ADDING MODE CHOICE ALTERNATIVE INTO TRANSPORTATION NETWORK REDUNDANCY ANALYSIS

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ABSTRACT

Redundancy analysis, as an important concept in resiliency analysis, has been recently considered by Xu et al. (2018) by providing two network-based measures to systematically characterize the redundancy of transportation networks. The route diversity measure evaluates the existence of multiple routes between each origin-destination (O-D) pair for travelers, and the network spare capacity measure quantifies the network-wide residual capacity with consideration of congestion for planners. In this paper, mode choice option is also considered in the redundancy analysis for the multi-modal transportation network. Hence, the route diversity measure is extended to the travel alternative diversity measure to consider both route and mode availability, while the network spare capacity measure is expanded from a single mode transportation network with only route choice to a multi-modal transportation network with both mode choice and route choice in a combined modal split and traffic assignment (CMSTA) problem. Numerical results will be presented to demonstrate how adding mode option can significantly increase the transportation network redundancy.

Keywords: network redundancy, network capacity, multi-mode, nested logit, mode similarity

1. INTRODUCTION

Currently, most transportation networks were designed with efficient and economical purposes, in which the transportation projects use money, time, goods, etc. carefully and without wasting any. An economical design is preferred when the budget is of concerned. However, in consideration of the unexpected natural and man-made disasters, “plan some more” is needed to provide adequate redundancy to make our transportation networks robust and resilient. To be more specific, redundancy is vital for transportation networks when facing the planned and unplanned disruptions (see Xu et al., 2018). Disruptions to transportation networks may cut off the critical lifelines in modern society, which can damage regional and national economic competitiveness and make peoples’ lives difficult.

The network redundancy can be quantitatively measured from two aspects: travel alternative diversity and network spare capacity, which capture the considerations of both travelers and planners. The travel alternative diversity dimension is to evaluate the number of effective connections between a specific origin-destination (O-D) pair; while the network spare capacity dimension is to quantify the network-wide residual capacity an explicit consideration of travelers’ mode and route choice behaviors as well as congestion effect (Xu et al., 2018). The travel alternative diversity can be evaluated straightforward by counting how many efficient routes (excluding unreasonable routes) or modes connecting origins and destinations. However, the network capacity should be quantitively evaluated not only by integrating all components’ capacities with complex topological connection, but also by modelling the complicated travelers’ behavior in transportation network. Despite that there are a few studies on the network capacity problem, considering both route choice and mode choice together is still lacking, particularly for the transportation network redundancy analysis. Since the diversity of urban transport modes and the growth of public transport trips, it is essential to consider the mode choice option in the network capacity problem. In this study, we extend the current network capacity models by considering both mode choice and route choice to provide a more comprehensive measurement for the multi-modal transportation network redundancy analysis.

For passenger networks in urban transportation system, it is well known that multiple O-D pairs exist and demands between different O-D pairs are not exchangeable or substitutable. Thus, the road network capacity models are usually formulated as a mathematical programming with equilibrium constraints. A widely used model is the reserve capacity model, which is proposed as the largest multiplier μ applied to a given O-D demand matrix that can be allocated to a transportation network without violating any individual link capacity (Wong and Yang, 1997). The product of the largest multiplier μ and the existing O-D demand (represented by vector $\mathbf{q}$) gives the maximum travel demand can be loaded to the network, which indicates whether the current network has spare capacity or not. So the *network spare capacity* is generally explained as: If $\mu > 1$, then the network can be loaded more travel demand, and the additional demand can be accommodated by the network is $(\mu - 1)\mathbf{q}$; otherwise, i.e. $\mu < 1$, the network is overloaded and the existing O-D demand should decrease by $(1 - \mu)\mathbf{q}$ to satisfy the capacity constraints.

