tilde{O}(t)=\rho(t)P(t)\tilde{H}_{CAV}(t)+\rho(t)\left(1-P(t)\right)\tilde{H}_{RHV}(t) \tag{10}$$

When $\rho(t)\geq\rho_c(t)$, the speed is determined by SSL $\tilde{v}_s(t)$ according to equation (4), the total space $\tilde{O}(t)$ reserved for the minimum-space headways fills the total spaces. That is, the relationship between the traffic speed and density attains a critical value, as illustrated in Figure . The flow speed cannot be increased so as to maintain safety. However, when $\rho(t)<\rho_c(t)$, ISL $\tilde{v}_l(t)$ can be lower than SSL $\tilde{v}_s(t)$, thus allowing extra space.

$$\tilde{O}(t)\begin{cases}<1 & if\ \rho(t)<\rho_c(t)\\ =1 & if\ \rho(t)\geq\rho_c(t)\end{cases}$$

That is to say, if there is no ISL $\tilde{v}_l(t)$, vehicles can actually travel faster under light traffic conditions. The free space 1-$\tilde{O}(t)$ not occupied by vehicles for safe traveling can be randomly distributed among them. For simplification, we assume the safe space headway criteria are magnified by $\frac{1}{\tilde{O}(t)}$, i.e.,

$$\begin{aligned}H_{CAV}(t)&=\tilde{H}_{CAV}(t)/\tilde{O}(t)\\ H_{RHV}(t)&=\tilde{H}_{RHV}(t)/\tilde{O}(t)\end{aligned}\qquad if\ \rho(t)<\rho_c(t)$$

To sum up, the space headway distributions are thus calculated:

$$H_{CAV}(t)=\begin{cases}\dfrac{v\ (t)\Delta T_{CAV}+l+C}{\rho(t)\cdot(l+C)+\rho(t)\cdot v\ (t)\cdot\left(P(t)\ \Delta T_{CAV}\ +\left(1-P(t)\right)\ \Delta T_{RHV}\right)} & if\ \rho(t)<\rho_c(t)\\ v\ (t)\cdot\Delta T_{CAV}+l+C & if\ \rho(t)\geq\rho_c(t)\end{cases}$$
$$H_{RHV}(t)=\begin{cases}\dfrac{v\ (t)\Delta T_{RHV}+l+C}{\rho(t)\cdot(l+C)+\rho(t)\cdot v\ (t)\cdot\left(P(t)\ \Delta T_{CAV}\ +\left(1-P(t)\right)\ \Delta T_{RHV}\right)} & if\ \rho(t)<\rho_c(t)\\ v\ (t)\cdot\Delta T_{RHV}+l+C & if\ \rho(t)\geq\rho_c(t)\end{cases}\qquad(11)$$

As stated in Green (2000), the brake response time of a human driver is composed of three components: mental processing, muscle movement, and brake engagement time. On average, the mental processing takes about 1.3 sec for unexpected occasions, the average muscle movement takes 0.2 sec (Wierwille and Casali, 1983; Lerner et al., 1995; Green 2000), and the brake engagement time takes 0.35 sec under emergency conditions. Therefore, the braking response time of RHVs to an unexpected occasion is 1.85 sec. However, CAVs do not have mental reactions nor muscle movement; therefore, the response time of CAVs is considered to be 0.35 sec for the brake engagement process. This value is consistent with the 0.6sec time headway between successive CAVs (Chen et al., 2017).

3. Capacity increment and headway distribution

Figure 3 demonstrates the impact of the penetration rate of CAVs[1] and the compulsory speed limit on road capacity. In this example, all vehicles are considered passenger car equivalent

[1] In this case, the penetration rate is equivalent to proportion as we interest in cell-lane capacity.

(P.C.E.) or passenger car unit (P.C.U.) vehicles 20 ft in length, and the safe constant gap is 6.5 ft (when the related vehicles are at a standstill). As stated above, the response times of RHVs and CAVs are set to be 1.85 s and 0.35 s, respectively. As shown in Figure 3, the capacity monotonically increased with the increasing penetration rate of CAVs and the ISL.

The black solid line quantifies the variation of capacity with respect to the penetration rate of CAVs varying from 0% to 100% by fixing the compulsory speed limit at 70 miles/h. As demonstrated in this figure, the capacity varies significantly from 1719 P.C.U./h/lane (with 100% RHVs) to 6055 P.C.U./h/lane (with 100% CAVs). On the other hand, by typical macroscopic traffic flow theory, the increase in free-flow speed (or the compulsory speed limit in our case) would introduce an increase in capacity. The red dotted line presents an example for this using a fixed 66.7% penetration rate of CAVs, while the pink dot dash line presents the 33.3% one.

	Scenario: 33.3% Penetration rate			Scenario: 66.7% Penetration rate		
Density (P.C.U./mile)	Speed (miles/h)	$H_{CAV}(t)$ (ft)	$H_{RHV}(t)$ (ft)	Speed (miles/h)	$H_{CAV}(t)$ (ft)	$H_{RHV}(t)$ (ft)
10	**70**	**196**	**694**	**70**	**286**	**1012**
32	**70**	**61**	**217**	**70**	**89**	**316**
47	49	47	145	**70**	**61**	**215**
100	14	32	63	20	36	86
200	0.8	25	27	1	25	28

