Modeling the Impacts of Speed Limits on Uncertain Road Networks

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Abstract

Road speed limits have been widely implemented in many countries to improve traffic safety and environmental quality. In this paper, we provide a different perspective on the network-wide impacts of speed limits: managing network uncertainty. Specifically, a multi-dimensional modeling approach is proposed to investigate the impacts of speed limits on link travel time uncertainty profile, travelers' risk-averse route choice decisions, and network flow reallocation under uncertainty. Methodologically, a truncated travel time probability distribution is proposed and its moment expression is derived to characterize the impact of speed limit on link travel time uncertainty profile. The mean-excess travel time (METT) is then adopted as a risk measure to account for the travelers' risk-averse route choice adjustment and network flow reallocation induced by speed limits. We find that a speed limit scheme has different impacts on various dimensions of travel time uncertainty. After imposing a speed limit, the mean travel time on the directly affected link is increased while the travel time variance is reduced. However, both the skewness (i.e., asymmetry indicator) and kurtosis (i.e., fatness indicator of distribution tail) are significantly increased as the speed limit becomes tighter. Also, a speed limit increases both the travel time budget and METT on the directly affected links. The shift in travel time uncertainty profile of the directly affected links further adjusts the traveler's risk-averse route choice decisions, the travel time uncertainty profile of the indirectly affected links, and consequently the network flow reallocation.

Keywords: speed limit; network uncertainty; route choice; mean-excess travel time

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1 INTRODUCTION

Recent empirical studies have pointed out that travel time uncertainty plays an important role in travelers' travel choice decisions (e.g., route choice and departure time choice). Travelers treat travel time uncertainty as a risk in their travel choices, because it introduces uncertainty for an on-time arrival at the destination and travel schedule. Transportation systems uncertainty could be managed or handled from travel demand side (i.e., O-D demand), traffic supply side (i.e., link capacity and road speed control), or travelers' behavior side (i.e., travelers' perception on network conditions). The typical strategies of managing or hedging against uncertainty include network capacity enhancement (e.g., Chen *et al.*, 2011a), travelers' information provision (e.g., Siu and Lo, 2006; Xu *et al.*, 2013), and road pricing (e.g., Li *et al.*, 2008; Gardner *et al.*, 2008; Dong and Mahmassani, 2013). In this paper, we address the possibility of adopting speed limit to explicitly manage network uncertainty.

Road speed limits are widely implemented in most countries to regulate the maximum permissible vehicular speed. The main motivation of speed limits is to improve road traffic safety and to reduce the number of road traffic casualties from traffic collisions. The World Health Organization (WHO, 2004) identifies speed control as one of various interventions likely to reduce road casualties. Other than the safety consideration, speed limits can also be used to improve local air quality issues or other factors affecting environmental quality (e.g., vehicular noise, vibration, and emission). Some cities have reduced speed limits to as low as 30 km/h for improving both safety and efficiency (Archer *et al.*, 2008). For a more comprehensive review on the impacts of speed limits on accidents, CO_2 and air pollution emissions, noise emissions and speed choice behaviors, interested readers are directed to Nitzsche and Tscharaktschiew (2013).

Most existing studies focused on the impact of speed limits from a local perspective, while the network flow reallocation effect due to speed limits has not been explored in depth. Recently, Yang *et al*. (2012) made the first attempt to investigate the impact of a link-specific speed limit law on the *network-wide* traffic flow reallocation. Yang *et al*. (2013) extended the speed limit problem to a tri-objective bi-level programming model to design optimal link-specific speed limits with the minimization of total travel time (TTT), number of expected accidents and vehicular emissions simultaneously, while Wang (2013) considered designing the optimal speed limit scheme to maximize network efficiency (i.e., minimize TTT) with consideration of network equity (i.e., travel time change after imposing a speed limit scheme among road users from different origin-destination (O-D) pairs). Both studies found that TTT may decrease or increase after imposing a speed limit scheme. In addition, Nitzsche and Tscharaktschiew (2013) provided a more general/overall assessment of speed limits on an urban economy. They employed a simulation-based spatial computable general equilibrium model to an 'average' German metropolitan area and its residents, while considering the behavioral changes (e.g., route choice, mode choice, adjustments in time allocation, and changes in demand patterns) and economic, environmental and spatial effects. Yang *et al*. (2015) further considered road users' non-obedient behavior in speed selection. Road users determine their actual speeds on each link according to the subjective travel time cost, perceived crash risk, and perceived ticket risk. Heterogeneous travelers interact with each other and choose their own optimal speed, resulting in a Nash equilibrium speed pattern. To the best knowledge of the authors, there are very few studies that explicitly investigate the impact of speed limits on traveler's *risk-taking* route choice decisions and the resultant network flow reallocations *under uncertainty*. An exception is that Yan *et al*. (2015) examined the impacts of speed limits on the performance of link capacity degradable transportation networks via the mean and variance of link and route travel time as well as total travel time. They found that imposing some speed limits can reduce the mean and standard deviation of total travel time simultaneously for some networks, but cannot always reduce the total travel time budget of a network.

Intuitively, after imposing a speed limit on a link, the link (average) travel time will be increased, and the travel time distribution may be squeezed with a larger leftmost point (i.e., larger mean and smaller variability). Speed limits have a direct impact on travel time uncertainty. *How does a speed limit change the travel time uncertainty profile*? *Does the speed limit help to reduce travel time uncertainty?* The changes in travel time uncertainty profile may further affect the travelers' risk-taking route choice decisions. The risk-averse route choice adjustment caused by speed limits will lead to the traffic flow reallocation, further affecting the network uncertainty profile. Travelers have different risk-aversion attitudes towards travel time uncertainty. *Which group of travelers (more risk-averse or less risk-averse) will be more affected by speed limits in their route choice decisions*? Addressing these issues is critical in understanding travelers' responses to a speed limit scheme and assessing the effect of a speed limit scheme under an uncertain environment.

In this paper, we address the possibility of using speed limits to manage network uncertainty, which is significantly different from prior studies on investigating the network-wide impacts of speed limits (e.g., Yang *et al*., 2012, 2013; Wang, 2013). To this end, we explicitly model the multi-dimensional impacts of speed limits on link travel time uncertainty, travelers' risk-averse route choice decisions, and network flow reallocation under uncertainty. Specifically, the modeling approaches include: (1) a truncated link travel time probability distribution is proposed and its moments expression is derived to characterize the effect of speed limits on link travel time uncertainty profile; and (2) the risk measure of mean-excess travel time (METT) introduced by Chen and Zhou (2010) is adopted as a risk-averse route choice criterion with a complete consideration of both on-time arrival reliability and late arrival unreliability. By solving the METT-based traffic equilibrium model, we are able to uncover the impacts of a speed limit scheme on the travel time uncertainty profiles of both directly and indirectly affected links as well as the resultant network flow reallocation from collective route choice decisions under an uncertain environment.

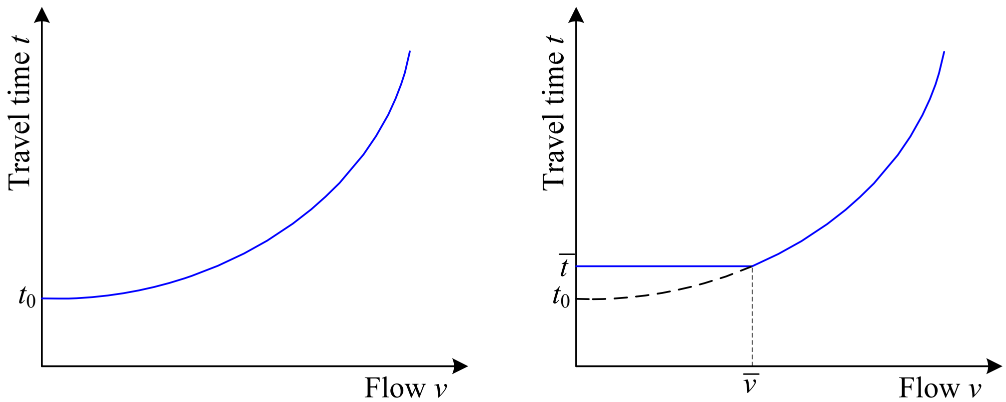
The remainder of this paper is organized as follows. Section 2 models the effect of speed limit on link travel time uncertainty profile. Section 3 presents the METT-based route choice model under uncertainty. To further consider congestion effect, Section 4 presents the METT-based traffic equilibrium model. Then, Section 5 provides some numerical examples to illustrate the effect of speed limit on network uncertainty. Finally, some concluding remarks are summarized in Section 6. It is worth noting that to lay out the modeling and analysis step by step, Section 2 and Section 3 model the *direct impacts* of speed limit on the travel time distribution and METT of the link that speed limit is directly imposed on (i.e., travel time uncertainty pattern is given without rerouting consideration). Section 4 then models the *indirect impacts* of speed limit on network uncertainty through rerouting and network flow reallocation.

