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# A review on dynamic substructuring methods for model updating and damage detection of large-scale structures

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**Abstract**

Substructuring methods possess many merits in model updating and damage identification of large-scale structures. With substructuring methods, a global structure is divided into a number of independent substructures. Only the substructures are repeatedly analyzed and the re-analysis of the global structure is thereby avoided. This paper reviews widely-used dynamic substructuring methods for model updating and damage identification of large-scale structures. These methods can be categorized into forward and inverse substructuring approaches. The former is a conventional process that assembles the vibration properties of each substructure to obtain the vibration properties of the global structure. The latter, on the other hand, disassembles the vibration properties of the global structure into those of the substructures. In each category, both frequency and time domain methods have been developed and will be reviewed.

**Key words:** substructuring method, model updating, vibration method, damage detection, large-scale structures.

## 1. INTRODUCTION

Vibration-based structural health monitoring (SHM), including vibration response analysis, model updating and damage identification, has been investigated worldwide during the past decades (Ni et al., 2012; Xia et al., 2011; Mottershead et al., 2011; Brownjohn, 2007). Civil structures are often large-scale, and their finite element (FE) models consist of a large number of degrees-of-freedom (DOFs) and uncertain parameters. The response analysis, model updating and damage identification of large-scale structures are expensive in terms of computation time and computer memory because the analysis may need a number of iterations (Brownjohn et al., 2001; Bakir et al., 2007). For example, Xia et al. (2008) performed a model updating for the three-span Balla Balla Bridge in Western Australia and took 155 iterations and approximately 420 hours for convergence. Duan et al. (2011) constructed a fine FE model of the Tsing Ma Suspension Bridge which consists of about 300, 000 nodes, 450, 000 elements, and 1.2 million DOFs. About five hours were spent to obtain the first 100 eigensolutions using a 64-bit Itanium server with eight CPUs of 1.5 GHz each. In such a case, updating the FE model using the conventional approach is very difficult, even with a powerful computer.

Dynamic substructuring methods have played a significant role in the field of structural dynamics. With the substructuring methods, a global structure is divided rationally into smaller substructures to be analyzed independently (Craig, 2000). Analyzing a structure component-wisely has several advantages over the global methods which handle the entire problem at once. First, independent substructures can be analyzed much easier and

quicker than the large-size global structure. Second, local behaviors/parameters can be identified more accurately than the entire system as a substructure contains considerably fewer uncertain parameters than the global structure. As such, the convergence of the optimization process is more rapid and ill-condition problems are alleviated. Third, substructuring methods allow sharing and combining substructures from different groups, or from numerical simulations and experimental measurements. This feature allows the combination of different substructures and parallel computation. Finally, if a local area of a structure is of interest, only the local area is required to be measured or calculated, and the testing or computation of the entire structure can be avoided.

Substructuring methods can be carried out in two manners. In the conventional substructuring approaches, a global FE model is partitioned into independent substructural models. The vibration properties of the substructures are then *assembled* to recover the vibration properties of the global structure. Herein the vibration properties can be the frequency domain properties such as natural frequencies, mode shapes, frequency response functions (FRFs) and their variants or the time domain properties such as vibration responses, impulse response functions (IRFs) and their variants. The obtained vibration properties can be used for model updating or damage identification by comparing with the experimental counterparts of the global structure through a model updating process. This process is referred to as the forward substructuring method in the context. On the other hand, the inverse method *disassembles* the vibration properties of the global structure into those of substructures, which can be used to update the

independent substructures by treating them as independent structures. A main merit of this approach is that the damage identification can be more effectively performed since the vibration properties of the substructures are more sensitive to the local damage. On the basis of this classification, the forward and inverse substructuring methods will be introduced and reviewed in Sections 2 and 3, respectively. The methods in time and frequency domain will be presented.

## 2. FORWARD SUBSTRUCTURING METHODS

The forward substructuring methods have existed for a relatively long period (Kron, 1963; Craig, 2000; Hurty, 1965). A global complex structure is divided into substructures to increase the efficiency, and the solutions of the substructures are coupled to calculate the solutions of the global structure. If the global structure with $N$ DOFs is divided into $N_S$ substructures, the equation of motion is coupled for the $N_S$ substructures and can be rewritten in a block-diagonal format as

$$\mathbf{M}^p\{\ddot{x}^p\}+\mathbf{C}^p\{\dot{x}^p\}+\mathbf{K}^p\{x^p\}=\{f^p\}+\{g^p\} \tag{1}$$

with the compatibility condition and equilibrium condition

$$\mathbf{D}\{x^p\}=\mathbf{0},\ \ \mathbf{L}^T\{g^p\}=\mathbf{0} \tag{2}$$

where $\{x^p\}$, $\{\dot{x}^p\}$, and $\{\ddot{x}^p\}$ are the assembled global displacement, velocity and acceleration responses, respectively, $\mathbf{M}^p$, $\mathbf{C}^p$ and $\mathbf{K}^p$ are the block-diagonal assembly of substructural mass matrices, damping matrices and stiffness matrices, respectively, matrix $\mathbf{D}$ includes constraints to guarantee the nodes at the interface identical

displacement, matrix **L** localizes the interface DOFs of the substructures in the global structure, $\{f^p\}$ is the external forces, and $\{g^p\}$ is the interface forces. Hereinafter, superscript *p* denotes the primitive assembly of all substructural variables. In particular, a primitive matrix is assembled by placing all substructural matrices in a block-diagonal form, and a primitive vector is assembled by including the substructural vectors one after another. The equation of motion (Eq. (1)) can be transformed and simplified in the frequency or time domain. In the former, the eigensolutions (eigenavlues and eigenvectors) and the associated eigensensitivities are calculated. In the latter, the time history responses and the associated sensitivities are sought. The methods in the two domains are described separately in the following sections.

*2.1 Substructuring methods for eigensolutions and eigensensitivity in frequency domain*

2.1.1 Eigensolutions by substructuring methods

Common frequency domain substructuring methods include Kron's substructuring method and component mode synthesis (CMS) method. Performing the translation of $\{x,\tau\}^T = \{\bar{\phi},\tau\}^T \exp\left(i\sqrt{\bar{\lambda}}t\right)$ (*i* stands for the imaginary part) on Eq. (1), the eigensolutions of the global structure are recovered from the eigensolutions of $N_s$ substructures (Simpson, 1973; Sehmi, 1989)

$$\begin{bmatrix} \mathbf{\Lambda}^p - \bar{\lambda}\mathbf{I} & -\mathbf{\Gamma} \\ -\mathbf{\Gamma}^T & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{z} \\ \tau \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \tag{3}$$

where

$$\boldsymbol{\Lambda}^p = \begin{bmatrix} \boldsymbol{\Lambda}^{(1)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Lambda}^{(2)} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\Lambda}^{(N_s)} \end{bmatrix}, \quad \boldsymbol{\Phi}^p = \begin{bmatrix} \boldsymbol{\Phi}^{(1)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Phi}^{(2)} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\Phi}^{(N_s)} \end{bmatrix}, \quad \boldsymbol{\Gamma} = \left[ \mathbf{D}\boldsymbol{\Phi}^p \right]^T \tag{4}$$

$\boldsymbol{\Lambda}^p$ is the block-diagonally assembled eigenvalues of all substructures, $\boldsymbol{\Phi}^p$ is block-diagonally assembled eigenvectors of all substructures, $\{\tau\}$ represents the interface forces between the adjacent substructures, $\{\mathbf{z}\}$ indicates the participation factor of substructural modes in frequency domain, and $\bar{\lambda}$ denotes eigenvalues of the global structure to be determined. The mode shapes of the global structure can be recovered by $\bar{\boldsymbol{\Phi}} = \boldsymbol{\Phi}^p \{\mathbf{z}\}$.

1) Eigensolutions by Kron's substructuring method

Kron first proposed a dynamic substructuring method in *Diakoptics* (Kron, 1963) to obtain eigensolutions of a system with numerous variables in a piece-wise manner. The independent substructures are constrained by imposing displacement constraints at the interface coordinates of the adjacent substructures via the Lagrange multiplier technique and virtual work theorem. Simpson and Tabarrok (1968) initiated Kron's complicated electrical notation into the receptance form of a structure and searched the eigenvalues via the bisection scanning and sign count algorithms. Simpson (1973, 1982) replaced the receptance form with a transcendental dynamic stiffness matrix. The Newtonian process was utilized to accelerate the computational speed. Williams and Kennedy (1991) proposed a multiple-determinant parabolic interpolation method to ensure the successful

convergence on the required eigenvalues in all circumstances. Sehmi (1986, 1989) solved Kron's receptance matrix by the subspace iteration method and Lanczos method.