The concept of network capacity was generally discussed for the road network capacity by the car mode only. In Cheng et al. (2014) and Xu et al. (2018), an additional mode, rail transit, is involved to consider the travelers' mode choice in urban transportation systems. Both models assumed that the rail transit is totally independent with the road traffic, as the railway uses physically separate transport facilities and devices. However, some other travel modes, such as buses, may share the same physical links to the car mode, or even have interactions with car flows on the travel costs (Watling, 1996). Adding of such modes could have an impact to the total capacity of the whole system. Except for sharing physical components, other external factors also cause correlations among different travel modes. For example, the public transport priority policy may correlate buses and subway; the travelers' attitude to traffic congestion may corelate bus and auto (e.g., private car or taxi), which are ground transportation modes. Empirically, the correlation of travel modes, referred as the *mode similarity*, affects the travel mode choice probabilities, and further affects the evaluation of the network capacity.

2. FORMULATION OF THE MULTI-MODAL NETWORK CAPACITY PROBLEM

In this study, aiming to measure the redundancy of transportation systems with multiple travel modes, a multi-modal transportation network capacity model will be formulated. The model will be mathematically formulated as a bilevel program, in which the upper-level problem aims to maximize the total travel demand by all modes and the lower-level problem regulates the flow pattern in the multi-modal transportation system. Specifically, the lower-level mathematical program (MP) is formulated as a combined mode split and traffic assignment (CMSTA) problem, in which the total travel demand is given under the capacity constraints from the upper-level model. In the CMSTA model, a nested logit (NL) model will be adopted to model the mode similarity, while the path-size logit (PSL) model is used to account for route overlapping.

2.1 Upper-level model

Based on the *reserve capacity* model for road network (Wong and Yang, 1997), the upper-level problem of the multimodal transportation network capacity model can be formulated as:

$$\max_{\mu} \mu \tag{1}$$

$$s.t.\ v_a^m(\mu) \le \varphi_a^m C_a^m, \forall a \in A, m \in M, \tag{2}$$

$$q_{rs}^m(\mu) \le Q_{rs}^m, \forall m \in M, r \in R, s \in S, \tag{3}$$

where v_a^m is the traffic flow on road link a by mode m; φ_a^m is the maximum flow-to-capacity ratio on link a by mode m; C_a^m is the capacity of link a serving mode m; Q_{rs}^m is the capacity of travel mode m between O-D pair (r, s), which can be related to the maximum rate of the transit service, such as buses or subway. Note that the constraints (2) and (3) exhibit a non-linear relationship, as the link flows and mode specific O-D demands are an implicit function of the multiplier μ, of which the relationship is given in the lower-level problem of the bi-level program. The formulation of the lower-level problem is presented in the following section.

2.2 Lower-level model

This section develops the combined NL-PSL model for the CMSTA problem. Before formulating the NL-PSL model, this section starts with a generalization of the stochastic user equilibrium (SUE) model with elastic demand (or SUE-ED model). Using the excess demand formulation to transform the elastic demand into a modal split (Sheffi 1985), the combined model with bi-modal choice is obtained. Finally, we incorporate the nested tree structure of the NL model in the mode choice level to formulate the proposed NL-PSL model, in which the PSL uses a path-size factor to deal with the route overlapping problem (or route similarity). For each formulation, the network capacity problem will be resolved and discussed in the numerical section, and thus the advantages of the nested structure for multi-modal transportation network can be shown.

2.2.1 Elastic Demand Model for Car Network

This section presents a formulation of the single-modal elastic demand model with the logit-based SUE, where automatically a route similarity factor should be involved to handle the route overlapping problem. Unlike the standard UE model assuming the O-D travel demand is fixed, the elastic demand model relaxes the fixed O-D demand assumption by considering it as a function of the minimum O-D travel costs (Sheffi, 1985). A demand function is consequently defined, in which the amount of O-D demand, q_{rs}, is decreasing with respect to the O-D travel cost, π_{rs}, i.e.,

$$q_{rs} = D_{rs}(\pi_{rs}), \forall r \in R, s \in S. \tag{4}$$

With its inverse function, i.e., $\pi_{rs} = D_{rs}^{-1}(q_{rs})$, the equilibrium of O-D demands is often described as the balance of supply and demand in the network level (Sheffi, 1985). When extended to stochastic user equilibrium (SUE), the minimum O-D cost, π_{ij}, is replaced by the O-D specific expected perceived travel cost (EPC), w_{ij}, which is consistent with the SUE travelers' choice behavior based on the extreme value theory. The SUE-ED model can be formulated as the following mathematical programming (MP) by assuming the logit-based random utilities (Xu and Chen, 2013).