2 Modeling Effect of Speed Limit on Link Time Uncertainty

In this section, we model the effect of speed limit on link travel time uncertainty by using a truncated link travel time probability distribution and its moments. Consider a strongly connected network $G=[N, A]$, where N and A denote the sets of nodes and links, respectively. Let W denote the set of O-D pairs, and P^w denote the set of routes connecting O-D pair $w \in W$.

2.1 Truncated Link Travel Time Distribution and Moments Derivation

The link travel time functions without and with speed limit are shown in Figure 1. Without a speed limit, the minimum travel time is the free-flow travel time t_0. After imposing a speed limit $\bar{s}$, the minimum travel time ($\bar{t}$) equals the link length (L) divided by the speed limit ($\bar{s}$), i.e., $\bar{t} = L/\bar{s}$.



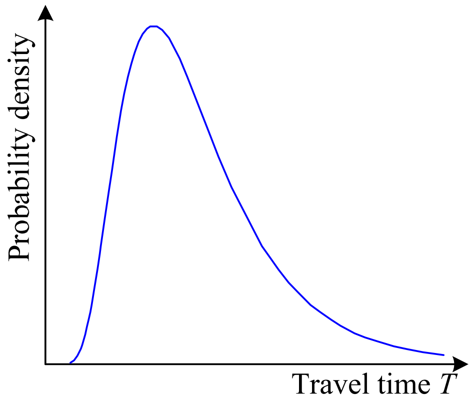
(a) Without speed limit (b) With speed limit

Figure 1 Link performance (or travel time) functions (Yang *et al.*, 2012)

The above link travel time functions provide an aggregate relationship between a deterministic traffic flow and a deterministic travel time. To deal with the travel time uncertainty, we look at the probability density function (PDF) or cumulative distribution function (CDF) of link travel time. Without loss of generality, Figure 2(a) illustrates an asymmetric PDF of link travel time without speed limit. After imposing a speed limit scheme $\bar{s}$, the original fast drivers need to slow down with the minimum travel time of $\bar{t} = L/\bar{s}$, while the slow drivers on the right side of the distribution tail are not much affected. Hence, we can model the link travel time distribution with speed limit as a truncation of the original distribution without speed limit, as shown in Figure 2(b). However, a direct truncation of the original PDF does not provide a valid PDF. Instead, we need to scale it so that it can be properly integrated to one to fulfill the conservation property of a valid PDF. Mathematically, the new PDF of link travel time with speed limit can be expressed as

$$f\left(T \middle| T > \bar{t}\right) = \frac{f(T)}{1 - F(\bar{t})}, \tag{1}$$

where $f(.)$ and $F(.)$ are the PDF and CDF of the random travel time T, respectively; and $F(\bar{t}) = \Pr(T \le \bar{t})$. Note that for a given speed limit, the fixed denominator (a fraction) makes the distribution taller. However, different parts of the PDF (e.g., the part immediate to the right of the speed limit, and the part toward the tail) have different proportions, making the scaling effect visually different.



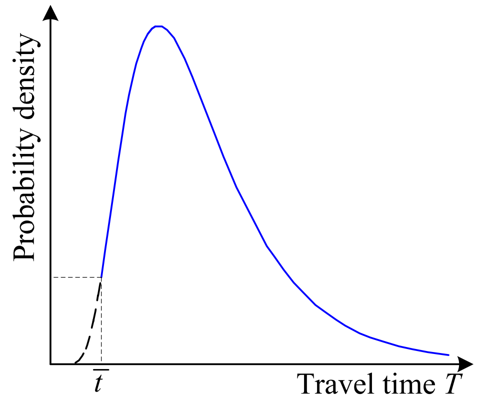


Figure 2 Link travel time probability distributions

Remark: We recognize that the truncated or rescaled travel time distribution may not be a realistic way for modeling the unknown impact of speed limit on travel time uncertainty. Other than the truncated/rescaled travel time distribution suggested in this paper, one may consider other ways of modeling the impacts of speed limit under uncertainty. The selection of modeling approach needs to consider the tradeoff between behavioral realism and mathematical tractability. For example, rather than truncated continuous distributions, we could also use discrete distributions by assuming all previous speeding travelers will drive around the newly posted speed limit. This approach may be more realistic and understandable. However, it loses some mathematical tractability due to its discontinuity when embedding it to the network equilibrium model. Considering the truncation or rescaling, our proposed approach underestimates the probability around the speed limit and overestimates the probability of the distribution tail greater than the speed limit, and accordingly overestimates the travel time budget (TTB) and mean-excess travel time (METT). Alternatively, one may use shifted travel time distributions. However, a direct shift of travel time distribution (e.g., lognormal) leads to a low frequency/probability near the speed limit point (i.e., the location parameter of travel time distribution). A (new) speed limit not only changes the location parameter of travel time distribution, but also affects the entire distribution profile (e.g., the variance, skewness and kurtosis). A straightforward example is that a tighter speed limit reduces the travel time variability; while a simple distribution shift does not change the variance. After imposing a tighter speed limit scheme, the original fast drivers (i.e., the short travel times between the two location parameters of the before and after distributions) need to slow down with a larger minimum travel time, while the slow drivers on the right-hand side of the distribution tail are not much affected. The original fast drivers usually do not slow down too much; instead, their driving preference may make them drive just around the new speed limit. Accordingly, the travel times around the new speed limit will have a much greater frequency than that in the simple shifted distribution. This change of travel time distribution around the new speed limit can be better captured by our proposed approach, to be shown in Figure 3 and Figure 7. In summary, the above difficulties and complexities motivate us to use a truncated/rescaled travel time distribution in this paper.

Let us consider the lognormal distributed link travel time $T \sim LN(\mu, \sigma)$. Note that lognormal distribution has been extensively adopted in general reliability applications to model failure times. It can capture the asymmetric and skewed characteristics of a random variable. In the context of transportation systems, it has been used to characterize travel demand and travel time uncertainties (e.g., Zhao and Kockelman, 2002; Zhou and Chen, 2008). Furthermore, recent empirical studies have also justified the use of lognormal distribution to characterize travel time uncertainty. For example, Arezoumandi (2011) fitted the 24-hour travel time data across Interstate 255/270 in St. Louis County, Missouri, USA to several distributions, including gamma, largest extreme value, log-logistic, lognormal, and Weibull distributions. He concluded that the lognormal distribution provided the best fit for post-variable speed limit conditions. Chen *et al*. (2014) used travel time estimates of 242 weekdays from the real-time travel information system in Hong Kong to generate travel time distribution for each link (totally 3655 links) and each 5-minute interval. The link travel time distributions were fitted to either a lognormal or a normal distribution using chi-square tests (83.4% were lognormal distributions and 16.6% were normal distributions). Thus, lognormal distribution is a reasonable PDF for characterizing travel time distribution. However, other valid distributions can also be used in the proposed analysis framework.

The n-th generic moment of T without speed limit can be expressed as follows (Johnson *et al*., 1994):

$$E\left[T^n\right] = \exp\left(n \cdot \mu + \frac{n^2}{2}\sigma^2\right). \tag{2}$$

The two parameters μ and σ can be calculated from the mean and variance of travel time (i.e., t and ε):

$$\mu = \ln(t) - \frac{1}{2}\ln\left(1 + \varepsilon/(t)^2\right), \tag{3}$$

$$\sigma^2 = \ln\left(1 + \varepsilon/(t)^2\right). \tag{4}$$

With a speed limit $\bar{s}$ (i.e., the minimum travel time is $\bar{t} = L/\bar{s}$), the truncated travel time has the following PDF:

$$f\left(T \middle| T > \bar{t}\right) = \frac{\frac{1}{T\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(\ln T - \mu)^2}{2\sigma^2}\right]}{1 - \Phi\left(\frac{\ln \bar{t} - \mu}{\sigma}\right)}. \tag{5}$$

The n-th generic moment of the truncated travel time can be written as follows:

$$E\left[T^n \middle| T > \bar{t}\right] = \frac{\int_{\bar{t}}^{\infty} T^n \frac{1}{T\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(\ln T - \mu)^2}{2\sigma^2}\right]dT}{\Phi\left(-\frac{\ln \bar{t} - \mu}{\sigma}\right)}. \tag{6}$$

