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Supply Diversification under Random Yield: The Impact of Price Postponement

Lingxiu Dong*

Olin Business School, Washington University in St. Louis, St. Louis, Missouri, USA, dong@wustl.edu

Guang Xiao

PolyU Faculty of Business, The Hong Kong Polytechnic University, Kowloon, Hong Kong, China, guang.xiao@polyu.edu.hk

Nan Yang

Miami Herbert Business School, University of Miami, Coral Gables, Florida, USA, nyang@bus.miami.edu

Supply diversification and pricing are two common mechanisms for dealing with supply yield uncertainty. This paper characterizes a firm's pricing and sourcing decisions under supply yield uncertainty, and studies the interplay between the two mechanisms. We compare the optimal sourcing decisions under two distinct pricing schemes: (1) ex ante pricing - the firm simultaneously makes the sales price and sourcing decisions before production takes place; (2) responsive pricing - the pricing decision is postponed until after the production yield realization. Although the firm is better off with responsive pricing, the effect of price postponement on the optimal sourcing decision and the firm's need for supply diversification varies and is influenced by factors such as unit procurement cost, supply portfolio, and supply reliability. For the case of one unreliable supplier, we show that responsive pricing mitigates the overage and underage risks imposed by yield uncertainty, and results in a lower optimal order quantity than that under ex ante pricing when the procurement cost is low, and a higher quantity otherwise. For the case of two unreliable suppliers, we show that responsive pricing promotes supply diversification when the sole-sourced supplier's reliability is low, and discourages when it is high. By contrast, when the sole-sourced supplier's reliability is moderate, responsive pricing promotes supply diversification when its unit procurement cost is low and discourages otherwise. The composition of supply portfolio also has a fundamental impact on such strategic interplay: When the supply portfolio consists of one unreliable and one reliable supplier, diversified sourcing is never optimal under ex ante pricing, but may be optimal under responsive pricing. Thus, responsive pricing promotes dual sourcing in this setting. We also conduct comprehensive numerical experiments to both confirm the robustness of the main results and derive additional insights.

Key words: responsive pricing, ex ante pricing, supply diversification, random yield

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1. Introduction

Supply risk persists in today's global supply chains. In a McKinsey Global Survey (Muthukrishnan and Shulman 2006), nearly two thirds of the executives indicated that the risks of their supply

* Corresponding Author

chain have increased over the past five years, out of which supply uncertainty is listed as the 3rd significant risk by the survey participants. One major supply risk, common in many industries, is production yield risk, one of the primary threats that firms must consider when planning their production (Jones et al. 2003). For example, in the semiconductor industry, the production output of the complex production processes of wafer fabrication (consisting of 300 individual production steps) is highly uncertain and is often less than the initial batch size (Kempf 2004). In agribusiness, the per-acre crop yield depends on many uncontrollable factors, such as temperature, weather condition, irrigation levels, etc. In the vaccine manufacturing, the large-scale production of COVID-19 vaccine by AstraZeneca suffers from significant manufacturing challenges due to low and unpredictable production yield, especially in the initial stage, leading a failure to deliver tens of millions of promised doses of the COVID-19 vaccine to the European Union (Ring and Low 2022). Indeed, low initial production yields are characteristic not only of the COVID-19 vaccine but of other new vaccines as well (Angelus and Özer 2022).

Supply diversification has, among various operational strategies adopted to mitigate the supply yield risk, found its application in many industries. When facing production yield risk, firms usually spread their procurement orders among multiple suppliers to achieve risk pooling at reasonable cost. For example, Yousuf (2012) documents that besides procuring oranges from Florida, the largest citrus-growing area in the United States, PepsiCo Inc. also spreads its orange sourcing and juice production between California and Brazil to hedge against the potential yield loss due to clement weather. Similarly, as indicated by the 2013 Annual Report, Monsanto, the world largest seed producer, attempts to manage the weather-induced yield risk by producing seeds at multiple growing locations, such as North and South America, etc.

On demand side, pricing can serve as an effective revenue generating and risk mitigating tool. In response to the realized yield loss, a price-setting firm can postpone the pricing decision to maximize the revenue from the limited supply. For example, in 2010, after a deep freeze that hurt much of Florida's citrus crop, Tropicana raised orange juice price by as much as 8%. According to a news article in New York Times, "[Tropicana] spent a while examining the impact of the freeze and wanted to make [price] changes without affecting people's grocery bills too much." (The Associated Press 2010). In the 2015 U.S. bird-flu outbreak, the wholesale price of "liquid" eggs nearly tripled within a month of the start of the outbreak. The wholesale price of large shell eggs sold at grocery stores went up 85%, while turkey eggs went up 4.5% higher than the price a year ago (Gee 2015).

Supply diversification and price postponement are two powerful tools for mitigating supply yield risk, with the former operating on the supply side and the latter handling the demand side. It is not uncommon that companies deploy both tools to actively control the negative impact of yield uncertainty. Due to the difference in their nature, supply diversification and responsive pricing are often deployed by different functional units/departments within a firm, e.g., procurement unit responsible for supply diversification and marketing/sales unit in charge of responsive pricing. Thus, an important question faced by firms engaging in more than one risk-mitigating measure is how to

effectively integrate the deployment of multiple measures, whether the use of one measure offsets or enhances the value of using the other measure. The answer is not immediately clear. On the one hand, when the realized yield is low, responsive pricing can manage the supply underage by increasing the selling price, and the firm may find it more appealing to include a second source to benefit from the revenue enhancement effect of responsive pricing. On the other hand, when the realized yield is high, responsive pricing can reduce the supply overage by lowering the selling price, and the firm may find including a second source less desirable.

The objective of the paper is to understand the impact of a firm's pricing scheme on its sourcing strategy: How should a price-setting firm optimally diversify its supply base? How do pricing schemes affect the firm's need for supply diversification? More specifically, we study the sourcing and pricing decisions of a monopoly firm which procures from two potential suppliers to meet its deterministic, price-sensitive demand. The two suppliers can be external suppliers or internal production facilities of the firm. The suppliers' production processes are unreliable and are modeled by proportional random yields. Two pricing schemes, distinguished based on the timing of the pricing decision, are studied: (1) *ex ante* pricing scheme, under which the firm sets price and quantity simultaneously before production takes place; and (2) responsive pricing scheme, under which the firm postpones the pricing decision until after the yield realization. Within each pricing scheme, we characterize the firm's optimal sourcing and pricing decisions in three scenarios of supplier portfolio: (1) single unreliable supplier model; (2) two unreliable supplier model; and (3) one unreliable and one reliable supplier model. We then investigate the impact of responsive pricing on the firm's diversification strategy by comparing the results under the two pricing schemes.

Intuition suggests an ambiguous impact of responsive pricing on the optimal sourcing quantity: Responsive pricing increases the value of sourcing quantity when yield is low, and decreases otherwise. We show that whether responsive pricing increases or decreases the optimal sourcing quantity depends on the unit procurement cost. Whether responsive pricing promotes or discourages supply diversification depends not only on the unit procurement cost but also on the supply reliability. Among other results, we find that, with two unreliable suppliers, responsive pricing promotes supply diversification when the sole-sourced supplier's reliability is low, and discourages when the sole-sourced supplier's reliability is high. By contrast, when the sole-sourced supplier's reliability is at a moderate level, responsive pricing promotes supply diversification when its unit procurement cost is low, and discourages otherwise. In addition, the composition of supply portfolio also has a fundamental impact on the above strategic interplay. With a perfectly reliable and an unreliable supplier, diversified sourcing is never optimal under *ex ante* pricing, but may be optimal under responsive pricing. That is, responsive pricing always promotes supply diversification in this case. We further summarize our key results regarding the optimal sourcing decisions and impact of price postponement under different supplier portfolio in Table 1.

¹ "Sole or dual sourcing" in Table 1 means that the firm sources from both suppliers when their cost difference is small, and single sources from the cheaper supplier with lower effective cost otherwise.

Supply base	Sourcing under ex ante pricing	Sourcing under responsive pricing	Impact of price postponement
One unreliable supplier	Sole sourcing	Sole sourcing	Price postponement increases order quantity when the cost is high, and decreases otherwise.
One unreliable and one reliable supplier	Sole sourcing	Sole or dual sourcing ¹	Price postponement gives rise to the adoption of supply diversification.
Two unreliable suppliers	Sole or dual sourcing	Sole or dual sourcing	Price postponement promotes dual sourcing when the sole-sourced supplier is either less reliable or moderately reliable with a low cost.

Table 1 Summary of Key Results on Optimal Sourcing Decisions and Impact of Price Postponement

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 sets up the model. The optimal sourcing and pricing decisions for the ex ante and responsive pricing schemes are characterized in Sections 4 and 5, respectively. Section 6 investigates the impact of price postponement on the firm's sourcing and supply diversification decisions. Section 7 conducts extensive numerical study to confirm the robustness of our main results and derive additional insights. Section 8 concludes the paper. All proofs and additional results are relegated to Appendices A-C.

2. Literature Review

The literature of supply risk management has studied a variety of supply uncertainty types reflecting different nature of unreliable supply in different production/supply systems, including, among others, the stochastic proportional yield (Henig and Gerchak 1990), the random capacity (Ciarallo et al. 1994), and the binomial supply risk in the context of disruptions or quality risk (Chen et al. 2001, Lu et al. 2015, Lu and Wang 2021). In a seminal review of the random yield literature, Yano and Lee (1995) comment on key modeling issues and research questions. Naturally, dynamic production/inventory decisions in the presence of both supply uncertainty and demand uncertainty are among the first operational decisions investigated. Henig and Gerchak (1990) characterize the optimal sole sourcing inventory policy as a threshold policy under which the firm places an order when the inventory falls below the threshold, and both the order quantity and the threshold are functions of the yield distribution. Li et al. (2008), Huh and Nagarajan (2010) extend previous research by providing heuristics and bounds to effectively calculate the reorder point and reorder quantity. On the other hand, emergency production (e.g., Kouvelis and Li 2013) and backup supply (e.g., Pan et al. 2022) can also be adopted to mitigate potential supply yield shortage.

Besides inventory, supply diversification is another effective way to mitigate various forms of supply risk and is widely studied in the existing literature (see, e.g., Tang and Kouvelis 2011). For example, Li et al. (2013) and Feng et al. (2019) investigate a firm's multi-sourcing decision under supply random capacity in a single period and a multi-period setting, respectively. Hu and Kostamis (2015) analyze supply diversification under disruption risks. In the context of random yield, Anupindi and Akella (1993) study the optimal sourcing quantity decision when the firm

procures from two unreliable suppliers. They find that the optimal ordering policy follows a threshold structure which results in the less expensive supplier always being selected before the more expensive supplier regardless of the yield distributions. This simple and intuitive rule is referred to as “cost is an order qualifier” in the literature, and has been validated in various model settings where supply diversification is the focal risk mitigation strategy. Dada et al. (2007) show that the rule is valid in a single period model where a newsvendor sources from multiple suppliers with a general construct of supply uncertainty. Federgruen and Yang (2008, 2009, 2011, 2014) prove that the rule holds in single-period, multi-period, and infinite horizon settings with multiple unreliable suppliers, and provide efficient algorithms to compute the optimal policy. Swaminathan and Shanthikumar (1999) and Dong et al. (2022), on the other hand, find that when either random demand follows a discrete distribution or supply yields are correlated, it can be optimal that the firm sole sources from the more reliable but more expensive supplier.

On the other hand, for firms facing price dependent demand, pricing can serve as a powerful revenue generating/risk mitigating tool. Joint pricing and inventory decisions are first studied for systems subject to demand uncertainty only. For multi-period setting, Federgruen and Heching (1999) show that a base-stock/list-price policy is optimal. When incorporating supply uncertainty, Li and Zheng (2006) and Feng (2010) revisit this problem with stochastic proportional yield and stochastic capacity, respectively. For a comprehensive review along this line of research, see Yano and Gilbert (2003), Chan et al. (2004), and Chen and Simchi-Levi (2012) for details.

For single period setting, Whitin (1955) and Mills (1959) are among the first to study the inventory planning problem under price dependent demand. Petruzzzi and Dada (1999) review and extend the single period inventory control problem under price dependent demand. Kocabiyıkoğlu and Popescu (2011) offer a unifying perspective on the price-setting newsvendor problem by introducing a measure of elasticity of stochastic demand, and characterize the structural results under general assumptions of such measure. Instead of directly analyzing the price induced demand, one stream tackles the pricing problem via consumer choice model, and studies its impact on product design, positioning and/or assortment related operational decisions (McFadden 1974). For example, Dong et al. (2009) and Li and Huh (2011) investigate the pricing problem and the property of the associated profit function under multinomial logit model and nested logit model, respectively. This pricing model has been widely adopted to study assortment pricing problem involving with additional complexities, including network effect (Wang and Wang 2017), reference prices (Wang 2018), bounded rationality (Wang 2022), etc. For the above streams of literature, the pricing decision is made before demand uncertainties are resolved, and we differ by investigating the impact of price postponement in hedging against supply yield risk.

