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Benefits of Backup Sourcing for Components in Assembly Systems Under Supply Uncertainty

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Abstract

We consider an assembly system for a final product consisting of multiple components. The assembler orders each component from its primary supplier, and one of the components is subject to supply uncertainty in which the actual available quantity for assembly is only equal to a random fraction of the order quantity. After this actual available quantity is realized but before assembly for meeting final demand, the assembler can procure additional components from some backup suppliers. We derive the optimal ordering policy under this backup sourcing strategy and analyze the benefits of using backup suppliers in mitigating the adverse effect of supply uncertainty in component procurement from the primary suppliers. Our result shows that the availability of backup supplier for the component with supply uncertainty can provide very substantial benefits to the assembler, especially when the unit cost of this component from the primary supplier is high relative to the extra unit component cost from the backup supplier for high-margin products under an operating environment of high supply uncertainty and low demand uncertainty. We further show that the availability of backup suppliers for the other components with no supply uncertainty can provide additional significant benefits to the assembler when extra unit component cost of backup supplier for the unreliable component supplier is high for low-margin products with high demand uncertainty.

Keywords: Supply Chain Management, assembly systems, procurement strategies, backup sourcing, supply uncertainty.

1. Introduction

Global sourcing has become increasingly important for many manufacturers around the world, since offshore suppliers can often offer lower prices than domestic suppliers. However, global sourcing is usually accompanied by longer lead times and a higher risk of supply uncertainty due to unexpected glitches in a complex global supply process. In a survey by Muthukrishnan and Shulman (2006), 65 percent of firms stated that there has been an increase in supply chain risk in the past five years and listed supplier reliability as one of the top three major challenges. Another survey by Cohen et al. (2008) also found that firms face major risks of on-time delivery and supply uncertainty upon globalizing their supply chains.

In this paper, we define supply uncertainty as the situation when the actual quantity delivered is less than the quantity ordered from a supplier. In particular, we analyze the situation when supply uncertainty results in the firm receiving only a random fraction of the quantity ordered from the supplier. This random uncertainty in the supply process (may also be referred to as random yield) commonly occurs in the electrical and mechanical manufacturing industries due to the complexity of their production processes. For example, the yield rate for Samsung's curved glass production for cell phones is less than 50%, and the common yield rate in micro-chip production in the semiconductor industry is between 0% and 100%, depending on the effectiveness of quality inspection; see Sonntag and Kiesmuller (2017). Additional uncertainty in the supply process can occur during transportation and storage due to various factors impacting the quality of the product such as temperature control, product handling, control of the air flow within the container. According to Voelkel et al. (2020), the range of the random yield is between 70% to 80% for perishable food, and is about 75% for vaccines.

The adverse effect of component supply uncertainty is especially acute for products of short shelf life such as high technology consumer products or fashion apparel. For these products, manufacturers often need to make a forecast of demand for the coming season, purchase the required components and assemble the products ahead of the selling season. Due to the rapidly changing nature of these markets, manufacturers must be able to procure all required components and assemble products in a timely fashion to meet demand only for the next selling season with no intention of carrying extra components and products to subsequent seasons to avoid obsolescence.

Compounding high demand uncertainty with a high degree of uncertainty in the component supply process, the manufacturers need to carefully hedge against both supply and demand uncertainty to avoid carrying excessive inventory or facing supply shortage under such operating environments. With some of the required components facing supply uncertainty, manufacturers must also carefully coordinate the order quantity of all required components for the final assembly process and develop an effective component procurement strategy to mitigate the associated risk of inherent component supply uncertainty in global sourcing.

One common procurement strategy of tackling this challenge is backup sourcing, where manufacturers can procure their bulk amount of components from offshore suppliers as their primary suppliers to take advantage of a high volume at a low unit cost, and utilize local suppliers as backup suppliers to make up for any supply shortage in a timely manner. Thus, backup sourcing of components provides manufacturers with the option to obtain additional units when some of the component suppliers are unable to deliver the order quantity of components due to various factors of uncertainty or disruption in their component supply process. A survey by Rigby and Bilodeau (2009) suggests that 63% of firms in 2008 considered backup sourcing as a key procurement strategy.

Our modeling framework is applicable to manufacturers in industries such as consumer electronics and fashion apparel facing rapidly changing market demand while some major components of their products are subject to supply uncertainty. In particular, many consumer electronics products require advanced semi-conductors which are subject to a high degree of supply uncertainty due to the underlying complex manufacturing operations. Similarly, fashion apparel requires high-quality fabrics, which are subject to uncertain production yield due to its need for careful control of chemicals and material processing operations. For example, Nokia decided to source required components for each of its product categories from multiple suppliers due to the unpredictable nature of microchip production and relied on backup suppliers to supply the additional units that the firm requires to meet unexpected demand; see the reference in Sheffi (2005). We refer to Merzifonluoglu (2015), Zeng and Xia (2015) and Namdar et al. (2017) for some additional examples in different industries where firms had deployed backup sourcing to tackle supply uncertainty of components for improving supply chain performance.

In this paper we develop an analytical framework for studying the benefits of backup sourcing for components in an assembly system under supply uncertainty. Specifically, we consider an assembly system for a product consisting of multiple components, with one of the components facing supply uncertainty. The assembler needs to procure these components and assemble a product for the next selling season. The assembler sources each required component from a primary supplier ahead of the selling season, and has also secured a backup supplier for each of these components. The primary suppliers, usually offshore suppliers, offer lower unit component prices, but have much longer order lead times. In contrast, the backup suppliers, usually local suppliers, can deliver the components very quickly, but at higher unit component prices than the respective primary suppliers. As such, the assembler can utilize these backup suppliers to compensate for any potential shortfall in components due to supply uncertainty upon realizing the actual amount of components delivered by the primary suppliers. We evaluate the benefits of using this backup sourcing strategy in mitigating the adverse effect due to the supply uncertainty of the primary supplier.

We use two performance measures in evaluating the benefits of backup sourcing to the assembler. First, we compute the increase in the expected profit of the assembler due to the availability of backup suppliers. Second, we define the threshold price of a product as the minimum unit price that the assembler is willing to sell the product for a positive expected profit. We analyze how the availability of backup suppliers can lower the threshold price of the product.

Using our modeling framework, we first determine the optimal ordering quantities of each component for the primary suppliers as well as the subsequent order quantities for the backup suppliers based on the actual amount of components delivered by the primary suppliers. Our results show that the optimal order quantities can be complex and depend critically on the underlying cost and operating parameters of the system. Using our analytical results, we further conducted a comprehensive set of numerical experiments to evaluate how the various model parameters including the unit component costs, product price, yield and demand variability can affect the benefits of backup sourcing. Based on our numerical results, we characterize specific operating conditions under which the use of backup suppliers can provide substantial benefits to the assembler in terms of increasing his expected profit and reducing the threshold price of the product.

Overall, our result shows that the availability of backup supplier for the component with supply uncertainty can provide substantial benefits to the assembler, especially when the unit cost of this component from the primary supplier is high relative to the extra unit component costs from the backup suppliers for high-margin products under an operating environment of high uncertainty of primary supply and low demand uncertainty. We further show that the availability of backup suppliers for the other components with no supply uncertainty can provide additional benefits to the assembler. While this additional benefit would generally be much smaller than the primary benefit of hedging against the unreliable supplier, this additional hedging against the components with no supply uncertainty can still be significant when the unit cost from the backup supplier for the unreliable component is high for low-margin products with high demand uncertainty.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes our problem setting. Section 4 presents the benefits of having a backup supplier for the component with supply uncertainty. Section 5 presents the benefits when all the backup suppliers are available. Section 6 discusses the special case where only backup suppliers for the components with no supply uncertainty are available. Section 7 summarizes the major managerial insights. All mathematical proofs are provided in the Appendix A.2.

2. Literature Review

This paper is concerned with supply risk management. In the literature, there are many ways to model supply risks based on their distinct natures. For example, supply shortage may be driven by unexpected capacity deterioration, which is captured by random capacity model (see Ciarallo et al., 1994). Supply shortage may also be caused by some disruptive events (e.g. nature disaster, labor strike, etc) and is modeled by binomial disruption (see Chen et al., 2001; Gurnani et al., 2012). Our paper captures supply risks as a supply random yield, which models the scenario that the production process suffers from inherent risk and leads to shortage of output as a random fraction of the input quantities. For a comprehensive review regarding supply random yield, see the seminal papers by Yano and Lee (1995); Grosfeld-Nir and Gerchak (2004). An important research stream is on the effective design of operational strategies to hedge against supply yield risk (see, e.g., Tang, 2006, for general discussions). These strategies include, but are not limited to, production inflation and inventory (Henig and Gerchak, 1990; Inderfurth and Kiesmüller, 2015; Song and Wang, 2017), supplier improvement (Wang et al., 2009, 2014; Tang et al., 2014), pricing (Tang and Yin, 2007; Eskandarzadeh et al., 2016; Dong et al., 2016), and contract design (Chick et al., 2008; He and Zhang, 2010; Tang and Kouvelis, 2014; Anderson and Monjardino, 2019).