In the Kron's receptance matrix, the substructural eigenvalues and eigenvectors are calculated from the stiffness and mass matrices of substructures directly without constraining on the substructural boundaries. It renders the substructural eigensolutions calculated in its original boundary condition. Consequently, the Kron's substructuring method has distinct advantages in handling large-scale systems due to its high accuracy under complicated interface conditions.

In the Kron's substructuring method, all modes of each substructure are calculated to form $\mathbf{\Lambda}^p$ and $\mathbf{\Phi}^p$. Calculation of all substructural modes is often inefficient and unworthy because SHM generally needs the first several modes of the entire structure only. Weng et al. (2009) constructed the eigenequation of the global structure by the first few master modes of each substructure, and the residual slave modes are compensated by the residual flexibility. The eigenequation (Eq. (3)) is then rewritten in accordance with the master modes and residual flexibility as (Weng et al., 2009)

$$\left[\left(\mathbf{\Lambda}_m^p - \bar{\lambda}\mathbf{I}\right) + \mathbf{\Gamma}_m \zeta^{-1} \mathbf{\Gamma}_m^T\right]\{\mathbf{z}_m\} = \mathbf{0} \tag{5}$$

where $\mathbf{\Gamma}_m = \left[\mathbf{D}\mathbf{\Phi}_m^p\right]^T$, and subscripts $m$ and $s$ represent "master" and "slave" eigensolutions, respectively. The master substructural eigenmodes are diagonally assembled as

$$\mathbf{\Lambda}_m^p = Diag\left[\mathbf{\Lambda}_m^{(1)}, \mathbf{\Lambda}_m^{(2)}, ..., \mathbf{\Lambda}_m^{(N_s)}\right], \quad \mathbf{\Phi}_m^p = Diag\left[\mathbf{\Phi}_m^{(1)}, \mathbf{\Phi}_m^{(2)}, ..., \mathbf{\Phi}_m^{(N_s)}\right] \tag{6}$$

The contribution of the slave modes is compensated by the first-order residual flexibility $\zeta = \mathbf{\Gamma}_s^T \left( \mathbf{\Lambda}_s^p \right)^{-1} \mathbf{\Gamma}_s$, which is calculated from the master modes as

$$\mathbf{\Gamma}_s^T \left( \mathbf{\Lambda}_s^p \right)^{-1} \mathbf{\Gamma}_s = \mathbf{D} \mathbf{\Phi}_s^p \left( \mathbf{\Lambda}_s^p \right)^{-1} \left[ \mathbf{\Phi}_s^p \right]^T \mathbf{D}^T$$

$$\mathbf{\Phi}_s^p \left( \mathbf{\Lambda}_s^p \right)^{-1} \left[ \mathbf{\Phi}_s^p \right]^T = \begin{bmatrix} \left( \mathbf{K}^{(1)} \right)^{-1} - \mathbf{\Phi}_m^{(1)} \left( \mathbf{\Lambda}_m^{(1)} \right)^{-1} \left[ \mathbf{\Phi}_m^{(1)} \right]^T & & \\ & \ddots & \\ & & \left( \mathbf{K}^{(N_s)} \right)^{-1} - \mathbf{\Phi}_m^{(N_s)} \left( \mathbf{\Lambda}_m^{(N_s)} \right)^{-1} \left[ \mathbf{\Phi}_m^{(N_s)} \right]^T \end{bmatrix} \tag{7}$$

The size of the simplified eigenequation (Eq. (5)) is equal to the number of all master modes, which is considerably smaller than the original one in Eq. (3). The computational load is thus considerably reduced.

On the basis of this concept of modal truncation and the compensation of the residual flexibility, Weng et al. (2009) proposed a more accurate substructuring method by including the second-order residual flexibility. In addition, Weng et al. (2011a) included the inertia effect of the slave modes by the high-order residual flexibility through an iterative procedure. After several iterations, the eigensolutions converge to the exact values using few master modes only.

2) Eigensolutions by Component Mode Synthesis (CMS)

Although the CMS technique also calculates the eigensolutions from the substructural eigenmodes, it constrains independent substructures through four kinds substructural modes such as the normal modes, rigid body modes, constraint modes and attachment

modes (Craig, 2000; Hurty, 1965; MacNeal, 1971; Rubin, 1975). Two or more kinds of substructural modes are required to be calculated in advance, which makes the CMS method more accurate. However, the method cannot be used when one or more kinds of substructural modes are not available under complicated interface conditions.

The normal modes are obtained using the general eigenequation for both the fixed boundary condition and free boundary condition. Usually, only a few normal modes are retained for improving the computational efficiency while the other modes are discarded. The rigid body modes (RBMs) describe the rigid body movement of a free-free substructure. For a two-dimensional structure having $N$ nodes, the three independent RBMs are the $x$ translation ($R_x = 1$, $R_y = 0$), the $y$ translation ($R_x = 0$, $R_y = 1$) and the $z$ rotation ($R_x = -y$, $R_y = x$), i.e.,

$$\mathbf{R}^T = \begin{bmatrix} 1 & 0 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 1 & 0 & 0 & \cdots & 1 & 0 \\ -y_1 & x_1 & 1 & -y_2 & \cdots & x_N & 1 \end{bmatrix} \tag{8}$$

A constraint mode is defined as the force to generate a unit displacement to one constraint coordinate, while the remaining constraint coordinates are restrained and the remaining DOFs of the structure are free. The force equilibrium satisfies

$$\begin{bmatrix} \mathbf{K}_{II} & \mathbf{K}_{IO} \\ \mathbf{K}_{OI} & \mathbf{K}_{OO} \end{bmatrix} \begin{bmatrix} \mathbf{\Phi}_{IO} \\ \mathbf{I}_{OO} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ f_{OO} \end{bmatrix} \tag{9}$$

The constraint mode matrix is given by

$$\underset{N\times N_b}{\mathbf{\Phi}_c} = \begin{bmatrix} \mathbf{\Phi}_{IO} \\ \mathbf{I}_{OO} \end{bmatrix} = \begin{bmatrix} -\mathbf{K}_{II}^{-1}\mathbf{K}_{IO} \\ \mathbf{I}_{OO} \end{bmatrix} \tag{10}$$

where subscript $I$ represents the inner DOFs and the subscript $O$ represents the interface DOFs. $N_b$ is the number of constraint modes and equal to the number of the inner DOFs.

An attachment mode is regarded as the displacement vector due to a single unit force applied at one of the given coordinates, which is therefore acquired by

$$\begin{bmatrix} \mathbf{K}_{II} & \mathbf{K}_{IO} \\ \mathbf{K}_{OI} & \mathbf{K}_{OO} \end{bmatrix} \begin{bmatrix} \mathbf{\Phi}_{IO} \\ \mathbf{\Phi}_{OO} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_{OO} \end{bmatrix} \tag{11}$$

Based on the above four groups of mode components, CMS methods are mainly classified into the fixed interface, free interface and hybrid interface (or mixed interface) CMS methods.

The fixed interface CMS method was first proposed by Hurty (1965), in which the global eigensolutions are expressed by the fixed normal modes and constraint modes. Craig and Bampton (1968) simplified this method without partitioning the interface forces into statically determinate and indeterminate ones. Their simplified method is successful and widely known for its brevity, accuracy and robustness. Suarez and Singh (1992) considered the effect of truncated normal modes with the mode superposition method to improve the computational precision. Shyu et al. (1997) accounted for the higher-order terms of the static constraint modes to improve the accuracy. Quasi-static modes rather than static constraint modes are used to compensate for the inertial effect of truncated modes. Takewaki and Uetani (2000) extended the fixed interface CMS method to the damped system. Rixen (2004) proposed an efficient dual Craig-Bampton method, in which the substructures are assembled dually with interface forces and a transformation matrix is defined to reduce the size of the primitive global system. Kim and Lee (2015) developed an enhanced Craig-Bampton method to improve the accuracy subsequently. This method derives an enhanced transformation matrix to account for the higher-order

effect of substructural modes with the residual flexibility. Bennighof and Lehoucq (2004) introduced an automatically multi-level substructuring method for the fast eigenanalysis of linear elastodynamics, which is accomplished via recursive partitioning and reordering process. Kim et al. (2015) derived a new transformation matrix to improve the accuracy of this multi-level substructuring method by considering the effect of residual modes. Koutsovasilis et al. (2010) and Kim et al. (2017) combined the fixed interface CMS method with the condensation method to reduce the dimension of the eigenequation further.

