

Peer-to-Peer Energy Trading in Energy Local Area Network Considering Decentralized Energy Routing

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Abstract

Peer-to-peer (P2P) energy trading is perceived as a promising energy management mechanism for decentralized renewable energy sources. The concept of energy local area network (e-LAN) is proposed as a new system framework for future microgrids to facilitate P2P trading where the end-to-end power delivery is achieved by energy routing. Decentralized P2P trading in e-LAN requires a perfectly competitive market and flexible end-to-end power delivery. In this paper, a new P2P trading model incorporating decentralized energy routing is proposed for e-LAN. The proposed model is composed of a P2P energy market and a loss compensation market. In the P2P energy market, a tripartite graph model is proposed to represent transactions where trading decisions and energy routing are jointly considered for each participant to pursue its optimal benefit. Considering the size problem and nonlinear weights of the tripartite graph, an extended depth-first branch-and-bound algorithm is proposed to prune low-efficiency paths. Based on the pruned tripartite graph, the competitive P2P market is captured by a multi-leader multi-follower Stackelberg game model. A loss compensation market is established to coordinate energy routing paths planned in a decentralized manner. By means of setting the price for utilizing each transmission line, the loss compensation market avoids network congestion and ensure a normal operation of e-LAN. Simulation results show that the proposed model can bring higher utilities for P2P participants compared with the existing P2P trading models in e-LAN. Additionally, the effectiveness of the proposed model in resolving path conflicts in a decentralized fashion and improving computation efficiency, are verified.

Keywords: P2P energy trading, energy local area network, decentralized energy routing.

Nomenclature

P2P	peer-to-peer	RES	renewable energy source
e-LAN	energy local area network	ER	energy router
DFBB	depth-first branch-and-bound	E-DFBB	extended depth-first branch-and-bound
SE	Stackelberg equilibrium	KKT	Karush-Kuhn-Tucker
NSO	Network system operator		

1. Introduction

Due to the increasingly penetration of renewable energy sources (RESs), microgrid has been viewed as a fundamental component for modern power system to facilitate the transformation from the conventional centralized grid to a decentralized smart grid [1], [2]. For more effective operation and energy management of microgrid, a novel market mechanism, peer-to-peer (P2P) energy trading [3], [4], [5], is promoted where users can play both roles of energy producers and consumers. Within each microgrid, local users are allowed to trade their energy surplus or deficiency with other local users for generating revenues or reducing energy cost. However, unlike traditional top-down electricity market, the decentralized P2P energy trading requires flexible end-to-end power delivery between energy producers and energy consumers, which poses the challenges to the architecture design of microgrid.

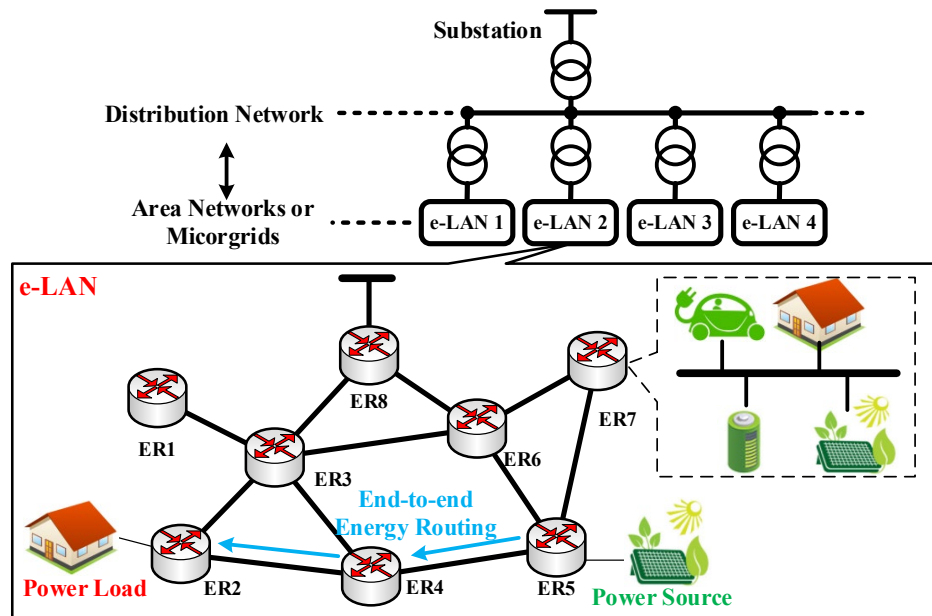


Fig.1. Diagram of e-LAN.

In recent years, [6], [7], [8] propose the concept of energy local area network (e-LAN) as an innovative and promising framework for the future microgrid to promote P2P energy trading. As shown in Fig.1, the e-LAN is a local energy grid where all electrical devices, including generators, loads and energy storages, are interconnected in the form of mesh network by using energy routers (ERs). In e-LAN, electric energy is treated similar to a packet of mail tagged with information about the sender and receiver [9]. When an energy producer and an energy consumer are matched as a trading pair, the traded power can be controlled to flow from the producer to the consumer by the collaborative work of ERs, which is called energy routing [9]. More details about energy routing are presented in Section 2.1.

Although the flexibility of end-to-end power dispatching and transmission is significantly improved in e-LAN, it gives rise to a new problem of energy routing, which aims to find the optimal routing paths for transmitting power between trading pairs. Note that the energy routing problem in e-LAN differs from the optimal power flow (OPF) problem in traditional power grid in a number of aspects. At first, the decision variables for the two problems are different. While OPF searches for the optimum operation parameters such as voltage and injected power, energy routing finds the optimum routing paths for trading pairs. Secondly, the two problems should be solved in different manner as they exist for different market mechanisms. In the traditional electricity market, utility companies are required to bear the cost of transmission loss, whereas in local energy trading the cost of transmission loss has to be borne by the trading pairs. Therefore, OPF is to directly optimize the operation efficiency of the whole power system no matter in centralized or decentralized way. However, the energy routing problem should be solved in a decentralized environment where each trading pair should be allowed to pursue their minimum-cost routing path.

Due to the difference between active energy routing and traditional passive power flow, the market design of P2P energy trading in e-LAN becomes a challenging task. Actually, considerable efforts have been made by researchers to design a competitive P2P energy market. Several centralized P2P energy trading models are presented in [10], [11], [12] where a centralized operator is established to manage energy sharing between local users and achieve supply-demand balance. Considering that the centralized P2P trading models may cause data privacy violation of participants, [13], [14] designs a fully decentralized P2P energy market where a dual decomposition based approach is proposed to decouple the social welfare maximization problem into a series of personal welfare maximization problems. Similarly, other standard decentralized computation methods such as alternating direction method of multipliers (ADMM) [15], [16], [17] and consensus-based methods [18], [19] are also employed to achieve the P2P energy market in a decentralized fashion. Besides the optimization methods, various game theory methods such as Stackelberg game theory [20], [21], [22], Nash bargaining [23], [24], [25], [26] and auction theory [27], [28], are also used to model a competitive P2P energy market, in which each participant pursues its benefit by playing a non-cooperative or cooperative game.

However, these state-of-the-art P2P energy trading models above cannot deal with the problem of energy routing and thus is not appropriate for e-LAN. It is notable that the power transfer distance (PTD) in [13], [19] and transmission cost assignment in [24] are the cost estimation for a determined power flow path rather than planning energy routing

path. To bridge the gap, [6], [29] proposes an energy transaction model incorporating a graph based energy routing algorithm for e-LAN. In this model, the energy routing in e-LAN is defined as the shortest path optimization problem in graph theory. When a trading pair has agreed to enter into a transaction, the graph based energy routing algorithm is invoked to find the minimum-loss transmission path for the trading pair. [30] uses the shortest-path Dijkstra algorithm to plan routing path and measure the minimum electrical distance between two participants. Based on the obtained electrical distances, a peer matching model is proposed to arrange the participants electrically close to each other as a trading pair. In [31], a real-time transaction market is presented for energy management in Energy Internet where the price and quantity of an energy transaction are determined by using a mid-point rule. After all transactions are confirmed, a minimum loss routing (MLR) algorithm is proposed for each trading pair to minimize their transmission cost.

In addition, some researchers focus their attention on the design of the energy routing algorithms. A biased min-consensus-based approach [32] and a deep reinforcement learning based method [33] are presented for the power delivery planning in the scenario of multi-source and multi-load. Considering that these energy routing methods can only generate single-path solution, [34], [35] formulates the minimum loss power transmission problem as a minimum-cost flow problem where the traded power can be split into packets and transmitted through multiple paths. By doing such, the small-capacity transmission lines can be fully utilized. In [36], a minimum loss multipath transmission (MLMT) is proposed for each determined trading pair to effectively reduce the energy transmission pressure of lines and makes the energy distribution of different lines more uniform. In [37], we propose a semi-decentralized energy routing algorithm to improve the computation efficiency and achieve congestion management in the scenario of simultaneous power transmission by multiple trading pairs.