Our paper focuses specifically on the use of multi-sourcing as a risk mitigation strategy to hedge against supply risk of random yield. Multi-sourcing has also been used to mitigate other supply risks including lead-time uncertainty (Allon and Van Mieghem, 2010), procurement cost uncertainty (Xiao et al., 2015), and exchange rate uncertainty (Ding et al., 2007). In the context of random yield, Anupindi and Akella (1993) are among the first to study a newsvendor's sourcing decision with two suppliers, and show that it is never optimal for the firm to sole source from the more expensive supplier alone. Dada et al. (2007) study a newsvendor's procurement problem when sourcing from multiple unreliable suppliers, and summarize the optimal selection rule as "cost is an order qualifier, reliability is an order winner". Federgruen and Yang (2008, 2009, 2011, 2014) extend this rule for single-period, multi-period, and infinite horizon settings with a general pool of random yield suppliers. However, Swaminathan and Shanthikumar (1999) and Dong et al. (2020) show that this rule may be violated when demand follows discrete distributions or the yield distributions are highly positively correlated, respectively. In addition, it is noteworthy that there is a stream within the multi-sourcing literature that studies the adoption of backup/contingency supply as alternative sources, and argues that it could effectively mitigate yield loss and ensure perfect match between supply and demand (see, e.g., Xu and Lu, 2013; Chen and Yang, 2014; Merzifonluoglu, 2015).

All the above papers focus on either a single firm or a bilateral supply chain setting to evaluate the effectiveness of adopted operational strategies in hedging against random yield. In contrast, our paper studies a multi-component assembly system, and contributes to the research literature in multi-sourcing through a thorough investigation of the value of backup sourcing of components in mitigating supply yield risk in assembly systems.

On the other hand, our paper also contributes to the broad literature on managing component procurement for assembly systems under supply uncertainty. There are several research papers that study the optimal inventory policies in an assembly system under various kinds of supply uncertainty. For example, Gallien and Wein (2001), Gurnani et al. (1996), Song et al. (1999, 2000), and Song and Yao (2002) examine the component inventory policies in an assembly system with uncertain procurement lead-times, while Bollapragada et al. (2004) study the impact of component capacity risk in assembly systems. Our research is most relevant to research work that analyzes the component inventory policies for assembly systems with random yields. For instance, Yao (1988) studies the optimal run quantities for an assembly system with component supply yield uncertainty. Singh et al. (1990) analyze the impact of random yield losses of individual components in a semi-conductor manufacturing environment. Gerchak et al. (1994) study a single-period lot sizing problem for a random yield assembly system. Gurnani et al. (2000) study the optimal inventory decision in a two-component assembly system with stochastic demands and random yields. Gurnani and Gerchak (2007) and Güler and Bilgiç (2009) investigate the coordination issue for a decentralized two-component and N-component assembly systems, respectively. Pan and So (2010) characterize the structure of the optimal ordering quantities, in which one of the required components is subject to supply uncertainty. Pan and So (2016) further analyze the use of vendor-managed inventory (VMI) contracts in managing component procurement in a decentralized assembly system under supply uncertainty.

All the above papers focus on either inventory production decision or contract design issue with a single supplier for each component. In contrast, our paper studies a backup sourcing strategy for improving the performance of an assembly system with random yields, in which each component has a backup supplier. We contribute to the research literature in managing component procurement for assembly systems under supply uncertainty through an analysis on how backup sourcing can be used to hedge against supply yield risk of components in an assembly system.

In summary, none of the aforementioned streams of literature consider backup sourcing of components in an assembly system under supply uncertainty. Our paper aims to bridge this gap and contributes to the literature by not only confirming the value of backup sourcing in an assembly system, but also quantifying how such value is affected by the characteristics of the operating environment such as the associated component costs as well as the underlying yield and demand distributions. Specifically, we show that the value of backup supplier for the component with supply uncertainty is most substantial when the unit cost of this component from the primary supplier is high relative to the extra unit component cost from the backup supplier for high-margin products under an operating environment of high supply uncertainty and low demand uncertainty. Our result also shows that a backup supplier for the component without supply uncertainty can still provide significant benefits when the extra unit cost of the backup supplier for the unreliable component is high for low-margin products with high demand uncertainty. This finding is unique to the assembly environment, and cannot be inferred directly from the results in the existing literature since backup supply for a reliable product will normally bring no value. Thus, our research generates useful managerial insights into how the adoption of backup suppliers for all components would improve both the profitability and business feasibility of an assembler.

3. Model Formulation

We consider the assembly of a product consisting of two components, and one unit of each component is required to assemble the final product. There is uncertainty in the supply of component 1 in which the actual available quantity for assembly is only equal to a random fraction of the order quantity. There is no uncertainty in the supply of component 2. We note that it is straightforward to extend our results to a more general case in which the product may consist of more than two components, with only one of these components has supply uncertainty. For ease of exposition, we restrict our discussions to the two-component product only.

An assembler faces a one-time future uncertain demand D with a fixed selling price of p for the final product. In this paper, the assembler is a price-taker facing exogenous market price for the final product. In other words, we consider the product price p as a fixed model parameter, and analyze how this model parameter would affect the system performance.

The assembler procures component 1 from primary supplier 1 and component 2 from primary supplier 2 at unit costs of c_1 and c_2, respectively. These unit costs represent the total cost to procure one unit of component from the primary suppliers which are usually offshore suppliers, and may include purchase price, shipping cost, in-transit inventory cost, and possibly tariffs. Components from primary supplier 1 are subject to supply uncertainty in which only a (random) fraction of the ordered components Q would be available for assembly and is equal to ϵQ, where ϵ is a random variable representing the supply reliability factor with positive support on $[0, 1]$. Components from primary supplier 2 have no supply uncertainty in which all ordered components are available for assembly.

Ahead of the selling season, the assembler orders and receives the components from the two primary suppliers. However, both primary suppliers have long order lead times such that the assembler would not have sufficient time to procure any additional components from either primary supplier if needed. However, upon realizing the actual amount of components available for assembly from the primary suppliers, the assembler can procure additional component 1 from backup supplier 1 and component 2 from backup supplier 2 at (higher) unit costs of $\tilde{c}_1$ and $\tilde{c}_2$, respectively. We assume that these two backup suppliers are usually local supplier with very short order lead times such that they are able to deliver any additional components to the assembler in time for use in assembly before the final demand is realized. In other words, the main feature of the backup suppliers captured in our model is their ability to deliver additional components quickly, but at higher unit costs as compared to their primary counterparts.

The sequence of events are as follows. First, the assembler places an order for components 1 and 2 from the two primary suppliers, denoted by (Q_1, Q_2). Then, the actual amount of component 1 available for assembly is observed, and if needed, the assembler can place an order to the two backup suppliers for some additional components, denoted by $(\tilde{Q}_1, \tilde{Q}_2)$. These two backup suppliers deliver the additional components in time for final assembly before the start of the selling season. The firm assembles the product, and then the final demand D is realized. The objective of the assembler is to determine the optimal order quantity of components (Q_1, Q_2) from the two primary suppliers and the subsequent order quantity of additional components $(\tilde{Q}_1, \tilde{Q}_2)$ from the two backup suppliers upon realizing the random fraction ϵ in order to maximize his expected profit.

To allow for a direct comparison of our results with those by Pan and So (2010), we assume that the assembler pays for all ordered units of component 1 regardless of the actual amount that is available for assembly. Our analysis can be easily adapted for the case when the assembler would only need to pay for the available amounts. For a simpler exposition, we further assume that any excess customer demand is lost and all excess components are scrapped. Without loss of generality, we assume that there are no shortage penalty for unmet demand and no salvage value for any leftover components or products, which are commonly adopted in standard newsvendor analysis to simplify exposition. However, we can extend our analysis and results in a straightforward manner to the case in which the leftover components or products could have some salvage values to reflect practical situations where the excess components or products incur some holding costs and possibly price erosion, but can be salvaged for use in the future.

We introduce some basic notations and assumptions. Let $g(.)$ and $G(.)$ denote the density function and cumulative distribution function of supply reliability factor ϵ, respectively. Denote $\bar{G}(.) = 1 - G(.)$ and let μ be the expected value of ϵ. We assume that $\frac{c_1}{\mu} < \tilde{c}_1$ and $c_2 < \tilde{c}_2$, such that the two backup suppliers can respond faster than the two primary suppliers at the expense of a higher average unit cost. We can also assume that the unit component costs, c_1 and c_2, are less than the unit product price p; otherwise, there is no incentive to order any components from the primary suppliers to assemble the final product for a positive profit. To simplify exposition, we assume that the assembly operation is defect free and all other associated assembly costs are negligible.

We conduct our analysis as follows. As there is underlying supply uncertainty for component 1 from primary supplier 1, it is intuitive for the assembler to secure a backup supplier for component 1 to mitigate the adverse impact and improve profitability. As such, we first focus our analysis on quantifying the benefits of having backup supplier 1 only, i.e., we assume that backup supplier 2 is not available. We present our result for this case in Section 4. Since component 2 from primary supplier 2 has no supply uncertainty, it raises an interesting question as whether the assembler can benefit from having backup supplier 2 also. We address this question in Section 5 by analyzing the more general situation where both backup suppliers are available. Finally, it would be interesting to investigate on how the use of backup supplier 2 would benefit the assembler even when backup supplier 1 is not available. We address this question by analyzing the special case where only backup supplier 2 is available in Section 6.