By setting $x = (\ln T - \mu)/\sigma$, i.e., $T = \exp(\mu + x\sigma)$, we have

$$\begin{aligned}
&\int_0^{\bar{t}} T^n \frac{1}{T\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(\ln T - \mu)^2}{2\sigma^2}\right]dT \\
&= \int_{-\infty}^{\frac{\ln \bar{t} - \mu}{\sigma}} \exp(n\mu + nx\sigma)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)dx \\
&= \exp(n\mu)\int_{-\infty}^{\frac{\ln \bar{t} - \mu}{\sigma}} \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2} + nx\sigma\right)dx \\
&= \exp\left(n\mu + \frac{n^2\sigma^2}{2}\right)\int_{-\infty}^{\frac{\ln \bar{t} - \mu}{\sigma}} \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{(x - n\sigma)^2}{2}\right]dx.
\end{aligned} \tag{7}$$

By setting $y = x - n\sigma$, we have

$$\begin{aligned}
&\int_0^{\bar{t}} T^n \frac{1}{T\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(\ln T - \mu)^2}{2\sigma^2}\right]dT \\
&= \exp\left(n\mu + \frac{n^2\sigma^2}{2}\right)\int_{-\infty}^{\frac{\ln \bar{t} - \mu}{\sigma} - n\sigma} \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{y^2}{2}\right)dy \\
&= \exp\left(n\mu + \frac{n^2\sigma^2}{2}\right)\Phi\left(\frac{\ln \bar{t} - \mu}{\sigma} - n\sigma\right) = E\left[T^n\right]\Phi\left(\frac{\ln \bar{t} - \mu}{\sigma} - n\sigma\right).
\end{aligned} \tag{8}$$

where $E\left[T^n\right] = \exp\left(n\mu + \frac{n^2}{2}\sigma^2\right)$. Finally, we have

$$E\left[T^n \middle| T > \bar{t}\right] = \frac{E\left[T^n\right] - \int_0^{\bar{t}} T^n \frac{1}{T\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\left(\ln T - \mu\right)^2}{2\sigma^2}\right] dT}{\Phi\left(-\frac{\ln \bar{t} - \mu}{\sigma}\right)}$$

$$= \frac{E\left[T^n\right] - E\left[T^n\right]\Phi\left(\frac{\ln \bar{t} - \mu}{\sigma} - n\sigma\right)}{\Phi\left(-\frac{\ln \bar{t} - \mu}{\sigma}\right)} = \frac{E\left[T^n\right]\Phi\left(n\sigma - \frac{\ln \bar{t} - \mu}{\sigma}\right)}{\Phi\left(-\frac{\ln \bar{t} - \mu}{\sigma}\right)}. \tag{9}$$

With the n-th generic moment expression, we can further calculate the variance, skewness, and kurtosis, which are related to the second, third and fourth moments, respectively. To simplify the notation, we use T' to denote the random travel time under speed limit (i.e., with the lower bound $\bar{t}$), $E\left[T^n \middle| T > \bar{t}\right] = E\left[T'^n\right]$.

$$\text{Variance} = E\left[T'^2\right] - \left(E\left[T'\right]\right)^2, \tag{10}$$

$$\text{Skewness} = \frac{2\left(E\left[T'\right]\right)^3 - 3E\left[T'\right] \cdot E\left[T'^2\right] + E\left[T'^3\right]}{\left(E\left[T'^2\right] - \left(E\left[T'\right]\right)^2\right)^{\frac{3}{2}}}, \tag{11}$$

$$\text{Kurtosis} = \frac{-6\left(E\left[T'\right]\right)^4 + 12E\left[T'^2\right]\left(E\left[T'\right]\right)^2 - 3\left(E\left[T'^2\right]\right)^2 - 4E\left[T'\right]E\left[T'^3\right] + E\left[T'^4\right]}{\left(E\left[T'^2\right] - \left(E\left[T'\right]\right)^2\right)^2}. \tag{12}$$

Note that most PDFs have at most three or four parameters to be estimated (e.g., location, scale, and shape parameters). The first four moments are generally sufficient to fit/estimate an asymmetric PDF (Hill *et al*., 1976; Clark and Watling, 2005).

2.2 Illustrative Example of Truncated Link Travel Time Distribution

For illustration purposes, a single-link network is considered. Assume the travel time follows the lognormal distribution with the mean of 15 and the coefficient of variation (COV) of 0.30. The link length is 10 km. Figure 3 shows the probability density profiles of travel time without speed limit and with the speed limit of 50 km/h. The speed limit of 50 km/h corresponds to the minimum travel time of 12 min. One can see that the travel time distribution above the minimum travel time of 12 min (i.e., solid curve) is significantly different from the case with speed limit (i.e., dashed curve). The truncated distribution needs to be scaled so that it can be integrated to one as a valid PDF.

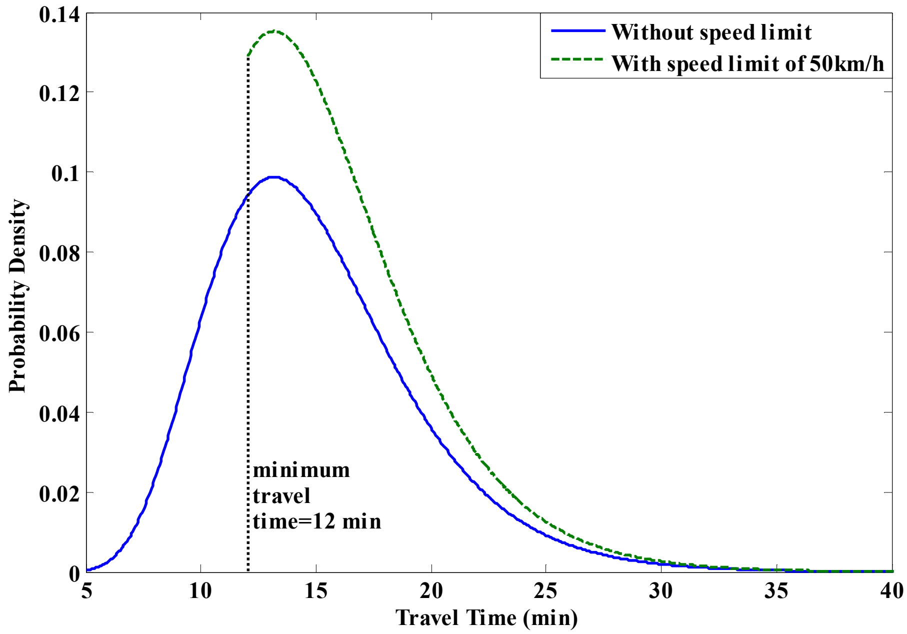


Figure 3 Probability density functions of link travel time with and without speed limit

To be more precise, Table 1 presents the common statistics (i.e., mean, standard deviation (SD), COV, skewness, and kurtosis) of link travel time under different speed limit schemes. These statistics correspond to the first four moments of a random variable. Hence using them together could provide a relatively accurate and complete characterization of travel time uncertainty compared to the mean and SD alone. SD and COV are usually adopted to measure the absolute and relative dispersion, respectively; skewness characterizes the asymmetry of a probability distribution, while kurtosis characterizes the flatness or peakedness of a distribution relative to the normal distribution. A normal distribution has zero skewness and zero excess kurtosis (or kurtosis of 3). If the skewness S is less than -1 or greater than +1, the distribution is highly skewed. Also, if the (excess) kurtosis K is greater than 0, the distribution is called leptokurtic. Compared to a normal distribution, its central peak is higher and sharper, and its tail is longer and fatter.

We would expect an increase of the expected travel time and a reduction of the variance due to the truncation of speed limit. The reason is that a speed limit scheme makes the original fast drivers (i.e., short travel times) slow down while the original slow drivers are not much affected. These changes can be observed by the mean, SD and COV columns of Table 1. Speed limits can contribute to the travel time variability reduction on the imposed links. Small travel time variability is generally desirable, since it could enhance the predictability of actual travel times. To guarantee a high reliability of punctual arrival at the destination, some travelers even prefer a route with high mean but low travel time variance.

Table 1 Effect of speed limits on link travel time distribution characteristics

Speed limit	Mean		SD		COV		Skewness		Kurtosis	
No	15.00		4.50		0.30		0.93		1.57	
70	15.30		4.33		0.28		1.08		1.85	
65	15.48		4.25		0.27		1.14		2.01	
60	15.76		4.15		0.26		1.22		2.23	
55	16.18		4.03		0.25		1.31		2.54	
50	16.80	↑	3.88	↓	0.23	↓	1.41	↑	2.94	↑
45	17.69		3.71		0.21		1.53		3.44	
40	18.96		3.54		0.19		1.65		4.03	
35	20.77		3.38		0.16		1.77		4.68	
30	23.36		3.24		0.14		1.88		5.35	

Note: ↑: increase from No to 30; ↓: decrease from No to 30.

