An energy-efficient rescheduling approach under delay perturbations for metro systems

Xin Yang [1], Anthony Chen [2, 3] *, Jianjun Wu [1], Ziyou Gao [1] and Tao Tang [1]

[1] State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University
Beijing 100044, China

[2] Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong

[3] Key Laboratory of Road and Traffic Engineering, Tongji University
Shanghai 201804, China

Abstract

Due to the increasing energy prices and environmental issues, energy conservation for metro trains has become an important research topic in recent years. The static timetable optimization approach can reduce the trains' energy consumption to a certain degree by scheduling multiple trains' movements to improve the utilization of regenerative braking energy. However, this optimization approach is prone to fail when one or some trains are delayed at stations. In this paper, we develop an energy-efficient rescheduling approach under delay perturbations for metro trains, which aims to minimize the net energy consumption under the premise of reducing or eliminating the delay altogether. Firstly, we provide an illustrative example to describe the rescheduling problem. Secondly, we formulate an integer programming model to determine the new energy-efficient schedule. Furthermore, we design an allocation algorithm to find the optimal solution. Finally, a numerical example based on the real-world operation data from the Beijing Metro Yizhuang Line is presented to illustrate the practicability and effectiveness of the developed approach. The results show that the energy-efficient rescheduling approach can reduce the net energy consumption by 8.19% in comparison with the traditional rescheduling approach.

Keywords: metro trains; rescheduling approach; regenerative braking; energy consumption

---

* Correspondence to: Anthony Chen, Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University, Hong Kong. E-mail: anthony.chen@polyu.edu.hk

1. INTRODUCTION

Metro system has received rapid development all over the world because of its high reliability and large transport capacity (González-Gil et al. 2014). As an example in mainland of China, a total of one hundred and thirty-four metro lines are in operation in thirty cities and the total distance is over 4100 kilometers by the end of 2016, expanding from three cities with five lines in 2000. Although the energy cost per passenger of metro is cheaper than other transport tools, the total energy consumption and carbon emission are considerable due such a large scale. Due to the rising environmental issues and energy prices, energy conservation for metro operations has attracted much attention in recent years (Yang et al. 2016a).

Before the application of regenerative braking technology, researchers focused on optimizing the speed profile of a train to reduce its tractive energy consumption. A number of research achievements have published on the speed profile optimization topic (Howlett and Pudney 1995; Kim and Chien 2010a, 2010b; Ke et al. 2009, 2012). Regenerative braking is an energy recovery technique applied to metro trains that uses an electric motor as an electric generator during the braking phase of trains to recover the kinetic energy as electrical energy (Yang et al. 2014; Yang et al. 2015b). The illustration of regenerative braking principle used on metro trains is shown in Fig. 1.

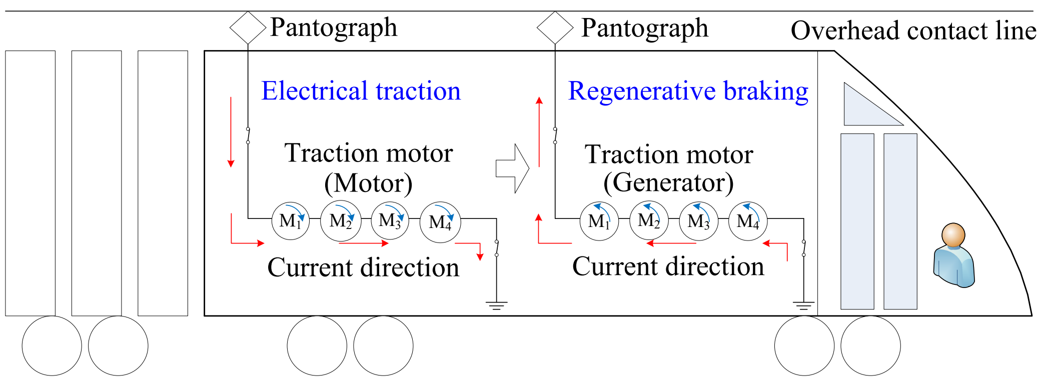


Fig. 1. Illustration of regenerative braking principle used on metro trains.

After the regenerative braking technique was widely applied to metro trains, the static timetable optimization problem has become an important approach to improve the utilization of regenerative braking energy, such that the energy consumption of trains can be reduced. For example, Ramos *et al.* (2007) firstly proposed a timetable optimization method for maximizing the overlapping time between adjacent trains such that the utilization of regenerative braking energy can be increased. Peña-Alcaraz *et al.* (2012) extended the timetable optimization method for coordinating the acceleration and braking processes of all trains in the same electrical section to reduce the total energy consumption. They also developed an energy flow model to evaluate the regeneration saving factor within the same electrical section. Domínguez *et al.* (2012) further developed both power flow model and speed profile model to minimize the net energy consumption (i.e., the tractive energy consumed by the train removing any energy recovered from regenerative braking). Based on the real-world operation data from the Beijing Metro Yizhuang Line in China, Yang *et al.* (2013) proposed a cooperative scheduling model to synchronize the accelerating and braking processes of adjacent trains such that the regenerative braking energy from braking trains can be immediately utilized by accelerating trains. Furthermore, Li and Yang (2013) extended the model to consider the randomness of departure time delay for trains at busy stations with minimum of energy consumption. Yang *et al.* (2015a) developed an energy-efficient scheduling approach to maximize the utilization regenerative braking energy, where the number of trains and the cycle time were kept unchanged in the optimization model. After a comprehensive review, González-Gil *et al.* (2013) concluded that timetable optimization is a direct and efficient approach to increase the regenerative braking energy exchange between trains. Yang *et al.* (2016a) provided a survey of timetable and speed profile optimization methods on energy efficiency for metro systems.

In real-world metro systems, small delays often occur at busy stations due to the crowded passengers during peak hours (Zhang 2014a). The above static timetable optimization approaches are prone to fail when one or more trains are delayed at stations during daily operations. To reduce the energy consumption for metro trains under such delay perturbations, this paper develops an energy-efficient rescheduling approach for the delayed trains to reduce or eliminate the delay altogether with the minimum net energy consumption. Compared to previous studies (Yang et al. 2013, 2014, 2015a, 2015b, 2016b), this paper has the following contributions and differences:

(1) This paper considers the dynamic rescheduling problem to handle the delay during the train running phase, while the previous studies mainly focus on the static timetable optimization problem during the timetable planning phase.

(2) This paper designs a fast allocation algorithm to deal with real-time computing requirements during the train running phase, while the genetic algorithm used in previous studies takes a long time to find the optimal solution.

Figure 2 provides a summary of the research studies for the timetable planning phase and the train running phase. The timetable planning phase mainly focus on the speed optimization problem and the static timetable optimization problem, while the train running phase emphasizes on the dynamic rescheduling problem given a small perturbation. Together with Table 1, the differences and contributions made in this paper and those in the literature are highlighted.