It can be seen from literature review that there already exist various studies to design P2P energy trading based on e-LAN and energy routing algorithms. However, they are still found to have the following problems:

a) Trading decision-making and energy routing path optimization are treated as two independent problems in these studies. In other words, the trading decision and energy routing path of each P2P participant are optimized separately. It is worth noting that the benefit or utility of a P2P energy transaction is composed of trading utility obtained by optimizing trading decisions and transmission cost obtained by optimizing energy routing paths. Transactions with optimal trading utility may lead to low-efficiency power delivery. Therefore, to ensure each participant can benefit from P2P energy market, the designed P2P energy trading models should allow them to consider trading decision-making and energy routing path optimization as a single problem.

b) There lacks a proper network management method to resolve path conflict in a fair and decentralized way so as to ensure a normal operation of e-LAN. As each trading pair intends to utilize their preferred transmission lines, energy routing for all P2P transactions may not be achieved simultaneously to avoid line congestion, which leads to conflicting interests between some trading pairs. Although [29], [30], [31], [36], [37] provides various congestion management methods, they are not suitable to manage the decentralized energy routing in e-LAN. The congestion management in [29], [30], [31] is achieved from the

local perspective of each trading pair and it only works in scenario of a single trading pair. Its effectiveness cannot be guaranteed in simultaneous power transmission by multiple trading pairs. In [36], [37], line congestion is eliminated by using centralized methods. As a result, the preferences and proactiveness of trading pairs are ignored, which may discourage trading pairs to participate in P2P energy trading. Moreover, [36] ignores the fairness of resolving path conflicts by giving the trading pairs with higher trading quantities priority to use transmission line in e-LAN. Therefore, a decentralized network management method should be designed to fairly achieve congestion-free power delivery in e-LAN.

To address the aforementioned problems, this paper proposes a two-market P2P energy trading model incorporating decentralized energy routing. The proposed model includes two markets: (i) a P2P market between local users in e-LAN, and (ii) a loss compensation market. In the proposed P2P market, each participant can formulate its optimal transaction strategy and always obtains benefit from P2P energy trading by jointly optimizing trading decisions and energy routing path. Considering the potential path conflict caused by decentralized energy routing, the loss compensation market is established to achieve congestion management in a fair fashion. Compared with the previous works, the novelty and main contributions of this paper are summarized as follows:

- A tripartite graph model is proposed to capture the peer-to-many-to-peer trading relationship where sellers (source nodes) and buyers (sink nodes) are connected via all possible transmission paths (bridge nodes). By doing such, the energy routing and trading decision-making are formulated as a problem for each P2P participant.
- An extended depth-first branch-and-bound (E-DFBB) algorithm is proposed to prune low-efficiency paths and reduce the size of the proposed tripartite graph with nonlinear weights. Based on the pruned tripartite graph, Lagrange multiplier method and Karush-Kuhn-Tucker (KKT) conditions are used for each P2P participant to formulate its optimal transaction strategy.
- A multi-leader multi-follower Stackelberg game is used to model a competitive P2P energy market involving multiple buyers and multiple sellers. It is rigorously proved that there always exists Stackelberg equilibrium (SE) in the proposed Stackelberg game model.
- A loss compensation market is proposed to manage the operation of e-LAN with P2P energy trading and the problem of path conflicts caused by decentralized decisions is resolved by a dynamic price-adjustment method.

The rest of the paper is organized as follows. Section 2 gives a detailed description about energy routers and energy routing. In Section 3, the framework for proposed two-market P2P energy trading model is presented. In Section 3, a Stackelberg game model considering decentralized energy routing is proposed and its equilibrium analysis is presented. Section 4 formulates a loss compensation market for managing energy routing in e-LAN such that P2P energy trading in e-LAN can be physically realized without network congestion. The effectiveness of the proposed P2P energy trading model is verified in Section 5 by numerical simulations. Finally, conclusions are drawn in Section 6.

2. Energy Router and Energy Routing

ER and energy routing are the key device and technique in e-LAN, respectively, which makes e-LAN different from traditional power network. In this section, a detailed description about ER and energy routing is presented.

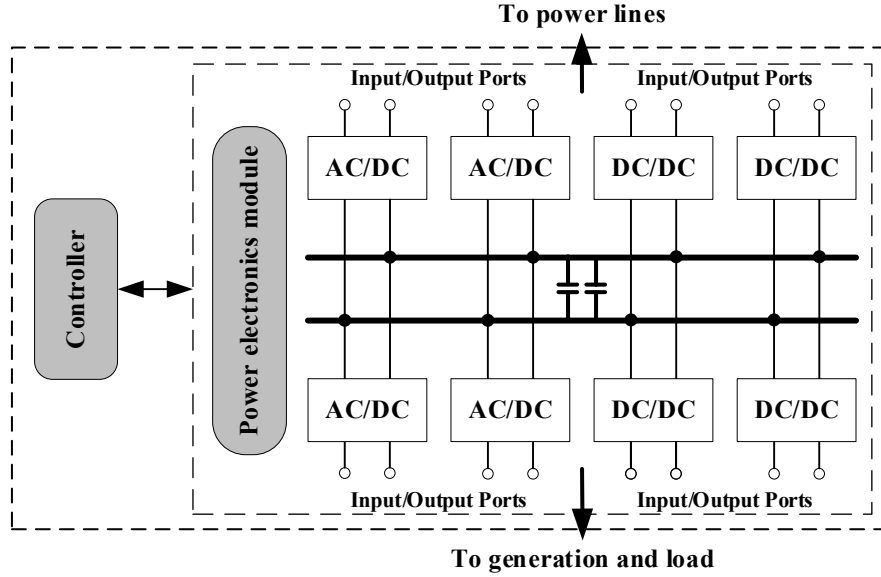


Fig.2. Basic structure of energy router [6], [29].

The typical structure of ERs is shown in Fig.2 [6], [29], where an ER is composed of a controller and a power electronics module. The power electronics module is a multi-port power converter system where each port is essentially a bidirectional AC/DC or DC/DC converter that can output or receive power. Energy generation, storage devices and loads with different terminal characteristics are interfaced to a common internal dc bus via suitably designed power converters, and exchange energy with one another via the dc bus.

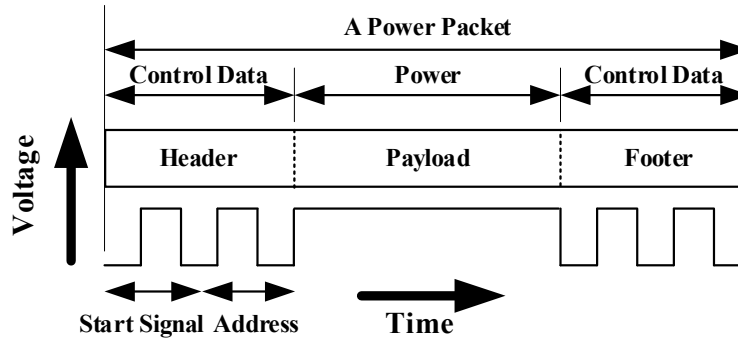


Fig.3. Structure of power packet [9].

Unlike the power flow in traditional grid, electrical energy in e-LAN is routed based on the power packet [7], [8], [9]. As shown in Fig.3, a power packet consists of a header, the payload and a footer. The payload is the transmitted power from the power source to the load. According to [9], the amount of transmitted power can be regulated by changing

the length of the payload, which is achieved through pulse width modulation. The header and footer are regarded as information tags attached to the payload. The header includes some information such as the address of source and load and the footer contains a mark at the end of packet. In doing so, the power and information can be transmitted together, which avoids contradiction between the quantity of shared power and its information.

Energy routing in e-LAN is realized by using multiple ERs to forward the transmitted power from the power source to the load. Fig.4 shows a simplified energy router realized using simple switches instead of power converters for illustrating the idea. The main function of the switches is to reconfigure the connection between an input port and an output port for realizing omni-directional power flow. When the transmitted power is input to an ER, the controller reads the tag information of power packets on the lines through an isolator. Then, the controller outputs signals for ON/OFF regulation to the switches on the switch circuit via the gate drivers. The former switches, i.e., A1, B1, C1, and D1 in Fig.4, guide the received power packet to the specified storage area according to its tag information. The storage areas are separated by source to avoid any loss of origin information of power. The latter switches, i.e., A2, B2, C2, and D2, select the forwarding destination of the power packet, i.e., the objective load or the other router. In addition, the latter switches must also reproduce power packet information. They attach the information tag to the power payload based on the original tag information given by the controller.