However, this is **not a complete picture** about the impacts of speed limits. From Table 1, we can also see that both the skewness and kurtosis values increase as the speed limit scheme becomes tighter. Particularly, the kurtosis value has a significant increase. In the context of travel time uncertainty, large skewness and kurtosis values are generally not preferable. A *larger skewness value* indicates that the travel time distribution become more asymmetric, and there is a *higher probability of encountering extremely long travel times* (relative to the mean travel time). The kurtosis value quantifies the weight of distribution tail relative to the rest of a distribution. A *larger kurtosis value* indicates that the right tail of travel time distribution (i.e., *the slower travel times*) becomes fatter. To sum up, a speed limit scheme has different impacts (positive or negative) on different dimensions of travel time uncertainty (e.g., centrality, variability, asymmetry, and peakedness). Hence, both the *positive* impact (in terms of the variability) and the *negative* impact (in terms of the asymmetry and peakedness) need to be considered simultaneously in the assessment and design of a speed limit scheme. After imposing speed limits, the travel time distribution exhibits a strong positive skewness and a significantly long and fat upper tail. From the travelers' viewpoint, a longer and fatter travel time distribution tail corresponds to a higher degree of uncertainty, particularly a larger unreliability. This distribution profile shift may further have a significant impact on traveler's risk-taking route choice decisions (to be shown in Sections 3.2 and 5).

3 METT-based Route Choice Model under Uncertainty

This section presents the concept of mean-excess travel time (METT) and illustrates the impact of speed limits on METT-based route choice decisions under uncertainty. In the literature, many risk criteria have been proposed for capturing the travelers' route choice decisions under travel time uncertainty, such as travel time budget (Lo *et al*., 2006), percentile travel time (Nie and Wu, 2009), and METT (Chen and Zhou, 2010). We refer to Xu *et al*. (2017) for a more detailed summary on the existing traffic equilibrium models under uncertainty, including expected utility, reliability, prospect theory, game theory, ambiguity-aware CARA (constant absolute risk aversion) travel time model, stochastic dominance, and multi-objective optimization, etc.

3.1 Mean-Excess Travel Time (METT)

Definition (Chen and Zhou, 2010): *The* ***mean-excess travel time*** (METT) $\eta_p^w(\alpha)$ *on route p between O-D pair w with respect to a predefined confidence level α is defined as the conditional expectation of route travel time* T_p^w *exceeding the corresponding travel time budget* (TTB) $\xi_p^w(\alpha)$, i.e.,

$$\eta_p^w(\alpha) = E\left[T_p^w \middle| T_p^w \geq \xi_p^w(\alpha)\right], \ \forall p \in P^w, w \in W, \tag{13}$$

where $E[\cdot]$ is the expectation operator; and $\xi_p^w(\alpha)$ is endogenously determined by the following travel time reliability chance-constrained model:

$$\xi_p^w(\alpha) = \min\left\{\xi \middle| \Pr\left(T_p^w \leq \xi\right) \geq \alpha\right\}, \tag{14}$$

$$= E\left[T_p^w\right] + \gamma_p^w(\alpha) \ , \ \forall p \in P^w, w \in W, \tag{15}$$

where $\gamma_p^w(\alpha)$ is a buffer time added to the expected travel time $E\left[T_p^w\right]$ to ensure the travel time reliability requirement for on-time arrivals at the confidence level (CL) α. Given a CL α, the TTB is the minimum threshold allowed by travelers such that the cumulative probability of actual travel time less than this threshold is at least α.

Meanwhile, Eq. (13) can be decomposed as:

$$\eta_p^w(\alpha) = \xi_p^w(\alpha) + E\left[\left(T_p^w - \xi_p^w(\alpha)\right) \middle| T_p^w \geq \xi_p^w(\alpha)\right], \ \forall p \in P^w, w \in W, \tag{16}$$

where the first and second terms represent the reliability (in terms of TTB) and unreliability (in terms of expected excess delay (EED)) aspects of travel time uncertainty, respectively. These two terms explicitly characterize the left region of the travel time distribution with the α-percentile reliability requirement and the right region with the (1-α) percentile of unreliability in the distribution tail, respectively. In this sense, METT considers both on-time arrival reliability requirement (via TTB) and late arrival unreliability (via EED), while TTB only considers on-time arrival reliability requirement. We should point out that the decomposition of METT into two terms (i.e., TTB and EED) is only for the convenience of explicitly presenting the composition of METT. However, these two terms are interdependent through the CL.

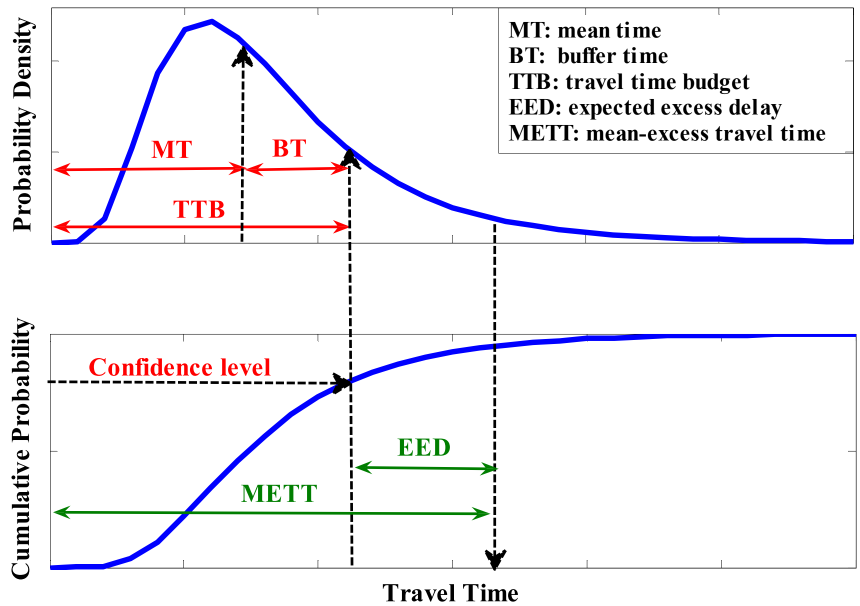


Figure 4 Illustration of mean-excess travel time (Chen and Zhou, 2010)

3.2 Effect of Speed Limit on the METT-based Route Choice Decisions

(1) Theoretical Analysis

First of all, we provide a theoretical analysis on the METT change of the imposed link before and after implementing a speed limit scheme.

Proposition 1: *Without considering network flow reallocation, speed limit increases both TTB and METT on the directly affected links.*

Proof. Recall that METT is defined according to TTB as shown in Eq. (13). Also, the CDF is monotonically increasing for continuous travel time distributions. Then, the cumulative probability α (i.e., travel time reliability (TTR)) can be uniquely mapped to the corresponding travel time value (i.e., TTB). With this relationship, we examine the change of TTR first. The difference (with and without speed limit) in the reliability of finishing this trip within the threshold b ($b > \bar{t}$) can be expressed as:

$$
\begin{aligned}
TTR_{\text{with}}(b) - TTR_{\text{without}}(b) &= \int_{\bar{t}}^{b} \frac{f(T)}{1-F(\bar{t})} dT - \int_{0}^{b} f(T)\, dT \\
&= \frac{F(b) - F(\bar{t})}{1-F(\bar{t})} - F(b) \\
&= \frac{F(b) - F(\bar{t}) - F(b) + F(b)F(\bar{t})}{1-F(\bar{t})} \\
&= \frac{F(b)F(\bar{t}) - F(\bar{t})}{1-F(\bar{t})} < 0,
\end{aligned}
\tag{17}
$$

where $F(b) < 1$ is used in the inequality. The CDF curve with speed limit is below the CDF curve without speed limit. For the same travel time threshold, the reliability of on-time arrival is reduced after imposing a speed limit scheme. Considering the relationship between TTR and TTB, we have:

$$
TTB_{\text{with}}(\alpha) - TTB_{\text{without}}(\alpha) > 0. \tag{18}
$$

In other words, the TTB on the imposed link of speed limit is increased. According to Chen *et al*. (2011b), METT can be expressed in terms of TTB as follows:

$$
METT(\alpha) = \frac{1}{1-\alpha} \int_{\alpha}^{1} TTB(\tau)\, d\tau. \tag{19}
$$

By substituting Eq. (18) into Eq. (19), we can obtain:

$$
\begin{aligned}
& METT_{\text{with}}(\alpha) - METT_{\text{without}}(\alpha) \\
&= \frac{1}{1-\alpha} \int_{\alpha}^{1} TTB_{\text{with}}(\tau)\, d\tau - \frac{1}{1-\alpha} \int_{\alpha}^{1} TTB_{\text{without}}(\tau)\, d\tau \\
&= \frac{1}{1-\alpha} \int_{\alpha}^{1} \left(TTB_{\text{with}}(\tau) - TTB_{\text{without}}(\tau) \right) d\tau > 0.
\end{aligned}
\tag{20}
$$