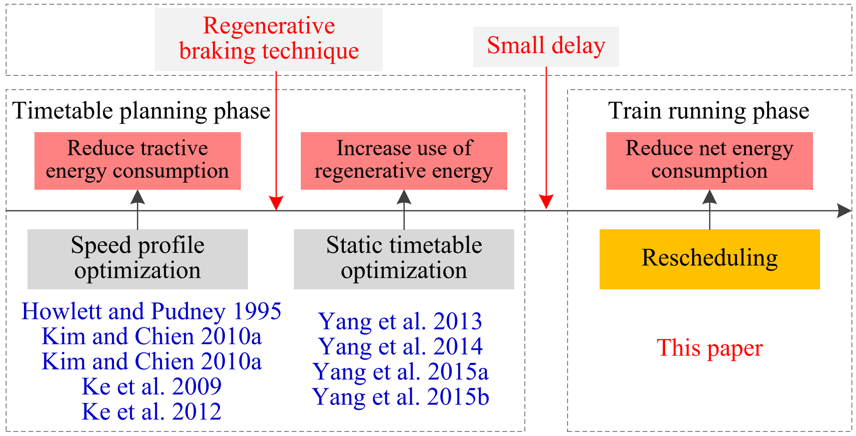


Fig. 2. Research highlights in this paper.

Table 1. Recent publications on energy-efficient timetable optimization and our work.

Publication	Problem type	Objective	Algorithm	Considering the delay or not
Ramos *et al.* (2007)	Static	Overlap time	CPLEX Solver	No
Peña-Alcaraz *et al.* (2012)	Static	Regenerative energy	SBB Solver	No
Domínguez *et al.* (2012)	Static	Net energy consumption	Numerical simulation	No
Yang *et al.* (2013)	Static	Overlap time	Genetic algorithm	No
Li and Yang (2013)	Static	Regenerative energy	Genetic algorithm	Yes
Yang *et al.* (2015a)	Static	Regenerative energy	Genetic algorithm	No
Yang *et al.* (2015b)	Static	Regenerative energy	Adaptive genetic algorithm	No
Our work	Dynamic	Net energy consumption	Allocation algorithm	Yes

The remainder of this research is organized as follows. In section 2, we provide the problem statement. In Section 3, we formulate an integer programming model to determine the new energy-efficient schedule. In Section 4, we design an allocation algorithm to find the optimal solution. In Section 5, a numerical example based on the real-world data from the Beijing Metro Yizhuang Line are presented. Finally, conclusions are given in Section 6.

2. PROBLEM STATEMENT

This section provides an illustrative example to describe the rescheduling problem in a metro line shown in Fig. 3, which includes 4 stations and 3 sections. Station 1 and station 4 are the origin and the destination, respectively, and the delay of train i occurs at station 2.

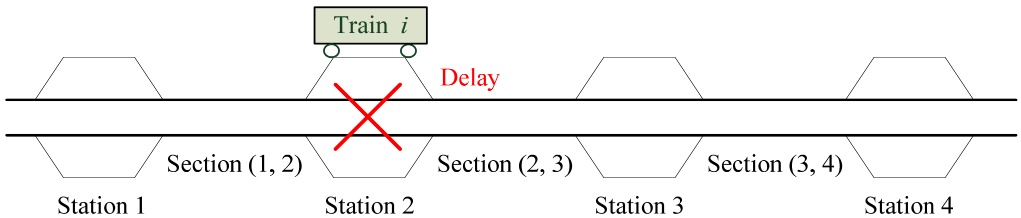


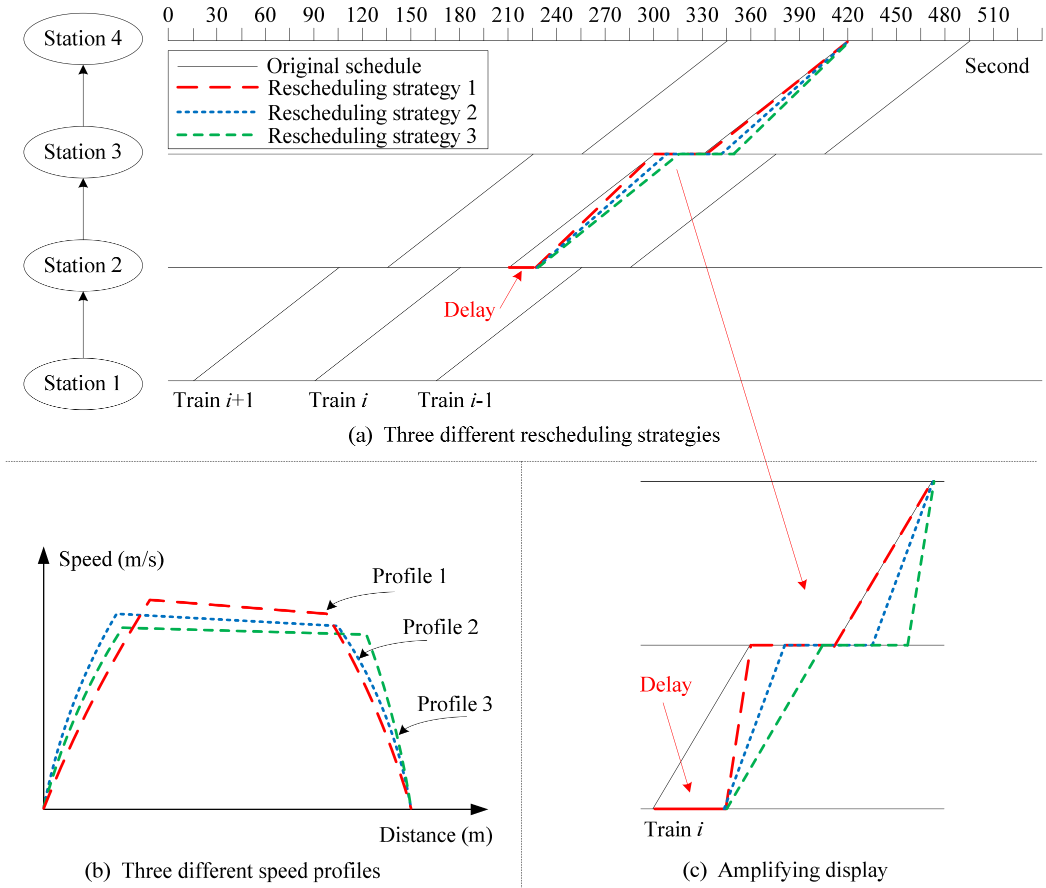
Fig. 3. Illustration of a delayed train in a metro line.

After the delay has occurred, train i needs to adjust its original schedule in order not to affect other trains for arriving at the destination on time. As shown in Fig. 4, three rescheduling strategies are provided to recover the schedule of train i:

a) Strategy 1 aims to eliminate the total delay at section (2, 3) by significantly increasing the train's speed at this section in order to recover the original schedule at station 3. Due to the significant increase in speed, more energy consumption is needed.

b) Strategy 2 aims to reduce part of the delay at section (2, 3) and eliminate the remaining delay at section (3, 4). This rescheduling strategy avoids significant increase in the train's speed, but it takes longer to recover the original schedule at station 4 compared to Strategy 1. In this strategy, delay will also occur in station 3.

c) Strategy 3 keeps the same running speed at section (2, 3) and eliminates the delay at section (3, 4). This rescheduling strategy pushes the delay to station 3 when the train has no potential of increasing the speed at section (2, 3).