4. Benefits of Having Backup Supplier 1 Only

Assume that $\tilde{c}_1 < p$; otherwise, the assembler would never utilize backup supplier 1 to assemble the final product for a positive profit. Let x denote the random demand with distribution $F(.)$. To simplify exposition, assume that $F(.)$ is continuous and differentiable with density $f(.)$. Let e denote the realized value of the (random) supply reliability factor ϵ, and $(x)^+ = \max\{0, x\}$. Under the assumptions of zero shortage penalty for unmet demand and zero salvage value for any leftover components or products, the assembler's expected profit function can be written as

$$\pi_1(Q_1, Q_2, \tilde{Q}_1) = E_x[p \min\{eQ_1 + \tilde{Q}_1, Q_2, x\} - c_1 Q_1 - c_2 Q_2 - \tilde{c}_1 \tilde{Q}_1] \tag{1}$$

for any given order quantity for the primary suppliers (Q_1, Q_2) and an additional order of $\tilde{Q}_1$ from backup supplier 1 with the supply reliability realization e.

Without backup supplier 1, it has been shown by Pan and So (2010) that the optimal order quantities for the two primary suppliers is given by $(\frac{D}{\delta_p}, D)$ when the product price p exceeds some threshold level p_0, where δ_p is defined by

$$\int_0^{\delta_p} t g(t) dt = \frac{c_1}{p}, \tag{2}$$

and p_0 is the unique solution to

$$p_0 = \frac{c_2}{\bar{G}(\delta_0)} \quad \text{and} \quad \int_0^{\delta_0} tg(t)dt = \frac{c_1}{p_0}. \tag{3}$$

When the product price p is below the threshold p_0, it is not profitable to assemble the product.

We shall derive a parallel result for the case when there is a backup supplier for component 1. First, define p_1 and δ_1 by

$$p_1 = \tilde{c}_1 G(\delta_1) + c_2 \quad \text{and} \quad \int_0^{\delta_1} tg(t)dt = \frac{c_1}{\tilde{c}_1}. \tag{4}$$

Comparing (3) and (4), it is clear that $\delta_0 \le \delta_1$ if and only if $\tilde{c}_1 \le p_0$. Furthermore, we can derive the following properties about p_1.

Lemma 1 *(i) p_1 is strictly increasing in $\tilde{c}_1$, with $p_1 = p_0$ when $\tilde{c}_1 = p_0$.*
(ii) $\tilde{c}_1 \le p_1$ if and only if $\tilde{c}_1 \le p_0$.

Let (Q_1^*, Q_2^*) denote the optimal order quantities for the two primary suppliers, and let $\tilde{Q}_1^*$ denote the subsequent optimal order quantity from backup supplier 1. Define M_1 by

$$M_1 = F^{-1}(\frac{p - \tilde{c}_1}{p}). \tag{5}$$

Note that the quantity M_1 corresponds to the optimal solution to a newsvendor problem with an understocking cost of $p - \tilde{c}_1$ and an overstocking cost of $\tilde{c}_1$. Thus, M_1 represents the maximum order-up-to level for ordering component 1 from the backup supplier when there is sufficient amount of matching component 2.

Define $Q_2^a = F^{-1}(\frac{p-p_1}{p})$ and $Q_1^a = \frac{Q_2^a}{\delta_1}$. Also, define (Q_1^b, Q_2^b) as the unique solution of (Q_1, Q_2) to the following two equations:

$$\begin{aligned} p\int_{\frac{M_1}{Q_1}}^{\frac{Q_2}{Q_1}} \bar{F}(tQ_1)tg(t)dt + \tilde{c}_1 \int_0^{\frac{M_1}{Q_1}} tg(t)dt - c_1 &= 0, \\ p\bar{F}(Q_2)\bar{G}(\frac{Q_2}{Q_1}) - c_2 &= 0. \end{aligned}$$

We can show that $Q_2^a < M_1$ and $Q_2^b \ge M_1$.

The next proposition provides the optimal order quantities for the primary and backup suppliers:

Proposition 1 *(i) When $\tilde{c}_1 < p_0$,*

$$\begin{cases} (Q_1^*, Q_2^*) = (Q_1^a, Q_2^a) \text{ and } \tilde{Q}_1^* = (Q_2^a - eQ_1^a)^+ & \text{if } p > p_1, \\ (Q_1^*, Q_2^*) = (0, 0) \text{ and } \tilde{Q}_1^* = 0 & \text{if } p \leq p_1. \end{cases}$$

(ii) When $\tilde{c}_1 \geq p_0$,

$$\begin{cases} (Q_1^*, Q_2^*) = (Q_1^b, Q_2^b) \text{ and } \tilde{Q}_1^* = (M_1 - eQ_1^b)^+ & \text{if } p > p_0, \\ (Q_1^*, Q_2^*) = (0, 0) \text{ and } \tilde{Q}_1^* = 0 & \text{if } p \leq p_0. \end{cases}$$

Proposition 1 demonstrates one major benefit of having backup supplier 1 for the assembler is to reduce the threshold price, i.e., the minimum product price for which it is profitable for the assembler to sell the product. Specifically, the availability of backup supplier 1 can reduce the threshold price from p_0 to p_1 when $\tilde{c}_1 < p_0$. Lemma 1 further shows that this threshold price p_1 increases as the unit component cost $\tilde{c}_1$ increases, and it is always less than the unit component cost $\tilde{c}_1$ when $\tilde{c}_1 < p_0$.

Another benefit of having backup supplier 1 for the assembler is to increase his expected profit. We can derive analytical results under deterministic demand; see Appendix A.1. However, we are unable to derive a closed-form analytical expression for the expected profit function of the assembler under stochastic demand. Therefore, we conducted a numerical study to understand how various model parameters would affect the expected profit increase for the assembler. The results observed in our numerical study can further be established analytically under deterministic demand; see Propositions 7 and 8 in Appendix A.1.

While the availability of backup supplier 1 would always benefit the assembler by reducing the threshold price and increasing his expected profit, it is unclear as whether having backup supplier 1 would benefit or hurt the two primary suppliers when it is profitable for the assembler to sell the product without backup supplier 1, i.e., $p > p_0$. We can answer this question by comparing the optimal order quantities for the primary suppliers with and without backup supplier 1 when $p > p_0$.

There are intricate relationships between the two primary suppliers with the availability of backup supplier 1, and the assembler needs to balance several cost factors in deciding the order quantities for the two primary suppliers. First, the assembler could order less units of component 1 from the primary supplier with the availability of backup supplier 1 to hedge against the supply uncertainty. On the other hand, the assembler could afford to order more units of component 2 with backup supplier 1 to hedge against any shortfall of component 1 from the primary supplier, which in turn, would match with a larger order quantity of component 1 from the primary supplier. Also, the product price p and unit component cost $\tilde{c}_1$ affect the resulting profit margin, which would determine the extent to how the assembler relies on backup supplier 1 to hedge against supply uncertainty. We shall derive some analytical result by taking into account of all these interactions under the following class of demand distributions.

Definition 1: Let Φ be a distribution function with corresponding density function ϕ. We say Φ is IGFR (increasing generalized failure rates) if $\frac{x\phi(x)}{1-\Phi(x)}$ is increasing for all x such that $\Phi(x) < 1$. It is well known that many commonly used distributions including uniform, exponential, gamma and normal distributions, are IGFR. For additional details, see Lariviere and Porteus (2001) and Lariviere (2006).

Let $(\bar{Q}_1, \bar{Q}_2)$ denote the optimal order quantities for the two primary suppliers without backup supplier 1. We can obtain the following result that compares the optimal order quantities for the primary suppliers with and without backup supplier 1.

Proposition 2 *Suppose that the demand distribution $F(.)$ is IGFR.*
(i) When $\tilde{c}_1 < p_0$, there exist two product prices (denoted by $\hat{p}_1$ and $\hat{p}_2$) such that

$$\begin{cases} Q_1^* > \bar{Q}_1, & Q_2^* > \bar{Q}_2, \quad \text{if } p_0 < p < \hat{p}_1 \\ Q_1^* \leq \bar{Q}_1, & Q_2^* > \bar{Q}_2, \quad \text{if } \hat{p}_1 \leq p < \hat{p}_2 \\ Q_1^* < \bar{Q}_1, & Q_2^* \leq \bar{Q}_2, \quad \text{if } p \geq \hat{p}_2. \end{cases}$$

(ii). When $\tilde{c}_1 \geq p_0$, $Q_1^* < \bar{Q}_1$ and $Q_2^* < \bar{Q}_2$.

The results in Proposition 2 highlight the intricate interactions for the ordering decisions under stochastic demand with the availability of backup supplier 1. With a high value of $\tilde{c}_1$ ($\geq p_0$), the assembler would always reduce the optimal order quantities for the two primary suppliers when backup supplier 1 is available. Thus, the availability of backup supplier 1 would always hurt the profitability of both primary suppliers in this case. With a low value of $\tilde{c}_1$ ($< p_0$), the availability of backup supplier 1 could either increase or decrease the profitability of the two primary suppliers, depending on the product price p. For products with a low price ($p_0 < p < \hat{p}_1$), the availability of backup supplier 1 benefits both primary suppliers and increases their profits. For products with a high price ($p \geq \hat{p}_2$), the availability of backup supplier 1 hurts both primary suppliers and decreases their profits. For product price between $\hat{p}_1$ and $\hat{p}_2$, it benefits primary supplier 2, but hurts primary supplier 1.