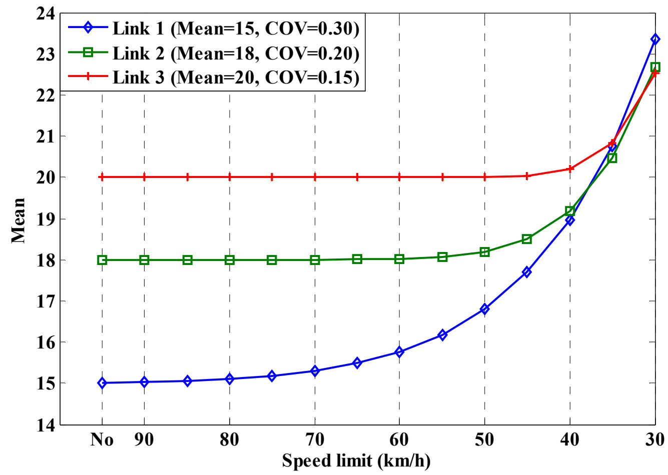
The above theoretical analysis indicates that speed limits always increase both the TTB and METT on the *directly* affected links. However, the impacts on the *indirectly* affected links are not apparent due to the network flow reallocation based on the METT route choice criterion under network uncertainty.

(2) Numerical Illustration

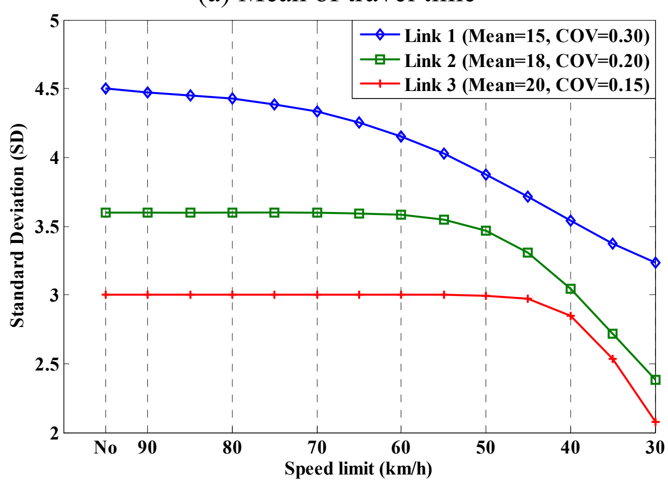
Below we consider a simple network with one O-D pair connected by three parallel links to illustrate the effect of speed limit on the METT-based route choice model. Specifically, we examine: (a) the impact of speed limits on travelers' risk-averse route choice criteria in terms of mean, SD and METT, (b) the impact of different speed limit schemes on the travelers' route choice shift, and (c) the impact of speed limits on different groups of travelers classified by their risk-aversion attitudes.

The three links have the same length of 10 km. The link travel times follow the lognormal distribution with the mean of 15, 18, and 20 and the COV of 0.30, 0.20, and 0.15, respectively. Link 1 has the smallest mean travel time but the largest travel time variability; while link 3 has the largest mean travel time but the smallest travel time variability. Figure 5(a) and Figure 5(b) show different impacts of speed limits on the mean and SD of travel time on each link. Specifically, the mean of travel time on three links is always increasing as the speed limit becomes tighter; whereas the SD of travel time on three links is always decreasing in this process. It seems that there is a tradeoff on the impact of speed limits between the mean and SD of travel time. This indicates the need to explicitly consider the traveler's risk-taking route choice decisions (i.e., with a risk strategy towards uncertainty) when modeling the travelers' behaviors and assessing the impact of a speed limit scheme. Different risk criteria (e.g., mean, mean-variance, TTB, and METT) measure the relationship between mean and SD differently, leading to different route choice decisions. In this example, we adopt the METT as a risk-averse route choice criterion to consider the travelers' on-time arrival reliability and late arrival unreliability simultaneously. All travelers are assumed to have the same confidence level of 0.85.

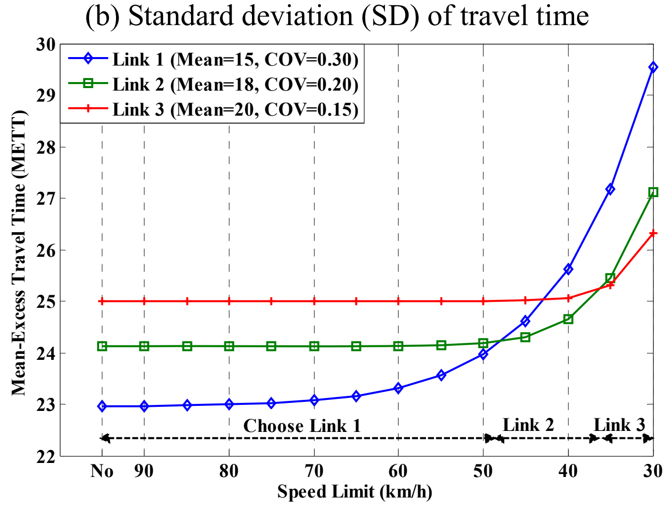
Before implementing any speed limit scheme (i.e., do-nothing case), link 1 has the smallest METT as shown in Figure 5(c). Accordingly, without congestion consideration, all travelers will choose link 1 due to the lowest 'cost' or 'disutility'. After imposing a speed limit scheme, the METTs of all three links are increased but with different extents, leading to different route choice decisions. From link 1 to link 3, the METT curves become steadier. The reason is that link 1 has the smallest mean travel time and the largest variance, whose stochastic characteristics are more affected by the truncation of speed limits. In addition, the turning point of METT curve between the steady portion and the increasing portion is different for the three links. When the uniform speed limit is larger than 50 km/h, the METT relationship of three links is unchanged relative to the do-nothing case. When the speed limits are set at 35-45 km/h, link 2 has the smallest METT; whereas if the speed limits are further tighter, link 3 becomes the optimal choice with the smallest METT. With a strict speed limit scheme (i.e., a large minimum travel time), the truncation effect is more significant to links with relatively short travel times, as shown by the sharp METT increase of link 1 and link 2. Hence, the METTs of links 1 and 2 are larger than that of link 3 under a strict speed limit. On the other hand, if the speed limit scheme is only imposed on link 1, we observe different route choice decisions compared to the above uniform speed limit scheme. Specifically, as long as the speed limit on link 1 is lower than 50 km/h, link 2 is always the best choice; whereas link 3 will never be the best alternative.



(a) Mean of travel time



(b) Standard deviation (SD) of travel time



(c) Mean-excess travel time (METT)

Figure 5 Mean, SD and METT of three links under different speed limits

From the above observations, a speed limit scheme imposes different impacts on the travel time uncertainty among the links. Implementing speed limits could have a substantial impact on travelers' risk-averse route choice decisions and consequently the network equilibrium state. The impact of speed limits on travelers' risk-averse route choice behaviors should also be explicitly considered in designing the optimal speed limit scheme.

To examine which group of travelers is more affected by speed limit implementation, we continue to look at the example used in Section 2.2. Confidence level (CL) represents the travelers' risk-aversion attitude. The larger the confidence level, the more risk-averse are the travelers. Figure 6 shows the travel time budget (TTB), the expected excess delay (EED), and the mean-excess travel time (METT) under different combinations of CLs and speed limits. We can see that the increase of TTB and METT and the decrease of EED are more significant for lower CLs. In particular, the EED is substantially reduced under lower CLs. More risk-averse travelers with a higher CL are relatively less affected by speed limits. In general, TTB and METT are both increasing and EED is decreasing as the speed limit becomes tighter. With a tighter speed limit (i.e., larger skewness and kurtosis as shown in Table 1), travelers need to budget a longer buffer time to ensure a certain reliability requirement. However, the benefit is that the unreliability (or tardy time, in terms of the EED) beyond the reliability requirement can be significantly reduced, especially for less conservative travelers.