$$\min_{(v_a, q_{rs})} \sum_a \int_0^{v_a} t_a(x)\, dx + \frac{1}{\theta_K} \sum_{rs} \sum_k f_k^{rs} \ln f_k^{rs} - \sum_{rs} \int_0^{q_{rs}} D_{rs}^{-1}(v)\, dv - \frac{1}{\theta_K} \sum_{rs} q_{rs} \ln q_{rs} \tag{5}$$

$$s.t.\ \sum_k f_k^{rs} = q_{rs}, \forall k \in K_{rs}, r \in R, s \in S, \tag{6}$$

$$v_a = \sum_{rs} \sum_k f_k^{rs} \delta_{ak}^{rs}, \forall a \in A, \tag{7}$$

$$f_k^{rs} \geq 0, \forall k \in K_{rs}, r \in R, s \in S, \tag{8}$$

$$0 < q_{rs} \leq \mu \cdot \bar{q}_{rs}, \forall r \in R, s \in S. \tag{9}$$

where f_k^{rs} is the flow on route *k* between O-D pair (*r, s*); θ_K is the scale parameter of multinomial logit (MNL) model associated with stochastic route choice; δ_{ak}^{rs} is the link/route incidence parameter, and equal to 1 if route *k* (from *r* to *s*) travels through link *a*; 0 otherwise. $\mu \cdot \bar{q}_{rs}$ is the maximum potential demand that can be raised from *r* to *s*, as the O-D demand must be finite if the EPC drops to zero. $\{\bar{q}_{rs}\}$ is given as a prescribed O-D matrix for the computation of network capacity, while the multiplier μ is determined in the upper-level problem.

Based on (Maher, 2001), the elastic demand model assumed the O-D travel demand is endogenous, which generates a flexible O-D pattern compared with the standard UE model with fixed O-D demand. With the upper-limit of demand for each O-D pair, the gap between the maximum and the equilibrated demand can be regarded as the portion of travelers that do not choose to travel on the network. The not travel demand is also referred as the excess demand, denoted by e_{rs}. The elastic demand model represented with excess demand can be given by

$$\min_{(v_a, e_{rs})} \sum_a \int_0^{v_a} t_a(w)\, dw + \frac{1}{\theta_K} \sum_{rs} \sum_k f_k^{rs} \ln f_k^{rs} + \sum_{rs} \int_0^{e_{rs}} W_{rs}(v)\, dv - \frac{1}{\theta_K} \sum_{rs} \sum_k f_k^{rs} \ln \sum_l f_l^{rs} \tag{10}$$

where $W_{rs}(e_{rs})$ equals to $D_{rs}^{-1}(q_{rs})$ by definition, and $\sum_k f_k^{rs} + e_{rs} = \mu \cdot \bar{q}_{rs}$. Besides, the above objective function subjects to the constraints (7) and (8) and the nonnegative constraint, $e_{rs} \geq 0$ ($\forall r, s$). It is worth to mention that, for a single modal network, alternative routes can be introduced by adding new links. However, in some cases, the total network capacity may not increase as expected, which is the so-called "*network capacity paradox*", see Yang and Bell (1998) for details. Hence, our study suggests that adding alternative modes could be a better option if the decision makers need to promote the transportation system capacity. The formulation for the multi-modal network assignment will be derived in next sections.

2.2.2 Combined Model for Bi-modal Network

2.2.2.1 Separate network. Based on the elastic demand model with respect to the excess demand, Sheffi (1985) extended the UE model with elastic demand by replacing the demand function with a binary logit function. In the bi-modal model, the excess demand is interpreted as the alternative travel mode (for example, public transit) which is independent to the original road network. The excess demand of the primary travel mode (for example, car) will transfer to the alternative mode. To properly balance the demand between the two modes, extra parameters are introduced, such as, the expected travel cost and the attractiveness of the alternative mode. A typical formulation is given as follows:

$$W_{rs}(q_{rs}^{t}) = \frac{1}{\theta_M}\ln\left(\frac{q_{rs}^{t}}{q_{rs}^{total} - q_{rs}^{t}} + w_{rs}^{t} - \Psi_{rs}^{t}\right) \tag{11}$$