4.1 Numerical Results

We first provide an illustrative example to demonstrate the results from our analysis. In this illustrative example, we set the unit product selling price $p = 16$, unit component costs $c_1 = 2$ and $c_2 = 4$, with the supply reliability ϵ uniformly distributed in $[0, 1]$ and demand distributed D uniformly distributed in $[0, 10000]$. With no backup suppliers, we can compute the threshold price given in (3), and $p_0 = 10.47$. The optimal order quantities are given by $(\bar{Q}_1^*, \bar{Q}_2^*) = (6842, 3994)$ and the optimal expected profit of the assembler is equal to $\bar{\pi}_0^* = 7796$.

Suppose that backup supplier 1 is available with $\tilde{c}_1 = 5$. Proposition 1 shows that the threshold price can be reduced since $\tilde{c}_1 < p_0 = 10.47$. In particular, the threshold price is reduced to $p_1 = 8.47$ as given in (4). Furthermore, it follows from Proposition 1 that $(Q_1^*, Q_2^*) = (Q_1^a, Q_2^a) = (5260, 4704)$ since $\tilde{c}_1 < p_0$. Comparing with the case with no backup suppliers, the assembler would now order less component 1 from primary supplier 1 and more component 2 from primary supplier 2 due to the fact that the assembler can order additional amount of component 1 from backup supplier 1 upon realization of the actual available amount of component 1 from primary supplier 1. With backup supplier 1, the optimal expected profit of the assembler is increased to $\pi_1^* = 17709$.

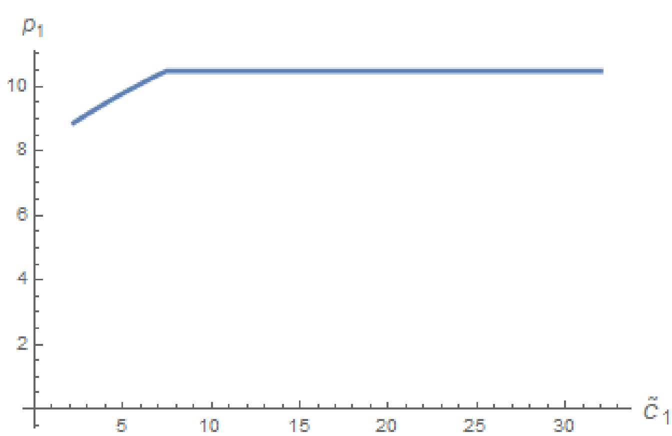

Figure 1: Threshold Price With Backup Supplier 1 Only.

We next provide some additional numerical results based on the above illustrative example. Figure 1 shows the threshold price $\hat{p}$ when the unit cost of backup supplier 1, $\tilde{c}_1$, increases from $\frac{c_1}{\mu} = 4$ to $p_0 = 10.47$. Observe that $\hat{p} = p_1$ and increases as $\tilde{c}_1$ increases (see Lemma 1(i)) and stays

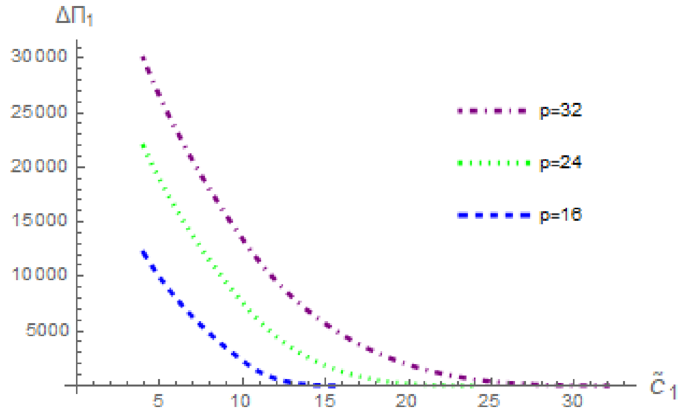

Figure 2: Profit Increase With Backup Supplier 1 Only.

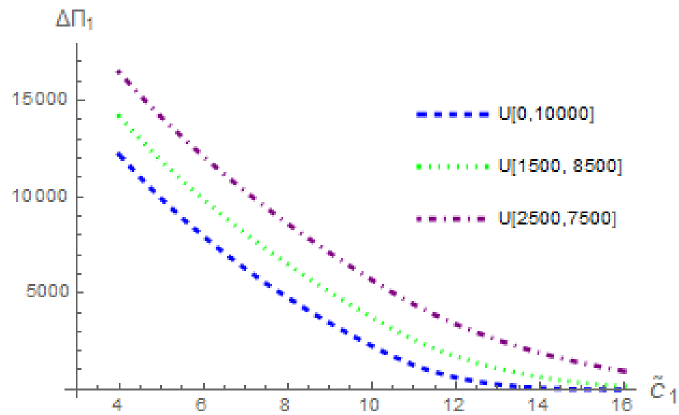

Figure 3: Impact of Demand Uncertainty With Backup Supplier 1 Only.

at $p_0 = 10.47$ when $\tilde{c}_1 \geq p_0 = 10.47$. Figure 2 shows that the expected profit increase due to the availability of backup supplier 1, $\Delta\pi_1$, decreases as $\tilde{c}_1$ increases and that $\Delta\pi_1$ is higher when the product price p increases. Figure 3 provides the values of $\Delta\pi_1$ as $\tilde{c}_1$ increases under three different demand distributions, $U[0, 10000]$, $U[1500, 8500]$ and $U[2500, 7500]$. They show that the expected profit increase due to the availability of backup supplier 1 is higher when the underlying demand variability decreases. Finally, Figure 4 shows the values of $\Delta\pi_1$ when the yield factor ϵ follows a power distribution with cumulative distribution function of $G(t) = t^n$ where $t \in [0, 1]$ for $n = 1, 2, 3$. Observe that $\Delta\pi_1$ is higher for a smaller value of n. This shows that the expected profit increase for the assembler due to the availability of backup supplier 1 is higher when the yield factor ϵ is stochastically less reliable.

Overall, our numerical study shows that the expected profit increase for the assembler due to the availability of backup supplier 1 is higher when p increases, c_1 increases, $\tilde{c}_1$ decreases,

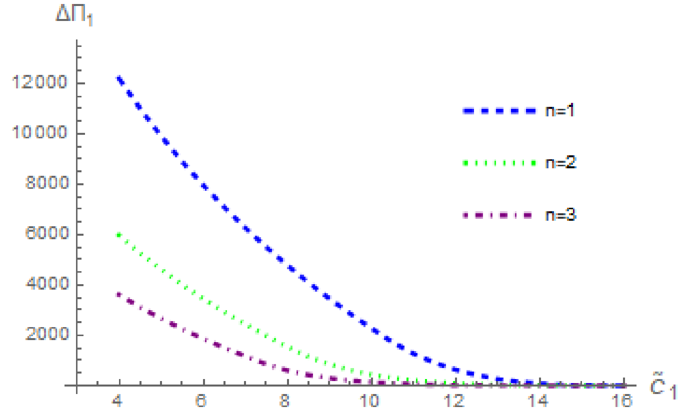

Figure 4: Impact of Supply Uncertainly With Backup Supplier 1 Only.

supply reliability becomes more variable, or the underlying demand variability decreases. (These results can further be established analytically under deterministic demand; see Propositions 7-8 in Appendix A.1.) In other words, it is most beneficial for the assembler to have backup supplier 1 when the unit cost of component 1 from primary supplier 1 is high (large c_1) relative to the extra unit component cost from backup supplier 1 (low $\tilde{c}_1$) for high-margin products (high p), especially under an operating environment of low demand uncertainty and high supply uncertainty from primary supplier 1.

5. Benefits of Having Both Backup Suppliers

In this section, we extend our analysis to the situation in which a backup supplier for component 2 is also available. We analyze the conditions under which the availability of backup supplier 2 can provide additional benefits to the assembler in mitigating the adverse effect due to supply uncertainty of component 1 from the primary supplier. We can assume that $\tilde{c}_1 < p$ and $\tilde{c}_2 < p$; otherwise, the assembler would never utilize either backup suppliers to assemble the final product for a positive profit.

For any primary order (Q_1, Q_2) and subsequent backup order $(\tilde{Q}_1, \tilde{Q}_2)$, the assembler's expected profit function can be written as

$$\pi_2(Q_1, Q_2, \tilde{Q}_1, \tilde{Q}_2) = pE_x[\min\{eQ_1 + \tilde{Q}_1, Q_2 + \tilde{Q}_2, D\}] - c_1Q_1 - c_2Q_2 - \tilde{c}_1\tilde{Q}_1 - \tilde{c}_2\tilde{Q}_2. \qquad (6)$$

The analysis becomes more technically involved when both backup suppliers are available. As such, we conduct our analysis and present our results in two mutually exclusive scenarios: 1) $\tilde{c}_1 \leq \tilde{c}_2$;

and 2) $\tilde{c}_1 > \tilde{c}_2$.

5.1 Scenario 1: $\tilde{c}_1 \leq \tilde{c}_2$

Similar to the definition of M_1 given in (5), we define

$$M_2 = F^{-1}(\frac{p - \tilde{c}_2}{p}). \tag{7}$$

Here, the quantity M_2 corresponds to the optimal solution to a newsvendor problem with an understocking cost of $p - \tilde{c}_2$ and an overstocking cost of $\tilde{c}_2$. In other words, M_2 represents the maximum order-up-to level for ordering component 2 from the backup supplier when there is sufficient amount of matching component 1. For any given primary order (Q_1, Q_2) and supply reliability realization e, the assembler would order from backup supplier 2 to bring the inventory level of component 2 to either eQ_1 or M_2, whichever is smaller, i.e., $\tilde{Q}_2^* = (\min\{eQ_1, M_2\} - Q_2)^+$.