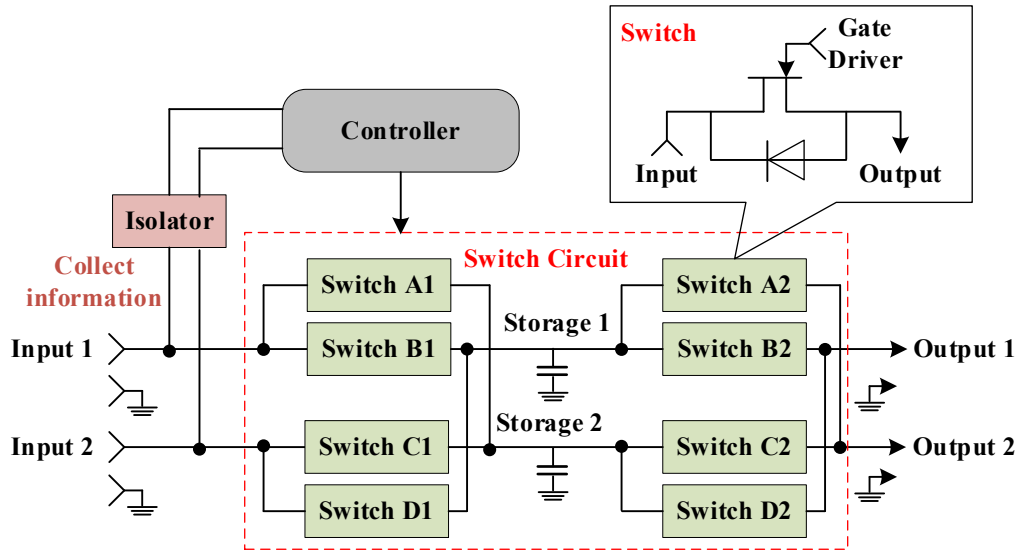


Fig.4. Diagram of power forwarding by a simplified energy router.

3. Framework for Two-Market P2P Energy Trading

An e-LAN can be visualized as a community-scale power network through which a small group of prosumers are interconnected. It is assumed that each user in the e-LAN is equipped with some forms of energy generation and storage devices and local loads. As depicted in Fig.1, each user is interfaced to the e-LAN via an ER which enables energy exchanges between its energy generation and storage devices and local loads as well as to allow them to exchange energy with other users in the e-LAN.

Under such e-LAN, each user can trade its energy surplus or deficiency with other users. To model this, we define B as the set of buyers and S as the set of sellers in a given energy trading period, where B and S are two disjoint sets. Since the market clearing model of a single trading period can be extended to multiple trading periods by adding time-coupled constraints, this paper focuses on the market design of a single trading period to more explicitly demonstrate the performance of the proposed model.

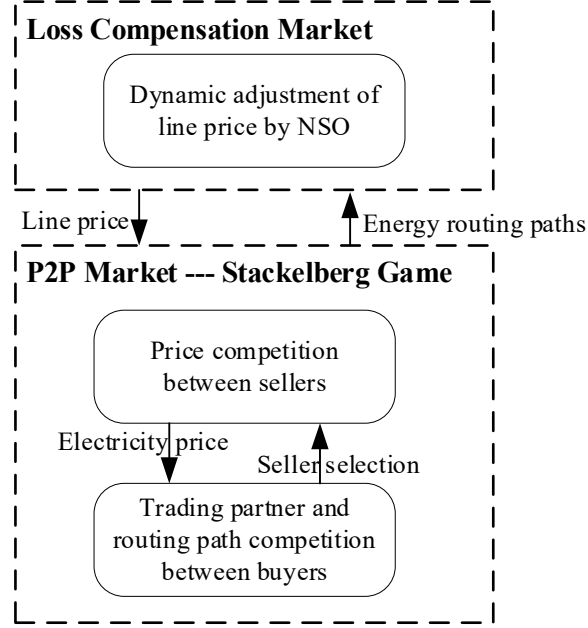


Fig.5. The proposed two-market P2P energy trading model.

As shown in Fig.5, the proposed P2P energy trading model consists of a P2P market and a loss compensation market. The aim of the P2P market is to maximize the utilities of both buyers and sellers. Considering the general situation where there are multiple participants with conflicting interests, a multi-leader multi-follower Stackelberg game is formulated to model the competitive P2P market in an e-LAN, in which sellers and buyers act as leaders and followers, respectively. At the leader level, each seller independently formulates its strategy by setting its electricity price. Knowing strategies of each seller, each buyer at the follower level establishes its tripartite graph to enumerate all possible transactions and uses E-DFBB algorithm to prune the low-efficiency branches in the tripartite graph. Based on pruned tripartite graph, each buyer plays its best response to the sellers' strategies by using Lagrange multiplier method and KKT conditions. The interactions between leaders and followers are captured by iterations. A detailed analysis of the proposed Stackelberg game model will be discussed in Section 4.

In the proposed loss compensation market, the network system operator (NSO) charges each trading pair a certain transmission cost for compensating the power loss arising from the end-to-end power transmission in P2P energy trading and for network maintenance. As previously discussed, decentralized energy routing may result in competitions between P2P trading pairs on the utilization of e-LAN. To resolve these competitions, NSO should set a different price for each transmission line and adopt a dynamic price-adjustment method for managing energy routing in e-LAN to avoid network congestion. More details about the

proposed loss compensation market will be discussed in Section 5.

The whole market starts by NSO publishing an initial price of each transmission line as input to the P2P energy market. After undergoing finite trading negotiations among participants, the P2P energy market will finally converge to a stable market outcome. The energy routing paths planned for achieving transactions in the market outcome will be submitted to NSO for feasibility checking. Based on the utilization of each transmission line in the submitted energy paths, NSO will update the price of each transmission line and broadcast them as new input to the P2P market. Such iterations of the loss compensation market will repeat until the status of each transmission line converges to a steady state.

4. Stackelberg Game Model in e-LAN

4.1. Tripartite Graph for P2P Energy Trading in e-LAN

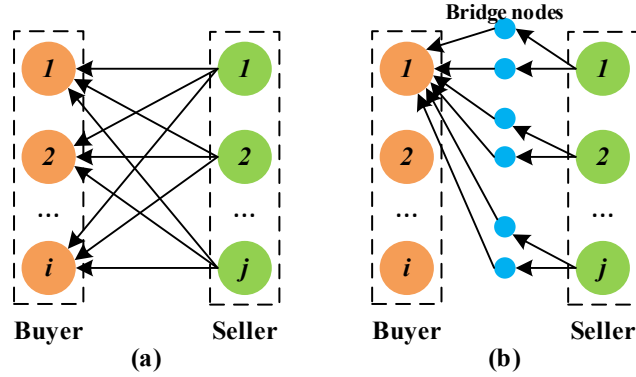


Fig.6. (a) P2P energy market model proposed in [13]. (b) P2P energy market model proposed in this paper.

In the P2P energy market proposed by other researchers, each buyer $i \in B$ can negotiate with each seller $j \in S$ and purchase energy from multiple sellers at the same time, as shown in Fig.6(a) [13]. When transmission paths or energy routing paths are included, energy transactions in the proposed P2P energy market can be represented by the tripartite graph shown in Fig.6(b) (taking buyer 1 as an example), where transmission paths are denoted by bridge nodes. Each bridge node represents a possible energy transaction between the two participants connected to it. Thus, each participant in the P2P energy market will attempt to optimize its actions by evaluating the profit gained from each bridge node connecting to itself.

For the ease of analysis, e-LAN is commonly modeled as a graph. Fig.7 shows an example of a 9-node e-LAN represented as a directional network graph G where each node denotes an ER and the transmission lines that interconnect adjacent ERs are depicted by the edges of the graph. The direction of an edges represents the direction of power flow on the transmission line and the edge weight W denotes the transmission cost arising from transmission loss. Based on the described graph model, the following part of this section formulates the utility functions of buyer and seller and analyzes the equilibrium of the

proposed Stackelberg game model.

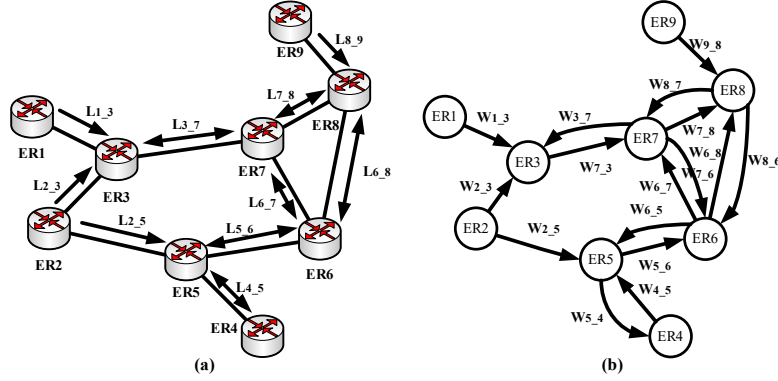


Fig.7. (a) An example e-LAN. (b) Graph model of the e-LAN in Fig.5(a).

4.2 Utility Function

4.2.1 Utility Function of Buyer

As different energy routing paths may be charged different transmission cost and connect to sellers offering different electricity quality, a buyer has different utilities or valuation over an energy transaction through each path. Therefore, the utility function U_i of buyer $i \in B$ can be given by equation (1), which consists of three terms: (1) u_{i-p} -- satisfaction derived from receiving a certain amount of electricity through the p th path [13], (2) C_{i-p} -- cost of purchasing electricity through the p th path, and (3) T_{i-p} -- transmission cost associated with the p th path. Note that in this paper buyers are assumed to bear the transmission cost involved in energy transactions. Due to intermittency and fluctuation of renewable generation, each prosumer cannot always be a producer or a consumer. Therefore, it is fair to always require consumers to bear the transmission cost.

$$U_i = \sum_{p=1}^{|\Omega_i|} (u_{i-p} - C_{i-p} - T_{i-p}) \quad (1)$$

where Ω_i denotes the set of all transmission paths between buyer i and all sellers in S .