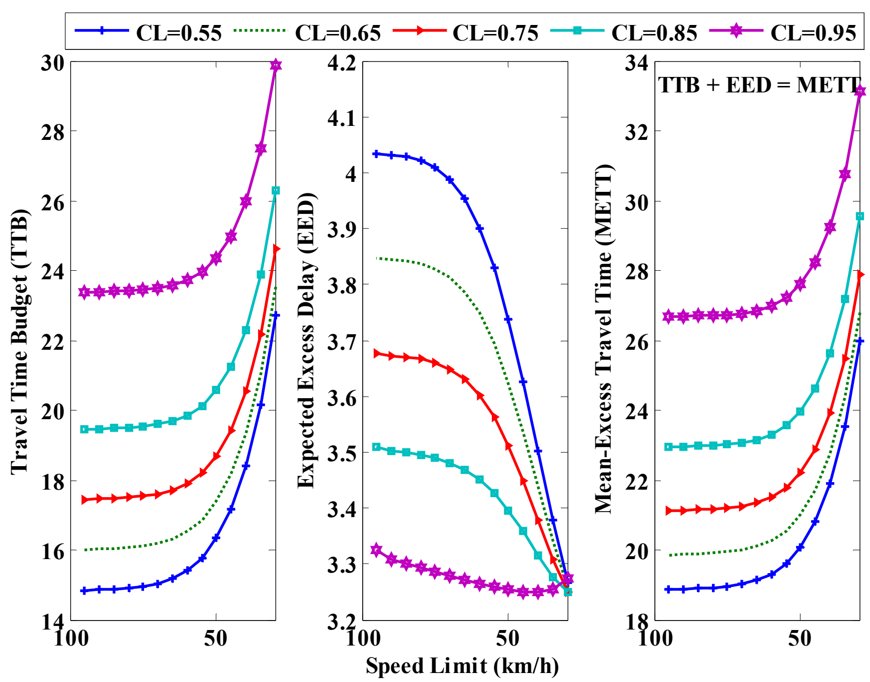


Figure 6 Effect of speed limits on METT under different confidence levels (CLs)

4 MEAN-EXCESS TRAFFIC EQUILIBRIUM (METE)

Note that the modeling and illustrative examples presented in Sections 2 and 3 are for a given flow and travel time uncertainty pattern without rerouting consideration (i.e., may not be in equilibrium). The traffic flow reallocation effect due to speed limits and congestion effect have not been captured. With the above risk-averse route choice criterion of METT and also the congestion effect, we have the following mean-excess traffic equilibrium (METE) conditions:

$$\eta_p^w\left(\mathbf{f}^*\right)\begin{cases}=u^w, \text{if } f_p^{w*}>0\\ \geq u^w, \text{if } f_p^{w*}=0\end{cases}, \quad \forall p\in P^w, w\in W, \tag{21}$$

where f_p^w is the flow on route *p* between O-D pair w; u^w is the minimum METT between O-D pair w, i.e., $u^w=\min\left\{\eta_p^w, p\in P^w\right\}$. Here the route and O-D cost variables are associated with the travel time uncertainty under a speed limit scheme. The METE state is reached by allocating O-D demands to the network such that for each O-D pair, all used routes have equal and minimal METT. Mathematically, the above 'if-then' condition can be equivalently formulated as a variational inequality problem, which is to find a route flow pattern $\mathbf{f}^*\in\Omega$, such that

$$\boldsymbol{\eta}\left(\mathbf{f}^*\right)^T\left(\mathbf{f}-\mathbf{f}^*\right)\geq 0, \ \forall\mathbf{f}\in\Omega, \tag{22}$$

where Ω is the constraint set defined below:

$$\sum_{p\in P^w} f_p^w = q^w, \ \forall w\in W, \tag{23}$$

$$f_p^w \geq 0, \ \forall p\in P^w, w\in W, \tag{24}$$

where q^w is the demand of O-D pair *w*. Eq. (23) is the demand conservation constraint; and Eq. (24) is a non-negativity constraint on the route flows.

The modeling approach presented in Section 2 and Section 3 is general since it directly works with the travel time distribution without specifying the uncertainty sources yet. Day-to-day demand fluctuation and capacity degradation are two main sources of uncertainty in transportation systems. Also, most traffic equilibrium models under uncertainty considered either demand uncertainty or capacity uncertainty or both. In the numerical examples of Section 5, we will consider travel demand fluctuation as a representative uncertainty source. The lognormal distribution will be used to characterize travel demand, traffic flow, and travel time uncertainties. With the modeling of uncertainty propagation (travel demand-route flow-link flow- link travel time) and the lognormal distribution characterization of flows and travel times, we are able to calculate the moments of link travel time *without or with speed limits* as in Eq. (2) or Eq. (9). The concept of cumulants is then used to aggregate the moments of link travel times to route travel times. The first four moments of route travel times are further used to analytically approximate the TTB and METT used in Eq. (22). The detailed process can be found in Chen *et al*. (2011b) and Xu *et al*. (2013). The uncertainty propagation from link travel time to route travel time is critical in the above modeling process. When we use the route-based METT model (Chen and Zhou, 2010), we need to reply on the Central Limit Theorem for propagating link travel time distribution to route travel time distribution under the assumption of independent link travel times and normally distributed route travel times. However, we do not need this assumption in the link-based METT model (Xu *et al*., 2017). By solving the above METE model, we can capture the impacts of a speed limit scheme on travel time uncertainty profiles of both directly and indirectly affected links as well as the resultant network flow reallocation.

5 NUMERICAL EXAMPLES

In this section, we provide numerical examples to demonstrate the impacts of speed limits on traffic flow reallocation and the shift of travel time uncertainty profile on both directly and indirectly affected links. A simple network with one O-D pair and three parallel links/routes is used for ease of results exposition. We use the Bureau of Public Road (BPR) function with the parameters of 0.15 and 1. The free-flow travel times of the three links are, respectively, 20, 15, and 20 minutes, and their capacities are all 5 vehicles per minute (veh/min). The O-D demand follows the lognormal distribution with the mean of 25 flow units and the COV of 0.30. All travelers have the confidence level of 80%. A speed limit scheme is only imposed on link 2. The maximum allowed speed is 45 km/h on this link. This setting is used to examine the shift of risk-averse route choice decisions.

5.1 Traffic Equilibrium under a Given Speed Limit Scheme

Table 2 presents the METE equilibrium flows without and with speed limit, as well as the corresponding travel time distribution characteristics and risk measures. As expected, both the equilibrium conditions and conservation constraints are fully satisfied. Figure 7 further depicts the travel time PDFs on link 2 without and with speed limit associated with the respective (different) METE flow pattern. Without speed limit, more than 60% of travelers use link 2 due to its short free-flow travel time, even though all three routes have equal METT. However, the standard deviation (SD) and COV on link 2 are much larger than those on link 1 and link 3. Generally, larger travel time variability is not preferable, since it adds uncertainty or risk for an on-time arrival at the destination. Some travelers may prefer to choose a route with high average travel time but low travel time variance, in order to guarantee a high reliability and predictability.

Table 2 METE states without and with speed limit

Speed Limit	Route	Flow	Travel Time Characteristics					TTB	EED	METT
			Mean	SD	COV	Skew	Kurt			
Without	1	4.84	22.90	1.98	0.086	2.36	11.33	23.47	2.57	26.03
	2	15.32	21.89	2.64	**0.121**	1.21	2.69	23.71	**2.32**	26.03
	3	4.84	22.90	1.98	0.086	2.36	11.33	23.47	2.57	26.03
With	1	5.10 (↑)	23.06 (↑)	2.03 (↑)	0.088 (↑)	2.29	10.54	23.71 (↑)	2.57	26.28 (↑)
	2	14.80 (↓)	22.79 (↑)	2.02 (↓)	**0.089** (↓)	3.09 (↑)	7.61 (↑)	24.57 (↑)	**1.71** (↓)	26.28 (↑)
	3	5.10 (↑)	23.06 (↑)	2.03 (↑)	0.088 (↑)	2.29	10.54	23.71 (↑)	2.57	26.28 (↑)

Note: ↑: increase; ↓: decrease.

The impacts of speed limits are twofold: direct and indirect change of travel time uncertainty profiles on each link, and traffic flow reallocations among the links.

(a) **Link 2**: The speed limit scheme *directly* affects some fast drivers using link 2 to slow down. This increases the mean travel time from 21.89 to 22.79, the TTB from 23.71 to 24.57, and the METT from 26.03 to 26.28, respectively. Accordingly, link 2 does not provide sufficient incentive to attract travelers from selecting it. Some travelers will switch to link 1 and link 3, *directly* leading to the flow reduction on link 2 from 15.32 to 14.80 (i.e., 0.52). Due to the 'squeezing' effect of speed limit, the COV on link 2 is reduced from 0.121 to 0.089. The unreliability of exceeding the TTB (i.e., EED) is also reduced from 2.32 to 1.71. However, the skewness (i.e., asymmetry) and kurtosis (i.e., peakedness) are significantly increased after the speed limit implementation, as shown in both Table 2 and Figure 7. In the context of travel time uncertainty, large skewness and kurtosis values are generally not preferable. There is a relatively higher chance of encountering extremely long travel times.