Define (Q_1^c, Q_2^c) as the unique solution of (Q_1, Q_2) to

$$\begin{aligned} p\int_{\frac{Q_2}{Q_1}}^{\frac{M_2}{Q_1}} \bar{F}(tQ_1)tg(t)dt + \tilde{c}_1 \int_0^{\frac{Q_2}{Q_1}} tg(t)dt - \tilde{c}_2 \int_{\frac{Q_2}{Q_1}}^{\frac{M_2}{Q_1}} tg(t)dt - c_1 &= 0, \\ pG(\frac{Q_2}{Q_1})\bar{F}(Q_2) - \tilde{c}_1 G(\frac{Q_2}{Q_1}) + \tilde{c}_2 \bar{G}(\frac{Q_2}{Q_1}) - c_2 &= 0. \end{aligned} \tag{8}$$

Also, define (Q_1^d, Q_2^d) as the unique solution of (Q_1, Q_2) to

$$\begin{aligned} p\int_{\frac{Q_2}{Q_1}}^{1} \bar{F}(tQ_1)tg(t)dt + \tilde{c}_1 \int_0^{\frac{Q_2}{Q_1}} tg(t)dt - \tilde{c}_2 \int_{\frac{Q_2}{Q_1}}^{1} tg(t)dt - c_1 &= 0, \\ pG(\frac{Q_2}{Q_1})\bar{F}(Q_2) - \tilde{c}_1 G(\frac{Q_2}{Q_1}) + \tilde{c}_2 \bar{G}(\frac{Q_2}{Q_1}) - c_2 &= 0. \end{aligned} \tag{9}$$

(It can be shown that $Q_1^c > M_2$ and $Q_2^c \leq M_2 \leq M_1$, and $Q_2^d < Q_1^d \leq M_2 \leq M_1$; see the Appendix). Note that the above two sets of equations, (8) and (9), are the same when $Q_1 = M_2$. We define p_α as the specific value of p such that $Q_1^c = Q_1^d = M_2$.

Suppose that p_2 and δ_2 satisfy the following two equations:

$$p_2 \int_{\delta_2}^{1} tg(t)dt + \tilde{c}_1 \int_0^{\delta_2} tg(t)dt - \tilde{c}_2 \int_{\delta_2}^{1} tg(t)dt - c_1 = 0, \tag{10}$$

$$p_2 G(\delta_2) - \tilde{c}_1 G(\delta_2) + \tilde{c}_2 \bar{G}(\delta_2) - c_2 = 0. \tag{11}$$

The next result presents the condition when p_2 exists and some properties of p_2.

Lemma 2 *Suppose that $\tilde{c}_1 \leq \tilde{c}_2$. When $\tilde{c}_2 \leq p_1$, p_2 exists and $\tilde{c}_2 \leq p_2 \leq p_1 \leq p_0$.*

The next proposition summarizes the optimal order quantities for the primary and backup suppliers, (Q_1^*, Q_2^*), and $(\tilde{Q}_1^*, \tilde{Q}_2^*)$, which depend on the values of $\tilde{c}_2$ and p:

Proposition 3 *(i) When $\tilde{c}_2 < p_1$, the optimal order quantities are as follows:*

$$\begin{cases} (Q_1^*, Q_2^*) = (Q_1^c, Q_2^c), \tilde{Q}_1^* = (Q_2^c - eQ_1^c)^+, \tilde{Q}_2^* = (\min\{eQ_1^c, M_2\} - Q_2^c)^+, & \text{if } p > p_\alpha; \\ (Q_1^*, Q_2^*) = (Q_1^d, Q_2^d), \tilde{Q}_1^* = (Q_2^d - eQ_1^d)^+, \tilde{Q}_2^* = (eQ_1^d - Q_2^d)^+, & \text{if } p_2 < p \le p_\alpha; \\ (Q_1^*, Q_2^*) = (0, 0), \tilde{Q}_1^* = \tilde{Q}_2^* = 0, & \text{if } p \le p_2. \end{cases}$$

(ii) When $\tilde{c}_2 \geq p_1$, $\tilde{Q}_2^* = 0$.

Proposition 3(i) shows that when $\tilde{c}_2 < p_1$, the assembler can further reduce the threshold price from p_1 to p_2 by utilizing both backup suppliers to hedge against supply uncertainty of component 1. When the product price p is between p_2 and p_α, the optimal order quantity for primary supplier 2 (i.e., $Q_2^* = Q_2^d$) is less than M_2. If the available amount of component 1 is less than that of component 2 upon realization of supply uncertainty, the assembler would order from backup supplier 1 to match all available units of component 2. Also, the optimal order quantity for primary supplier 1 (i.e., $Q_1^* = Q_1^d$) is less than M_2. If the available amount of component 1 is more than that of component 2 upon realization of supply uncertainty, the assembler would order sufficient units of component 2 from backup supplier 2 to match all available units of component 1. When the product price p is above p_α, the optimal order quantity for primary supplier 2 (i.e., $Q_2^* = Q_2^c$) is still less than M_2, and the assembler would order sufficient units of component 1 from backup supplier 1 to match all available units of component 2. However, the optimal order quantity of primary supplier 1 (i.e., $Q_1^* = Q_1^c$) is now more than M_2. In this case, depending on the supply realization of component 1, the assembler would order additional units of component 2 from backup supplier 2 to the minimum of eQ_1 and M_2.

Finally, Proposition 3(ii) shows that the assembler would never utilize backup supplier 2 to hedge against supply uncertainty of component 1 when $\tilde{c}_2 \geq p_1$. In this case, the result reduces to the previous case with only backup supplier 1; see the proof of Proposition 1.

5.2 Scenario 2: $\tilde{c}_1 > \tilde{c}_2$

To present the results for the optimal order quantities in this case, we first define a set of cost and price levels. Let

$$\delta = \bar{G}^{-1}(\frac{c_2}{\tilde{c}_2}) < 1, \tag{12}$$

and define

$$c_\theta = \frac{\tilde{c}_2 \int_\delta^1 tg(t)dt + c_1}{\mu}, \tag{13}$$

$$c_\phi = \frac{c_1}{\int_0^\delta tg(t)dt}. \tag{14}$$

We can derive some properties on c_θ as follows:

Lemma 3 *Suppose that $\tilde{c}_1 > \tilde{c}_2$.*

(i) c_θ is strictly increasing in $\tilde{c}_2$, with $c_\theta = p_0$ when $\tilde{c}_2 = p_0$.

(ii) $\tilde{c}_2 < c_\theta$ if and only if $\tilde{c}_2 < p_0$.

(iii) When $\tilde{c}_1 \leq c_\theta$, p_2 exists and $\tilde{c}_1 \leq p_2 \leq c_\theta \leq p_0$.

We can show (see Lemma 4 in the Appendix) that for any given $\tilde{c}_1$, there exists a unique value of p such that $\frac{M_1}{M_2} = \delta$. Define c_α as the (unique) value of $\tilde{c}_1$ satisfying the following equation

$$p \int_\delta^1 \bar{F}(tM_2)tg(t)dt + \tilde{c}_1 \int_0^\delta tg(t)dt - c_\theta \mu = 0.$$

Let $\tilde{M}_2 = F^{-1}(\frac{\tilde{c}_1 - \tilde{c}_2}{\tilde{c}_1})$. Define c_β be the (unique) value of $\tilde{c}_1$ satisfying the following equation

$$\tilde{c}_1 \int_0^1 \bar{F}(t\tilde{M}_2)tg(t)dt - c_\theta \mu = 0,$$

and c_γ be the (unique) value of $\tilde{c}_1$ satisfying the following equation

$$\tilde{c}_1 \int_0^\delta \bar{F}(\frac{t\tilde{M}_2}{\delta})tg(t)dt - c_1 = 0.$$

We can also show (see Lemma 4(iii) in the Appendix) that c_α, c_β and c_γ all exist, $c_\theta < c_\alpha < c_\beta$ and $min\{c_\phi, c_\beta\} < c_\gamma$ when $\tilde{c}_2 < p_0$.

Define (Q_1^e, Q_2^e) as the unique solution of (Q_1, Q_2) to

$$\begin{aligned} p \int_{\frac{M_1}{Q_1}}^{\frac{M_2}{Q_1}} \bar{F}(tQ_1)tg(t)dt + \tilde{c}_1 \int_0^{\frac{M_1}{Q_1}} tg(t)dt - \tilde{c}_2 \int_{\frac{Q_2}{Q_1}}^{\frac{M_2}{Q_1}} tg(t)dt - c_1 &= 0, \\ \tilde{c}_2 \bar{G}(\frac{Q_2}{Q_1}) - c_2 &= 0. \end{aligned}$$

Also, define (Q_1^f, Q_2^f) as the unique solution of (Q_1, Q_2) to

$$\begin{aligned} p \int_{\frac{M_1}{Q_1}}^{1} \bar{F}(tQ_1)tg(t)dt + \tilde{c}_1 \int_0^{\frac{M_1}{Q_1}} tg(t)dt - \tilde{c}_2 \int_{\frac{Q_2}{Q_1}}^{1} tg(t)dt - c_1 &= 0, \\ \tilde{c}_2 \bar{G}(\frac{Q_2}{Q_1}) - c_2 &= 0. \end{aligned}$$

(It can be shown that $M_1 \le Q_2^e \le M_2 \le Q_1^e$ and $M_1 \le Q_2^f \le Q_1^f \le M_2$; see the Appendix). Finally, we define 4 different price levels as follows:

(i) p_θ: value of p when $Q_2^d = Q_2^f = M_1$;

(ii) p_ϕ: value of p when $Q_1^e = Q_1^f = M_2$;

(iii) p_β: value of p when $Q_2^c = Q_2^e = M_1$ and $\frac{M_2\delta}{M_1} < 1$;

(iv) p_γ: value of p when $Q_2^e = M_2$.