(a) Three different rescheduling strategies

(b) Three different speed profiles

(c) Amplifying display

Fig. 4. Illustration of different rescheduling strategies and speed profiles.

From the illustration, all three rescheduling strategies can recover the original schedule of the delayed train i by using different speed profiles as shown in Fig. 4b. However, the net energy consumption of train i (i.e., the difference between the tractive energy consumed by train i and the utilization of regenerative braking energy) is different under each rescheduling strategy (i.e., amount of regeneration braking energy that can be reused depends on the rescheduled timetable) and speed profile implementation (i.e., amount of tractive energy required depends on the speed profile adopted).

The purpose of this paper is to determine a rescheduling strategy that can reduce or eliminate the delay with the minimum net energy consumption. Note that the rescheduling approach developed in this paper is applicable for tackling small delays that do not disturb the normal operation of other trains. Moreover, if the delay cannot be completely eliminated, the delayed train should run with the maximum allowable speed at the following sections to reduce the delay as much as possible. In addition, for some special delays caused by explosions, earthquakes, floods, etc., the operations must comply with the emergency response plans provided by the metro company, which are beyond the scope of this paper.

3. MODEL FORMULATION

This section develops an energy-efficient rescheduling approach for the delayed train to reduce or eliminate the delay altogether with the minimum net energy consumption. The following discussions focus on detailing each part of the formulation, i.e., notions, model assumptions, energy-efficient profile, objective function and optimization model.

3.1. Notations

We define the origin and the destination of a metro line as station 1 and station N, respectively. The delay of train i occurs at station k.

Notations used throughout the paper are listed as follows and all boldface letters denote the corresponding vectors.

(1) Indices

i	train index, $i = 1, 2, \dots, I$
n	station index, $n = 1, 2, \dots, N$

(2) Parameters

M	train mass
h	time headway
h_0	minimum allowed time headway
f_a	maximum tractive force
f_b	maximum braking force
r	basic running resistance
g	additional running resistance caused by gradient and curve
t_n	dwell time at station n
a_{0in}	original arrival time of train i at station n, and $a_{0in} = a_{0(i+1)n} + h$
d_{0in}	original departure time of train i at station n, i.e., $d_{0in} = a_{0in} + t_n$
$s_{(n-1,\,n)}$	length of section $(n\text{-}1, n)$
η_1	conversion efficiency of the accelerating process
η_2	conversion efficiency of the braking process
β	transmission loss coefficient on regenerative braking energy
δ	maximum allowed reduction of running time at each section from the original schedule

(3) Decision Variables

a_{in}	rescheduled arrival time of the delayed train i at station n, where $n = k+1, k+2, \dots, N$

(4) Intermediate Variables

d_{in}	rescheduled departure time of the delayed train i at station n, i.e., $d_{in} = a_{in} + t_n$, where $n = k+1, k+2, \dots, N\text{-}1$
$\mathbf{v}_{i(n-1)}$	rescheduled speed profile of the delayed train i at section $(n\text{-}1, n)$, where $n = k+1, k+2, \dots, N$
$c_{i(n-1)}$	rescheduled switching time from accelerating phase to coasting phase of the delayed train i at section $(n\text{-}1, n)$, where $n = k+1, k+2, \dots, N$
$b_{i(n-1)}$	rescheduled switching time from coasting phase to braking phase of the delayed train i at section $(n\text{-}1, n)$, where $n = k+1, k+2, \dots, N$
$v_{ci(n-1)}$	speed at the switching point from accelerating phase to coasting phase at section $(n\text{-}1, n)$
$v_{bi(n-1)}$	speed at the switching point from coasting phase to braking phase at section $(n\text{-}1, n)$

3.2. Model assumptions

According to the engineering requirements and operation characteristics of metro trains, the following assumptions are made to simplify the model formulation.

(*A1*). There are at least two sections remained from the delay occurred station to the destination.

(*A2*). To avoid disturbing the normal operation of train i+1, the delay time should be less than h-h_0 when the delay of train i occurs at station k.

(*A3*). Train mass, maximum tractive force, maximum braking force, basic running resistance and additional running resistance are considered as constant parameters.

(*A4*). The conversion efficiencies of the accelerating process (from electrical energy to mechanical energy) and braking process (from mechanical energy to electrical energy) are assumed as constant parameters.

(*A5*). The transmission loss coefficient on regenerative braking energy is also assumed as a constant parameter.

3.3. Energy-efficient Speed Profile

The research on energy-efficient speed profile has been studied for a long time. For example, Kim and Chien (2010a) developed a train simulation model to emulate the movement of a train, which can estimate travel time, calculate energy consumption and generate an energy-efficient speed profile. Kim and Chien (2010b) developed an optimal train control model to determine the speed profile with minimum energy consumption considering speed limit, track alignment and schedule adherence. Ke et al. (2009) designed an effective ant colony algorithm to generate the energy-efficient speed profiles for trains in mass rapid transit systems. Then Ke et al. (2012) improved the ant colony algorithm based on the MAX-MIN principle to optimize the speed profile that made a significant reduction on the computation time. For metro trains with short section travel distance in this paper, based on the discussions of the optimal train control theory in studies (Howlett and Pudney 1995; Li and Lo 2014a; Li and Lo 2014b; Yang et al., 2016b), we consider the energy-efficient speed profile consisting of three phases, i.e., maximum acceleration phase, coasting phase and maximum braking phase.

Given a section (n-1, n), the rescheduled departure time of the delayed train i at station n-1 is $d_{i(n-1)}$, and the rescheduled arrival time of the delayed train i at station n is a_{in}. For simplicity, we define $\boldsymbol{a} = \{a_{in} \mid n = k+1, k+2, \dots, N\}$. Based on the equation of motion, the speed of train i at time t is

$$v_{i(n-1)}(\boldsymbol{a},t) = \begin{cases} (f_a - r + g)(t - d_{i(n-1)})/m, & \text{if } d_{i(n-1)} \le t < c_{i(n-1)}, \\ v_{ci(n-1)} - (r-g)(t - c_{i(n-1)})/m, & \text{if } c_{i(n-1)} \le t < b_{i(n-1)}, \\ v_{bi(n-1)} - (f_b + r - g)(t - b_{i(n-1)})/m, & \text{if } b_{i(n-1)} \le t < a_{in}, \end{cases} \quad (1)$$

where the first row denotes the speed profile during the maximum accelerating phase; the second row denotes the speed profile during the coasting phase; and the third row denotes the speed profile during the maximum braking phase. Therefore, the speed profile of train i at section (n-1, n) is formulated as $\mathbf{v_{i(n-1)}} = \{v_{i(n-1)}(\boldsymbol{a}, t) \mid d_{i(n-1)} \le t < a_{in}\}$.