(b) **Links 1 and 3**: Note that link 1 and link 3 have the same characteristics, thus they have the same flow and travel time uncertainty profile. The flow of 0.52 originally on link 2 will be split to link 1 and link 3 equally, leading to the flow increase on links 1 and 3 from 4.84 to 5.10 (i.e., 0.26). The flow increase further makes their mean travel time, SD, COV, TTB and METT larger. The skewness and kurtosis are slightly reduced. Since the examined speed limit scheme has no direct truncation on link 1 and link 3, their travel time uncertainty profiles are not changed significantly.

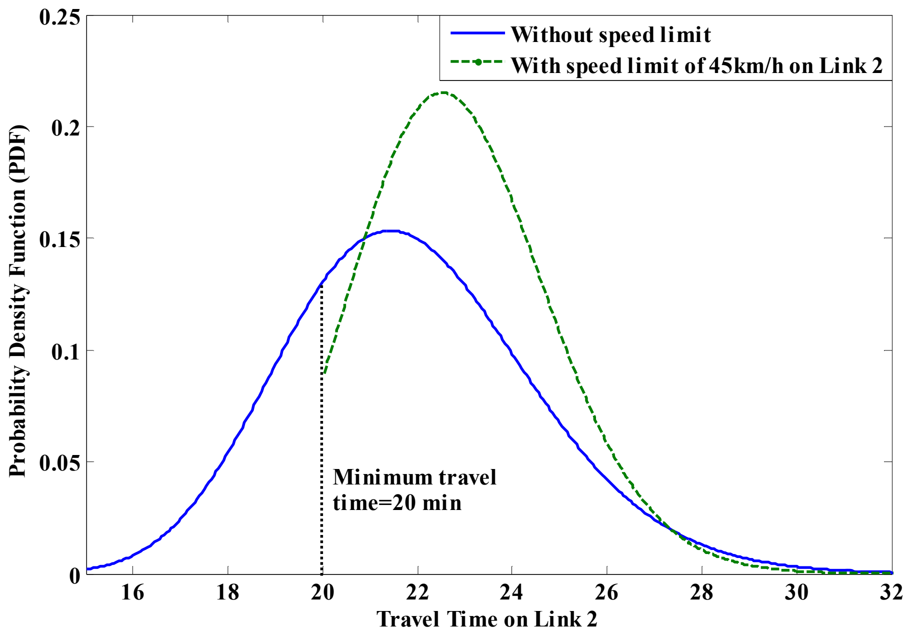


Figure 7 Travel time PDF of link 2 without and with speed limit

In summary, even though the speed limit is link-specific, it has both direct and indirect impacts on all network links with different extents. The change of travel time uncertainty profiles on the network links lead to the network flow reallocations. Speed limits have both positive impact (in terms of COV and EED reduction) and negative impact (in terms of the mean travel time, TTB and METT increase) on the travel time uncertainty profile of the directly affected links. Hence, the risk measures that we use to model travelers' route choice decisions should have a complete uncertainty characterization, including the multi-dimensional statistical characteristics (e.g., variability, asymmetry and peakedness) and the risk performance (e.g., reliability and unreliability under a given risk-aversion attitude).

To demonstrate the necessity of considering the uncertainty-related impact of speed limits, we compare the UE and METE results in Table 3. Both models correspond to the speed limit of 45 km/h on link 2. Different route choice criteria provide different optimal routes, which reflect travelers' different risk preferences and considerations towards travel time uncertainty. These lead to different traffic flow patterns as well as different travel time uncertainty profiles on each link. Under the equilibrium state of the UE model with equal mean travel time, though route 2 is used by most travelers, it actually has the highest TTB and METT. The reason is that the UE model only considers the expected value of travel time uncertainty and therefore does not account for the positive/negative impacts of speed limits on the travel time variability, skewness, and fatness of distribution tail. Under the considerations of on-time arrival reliability and late arrival unreliability, travelers on route 2 are willing to switch to route 1 and route 3. A new equilibrium pattern will then reach with equal METT on all three routes. Therefore, a risk measure with a tailored uncertainty consideration (e.g., METT) is needed in order to comprehensively account for the impacts of speed limits on travelers' risk-taking route choice decisions and network flow reallocations.

Table 3 Comparison between UE and METE states (with speed limit)

Speed Limit	Route	Flow	Travel Time Characteristics					TTB	EED	METT
			Mean	SD	COV	Skew	Kurt			
UE*	1	4.87	22.92	1.99	0.087	2.35	11.23	23.50	2.57	26.06
	2	**15.26**	22.92	2.09	0.091	2.89	6.89	**24.70**	1.81	**26.50**
	3	4.87	22.92	1.99	0.087	2.35	11.23	23.50	2.57	26.06
METE	1	5.10	23.06	2.03	0.088	2.29	10.54	23.71	2.57	26.28
	2	14.80 (↓)	22.79 (↓)	2.02	0.089	3.09	7.61	24.57 (↓)	1.71	26.28 (↓)
	3	5.10	23.06	2.03	0.088	2.29	10.54	23.71	2.57	26.28

*: The UE is based on the expected route travel time, not simply the deterministic travel time

5.2 Comparison among Various Speed Limit Schemes

The above analyses correspond to the speed limit of 45 km/h on link 2. Figure 8 shows the equilibrium METT and flows under various speed limits. We can see that with a tighter speed limit on link 2, the truncation effect on travel time uncertainty and network flow reallocation becomes more significant. The METT of link 2 becomes larger, and some travelers will give up link 2 and switch to link 1 or link 3. The flow increase on link 1 and link 3 also increases their METTs. Finally, the equilibrium METT of this O-D pair is also enlarged with an increasing rate.

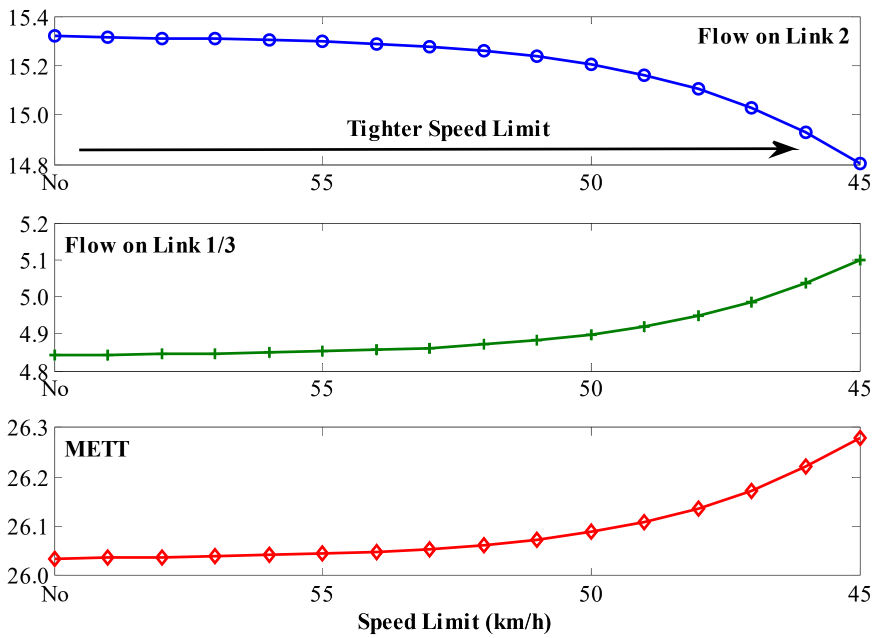


Figure 8 Equilibrium METT and flows under various speed limits

5.3 Sioux Falls Network Example

In this section, we use the well-known Sioux Falls network to demonstrate the solvability of the proposed model with speed limit under uncertainty. This network has 24 nodes, 76 links and 550 O-D pairs. The O-D demands are assumed to follow the lognormal distribution with the variance-to-mean ratio (VMR) of 0.30. A behaviorally generated route set (Bekhor *et al.*, 2008) is used to compare the flow pattern without and with speed limit. In this route set, each O-D pair has at most 13 routes and 6.3 routes on average. We run the route-based METE model (Chen and Zhou, 2010) with the on-time arrival reliability requirement of 80%, and then identify the 30 links with the coefficient of variation (COV) of link travel time greater than 0.50, as highlighted in Figure 9. A speed limit is imposed on these 30 links by increasing their respective free-flow travel time by 1.25 times.

Without loss of generality, we choose O-D pair (3, 8) to demonstrate the equilibrium results. This O-D pair consists of 6 routes in the route set. The METE equilibrium results without and with the above speed limit scheme are shown in Table 4. One can readily verify that both equilibrium results satisfy the equilibrium conditions. Before imposing the speed limit, routes 1 and 2 are used despite that route 3 also has the identical equilibrium cost. After imposing the speed limit particularly on links 12 and 16 with a large CV, the travelers originally chosen route 1 will move to route 3.