Another stream of research recognizes that pricing can play a more powerful role in mitigating the mismatch of supply and demand when price commitment is postponed until after all uncertainties are resolved. Van Mieghem and Dada (1999) are the first to study the impact of the pricing timing on the inventory decision when demand is uncertain. Chod and Rudi (2005) study the impact of

responsive pricing on the usage of flexible resources. In the context of uncertain yield, Tang and Yin (2007) and Kouvelis et al. (2021) study the impacts of responsive pricing for a risk neutral and a risk averse firm, respectively. Kazaz and Webster (2011) study the impact of yield dependent trading cost on the firm's pricing and quantity decisions under responsive pricing. Li et al. (2017) study the supply diversification problem under different pricing schemes under random capacity model. Our work differs from Li et al. (2017) as we model supply uncertainty in the form of proportional random yield, which induces production inflation, brings additional tensions and tradeoffs into the model, and leads to different implications for the optimal sourcing and diversification strategy.

Besides supply diversification and pricing, other supply risk mitigating strategies have been studied. See Tang (2006) for a comprehensive review of these strategies. Understanding the strategic interplay between different strategies serves as a first step for decision makers to choose the best set of strategies. Our paper and the following papers share a common theme of exploring the strategic interplay of different risk mitigating tools. Dong and Tomlin (2012) and Dong et al. (2018) investigate the interplay between operational hedging and insurance in managing disruption risk in single firm and production chain settings, respectively. Rui and Lai (2015) study how deferred payment and inspection interact in hedging against supplier product adulteration risk. Geng et al. (2022) investigate the interplay between payment and price postponement in hedging against yield risk in a decentralized supply chain.

To sum up, our work contributes to the random yield literature by studying the interplay of a firm's pricing scheme and its supply diversification decision. Supply diversification is one of the common proactive risk mitigating tools which aims at improving supply reliability via risk pooling, whereas pricing is the main customer-facing tool that influences the demand level to reduce its mismatch with supply. The timing of the pricing decision determines whether it serves as a proactive tool (ex ante pricing) or a recourse tool (responsive pricing). Focusing on the case of stochastic proportional yield, we find that the pricing scheme has a significant impact on the value of inventory to the firm. Hence, whether responsive pricing increases or decreases a firm's need for supply diversification depends on a number of factors, among others, unit procurement costs, the composition of supplier portfolio, and supply reliability.

3. Model Setup

We study the sourcing and pricing problem of a firm, which procures from two potential suppliers to meet its price sensitive demand. The two suppliers can be external suppliers or internal production facilities of the firm. The supply processes are unreliable in the sense that, for each supplier $i \in \{1, 2\}$, the delivered quantity is a random fraction ξ_i of the order quantity. We assume that the yield factors are continuous random variables with support on the interval $[0, 1]$. For supplier $i \in \{1, 2\}$, let $g_i(\xi_i)$, $G_i(\xi_i)$, and $\bar{G}_i(\xi_i)$ denote the probability density function (p.d.f.), the cumulative distribution function (c.d.f.), and the complementary cumulative distribution function (c.c.d.f.) of its yield distribution. Let μ_i and σ_i be the mean and standard deviation of the yield distribution of supplier i . In the following analysis, we assume the yield processes are independent. This assumption

is commonly adopted in the literature, see, e.g., Anupindi and Akella (1993), Federgruen and Yang (2009, 2011), and applies to the scenarios where the two suppliers have little interaction. For example, the suppliers are geographically apart and/or don't have common raw material supply. We relax this assumption in Section 7.3 to discuss the impact of yield correlation.

Throughout the analysis, we assume that the firm pays for the entire order quantity regardless of the yield realization at its suppliers. More specifically, the unit procurement cost from supplier i is c_i , and the effective unit procurement cost is c_i/μ_i . This payment scheme is commonly adopted in industries where supply yield is affected by uncontrollable factors, e.g., vaccine or agriculture industries. This payment scheme is also commonly used in the literature, see Federgruen and Yang (2009) and Tang and Kouvelis (2011) for detailed discussions. In the case where the two suppliers are internal production facilities of the firm, the unit procurement cost c_i refers to the unit production cost at facility i , and is incurred for every unit of input production quantity. Without loss of generality, we assume there is no salvage value or goodwill cost, as these costs can be easily incorporated without changing the main results qualitatively.

Our focal firm is a monopoly in the retail market and faces price sensitive demand $d(p)$. Let $p_0 < +\infty$ be the maximal price that induces zero demand, i.e., $d(p_0) = 0$. Moreover, we assume that $d(p)$ strictly decreases in p and it has inverse demand function $p(d) = d^{-1}(p)$. To focus on the supply risk, we assume the demand curve is deterministic, which is commonly adopted in the literature on pricing under supply uncertainty (e.g., Fang and Shou 2015, Kouvelis et al. 2018). We relax this assumption in Section 7.2 to investigate the effect of demand uncertainty.

We study and compare the firm's sourcing decisions under two pricing schemes: (1) the ex ante pricing scheme, under which the firm makes the pricing and sourcing decisions simultaneously before yield uncertainties realize; and (2) the responsive pricing scheme, under which the firm postpones the pricing decision after yield uncertainties are resolved. For each pricing scheme, we characterize the firm's optimal sourcing and pricing decisions for any given pair of unit procurement costs (c_1, c_2) . We now introduce the problem formulation under each pricing scheme.

Ex ante pricing scheme. The firm faces a one-stage problem of setting price and order quantities simultaneously before yields realize. Let p, q_1, q_2 be the market price and the order quantity for each supplier, respectively. The firm's objective function Π_a is given by the following equation, where subscript a denotes the ex ante pricing scheme:

$$\Pi_a(p, q_1, q_2) = pE_{\xi_1, \xi_2}[\min\{d(p), q_1\xi_1 + q_2\xi_2\}] - c_1q_1 - c_2q_2. \quad (1)$$

Responsive pricing scheme. The firm engages in a two-stage decision making process. At the first stage, the firm decides the order quantity from each supplier. At the second stage, after yield realization, the firm sets market price to maximize its revenue. The firm's objective function Π_r is given by the following equation, where subscript r denotes the responsive pricing scheme:

$$\begin{aligned} \Pi_r(q_1, q_2) &= E_{\xi_1, \xi_2}[\pi_r(q_1\xi_1 + q_2\xi_2)] - c_1q_1 - c_2q_2, \\ \text{where } \pi_r(q_1\xi_1 + q_2\xi_2) &= \max_{p \in [0, p_0]} p \min\{d(p), q_1\xi_1 + q_2\xi_2\}. \end{aligned} \quad (2)$$

The comparison of the objectives in Equations (1) and (2) clearly shows that responsive pricing improves the firm's expected profit due to the flexibility of price postponement. However, in reality, price postponement is usually not achieved without a cost due to various reasons, such as goodwill/reputation loss, delayed announcement, and costly price adjustment. To capture such a cost, we assume that there is a fixed cost $K \geq 0$ when the firm chooses or switches to the responsive pricing scheme. As such, the decision of adopting the postponed pricing strategy critically depends on whether the gain from pricing flexibility outweighs the associated fixed cost. Depending on various market environment and yield conditions, it is not uncommon that different firms may adopt distinct pricing strategies to better match supply with demand. Nevertheless, how to effectively coordinate the sourcing and supplier selection decisions with the chosen pricing schemes, and how such strategic decisions get affected by the supplier portfolios remain unclear, which serve as the major focus of the subsequent analysis. To this end, we proceed to characterize and compare the optimal sourcing decisions between the ex ante pricing and the responsive pricing schemes for three scenarios of supplier portfolio: (1) single unreliable supplier model (U); (2) two unreliable supplier model (UU); and (3) one unreliable and one reliable supplier model (UR). For expositional brevity, we use the notation in Table 2 to represent the six supplier-portfolio-pricing-scheme combinations.

Pricing Scheme	One unreliable supplier	Two unreliable suppliers	One unreliable and one reliable supplier
Ex-ante pricing	U-a	UU-a	UR-a
Responsive pricing	U-r	UU-r	UR-r

Table 2 Notation for Supplier Portfolio and Pricing Scheme Combinations

4. Ex-ante Pricing Scheme

In this section, we characterize the firm's optimal pricing and sourcing decisions under the ex ante pricing scheme, which serve as a benchmark to be compared with that under responsive pricing.

4.1. Single Unreliable Supplier Model (U-a)

We first consider the case of a single unreliable supplier (U-a model) to develop methodology and insights for the two-supplier models. We drop the supplier index i , and let c be the unit procurement cost from the unreliable supplier. If $c > p_0\mu$, it is optimal for the firm not to order from the supplier. Hence, the feasible range of the unit procurement cost is $\mathbb{C}_0 = \{c | c \in [0, p_0\mu]\}$. For any given procurement cost $c \in \mathbb{C}_0$, the feasible range of the retail price p is the interval $[c/\mu, p_0]$, because the firm would earn a negative expected profit if $p < c/\mu$, and would have no demand if $p > p_0$. Let q_a and p be the firm's order quantity and retail price, respectively. In general, the objective function $\Pi_a(p, q_a)$ is not jointly concave in (p, q_a) . However, $\Pi_a(p, q_a)$ is concave in p for any given q_a , and concave in q_a for any given p . We adopt a sequential optimization approach that first solves for $q_a^*(p)$ for a given p and then solves for the optimal p_a^* . First, the optimal order quantity as a function of p is given by the following lemma.

LEMMA 1. For any sales price $p \in [c/\mu, p_0]$, the optimal order quantity is $q_a^*(p) = \frac{d(p)}{x(p)}$, where $x(p)$ is implicitly defined by:

$$\int_0^x \xi g(\xi) d\xi = \frac{c}{p}. \quad (3)$$

With the price set, the market demand for the product is determined. To cope with the supply risk, the firm will order more than the demand. Lemma 1 shows that the optimal order quantity $q_a^*(p)$ is a linear inflation of the price-induced demand $d(p)$. The optimal inflation rate $1/x(p)$ reflects the tradeoff between *overage* and *underage risks* due to uncertain supply. When a high yield realization leaves the firm with unsold inventory, the firm suffers from an overage risk and incurs the overage cost, c , for every unit of unsold inventory. When a low yield realization leaves the firm with insufficient inventory to meet the price-induced demand, the firm suffers from an underage risk and incurs the underage cost, $p - c$, for every unit of unsatisfied demand. The optimal inflation rate $1/x(p)$ is dependent on the cost-to-price ratio and the distribution of the random yield factor, but is independent of the price-induced demand.

To solve for the optimal price, we plug $q_a^*(p)$ into the firm's ex ante expected profit function and obtain $\Pi_a(p, q_a^*(p)) = pd(p)(1 - G(x(p)))$. In general, the expected profit function is not concave in p (see Pan and So 2010). To ensure its quasi-concavity, we impose the following set of assumptions.

ASSUMPTION 1. (a) $\frac{x\bar{G}(x)}{\int_0^x \xi g(\xi) d\xi}$ decreases in $x \in [0, 1]$.
(b) The demand function $d(p)$ satisfies the increasing price elasticity (IPE) property, i.e., the demand elasticity function $\eta(p) = -\frac{pd'(p)}{d(p)}$ increases in p .

Assumption 1(a) is quite general and is satisfied by all increasing generalized failure rate (IGFR) distributions and their truncations within $[0, 1]$ (see Kouvelis et al. 2018, for detailed discussions). Assumption 1(b) requires the IPE property, which is heavily adopted in the literature and is satisfied by many demand functions (e.g., linear, iso-elastic, concave, and log-concave demand functions, see Lemma 12 in Appendix B.4). With Assumption 1, the following proposition establishes the unimodality of $\Pi_a(p, q_a^*(p))$ and the uniqueness of the optimal price p^* :

PROPOSITION 1. For the U-a setting with $c \in \mathbb{C}_0$, under Assumption 1, $\Pi_a(p, q_a^*(p))$ is quasi-concave in p , and the optimal price p^* is uniquely defined by the solution of the first order condition. Moreover, $p^* \geq p_d$, where $p_d = \arg \max_{p \in [0, p_0]} (p - c)d(p)$ is the optimal price under reliable supply.

Proposition 1 characterizes the unique optimal price under the ex ante pricing scheme. Compared to the case of reliable supply, supply uncertainty leads to a higher retail price and a lower demand.

4.2. Two Unreliable Supplier Model (UU-a)

For the UU-a model, we focus on the procurement cost pair (c_1, c_2) in set $\mathbb{C}_1 = \{(c_1, c_2) | [0, p_0\mu_1] \times [0, p_0\mu_2]\}$ to exclude the uninteresting case that the firm does not order from either supplier. Similarly, we adopt the sequential optimization approach. We first characterize the optimal sourcing decision for any given price $p \in [0, p_0]$ and cost pair $(c_1, c_2) \in [0, p\mu_1] \times [0, p\mu_2]$ below.

LEMMA 2. For the UU-a setting, given any retail price $p \in [0, p_0]$ and procurement cost pair $(c_1, c_2) \in [0, p\mu_1] \times [0, p\mu_2]$:

(i) The objective function $\Pi_a(q_1, q_2|p)$ is twice continuously differentiable and jointly concave in (q_1, q_2) . The characterization of $(q_1^*(p), q_2^*(p))$ is relegated to Appendix B.1.

(ii) There exist two continuously increasing cost threshold functions $\tilde{A}^1(c_1|p)$ and $\tilde{A}^2(c_2|p)$ with boundary values $\tilde{A}^i(0|p) = 0$ and $\tilde{A}^i(p\mu_i|p) = p\mu_{3-i}$, and $\tilde{A}^i(c_i|p) \geq \frac{c_i\mu_{3-i}}{\mu_i}$, $i \in \{1, 2\}$. If $c_2 \in [\tilde{A}^1(c_1|p), p\mu_2]$, then the firm sole sources from supplier 1; if $c_1 \in [\tilde{A}^2(c_2|p), p\mu_1]$, then the firm sole sources from supplier 2; otherwise, it dual sources.