The switching speeds $v_{ci(n\text{-}1)}$ and $v_{bi(n\text{-}1)}$ are determined as follows:

$$\begin{cases} v_{ci(n-1)} = (f_a - r + g)(c_{i(n-1)} - d_{i(n-1)}) / m, \\ v_{bi(n-1)} = (f_a - r + g)(c_{i(n-1)} - d_{i(n-1)}) / m - (r-g)(b_{i(n-1)} - c_{i(n-1)}) / m. \end{cases} \tag{2}$$

Furthermore, the switching speeds $v_{ci(n\text{-}1)}$ and $v_{bi(n\text{-}1)}$ should satisfy the following equations:

$$\begin{cases} v_{bi(n-1)} - (f_b + r - g)(a_{i(n-1)} - b_{i(n-1)}) / m = 0, \\ v_{ci(n-1)}^2 / (f_a - r + g) + (v_{ci(n-1)}^2 - v_{bi(n-1)}^2) / (r-g) + v_{bi(n-1)}^2 / (f_b + r - g) = 2s_{(n\text{-}1,n)} / m, \end{cases} \tag{3}$$

where the first equation denotes that the speed of train i should reduce to zero when it arrives at station n; and the second equation denotes that the speed profile should satisfy the travel distance constraint. Equations (2) and (3) imply that the intermediate variables $c_{i(n\text{-}1)}$, $b_{i(n\text{-}1)}$, $v_{ci(n\text{-}1)}$ and $v_{bi(n\text{-}1)}$ can be expressed in terms of the decision variables a_{in} and given parameters. Combined them with Equation (1), we see that the speed profile $\mathbf{v}_{i(n\text{-}1)}$ can also be expressed in terms of the parameters and decision variables.

3.4. Objective function

After the delay of train i (delay time is T_d) occurs at station k, train i needs to reduce or eliminate the delay at the following sections from section (k, k+1) to section (N-1, N). As described in the *Problem Statement Section*, we aim to determine a rescheduling strategy that can reduce or eliminate the delay with the minimum net energy consumption. Therefore, the objective function is to minimize the net energy consumption of the delayed train i from section (k, k+1) to section (N-1, N).

As shown in Fig. 5, we need to generate new a speed profile for the delayed train i at the following sections with shorter running times to reduce or eliminate the delay time incurred at station k.

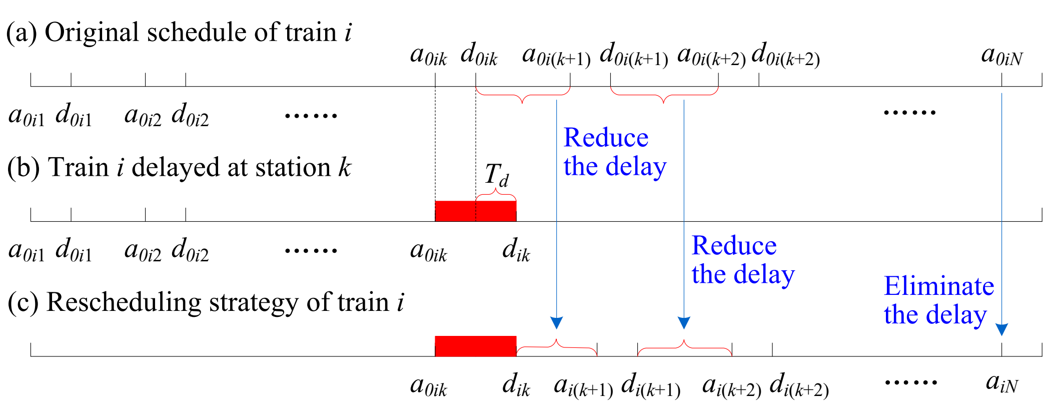


Fig. 5. Rescheduling strategy for reducing or eliminating delay.

Once the speed profile of train i is changed, the tractive energy consumption of train i will also change. In addition, the relative position of train i with its adjacent trains (see trains i-1 and i+1 shown in Fig. 4a) will change, which will affect the utilization of regenerative braking energy from train i. Therefore, the net energy consumption of train i (i.e., the difference between the tractive energy consumed by train i and the utilization of regenerative braking energy) will be different.

To formulate the final net energy consumption, we first calculate the tractive energy consumption of train i. For each $1 \le i \le I$ and $k+1 \le n \le N$, the tractive energy consumption of train i at section (n-1, n) at time t is

$$F_{i(n-1)}(\boldsymbol{a},t) = \begin{cases} f_a v_{i(n-1)}(\boldsymbol{a},t)/\eta_1, & \text{if } d_{i(n-1)} \le t < c_{i(n-1)}, \\ 0, & \text{if } c_{i(n-1)} \le t < a_{in}, \end{cases} \tag{4}$$

where the first condition denotes the accelerating phase, and the second condition denotes the coasting and braking phases (i.e., no tractive energy is needed in the coasting and braking phases). Hence, the total tractive energy consumption of train i from section (k, k+1) to section (N-1, N) is

$$J_F(\boldsymbol{a}) = \sum_{t=d_{ik}}^{a_{iN}} F_{i(n-1)}(\boldsymbol{a},t). \tag{5}$$

Furthermore, we show how to calculate the total utilization of regenerative braking energy of train i from section (k, k+1) to section (N-1, N) by coordinating adjacent trains i-1 and i+1 during the accelerating phase as follows. For each $1 \le i \le I$ and $k+1 \le n \le N$, the energy regenerated from braking of train i at section (n-1, n) at time t is

$$B_{i(n-1)}(\boldsymbol{a},t) = \begin{cases} 0, & \text{if } d_{i(n-1)} \le t < b_{i(n-1)}, \\ f_b v_{i(n-1)}(\boldsymbol{a},t)\eta_2, & \text{if } b_{i(n-1)} \le t < a_{in}, \end{cases} \tag{6}$$

where the first condition denotes the accelerating and coasting phases, and the second condition denotes the braking phase. As shown in Fig. 6a, the regenerative braking energy of braking train i can be utilized to accelerate its adjacent trains i-1 and i+1. For simplicity, we define

$$Ts = \{[b_{i(n-1)}, a_{in}) \cap [d_{(i+1)(n-1)}, c_{(i+1)(n-1)})\} \cup \{[b_{i(n-1)}, a_{in}) \cap [d_{(i-1)n}, c_{(i-1)n})\}, \tag{7}$$

where Ts denotes the total overlapping time; $[b_{i(n-1)}, a_{in}) \cap [d_{(i+1)(n-1)}, c_{(i+1)(n-1)})$ denotes the overlapping time between braking train i at section (n-1, n) and accelerating train i+1 at section (n-1, n) shown in Fig. 6b; and $[b_{i(n-1)}, a_{in}) \cap [d_{(i-1)n}, c_{(i-1)n})$ denotes the overlapping time between braking train i at section (n-1, n) and accelerating train i-1 at section (n, n+1) shown in Fig. 6c.