The next proposition summarizes the optimal order quantities for the primary and backup suppliers, (Q_1^*, Q_2^*), and $(\tilde{Q}_1^*, \tilde{Q}_2^*)$, which depend on the specific values of $\tilde{c}_1$ and p:

Proposition 4 *Suppose that $\tilde{c}_2 < p_0$. The optimal order quantities are given as follows:*

(i) When $\tilde{c}_1 < c_\theta$,

$$\begin{cases} (Q_1^*, Q_2^*) = (Q_1^c, Q_2^c), \tilde{Q}_1^* = (Q_2^c - eQ_{s1}^c)^+, \tilde{Q}_2^* = (\min\{eQ_{s1}^c, M_2\} - Q_2^c)^+, & \text{if } p \ge p_\alpha, \\ (Q_1^*, Q_2^*) = (Q_1^d, Q_2^d), \tilde{Q}_1^* = (Q_2^d - eQ_{s1}^d)^+, \tilde{Q}_2^* = (eQ_{s1}^d - Q_2^d)^+, & \text{if } p_2 < p < p_\alpha. \end{cases}$$

(ii) When $c_\theta \le \tilde{c}_1 < c_\alpha$,

$$\begin{cases} (Q_1^*, Q_2^*) = (Q_1^c, Q_2^c), \tilde{Q}_1^* = (Q_2^c - eQ_{s1}^c)^+, \tilde{Q}_2^* = (\min\{eQ_{s1}^c, M_2\} - Q_2^c)^+, & \text{if } p \ge p_\alpha, \\ (Q_1^*, Q_2^*) = (Q_1^d, Q_2^d), \tilde{Q}_1^* = (Q_2^d - eQ_{s1}^d)^+, \tilde{Q}_2^* = (eQ_{s1}^d - Q_2^d)^+, & \text{if } p_\theta \le p < p_\alpha, \\ (Q_1^*, Q_2^*) = (Q_1^f, Q_2^f), \tilde{Q}_1^* = (M_1 - eQ_{s1}^f)^+, \tilde{Q}_2^* = (eQ_{s1}^f - Q_2^f)^+, & \text{if } p < p_\theta. \end{cases}$$

(iii) When $c_\alpha \le \tilde{c}_1 < \min\{c_\phi, c_\beta\}$,

$$\begin{cases} (Q_1^*, Q_2^*) = (Q_1^c, Q_2^c), \tilde{Q}_1^* = (Q_2^c - eQ_{s1}^c)^+, \tilde{Q}_2^* = (\min\{eQ_{s1}^c, M_2\} - Q_2^c)^+, & \text{if } p \ge p_\beta, \\ (Q_1^*, Q_2^*) = (Q_1^e, Q_2^e), \tilde{Q}_1^* = (M_1 - eQ_{s1}^e)^+, \tilde{Q}_2^* = (\min\{eQ_{s1}^e, M_2\} - Q_2^e)^+, & \text{if } p_\phi \le p < p_\beta, \\ (Q_1^*, Q_2^*) = (Q_1^f, Q_2^f), \tilde{Q}_1^* = (M_1 - eQ_{s1}^f)^+, \tilde{Q}_2^* = (eQ_{s1}^f - Q_2^f)^+, & \text{if } p < p_\phi. \end{cases}$$

(iv) When $\min\{c_\phi, c_\beta\} \le \tilde{c}_1 < \max\{c_\phi, c_\beta\}$,

(a) if $c_\phi > c_\beta$,

$$\begin{cases} (Q_1^*, Q_2^*) = (Q_1^c, Q_2^c), \tilde{Q}_1^* = (Q_2^c - eQ_{s1}^c)^+, \tilde{Q}_2^* = (\min\{eQ_{s1}^c, M_2\} - Q_2^c)^+, & \text{if } p \ge p_\beta, \\ (Q_1^*, Q_2^*) = (Q_1^e, Q_2^e), \tilde{Q}_1^* = (M_1 - eQ_{s1}^e)^+, \tilde{Q}_2^* = (\min\{eQ_{s1}^e, M_2\} - Q_2^e)^+, & \text{if } p < p_\beta. \end{cases}$$

(b) if $c_\phi \le c_\beta$,

$$\begin{cases} \tilde{Q}_2^* = 0, & \text{if } p \ge p_\gamma, \\ (Q_1^*, Q_2^*) = (Q_1^e, Q_2^e), \tilde{Q}_1^* = (M_1 - eQ_{s1}^e)^+, \tilde{Q}_2^* = (\min\{eQ_{s1}^e, M_2\} - Q_2^e)^+, & \text{if } p_\phi \le p < p_\gamma, \\ (Q_1^*, Q_2^*) = (Q_1^f, Q_2^f), \tilde{Q}_1^* = (M_1 - eQ_{s1}^f)^+, \tilde{Q}_2^* = (eQ_{s1}^f - Q_2^f)^+, & \text{if } p < p_\phi. \end{cases}$$

(v) When $\max\{c_\phi, c_\beta\} \le \tilde{c}_1 < c_\gamma$,

$$\begin{cases} \tilde{Q}_2^* = 0, & \text{if } p \ge p_\gamma, \\ (Q_1^*, Q_2^*) = (Q_1^e, Q_2^e), \tilde{Q}_1^* = (M_1 - eQ_{s1}^e)^+, \tilde{Q}_2^* = (\min\{eQ_{s1}^e, M_2\} - Q_2^e)^+, & \text{if } p < p_\gamma. \end{cases}$$

(vi) When $\tilde{c}_1 \ge c_\gamma$, $\tilde{Q}_2^* = 0$.

Proposition 4 demonstrates the impact on the threshold price of the product when both backup suppliers are available under the conditions that $\tilde{c}_1 > \tilde{c}_2$ and $\tilde{c}_2 < p_0$. When $\tilde{c}_1 < c_\theta$, the assembler can reduce the threshold price from p_0 to p_2 with the availability of both backup suppliers. When $\tilde{c}_1 \ge c_\theta$, the threshold price is equal to $\tilde{c}_1$ if $\tilde{c}_1 < p_0$, and is equal to p_0 if $\tilde{c}_1 \ge p_0$.

The results of Proposition 4 further illustrate the optimal order quantities for the primary suppliers relative to the maximum order-up-to levels M_1 and M_2 at different levels of $\tilde{c}_1$ and p under the conditions that $\tilde{c}_1 > \tilde{c}_2$ and $\tilde{c}_2 < p_0$. In particular, we show that $Q_2^* = 0$ only under three conditions: (a) $c_\phi \le \tilde{c}_1 < c_\beta$ and $p \ge p_\gamma$, (b) $\max\{c_\phi, c_\beta\} \le \tilde{c}_1 < c_\gamma$ and $p \ge p_\gamma$; and (c) $\tilde{c}_1 \ge c_\gamma$. Under these three conditions, the assembler would never utilize backup supplier 2, so the result reduces to the previous stochastic case with only backup supplier 1. Otherwise, it would be beneficial for the assembler to have a backup supplier for component 2.

5.3 Numerical Results

We next provide some numerical results to demonstrate the results from our analysis. Using the same illustrative example given in Section 4.1, with $p = 16$, $c_1 = 2$ $c_2 = 4$, $\tilde{c}_1 = 5$, ϵ uniformly distributed in $[0, 1]$ and demand distributed D uniformly distributed in $[0, 10000]$, suppose that backup supplier 2 is also available with $\tilde{c}_2 = 6.5$. This corresponds to Scenario 1 with $\tilde{c}_1 \le \tilde{c}_2$. In this case, since $\tilde{c}_2 < p_1 = 8.47$, Lemma 2 shows that the threshold price p_2 exists, and it can be found that $p_2 = 8.45$ using (10) and (11). The optimal order quantities of both the primary and backup suppliers are given by the results in Proposition 3, and it can be found that $p_\alpha = 22.12$ from its definition, and so $(Q_1^*, Q_2^*) = (Q_1^d, Q_2^d) = (5468, 4647)$ since $p_2 = 8.45 < p \le p_\alpha = 22.12$. Comparing with the case with backup supplier 1 only, the assembler would now order more component 1 from primary supplier 1 and less component 2 from primary supplier 2 due to the fact that the assembler can order additional amount of component 2 from backup supplier 2 later. With both backup suppliers, the optimal expected profit of the assembler is increased to $\pi_2^* = 17779$.