where w_{rs}^{t} is a constant representing the O-D travel cost by public transit; Ψ_{rs}^{t} is the exogenous attractiveness of the transit mode; q_{rs}^{t} is referred as the demand of traveling by transit mode (in place of the excess demand e_{rs}); θ_M is the scale parameter for mode choice, and $\theta_M < \theta_K$. With equation (11), the bi-modal model can be formulated as:

$$\min_{(v_a, q_{rs}^t)} \sum_a \int_0^{v_a} t_a(w)\,dw + \frac{1}{\theta_K}\sum_{rs}\sum_k f_k^{rs}\ln f_k^{rs} - \frac{1}{\theta_K}\sum_{rs}\sum_k f_k^{rs}\ln\sum_l f_l^{rs}$$
$$+ \sum_{rs}\int_0^{q_{rs}^t}\left(\frac{1}{\theta_M}\ln\frac{w}{q_{rs}^{total} - w} + w_{rs}^{t} - \Psi_{rs}^{t}\right)dw, \tag{12}$$

$$s.t.\ \sum_k f_k^{rs} + q_{rs}^{t} = q_{rs}^{total}, r \in R, s \in S, \tag{13}$$

$$\mu \cdot \bar{q}_{rs} = q_{rs}^{total} \tag{14}$$

$$0 < q_{rs}^{t} \le q_{rs}^{total}, \forall r \in R, s \in S, \tag{15}$$

Equations (7) and (8).

2.2.2.2 Shared network. Different travel modes may not always operate on independent physical networks, such as the regular bus and car. In such case, the multiple modes share the same physical network. Other than the interaction among the multi-modal traffic flows (we did not consider this feature yet in this paper), the effect of sharing same facilities will certainly influence the flow pattern of the maximum total demand. Here, we consider the modes of bus and car, which shares the same road network. The travel demand for the two modes can be convert to the standard unit of the traffic flow, i.e., passenger car unit. Therefore, the following formulation can be conducted.

$$\min_{(v_a, q_{rs}^m)} \sum_a \int_0^{v_a} t_a(x)\,dx + \frac{1}{\theta_K}\sum_{rs}\sum_{m=1}^{2}\sum_k f_{mk}^{rs}\ln f_{mk}^{rs} + \left(\frac{1}{\theta_M} - \frac{1}{\theta_K}\right)\sum_{rs}\sum_{m=1}^{2} q_{rs}^{m}\ln q_{rs}^{m} + \sum_{rs}\sum_{m=1}^{2}\Psi_{rs}^{m} q_{rs}^{m}, \tag{16}$$

$$s.t.\ \sum_m q_{rs}^{m} = q_{rs}^{total}, r \in R, s \in S, \tag{17}$$

$$\sum_k f_{mk}^{rs} = q_{rs}^{m}, \forall m \in M_{rs}, r \in R, s \in S, \tag{18}$$

$$v_a = \sum_m \frac{PCE_m}{OCC_m}\sum_{rs}\sum_k f_{mk}^{rs}\delta_{a,mk}^{rs}, \forall a \in A, \tag{19}$$

Equations (8) and (14).

where PCE_m is the passenger car equivalent (PCE) factor of mode *m*. OCC_m is the average occupancy of mode *m*, which is calculated as the average number of persons occupying a vehicle of mode *m*. Note that, in the above formulation, the link travel times of bus and car are assumed to be same. In summary, figure 1 illustrates the process of developing the bi-modal demand model based on the elastic demand model.

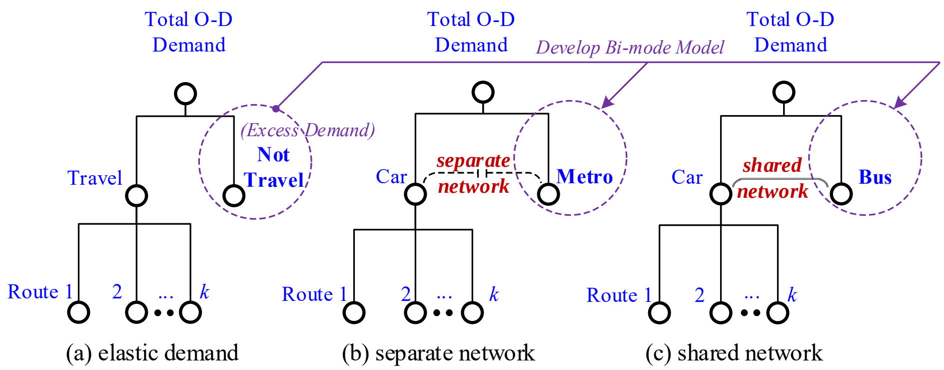


Figure 1. Similar structures for elastic demand model and bi-modal models.