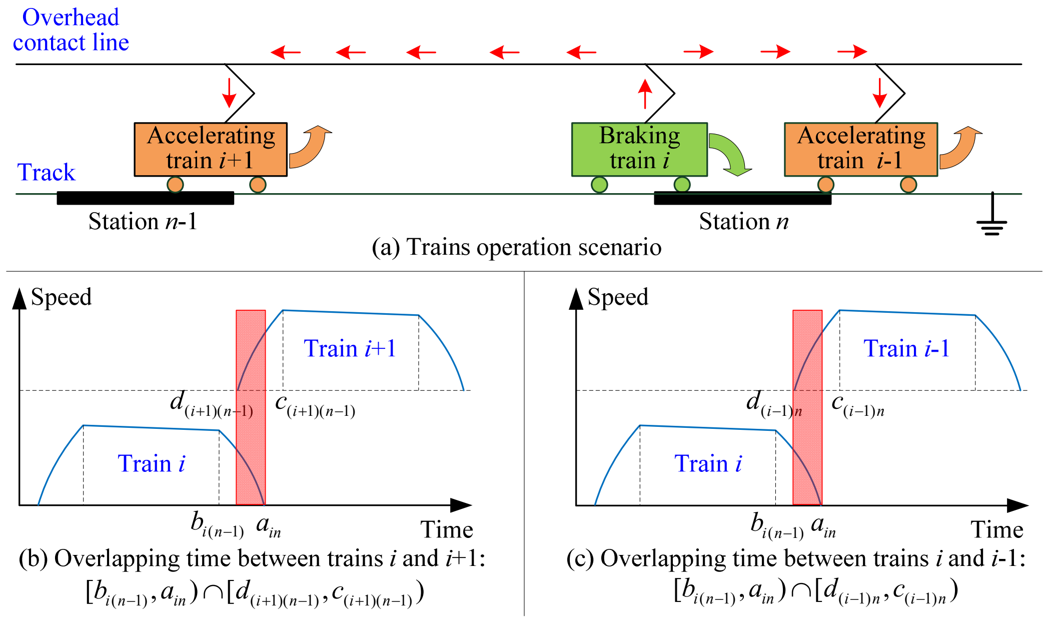


Fig. 6. Utilization of the regenerative braking energy from braking train i to adjacent trains i-1 and i+1.

Only during the total overlapping time, the regenerative braking energy of train i can be utilized by trains i+1 or i-1. Therefore, the total utilization of regenerative braking energy of train i from section (k, k+1) to section (N-1, N) is

$$J_B(\boldsymbol{a}) = \sum_{n=k+1}^{N} \sum_{t \in Ts} \min\left\{B_{i(n-1)}(\boldsymbol{a},t)(1-\beta), [F_{(i+1)(n-1)}(\boldsymbol{a},t) + F_{(i-1)n}(\boldsymbol{a},t)]\right\} + \sum_{t \in Ts} \min\left\{B_{iN}(\boldsymbol{a},t)(1-\beta), F_{(i+1)N}(\boldsymbol{a},t)\right\}, \tag{8}$$

which contains two terms: (1) the first term denotes that the utilization of regenerative braking energy of train i from section (k, k+1) to section (N-2, N-1); and (2) the second term denotes that the utilization of regenerative braking energy of train i at section (N-1, N), where train i-1 has already arrived at the destination station N, and is therefore not included. In both terms, the minimum operator is used to select between the regenerative braking energy and the tractive energy required to accelerate the train. If the tractive energy is larger than the regenerative braking energy, it means all the regenerative braking energy is being used and additional energy is needed to accelerate the adjacent trains according to the implemented speed profiles. On the other hand, if the tractive energy is smaller than the regenerative energy, it means the regenerative braking energy is more than sufficient to accelerate the adjacent trains, and the remaining regenerative braking energy not used will be wasted by heating resistors installed on the overhead contact lines.

Finally, the net energy consumption of train i from section (k, k+1) to section (N-1, N) is simply the difference between the tractive energy in Equation (5) and the regenerative braking energy in Equation (8):

$$J(\boldsymbol{a}) = J_F(\boldsymbol{a}) - J_B(\boldsymbol{a}). \tag{9}$$

Remark 2.

The equation $J_B(\boldsymbol{a})$ in the final objective function $J(\boldsymbol{a})$ contains a number of minimum operators, which make the $J(\boldsymbol{a})$ neither convex nor continuous (i.e., nonlinear and non-smooth). Therefore, it is difficult to evaluate the objective function using the traditional optimization method such as Newton algorithm and branch-and-bound algorithm.

3.5. *Optimization model*

Based on the above analysis, we formulate the energy-efficient rescheduling problem as the following integer programming model:

$$\begin{cases} \min \ \ J(\boldsymbol{a}) \\ s.t. \ \ a_{in} - d_{i(n-1)} \geq a_{0in} - d_{0i(n-1)} - \delta, \ \ n = k+1, k+2, \cdots, N, \\ \qquad \sum_{n=k+1}^{N} \left(a_{in} - d_{i(n-1)}\right) = \sum_{n=k+1}^{N} \left(a_{0in} - d_{0i(n-1)}\right) - T_d, \\ \qquad a_{in} \in \mathbf{Z}, \qquad\qquad\qquad\qquad n = k+1, k+2, \cdots, N, \end{cases} \tag{10}$$

where the first set of constraints ensures that the running time at each section of the rescheduling strategy should be not less than the minimum allowed running time; the second constraint ensures that the delay time at station k should be totally eliminated before train i arriving at station N; and the third set of constraints ensures that the decision variables are all integer according to the engineering requirements.

4. SOLUTION PROCEDURE

As explained in *Remark 2*, the final objective function in the optimization model (10) is neither convex nor continuous. The traditional optimization method may fail to find the optimal solution. Heuristic algorithms are often used to tackle these complex problems (Xu et al. 2014; Ramezani et al., 2016). Therefore, this section describes an allocation algorithm designed for solving the energy-efficient rescheduling problem formulated as an integer programming model. The allocation algorithm has been shown to be effective in solving the static timetable optimization problem (Yang et al. 2015a) with a genetic algorithm. The overall allocation algorithm procedure is depicted in Fig. 7, and summarized as follows:

Step 1. Initialize a large enough positive number λ and a number $\varepsilon = 0$.

Step 2. Input the delay time T_d and the delay occurred station k.

Step 3. Calculate the original running time $t_{(n\text{-}1,\, n)}$ at section (n-1, n), i.e., $t_{(n\text{-}1,\, n)} = a_{0in} - d_{0i(n\text{-}1)}$, where $n = k$+1, k+2, … , N.

Step 4. Set $T_d = T_d - \varepsilon$.

Step 5. If $T_d = 0$, stop and return the optimal solution. Otherwise, set $\varepsilon = 1$, $n = k + 1$, $j = 1$ and $U_j = \lambda$.

Step 6. Set $t_{(n\text{-}1,\, n)} = t_{(n\text{-}1,\, n)} - 1$.

Step 7. If $t_{(n\text{-}1,\,n)} \geq a_{0in}$ - $d_{0i(n\text{-}1)}$ - δ, calculate the net energy consumption $U_n = J(\boldsymbol{a})$ based on the equation (9). Otherwise, set $U_n = \lambda$ and $n = n + 1$, and go to step 6.

Step 8. If $U_n < U_j$, update $j = n$ and $U_j = U_n$. Otherwise, keep U_j.

Step 9. Set $t_{(n\text{-}1,\,n)} = t_{(n\text{-}1,\,n)} + 1$ and $n = n + 1$.

Step 10. If $n > N$, set $t_{(j\text{-}1,\,j)} = t_{(j\text{-}1,\,j)}$ -1 and go to step 4. Otherwise, go to step 6.