Specifically, the expressions of the three terms in (1) are given by (2), (3), (4), respectively.

$$u_{i-p} = \beta_{i-p} P_{i-p} - \frac{\theta_{i-p}}{2} P_{i-p}^2 \quad (2)$$

$$C_{i-p} = \pi_{i-p} P_{i-p} \quad (3)$$

$$T_{i-p} = \sum_{(m,n) \in \ell_p} \pi_{(m,n)} (W_{line-(m,n)} + W_{ER-(m,n)}) \quad (4)$$

where P_{i-p} is the amount of power transmitted through the p th path. β_{i-p} and θ_{i-p} in (2) are the preference parameters of buyer i which characterize the buyer's preferences over the p th path and the seller associated with it. π_{i-p} in (3) is the electricity price offered by

the seller associated with the p th path. In (4), (m, n) denotes the edge or transmission line (m, n) connecting ER m and ER n on the p th path, ℓ_p denotes the set of all edges on the p th path, $\pi_{(m,n)}$ is the unit price of the compensating power for the transmission loss in transmission line (m, n) , $W_{line-(m,n)}$ is the conduction loss in transmission line (m, n) given by (5), and $W_{ER-(m,n)}$ is the power conversion losses in ER m and ER n given by (6). T_{i-p} therefore denotes the total transmission loss summed over all transmission lines and ERs associated with the p th path.

$$W_{line-(m,n)} = \frac{R_{(m,n)}}{V_{(m,n)}^2} ((P_{i-p} + P_{ex(m,n)})^2 - P_{ex(m,n)}^2) \quad (5)$$

$$W_{ER-(m,n)} = [(1 - \eta_{out-m}) + (1 - \eta_{in-n})]P_{i-p} \quad (6)$$

where $R_{(m,n)}$ and $V_{(m,n)}$ is the resistance and voltage of transmission line (m, n) connecting ER m and ER n , respectively, and $P_{ex(m,n)}$ is the existing power on transmission line (m, n) before P_{i-p} is added to it. η_{out-m} and η_{in-n} is the efficiency of the output port of ER m and ER n , respectively.

4.2.2 Utility Function of Seller

In this paper, the sellers in the P2P energy market are assumed to assign higher priority to meeting their own load demands and only sell excess energy to buyers through P2P energy trading. The utility function U_j of seller $j \in S$ comprises two parts: (a) u_j -- satisfaction derived from self-consumption [13], and (b) ϕ_j -- revenue generated from P2P energy trading, which are given by (7)-(9).

$$U_j = u_j + \phi_j \quad (7)$$

$$u_j = \beta_j D_j - \frac{\theta_j}{2} D_j^2 \quad (8)$$

$$\phi_j = \pi_j \sum_{p=1}^{|\Psi_j|} P_{j-p} \quad (9)$$

where, similar to (2), β_j and θ_j are the preference parameters of seller j which characterize the seller's satisfaction of self-consumption. D_j is the power demand of seller j . Ψ_j is the set of all paths from buyers to seller j and P_{j-p} denotes the amount of power transmitted through the p th path to seller j . π_j is the unit electricity price offered by seller j . Note that $(D_j + \sum_{p=1}^{|\Psi_j|} P_{j-p})$ should not exceed the total energy G_j generated by seller $j \in S$.

4.3. Equilibrium Analysis of Stackelberg Game in e-LAN

When the electricity price of each seller is published, the optimization problem of buyer i can be formulated as follows.

$$\arg \max_{P_{i-p}} U_i \quad (10a)$$

$$\text{s.t. } P_{i-p} \geq 0, \forall \ell_p \in \Omega_i \quad (10b)$$

$$D_{i-\min} \leq \sum_{p=1}^{|\Omega_i|} P_{i-p} \leq D_{i-\max} \quad (10c)$$

where $\mathbf{P}_{i-\mathbf{p}} = [P_{i-1}, P_{i-2}, \dots, P_{i-|\Omega_i|}]$. $D_{i-\min}$ and $D_{i-\max}$ denote the non-flexible and total power demand of buyer i , respectively. The total power demand of each buyer consists of non-flexible demand and flexible demand, and each buyer can adjust its flexible power demand to maximize its utility according to the electricity prices published by the sellers.

Set Ω_i can be obtained by directly using depth-first-search (DFS) based algorithm in graph theory. However, due to the meshed topology of e-LAN as well as the existence of multiple sellers in the P2P energy market, there could be a large number of paths in Ω_i , which leads to a huge search space for the optimization problem (10). As a result, this paper proposes an extended depth-first branch-and-bound (E-DFBB) algorithm to prune the low-utility paths and reduce the invalid search space of the optimization problem (10).

Before introducing the proposed E-DFBB algorithm, the conventional DFBB algorithm will first be briefly introduced here. DFBB algorithm is used to find the shortest path between two nodes in a graph. It adopts DFS to traverse a graph. Different from DFS algorithm that enumerates all paths associated with each branch and find the shortest path from these paths, DFBB algorithm estimates the performance bounds of each branch during the path exploration process. If a branch under exploration is found to be worse than the optimal solution found so far, it will be pruned before enumerating all the paths associated with it. The search efficiency will thus be significantly improved. However, DFBB algorithm cannot be directly applied in our problem due to several reasons. First, DFBB algorithm requires the edges in a graph to have constant costs, whereas the edge costs in our e-LAN's graph model are quadratic functions of transmitted power. Second, DFBB algorithm ignores the influence of edge capacity in the search for optimal solution. However, capacity is an important physical constraint for transmission line and thus it cannot be ignored. Third, DFBB algorithm only produces single-path optimal solution, whereas the optimization problem (10) intends to pursue multi-path optimal solution.

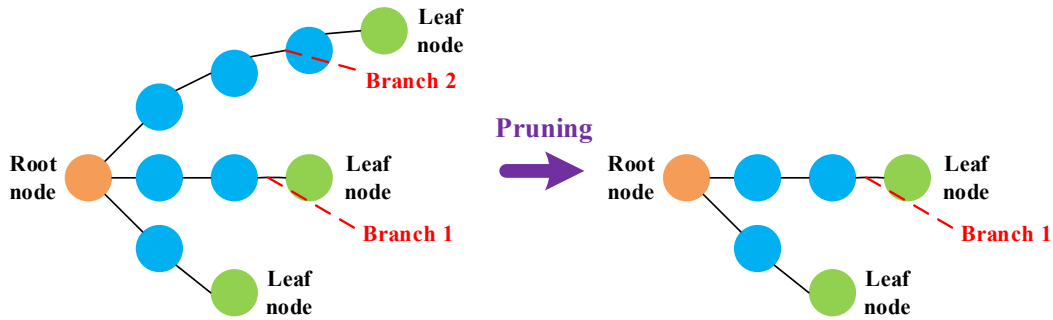


Fig.8. An example pruning by E-DFBB algorithm.

To deal with these problems, this paper proposes an E-DFBB algorithm. Since the cost of each edge in our e-LAN's graph model is a quadratic function of transmitted power, which is yet to be undetermined, each edge will be assigned an upper-bound cost (UB) and a lower-bound cost (LB). For each branch under exploration, its performance will be evaluated by its UB and LB (compared to a constant cost associated with DFBB algorithm).

For example, in E-DFBB algorithm, branch 1 is no doubt better than branch 2 if the UB of branch 1 is lower than LB of branch 2, as shown in Fig.8. Besides, to ensure that the available capacity of the pruned path set meets the requirement for transmitting the traded power, pruning will not be executed unless the available capacity of the enumerated paths is larger than the traded power. The pseudo-code of the proposed E-DFBB algorithm is shown in Algorithm 1.