The major limitation of the fixed interface CMS methods is its inability to obtain constraint modes from experiments conveniently. Conducting a modal test on a free-free structure is considerably more convenient than on one with fixed constraints. To overcome this limitation, the free interface CMS method was first proposed by MacNeal (1971) for structures with flexible boundary conditions, in which the substructural displacements are represented by free interface normal modes, attachment modes and rigid body modes. Rubin (1975) later extended MacNeal's method to consider the inertial effect of truncated modes with a second-order Maclaurin-series expansion. Arora and Nuyen (1980) combined the free interface CMS method with the subspace iteration method to calculate the structural eigensolutions. Tournour et al. (2001) took a deep insight into the validation, performance and convergence of the free interface CMS method, and provided a simple and effective convergence criterion for the selection of retained substructural modes. Rixen (2004) used the Lagrange multipliers along the interface for assembling substructures, and effectively reduced the the assembled model

into a form of quasi-diagonal matrices. Kim et al. (2018) improved the free interface CMS method by considering the higher-order effect of the residual substructural modes, and the computational efficiency is improved through the system equivalent reduction expansion process. Several researchers have improved the computational efficiency of the free interface CMS methods by combining with efficient methods such as dynamic condensation approaches (Liu et al., 2011) and parallel computation techniques (Yang et al., 2012).

The free interface CMS method is not as accurate as the fixed interface CMS method, as the free interface constraint is weaker than the fixed interface. However, the free interface method is considerably more efficient in handling complicated substructural interfaces (Liew et al., 1996). The complete eigenmodes of the substructures with changed interface have to be recomputed in the fixed interface method (Voormeeren et al., 2012). As such, researchers have developed the hybrid interface (mixed interface) CMS method to make full use of the advantages of the fixed and free interface CMS methods.

Liew et al. (1996) proposed a hybrid CMS method to investigate the dynamic properties of a plate by using mixed normal modes. The accuracy and feasibility of this method are verified using several plate structures. Qiu et al. (2003) proposed a hybrid CMS method to express the higher free interface vibration modes with several lower hybrid modes. They developed an accurate substructuring method to calculate displacements precisely. Shanmugam and Padmanabhan (2006) introduced an accurate and efficient hybrid interface CMS method for rotor dynamic analysis, in which the free/fixed interface CMS

method is used in each substructure depending on the number of interface and inner DOFs. Voormeeren et al. (2010) developed a hybrid substructuring method to divide the substructural DOFs into a series of internal DOFs, free interface DOFs and fixed interface DOFs. The free or fixed boundary conditions of each substructure are selected by comparing the diagonal of the stiffness matrix of different substructures.

Apart from the substructuring method developed on substructural eigenmodes (normal modes, rigid body modes, constraint modes and attachment modes), Meirovitch and Kwak (1990), Morales (2000), and Johnson (2003) proposed the substructure-based Rayleigh-Ritz method, where each substructure is approximated by a series of admissible trial vectors with an iterative process. Craig and Hale (1988), Bai (2002), Salimbahrami and Lohmann (2006) converted the equation of motion into a state space. The computational time is saved by projecting the higher-order space into the lower-order space using Krylov subspaces/vectors.

3) Eigenmode selection by substructuring method

The substructuring method attempts to retain a small number of substructural modes to recover the dominant dynamic properties of the global system. An important topic is to determine which modes to be retained in each substructure. One commonly used scheme is determined from the aspect of energy. Generally, lower modes contribute more energy and are often retained in the substructuring method (Weng et al., 2009; Craig, 2000). However, this concept is not always correct. A number of mode selection methods have

been developed. Kammer and Triller (1996) constructed an effective interface mass (EIM) matrix to indicate the mode contribution at the substructural interfaces. Liao et al. (2007) proposed the moment-matching principle to form a coupling matrix for estimating the substructural mode contribution ($CMS_\chi$ method). Park et al. (2012) derived an equivalent modal contributor (EMC) method for mode selection. It relates the modes of a global system and subsystems to a vector, whose norm is then used to evaluate the contribution of each substructure. Kim et al. (2016) compared the EIM method, $CMS_\chi$ method and EMC method in terms of computational accuracy and efficiency.

### 2.1.2 Eigensensitivity by substructuring method

Eigensensitivity is the derivative of eigensolution to a specific elemental parameter, indicating the effect of parameter variation on the eigensolutions. Eigensensitivity provides the searching direction in sensitivity-based model updating or optimization process. Since the eigensensitivity is usually calculated element by element in model updating or optimization, it dominates the computational time. Following the substructuring method for eigensolutions, Xia et al. (2010) extended the Kron's method to derive the eigensensitivity formulae. The eigensensitivity of the $i$th ($i$ = 1, 2, …, $N$) mode with respect to an elemental parameter $\alpha$ is derived by differentiating Eq. (5) with regard to the parameter as

$$\left[\left(\mathbf{\Lambda}_m^p - \bar{\lambda}_i\mathbf{I}\right) + \mathbf{\Gamma}_m\zeta^{-1}\mathbf{\Gamma}_m^T\right]\frac{\partial\{\mathbf{z}_i\}}{\partial\alpha} + \frac{\partial\left[\left(\mathbf{\Lambda}_m^p - \bar{\lambda}_i\mathbf{I}\right) + \mathbf{\Gamma}_m\zeta^{-1}\mathbf{\Gamma}_m^T\right]}{\partial\alpha}\{\mathbf{z}_i\} = \{\mathbf{0}\} \tag{12}$$

By pre-multiplying $\{\mathbf{z}_i\}^T$ on both sides of Eq. (12) and noting $\left[\left(\mathbf{\Lambda}_m^p - \bar{\lambda}\mathbf{I}\right) + \mathbf{\Gamma}_m \zeta^{-1} \mathbf{\Gamma}_m^T\right]\{\mathbf{z}_m\} = \mathbf{0}$ in Eq. (5), the *i*th eigenvalue derivative with respect to $\alpha$ is

$$\frac{\partial \bar{\lambda}_i}{\partial \alpha} = \{\mathbf{z}_i\}^T \left[\frac{\partial \mathbf{\Lambda}_m^p}{\partial \alpha} + \frac{\partial\left(\mathbf{\Gamma}_m \zeta^{-1} \mathbf{\Gamma}_m^T\right)}{\partial \alpha}\right]\{\mathbf{z}_i\} \tag{13}$$

where

$$\frac{\partial\left(\mathbf{\Gamma}_m \zeta^{-1} \mathbf{\Gamma}_m^T\right)}{\partial \alpha} = \frac{\partial \mathbf{\Gamma}_m}{\partial \alpha} \zeta^{-1} \mathbf{\Gamma}_m^T - \mathbf{\Gamma}_m \zeta^{-1} \frac{\partial \zeta}{\partial \alpha} \zeta^{-1} \mathbf{\Gamma}_m^T + \mathbf{\Gamma}_m \zeta^{-1} \frac{\partial \mathbf{\Gamma}_m^T}{\partial \alpha} \tag{14}$$