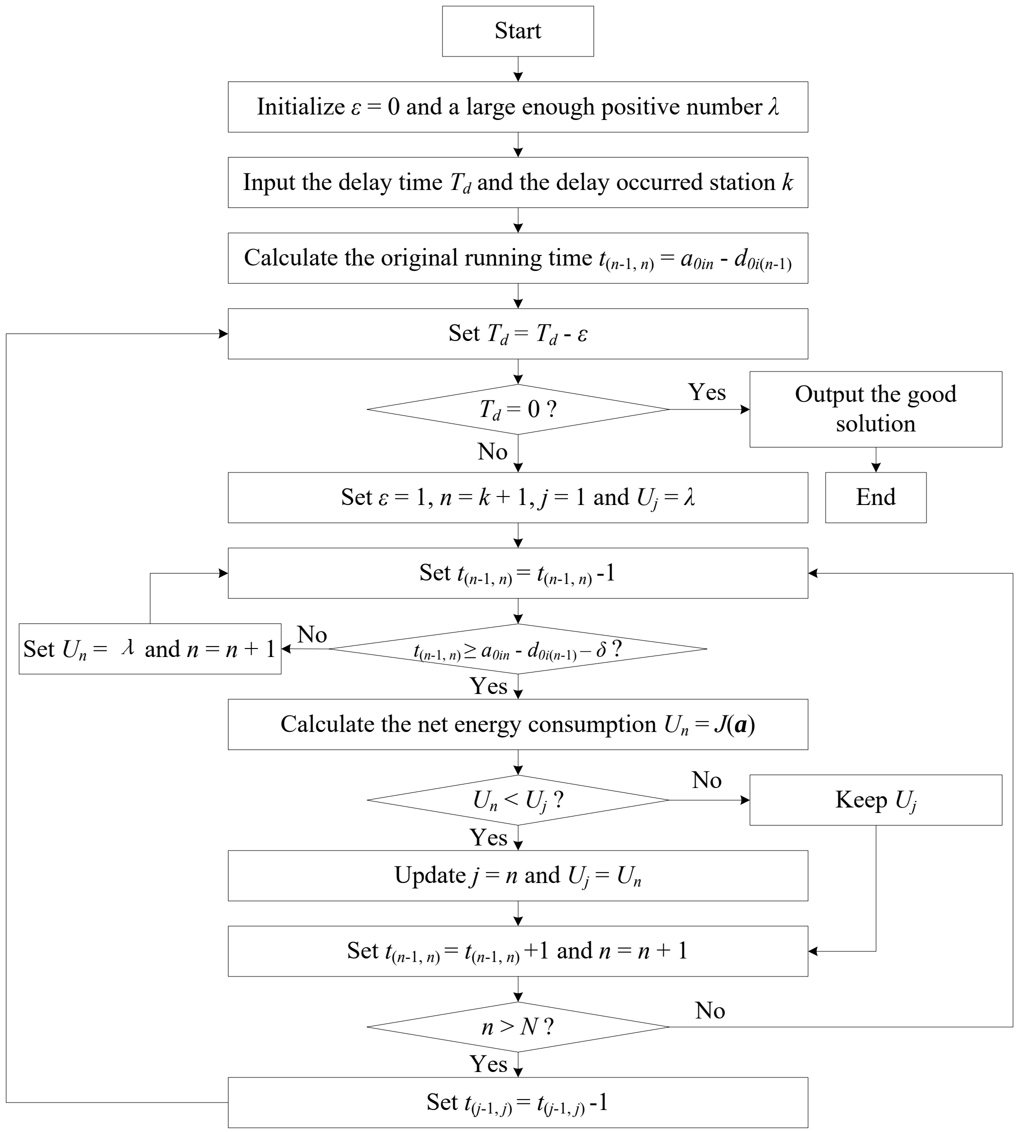


Fig. 7. Flow chart of the allocation algorithm.

5. NUMERICAL EXPERIMENTS

This section presents some numerical results using the real-world operation data from the Beijing Metro Yizhuang Line to illustrate the effectiveness of the developed energy-efficient rescheduling approach. As shown in Fig. 8, the Yizhuang Line consists of 14 stations from Songjiazhuang station to Yizhuang station. Based on the historical data, delays often occur at the Tongjinan station due to crowded passengers.

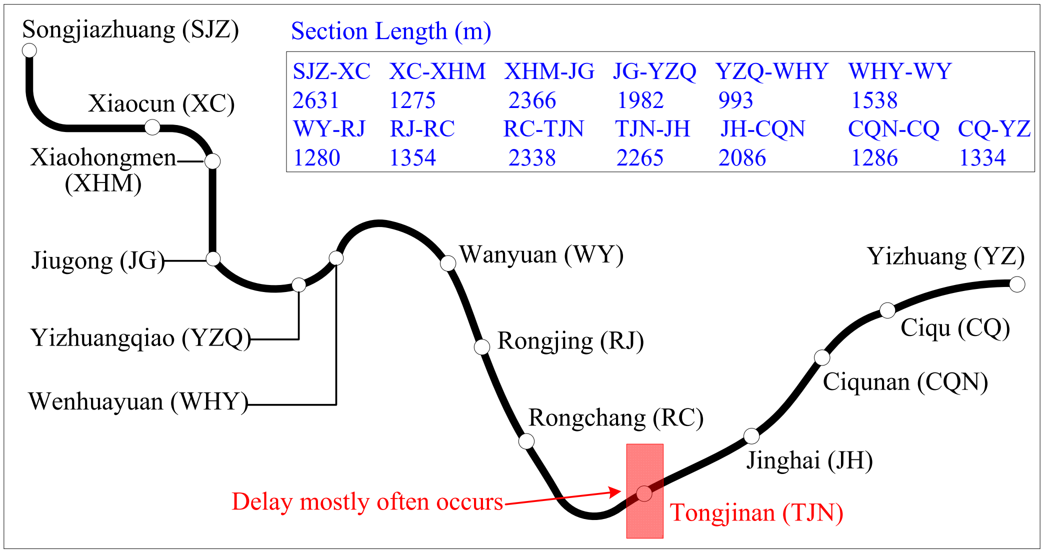


Fig. 8. Illustration of the Beijing Metro Yizhuang Line (Zhang 2014b).

We got the current real-life operation data of the Beijing Metro Yizhuang Line from the Beijing Metro Operations Company (Zhang 2014b). The original schedule is provided in Table 2, and the remaining parameters values are listed in Table 3.

Table 2. Original schedule for the Beijing Metro Yizhuang Line.

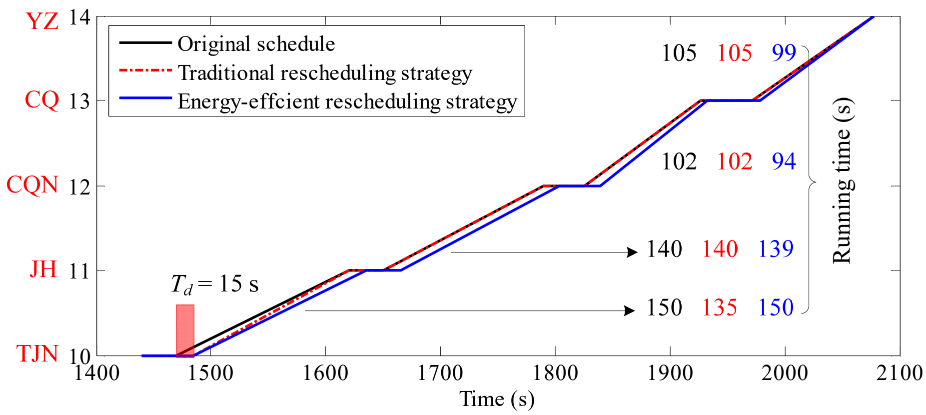
Station	SJZ	XC	XHM	JG	YZQ	WHY	WY
Arrival time (s)	0	220	358	545	710	835	979
Departure time (s)	30	250	388	575	745	865	1009
Station	RJ	RC	TJN	JH	CQN	CQ	YZ
Arrival time (s)	1112	1246	1440	1620	1790	1927	2077
Departure time (s)	1142	1276	1470	1650	1825	1972	--

Table 3. Value and unit of some parameters.