When the supply base consists of two unreliable suppliers, sourcing from both suppliers can change the combined yield distribution of the output quantity. Hence, by carefully choosing the order quantities from both suppliers, the firm can reduce the variability of the output quantity and achieve risk pooling, at an effective unit procurement cost higher than that of the cheaper supplier. Therefore, for any given price p , dual sourcing is optimal when the procurement costs of the two suppliers are relatively close, as shown in part (ii) of Lemma 2.

Now, we analyze the firm's pricing decision that maximizes its ex ante profit, $\Pi_a(p, q_1^*(p), q_2^*(p))$. As discussed above, for any given $(c_1, c_2) \in \mathbb{C}_2$, as the retail price p increases from 0 to p_0 , the optimal sourcing policy varies from no sourcing to sole sourcing from the cheaper supplier, and possibly, to dual sourcing. Consequently, the objective function $\Pi_a(p, q_1^*(p), q_2^*(p))$ is a piecewise function of p and assumes different functional forms in the no-sourcing, sole-sourcing, and dual-sourcing intervals. Given the complexity, it is hard to establish that the piecewise objective function is unimodal in general. It can be shown, however, that $\Pi_a(p, q_1^*(p), q_2^*(p))$ is a continuous function of p , and thus, by the extreme value theorem, there exists a p^* that maximizes the profit function. Based on this argument, we can show that there exist continuously increasing threshold functions for c_1 and c_2 , respectively, such that the firm dual sources if both c_1 and c_2 fall below their corresponding thresholds. A depiction is given in Figure 1(a).

PROPOSITION 2. Assume Assumption 1 holds. For the UU-a setting with $(c_1, c_2) \in \mathbb{C}_1$, there exist two continuously increasing cost threshold functions, $\tilde{A}^1(c_1)$ and $\tilde{A}^2(c_2)$, with boundary values $\tilde{A}^i(0) = 0$ and $\tilde{A}^i(p_0\mu_i) = p_0\mu_{3-i}$, and $\tilde{A}^i(c_i) \geq \frac{c_i\mu_{3-i}}{\mu_i}$, $i \in \{1, 2\}$, such that the firm dual sources if $c_1 < \tilde{A}^2(c_2)$ and $c_2 < \tilde{A}^1(c_1)$. Moreover, it is never optimal for the firm to sole source from the supplier with higher effective unit procurement cost.

Proposition 2 provides a sufficient condition on the procurement cost pairs under which the firm dual sources. The cost threshold is defined as: $\tilde{A}^i(c_i) = \frac{\partial E[\pi_a(p, q_1\xi_1 + q_2\xi_2)]}{\partial q_j} \big|_{p=p_a^*, q_i=q_a^*, q_j=0, i \in \{1, 2\}, j = 3-i}$, where $\pi_a(p, q_1\xi_1 + q_2\xi_2) = p \min\{d(p), q_1\xi_1 + q_2\xi_2\}$, and q_a^* and p_a^* are defined in Proposition 1. The cost threshold has a clear, insightful interpretation: It is the marginal revenue of procuring from supplier j given that the firm currently sole sources from supplier i at the optimal quantity q_a^* and optimal retail price p_a^* , and measures the marginal risk pooling benefit introduced by a second supplier. If the marginal procurement cost of supplier j is below the marginal revenue $\tilde{A}^i(c_i)$, then the

firm should include supplier j into its active supply base. It is worth noting that, if $\Pi_a(p, q_1^*(p), q_2^*(p))$ is unimodal in p , then $c_1 < \tilde{A}^2(c_2)$ and $c_2 < \tilde{A}^1(c_1)$ is both the necessary and the sufficient condition for dual sourcing. Our extensive numerical study shows that for common yield distributions such as Uniform, Normal, and Beta, $c_j = \tilde{A}(c_i)$ is indeed the curve that separates the sole sourcing region and the dual sourcing region in the (c_1, c_2) space. In addition, Proposition 2 also extends the insight that “cost is an order qualifier” to the case of a price-setting firm with deterministic demand curve and two unreliable suppliers under continuous yields.

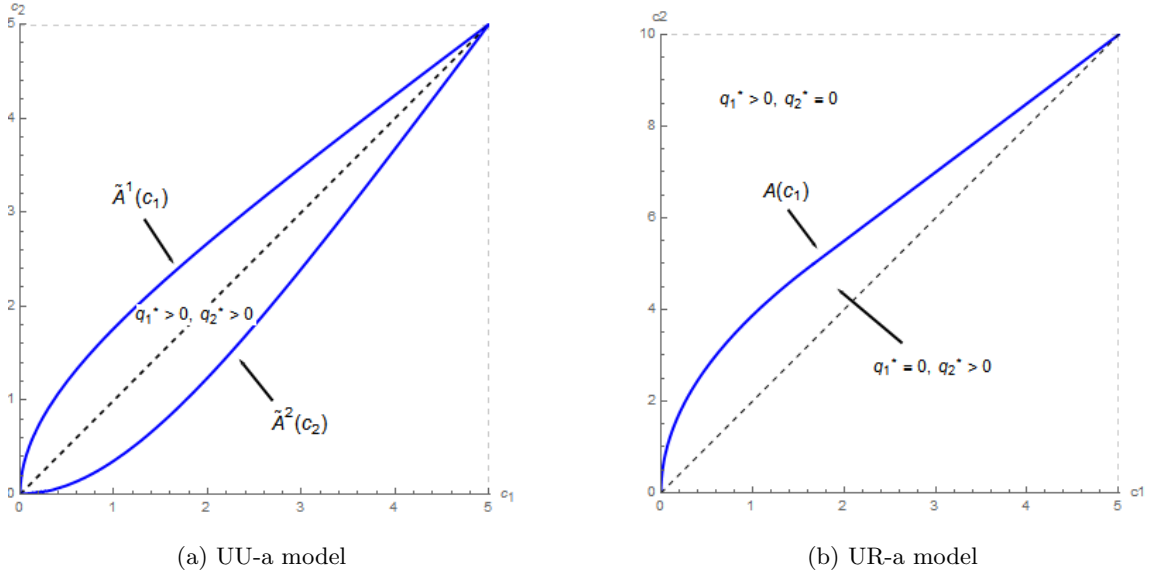


Figure 1 Optimal Sourcing Decision under Linear Demand Function and Ex ante Pricing Scheme in UU-a Model (Left) and UR-a Model (Right), $d(p) = 10 - p$, $\xi_1 \sim \text{Uniform}[0, 1]$

4.3. One Unreliable and One Reliable Supplier Model (UR-a)

In this section, we focus on the UR-a model by showing that it is not a limiting case of the UU-a model and revealing the significant impact of having a reliable supplier in the supply base on the ex-ante pricing firm’s optimal sourcing decision.

We assume that supplier 1 is unreliable and supplier 2 is reliable, and focus on cost pairs (c_1, c_2) in set $\mathbb{C}_2 = \{(c_1, c_2) | (c_1, c_2) \in [0, p_0\mu_1] \times [c_1/\mu_1, p_0]\}$. The condition $c_2 \geq c_1/\mu_1$ is imposed to exclude the uninteresting case that the reliable supplier is cheaper than the unreliable supplier, and the firm sole sources from the reliable supplier. Similarly, we adopt the sequential approach that first solves for the optimal q_1^* and q_2^* for a given p , and then solves for the optimal p^* . The following proposition characterizes the firm’s optimal sourcing decision with a depiction in Figure 1(b).

PROPOSITION 3. Assume Assumption 1 holds. For the UR-a setting with $(c_1, c_2) \in \mathbb{C}_2$, there exists a continuously increasing cost threshold $A(c_1) (\geq c_1/\mu_1)$ with boundary values $A(0) = 0$ and $A(p_0\mu_1) = p_0$. If $c_2 < A(c_1)$, the firm sole sources from the reliable supplier with optimal price and quantity: $p^* = \arg \max_{p \in [0, p_0]} (p - c_2)d(p)$ and $q^* = d(p^*)$. Otherwise, the firm sole sources from the unreliable

supplier with optimal price and quantity given by Proposition 1. At $c_2 = A(c_1)$, the firm is indifferent between sole sourcing from either of the two suppliers.

Proposition 3 states that sole sourcing is optimal when the supply base consists of one unreliable supplier and one reliable supplier. This is in sharp contrast to the optimal sourcing strategy under the UU-a model, where dual-sourcing is optimal when the two suppliers' cost difference is small.² To understand why under ex ante pricing dual sourcing does not add value in the presence of a reliable supplier, let us consider the sourcing quantity decision at any given retail price p . The induced demand at this price is $d(p)$. If the firm pursues dual sourcing and sources q_2 from the reliable supplier, then the effective demand for the unreliable supplier is $d(p) - q_2$. By Lemma 1, the optimal sourcing quantity from the unreliable supplier is a linear inflation of $d(p) - q_2$, i.e., $(d(p) - q_2)/x(p)$, where $x(p)$ is independent of q_2 . It can be established that, for any given retail price p , the marginal value of sourcing from the reliable supplier, $\frac{\partial \Pi_a(p, \frac{d(p)-q_2}{x(p)}, q_2)}{\partial q_2}$, is constant in q_2 : A positive value leads to sole sourcing $d(p)$ from the reliable supplier, and a negative value results in sole sourcing $d(p)/x(p)$ from the unreliable one. Consequently, sole sourcing is optimal at the optimal price p_a^* . At $c_2 = A(c_1)$, sole sourcing from the unreliable supplier and sole sourcing from the reliable supplier offer the same expected profit. Intuitively, price commitment leads to demand commitment, and sourcing part of the demand from a reliable supplier would simply decrease the demand for the unreliable supplier, but the firm still copes with the same underlying uncertain yield as it does under sole sourcing from the unreliable supplier. Dual sourcing does not affect the underlying supply risk associated with the ex ante profit of the firm, and, thus, does not deliver the benefit of (supply) risk pooling.

Proposition 3 represents a departure from the well-known result of “cost is an order qualifier,” i.e., it is never optimal to skip a cheap supplier and source from an expensive supplier. This rule has been shown to hold for the case of stochastic demand with continuous distribution (see, e.g., Dada et al. 2007, Federgruen and Yang 2008). Swaminathan and Shanthikumar (1999) and Dong et al. (2022) have found that the rule may not hold when either demand has discrete distribution or the supply yields are correlated, respectively. We show that it also does not hold when demand function is deterministic and the price is set ex ante, and provide the underlying intuition.

5. Responsive Pricing Scheme

In this section, we characterize the firm's optimal pricing and sourcing decisions under the responsive pricing scheme. Our analysis not only extends the supply diversification theory to a responsive pricing environment under different supply portfolios, but also provides a benchmark to be compared with that under ex-ante pricing.

² Mathematically, the UR-a model cannot be treated as a limiting case of the UU-a model with the yield of the expensive supplier approaching 1.

5.1. Single Unreliable Supplier Model (U-r)

Similar to the U-a model, we assume $c \in \mathbb{C}_0$ to avoid the trivial case of no production. Recall that $p(d) := d^{-1}(p)$ defined in Section 3 is the inverse demand function. To facilitate the analysis, we make the following assumption on the revenue function $p(d)d$:

ASSUMPTION 2. *The revenue function $p(d)d$ is concave and twice differentiable in d .*

Assumption 2 is a standard assumption in joint pricing and inventory management literature and is satisfied by many commonly used demand functions (see, e.g., Federgruen and Heching 1999). The concavity of $p(d)d$ in d suggests a decreasing marginal revenue with respect to demand.

As is standard, we solve the two-stage problem defined in (2) by backward induction. Let d^* be the revenue maximizing quantity, which is the unique solution of the first order condition $p'(d^*)d^* + p(d^*) = 0$. At the second stage, under Assumption 2, for any on-hand inventory $q_r\xi$, the firm's optimal price is $p_r^* = p(\min\{q_r\xi, d^*\})$. When inventory is abundant (i.e., greater than d^*), the firm sets the price at $p(d^*)$ to sell just d^* to achieve the unconstrained revenue maximization albeit left with surplus inventory; when inventory is scarce (i.e., less than d^*), the firm sets the inventory clearing price. Thus, the expected profit function $\Pi_r(q_r) = E\pi_r^*(q_r\xi) - cq_r$ has different expressions over the intervals $[0, d^*)$ and $[d^*, \infty)$. The following proposition shows that the objective function is well behaved and that the firm orders more than the revenue maximizing quantity d^* if and only if its unit procurement cost is less than a threshold, c_r .