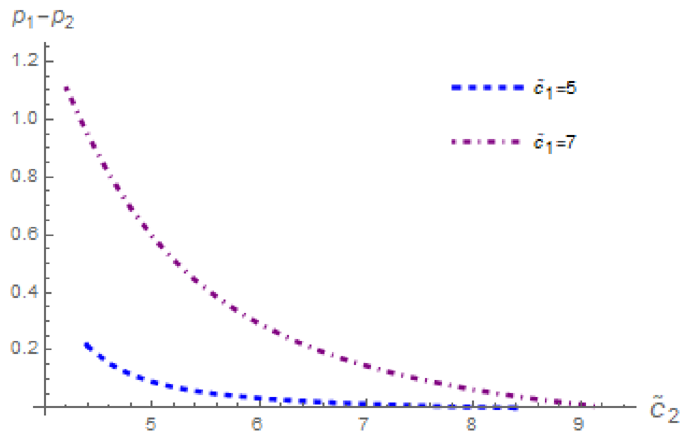

Figure 5: Threshold Price Reduction With Both Backup Suppliers.

The next set of numerical results illustrates the additional benefits of having backup supplier 2 to the assembler in addition to the availability of backup supplier 1. Again, consider the illustrative example discussed earlier with $\tilde{c}_1 = 5$ and $\tilde{c}_1 = 7$. Figure 5 shows the additional reduction in threshold price from p_1 (with backup supplier 1 only) to p_2 (with both backup suppliers) when backup supplier 2 is also available with $\tilde{c}_2 \geq c_2 = 4$. Since $(p_0 - p_1) = 10.47 - 8.47 = 2$ when $\tilde{c}_1 = 5$ and $(p_0 - p_1) = 10.47 - 9.29 = 1.18$ when $\tilde{c}_1 = 7$ in this example, we can observe from Figure 5 that this additional reduction in threshold price $(p_1 - p_2)$ is relatively small as compared to the corresponding reduction from p_0 (with no backup suppliers) to p_1 (with backup supplier 1 only) when $\tilde{c}_1$ is small (i.e., $\tilde{c}_1 = 5$), but is rather substantial for small values of $\tilde{c}_2$ when $\tilde{c}_1$ is large (i.e., $\tilde{c}_1 = 7$). Furthermore, $(p_1 - p_2)$ decreases to zero when $\tilde{c}_2$ increases to $p_1 = 8.47$ when $\tilde{c}_1 = 5$ and $(p_1 - p_2)$ decreases to zero when $\tilde{c}_2$ increases to $p_1 = 9.29$ when $\tilde{c}_1 = 7$; see Proposition 3.

We next study the impact on the value of expected profit increase due to the availability of backup supplier 2 in addition to backup supplier 1. Let π_2^* denote the optimal expected profit when both backup suppliers are available, and so the value of expected profit increase due to backup supplier 2 is given by $\Delta\pi_2 = \pi_2^* - \pi_1^*$. Figure 6 shows the values of $\Delta\pi_2$ for $\tilde{c}_2 \geq 4$ with two different values of $\tilde{c}_1$ ($\tilde{c}_1 = 5, 7$). Observe that $\Delta\pi_2$ decreases as $\tilde{c}_2$ increases, and the magnitude of this expected profit increase is relatively small as compared to that of $\Delta\pi_1$ as shown in Figure 2.

Figure 7 shows the impact of $\tilde{c}_1$ on the values of $\Delta\pi_2$ at two different values of $\tilde{c}_2$. In both graphs,

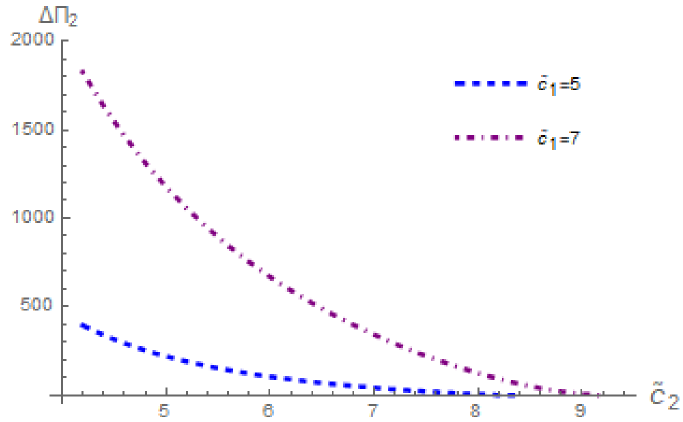

Figure 6: Profit Increase With Both Backup Suppliers.

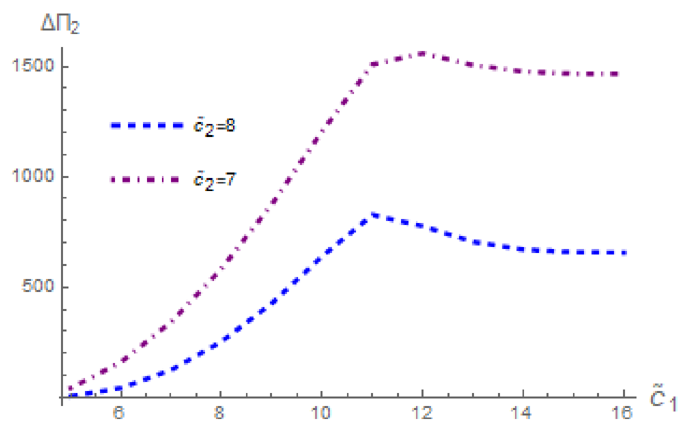

Figure 7: Impact of $\tilde{c}_1$ With Both Backup Suppliers.

we can observe that $\Delta\pi_2$ first increases as $\tilde{c}_1$ increases, but then decreases slightly as $\tilde{c}_1$ increases further above a certain level. This suggests that when the assembler needs to pay a higher unit component cost to backup supplier 1 to hedge against the unreliable primary component supplier, such a backup strategy becomes less effective. As a result, the availability of backup supplier 2 would provide substantial value to the assembler. This occurs since the assembler would likely opt to order less amounts of both components from the primary suppliers and wait for the realization of supply uncertainty of component 1 to make further adjustments. This is where the assembler obtains the value of backup supplier 2. However, when the unit cost of backup supplier 1 gets too high, the order amounts of both components from the primary suppliers would increase. As the order quantity of component 2 from the primary supplier gets closer to M_2, backup supplier 2 becomes less valuable to the assembler.

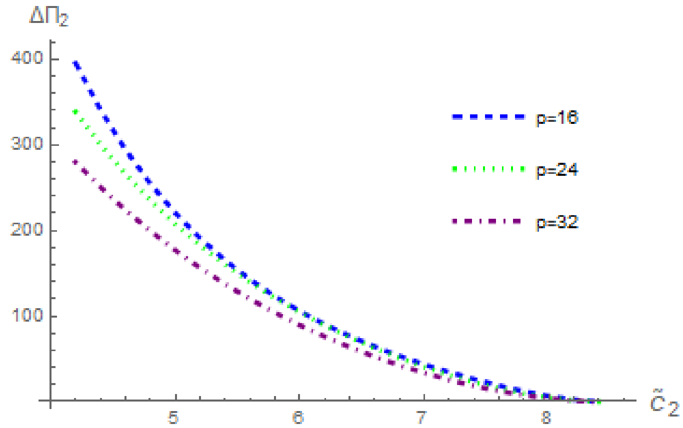

Figure 8: Impact of p With Both Backup Suppliers.

Figure 8 shows the values of $\Delta\pi_2$ for $\tilde{c}_2 \geq 4$ at three different values of p ($p = 16, 24, 32$). Observe that the corresponding values of $\Delta\pi_2$ are higher at a lower value of p, i.e, $p = 16$. This implies that the benefit of having backup supplier 2 is larger for products with smaller profit margins. In other words, the additional hedge against the unreliable primary supplier 1 with backup supplier 2 allows the assembler to better manage its component cost, which becomes increasingly significant due to the low product margin of the product.

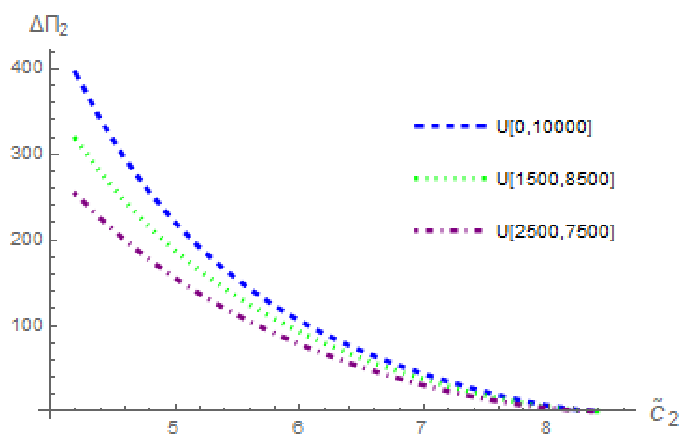

Figure 9: Impact of Demand Uncertainty With Both Backup Suppliers.

Figure 9 shows the values of $\Delta\pi_2$ for $\tilde{c}_2 \geq 4$ with three different demand distributions: $U[0, 10000]$, $U[1500, 8500]$ and $D = U[2500, 7500]$. Observe that the corresponding values of $\Delta\pi_2$ are higher for a higher degree of demand uncertainty, e.g., $U[2500, 7500]$ versus $U[0, 10000]$, which suggests that the benefit of having backup supplier 2 is larger for products with a higher demand uncertainty.