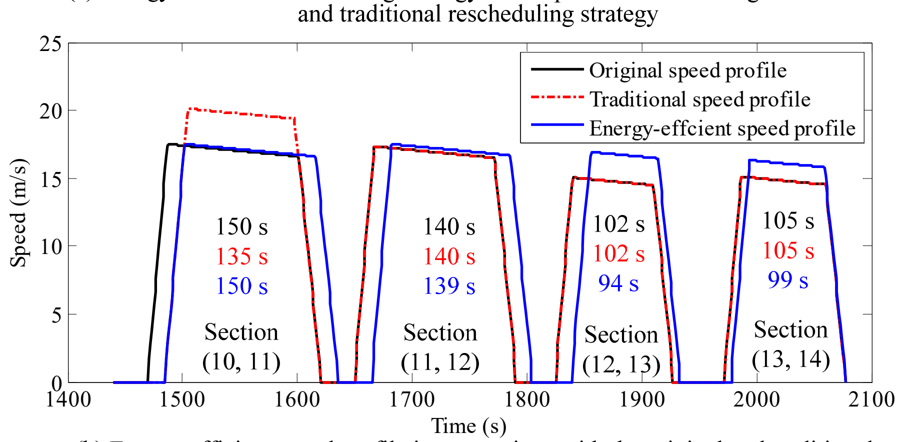
Parameter	m	h	h_0	f_a	f_b	r	g	η_1	η_2	β	δ
Value	311800	150	90	315000	258000	2000	500	0.7	0.8	0.05	20
Unit	kg	s	s	N	N	N	N	--	--	--	s

We assume the delays occurred at TJN station (i.e., $k = 10$), and the delay time is 15 seconds (i.e., T_d = 15 s). Based on the traditional rescheduling strategy (i.e., rescheduling strategy 1 in the *Problem Statement Section*), train i will speed up at the following section (10, 11) to eliminate the delay. In this case, the utilization of regenerative braking energy of train i is 0 kW·h, and the net energy consumption of train i is 72.0939 kW·h. By performing the allocation algorithm on a Windows 8.1 platform of personal computer with processor frequency of 2.4 GHz and memory size of 8 GB, we obtain the energy-efficient rescheduling strategy (including the speed profiles) shown in Fig. 9 in comparison with the original schedule and the traditional rescheduling strategy. The solution time is 853 ms.

Fig. 9a shows that: (1) the traditional rescheduling strategy eliminates the delay (15 s) by solely reducing the running time at section (10, 11) shown in Fig. 9b; (2) the energy-efficient rescheduling strategy distributes the delay into three sections (11, 12), (12, 13) and (13, 14) shown in Fig. 9b. We also obtain that the utilization of regenerative braking energy of train i is 4.9017 kW·h and the net energy consumption of train i is 66.1876 kW·h. The results suggest that the energy-efficient rescheduling strategy can reduce the net energy consumption of train i by (72.0939 - 66.1876) / 72.0939 = 8.19% in comparison with the traditional rescheduling strategy.



(a) Energy-efficient rescheduling strategy in comparison with the original schedule and traditional rescheduling strategy



(b) Energy-efficient speed profile in comparison with the original and traditional speed profile

Fig. 9. Energy-efficient rescheduling strategy in comparison with the original schedule and traditional rescheduling strategy.

To show how the energy-efficient rescheduling strategy improves the utilization of regenerative braking energy compared to the traditional rescheduling strategy, we present the overlapping time between train i and its adjacent trains i-1 and i+1 with both traditional and energy-efficient speed profiles as shown in Fig. 10. Using the time of train i as the standard time axis, Fig. 10a shows the speed profile of train i-1; Fig. 10b shows the traditional and energy-efficient speed profiles of train i; and Fig. 10c shows the speed profile of train i+1. The results show that: (1) there is no overlapping time between train i and its adjacent trains i-1 and i+1 when train i operates under the traditional speed profile; and (2) there is 17 s of overlapping time between train i and train i+1 (no overlapping time between train i and train i+1 in this case) when train i follows the energy-efficient speed profile. The increased 17 s of overlapping time (i.e., 14 s from train i braking at section (10, 11) and 3 s from train i braking at section (11, 12)) makes the utilization of regenerative braking energy of train i to be improved by 4.9017 kW·h and the net energy consumption of train i to be reduced by 8.19%.

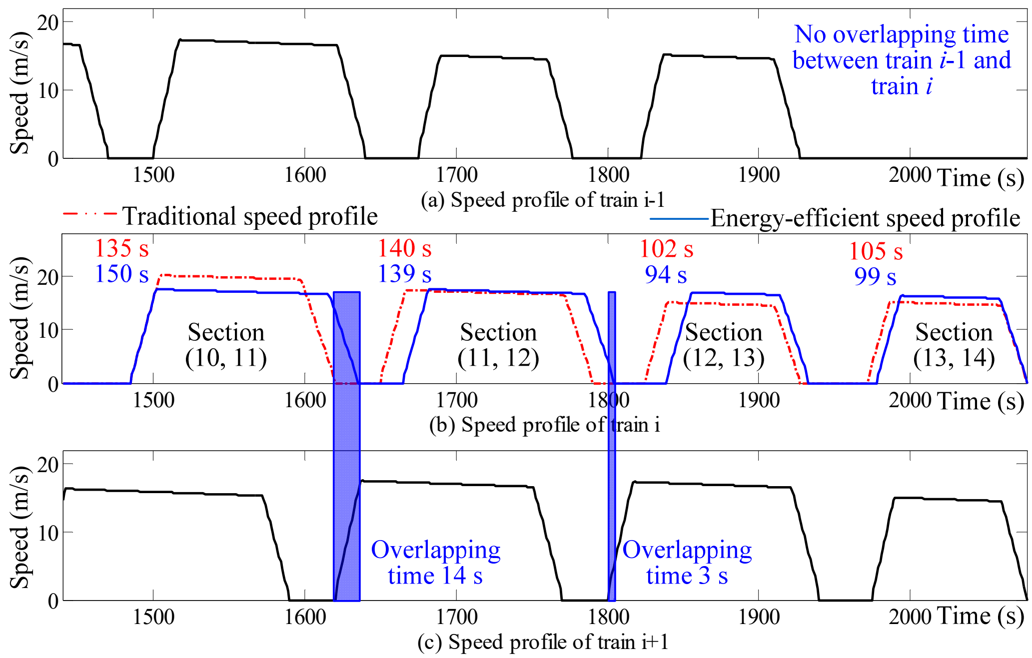


Fig. 10. Overlapping time between train i and its adjacent trains i-1 and i+1 with traditional and energy-efficient speed profiles.