PROPOSITION 4. *Assume Assumption 2 holds. For the U-r setting with $c \in \mathbb{C}_0$, the objective function $\Pi_r(q_r)$ is continuously differentiable and concave in the order quantity q_r . Let*

$$c_r \triangleq E[(p'(d^*\xi)(d^*\xi) + p(d^*\xi))\xi]. \quad (4)$$

The optimal order quantity is given by:

$$q_r^* \text{ solves } \begin{cases} E[(p'(q_r^*\xi)(q_r^*\xi) + p(q_r^*\xi))\xi] = c & c \geq c_r \\ \int_0^{q_r^*} (p'(q_r^*\xi)(q_r^*\xi) + p(q_r^*\xi))\xi g(\xi) d\xi = c & \text{otherwise.} \end{cases}$$

5.2. Two-Supplier Models (UU-r and UR-r)

In this section, we derive the unified sourcing policy for both the UU-r and UR-r models. Analogous to the ex-ante pricing case, the firm's objective function, $\Pi_r(q_1, q_2)$, has different expressions in the (q_1, q_2) space, $[0, \infty) \times [0, \infty)$; each region needs to be analyzed individually. We relegate the detailed description of the splitted regions and the corresponding analysis of $\Pi_r(q_1, q_2)$ over each region under UU-r and UR-r models to Appendices B.2 and B.3, respectively. It can be shown that, for both models, the objective function is jointly concave in (q_1, q_2) over the feasible space of (q_1, q_2) . Thus, we can use the K.K.T. condition to analyze the constrained optimization in each region, and combine them to characterize the optimal sourcing decisions in the following proposition.

PROPOSITION 5. *Assume Assumption 2 holds. For both the UU-r model with $(c_1, c_2) \in \mathbb{C}_1$ and the UR-r model with $(c_1, c_2) \in \mathbb{C}_2$:*

(i) The firm's objective function, $\Pi_r(q_1, q_2)$, is twice continuously differentiable, jointly concave and submodular in (q_1, q_2) . The characterization of (q_1^*, q_2^*) is relegated to Appendix B.2 for the UU-r model and Appendix B.3 for the UR-r model.

(ii) For the UU-r model, there exist two continuously increasing cost threshold functions, $\tilde{R}^1(c_1)$ and $\tilde{R}^2(c_2)$, with boundary values $\tilde{R}^i(0) = 0$ and $\tilde{R}^i(p_0\mu_i) = p_0\mu_{3-i}$, and $\tilde{R}^i(c_i) > \frac{c_i\mu_{3-i}}{\mu_i}$, $i \in \{1, 2\}$. If $c_2 \in [\tilde{R}^1(c_1), p_0\mu_2]$, the firm sole sources from supplier 1; if $c_1 \in [\tilde{R}^2(c_2), p_0\mu_1]$, the firm sole sources from supplier 2; otherwise, it dual sources.

(iii) For the UR-r model, there exists a continuously increasing cost threshold $R(c_1) \geq c_1/\mu_1$ with boundary values $R(0) = 0$ and $R(p_0\mu_1) = p_0$. If $c_2 \in [R(c_1), p_0\mu_2]$, the firm sole sources from the unreliable supplier (supplier 1); otherwise, it dual sources.

Proposition 5 identifies a unified sourcing structure under both the UU-r and UR-r models: Dual sourcing is optimal when the cost difference between the two suppliers are small. Part (ii) characterizes the threshold functions $\tilde{R}^i(c_i)$ in the UU-r model, which have similar interpretation as $\tilde{A}^i(c_i)$ in Proposition 2. Specifically, $\tilde{R}^i(c_i) = \frac{\partial E[\pi_r(q_1\xi_1 + q_2\xi_2)]}{\partial q_j} \big|_{q_i=q_r^*, q_j=0}$, where q_r^* is defined in Proposition 4. It measures the marginal risk pooling benefit from a second supplier, represented by the marginal revenue of procuring from supplier j given that the firm currently sole sources q_r^* from supplier i . If this marginal revenue is greater than supplier j 's marginal cost c_j , then it is beneficial for the firm to include supplier j into the active supply base; otherwise, the firm should maintain sole sourcing from supplier i , $i \in \{1, 2\}$ and $j = 3 - i$. The facts that $\tilde{R}^1(c_1) > \frac{c_1\mu_2}{\mu_1}$ and $\tilde{R}^2(c_2) > \frac{c_2\mu_1}{\mu_2}$ imply that “cost is an order qualifier” holds under UU-r model.

Part (iii) characterizes the threshold function $R(c_1)$ in the UR-r model as the marginal revenue of procuring from the reliable supplier, given that the firm currently sources from the unreliable supplier. The firm will dual source if the marginal revenue overweighs the marginal cost of the reliable supplier. Moreover, the fact $R(c_1) \geq c_1/\mu_1$ means the firm sole sources from the unreliable supplier when the reliable supplier is much more expensive, confirming that “cost is an order qualifier” holds under the responsive pricing scheme. A comparison between Propositions 3 and 5(iii) suggests that the UR-supply portfolio does not necessarily invalidate the “cost is an order qualifier” result, and it is the pricing timing that makes the differences. See Section 6.3 for detailed discussions. Figure 2 depicts the optimal sourcing structure for both the UU-r and UR-r models under Uniform yield distributions and a linear demand function.

6. Impact of Price Postponement

In this section, we investigate the impact of price postponement on the firm's optimal sourcing and supply diversification decision. We first analyze how price postponement affects the firm's optimal sourcing quantity in the single unreliable supplier setting, which reveals the fundamental impacts of price postponement. Then we investigate how supply diversification and price postponement interact with each other in mitigating supply yield risks. We also distill the unique insight regarding how the supply portfolio with a fully reliable supplier alters such strategic interplay.

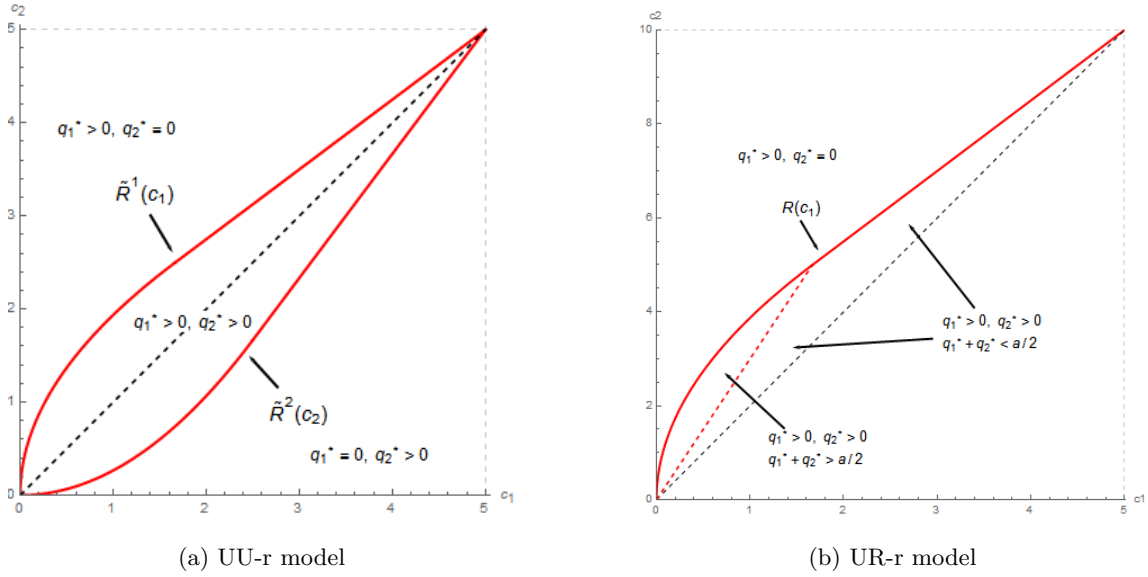


Figure 2 Optimal Sourcing Decision under Linear Demand Function and Responsive Pricing Scheme in UU-r Model (Left) and UR-r Model (Right), $d(p) = 10 - p$, $\xi_1 \sim \text{Uniform}[0, 1]$

6.1. Impact on the Optimal Sourcing Strategy: U Models

The firm is always better off with responsive pricing, as postponing the pricing decision till after the yield realization offers the firm operational flexibility to adjust demand to match supply. How such flexibility affects the firm's optimal sourcing decisions will be revealed by comparing the optimal order quantities under the two pricing schemes in the U models. To facilitate the analysis, we assume that both Assumption 1 and Assumption 2 hold whenever we compare the responsive pricing scheme with the ex-ante pricing scheme. Assumption 1(b) and Assumption 2 are for the demand function, and are satisfied by many commonly used demand functions, e.g., linear, iso-elastic, linear-power, concave demand functions. See Lemma 12 in Appendix B.4 for detailed discussions.

When abundant supply is available due to a high yield realization, the firm sets the unconstrained revenue maximizing price to stimulate demand and decrease unsold inventory. In this case, responsive pricing mitigates the overage risk from the high yield realization, and reduces the ex post marginal revenue of inventory. When inventory is insufficient due to a low yield realization, the firm sets the inventory clearing price to increase the revenue from the scarce supply. In this case, responsive pricing mitigates the underage risk from the low yield realization, and increases the ex post marginal revenue of inventory. The two effects of responsive pricing, *overage risk mitigation effect* and *underage risk mitigation effect*, drive the marginal revenue of inventory in opposite directions. The dominating effect of the two determines whether responsive pricing decreases or increases the expected marginal revenue of inventory, which, in turn, determines whether responsive pricing decreases or increases the optimal sourcing quantity.

When the unit procurement cost is low, the firm is incentivised to order a large quantity, and, thus, more exposed to the overage risk. In this case, the overage risk mitigation effect of responsive pricing dominates. To mitigate the overage risk, the firm is more likely to reduce price after yield realization

to stimulate demand, resulting in a lower expected marginal revenue of inventory than that under ex ante pricing. Therefore, when the unit procurement cost is low, the firm orders less under responsive pricing than it does under ex ante pricing. On the other hand, when the unit procurement cost is high, the firm orders a small quantity and is more exposed to the underage risk. In this case, the underage risk mitigation effect of responsive pricing dominates. To mitigate the underage risk, the firm is more likely to increase price after yield realization to limit demand, resulting in a higher expected marginal revenue of inventory than that under ex ante pricing. Therefore, when the unit procurement cost is high, the firm orders more under responsive pricing than it does under ex ante pricing. The above intuition is formalized in the following proposition.

PROPOSITION 6. *Compare U-a and U-r:*

(i) *For a linear demand function $d(p) = a - bp$ with $a > 0$ and $b > 0$ ³ and Uniform $[x, y]$ yield distribution with $0 \leq x < y \leq 1$, there exists a threshold cost $\bar{c} \in (0, a\mu/b)$, s.t., $q_r(c) \leq q_a(c)$ if $c \leq \bar{c}$ and $q_r(c) > q_a(c)$ otherwise.*

(ii) *For a general demand function and a general yield distribution under Assumptions 1 and 2, there exists $\delta_i > 0$, $i \in \{1, 2\}$, such that $q_r(c) < q_a(c)$ if $c \in (0, \delta_1)$, and $q_r(c) > q_a(c)$ if $c \in (p_0\mu - \delta_2, p_0\mu)$.*

Proposition 6(i) establishes a threshold for the unit procurement cost under linear demand function and Uniform yield distribution: The firm orders more under responsive pricing if and only if the unit procurement cost is higher than the threshold cost. Part (ii) suggests that the insight continues to hold under a general demand function and a general yield distribution: Responsive pricing decreases the optimal sourcing quantity when the unit procurement cost is sufficiently low and increases when the cost is sufficiently high. Indeed, our extensive numerical studies confirm that the insight from Proposition 6(i) is robust under general demand functions and yield distributions.

6.2. Impact on the Optimal Sourcing Strategy: UU Models

In this section, we study how price postponement affects supply diversification. As shown in the UU models, the firm could adopt dual sourcing under both pricing schemes. To investigate the impact of price postponement on the firm's diversification decisions, we compare cost threshold functions $\tilde{A}^1(c_1)$ and $\tilde{R}^1(c_1)$. For expositional brevity, we omit the comparison of $\tilde{A}^2(c_2)$ and $\tilde{R}^2(c_2)$, which offers similar insights. Recall that $\tilde{A}^1(c_1)$ and $\tilde{R}^1(c_1)$ represent the marginal revenue of procuring from supplier 2 given that the firm currently sole sources from supplier 1 at unit cost c_1 , under ex ante pricing and responsive pricing, respectively. For any given c_1 , $\tilde{R}^1(c_1) < \tilde{A}^1(c_1)$ means that the marginal revenue of a second supplier under responsive pricing is lower than under ex ante pricing, which implies that responsive pricing discourages the adoption of dual sourcing. In a similar vein, $\tilde{R}^1(c_1) > \tilde{A}^1(c_1)$ implies that responsive pricing promotes supply diversification.