$\frac{\partial \mathbf{\Lambda}_m^p}{\partial \alpha}$ and $\frac{\partial \mathbf{\Gamma}_m}{\partial \alpha}$ are the eigenvalue derivatives and eigenvector derivatives of the substructures, respectively, and $\frac{\partial \zeta}{\partial \alpha} = \frac{\partial\left(\mathbf{\Gamma}_s^T \left(\mathbf{\Lambda}_s^p\right)^{-1} \mathbf{\Gamma}_s\right)}{\partial \alpha}$ denotes the derivative of the residual flexibility of the substructures. Since the substructures are independent and unrelated, these derivative matrices are calculated only within the *j*th substructure containing the parameter $\alpha$, and those in other substructures are zero matrices, i.e.,

$$\frac{\partial \mathbf{\Lambda}_m^p}{\partial \alpha} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\partial \mathbf{\Lambda}_m^{(j)}}{\partial \alpha} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \frac{\partial \mathbf{\Gamma}_m^T}{\partial \alpha} = \mathbf{D} \frac{\partial \mathbf{\Phi}_m^p}{\partial \alpha} = \mathbf{D} \times \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\partial \mathbf{\Phi}_m^{(j)}}{\partial \alpha} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{15}$$

$$\frac{\partial \zeta}{\partial \alpha} = \frac{\partial\left[\left(\mathbf{\Gamma}_s^T \left(\mathbf{\Lambda}_s^p\right)^{-1} \mathbf{\Gamma}_s\right)^{-1}\right]}{\partial \alpha} = \mathbf{D} \times \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\partial\left(\left(\mathbf{K}^{(j)}\right)^{-1} - \mathbf{\Phi}_m^{(j)} \left(\mathbf{\Lambda}_m^{(j)}\right)^{-1} \left[\mathbf{\Phi}_m^{(j)}\right]^T\right)}{\partial \alpha} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \times \mathbf{D}^T$$

where $\frac{\partial \mathbf{\Lambda}_m^{(j)}}{\partial \alpha}$ and $\frac{\partial \mathbf{\Phi}_m^{(j)}}{\partial \alpha}$ can be calculated conveniently by treating the *j*th substructure as an independent structure (Nelson, 1976).

The eigenvector derivative is computed by rewriting $\left\{\frac{\partial \mathbf{z}_i}{\partial \alpha}\right\}$ in terms of a residual vector $\{\varphi_i\}$ and the vector $\{\mathbf{z}_i\}$ as

$$\left\{\frac{\partial \mathbf{z}_i}{\partial \alpha}\right\} = \{\varphi_i\} + c_i \{\mathbf{z}_i\} \tag{16}$$

where $c_i$ is the participation factor of vector $\{\mathbf{z}_i\}$ and $\{\varphi_i\}$ represents a residual vector. Substituting Eq. (16) into Eq. (12), the residual vector $\{\varphi_i\}$ is solved from equation

$$\left[\mathbf{\Lambda}_m^p + \mathbf{\Gamma}_m \zeta^{-1} \mathbf{\Gamma}_m^T - \bar{\lambda}_i \mathbf{I}\right]\{\varphi_i\} = -\frac{\partial\left[\mathbf{\Lambda}_m^p + \mathbf{\Gamma}_m \zeta^{-1} \mathbf{\Gamma}_m^T - \bar{\lambda}_i \mathbf{I}\right]}{\partial \alpha}\{\mathbf{z}_i\} \tag{17}$$

Vector $\{\mathbf{z}_i\}$ satisfies the orthogonal condition of $\{\mathbf{z}_i\}^T \{\mathbf{z}_i\} = 1$, which is differentiated with respect to $\alpha$ as

$$\frac{\partial \{\mathbf{z}_i\}^T}{\partial \alpha}\{\mathbf{z}_i\} + \{\mathbf{z}_i\}^T \frac{\partial \{\mathbf{z}_i\}}{\partial \alpha} = 0 \tag{18}$$

Substituting Eq. (16) into Eq. (18), the participation factor $c_i$ is thus obtained as

$$c_i = -\frac{1}{2}\left(\{\varphi_i\}^T \{\mathbf{z}_i\} + \{\mathbf{z}_i\}^T \{\varphi_i\}\right) \tag{19}$$

Finally, the $i$th eigenvector derivative with regard to the parameter $\alpha$ is calculated by

$$\frac{\partial \bar{\mathbf{\Phi}}_i}{\partial \alpha} = \frac{\partial \mathbf{\Phi}_m^p}{\partial \alpha}\{\mathbf{z}_i\} + \mathbf{\Phi}_m^p \left\{\{\varphi_i\} - \frac{1}{2}\left(\{\varphi_i\}^T \{\mathbf{z}_i\} + \{\mathbf{z}_i\}^T \{\varphi_i\}\right)\{\mathbf{z}_i\}\right\} \tag{20}$$

In Eq. (20), only the derivative matrix of the substructure containing parameter $\alpha$ is computed to recover the eigensensitivity of the global structure. Since the size of a substructure is considerably smaller than that of the global structure, the substructuring method is efficient in calculating the eigensensitivity. Lallemand (1999) derived the eigensensitivity matrix of the fixed interface CMS method. To ensure the computational precision and enhance the computational efficiency of this substructuring method

simultaneously, Weng et al. (2011a) proposed an iterative substructuring method for determining the exact eigensensitivity using a few master modes. In addition, Weng et al. (2013b) developed the substructuring method for deriving the higher-order eigensensitivity of structures with close or repeated eigenfrequencies.

### 2.1.3 FRF by substructuring method

Apart from substructuring methods for eigensolutions, substructure-based FRFs are widely studied in the literature. D'Ambrogio and Sestieri (2004) proposed the FRF-based substructuring method (FBS) to derive the global FRFs through the primal assembly of substructural FRFs. Performing a Fourier transform on Eq. (1) and Eq. (2) gives the governing equations for FRFs (Gordis et al., 1991).

$$\begin{cases} \mathbf{H}^p(\omega)\left\{x^p(\omega)\right\} = \left\{f^p(\omega)\right\} + \left\{g^p(\omega)\right\} \\ \mathbf{D}\left\{x^p(\omega)\right\} = 0 \\ \mathbf{L}^T\left\{g^p(\omega)\right\} = 0 \end{cases} \tag{21}$$

where $\left\{x^p(\omega)\right\}$, $\left\{f^p(\omega)\right\}$, $\left\{g^p(\omega)\right\}$ represent the amplitude of the harmonic response, external forces and interface forces, and $\mathbf{H}^p$ is a block-diagonal matrix of the substructural FRFs.

Klerk et al. (2006, 2008) summarized and classified the FBS methods into three groups: impedance coupling method (D'Ambrogio and Sestieri, 2004), admittance coupling method (Jetmundsen et al., 1988; Gordis et al., 1991), and Lagrange multiplier coupling method (Crowley et al., 1984). Lim and Li (2000) used the least squares method and

truncated singular value decomposition in the FBS method to improve its accuracy. Lee and Eun (2014) introduced an FBS method for local damage detection using a limited set of measurement data only. The substructure-based FRFs were used to calculate the constraint forces, which were then utilized for damage detection. Law and Ihlenfeldt (2015) developed an FBS method to model the position-dependent dynamic behaviors on the basis of the coupling substructure analysis of the multiple-point receptance.

*2.2 Substructuring methods for dynamic responses and response sensitivity in time domain*

The substructuring method is initially developed for the fast calculation of eigensolutions, where the global eigenmodes are calculated by superposition of a few substructural eigenmodes. By decoupling the global eigenmodes onto the space of substructural eigenmodes, the eigenequation of the large structure is reduced significantly. The substructuring concept can also be generalized to calculate dynamic responses in time domain, by reconstructing the dynamic response from the substructural eigenmodes. As the calculation of dynamic reponse includes a large number of time steps, the substructuring method can considerably improves the computational efficiency.