2.2.3 CMSTA Model for Multi-modal Network

2.2.1.1 MNL structure with route overlapping consideration. The derivation of the multi-modal demand model is ready by following the formulation of the bi-modal demand model for the shared network. Only need to change the upper limit from 2 to |M| (the number of modes) in the summation of the O-D demand by modes. Nevertheless, considering the independently distributed assumption, i.e. route overlapping problem, imposed on the logit-based SUE model, we introduce a path-size factor ϖ_k^{rs} to correct the EPC of the overlapped routes. The path-size factor $\varpi_k^{rs} \in (0,1]$ is defined for each route k, and decided according to the length of links within a route and the relative lengths of routes that share a link. A typical form is as follows (Ben-Akiva and Bierlaire, 1999):

$$\varpi_k^{rs} = \sum_{a \in \Gamma_k} \frac{l_a}{L_k^{rs}} \frac{1}{\sum_{l \in K_{rs}} \delta_{al}^{rs}}, \forall k \in K_{rs}, r \in R, s \in S,$$

where l_a is the length of link a, L_k^{rs} is the length of route r connecting O-D pair rs, Γ_k is the set of all links in route r between O-D pair (r, s), and δ_{al}^{rs} is equal to 1 for link a on route r between O-D pair (r, s) and 0 otherwise. Hence, the combined multi-modal demand model can be formulated as

$$\min_{(v_a, q_{rs}^m)} \sum_a \int_0^{v_a} t_a(x)\,dx + \frac{1}{\theta_K} \sum_{rs} \sum_{mk} f_{mk}^{rs} \ln f_{mk}^{rs} - \frac{1}{\theta_K} \sum_{rs} \sum_{mk} f_{umk}^{rs} \ln \varpi_k^{rs}$$
$$+ \left(\frac{1}{\theta_M} - \frac{1}{\theta_K}\right) \sum_{rs} \sum_m q_{rs}^m \ln q_{rs}^m - \sum_{rs} \sum_m \Psi_{rs}^m q_{rs}^m, \quad (20)$$

s.t. Equation (8), (14), and (17)-(19).

2.2.3.2 Nested structure accounting for mode similarity. This section extends the multi-mode model with MNL to the formulation with a two-level structured NL (Marzano and Papola, 2008). With the nested structure, the travel modes with similarity (such as, bus and metro are recognized as public transit) are modeled in the same nest, which allows consider the correlations of the similar modes. Consequently, the CMSTA model with NL is formulated as

$$\min_{(f_{umk}^{rs}, q_{rs}^{um})} \sum_a \int_0^{v_a} t_a(x)\,dx + \frac{1}{\theta_K} \sum_{rs} \sum_{mk} f_{umk}^{rs} \ln f_{umk}^{rs} - \frac{1}{\theta_K} \sum_{rs} \sum_{mk} f_{umk}^{rs} \ln \varpi_k^{rs} +$$
$$\sum_{rs} \sum_{um} \Psi_{rs}^{um} \cdot q_{rs}^{um} + \sum_{rs} \sum_{um} \left(\frac{\theta_M}{\theta_U} - \frac{1}{\theta_K}\right) q_{rs}^{um} \ln q_{rs}^{um} + \sum_{rs} \sum_u \left(\frac{1-\theta_M}{\theta_U}\right) q_{rs}^u \ln q_{rs}^u \quad (21)$$

$$s.t. \sum_{um} q_{rs}^{um} = q_{rs}^{total}, r \in R, s \in S, \quad (22)$$

$$\sum_m q_{rs}^{um} = q_{rs}^u, \forall u \in U_{rs}, r \in R, s \in S, \quad (23)$$

$$\sum_k f_{umk}^{rs} = q_{rs}^{um}, \forall m \in M_{rs}^u, u \in U_{rs}, r \in R, s \in S, \quad (24)$$