Algorithm 1: E-DFBB Algorithm

Input: graph G , source node i , sink node j

Output: pruned path set $\tilde{\Omega}_i$

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1: while DFS rooted tree is not traversed do
2:   Select expansion node  $\{n_k\}$  according to DFS tree;
3:   if  $UB(< n_0, n_1, \dots, n_{k-1} >) + UB(n_k) > LB(\tilde{\Omega}_i) \parallel NotEnough(Cap(\tilde{\Omega}_i)) :$ 
4:     if IsSink( $n_k$ ):
5:       Add  $< n_0, n_1, \dots, n_{k-1}, n_k >$  into  $\tilde{\Omega}_i$ ;
6:       Update  $LB(\tilde{\Omega}_i)$  and  $Cap(\tilde{\Omega}_i)$  and clear low-utility paths saved in  $\tilde{\Omega}_i$ ;
7:     end if
8:   end if
9:   Update visit status of DFS tree;
10: end while
11: return  $\tilde{\Omega}_i$ 

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By applying E-DFBB algorithm on Ω_i , $\tilde{\Omega}_i$ is obtained and the optimization problem (10) is transformed to problem (11).

$$\arg \max_{\mathbf{P}_{i-p}} \tilde{U}_i \tag{11a}$$

$$\text{s.t. } P_{i-p} \geq 0, \forall \ell_p \in \tilde{\Omega}_i \tag{11b}$$

$$D_{i-min} \leq \sum_{p=1}^{|\tilde{\Omega}_i|} P_{i-p} \leq D_{i-max} \tag{11c}$$

Since the objective function (11a) is a convex function and the constraint set (11b) and (11c) is a convex set, the best response of buyer i can be solved by Lagrange multiplier method. The Lagrange function of (11) is given by (12), which is solved using the KKT

conditions given by (13)-(15), where λ_p , μ_1 and μ_2 the Lagrange multipliers [38] [39].

$$\begin{aligned} \xi(\tilde{\mathbf{P}}_{i-p}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = & \tilde{U}_i + \mu_1 \left(D_{i-min} - \sum_{p=1}^{|\tilde{\Omega}_i|} P_{i-p} \right) \\ & + \mu_2 \left(\sum_{p=1}^{|\tilde{\Omega}_i|} P_{i-p} - D_{i-max} \right) + \sum_{p=1}^{|\tilde{\Omega}_i|} \lambda_p P_{i-p} \end{aligned} \quad (12)$$

$$\frac{\partial \xi}{\partial P_{i-p}} = 0, \forall \ell_p \in \tilde{\Omega}_i; \lambda_p P_{i-p} = 0, \forall \ell_p \in \tilde{\Omega}_i \quad (13)$$

$$\mu_1 \left(D_{i-min} - \sum_{p=1}^{|\tilde{\Omega}_i|} P_{i-p} \right) = 0 \quad (14)$$

$$\mu_2 \left(\sum_{p=1}^{|\tilde{\Omega}_i|} P_{i-p} - D_{i-max} \right) = 0 \quad (15)$$

The optimal solution for P_{i-k} to be transmitted through the k th path of buyer i must take one of the following forms.

Case 1:

$$P_{i-k} = 0 \quad (16)$$

Case 2: $P_{i-k} \neq 0$, $D_{i-min} < \sum_{p=1}^{|\tilde{\Omega}_i|} P_{i-p} < D_{i-max}$, then $\lambda_{i-k} = 0$, $\mu_1 = 0$ and $\mu_2 = 0$. Substituting λ_{i-k} , μ_1 and μ_2 into (13)-(15) yields

$$P_{i-k} = \frac{\beta_{i-k} - \pi_{i-k} - \hat{L}_{i-k}}{\theta_{i-k} + L_{i-k}} \quad (17)$$

$$L_{i-k} = \sum_{(m,n) \in \ell_k} \pi_{(m,n)} \frac{2R_{(m,n)}}{V_{(m,n)}^2} \quad (18)$$

$$\hat{L}_{i-k} = \sum_{(m,n) \in \ell_k} \pi_{(m,n)} \left[\frac{2R_{(m,n)}}{V_{(m,n)}^2} P_{ex(m,n)} + (1 - \eta_{out-m}) + (1 - \eta_{in-n}) \right] \quad (19)$$

Case 3: $P_{i-k} \neq 0$, $\sum_{p=1}^{|\tilde{\Omega}_i|} P_{i-p} = D_{i-min}$, then $\lambda_{i-k} = 0$, $\mu_1 \neq 0$ and $\mu_2 = 0$. We assume $\tilde{\Omega}_{i+} = \{p | P_{i-p} \neq 0, \forall \ell_p \in \tilde{\Omega}_i\}$ denote the set of all paths having nonzero power. Obviously, $\sum_{p=1}^{|\tilde{\Omega}_{i+}|} P_{i-p} = D_{i-min}$ holds. Substituting these equations into (13), we obtain

$$\begin{aligned} P_{i-k} = & \frac{\beta_{i-k} - \pi_{i-k}}{\theta_{i-k} + L_{i-k}} \left(1 - \frac{1}{1 + \sum_{p=1}^{|\tilde{\Omega}_{i+} \setminus \ell_k|} \frac{1}{\theta_{i-p} + L_{i-p}}} \right) - \frac{\hat{L}_{i-k}}{\theta_{i-k} + L_{i-k}} + \\ & \frac{D_{i-min} - \sum_{p=1}^{|\tilde{\Omega}_{i+} \setminus \ell_k|} \frac{\beta_{i-p} - \pi_{i-p} + \hat{L}_{i-p}}{\theta_{i-p} + L_{i-p}} + \frac{\hat{L}_{i-k}}{\theta_{i-k} + L_{i-k}}}{1 + \sum_{p=1}^{|\tilde{\Omega}_{i+} \setminus \ell_k|} \frac{1}{\theta_{i-p} + L_{i-p}}} \end{aligned} \quad (20)$$

Case 4: $P_{i-k} \neq 0$, $\sum_{p=1}^{|\tilde{\Omega}_i|} P_{i-p} = D_{i-max}$, then

$$P_{i-k} = \frac{\beta_{i-k} - \pi_{i-k}}{\theta_{i-k} + L_{i-k}} \left(1 - \frac{1}{1 + \sum_{p=1}^{|\tilde{\Omega}_i + \ell_k|} \frac{1}{\theta_{i-p} + L_{i-p}}} \right) - \frac{\hat{L}_{i-k}}{\theta_{i-k} + L_{i-k}} + \frac{D_{i-max} - \sum_{p=1}^{|\tilde{\Omega}_i + \ell_k|} \frac{\beta_{i-p} - \pi_{i-p} + \hat{L}_{i-p}}{\theta_{i-p} + L_{i-p}} + \frac{\hat{L}_{i-k}}{\theta_{i-k} + L_{i-k}}}{1 + \sum_{p=1}^{|\tilde{\Omega}_i + \ell_k|} \frac{1}{\theta_{i-p} + L_{i-p}}} \quad (21)$$

Based on the obtained optimal solutions, the following theorem exists.

Theorem 1: Stackelberg equilibrium (SE) exists in the proposed Stackelberg game.

Proof: Given the electricity prices $\{\pi_j, \forall j \in S\}$, the optimal response $\{P_{i-k}, \forall \ell_k \in \tilde{\Omega}_i\}$ of buyer i is given by the previous four cases. Based on the strategies of buyers, there exists Nash equilibrium (NE) for the price setting game between sellers if the following conditions hold [40] [41] [42].

Condition 1: The strategy set is a nonempty, convex, compact subset of some Euclidean space \mathfrak{R}^n .

Condition 2: $\{U_j, \forall j \in S\}$ are continuous and concave in the strategy space.

There are limits on π_j such that $\pi_j \in [\pi_{j-min}, \pi_{j-max}]$, which are determined by government policy. Thus, the strategy set is a nonempty, convex, and compact subset of the Euclidean space \mathfrak{R}^n . According to (7)-(9), U_j is continuous in $\{\pi_j, \forall j \in S\}$ and the second-order derivative of U_j with respect to π_j is

$$\frac{\partial^2 U_j}{\partial \pi_j^2} = \pi_j \sum_{p=1}^{|\Psi_j|} \frac{\partial^2 P_{j-p}}{\partial \pi_j^2} + 2 \frac{\partial P_{j-p}}{\partial \pi_j} \quad (22)$$

As discussed previously, each seller $j \in S$ receives power demand from all paths Ψ_j between each buyer and itself. Any P_{j-p} thus must take one of the four cases for P_{i-p} , hence the four forms of $\frac{\partial P_{j-p}}{\partial \pi_j}$ are given by

$$\frac{\partial P_{j-p}}{\partial \pi_j} = 0 \quad (23)$$

$$\frac{\partial P_{j-p}}{\partial \pi_j} = -\frac{1}{\theta_{i-p} + \hat{L}_{i-p}} \quad (24)$$

$$\frac{\partial P_{j-p}}{\partial \pi_j} = -\frac{1}{\theta_{i-p} + L_{i-p}} \left(1 - \frac{1}{1 + \sum_{p=1}^{|\tilde{\Omega}_i + \ell_k|} \frac{1}{\theta_{i-p} + L_{i-p}}} \right) \quad (25)$$

$\frac{\partial P_{j-p}}{\partial \pi_j}$ of case 3 and case 4 are the same, as given by (25). $\frac{\partial^2 P_{j-p}}{\partial \pi_j^2}$ is obviously 0.

Combining (22)-(25), $\frac{\partial^2 U_j}{\partial \pi_j^2} \leq 0$ is obtained and thus U_j is concave. NE will always exist in the price setting game between sellers. In addition, each buyer $i \in B$ can always find its

unique best response to the strategies of sellers. Therefore, there exists SE in the proposed Stackelberg game.