1) Dynamic responses and response sensitivity by substructuring method

If the responses of the structure in Eq. (1) is expressed by the sum of the substructural eigenvectors as $\left\{x^p\right\}=\mathbf{\Phi}_m^p\left\{q_m\right\}$ , $\left\{\dot{x}^p\right\}=\mathbf{\Phi}_m^p\left\{\dot{q}_m\right\}$ and $\left\{\ddot{x}^p\right\}=\mathbf{\Phi}_m^p\left\{\ddot{q}_m\right\}$ , the equation of

motion of the global structure can be written in terms of substructural master modes as (Zhu et al., 2019)

$$\left\{\ddot{q}_m\right\}+\left(a_1\mathbf{I}+a_2\boldsymbol{\Lambda}_m^p\right)\left\{\dot{q}_m\right\}+\boldsymbol{\Lambda}_m^p\left\{q_m\right\}+\boldsymbol{\Gamma}_m\zeta^{-1}\boldsymbol{\Gamma}_m^T\left\{q_m\right\}=\left[\boldsymbol{\Phi}_m^p\right]^T\left\{f^p\right\} \tag{22}$$

where $\left\{\ddot{q}_m\right\}$, $\left\{\dot{q}_m\right\}$ and $\left\{q_m\right\}$ represents the participation factors of the master modes in time domain, $a_1$ and $a_2$ are damping coefficients, and the residue item $\boldsymbol{\Gamma}_m\zeta^{-1}\boldsymbol{\Gamma}_m^{\mathrm{T}}\left\{q_m\right\}$ serves to compensate the effect of the slave modes. The participation factors can be solved using the classical Newmark method (Newmark, 1959). The size of Eq. (22) is equal to the number of retained master modes, which is considerably smaller than the original equation of motion of the global structure (Eq. (1)). This condition leads to the fast computation of the dynamic responses. The displacement, velocity, and acceleration of the global structure are then obtained from $\left\{x^p\right\}=\boldsymbol{\Phi}_m^p\left\{q_m\right\}$, $\left\{\dot{x}^p\right\}=\boldsymbol{\Phi}_m^p\left\{\dot{q}_m\right\}$ and $\left\{\ddot{x}^p\right\}=\boldsymbol{\Phi}_m^p\left\{\ddot{q}_m\right\}$. Gruber and Rixen (2018) extended this substructuring method to nonclassically damped linear systems in the state-space representation.

Zhu et al. (2019) derived the first derivative of the structural responses. Differentiating Eq. (22) with respect to parameter $\alpha$ leads to

$$\frac{\partial\left\{\ddot{q}_m\right\}}{\partial\alpha}+\left(a_1\mathbf{I}+a_2\boldsymbol{\Lambda}_m^p\right)\frac{\partial\left\{\dot{q}_m\right\}}{\partial\alpha}+\left(\boldsymbol{\Lambda}_m^p+\boldsymbol{\Gamma}_m\zeta^{-1}\boldsymbol{\Gamma}_m^T\right)\frac{\partial\left\{q_m\right\}}{\partial\alpha}=$$

$$\frac{\partial\left[\boldsymbol{\Phi}_m^p\right]^T}{\partial\alpha}\left\{f^p\right\}-\frac{\partial\left(a_1\mathbf{I}+a_2\boldsymbol{\Lambda}_m^p\right)}{\partial\alpha}\left\{\dot{q}_m\right\}-\left(\frac{\partial\boldsymbol{\Lambda}_m^p}{\partial\alpha}+\frac{\partial\boldsymbol{\Gamma}_m\zeta^{-1}\boldsymbol{\Gamma}_m^T}{\partial\alpha}\right)\left\{q_m\right\} \tag{23}$$

where $\partial\boldsymbol{\Lambda}_m^p/\partial\alpha$, $\partial\boldsymbol{\Phi}_m^p/\partial\alpha$, and $\partial\zeta/\partial\alpha$ can be calculated by treating the substructures as independent ones like Eq. (15). As previously stated, only the derivative matrix of the substructure containing parameter $\alpha$ is required, and those of remaining

substructures are zeros. $\frac{\partial\{\ddot{q}_m\}}{\partial\alpha}$, $\frac{\partial\{\dot{q}_m\}}{\partial\alpha}$ and $\frac{\partial\{q_m\}}{\partial\alpha}$ can be solved from Eq. (23) through the Newmark method. The response sensitivity is then calculated by

$$\frac{\partial\left\{x^p\right\}}{\partial\alpha}=\frac{\partial\boldsymbol{\Phi}_m^p}{\partial\alpha}\left\{q_m\right\}+\boldsymbol{\Phi}_m^p\frac{\partial\left\{q_m\right\}}{\partial\alpha} \tag{24}$$

$$\frac{\partial\left\{\dot{x}^p\right\}}{\partial\alpha}=\frac{\partial\boldsymbol{\Phi}_m^p}{\partial\alpha}\left\{\dot{q}_m\right\}+\boldsymbol{\Phi}_m^p\frac{\partial\left\{\dot{q}_m\right\}}{\partial\alpha} \tag{25}$$

$$\frac{\partial\left\{\ddot{x}^p\right\}}{\partial\alpha}=\frac{\partial\boldsymbol{\Phi}_m^p}{\partial\alpha}\left\{\ddot{q}_m\right\}+\boldsymbol{\Phi}_m^p\frac{\partial\left\{\ddot{q}_m\right\}}{\partial\alpha} \tag{26}$$

Similar to Eq. (22), the size of the sensitivity equation (Eq. (23)) is reduced greatly. Thus, calculating the response sensitivity using the substructuring method is computationally efficient.

2) IRF by substructuring method

The substructuring methods are also found in time domain for calculation of IRFs. Impulse-based substructuring (IBS) method is the counterpart of the FBS method in time domain, where the structural responses are assembled from the substructural IRFs. This method is effective when the excitation is a transient impact or shock-like load. The structural responses are calculated from those of substructures through Duhamel integral and mode superposition, where the interface compatibility is enforced at each time step. Gordis (1995), and Gordis and Radwick (1999) improved the computational efficiency of the IBS method by deriving an integral equation for structural transient analysis, and extended the IBS method to local nonlinear system. To improve the computational efficiency further, Gordis (2001) adopted the recursive block convolution to solve the

governing integral equation. The IBS method is also applied to structures with rigid-elastic hybrid joints or hybrid rigid and nonlinear-elastic joints (Dong, 2015; Zhou, 2016).

*2.3 Substructuring method for FE model updating*

Accurate FE models are frequently required in a large number of applications, such as optimization design, damage identification, structural control, and SHM (Brownjohn, 2007). Due to the uncertainties in geometry, material properties, and boundary conditions, the dynamic responses of a structure predicted from a highly idealized numerical model usually differ from the measurements obtained from the as-built structures. Therefore, an effective model updating is necessary to obtain a more accurate FE model for various applications. Many types of measurement data, such as frequencies, mode shapes, FRFs, and time domain responses can be employed for model updating (Mottershead and Friswell, 1993; Brownjohn et al., 2001; Bakira et al., 2007; Zivanovic et al., 2007; Brownjohn, 2007; Farrar and Worden, 2007; Friswell et al., 2007).

The iterative model updating modifies the elemental parameters of an FE model repeatedly to minimize an objective function, which is expressed as the residue of the dynamic properties of the FE model and experimental counterparts (Bakira et al., 2007; Zivanovic et al., 2007; Weng et al., 2011b)

$$J(\alpha)=\left(\left\{\chi^{A}(\alpha)\right\}-\left\{\chi^{E}\right\}\right)^{T}\mathbf{W}\left(\left\{\chi^{A}(\alpha)\right\}-\left\{\chi^{E}\right\}\right)+\gamma\left\|\left(\{\alpha\}-\{\alpha_{0}\}\right)\right\|_{2} \tag{27}$$

where $\left\{\chi^{A}\right\}$ is the dynamic properties (for example, frequencies, model shapes and dynamic responses) obtained from the FE model, $\left\{\chi^{E}\right\}$ represents the experimental

dynamic properties, $\mathbf{W}$ denotes the weighting matrix of different dynamic properties, and $\gamma$ stands for the regularization parameter. The objective function is minimized by adjusting the elemental parameters $\alpha$ starting from its initial values $\alpha_0$ in an optimal manner. Newton's methods (Bakira et al., 2007) can be used to solve the nonlinear optimization problem.

The conventional model updating approaches calculate the dynamic properties using the global FE model. The global method is very expensive in terms of computation time and computer memory, especially for large-scale structures. First, the global FE model contains large system matrices (stiffness and mass matrices), which takes up a large amount of storage space and computational time to analyze. Second, extracting dynamic properties from large system matrices is a time-consuming process, and calculating sensitivity matrices consumes significantly more computational resources. Third, many uncertain parameters need to be adjusted in a large-scale FE model. It takes a long time to calculate sensitivity matrices with respect to the large number of updating parameters that the model contains. Moreover, the large number of parameters are likely to result in the divergence of the large-scale optimization problem, thus leading to erroneous sensitivities.