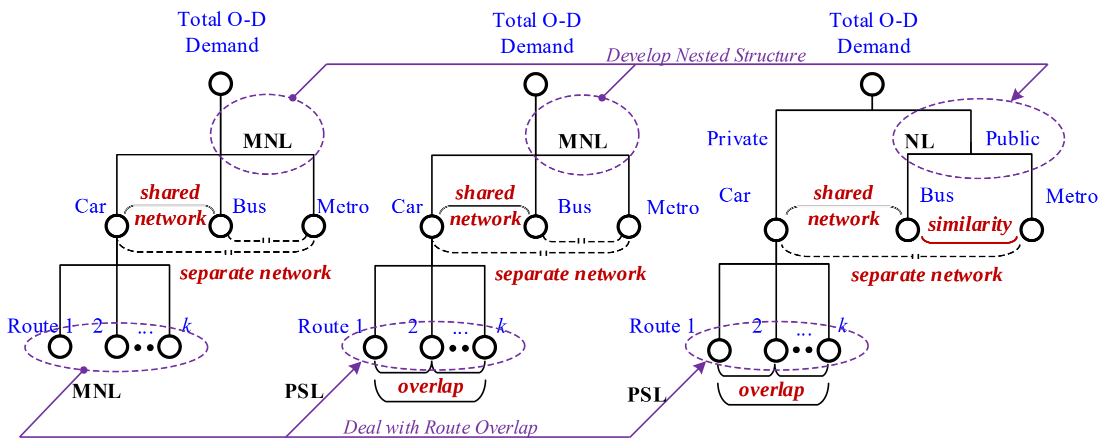
$$v_a = \sum_{um} \frac{PCE_{um}}{OCC_{um}} \sum_{rs} \sum_k f_{umk}^{rs} \delta_{a,umk}^{rs}, \forall a \in A, \quad (25)$$

$$f_{umk}^{rs} \geq 0, \forall k \subset K_{rs}^{um}, m \subset M_{rs}^u, u \subset U_{rs}, r \subset R, s \subset S, \quad (26)$$

$$0 < q_{rs}^{um} \leq q_{rs}^{total}, \forall u \in U_{rs}, r \in R, s \in S, \quad (27)$$

Equation (14).

where the label $u \in U_{rs}$ for the variables are the nest of the similar modes between (r, s). θ_U is the scale parameter for nest choice, and $\theta_U < \theta_M$. In summary, figure 2 illustrates the development of nested structure.



(a) non-nested structure with independent route choice (b) non-nested structure considering route overlap (c) nested structure considering route overlap

Figure 2. Structures for non-nested and nested multi-mode models.

3. SENSITIVITY ANALYSIS-BASED SOLUTION ALGORITHM

The multi-modal transportation network capacity model is a bi-level programming problem. The bi-level program usually can be solved by transferring it into a single level program, where the implicit relationship is approximated using the first-order Taylor expansion. That is,

$$v_a^m(\mu) \approx v_a^m(\mu^{(n)}) + \nabla_\mu v_a(\mu^{(n)}) \cdot (\mu - \mu^{(n)}), \forall a \in A,$$
$$q_{rs}^m(\mu) \approx q_{rs}^m(\mu^n) + \nabla_\mu q_{rs}^m(\mu^n) \cdot (\mu - \mu^n), \forall r \in R, s \in S.$$

With the above approximations, the nonlinear and implicitly defined constraints in Equation (2)-(3) can be approximated as

$$v_a^m(\mu^{(n)}) + \nabla_\mu v_a^m(\mu^{(n)}) \cdot (\mu - \mu^{(n)}) \le \varphi_a^m C_a^m, \forall a \in A, m \in M, \quad (2)$$
$$q_{rs}^m(\mu^{(n)}) + \nabla_\mu q_{rs}^m(\mu^{(n)}) \cdot (\mu - \mu^{(n)}) \le Q_{rs}^m, \forall m \in M, r \in R, s \in S. \quad (3)$$

where the derivatives $\nabla_\mu v_a^m(\mu^{(n)})$ and $\nabla_\mu q_{rs}^m(\mu^{m(n)})$ can be computed by using the sensitivity analysis method for the CMSTA model with respect to $\mu^{(n)}$. Below we present the sensitivity analysis-based algorithm for solving the bi-level programming formulated multi-modal network spare capacity.