4.4. Distributed Algorithm and Its Convergence

In P2P energy market, each participant is assigned a unique identity, namely, buyer or seller. To make the proposed model more practical, each buyer only has access to information about the electricity price and the amount of electricity available from each seller, while each seller only has access to information about power demand from each buyer. Direct communication is established between buyers and sellers to facilitate such information exchange. Each buyer/seller will not know any information about other buyers/sellers. Under this scenario, SE cannot be obtained by using analytical method. Therefore, a distributed algorithm is proposed to reach SE. As shown in Algorithm 2, the proposed P2P energy market will begin with price announcement by sellers. Buyers will adjust their power demands and send them to sellers. According to the collected power demand information, each seller j then updates his/her electricity price according to (26) and announces it to buyers again. The iterative process will repeat until the termination criterion specified in line 7 is satisfied.

Algorithm 2: Distributed Algorithm for SE

Input: Initial strategies of the sellers

Output: SE state

- 1: **Initialization:** Electricity price π_j of each seller j ; $l_p = 1$;
 - 2: **Repeat**
 - 3: $l_p = l_p + 1$;
 - 4: Each buyer $i \in B$ plays its best response by solving optimization problem (10);
 - 5: Transmit power demand of buyers $\sum_{p=1}^{|\Psi_j|} P_{j-p}(l_p)$ to each seller j ;
 - 6: Each seller j updates its price according to (26);
 - 7: **until** $|\pi_j(l_p + 1) - \pi_j(l_p)| < \varepsilon$
-

$$\pi_j(l_p + 1) = \pi_j(l_p) + \sigma_j(\sum_{p=1}^{|\Psi_j|} P_{j-p}(l_p) - P_{j,max}) \quad (26)$$

where l_p is the iteration counter of P2P market and Ψ_j is the electricity price adjustment parameter of seller j . $P_{j,max}$ is the total available power of seller j for sale.

In some cases, there may be a large number of buyers, leading to a large change of power demand. The electricity prices of sellers may thus oscillate strongly and it is difficult for Algorithm 2 to converge. Therefore, the step length control method is used to deal with

the effect of random exploration for better convergence of Algorithm 2 [43]. As given by (27), this technique is to limit the ramping rate of electricity price during iterations. This limit can be seen as an additional pruning rule in E-DFBB algorithm in which any branches with price changes exceeding the maximum and minimum ramping rates will be pruned.

$$\max(\pi_j(l_p) - \Delta, \pi_{min}) \leq \pi_j(l_p + 1) \leq \min(\pi_j(l_p) + \Delta, \pi_{max}), \Delta = rmp|\pi_j(l_p)| \quad (27)$$

where rmp represents the non-negative ramping rate.

5. Loss Compensation Market

5.1. Dynamic Price Adjustment

In P2P energy market, trading pairs are required to bear the cost of transmission loss. To ensure that each buyer receives its purchased power in full from P2P energy trading, transmission loss should be compensated by the e-LAN at a cost. In this paper, a loss compensation market is proposed to charge the trading pairs for compensation against transmission loss. Considering that energy routing paths planned in a decentralized manner in the P2P energy market may not meet the physical constraints of the e-LAN, a dynamic price-adjustment method is thus proposed to manage the power flow on each transmission line to avoid line congestion and to ensure a normal operation of the e-LAN.

After the P2P energy market submits the planned energy routing paths, the NSO of the loss compensation market will update the price of transmission line (m, n) according to (28) and (29). In case (a), transmission line (m, n) will be congested if the traded energy is transmitted according to the submitted plan at the l_l th iteration. There are conflicts between some trading pairs about the use of transmission line (m, n) . To resolve these conflicts, the NSO raises the price of transmission line (m, n) gradually in order to discourage some of the trading pairs involved from competing to use the transmission line (m, n) until there is no congestion in transmission line (m, n) . In case (b), although the transmission line is not congested, a larger $P_{(m,n)}$ will lead to a higher power loss due to the quadratic characteristic of the transmission loss function. Thus, the NSO will charge a higher line price for compensating the power loss.

(a) Congested:

$$\pi_{(m,n)}(l_l + 1) = \pi_{(m,n)}(l_l) + \delta_{(m,n)}(P_{(m,n)}(l_l) - C_{(m,n)})C_{(m,n)} \quad (28)$$

(b) Not congested

$$\pi_{(m,n)}(l_l + 1) = \max(\pi_{(m,n)}(l_l), \pi_{max} + \alpha_{(m,n)}P_{(m,n)}^2(l_l)) \quad (29)$$

where l_l is the iteration counter of the loss compensation market, $P_{(m,n)}$ is the total power carried on transmission line (m, n) , and $C_{(m,n)}$ is the capacity of transmission line (m, n) . $\alpha_{(m,n)}$ and $\delta_{(m,n)}$ are the price adjustment parameters for transmission line (m, n) with no congestion and congestion, respectively.

5.2. Convergence

To ensure convergence of loss compensation market, the line price $\pi_{(m,n)}(l_l + 1)$ at the $(l_l + 1)$ th iteration is to take the maximum value between $\pi_{(m,n)}(l_l)$ at the l_l th iteration and $\pi_{max} + \alpha_{(m,n)}P_{(m,n)}^2(l_l)$. By doing such, $\pi_{(m,n)}(l_l + 1)$ will always be no less than $\pi_{(m,n)}(l_l)$ and price oscillation can be avoided. As a result, $\pi_{(m,n)}$ either increases or remains unchanged regardless of players' trading strategies. In addition, $\pi_{(m,n)}$ is impossible to increase endlessly, since each buyer can select other transmission lines for energy routing in P2P transactions or directly purchase electricity from power grid when $\pi_{(m,n)}$ becomes unacceptably large. Therefore, the price of each transmission line must converge to a point and the convergence of loss compensation market is guaranteed.

6. Simulation results and analysis

In this section, the performance of the proposed P2P energy trading model will be evaluated by detailed numerical simulations in terms of P2P participants' market behaviors, P2P participants' utilities, path conflict resolving and the effect of the proposed E-DFBB algorithm. All numerical simulations are performed on Matlab R2014a on a PC with Intel i7-4790 3.60-GHz processor.

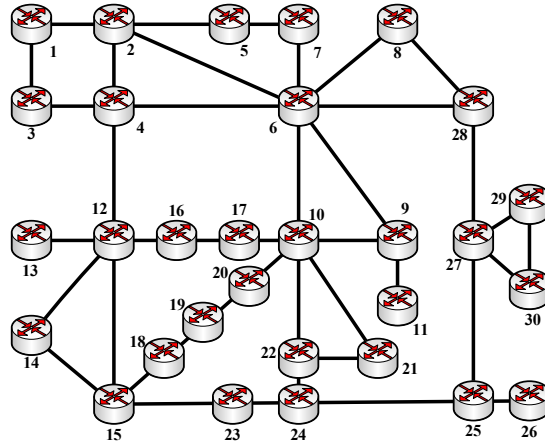


Fig.9. An e-LAN modified from IEEE 30-bus system [31].

6.1. Simulation Setup

All simulations are performed based on an e-LAN modified from IEEE 30-bus system [31], as shown in Fig.9. According to [6], the conversion efficiency of ER is set 0.98 and the transmission voltage between ERs is 400Vdc. The physical parameters of the transmission lines connecting the ERs in the e-LAN can be found in [31].

Table 1. P2P participants' parameters

	B_4	B_{23}	B_{25}		S_1	S_{22}	S_{27}
$D_{i-min}(\text{kW})$	7	6	6	$G_j(\text{kW})$	18	19	22

$D_{i-max}(\text{kW})$	11	10	10	$D_j(\text{kW})$	8	9	12
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For the P2P energy market model, buyer i 's preference parameter β_{i-p} is selected randomly from [1.5,2], and θ_{i-p} is taken as 0.1 [10,17]. In Hong Kong, the price of purchasing electricity from the grid π_{max} is 2.0 HKD/kWh, and the price of selling electricity to the grid is assumed to be 0.5 HKD/kWh. Price adjustment parameters $\alpha_{(m,n)}$ and $\delta_{(m,n)}$ for the loss compensation market are taken as 0.01 and 0.01, respectively. In this simulation study, it is assumed that there are three buyers (ER4, ER23 and ER25) and three sellers (ER1, ER22 and ER27) in the e-LAN shown in Fig.9. The power generation and power demand of these participants are given in Table 1.

To demonstrate the advantages of the proposed P2P energy trading model considering trading decision-making and energy routing jointly, four other P2P energy trading models which consider trading decision-making and energy path optimization separately are used for comparison. To ensure fair comparisons, Model 1, Model 2, Model 3 and Model 4 use the proposed Stackelberg game for trading decision-making but different energy routing methods, as shown in Table 2, where Model 5 is the proposed P2P energy trading model.

Table 2. Various Models for Comparative Simulations.