The substructuring method is efficient to calculate the dynamic properties and sensitivity matrices of the global structure, as the substructures are analyzed easier and quicker than the global structure. During the model updating process, some specific substructures can be re-analyzed and assembled with other unchanged substructures to recover the solutions

of the global structure, thereby avoiding repeated computation of the global structural properties. This will benefit the model updating process, where the dynamic properties and sensitivity matrices are iteratively required to match the experimental global properties in an optimal way. In addition, the substructures contain much fewer uncertain parameters than the global structure. This assists in accelerating the convergency of large-scale optimization problems.

The forward substructure-based model updating is illustrated in Figure 1. In each iteration, the vibration properties are calculated from substructures and then assembled by the interface displacement constraints to calculate the global properties. These global solutions are compared with the experimental counterparts to construct the objective function. The substructure-based sensitivity matrices with respect to an elemental parameter are calculated from one substructure that contains the elemental parameter, to indicate the searching direction in each step. The objective function is minimized by iteratively adjusting the elemental parameters $\alpha$ in accordance with the sensitivity matrices. The most advantageous feature of this substructure-based model updating method is that one substructure is handled independently without repeatedly analyzing the large-size matrices of the global structure when this substructure is changed.

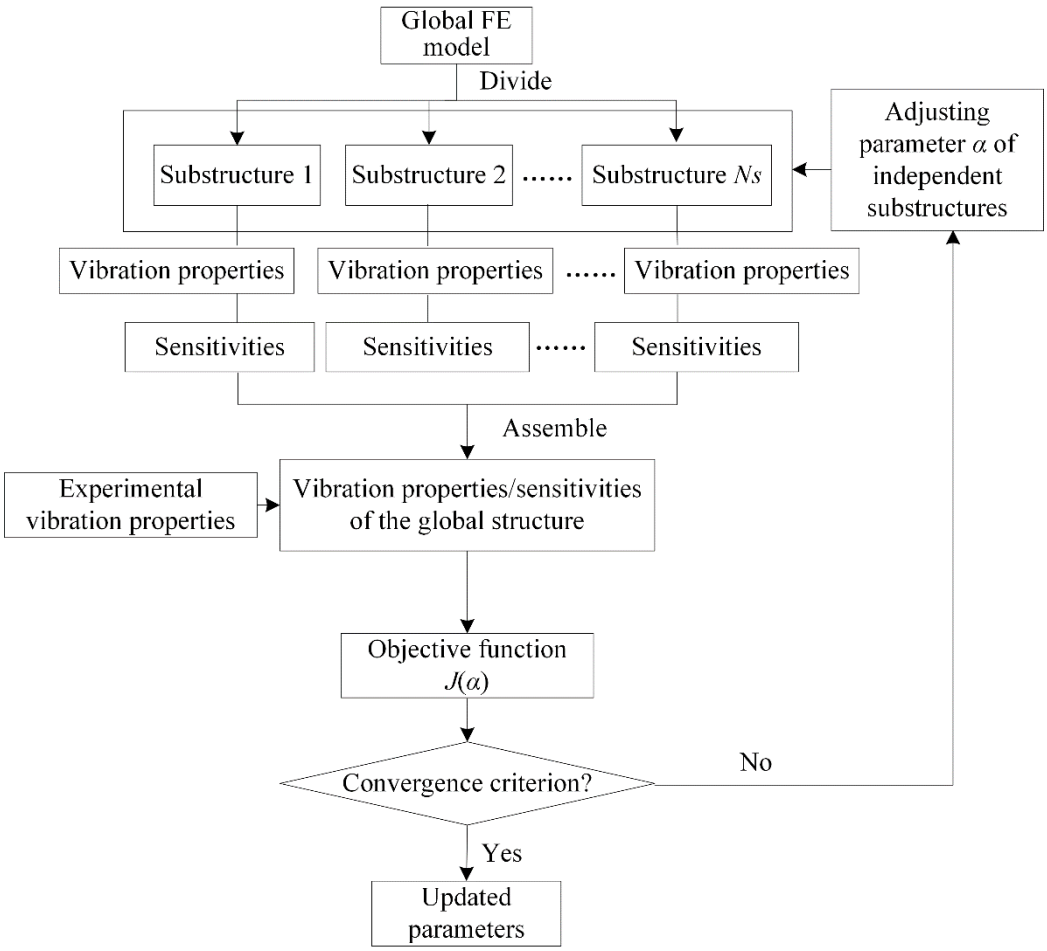


**Figure 1.** Forward substructuring method for model updating

Weng et al. (2011b) adopted the substructuring method to the model updating of a practical bridge. When a structure is damaged in a local area, only one or several substructures are repeatedly analyzed and the other substructures are unchanged. Yu et al. (2016) derived the eigensensitivity using the fixed interface CMS method and applied it to model updating process as well. Xu et al. (2018) proposed a multi-level damage identification of a bridge structure. The CMS method is used to condense the large-scale model, and model updating is applied along with the response reconstruction technique to identify the local damage.

## 3. INVERSE SUBSTRUCTURING METHOD

The majority of substructuring methods belong to the above forward substructuring method for the fast calculation of dynamic properties of the global structure. The forward substructure-based model updating requires the repeated assembly of the vibration properties (e.g., eigensolutions) of the substructural FE models into global vibration properties, and the assembled properties are then compared with the measurements on global structure. Although this assembly of substructural vibration properties is realized by a small-size equation (Eq. (5)), the repeated computation on the small-size equation is a heavy work as the model updating usually requires many iterations to converge.

Substructuring method can also be developed in an inverse manner, where the global dynamic properties are disassembled into substructural properties. Afterwards, a substructure can be used for model updating, optimization design, static/dynamic analysis, and vibration control by treating it as an independent structure (Weng et al., 2012). This approach, obtaining substructural properties from the global data, is in opposite direction of the conventional process and thus is referred to as the inverse substructuring method.

The inverse substructuring method involves the identification of substructural properties. When it is utilized in model updating, the global measurements are disassembled into the vibration properties of the substructures. Subsequently, the substructural vibration properties are used as references for updating the corresponding substructural FE models via the conventional model updating procedure. The objective function and sensitivity

analyisis are constructed directly on the independent substructures, without requiring the assembly procedure. In SHM, the change of global properties caused by damage (e.g., stiffness loss in the local area) is sometimes undetectable. Substructural properties are considerably more sensitive to local change than the global properties. It is promising to investigate the change of the substructural properties for model updating and damage identification. The FE models of independent substructures can also be updated in parallel to reproduce the substructural properties disassembled from the measurement of the global structure.

The inverse substructuring method can be further divided into frequency domain and time domain methods, according to the vibration properties disassembled.

*3.1 Extracting frequency domain properties of substructures*

In frequency domain, the challenging issue is to find the relation between the substructural properties and the global ones, and then extract the substructural properties from the global measurements. Doebling et al. (1998) extracted the substructural stiffness matrix through a disassembly procedure from the global stiffness and flexibility matrices on the basis of the presumed connectivity and strain energy distribution in substructures. The substructural modes are estimated based on the shape functions and geometry, and the substructural eigenvalues are solved from the well-determined linear least squares solution. Gordis (1997) and Felippa and Park (1997) proposed a force method to extract the substructural flexibility from the global flexibility by using the load transformation

matrix, which is limited to statically determinate beam-like structures. For a continuum structure, calculation of the transformation matrices is computationally demanding. Alvin and Park (1999) extended this method via a numerical algorithm without using the load transformation matrix. This method can be used for complicated structures. Park and Reich (1998) summarized three methods in extracting the substructural flexibility from the global measured data, namely, a free-free substructural flexibility method, a deformation-based flexibility method, and a strain-basis flexibility method. Hou et al. (2015) proposed a substructure isolation method on the basis of the virtual distortion method. They used force distortions for modeling the interfaces to isolate the target substructure from the influences of the remaining structure.

Weng et al. (2012) constructed the compatibility and equilibrium equations to extract the substructural flexibility from the experimental modal data of the global structure. An orthogonal projector is proposed to remove the rigid body modes of free-free substructures (Weng et al., 2016). The extracted substructural flexibility is then regarded as the reference for updating the substructural FE model at the substructural level. The substructural eigenvalues and eigenvectors decomposed from the substructural flexibility are utilized as indicators for damage detection (Weng et al., 2013a). These substructural properties are more sensitive to damage than the global ones because the damage often exists in the local area.