To facilitate the comparison of the marginal revenues, let us consider the firm switching its pricing scheme from ex ante pricing to responsive pricing. We can decompose the transition into two steps

³ In the subsequent analysis, for expositional convenience, we omit mentioning the condition $a > 0$ and $b > 0$ whenever using linear demand function $d(p) = a - bp$.

and compare the marginal revenue of a second supplier at each step. In the first step, the firm postpones the pricing decision without changing the sole-sourcing quantity that is optimal under ex ante pricing; this step teases out the *price effect* of the pricing scheme change. In the second step, already using responsive pricing, the firm adjusts the sole-sourcing quantity to the optimal level for responsive pricing; this step captures the *quantity effect* of the pricing scheme change. The difference of $\tilde{R}^1(c_1)$ and $\tilde{A}^1(c_1)$ can be written as the sum of the two effects on the marginal value of a second supplier:

$$\begin{aligned} \tilde{R}^1(c_1) - \tilde{A}^1(c_1) = & \underbrace{\frac{\partial E[\pi_r(q_1\xi_1 + q_2\xi_2)]}{\partial q_2} \Big|_{q_1=q_r^*, q_2=0} - \frac{\partial E[\pi_r(q_1\xi_1 + q_2\xi_2)]}{\partial q_2} \Big|_{q_1=q_a^*, q_2=0}}_{\text{Quantity Effect}} \\ & + \underbrace{\frac{\partial E[\pi_r(q_1\xi_1 + q_2\xi_2)]}{\partial q_2} \Big|_{q_1=q_a^*, q_2=0} - \frac{\partial E[\pi_a(p, q_1\xi_1 + q_2\xi_2)]}{\partial q_2} \Big|_{p=p_a^*, q_1=q_a^*, q_2=0}}_{\text{Price Effect}} \end{aligned} \quad (5)$$

The sign of $\tilde{R}^1(c_1) - \tilde{A}^1(c_1)$ can vary in c_1 , because, as we discuss below, the price effect and the quantity effect are often counterbalancing each other. Recall that a sole-sourcing firm under ex ante pricing is more exposed to the overage risk when the unit procurement cost c_1 is low. Responsive pricing mitigates the risk by pricing low when supply is abundant. Consequently, responsive pricing decreases the marginal value of inventory when the unit procurement cost c_1 is low (Proposition 6). When the sole-sourcing quantity is fixed at q_a^* , the price effect of responsive pricing decreases the marginal revenue of a second supplier, resulting in the price effect term in (5) being negative for low values of c_1 . Adjusting the sole-sourcing quantity to the optimal level for responsive pricing, the quantity effect of responsive pricing decreases the optimal sole-sourcing quantity, i.e., $q_r^* < q_a^*$ for low values of c_1 . Because the two supply sources are strategic substitutes under responsive pricing (i.e., $\Pi_r(q_1, q_2)$ is submodular in (q_1, q_2)), the adjustment of the optimal sole-sourcing quantity increases the marginal revenue of a second supplier, and the quantity effect in (5) is positive for low values of c_1 . Analogously, when the unit procurement cost c_1 is high, the underage risk dominates. In this case, the price effect is positive and the quantity effect is negative. Table 3 summarizes these two effects with respect to c_1 .

c_1	Price Effect	Quantity Effect
c_1 low	–	+
c_1 high	+	–

Table 3 Impact of c_1 on Price and Quantity Effects

In addition to the unit procurement cost, the interplay of the price and quantity effects depends heavily on the characteristics of the yield distribution. The following proposition first considers a linear demand function and uniform yield distributions with progressively increasing yield reliability for the sole sourced supplier, and characterizes the combined impact of cost and yield distribution

upon the interplay of these two effects. Then, it confirms the robustness of the results under general demand functions and yield distributions.

PROPOSITION 7. Comparing *UU-a* and *UU-r* with $(c_1, c_2) \in \mathbb{C}_1$,

(i) For a linear demand function $d(p) = a - bp$, assume $\xi_1 \sim \text{Uniform}[x_1, y_1]$ with $0 \leq x_1 < y_1 \leq 1$, and ξ_2 satisfies Assumption 1(a):

(a) If $x_1 = 0$ and $y_1 = 1$, then $\forall c_1 \in (0, \frac{a\mu_1}{b})$, $\tilde{R}^1(c_1) > \tilde{A}^1(c_1)$.

(b) If $x_1 > 0$, there exists a $\delta > 0$, s.t., $\forall c_1 \in (\frac{a\mu_1}{b} - \delta, \frac{a\mu_1}{b})$, $\tilde{R}^1(c_1) < \tilde{A}^1(c_1)$.

(c) If $x_1 > \frac{1}{7}$ and $y_1 = 1$, then $\forall c_1 \in (0, \frac{a\mu_1}{b})$, $\tilde{R}^1(c_1) < \tilde{A}^1(c_1)$.

(ii) For a general demand function and general yield distributions under Assumptions 1 and 2, the relation between $\tilde{R}^1(c_1)$ and $\tilde{A}^1(c_1)$ doesn't depend on the distribution of ξ_2 . Moreover, if $\sigma_1 < \mu_1 \sqrt{(1 - \mu_1)/(1 + \mu_1)}$ [resp. $\sigma_1 > \mu_1 \sqrt{(1 - \mu_1)/(1 + \mu_1)}$], there exists a $\delta > 0$ such that $\forall c_1 \in (p_0\mu_1 - \delta, p_0\mu_1)$, $\tilde{R}^1(c_1) < \tilde{A}^1(c_1)$ [resp. $\tilde{R}^1(c_1) > \tilde{A}^1(c_1)$]. In particular, the followings hold:

(a) Assume $\xi_1 \sim \text{Beta}(a, b)$. If $a > 1$ and $b > 0$, there exists a $\delta > 0$ such that $\forall c_1 \in (p_0\mu_1 - \delta, p_0\mu_1)$, $\tilde{R}^1(c_1) < \tilde{A}^1(c_1)$.

(b) Assume $\xi_1 \sim \text{Beta}(a, b)$. If $a \in (0, 1)$ and $b > 0$, there exists a $\delta > 0$ such that $\forall c_1 \in (p_0\mu_1 - \delta, p_0\mu_1)$, $\tilde{R}^1(c_1) > \tilde{A}^1(c_1)$.

Proposition 7(i)(a) shows that when the sole-sourced supplier has the standard uniform yield distribution over $[0, 1]$, responsive pricing increases the marginal revenue of a second supplier over the entire feasible range of the unit sole sourcing cost, c_1 . It implies that the positive quantity effect dominates the negative price effect for low values of c_1 , but the negative quantity effect is dominated by the positive price effect for high values of c_1 . In this case, responsive pricing promotes the adoption of dual sourcing. Proposition 7(i)(b) presents a case of an improved yield distribution for the sole sourced supplier: When the lower end of the yield distribution support is improved to be strictly greater than 0, the mean yield increases and the variance of the yield decreases. Under the improved yield distribution, responsive pricing discourages the use of dual sourcing when the unit sole-sourcing cost c_1 is high, because improved yield decreases the potential mismatch of supply and demand. For an ex ante pricing firm that sole sources at high cost c_1 , without changing the sole-sourcing quantity, switching to responsive pricing does not significantly increase the marginal revenue of sourcing from a second supplier. The weakened positive price effect at high values of c_1 is dominated by the negative quantity effect of responsive pricing. For low values of c_1 , where responsive pricing has a negative price effect, improved yield strengthens the negative price effect. The strengthened negative price effect may even offset the positive quantity effect, when yield distribution is markedly improved. Proposition 7(i)(c) provides a sufficient condition for this dominance to happen: If the lower end of the support is greater than $1/7$, responsive pricing discourages the use of dual sourcing over the entire feasible range of c_1 .

Part (ii) of Proposition 7 checks the robustness of the insights from part (i) under general demand function and general yield distributions. The first insight is that the relation between $\tilde{R}^1(c_1)$ and $\tilde{A}^1(c_1)$ only depends on the procurement cost c_1 and the yield distribution of ξ_1 , but independent

from the yield distribution of the more expensive supplier. For a sufficiently high cost c_1 , the relation between $\tilde{R}^1(c_1)$ and $\tilde{A}^1(c_1)$ depends on both the mean and the variability of supplier 1's yield distribution. More specifically, when the sole-sourced supplier's yield variability is low, the negative quantity effect dominates the positive price effect, and responsive pricing decreases the marginal revenue of a second supplier. By contrast, when the sole-sourced supplier's yield variability is high, the negative quantity effect is dominated by the positive price effect, and responsive pricing increases the marginal revenue of a second supplier, which partially confirms the findings in part (i). Moreover, Proposition 7(ii) allows us to obtain sharper insights on how the shape of yield distribution affects the strategic interplay in the high cost scenario. For example, parts (ii)(a) and (ii)(b) jointly show that for the Beta yield distributions, if the density function is either unimodal (i.e., $a > 1, b > 1$) or increasing (i.e., $a > 1, b < 1$), then responsive pricing always decreases the marginal revenue of a second supplier, which further discourages dual-sourcing. On the other hand, if the density function is either U-shaped (i.e., $a < 1, b < 1$) or decreasing (i.e., $a < 1, b > 1$), then responsive pricing always increases the marginal revenue of a second supplier.

In sum, the effect of price postponement on the supply diversification decision depends on the sole-sourced supplier's yield distribution and the unit procurement cost. Specifically, when the sole-sourced supplier is less reliable (i.e., the mean yield is low and/or the variance of the yield is high), responsive pricing promotes dual sourcing for all feasible unit sole-sourcing cost, whereas it discourages supply diversification when the sole-sourced supplier is more reliable (i.e., the mean yield is high and/or the variance of the yield is low). By contrast, when the sole-sourced supplier is moderately reliable (i.e., the mean and variance of the yield are at moderate levels), responsive pricing promotes dual sourcing when the unit sole-sourcing cost is low and discourages otherwise.

6.3. Impact on the Optimal Sourcing Strategy: UR Models

In this section, we study how the presence of a reliable supplier in the supplier portfolio alters the strategic interplay between price postponement and supply diversification in mitigating supply yield risks. Recall that, in the UR models, dual sourcing is not optimal under ex ante pricing but may be optimal under responsive pricing. Such distinction reveals that price postponement has a significant, structural impact on the firm's sourcing strategy: Responsive pricing promotes dual sourcing. To understand how responsive pricing gives rise to dual sourcing, one must recognize that postponing price commitment means postponing demand commitment. As the firm sets the retail price in response to the realized supply yield, it induces demand to change with the supply yield. The value of dual sourcing stems from the firm's need to cope with the demand uncertainty induced by the postponed price commitment.

Given the significant impact of the pricing timing on the firm's diversification decision in UR models, a subsequent question is how the firm should optimally adjust its sourcing strategy when switching its pricing scheme from ex ante pricing to the more profitable responsive pricing. The answer boils down to the position of the unit cost pair (c_1, c_2) relative to cost thresholds $A(c_1)$ and $R(c_1)$. Four scenarios are possible, see Table 4. Three out of the four regions require profound

changes in the firm's optimal sourcing strategy as the firm switches from ex ante pricing to responsive pricing: In regions 1 and 2 the firm switches from sole sourcing to dual sourcing; in region 3 the firm replaces the reliable supplier by the unreliable one. The firm maintains sole sourcing from the unreliable supplier in region 4, where the reliable supplier is very expensive. In addition, the firm always increases the total sourcing quantity when either adding the unreliable supplier into the supply base (i.e., region 1) or replacing the reliable supplier with a unreliable one (i.e., region 3). Proposition 8 compares the cost thresholds $A(c_1)$ and $R(c_1)$ and the optimal total order quantity under both Uniform and general yield distributions.

Region Index	Parameter Range	Sourcing under Ex ante Pricing	Sourcing under Responsive Pricing	Impact of Responsive Pricing on Supplier Selection	Impact of Responsive Pricing on Total Order Quantity
1	$c_2 < \min \{A(c_1), R(c_1)\}$	R	R & U	Adding U	Increase
2	$A(c_1) < c_2 < R(c_1)$	U	R & U	Adding R	Decrease
3	$R(c_1) < c_2 < A(c_1)$	R	U	Switching	Increase
4	$c_2 > \max \{A(c_1), R(c_1)\}$	U	U	No Change	No Change

Table 4 Comparison in the Sourcing Strategies under UR Models: $(c_1, c_2) \in \mathbb{C}_2$

PROPOSITION 8. Comparing UR-a and UR-r with $(c_1, c_2) \in \mathbb{C}_2$:

(i) For a linear demand function $d(p) = a - bp$, assume $\xi \sim \text{Uniform}[x, y]$ with $0 \leq x < y \leq 1$, the following statements hold:

(a) If $x = 0$ and $y = 1$, there exists a unique $\tilde{c} \in (0, \frac{a\mu_1}{b})$, such that $R(c_1) > A(c_1)$ iff $c_1 \in (0, \tilde{c})$ and $R(c_1) < A(c_1)$ iff $c_1 \in (\tilde{c}, \frac{a\mu_1}{b})$.

(b) If $0 < x < y \leq 1$, there exists a $\delta > 0$ such that $\forall c_1 \in (\frac{a\mu_1}{b} - \delta, \frac{a\mu_1}{b})$, $R(c_1) < A(c_1)$.

(ii) For a general demand function and a general yield distribution under Assumptions 1 and 2, if $\sigma_1 < \mu_1 \sqrt{\mu_1^{-1/2} - 1}$ [resp. $\sigma_1 > \mu_1 \sqrt{\mu_1^{-1/2} - 1}$], there exists a $\delta > 0$ such that $\forall c_1 \in (p_0\mu_1 - \delta, p_0\mu_1)$, $R(c_1) < A(c_1)$ [resp. $R(c_1) > A(c_1)$]. In particular, assuming $\xi_1 \sim \text{Beta}(a, b)$, the followings hold:

(a) If either $a > 1, b > 0$ or $a \in (0, 1], b > \frac{1-a^2}{a}$, there exists a $\delta > 0$ such that $\forall c_1 \in (p_0\mu_1 - \delta, p_0\mu_1)$, $R(c_1) < A(c_1)$.

(b) If $a \in (0, 1), b \in (0, \frac{1-a^2}{a})$, there exists a $\delta > 0$ such that $\forall c_1 \in (p_0\mu_1 - \delta, p_0\mu_1)$, $R(c_1) > A(c_1)$.