Groups	Models	Energy Routing Path Optimization Methods
Optimize trading decisions and energy routing paths separately	Model 1	Graph-based energy routing algorithm [6]
	Model 2	Minimum loss routing (MLR) algorithm [31]
	Model 3	Minimum loss multipath transmission (MLMT) [36]
	Model 4	Semi-decentralized energy routing algorithm [37]
Optimize trading decisions and energy routing paths jointly	Model 5 (Proposed)	-

6.2. Market Behavior

In this subsection, the market behaviors of P2P participants in the five models are compared. Table 3 shows the market outcomes of the five models. As Model 1, Model 2, Model 3 and Model 4 all adopt the proposed Stackelberg game to model trading decision-making in the P2P energy market, they obtain the same market outcome. A comparison of the electricity prices resulting from the five models is shown in the first row of Table 3. It is found that Model 5 generally leads to lower electricity prices than Model 1 to Model 4. Since the buyers in Model 5 need to consider the potential transmission costs for their trading decisions, they tend to reduce their flexible demand as compared with the buyers

in Model 1 to Model 4. This motivates the sellers in Model 5 to decrease their electricity prices in order to stimulate power demand of the buyers. As shown in the remaining rows of Table 3, buyer 4 decides to purchase more electricity from the P2P energy market even though it has to pay a higher transmission cost, whereas buyer 23 and buyer 25 choose to decrease their flexible demand for cost saving. In general, each participant exhibits different market behaviors in Model 1 – Model 4 and Model 5.

Table 3. Market outcomes of Model 1, Model 2, Model 3, Model 4 and Model 5.

	Model 1 – Model 4				Model 5			
	S_1	S_{22}	S_{27}	Total	S_1	S_{22}	S_{27}	Total
π_j	1.55	1.55	1.55	---	1.26	1.30	0.98	---
B_4	3.00	3.00	3.00	9.00	5.39	1.62	3.99	11.00
B_{23}	3.67	3.67	3.67	11.00	4.61	4.16	2.12	10.89
B_{25}	3.33	3.33	3.33	10.00	0	4.22	3.88	8.11

6.3. Utilities of P2P Participants

This subsection is to compare the utilities for P2P participants in the five models. As buyers are assumed to bear the cost of transmission loss, the utilities of sellers will not be significantly affected, as each seller can still generate more revenue from P2P trading than from traditional peer-to-grid (P2G) trading where participants can only trade electricity with the power grid. Therefore, this paper will focus more on the utilities of buyers and show that their utilities will be improved by the proposed model (Model 5).

Table 4. Comparison of buyers' utilities between Model 1, Model 2 and P2G trading.

	Model 1 and Model 2 (P2G Trading)		
	S_1	S_{22}	S_{27}
B_4	-0.14 (-0.90)	-0.34 (-0.90)	-0.52 (-0.90)
B_{23}	-0.88 (-0.86)	-0.07 (-0.86)	NaN (-0.86)
B_{25}	-1.14 (-0.72)	-0.003 (-0.72)	NaN (-0.72)

‘NaN’ means not a number

Model 1 and Model 2: Table 4 summarizes the actual utilities of the buyers in Model 1 and Model 2 after accounting for the cost of transmission loss. For comparison, the utilities of the same buyers when trading the same amount of electricity with the power grid are shown in brackets in the same table. For ease of understanding, the energy routing paths of Model 1 and Model 2 for each transaction is shown in Fig.10. The utilities of buyers 23 and 25 are reduced significantly by trading with seller 1 due to the long distances

between them and seller 1 which will incur a high cost of transmission loss. As a result, these buyers will prefer to trade electricity with the power grid instead of sellers. Note that the two transactions with NaN utility cannot be achieved as the available capacity of 6 kW of transmission line (27,25) cannot meet the total 7.00 kW simultaneous transmissions required by trading pairs {27,23} and {27,25}, and transmission line (27,25) will thus be congested.

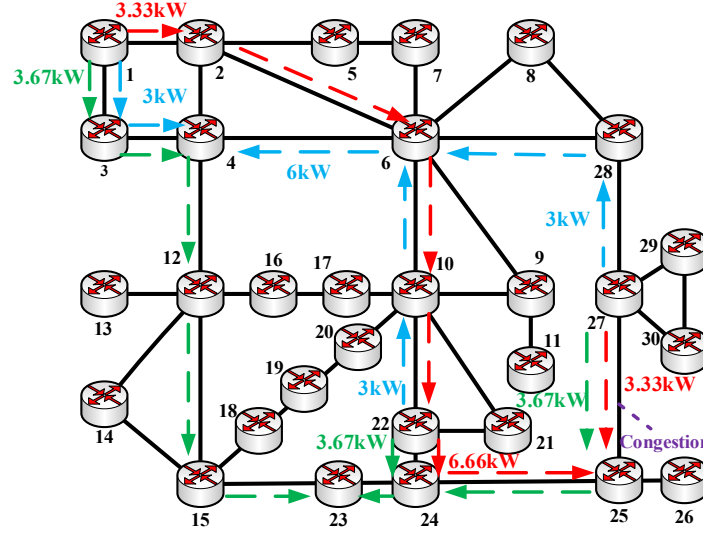


Fig.10. Energy routing paths of Model 1 and Model 2.

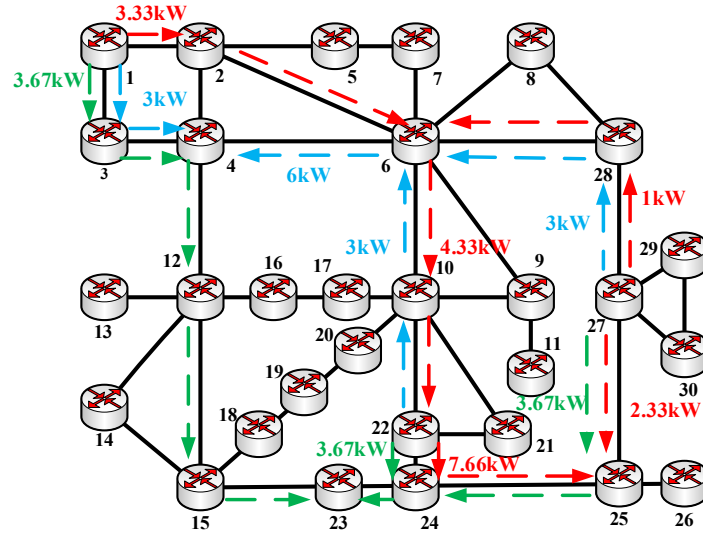


Fig.11. Energy routing paths of Model 3.

Model 3: Table 5 shows the utilities of buyers in Model 3. Also, the utilities of the same buyers when purchasing the same amount of electricity from the power grid are shown in brackets. The energy routing paths of Model 3 are depicted in Fig.11. Similarly, due to separate consideration of trading decision and energy routing path, buyer 23 and buyer 25 obtain lower utilities from trading with seller 1 than from P2G energy trading.

Although congestion in transmission line (27,25) is eliminated by using the centralized cooperative path planning in Model 3, buyer 25 is assigned a low-efficiency path $27 \rightarrow 28 \rightarrow 6 \rightarrow 10 \rightarrow 22 \rightarrow 24 \rightarrow 25$ and receives the traded power from seller 27, leading to the low utility of buyer 25 in the transaction with seller 27.

Table 5. Comparison of buyers' utilities between Model 3 and P2G trading.

	Model 3 (P2G Trading)		
	S_1	S_{22}	S_{27}
B_4	-0.14 (-0.90)	-0.34 (-0.90)	-0.52 (-0.90)
B_{23}	-0.88 (-0.86)	-0.07 (-0.86)	-0.25 (-0.86)
B_{25}	-1.14 (-0.72)	-0.003 (-0.72)	-1.24 (-0.72)

Model 4: The utilities of buyers in Model 4 and P2G energy trading is shown in Table 6 and the energy routing paths of Model 4 is depicted in Fig.12. Similar to Model 1, Model 2 and Model 3, buyer 23 and buyer 25 obtain lower utilities from energy transactions with seller 1 as they do not consider energy routing path optimization when making trading decisions. Moreover, to avoid congestion in transmission line (27,25), trading pair {27,23} has to delivery 1kW traded power through the path $27 \rightarrow 28 \rightarrow 6 \rightarrow 10 \rightarrow 22 \rightarrow 24 \rightarrow 23$, which incurs a high transmission cost for buyer 23. As a result, buyer 23 obtains a lower utility from this transaction than from trading with the power grid.

Table 6. Comparison of buyers' utilities between Model 4 and P2G trading.

	Model 4 (P2G Trading)		
	S_1	S_{22}	S_{27}
B_4	-0.14 (-0.90)	-0.34 (-0.90)	-0.52 (-0.90)
B_{23}	-0.88 (-0.86)	-0.07 (-0.86)	-1.04 (-0.86)
B_{25}	-1.14 (-0.72)	-0.003 (-0.72)	0.41 (-0.72)

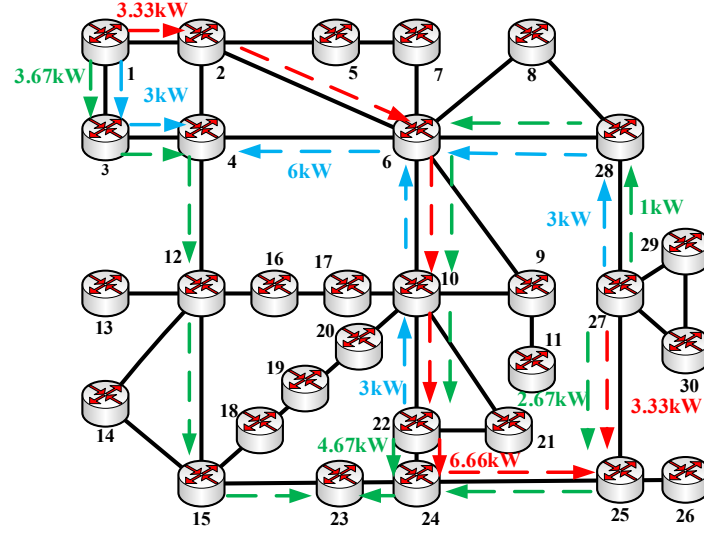


Fig.12. Energy routing paths of Model 4.