The inverse substructuring method is started by finding the displacement and force of a target substructure after it is isolated from the global structure. To be treated as an

independent structure, the displacement of a substructure is constituted by its deformational motion and rigid body motion (Weng et al., 2013a)

$$\left\{x^p\right\} = \mathbf{F}^p \left\{f^p\right\} + \mathbf{R}^p \left\{\beta^p\right\} \tag{28}$$

$$\mathbf{F}^p = Diag\left[\mathbf{F}^{(1)},...,\mathbf{F}^{(N_s)}\right], \quad \mathbf{R}^p = Diag\left[\mathbf{R}^{(1)},...,\mathbf{R}^{(N_s)}\right] \tag{29}$$

where $\mathbf{F}^p$ is the primitive matrix of the substructural flexibility, $\mathbf{R}^p$ represents the primitive matrix of orthogonal rigid body modes of substructures, and $\left\{\beta^p\right\}$ refers to the participation factors of rigid body modes.

The substructural displacements and forces in the primitive form are related to the global counterparts as

$$\left\{x^p\right\} = \mathbf{L}^p \left\{x_g\right\}, \quad \left[\mathbf{L}^p\right]^T \left\{f^p\right\} = \left\{f_g\right\} \tag{30}$$

where $\{x_g\}$ and $\{f_g\}$ stand for the nodal displacement and external force vectors of the global structure, respectively, and $\mathbf{L}^p$ is a Boolean matrix composed of 1 and 0 values relating the DOFs of the substructures and global structure.

As an independent structure, a substructure is loaded by the external force and interface forces from the adjacent substructures, which is

$$\left\{f^p\right\} = \left(\left[\mathbf{L}^p\right]^T\right)^+ \left\{f_g\right\} + \mathbf{D}^T \left\{\tau\right\} = \left\{\tilde{f}_g\right\} + \mathbf{D}^T \left\{\tau\right\} \tag{31}$$

Following the forward substructuring method, $\left\{\tau\right\}$ denotes the interface forces from the adjacent substructures, and matrix **D** implicitly defines the connections between the adjacent substructures.

By substituting Eq. (31) into Eq. (28) and Eq. (30), the global flexibility and substructural flexibility are related to the displacement as

$$\left\{x_g\right\}=\left[\mathbf{L}^p\right]^+\left\{x^p\right\}=\left[\tilde{\mathbf{L}}^p\right]^T\mathbf{F}^p\left(\left\{\tilde{f}_g\right\}+\mathbf{D}^T\left\{\tau\right\}\right)+\left[\tilde{\mathbf{L}}^p\right]^T\mathbf{R}^p\left\{\beta^p\right\}=\mathbf{F}_g\left\{f_g\right\} \tag{32}$$

Eq. (32) indicates that the primitive substructural flexibility matrix ( $\mathbf{F}^p$ ) can be calculated from the global flexibility matrix ($\mathbf{F}_g$) when $\left\{\tau\right\}$ and $\left\{\beta^p\right\}$ are given. The latter two variables are solved from the following force and displacement compatibility conditions:

1) The primitive substructural rigid body modes and forces satisfy the force equilibrium compatibility as

$$\left[\mathbf{R}^p\right]^T\left\{f^p\right\}=\left\{\mathbf{0}\right\}\ \text{ and }\ \left[\mathbf{R}^p\right]^T\left(\left\{\tilde{f}_g\right\}+\mathbf{D}^T\left\{\tau\right\}\right)=\left\{\mathbf{0}\right\} \tag{33}$$

2) From the physical point of view, matrix **D** constraints the displacement compatibility as

$$\mathbf{D}\left\{x^p\right\}=\left\{\mathbf{0}\right\}\ \text{ and }\ \mathbf{D}\left(\mathbf{F}^p\left\{f^p\right\}+\mathbf{R}^p\left\{\beta^p\right\}\right)=\left\{\mathbf{0}\right\} \tag{34}$$

After $\left\{\tau\right\}$ and $\left\{\beta^p\right\}$ are solved from Eqs. (33)-(34), the global flexibility matrix is related to the substructural flexibility matrix as (Weng et al., 2013a)

$$\mathbf{L}^p\mathbf{F}_g\left[\mathbf{L}^p\right]^T=\mathbf{F}^p-\mathbf{F}^p\mathbf{K}_C\mathbf{F}_R-\mathbf{F}_R\mathbf{K}_C\mathbf{F}^p-\mathbf{F}^p\mathbf{H}\mathbf{F}^p+\mathbf{F}_R \tag{35}$$

where $\mathbf{F}_R=\mathbf{R}^p\left(\left[\mathbf{R}^p\right]^T\mathbf{K}_C\mathbf{R}^p\right)^{-1}\left[\mathbf{R}^p\right]^T$ , $\mathbf{K}_C=\mathbf{D}^T\mathbf{F}_C^{-1}\mathbf{D}$ , $\mathbf{F}_C=\mathbf{D}\mathbf{F}^p\mathbf{D}^T$ , $\mathbf{H}=\mathbf{K}_C-\mathbf{K}_C\mathbf{F}_R\mathbf{K}_C$ . Consequently, the substructural flexibility matrix $\mathbf{F}^p$ can be extracted from $\mathbf{F}_g$. The extracted substructural flexibility is generally not normalized. An orthogonal projector has been developed such that the extracted substructural flexibility is mass-normalized (Weng et al., 2016). The substructural frequencies and model shapes

are decomposed from the substructural flexibility as indicators for damage detection (Weng et al., 2013a).

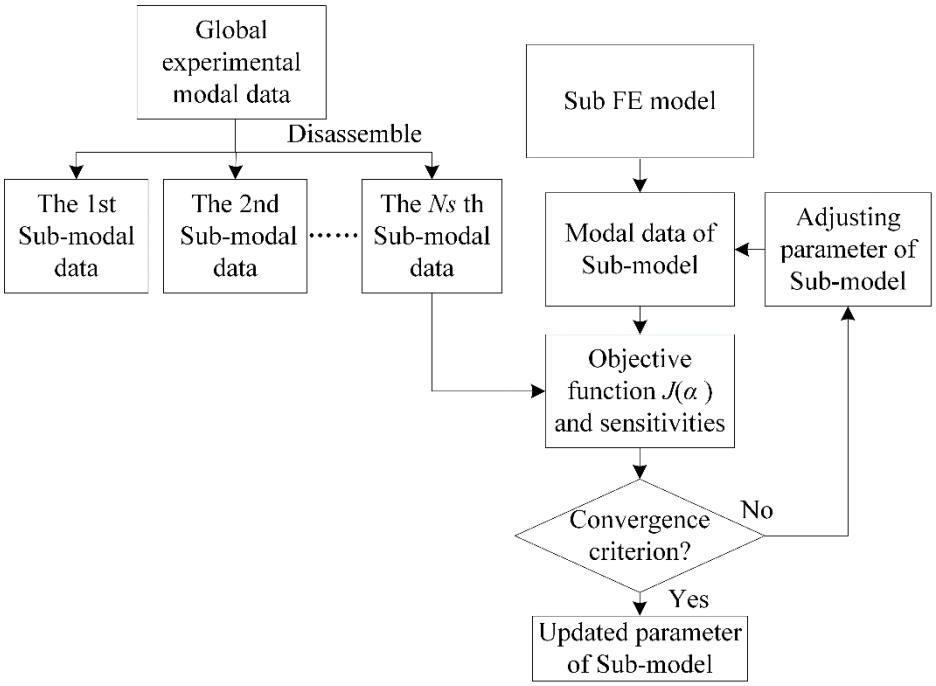


**Figure 2**. The inverse substructuring method for model updating

The substructural flexibility, frequencies and mode shaps extracted from the experimental global modal data can be used for updating the substructures, as illustrated in Figure 2. The FE model of one substructure is treated as an independent structure. The objective function is constructed from the residuals between the vibration properties (flexibility, frequencies and mode shaps) of the substructural FE model and the extracted data. In each iteration, the substructural vibration properties at the measured DOFs and sensitivity matrices are computed. The elemental parameters in the concerned substructure are updated by minimizing the objective function.