Step 0: *Initialization*. Determine an initial value $\mu^{(0)}$. Set n: = 0.

Step 1: *Solving lower-level problem.* Solve the CMSTA model in lower-level based on $\mu^{(n)}$, and obtain the equilibrium link flows $\{v_a^m\}$ and mode specific O-D demand $\{q_{rs}^m\}$.

Step 2: *Sensitivity analysis.* Calculate the partial derivatives, $\nabla_\mu v_a^m(\mu^{(n)})$ and $\nabla_\mu q_{rs}^m(\mu^{(n)})$ using the sensitivity analysis method for the CMSTA model.

Step 3: *Local linear approximation.* Formulate local linear approximations of the upper-level capacity constraints using the derivatives, and then solve the approximate linear programming problem to produce an auxiliary multiplier $\hat{\mu}^{(n)}$.

Step 4: *Updating solution.* Let $\mu^{(n+1)} = \mu^{(n)} + \alpha \cdot (\hat{\mu}^{(n)} - \mu^{(n)})$ where α is the step size.

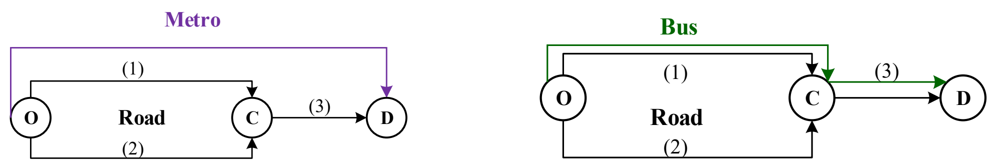
Step 5: *Convergence criterion.* If the convergence criterion is satisfied, then stop; otherwise, return to Step 1, and set $n := n + 1$.

4. NUMERICAL EXAMPLES

In this section, we provide two examples to illustrate: (i) the effect of adding new mode to the network redundancy; (ii) the effect of considering mode similarity (with nested structure) for evaluating the network capacity. The parameters for travel choices are assumed as $\theta_K = 1.0$, $\theta_M = 0.5$, and $\theta_U = 0.7$. The PCE factor, PCE_m, is set to 2.5 for bus, and 1 for passenger car. The average occupancy, OCC_m, is set to 30 for bus, and 1.2 for passenger car. The attractiveness of the car, bus and metro are set to 0, -3.5, and -1.5, respectively, which assume driving is more preferred in this area. Also, the travel cost of the metro is assumed to be a constant, and here let $w_{rs}^{metro} = 15$.

4.1 Effect of Adding Mode Choice Alternative

Consider the road network associated with only one O-D pair (from node O to node D), as figure 3 shows. The travel time functions of the three (road) links are given by: $t_1 = 10 + 2(v_1/C_1)$, $t_2 = 10 + 2(v_2/C_2)$, and $t_3 = 5 + (v_3/C_3)$. The link capacities are assumed as 800, 800 and 1300, respectively. It is easy to identify the network capacity is 1,300 from O to D, limited by the capacity of link 3. If the government wants to promote the systematic transport capacity by adding a new travel mode, the metro and bus can be two usual options. As figure 3a, to add metro to the transportation system, a separate new line should be constructed physically independent to the road network. As figure 3b, to add bus service, the existing road network can be utilized by sharing links with passenger cars. Here, we assume the capacity of the new metro line is 2,000, and the travel cost is fixed as 30. For the bus service, we assume the new bus line travels along links 1 and 2, and its capacity is 800. The capacity of the bi-modal network can be evaluated by using the models in section 3.2.2.