Accordingly, routes 2 and 3 become the equilibrium routes with the identical and minimum cost. In addition, the costs of all 6 routes get larger due to the effect of speed limit on the travel time uncertainty profile.

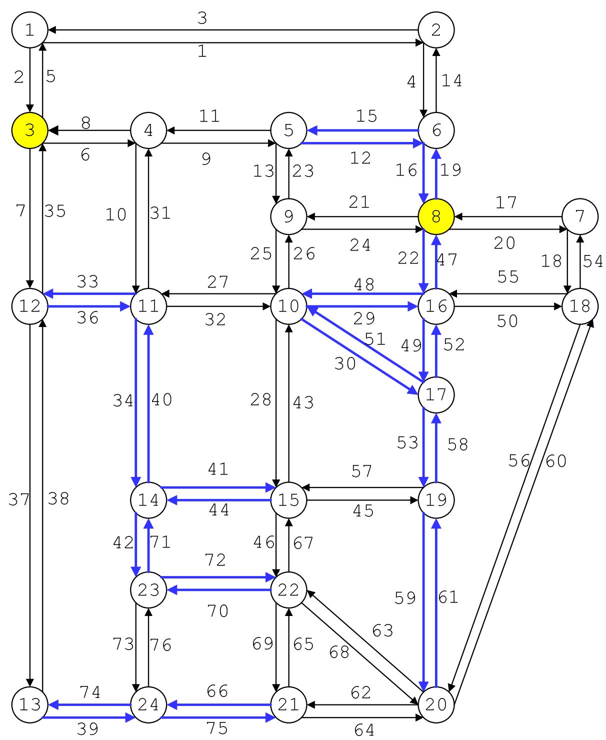


Figure 9 Sioux Falls network

Table 4 Equilibrium results of O-D pair (3, 8) without and with speed limit

Route	Route composition	Without speed limit		With speed limit	
		Flow	Cost	Flow	Cost
1	6-9-***12***-***16***	0.0288	0.6289	0	0.6333
2	6-9-13-24	0.1712	0.6289	0.1268	0.6331
3	5-1-4-***16***	0	0.6289	0.0732	0.6331
4	6-9-13-25-***29***-***47***	0	0.9203	0	0.9264
5	6-10-32-***29***-***47***	0	0.9294	0	0.9339
6	7-***36***-32-***29***-***47***	0	0.9770	0	0.9910

To further examine the effect of speed limit on travel time uncertainty profile, Figure 10 shows the relative difference of mean travel time, absolute difference of CV, relative difference of skewness and kurtosis after and before imposing the above speed limit scheme. One can see that majority of the 76 links have an increased expected travel time, a decreased CV, and an increased skewness and kurtosis. Similar to the small network in Section 5.1, a link speed limit has both direct and indirect impacts on all network links with different extents. It has a positive impact on the variability reduction at the cost of increasing the mean, skewness and kurtosis. The change of link travel time uncertainty profiles results in the network flow reallocations after imposing a speed limit. This further demonstrates the necessity of a tailored risk measure with a multi-dimensional consideration for characterizing the travelers' route choice decisions and planners' network performance assessment in designing a speed limit scheme.

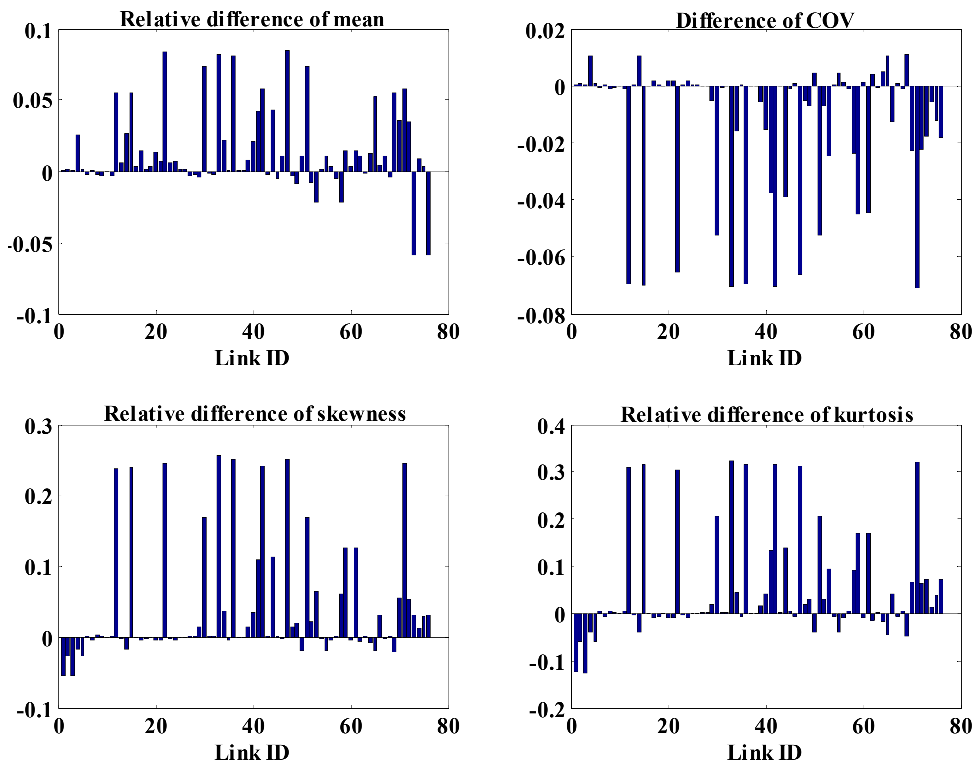


Figure 10 Change of link travel time uncertainty profile after imposing a speed limit

6 CONCLUDING REMARKS

In this paper, we provided a way to quantify the network-wide impacts of speed limits on road network uncertainty. Specifically, a multi-dimensional approach was used to model the impacts of speed limits on link travel time uncertainty, travelers' risk-averse route choice decisions, and traffic flow reallocation under uncertainty. First of all, a truncated travel time probability distribution was proposed and its moment expression was derived to characterize the multi-dimensional impacts of speed limit on link travel time uncertainty profile. Then, the METT was adopted as a risk measure to capture the travelers' risk-averse route choice adjustment and equilibrium flow reallocation caused by speed limits.

From the theoretical and numerical analyses, we have the following observations. A speed limit scheme imposes different impacts (either positive or negative) on different dimensions of travel time uncertainty profile, including the statistical characteristics (e.g., variability, asymmetry, and fatness of distribution tail) and the risk performances (e.g., reliability and unreliability). For the positive impact, speed limit increases the mean travel time on the *directly* affected link while reduces the travel time variance. For the negative impacts, speed limit significantly increases both the skewness (i.e., asymmetry indicator) and kurtosis (i.e., fatness indicator of distribution tail) of travel time uncertainty. Also, it increases the travel time budget and METT on the *directly* affected link. Travelers choose different optimal routes under different speed limit schemes. The shift in travel time uncertainty profile on the *directly* affected links further adjusts the traveler's risk-averse route choice decisions, the travel time uncertainty profile on the *indirectly* affected links, and consequently the equilibrium flow reallocation.

In view of the importance of speed limits on network uncertainty, it is valuable to develop an optimization model for designing the optimal speed limit scheme under uncertainty. This model should explicitly capture the impacts of speed limits on travelers' risk-averse route choice behaviors as well as the network-wide performance (e.g., Xu *et al*., 2014). To enhance the modeling realism, it is necessary to consider the joint effect of stochastic perception error (Chen *et al*., 2011b; Xu *et al*., 2013) and speed limit on network uncertainty, and to consider the relationship among design speed, posted speed limit and operating speed (Donnell *et al*., 2014). In this paper, the truncated continuous distribution was used to model the rescaled travel time distribution caused by a speed limit scheme, which may influence the specific results of route choice model and traffic equilibrium model. Future research should explore different ways (e.g., discrete distributions or stochastic speed processes (Kharoufeh and Gautam, 2004)) to deal with the speed adjustment as well as collecting empirical data to examine how drivers react to speed limits under an uncertain environment. Also, considering the safety motivation of implementing speed limits, we plan to assess its effectiveness on network uncertainty management specific to the capacity degradation caused by traffic incidents and work zones (e.g., Chen *et al*., 2002; Lo and Tung, 2003; Lo *et al*., 2006). Speed variance could be a predictor of crash occurrence probability. In this study, we implicitly assume that a link-specific speed limit is enforced for each whole link or at least a segment, which is quite common in freeways and urban expressways. However, the effect of a speed limit on actual travel times may be different depending on how the speed limit is enforced. It is of interest to see the impacts of different implementations of monitoring vehicular speed (e.g., speed cameras at one location, number plate or electronic toll recognition speed systems, and in-car speed-limiters).

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