This inverse substructuring method has been applied to the model updating of Canton Tower (Weng et al., 2012; Weng et al., 2013a). The relative change of substructural eigensolutions caused by a local damage is about 1.84%, while relative change of gobal eigensolutions caused by the same local damage is only 0.01% (Weng et al., 2013a). The substructural eigensolutions are more sensitive to the local damage than the global structure. In the inverse substructuring method, only one substructure concerned instead of the whole global structure is updated in each iteration. The size of the system matrices is reduced from 21,690×21,690 to 2,736×2,736. The computation time of the substructure-based model updating is less than 10% of the traditional global model updating (Weng et al., 2012). The computational efficiency is improved significantly.

*3.2 Identification of substructural parameters and interface forces in time domain*

In time domain, the concerned substructure is isolated from the remainder of the structure by accounting for the interaction forces at the interfaces. By treating the interface forces as the input, the equation of motion of an independent substructure is written as (Koh et al., 1991)

$$\mathbf{M}_{II}\{\ddot{x}_I\}+\mathbf{C}_{II}\{\dot{x}_I\}+\mathbf{K}_{II}\{x_I\}=\{f_I\}-(\mathbf{M}_{IO}\{\ddot{x}_I\}+\mathbf{C}_{IO}\{\dot{x}_I\}+\mathbf{K}_{IO}\{x_I\}) \tag{36}$$

where subscripts $I$ and $O$ represent the interior and interface DOFs of the target substructure, respectively; $\{x_I\}$, $\{\dot{x}_I\}$ and $\{\ddot{x}_I\}$ are the dynamic displacement, velocity, and acceleration of the interior DOFs, respectively. The right-hand-side of Eq. (36) comprises two parts: $\{f_I\}$ refers to the external force applied on the interior DOFs of the substructure, and the reminder denotes the interface forces from the adjacent

substructures.

The inverse substructural identification inherently requires the measured responses of interface nodes. The influences of the remaining part of the structure can be expressed as these measured interface responses. Koh et al. (1991) firstly used the substructuring method to identify structural parameters in time domain. The displacement, velocity, and acceleration of every interface node are needed to measure. The identification of substructual properties is performed on the independent substructure by extended Kalman filter. To enhance above method's noise robustness, auto-regressive and moving average with stochastic input (ARMAX) model is derived for the substructure by Yun and Lee (1997) to process the measurement data with noise. The sequential prediction error method is then used to identify the substructural properties. A back propagation neural network is combined for estimating the substructural parameters of a complex structural system (Yun and Bahng, 2000). Xing and Mita (2012), Su et al. (2012), and Mei et al. (2016) fitted ARMAX model to extract substructural vibration properties for damage detection.

It is usually impossible to measure all interface responses in practical engineering. Some research attempts to elimilate the unmeasured interface by mathematical transformation on the equation of motion. In consequence, complete measurement of interface responses is not required. Koh and Shankar (2003) utilized a receptance function to relate the response at one point to the excitation at the other point. They eliminated the interface forces by using different sets of measurement in the focused substructure under the same

excitation condition. Tee et al. (2009) incorporated a condensed model identification and recovery method into the substructural context to estimate substructural properties from incomplete measurements. Zhang and Johnson (2012, 2014) formulated a substructure identification method for shear structures and plane frame building structures. The cross power spectral densities between the floor accelerations and reference response are used to estimate the structural parameters without requiring measurement on interfaces. Hou et al. (2011, 2015) proposed a substructure isolation method on the basis of the virtual distortion method. They used force distortions for modeling the interfaces to isolate the considered substructure from the influences of the remaining structure. Hou et al. (2018) isolated a substructure through additional virtual mass for damage identification, and the probability distribution of damage factor is quantified via the Bayesian method.

In the right-hand-side of Eq. (36), if the external force and the interface force from the adjacent substructures are regarded as the equivalent external forces on the substructure, the equivalent external force and substructural parameters can be identified simultaneously. In consequence, another kind of substructuring method treats the incomplete measurement of interface responses as unknown paramters. The interface forces and substructural parameters are identified simultaneously via a model updating/system identification technique. Lei et al. (2015) presented substructural damage detection methods based on the sequential application of an extended Kalman estimator. The structural parameters and interface forces are identified simultaneously by the Kalman estimator. Li et al. (2012) and Li and Law (2012) combined the reconstruction technique and transmissibility function for identifying the concerned substructure.

Sensitivity-based model updating is applied to identify the structural damage and interface forces simultanously. Zhu et al. (2013) developed the substructural identification method for the system identification of the subway tunnel subject to moving train loads. The moving loads and interface forces are expressed by the Chebyshev polynomials. The dynamic response sensitivity with respect to the structural elemental parameters and the Chebyshev factors are derived in the state space domain. Sensitivity-based model updating and response reconstruction technique are used to identify the structural damage, external moving force and interface forces of adjacent substructures simultaneously. The equation of motion of an independent substructure subjected to moving loads is expressed as

$$\mathbf{M}_{II}\{\ddot{x}_I\}+\mathbf{C}_{II}\{\dot{x}_I\}+\mathbf{K}_{II}\{x_I\}=f_I\delta(l-vt)-(\mathbf{M}_{IO}\{\ddot{x}_I\}+\mathbf{C}_{IO}\{\dot{x}_I\}+\mathbf{K}_{IO}\{x_I\}) \tag{37}$$

where $f_I(t)\delta(l-vt)$ is the moving force with the speed of $v$ at location $l$. The moving force and the interface force from the adjacent substructures can be represented by Chebyshev polynomials $T$. The equation of motion of the target substructures is thus rewritten as

$$\mathbf{M}_{II}\{\ddot{x}_I\}+\mathbf{C}_{II}\{\dot{x}_I\}+\mathbf{K}_{II}\{x_I\}=\sum_{j=1}^{N1}b_jT_j\delta(l-vt)+\sum_{j=1}^{N2}b_{N1+j}T_j \tag{38}$$

In Eq. (38), $\sum_{j=1}^{N1}b_jT_j\delta(l-vt)+\sum_{j=1}^{N2}b_{N1+j}T_j$ is the Chebyshev polynomials that describe the moving force and interface force. $b$ and $T$ represent the orthogonal factors and orthogonal vectors of the equivalent external force, respectively. $N1$ denotes the order of the Chebyshev polynomials for the moving force, and $N2$ denotes the order of the Chebyshev polynomials for the interface force. As such, the substructural parameters and Chebyshev factors are updated to identify the structural damage and interface forces via sensitivity-

based model updating.

## 4. DISCUSSIONS ON FUTURE RESEARCH

Since the 1960s, the dynamic substructuring methods have been comprehensively developed in many aspects, including improving the accuracy and efficiency, extending to the sensitivity analysis and model updating, generalizing to time and frequency domains, and covering the forward and inverse manners. These studies exhibit the advantages of the substructuring methods in model updating and damage identification of large-scale structures over traditional global methods. Due to the inherent challenges and difficulties of civil engineering structures, the following two issues deserve further research to make the model updating and damage identification viable and practical.

For civil engineering structures, nonlinearity often exists in the local area, which may be introduced by connections or damage. The local nonlinearity will render the whole structure to be a nonlinear system. The conventional numerical approaches are global based and very computationally demanding for analyzing large-scale nonlinear structures. In this regard, the substructuring method has great potential by treating the nonlinear parts as independent substructures while the remainder as linear. The research on substructure-based nonlinear analysis and model updating method will significantly improve the efficiency of the nonlinear analysis for large-scale structures.

In addition, the model updating and damage identification of civil engineering structures

inevitably contain uncertainties or errors, such as the modeling error and measurement noise. Statistical analyses are often time consuming for civil structures with large system matrices and numerous updating parameters. The benefit of using the substructuring methods for the statistical analysis is that only the substructures involving large uncertainties need to be analyzed independently. The substructures with different uncertainties can be analyzed separately. As such, the substructuring methods deserve to be integrated into the uncertainty propagation for alleviating the computational burden.

## 5. CONCLUSIONS

The substructuring methods analyze the entire structure in a piecewise manner, and exhibit many advantages when applied to health monitoring of large-scale structures. This paper outlines the framework of dynamic substructuring methods for model updating, damage detection and related applications. A forward substructuring method in assembling the substructural solutions to achieve a fast calculation of the global solutions and an inverse substructuring method in disassembling the global properties to obtain the local properties are elaborated. Those substructuring methods in both frequency and time domains are presented. The substructuring methods significantly reduce the computational burden associated with the model updating and damage identification of large-size structures.

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