(a) construct a new metro line (b) operate a new bus service

Figure 3. Schemes of adding new mode to the road network

As the results of the model, by adding a new metro line, the network capacity will increase from 1,300 to 2,272; while by adding a new bus service, the network capacity will increase to 1,996. In both cases, the total travel demand is limited by link 3 from the road network. Same as experiences, adding a new metro line can expand the total network capacity more compared with adding a new bus service. One intuitive reason is that the metro usually owns independent network facilities and has a larger transport ability. However, in this example, the capacity of the bi-modal network is not restricted from the capacity of the new modes. Another persuasive reason is the EPC of the metro is lower than that of the bus, because the bus travel time is affected by the traffic congestion which is significantly increasing along with the growth of the travel demand. Also, the attractiveness of the metro is usually higher than the bus. Therefore, the probability of choosing metro (31%) is larger than that of choosing bus (25%) according to the results of the two capacity models.

4.2 Effect of Route Overlap and Mode Similarity

This section shows how considering route overlap and mode similarity will affect the mode share and further impact the network capacity evaluation. We consider the combination of all three modes from the previous section (see figure 4). The setting values of the parameters are same. The lengths of links 1, 2 and 3 are assumed to be 10, 10 and 20 unit, separately. The multi-modal network capacity is evaluated under three formulations: (1) with the MNL-MNL model (without considering route overlap and mode similarity), (2) with the MNL-PSL model (only dealing with route overlap), and (3) with the NL-PSL model (dealing with both route overlap and mode similarity).

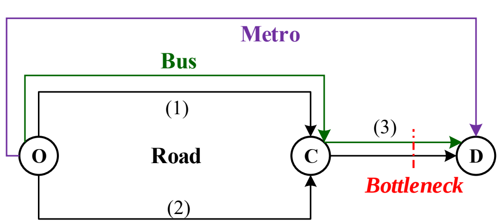


Figure 4. A network with three travel modes

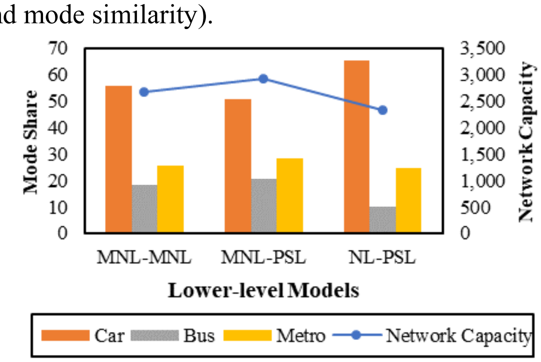


Figure 5. Results of three network capacity formulations with different lower-level models

From the results of the three models, the capacity on road link 3 is always the binding constraint to the total network capacity. Figure 5 demonstrates the change on total network capacity and the mode share rates when considering route overlap and mode similarity. From the MNL-MNL to MNL-PSL model, by which the route overlapping problem is tackled, the EPC of car mode is corrected by the path-size factor of PSL. It results the EPC of car increasing, and thus the travel demand transfers from the car mode to the other two modes. As there are much spare capacity of the bus and metro lines, more total travel demand can be raised until the road network reaches saturation again. Therefore, the network capacity evaluated by using MNL-PSL is obviously larger than that by MNL-MNL. Furthermore, from the MNL-PSL to NL-PSL model, the mode similarity is modeled by NL. The nest structure enlarges the EPC of the travel modes from the same nest by consider the correlations. It results the travel demand transferring from the public transit modes back to the car mode. Thus, the crowdedness in road network is increased, and then the whole system reaches its capacity with a lower level of demand because of the restriction from road network. In this specific case, without considering the mode similarity, the network capacity could be overestimated. Note that considering both route overlap and mode similarity may not necessary to increase the network capacity result.

5. CONCLUSIONS

This study extended the network redundancy analysis to further consider the multiple mode options. A combined mode split and traffic assignment (CMSTA) model is formulated step by step originating from the stochastic user equilibrium with elastic demand. In the CMSTA, the route overlapping problem is resolved by using the PSL model, while the mode similarity is embodied in the NL model. Numerical results showed the how adding mode options can significantly increase the network capacity, and further contribute to the transportation redundancy. The results also showed that considering either route overlap or mode similarity will result in the increasing of the EPC for the group of correlated route or mode alternatives. This changes the mode share rates under different formulations, and moreover evidently impacts the network capacity evaluation.

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