For our illustrative example with $\tilde{c}_1 = 5$ and $p = 16$, Figure 10 shows the values of $\Delta\pi_2$ for

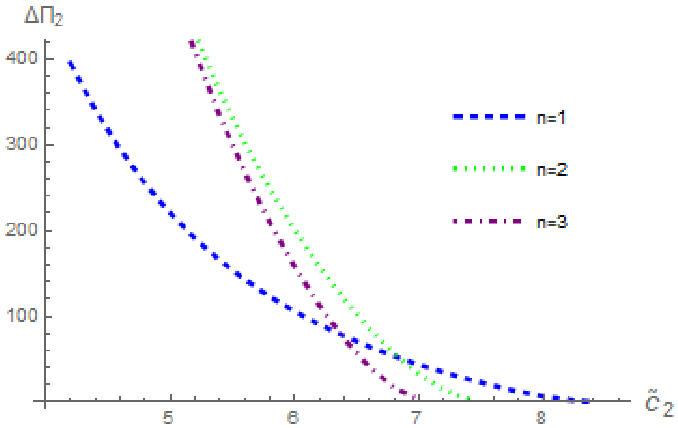

Figure 10: Impact of Supply Uncertainty With Both Backup Suppliers: $\tilde{c}_1 = 5$, $p = 16$.

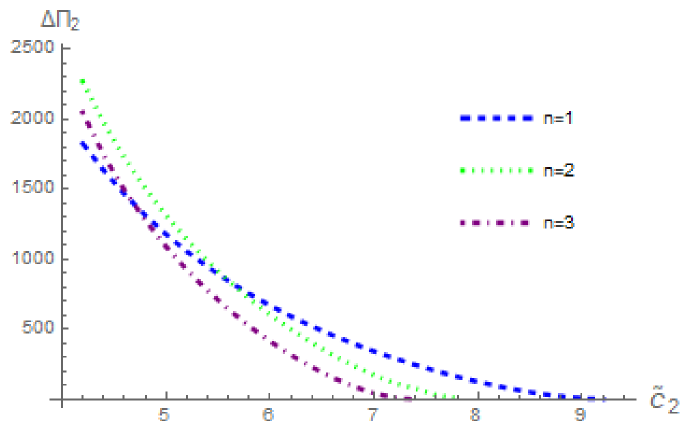

Figure 11: Impact of Supply Uncertainty With Both Backup Suppliers: $\tilde{c}_1 = 7$, $p = 16$.

$\tilde{c}_2 \geq 4$ when the yield factor ϵ follows a power distribution with cumulative distribution function of $G(t) = t^n$ where $t \in [0, 1]$ for $n = 1, 2, 3$. Figures 11 and 12 shows the corresponding results with a higher value of $\tilde{c}_1$ ($\tilde{c}_1 = 7$) and a higher value of p ($p = 32$), respectively. The results do not exhibit any specific patterns, which suggest that the impact of supply uncertainty on the additional expected profit increase due to the availability of backup supplier 2 would depend on the specific parameters of the model.

Overall, our results show that while the availability of backup supplier 1 provides the primary benefit of hedging against the unreliable primary supplier 1, the availability of backup supplier 2 can provide some additional benefits to the assembler with a lower threshold price and a higher expected profit. While this additional benefit is generally much smaller than the primary benefit due to the availability of backup supplier 1, it can be still significant, especially when extra unit

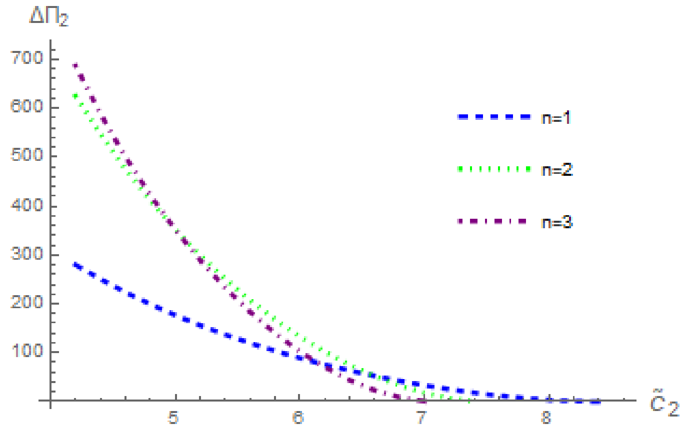

Figure 12: Impact of Supply Uncertainty With Both Backup Suppliers: $\tilde{c}_1 = 5$, $p = 32$.

component cost of backup supplier 1 ($\tilde{c}_1 - c_1$) is high for low-margin products under an operating environment of high demand uncertainty.

6. Case with backup Supplier 2 Only

To complete our analysis, we present some results for the case with backup supplier 2 only when the backup supplier 1 is not available (or alternatively, the unit cost of backup supplier 1 is too high, i.e., $\tilde{c}_1 \geq p$, such that the assembler would never utilize backup supplier 1). In particular, we provide the conditions under which the assembler would utilize backup supplier 2 for some additional benefits. We can obtain the following result by considering the limiting case in Section 5.2 with $\tilde{c}_1 = p$. Let (Q_1^g, Q_2^g) and (Q_1^h, Q_2^h) be the values of (Q_1^e, Q_2^e) and (Q_1^f, Q_2^f) defined in Section 5.2 when $\tilde{c}_1 = p$.

Proposition 5 *(i) When $\tilde{c}_2 < p_0$, the optimal order quantities are given as follows:*

$$\begin{cases} (Q_1^*, Q_2^*) = (\bar{Q}_1, \bar{Q}_2), \tilde{Q}_2^* = 0, & \text{if } p \geq c_\gamma, \\ (Q_1^*, Q_2^*) = (Q_1^g, Q_2^g), \tilde{Q}_2^* = (\min\{eQ_1^g, M_2\} - Q_2^g)^+, & \text{if } c_\beta < p < c_\gamma, \\ (Q_1^*, Q_2^*) = (Q_1^h, Q_2^h), \tilde{Q}_2^* = (eQ_1^h - Q_2^h)^+, & \text{if } c_\theta < p \leq c_\beta. \end{cases}$$

(ii) When $\tilde{c}_2 \geq p_0$, $\tilde{Q}_2^* = 0$.

Proposition 5 shows that when backup supplier 1 is not available, the assembler would utilize backup supplier 2 only when the unit product price is below c_γ and the unit component cost of backup supplier 2 is less than p_0. Otherwise, the availability of backup supply has no value to the assembler. Furthermore, Proposition 5 shows that with backup supplier 2 only, the assembler can reduce the threshold price from p_0 to c_θ when $\tilde{c}_2 < p_0$. We note from Lemma 3(i) that c_θ increases as $\tilde{c}_2$ increases and is always less than p_0 when $\tilde{c}_2 < p_0$.

Finally, we note that the expected profit increase due to the availability of backup supplier 2 only for our illustrative example discussed in the previous section can be deduced from the limiting case with $\tilde{c}_1 = p = 16$ given in Figure 7.

7. Conclusions

In this paper, we study the benefits of backup sourcing for an assembly system of a final product, in which one of the required components has unreliable supply from a primary supplier. We first study the use of a backup supplier for the component with unreliable supply as a hedging strategy against the unreliable primary supplier. We derive the optimal order quantities for all the components and conditions under which the assembler can reduce the threshold price of the product and increase his expected profit. We then extend our analysis to the situation when backup suppliers are available for all other components, and examine how the availability of either one or all backup suppliers can reduce the threshold price of the product and increase the expected profit of the assembler. To gain insights, we further conduct numerical experiments to quantify the magnitude of the associated benefits of these backup sourcing strategies.

Among our major findings, we demonstrate that it is most beneficial for the assembler to use backup supply to hedge against the component with unreliable primary supply when the unit cost of this component from the primary supplier is high relative to the extra unit component cost from backup supplier for high-margin products, especially under an operating environment of low demand uncertainty and high uncertainty from the unreliable primary supplier. Furthermore, the use of backup supply for the other (reliable) component can provide additional benefit to the assembler. While this additional benefit is much smaller than the primary benefit from the use of backup supply for the component with unreliable primary supply, it can still be significant when the extra unit cost for the component with unreliable supply is high for low-margin products with high demand uncertainty.

Leveraging the aforementioned results, our paper provides some important managerial insights regarding the role and adoption of backup supply of components in mitigating supply yield risk in an assembly system. In particular, we quantify the value of component backup sourcing through the perspectives of both profitability (as measured by expected profit) and business feasibility (as measured by threshold price), and further specify how such value is affected by the characteristics of the operating environment including the associated component costs as well as the yield and demand distributions. These insights could help practitioners in customizing the adoption of component backup sourcing based on their specific underlying system cost and risk features to achieve the largest benefit. One interesting result from our study is that it could be beneficial to keep a backup supplier option even for the component without supply risk.

We conclude by pointing out some limitations and potential directions for future research. First, we assume that the assembler is a price-taker facing exogenous market price for the final product. It would be interesting to extend our model to analyze component backup sourcing strategy for a price-setting assembler. Second, we focus on a centralized assembly system with exogenous component procurement costs. One can further consider a situation in which the assembler sources from profit-maximizing suppliers and study the impact of suppliers' wholesale price competition on the assembler's component sourcing decisions. Finally, it would also be interesting to adapt our model setting to explore other types of supply risk, such as random capacity and supply disruption.

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