Table 1. Space headway as function of penetration rate and traffic density

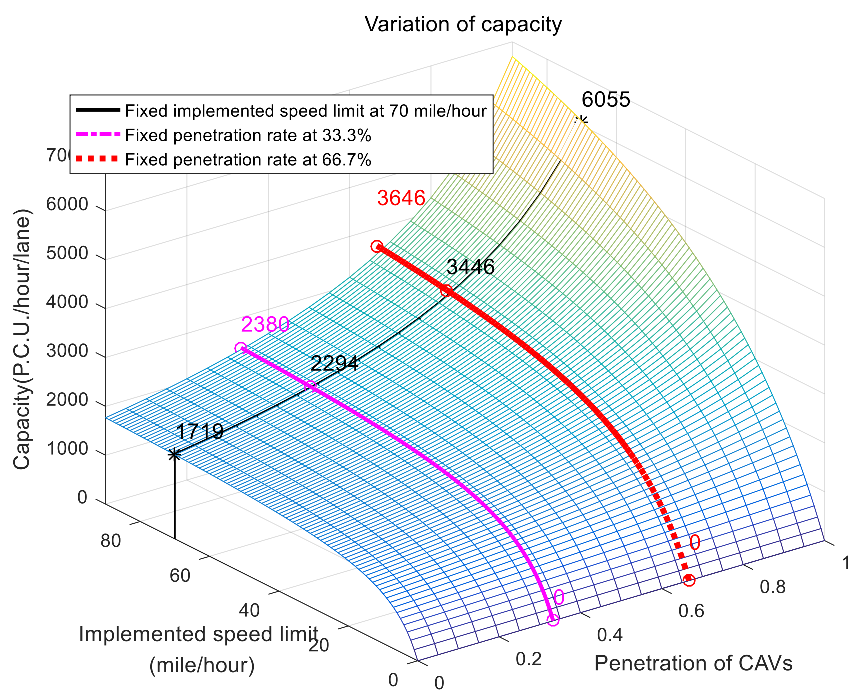

Figure 3 Capacity as function of penetration of CAVs and ultimate speed limit.

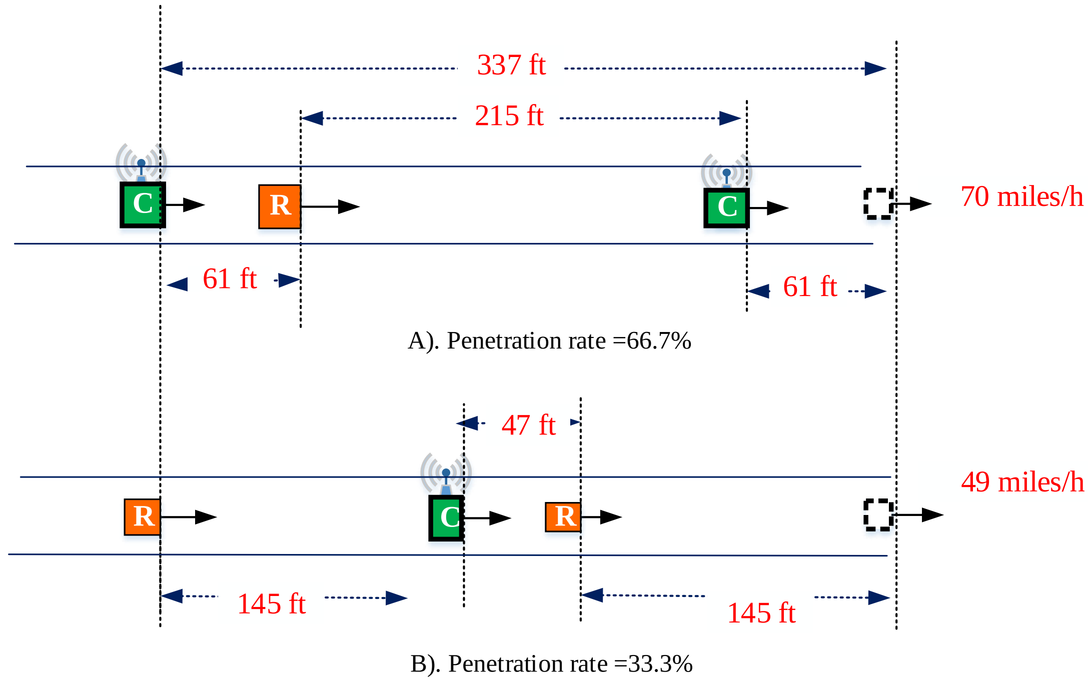

Figure 4 The variation of space headway with respect to penetration rate.

Error! Reference source not found. and **Error! Reference source not found.** present the space headways of RHVs and CAVs with respect to traffic density and penetration of CAVs. The space headway distributions with respect to different penetration rates depicted in Figure 4 were obtained by setting the traffic density to be 47 P.C.U./mile, i.e., 3 P.C.U./337ft, and assuming 66.7% and 33.3% penetration rates of CAVs, respectively. As it can be inferred from Figure 4, and **Error! Reference source not found.** that:

1) The space headway in front of an RHV is greater than that in front of a CAV under the same traffic conditions, and this difference is more significant under free-flowing conditions (the results related to free-flowing conditions use bold font in **Error! Reference source not found.**).

2) The increment of traffic density leads to smaller space headways for both RHVs and CAVs.

3) The penetration rate of CAVs affects the space headway distribution. In particular, the space headway in front of RHV increases 50% when the penetration rate of CAVs raises from 33.3% to 66.7%.

4). With the same traffic density, the traffic with higher penetration rate of CAVs might maintain a higher speed. As demonstrated in Figure 4, the traffic is 21 miles/h faster when the penetration rate of CAVs raises from 33.3% to 66.7%.

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