(iii) For a general demand function and a general yield distribution under Assumptions 1 and 2, the total order quantity change summarized in Table 4 holds.

Part (i)(a) of Proposition 8 shows that the comparison of $A(c_1)$ and $R(c_1)$ under Uniform $[0, 1]$ yield distribution is driven by the unit procurement cost of the unreliable supplier. To explain, recall our discussion of Proposition 6, when the firm has access to an unreliable supplier only (e.g., supplier 1): The responsive-pricing firm would order less than an otherwise identical ex-ante pricing firm if the unit procurement cost is low, and order more otherwise. Now, when a second, reliable supplier is accessible, the guaranteed supply from the reliable supplier is more valuable to the firm that orders less from its unreliable supplier, and less valuable otherwise. As such, $R(c_1)$ is higher than $A(c_1)$ for low values of c_1 , and is lower for high values of c_1 . Part (i)(b) verifies that the insight

from Uniform[0,1] distribution for the high cost scenario continues to hold for general Uniform[x, y] distribution within the interval $[0, 1]$.

Unit procurement cost tells only part of the story. Part (ii) of Proposition 8 shows that under the general yield distribution, when switching from the ex ante pricing to the responsive pricing, the firm's supplier selection also depends on the characteristics of supplier 1's reliability. When c_1 is sufficiently large, the relation between $R(c_1)$ and $A(c_1)$ depends on both the mean and the variability of the yield distribution, i.e., if $\sigma_1 < \mu_1 \sqrt{\mu_1^{-1/2} - 1}$, then $R(c_1) < A(c_1)$ in a neighborhood around $p_0 \mu_1$, and the opposite holds when $\sigma_1 > \mu_1 \sqrt{\mu_1^{-1/2} - 1}$. This indicates that when supplier 1 is more reliable, the firm will change from the reliable supplier to the unreliable one, i.e., region 3 appears when c_1 is high. However, when supplier 1 is less reliable, the firm will add the reliable supplier into the supply base when switching from ex ante pricing to responsive pricing, i.e., region 2 appears when c_1 is high. By applying Proposition 8(ii) to various yield distributions, we can obtain sharper insights regarding the impact of yield distribution on the firm's supplier selection. For example, for the Uniform yield distributions, $\sigma_1 < \mu_1 \sqrt{\mu_1^{-1/2} - 1}$ always holds, which leads to part (i)(b). For the Beta distributions, parts (ii)(a) and (ii)(b) show that if the density function is unimodal ($a > 1, b > 1$), increasing ($a > 1, b < 1$), or decreasing/U-shaped with the variance of the distribution being low ($a < 1, b > \frac{1-a^2}{a}$), then region 3 always appears when c_1 is large. However, if the density function is decreasing/U-shaped with the variance of the distribution being high ($a < 1, b < \frac{1-a^2}{a}$), region 2 occurs for large c_1 . Part (iii) further shows that when the unreliable supplier is either added to complement or used to replace the reliable supplier, the total order quantity always increases (i.e., regions 1 and 3), and vice versa (i.e., region 2).

In sum, the presence of a reliable supplier has a fundamental structural impact on the strategic interplay between price postponement and supply diversification: Responsive pricing always promotes dual sourcing. Consequently, the firm should carefully adjust its supplier selection with its pricing scheme to fully explore the potential benefits of risk pooling through supply diversification.

7. Numerical Study

In this section, we conduct extensive numerical study to derive additional insights. Specifically, we aim to: (1) study the value of dual sourcing and its contributing factors in Section 7.1; (2) incorporate demand uncertainty into the analysis to evaluate its corresponding impact in Section 7.2; and (3) investigate how the supply correlation between the two suppliers would affect the strategic interplay between price postponement and diversification in Section 7.3, respectively. In addition, we check and confirm the robustness of our main results under general yield distribution and provide additional numerical results in Appendix C. Without loss of generality, we focus on the case where supplier 1 is cheaper with a lower effective cost in the subsequent numerical analysis.

7.1. Value of Dual Sourcing

We quantify the value of dual sourcing in the relative sense under each pricing scheme, and examine how it is affected by various model parameters. That is, for cost parameters (c_1, c_2) , let $\Pi_i^U(c_1)$ and

$\Pi_i^{UU}(c_1, c_2)$ be the optimal profits in the U-model and UU-model, respectively, then the relative value of dual sourcing for pricing scheme i is defined as $RV_i(c_1, c_2) = \frac{\Pi_i^{UU}(c_1, c_2) - \Pi_i^U(c_1)}{\Pi_i^U(c_1)} \geq 0$, $i \in \{r, a\}$. For given cost parameters (c_1, c_2) , $RV_i(c_1, c_2) = 0$ indicates that firm opts to sole source from supplier 1 even when the dual sourcing option is available, because supplier 2 is too expensive to source. Otherwise, $RV_i(c_1, c_2) > 0$ measures the relative profit improvement that supplier 2 could bring to the firm when switching from sole sourcing to dual sourcing.

Now, we investigate the effect of model parameters on the relative value of dual sourcing. Since the firm never sole sources from the more expensive supplier 2, the parameters of supplier 2 only affect the profit of dual sourcing under each pricing scheme. In this case, it is straightforward to see that the relative value of dual sourcing increases when supplier 2 becomes more favorable, i.e., either cheaper with lower c_2 or more reliable with lower σ_2 and/or higher μ_2 , as shown below.

CLAIM 1. *For any given supplier 1, $RV_i(c_1, c_2)$ increases as supplier 2 becomes more reliable (i.e., with either higher mean or lower variance) or cheaper (i.e., lower cost), $i \in \{r, a\}$.*

Next, we study the impact of supplier 1, which is more involved since it affects both $\Pi_i^U(c_1)$ and $\Pi_i^{UU}(c_1, c_2)$, $i \in \{a, r\}$. We adopt a similar numerical setup as used in Section C.1 with a linear demand function $d(p) = 10 - p$ and Beta distributions to model random yields. We fix the mean yields of both suppliers at 0.5, the variance of supplier 2's yield at 0.03, and change supplier 1's yield variance within the set $\{0.03, 0.04, 0.05, 0.06, 0.07\}$. In addition, we choose three sets of cost parameters: (1) $c_2 = 1$, $c_1 \in \{0.5, 0.6, 0.7, 0.8, 0.9\}$; (2) $c_2 = 2$, $c_1 \in \{1.5, 1.6, 1.7, 1.8, 1.9\}$; and (3) $c_2 = 3$, $c_1 \in \{2.5, 2.6, 2.7, 2.8, 2.9\}$. Note that each pair of cost parameters (c_1, c_2) is chosen such that the firm opts to dual source under most of yield parameter combinations. For each combination of yield and cost parameters, we quantify $RV_i(c_1, c_2)$, $i \in \{a, r\}$, and depict the results in Figure 3, which reveals several interesting observations.

First, when supplier 1 becomes either more expensive or less reliable with a higher variance, the relative value of dual sourcing becomes larger.⁴ These two observations hold true for both ex ante and responsive pricing schemes. The intuition is as follow: When supplier 1 becomes less favorable with either a higher cost or a more risky yield, the firm's profit under both U and UU models reduce with the former decreasing in a larger magnitude than the latter. This is because the firm solely relies on supplier 1 in the U model, and suffers more as it becomes less favorable. By contrast, in the UU model, when supplier 1 is less favorable, supplier 2 tends to become more favorable. Hence, the firm could shift the orders to mitigate the negative impact from supplier 1, leading to a higher relative value of dual sourcing.

Second, comparing the relative value of dual sourcing between ex ante pricing and responsive pricing, i.e., $RV_a - RV_r$ (see Figure 9 in Appendix C.2), shows that dual sourcing, in general, tends to be more valuable under ex ante pricing than responsive pricing, except for some cost parameters where the firm adopts dual sourcing under responsive pricing and sole sourcing under ex ante pricing.

⁴ Fixing the supplier 1' yield variance and varying its yield mean lead to qualitatively same observations. Thus, we omit the details.

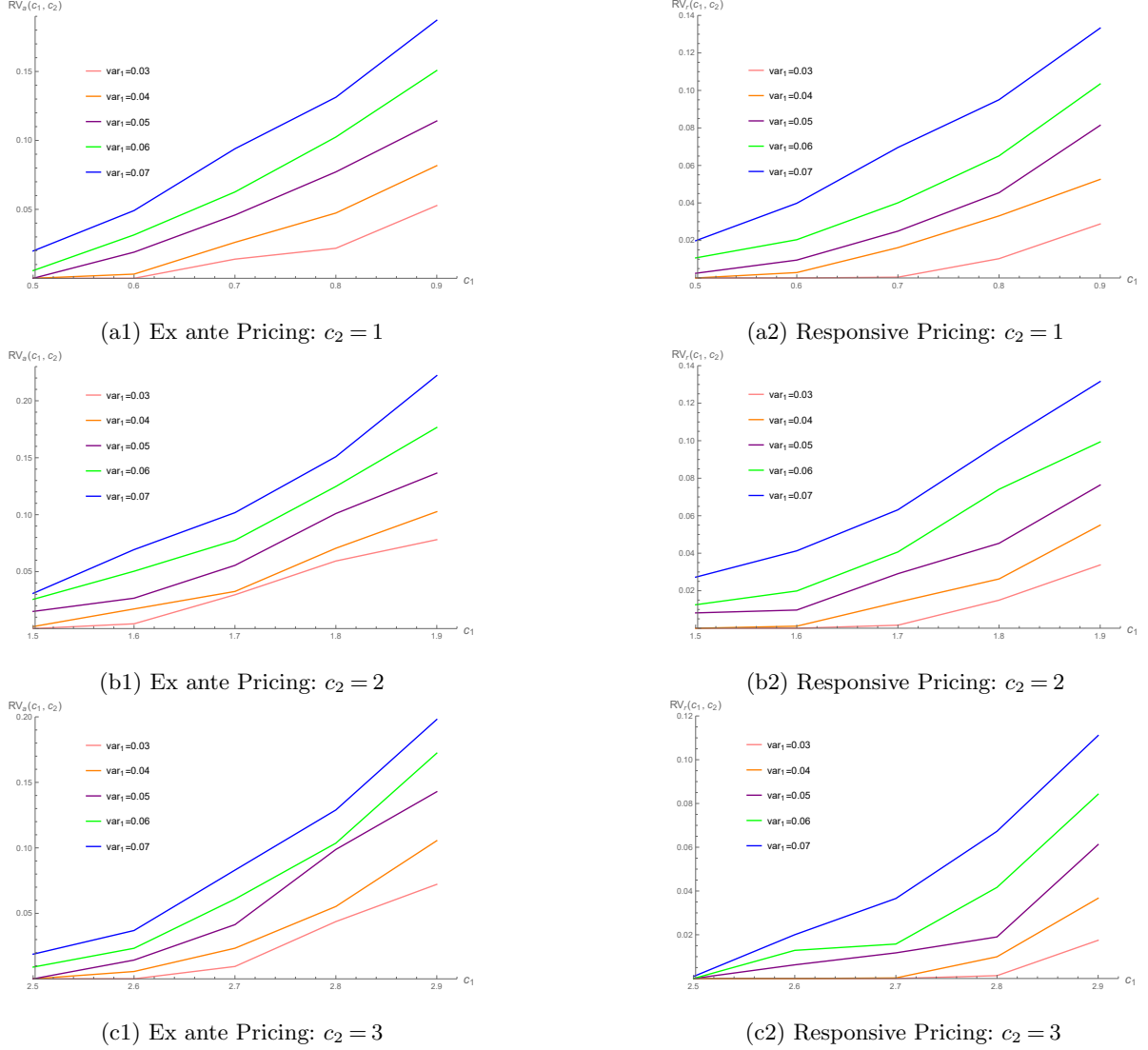


Figure 3 Relative Value of Dual Sourcing under Ex ante Pricing $RV_a(c_1, c_2)$ (Left) and Responsive Pricing $RV_r(c_1, c_2)$ (Right): $c_2 \in \{1, 2, 3\}$ and $Var_1 \in \{0.03, 0.04, 0.05, 0.06, 0.07\}$

This observation complements our main results regarding the impact of price postponement on the adoption of dual sourcing (i.e., comparing cost thresholds $\tilde{A}^1(c_1)$ and $\tilde{R}^1(c_1)$), and further implies that when dual sourcing is optimal under both pricing schemes (i.e., $c_2 < \min\{\tilde{A}^1(c_1), \tilde{R}^1(c_1)\}$), it tends to bring a higher value for the ex ante pricing firm, as diversified sourcing is the only instrument the ex ante pricing firm could utilize to mitigate supply yield risk.

To conclude, we remark that the above findings continue to hold qualitatively when we measure the value of dual sourcing in the absolute sense, i.e., $\Pi_i^{UU}(c_1, c_2) - \Pi_i^U(c_1)$, $i \in \{r, a\}$.

7.2. Random Demand

Our main models assume a deterministic demand curve to purely focus on mitigating supply yield risk. In this subsection, we incorporate demand uncertainty into our UU models and numerically

examine the impact of demand uncertainty on both the cost thresholds for dual sourcing and the strategic interplay between price postponement and diversification.

