Noise Correlation between Eigenvalues in Nonlinear Frequency Division Multiplexing

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Abstract: The noise effects on signals with multiple eigenvalues is studied numerically. Correlations between the noise of the eigenvalues are discovered for the first time, which allows a more compact pack of signal points.

OCIS codes: (060.1660) Coherent communications; (070.4340) Nonlinear optical signal processing.

1. Introduction

Advanced digital coding and signal processing techniques have become increasingly relevant to mitigate noises, signal distortions and interferences in fibre-optic communications. While these techniques can deliver significant benefits to optical systems, they are all based on off-the-shelf methods originally developed for linear channels (e.g., microwave channels), ignoring the nonlinear properties of the fibre. However, as data transmission rate (and also transmission power) is continuously increasing, the nonlinear effects of the fibre become too severe to a point that the data rate will be limited by an asymptotic fundamental bound, namely linear capacity limit or Shannon limit [1].

To overcome the linear capacity limit, it has been suggested to use "nonlinear normal modes" that can propagate along the fibre without any inter-modal interactions [2] in presence of nonlinear effects. Such a distinct feature can be exploited to design a new fibre optic communications system that separately encodes and transmits data for each individual nonlinear normal mode. The process of finding these nonlinear normal modes can be achieved via the use of nonlinear Fourier Transform (NFT) [3]. For any signal and nonlinear fibre channel, NFT leads to continuous and discrete spectrum (i.e., eigenvalues), which together with the spectral amplitude form the NFT domain representation of a time domain signal. For a non-realistic scenario, i.e., noise-less and loss-less fibre optic communication channel, the propagation of the NFT discrete and continuous spectrum are independent and determined by a simple phase factor. While the propagation of NFT spectrum in noise-less channells has been studied extensively, the effect of noise on the propagtaion of NFT spectrum has only been studied for limited cases. For example, soliton pulses are special class of pulses that can have either only discrete NFT spectrum or mixture of discrete and continuous NFT spectrum. There are only studies on the noise effects on the NFT spectrum of soliton with only one discrete NFT spectrum [4–6] (either with or without the NFT continuous spectrum).

Here, we study the noise effect on soliton pulses with multiple discrete NFT eigenvalues. We study the correlation between the discrete NFT eigenvalues after propagation through a noisy channel, and find that there is a positive correlation between the discrete NFT spectrum components. This is significant, considering that the NFT spectrum components propagate independently in a noise-less channel. In addition, we show that such a positive correlation can be exploited to design a better encoding and decoding scheme to achieve a much higher data transmission rate.

2. Simulation Results

In order to study the noise effects on a signal with more than one discrete eigenvalues, we propose a numerical approach by using nonlinear pulse propagation (NPP) [7] combined with numerical NFT. Simulations are carried out by propagating $A\mathrm{sech}(t)$ pulse for a length of 0.001, where A increases from 1 to 3 with step size 0.02. Gaussian white noise with noise power spectral density $\varepsilon = 0.01$ is presented in all simulations. For each A, simulations are ran 1000 times. Then, the eigenvalues of these 1000 outputs are calculated using the NFT. Fig. 1 shows the means, variances and covariances of the eigenvalues as functions of A. For A lies between 1 and 1.5, there is only one eigenvalue. Then the number of eigenvalue increases for each time A increases by 1. The blue, green and red circles in Fig. 1 show respectively the smallest, second smallest and third smallest (the largest) eigenvalues. The means of the eigenvalues match the prediction in [8]. The imaginary part of the eigenvalue $\text{Im}(\lambda_n) = A + 0.5 - n$, where $n = 1,2,3$. The variance of the eigenvalue matches with the theoretical model [3] (shown as a red horizontal line in the figure) when A is close 1. As A increases, the variance diverges from the prediction, and increases continuously. The variances of the second and third eigenvalues are similar to that of the smallest eigenvalue but slightly smaller. The most important observation is that there are positive covariances between the eigenvalues, and the values of the covariances are of the same order as the variances of the eigenvalues. This indicates that the fluctuations in all the eigenvalues are highly synchronised. This is a particularly useful feature for NFDM systems, since this positive correlation can be beneficial for designing telecommunication systems with a higher data rate. We elaborate this point as follows.

The positive correlation between the noises in the eigenvalues can be utilised to improve the transmission rate of a NFDM communication system. Fig. 1 (right) shows an illustration how it can be realized. A range of $A\text{sech}(Bt)$ pulses are chosen as channel inputs, which have two discrete eigenvalues with λ_1 ranging from 0.2 to 0.6, and λ_2 from 1.2 to 1.6. As one can see, the existence of positive covariance forms a ellipse pattern, which squeezes the distribution of eigenvalues in one direction. This effectively reduces the noise along the squeezed axis, and we can pack more signal points for higher rate communication compared to the scenario of uncorrelated noise, where the pattern is a circle.

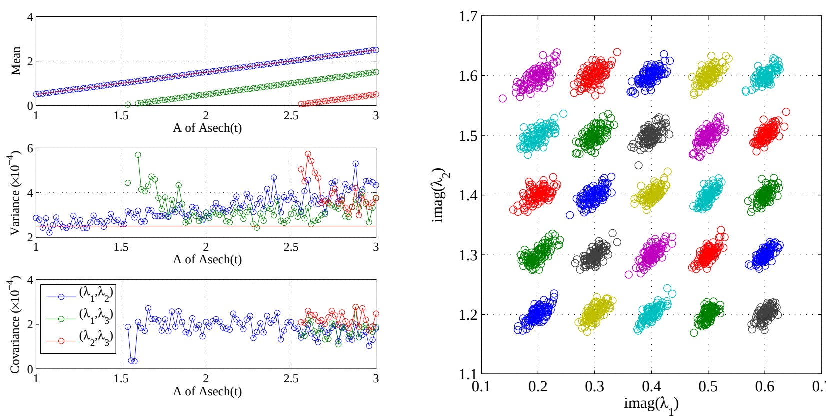

Fig. 1. (Left)Mean, variance and covariance of the NFT eigenvalues of $A\text{sech}(t)$ as functions of A. (Right) An example of application.

3. Conclusions

In conclusion, we developed a numerical approach to study the noise effects on signals with multiple eigenvalues. Correlations between the noise of the eigenvalues are discovered, which has an important implication. The correlation can be used to improve the data rate of NFDM communication system.

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