6. Conclusion

The key contribution of this paper is to develop an energy-efficient rescheduling approach under delay perturbations for metro trains, which can minimize the net energy consumption under the premise of reducing or eliminating the delay. A numerical example based on the real-world operation data from the Beijing Metro Yizhuang Line shows that the developed energy-efficient rescheduling approach can reduce the net energy consumption by 8.19% in comparison with the traditional rescheduling approach.

Note that keeping the train punctual is always a priority to the train operation, the proposed rescheduling strategy has the same effect on keeping the train punctual in comparison with the traditional strategy. Besides, the proposed rescheduling strategy can achieve a better energy performance.

One of our future research directions is to consider the variabilities of train mass, maximum braking force, maximum tractive force, basic running resistance and additional running resistance based on the real-world systems. In addition, the conversions between electrical energy and mechanical energy can be calculated by explicitly modeling the electrical network instead of using the constant conversion efficiencies.

7. Recommendation

To improve the performance of train operation, a number of optimization approaches can be considered.

- If the metro managers want to solely increase the utilization of regenerative braking energy during the timetable planning phase, please refer the developed approaches in Yang et al. (2013, 2015a).
- If the metro managers want to simultaneously reduce the energy consumption and passenger time during the timetable planning phase, please refer the developed approaches in Yang et al. (2014, 2015b).
- If the metro managers want to reduce the tractive energy consumption with consideration of uncertain passenger demand during the timetable planning phase, please refer the developed approach in Yang et al. (2016b).
- If the metro managers want to minimize the net energy consumption under the premise of reducing or eliminating the delay during the train running phase, please refer the developed approach in this paper.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Nos. 71701013, 71525002, 71621001), the Fundamental Research Funds for the Central Universities (No. 2016RC051), the State Key Laboratory of Rail Traffic Control and Safety (No. RCS2017ZJ002), the Chang Jiang Chair Professorship to Tongji University in Shanghai, the Research Grants Council of the Hong Kong Special Administrative Region (Project No. PolyU 15267116), and the Research Committee of the Hong Kong Polytechnic University (Project No. 1-ZVJV).

References

Domínguez, M., Fernández-Cardador, A., Cucala, A. P., and Pecharromán R. R. (2012). "Energy savings in metropolitan railway substations through regenerative energy recovery and optimal design of ATO speed profiles." *IEEE Transactions on Automation Science and Engineering*, 9(3), 496-504.

González-Gil, A., Palacin, R., and Batty, P. (2013). "Sustainable urban rail systems: Strategies and technologies for optimal management of regenerative braking energy." *Energy conversion and management*, 75, 374-388.

González-Gil, A., Palacin, R., Batty, P., and Powell, J. P. (2014). "A systems approach to reduce urban rail energy consumption." *Energy Conversion and Management*, 80, 509-524.

Howlett, P., and Pudney, P. (1995). *Energy-Efficient Train Control*. Springer, London.

Ke, B. R., Chen, M. C., and Lin, C. L. (2009). "Block-layout design using MAX–MIN ant system for saving energy on mass rapid transit systems." *IEEE Transactions on Intelligent Transportation Systems*, 10(2), 226-235.

Ke, B. R., Lin, C. L., and Yang, C. C. (2012). "Optimisation of train energy-efficient operation for mass rapid transit systems." *IET Intelligent Transport Systems*, 6(1), 58-66.

Kim, K., and Chien, S. I. J. (2010a). "Optimal train operation for minimum energy consumption considering track alignment, speed limit, and schedule adherence." *Journal of Transportation Engineering*, 137(9), 665-674.

Kim, K., and Chien, S. I. J. (2010b). "Simulation-based analysis of train controls under various track alignments." *Journal of Transportation Engineering*, 136(11), 937-948.

Li, X., and Lo, H. K. (2014a). "An energy-efficient scheduling and speed control approach for metro rail operations." *Transportation Research Part B: Methodological*, 64, 73-89.

Li, X., and Lo, H. K. (2014b). "Energy minimization in dynamic train scheduling and control for metro rail operations." *Transportation Research Part B: Methodological*, 70, 269-284.

Li, X., and Yang, X. (2013). "A stochastic timetable optimization model in subway systems." *International Journal of Uncertainty, Fuzziness & Knowledge-Based Systems*, 21(Supp. 1), 1-15.

Peña-Alcaraz, M., Fernández-Cardador, A., Cucala, A. P., Ramos, A., and Pecharromán, R. R. (2012). "Optimal underground timetable design based on power flow for maximizing the use of regenerative-braking energy." *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 226(4), 397-408.

Ramos, A., Peña-Alcaraz, M., Fernández-Cardador, A., and Cucala, A. P. (2007). "Mathematical programming approach to underground timetabling problem for maximizing time synchronization." *Proceedings of International Conference on Industrial Engineering and Industrial Management*, Madrid, Spain, 88-95.

Ramezani, M., Lamberterie, N. D., Skabardonis, A., and Geroliminis, N. (2016). "A link partitioning approach for real-time control of queue spillbacks on congested arterials." *Transportmetrica B: Transport Dynamics*, 1-14.

Xu, M., Lam, W. H. K., Gao, Z., and Grantmuller, S. (2016). "An activity-based approach for optimisation of land use and transportation network development." *Transportmetrica B: Transport Dynamics*, 4(2), 1-24.

Yang, X., Chen, A., Li, X., Ning, B., and Tang, T. (2015a). "An energy-efficient scheduling approach to improve the utilization of regenerative energy for metro systems." *Transportation Research Part C: Emerging Technologies*, 57, 13-29.

Yang, X., Li, X., Gao, Z., Wang, H., and Tang, T. (2013). "A cooperative scheduling model for timetable optimization in subway systems." *IEEE Transactions on*

Intelligent Transportation Systems, 14(1), 438-447.

Yang, X., Li, X., Ning, B., and Tang, T. (2015b). “An optimisation method for train scheduling with minimum energy consumption and travel time in metro rail systems.” *Transportmetrica B: Transport Dynamics*, 3(2), 79-98.

Yang, X., Li, X., Ning, B., and Tang, T. (2016a). “A survey on energy-efficient train operation for urban rail transit.” *IEEE Transactions on Intelligent Transportation Systems*, 17(1), 2-13.

Yang, X., Chen, A., Ning, B., and Tang, T. (2016b). “A stochastic model for the integrated optimization on metro timetable and speed profile with uncertain train mass.” *Transportation Research Part B: Methodological*, 91, 424–445.

Yang, X., Ning, B., Li, X., and Tang, T. (2014). “A two-objective timetable optimization model in subway systems.” *IEEE Transactions on Intelligent Transportation Systems*, 15(5), 1913-1921.

Zhang, T. (2014a). The annual report for train operations of the Beijing Metro, Technical Report, Beijing Mass Transit Railway Operation Corporation Limited. (in Chinese).

Zhang, L. (2014b). The operation data for the Beijing Metro Yizhuang Line, Technical Report, Beijing Mass Transit Railway Operation Corporation Limited. (in Chinese).