To these purposes, we adopt a similar numerical setup as used in subsection 7.1, i.e., the supplier 2's yield follows a Beta distribution with mean 0.5 and variance 0.03, and supplier 1's yield follows a Beta(k, k) distribution with mean 0.5 and variance decreasing in $k \in \{1, 2, 3, 4\}$. In addition, we pick up a linear demand function $d(p) = a - bp$, where $b = 1$ and a follows a Normal distribution with mean $\mu_a = 10$ and standard deviation chosen within the set $\sigma_a \in \{0.5, 1, 1.5, 2, 2.5, 3\}$. For the responsive pricing scheme, with the joint concavity of the objective function, we calculate the upper cost threshold $\tilde{R}_N^1(c_1)$ for any feasible c_1 , where the subscript "N" refers to Normal demand case. For the ex ante pricing scheme, the objective is not jointly concave and is challenging to calculate the upper cost threshold $\tilde{A}_N^1(c_1)$ for all feasible c_1 . As such, we pick up $c_1 \in \{0.1, 0.5, 1, 1.5, 2, 2.5, 3\}$, which is sufficient to show the impact of demand uncertainty on both the cost threshold $\tilde{A}_N^1(c_1)$ and the strategic interplay between price postponement and diversification.

To illustrate the impact of demand uncertainty on the firm's willingness-to-pay for dual sourcing, we plot the difference of the cost thresholds with and without demand uncertainty for responsive pricing, i.e., $\tilde{R}_N^1(c_1) - \tilde{R}^1(c_1)$, in Figure 4 and for ex ante pricing, i.e., $\tilde{A}_N^1(c_1) - \tilde{A}^1(c_1)$, in Figure 10 of Appendix C.3, respectively. There are two interesting observations. First, in general, for any given c_1 , the cost thresholds $\tilde{R}_N^1(c_1)$ and $\tilde{A}_N^1(c_1)$ decrease when the demand variability σ_a increases, indicating that for both pricing schemes, the firm's willingness-to-pay for dual sourcing (i.e., including the more expensive supplier 2 into the supply base) reduces as the demand becomes more uncertain. Second, we also observe that $\tilde{A}_N^1(c_1)$ decreases in a larger magnitude than that of $\tilde{R}_N^1(c_1)$, which implies that the ex ante pricing firm is more affected by demand uncertainty and less interested in diversification. The underlying intuition is given as follow: Recall that dual sourcing is usually adopted for the purpose of securing supply and enjoying the benefit of risk pooling. When the demand is deterministic, the firm knows exactly what is needed from the downstream and is, thus, more willing to pay for dual sourcing so as to fulfill demand as much as possible. When the demand is random and becomes more variable, securing supply via dual sourcing is less attractive since the firm still faces demand risk and the ensuing mismatch. As such, it may not opt to pay the same price for dual sourcing as the one paid with either no or lower demand uncertainty, which explains the first observation. On the other hand, although the responsive pricing firm faces demand risk, it can still leverage the postponed price to partially match realized supply with realized demand, and is less affected by demand uncertainty. By contrast, the ex ante pricing firm is fully exposed to demand risk and is less incentivized to ensure supply via expensive dual sourcing before demand realization, which results in our second observation.

To reveal the impact of demand uncertainty on the strategic interplay between price postponement and diversification, we plot $\tilde{R}_N^1(c_1) - \tilde{A}_N^1(c_1)$ as a function of σ_a for selected c_1 in Figure 5, which clearly shows that our main findings are robust when σ_a is relatively low. That is, for small σ_a , responsive pricing promotes dual sourcing when supplier 1 is less reliable, i.e., Figure 5(a), and

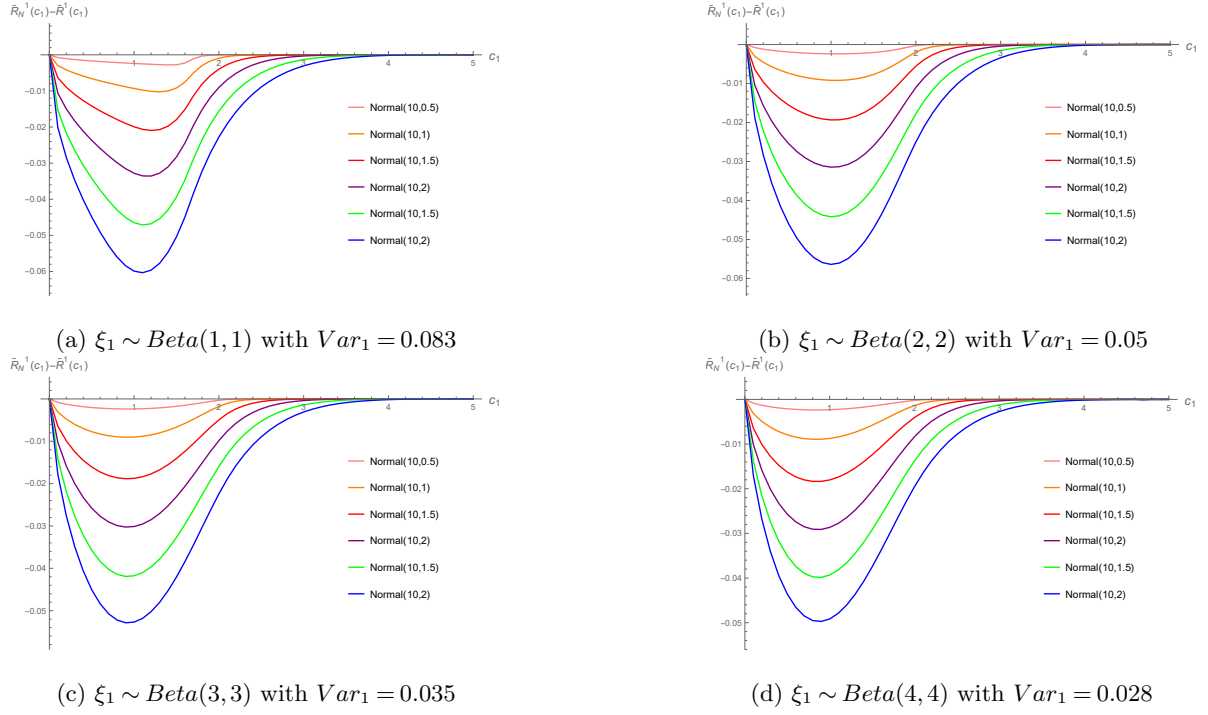


Figure 4 Difference between Responsive Pricing Cost Threshold under Normal Demand and Deterministic Demand

$\tilde{R}_N^1(c_1) - \tilde{R}^1(c_1)$: $\xi_1 \sim \text{Beta}(k, k)$ with $k \in \{1, 2, 3, 4\}$

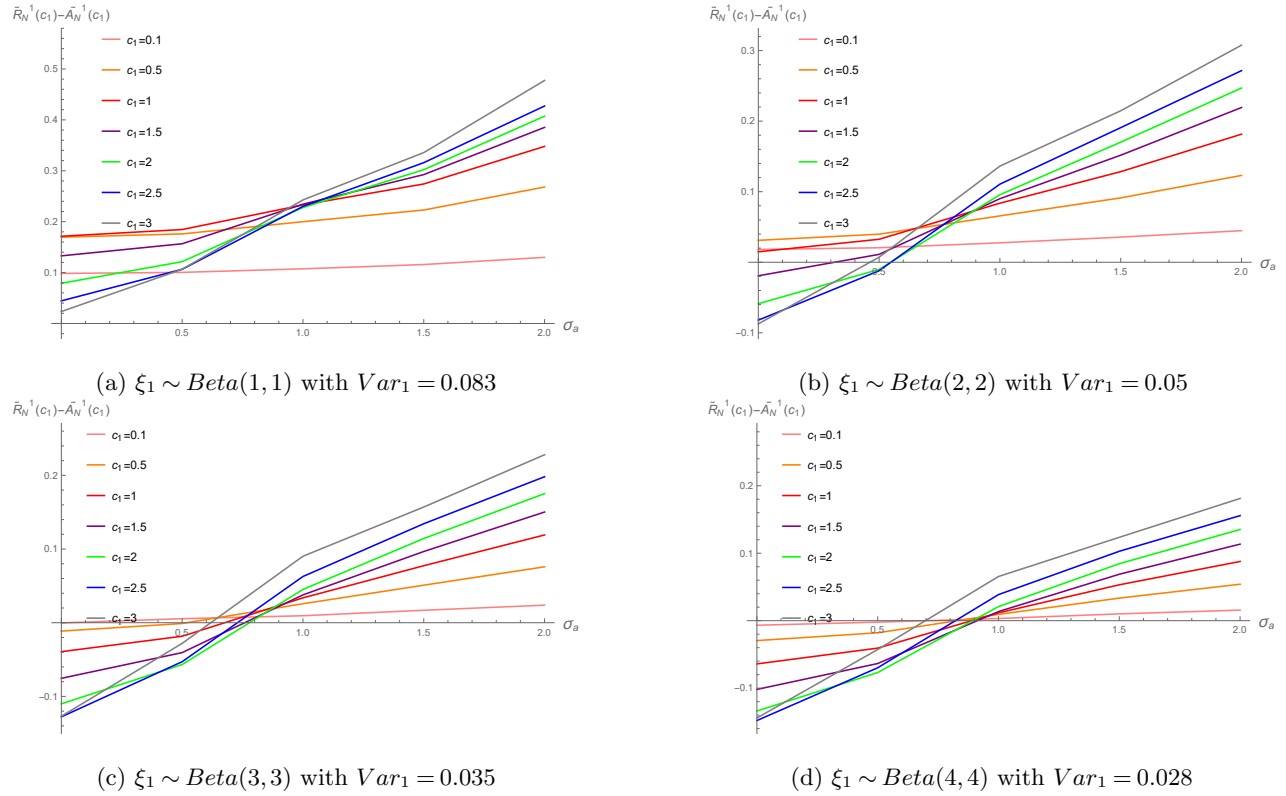


Figure 5 Difference between Responsive Pricing and Ex Ante Pricing Cost Thresholds under Normal Demand

$\tilde{R}_N^1(c_1) - \tilde{A}_N^1(c_1)$: $\xi_1 \sim \text{Beta}(k, k)$ with $k \in \{1, 2, 3, 4\}$

discourages when it is more reliable, i.e., Figures 5(c) and 5(d). When supplier 1 is moderately reliable, i.e., Figures 5(b), responsive pricing promotes diversification when c_1 is low and discourages otherwise. This observation is intuitive because the cost thresholds $\tilde{R}_N^1(c_1)$ and $\tilde{A}_N^1(c_1)$ continuously decrease in σ_a . Since deterministic demand is a boundary case with $\sigma_a = 0$, its main results naturally preserve when σ_a is low due to continuity. On the other hand, since $\tilde{A}_N^1(c_1)$ decreases in σ_a in a larger magnitude than $\tilde{R}_N^1(c_1)$, it is immediate to observe that if responsive pricing promotes dual sourcing under deterministic demand, it continues to promote when demand becomes random, see Figures 5(a) and 5(b) with $c_1 \in \{0.1, 0.5, 1\}$. By contrast, if ex ante pricing promotes dual sourcing under deterministic demand, i.e., Figures 5(b) with $c_1 > 1$, 5(c), and 5(d), the reversed scenario could happen when demand becomes more uncertain, i.e., responsive pricing starts to promote dual sourcing. These observations imply that with demand uncertainty, it is more possible for ex ante pricing firm to discourage diversification in the UU model unless the demand variability is relatively low, which can be understood by applying the rationale that the ex ante pricing firm is fully exposed to demand risk and is less incentivized to ensure supply via diversification without knowing the demand realization.

7.3. Correlated Supply Yields

In our UU models, we assume that the two unreliable suppliers' production yields are independent random variables. However, in reality, suppliers may face correlated yield risks due to various reasons, including being geographically close to each other or adopting the same input raw materials. In this subsection, we numerically investigate the impact of supply yield correlation on the strategic interplay between diversification and price postponement.

To begin with, we acknowledge that sourcing and supplier selection under correlated supply risks is an important research area, and there are many recent papers studying this issue under various supply risk models. Closely related to ours is Dong et al. (2022), which investigates a firm's supplier selection decision under correlated supply random yields and various demand functions. They show that the conventional cost-driven rule in supplier selection, i.e., "cost is an order qualifier and reliability is an order winner", is largely preserved when the yield correlation between the two unreliable suppliers is not too high (i.e., the correlation coefficient $\rho \leq \min\{\frac{\sigma_1 \mu_2}{\sigma_2 \mu_1}, 1\}$, where μ_i and σ_i is the mean and standard deviation of the supplier i 's yield distribution respectively, $i \in \{1, 2\}$). Otherwise, the firm may sole source from the more expensive but more reliable supplier alone.

Building upon their results, we contribute by investigating whether supply correlation would fundamentally alter the role of price postponement on the adoption of supply diversification. Instead of using Beta distribution as adopted in previous numerical subsections, we choose the multivariate normal distribution as the yield distribution, which allows us to capture the underlying correlation structure between the two suppliers with a single parameter of correlation coefficient ρ . In addition, to both keep the correlation structure and avoid truncating the distribution within the interval $[0, 1]$, we further require the yield mean μ_i and standard deviation σ_i to satisfy the following two conditions, $i \in \{1, 2\}$. First, the chance that each yield realization falls outside the interval $[0, 1]$ is negligible,

which is a common approach to deal with normal distribution in the literature. Specifically, we pick up the combination of yield mean and standard deviation such that $\mu_i - 3\sigma_i > 0$ and $\mu_i + 3\sigma_i < 1$, $i \in \{1, 2\}$, which requires the variability of the normal yield distribution to be not too large. Second, we assume supplier 1 is less reliable than supplier 2 with a higher coefficient of variation (i.e., $\frac{\sigma_1}{\mu_1} > \frac{\sigma_2}{\mu_2}$), which ensures $\frac{\sigma_1\mu_2}{\sigma_2\mu_1} > 1$ so that the conventional cost-driven rule in supplier selection continues to hold for any correlation coefficient $\rho \in (-1, 1)$.