Model 5 (the proposed P2P energy trading model): Table 7 compares the utilities of buyers in Model 5 and in P2G energy trading. The energy routing paths of Model 5 are depicted in Fig.13. It can be observed from Table 6 that the buyers in Model 5 always obtain higher utilities compared to trading with the power grid although they are required to bear the cost of transmission loss. Moreover, there exists no constraint violation in e-LAN. Therefore, it can be concluded that Model 5 overcomes the limitations of Model 1 to Model 4 by concurrently taking both factors into consideration during energy trading.

Table 7. Comparison of buyers' utilities between Model 5 and P2G trading.

	Model 5 (P2G Trading)		
	S_1	S_{22}	S_{27}
B_4	3.42(-2.26)	1.18(-0.37)	4.01(-1.39)
B_{23}	4.71(-1.29)	2.77(-1.07)	3.39(-0.33)
B_{25}	0(0)	2.77(-1.10)	5.16(-0.95)

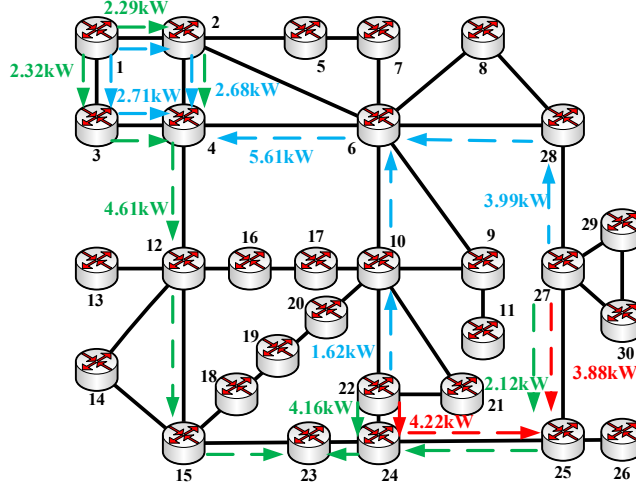


Fig.13. Energy routing paths of Model 5 (The proposed P2P energy trading model).

6.4. Path Conflict Resolving and The Effect of E-DFBB Algorithm

In this subsection, the effectiveness of the proposed P2P energy trading model (Model 5) in resolving path conflicts and the proposed E-DFBB algorithm in reducing invalid search space for P2P participants is demonstrated.

When the price of the transmission line (27,25) is 2 HKD/kW at iteration $l_i = 1$ of the loss compensation market, the P2P energy market converges to the SE state. Fig.14 depicts the selected energy paths corresponding to the SE state. Although the NSO charges the cost of transmission loss, transmission line (27,25) will still be overloaded due to having to transmit 7.61 kW power as required by trading pairs $\{27,23\}$ and $\{27,25\}$. To resolve the congestion problem, the NSO in the e-LAN increases the price of transmission line (27,25) step by step according to equation (27). The evolution of the price for transmission line (27,25) is shown in Fig.15 which increases from π_{max} to 9.92 HKD/kW. As the line price continues to increase, each trading pair adjusts its power transmitted through transmission line (27,25) until the congestion problem of transmission line (27,25) is resolved. Fig.11 depicts the adjusted energy paths in Model 5 where transmission line (27,15) is no longer congested.

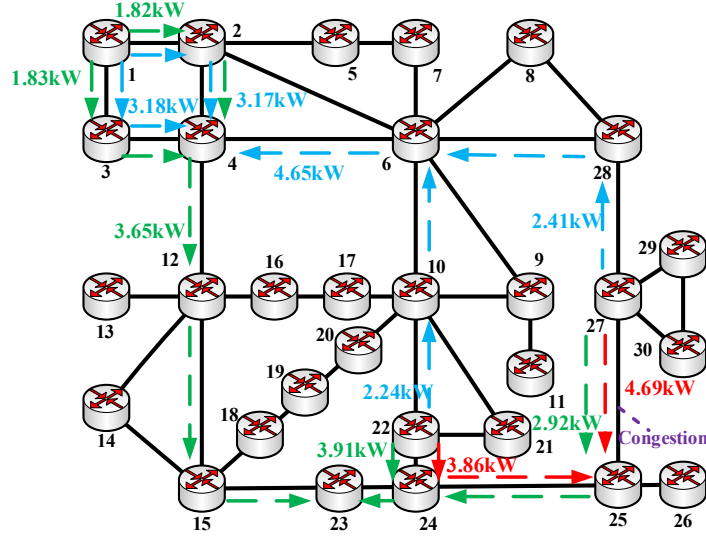


Fig.14. Energy routing paths of Model 5 at iteration $l_t = 1$.

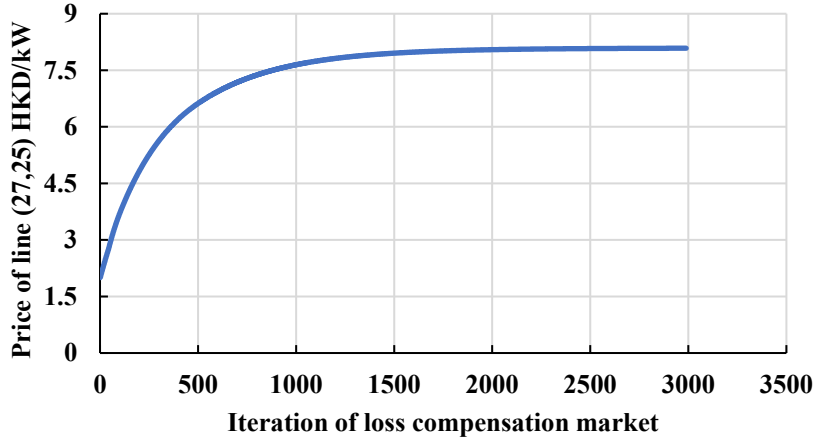


Fig.15. Evolution of the price of transmission line (27,25).

During the iterations of both the P2P energy market and the loss compensation market, E-DFBB algorithm is used to prune the low-utility paths for each buyer and its effect is shown in Table 8. There are in total 322, 518 and 432 paths from all sellers to buyer 1, buyer 15, and buyer 25, respectively. Since the electricity price and selected transmission lines are different at each iteration, the number of average valid paths per iteration is used to demonstrate the performance of E-DFBB algorithm. On average 311.13 paths are pruned per iteration for buyer 1. Similarly, there remains on average 7.00 and 2.72 valid paths for buyer 15 and 25, respectively after applying E-DFBB algorithm. Simulation results confirms the effectiveness of E-DFBB algorithm in removing the low-utility paths for each buyer. With the proposed E-DFBB algorithm, the proposed P2P energy market only takes 1 minute to converge. It is completely acceptable as it satisfies the time requirement of the two main market mechanisms, day-ahead market and hour-ahead market.

Table 8. Comparison of buyers' strategy space in Model 5 with and without applying E-DFBB algorithm.

	B_4	B_{23}	B_{25}
All Possible Paths	322	518	432
Valid Paths after Pruning	10.87	7.00	2.72

6. Conclusion

In this paper, a two-market P2P energy trading model comprising a P2P market and a loss compensation market is proposed to achieve P2P energy trading in e-LAN with decentralized energy routing. The proposed P2P energy market is formulated as a Stackelberg game model where a tripartite graph model is established to integrate trading decision-making problem and energy routing path optimization problem for each P2P participant to formulate its optimal transaction strategy. An E-DFBB algorithm is proposed to prune the low-efficiency paths in the proposed tripartite graph and improve computation efficiency. Any network congestion arising from the decentralized energy routing by the sellers/buyers in the P2P market is resolved by means of the market actions of a loss compensation market where the prices of transmission lines are set individually and adjusted dynamically depending on their loading conditions. The key findings of this paper are summarized as follows.

- (1) Joint optimization of trading decisions and energy routing paths in P2P energy trading leads to higher utilities than separate optimization of trading decisions and energy paths.
- (2) Path conflicts caused by decentralized energy routing can be effectively resolved by the proposed loss compensation market in a decentralized manner.
- (3) The proposed E-DFBB algorithm can significantly reduce the invalid search spaces of P2P participants and improve computation efficiency.

The proposed P2P energy trading model is for energy transactions within an e-LAN. Therefore, future work is required to extend this model by incorporating energy transactions between e-LANs.

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