With the above two conditions, we pick up three instances of supplier 1's cost $c_1 \in \{0.1, 1, 2\}$ and choose three sets of yield parameters $(\mu_1, \sigma_1, \mu_2, \sigma_2)$: (1) (0.5, 0.15, 0.5, 0.1); (2) (0.45, 0.15, 0.5, 0.1); and (3) (0.45, 0.15, 0.7, 0.1), under which the two suppliers are more reliable compared to the instances used under Beta distributions. For each set of parameters, we calculate the upper cost thresholds $\tilde{R}^1(c_1)$ and $\tilde{A}^1(c_1)$ by varying the correlation coefficient $\rho \in (-1, 1)$, and the results are shown in Figure 6. For all the tested numerical instances, we find that, due to choice of yield parameters, responsive pricing promotes dual sourcing when supplier 1 is relatively unreliable and cheap, i.e., Figures 6(b1) and (c1), and discourages otherwise. This is consistent with our analytical results in Section 6.2. Moreover, we verify that our main results on the strategic interplay are qualitatively robust under correlated supply yields, with the major observations summarized below.

First, for any given c_1 , the cost thresholds under ex ante pricing, i.e., $\tilde{A}^1(c_1)$, and responsive pricing, i.e., $\tilde{R}^1(c_1)$, both decrease when the correlation coefficient ρ increases, which indicates that the dual sourcing is less likely to be adopted under more correlated supply random yields (as shown in rows (a), (c), and (e) of Figure 11 in Appendix C.4). This finding is consistent with Dong et al. (2022) for the responsive pricing scheme and is further extended to the ex ante pricing scheme.

Second, for most of the tested numerical instances, the sign of $\tilde{A}^1(c_1) - \tilde{R}^1(c_1)$ remains unchanged whereas the absolute difference $|\tilde{A}^1(c_1) - \tilde{R}^1(c_1)|$ reduces as the correlation coefficient ρ increases, see Figure 6 excluding (c1). The two observations lead to some interesting implications: On the one hand, the former observation indicates the robustness of our results, i.e., the correlation structure, in general, will not fundamentally alter the role of price postponement on the adoption of supply diversification. In other words, when price postponement either promotes or discourages dual sourcing under independent random yields, such strategic interplay continues to happen when yields are either positively or negatively correlated, i.e., $\rho \in (-1, 1)$. On the other hand, the second observation reveals the quantitative effect of supply correlation on governing the strategic interplay. Specifically, when the yields become more positively correlated, the pricing scheme, which promotes dual sourcing under independent yields, becomes less active in promoting diversification (i.e., $|\tilde{A}^1(c_1) - \tilde{R}^1(c_1)|$ reduces as ρ increases to 1). By contrast, when the yields become more negatively correlated, the same pricing scheme tends to become more likely to advocate dual sourcing (i.e., $|\tilde{A}^1(c_1) - \tilde{R}^1(c_1)|$ enlarges as ρ decreases to -1).

Finally, we remark that although supply correlation, in general, does not alter the strategic interplay between price postponement and diversification, exceptions may happen around the crossing point where $\tilde{A}^1(c_1) = \tilde{R}^1(c_1)$. Among all the test instances, we find one such example that supplier

1 is less reliable and very cheap whereas supplier 2 is more reliable, i.e., Figure 6 (c1). In this case, responsive pricing promotes dual sourcing when $\rho = 0$, and continues to promote when ρ is either positive or weakly negative (i.e., $\tilde{A}^1(c_1) - \tilde{R}^1(c_1) < 0$). However, when the yields are highly negatively correlated, ex ante pricing would instead advocate diversification (i.e., $\tilde{A}^1(c_1) - \tilde{R}^1(c_1) > 0$). This is because the more reliable supplier 2 becomes even more attractive when it is highly negatively correlated with supplier 1, which increases the ex ante pricing firm's willingness to pay for dual sourcing. Despite such switch, the overall magnitude of the absolute difference $|\tilde{A}^1(c_1) - \tilde{R}^1(c_1)|$ is extremely small, i.e., in the range of $10^{-4} \sim 10^{-3}$.

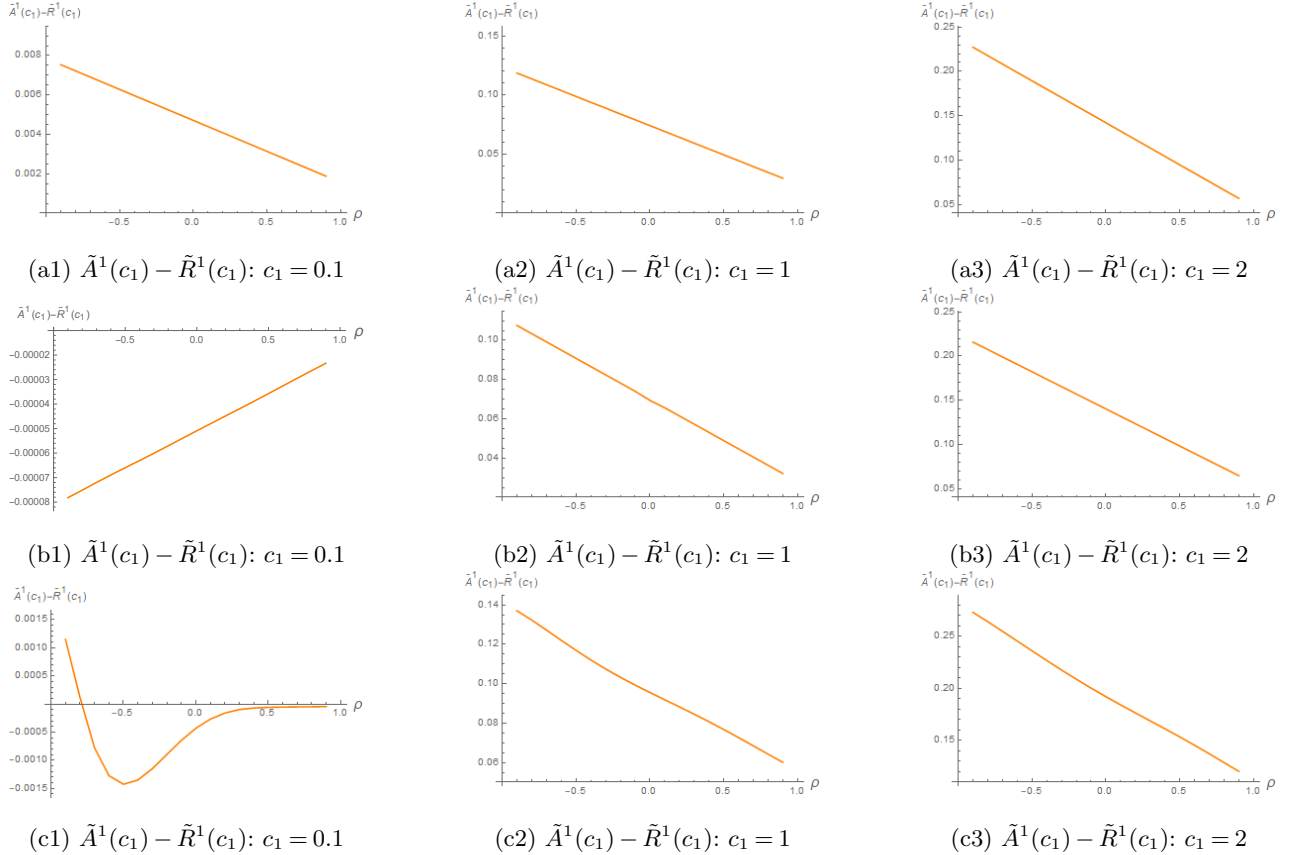


Figure 6 Impact of ρ on $\tilde{R}^1(c_1)$, $\tilde{A}^1(c_1)$, and $\tilde{A}^1(c_1) - \tilde{R}^1(c_1)$: $c_1 \in \{0.1, 1, 2\}$ and $(\mu_1, \sigma_1, \mu_2, \sigma_2) = (0.5, 0.15, 0.5, 0.1)$ (Row a), $(0.45, 0.15, 0.5, 0.1)$ (Row b), and $(0.45, 0.15, 0.7, 0.1)$ (Row c).

8. Conclusion

Supply diversification and pricing are two powerful tools to mitigate the negative impact of yield uncertainty, with the former operating on the supply side and the latter handling the demand side. This paper investigates an integrated approach to sourcing and pricing. In a stylized model, we compare two distinct pricing schemes, ex ante pricing and responsive pricing. The firm is better off with responsive pricing as the postponed pricing enables the firm to adjust demand to match supply. What remains unclear is how responsive pricing affects the firm's optimal sourcing decisions, with

respect to the sourcing quantity and the need for supply diversification. To answer these questions, we fully characterize the joint sourcing and pricing decisions for three scenarios of supply portfolio. Each scenario offers interesting insights on the intricate interplay of sourcing and pricing.

For a firm with access to a single unreliable supplier, its optimal order quantity under responsive pricing is lower than that under ex ante pricing when the procurement cost is low, and higher otherwise. This can be explained by the different levels of inventory risk under the two pricing schemes. Under ex ante pricing, to cope with the yield risk, it is optimal for the firm to order more than its price-induced demand, so the firm is exposed to the underage risk when the yield is low and the overage risk otherwise. Responsive pricing eliminates the underage risk, because when the realized yield is low, the firm can use high price to induce supply-matching demand and achieve revenue maximization. The firm is still exposed to some overage risk when the realized yield is so high that a revenue-maximizing demand would not clear the inventory. Therefore, when a low unit procurement cost induces an ex ante pricing firm to place a large order and thus exposes the firm more to the overage risk, an otherwise identical responsive-pricing firm, foreseeing the use of low price to maximize revenue, would place a smaller order, and the opposite holds true otherwise.

The characterization of the optimal sourcing strategy for cases with two suppliers entails specifying sole-sourcing and dual-sourcing regions in the space of unit procurement cost pair. For a firm with access to two unreliable suppliers, dual sourcing can be optimal under both pricing schemes. We say that responsive pricing promotes supply diversification if the marginal revenue of adding a second supplier to a sole-sourcing firm is higher than that under ex ante pricing, and discourages otherwise. The value of a second supplier stems from the improved supply reliability due to risk pooling. When the reliability of the sole-sourced supplier is high, i.e., supply-demand mismatch risk is low, responsive pricing is efficient in mitigating the risk, and thus, discourages supply diversification for the entire feasible range of unit sole-sourcing cost. By contrast, when the reliability of the sole-sourced supplier is low, responsive pricing is not as effective as it is in the high-reliability situation, and thus, promotes supply diversification. Interestingly, when the reliability of the sole-sourced supplier is moderate, responsive pricing promotes supply diversification when the unit sole-sourcing cost is low and discourages otherwise. This is because, the firm is more likely to be exposed to the overage risk when the unit sole-sourcing cost is low and underage risk when the cost is high. Responsive pricing values risk pooling more when it is more likely to mitigate the overage risk by decreasing the sole-sourcing quantity, and appreciates risk pooling less otherwise.

When one of the two suppliers is fully reliable, price postponement has a fundamental impact on supply diversification: Dual sourcing is never optimal for an ex ante pricing firm, but is optimal for a responsive pricing firm when the suppliers' unit procurement costs are close enough. The reason that an ex ante pricing firm does not benefit from supply diversification is that in satisfying the known price-induced demand, the reliable supplier in a dual sourcing arrangement simply shifts some of that demand away from the unreliable supplier, and the residue demand is addressed by an inflated order to the unreliable supplier with the same inflation rate as that in the case of a single

unreliable supplier. The firm is better off sole sourcing from one of the two suppliers. This is a case that “cost is an order qualifier” does not hold: The firm may sole source from the reliable supplier even when it is more expensive. For a responsive pricing firm, as postponed price commitment leads to postponed demand commitment, the firm faces uncertain demand at the time of the sourcing decision, a situation that the firm can benefit from risk pooling through diversified sourcing.

Finally, a comprehensive set of numerical experiments is conducted to confirm the robustness of our main results and derive additional insights. Specifically, we find that dual sourcing is more valuable when either the sole sourced supplier is less favorable (with higher cost or lower reliability) or the second supplier is more favorable, and the ex ante pricing firm tends to benefit more from dual sourcing. As for the impact of demand uncertainty, we find that a lower level of demand uncertainty preserves the strategic interplay between price postponement and diversification, whereas a higher level may discourage the ex ante pricing firm to adopt diversified sourcing when both suppliers are unreliable. In addition, supply correlation does not fundamentally alter the role of price postponement on the adoption of diversification. Yet, a more negatively correlated supply base may incentivize the pricing scheme that promotes dual sourcing to be a more active promoter, whereas a more positively correlated supply base may weaken the interest to promote dual sourcing.

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