https://dx.doi.org/10.1016/j.trb.2017.04.011

Alternate Weibit-Based Model for Assessing Green Transport Systems with Combined Mode and Route Travel Choices

Songyot Kitthamkesorn [a*], Anthony Chen [b]

[a] Excellence Center in Infrastructure Technology and Transportation Engineering, Department of Civil Engineering, Chiang Mai University, 50200, Thailand
[b] Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University, Hong Kong

Abstract

Reduction of vehicle emissions is a major component of sustainable transportation development. The promotion of green transport modes is a worthwhile and sustainable approach to change transport mode shares and to contribute to healthier travel choices. In this paper, we provide an alternate weibit-based model for the combined modal split and traffic assignment (CMSTA) problem that explicitly considers both similarities and heterogeneous perception variances under congestion. Instead of using the widely-adopted Gumbel distribution, both mode and route choice decisions are derived from random utility theory using the Weibull distributed random errors. At the mode choice level, a nested weibit (NW) model is developed to relax the identical perception variance of the logit model. At the route choice level, the recently developed path-size weibit (PSW) is adopted to handle both route overlapping and route-specific perception variance. Further, an equivalent mathematical programming (MP) formulation is developed for this NW-PSW model as a CMSTA problem under congested networks. Some properties of the proposed models are also rigorously proved. Using this alternate weibit-based NW-PSW model, different go-green strategies are quantitatively evaluated to examine (a) the behavioral modeling of travelers' mode shift between the private motorized mode and go-green modes and (b) travelers' route choice with consideration of both non-identical perception variance and route overlapping. The results reveal that mode shares and route choices from the NW-PSW model can better reflect the changes in model parameters and in network characteristics than the traditional logit and extended logit models.

Keywords: Nested weibit, path-size weibit, mathematical program, combined modal split and traffic assignment problem

---

*Corresponding author. Tel.: +66-95-661-1641; fax: +66-53-89-2376.
E-mail: songyot@eng.cmu.ac.th.

1 INTRODUCTION

Transportation is a major cause of vehicular emissions. Transportation consumes million liters of fossil fuel daily, resulting in not only severe congestion but also air pollution, greenhouse gas (GHG), and consequently global warming. These adverse impacts have prompted the national government in many countries to promote "go-green" transport modes such as non-motorized modes (e.g., bicycle) and public transit (e.g., metro, tram, bus, etc.) to keep the environmental costs low and to help travelers make healthier travel choices, while accommodating the increasing travel demands.

To quantitatively evaluate the effectiveness of go-green transport policies, we need a sound behavioral model of travelers' mode shift between the private motorized mode and go-green modes as well as travelers' route choice with consideration of both non-identical perception variance and route overlapping. A widely used approach is the combined travel demand model (e.g., Boyce, 2007; Briceño et al., 2008; Szeto *et al*., 2012; Kitthamkesorn et al., 2016), which provides a rigorous quantitative evaluation of different go-green promotion policies and a tractable computational tool in the network equilibrium framework. More specifically, the behavioral mode shift and route change can be effectively addressed by using the combined modal split and traffic assignment (CMSTA) model, which is a special case of the combined travel demand model that considers mode choice and route choice simultaneously. Based on different assumptions and applications, various CMSTA models have been developed in the transportation literature to model the mode choice and route choice made by travelers. A host of researchers (e.g., Florian, 1977; Florian and Nguyen, 1978; Abdulaal and LeBlanc, 1979; Oppenheim, 1995; Cantarella, 1997; Wu and Lam, 2003; García and Marín, 2005) has provided different modeling approaches to formulate the CMSTA problem. These formulations include mathematical programming (MP) (Florian and Nguyen, 1978; Abdulaal and LeBlanc, 1979; Oppenheim, 1995), variational inequality (VI) (Florian, 1977; Wu and Lam, 2003; García and Marín, 2005), and fixed point (FP) (Cantarella, 1997) for jointly determining the mode and route travel options. The early models (e.g., Florian, 1977; Florian and Nguyen, 1978; Abdulaal and LeBlanc, 1979) adopted a stochastic mode choice (i.e., random utility model) and combined it with a deterministic route choice (i.e., user equilibrium (UE) model). However, there seems to be an inconsistency between the two travel choices (i.e., using a deterministic UE to characterize route choice decisions while adopting a stochastic discrete choice model to describe mode choice decisions). To overcome this behavioral inconsistency, Cantarella (1997) and García and Marín (2005) provided the option to combine the stochastic mode choice model with either the UE model or the stochastic user equilibrium (SUE) model, while Oppenheim (1995) and Wu and Lam (2003) adopted the multinomial logit (MNL) model for modeling both mode choice and route choice decisions in the network equilibrium framework (i.e., integrating random utility model within the network equilibrium approach to model the congestion effect). The main difference among these models is the modeling approach. Oppenheim (1995) provided a MP formulation, Wu and Lam (2003) and García and Marín (2005) used a VI formulation, and Cantarella (1997) adopted a FP formulation.

Although the behavioral inconsistency problem has been resolved, the MNL model has two known drawbacks that stems from its independently and identically distributed (IID) assumptions with the Gumbel random error distribution: (1) its inability to handle similarities among alternatives and (2) its inability to handle non-identical perception variances among alternatives. At the mode choice level, the MNL model cannot handle the mode similarity (e.g., physical attributes and operating policies) (Ben-Akiva and Lerman, 1985) and the difference in mode perceived utility or disutility. At the route choice level, the MNL model cannot consider the route overlapping and route-specific perception variance (Sheffi, 1985). Recently, Kitthamkesorn et al. (2016) adopted the nested logit (NL) for mode choice and the cross nested logit (CNL) model for route choice model to handle the mode similarity and route overlapping, respectively. Both NL and CNL models used a two-level tree structure to handle the independence assumption (i.e., similarity among the available modes that share the same upper nest in the NL model and route overlapping in the CNL model). However, both NL and CNL models used the Gumbel distribution as the random perception error term, which requires the identical variance assumption in order to obtain an analytical probability expression. Hence, the CMSTA model developed by Kitthamkesorn et al. (2016) still cannot consider the non-identical perception variance in both mode choice and route choice levels. One possibility is to adopt the multinomial probit (MNP) model to overcome both shortcomings inherited by the IID Gumbel distribution (e.g., Meng and Liu, 2012). However, the MNP model does not have a closed-form probability expression, which poses computational difficulty since solving the MNP model requires intensive computation, e.g., Monte Carlo simulation (Sheffi and Powell, 1982), Clark's approximation method (Maher, 1992), or numerical method (Rosa and Maher, 2002).

In this paper, we develop an alternate weibit-based CMSTA model. Instead of the widely used Gumbel random error distribution, the proposed CMSTA model is based on the Weibull random error distribution. At the mode choice level, a nested weibit (NW) model is developed from the copula framework (Nelsen, 2006). Its nested structure handles the mode similarity while the Weibull distributed random error considers the mode-specific perception variance. At the route choice level, the recently developed path-size weibit (PSW) model is adopted to handle both route overlapping and route-specific perception variance. An equivalent mathematical programming (MP) formulation for the combined NW-PSW model is provided with some solution properties. It should be noted that MP formulation requires more assumptions (e.g., separability, differentiability, and symmetry of link cost functions, additivity of route cost structure, separable demand functions, etc.) compared to VI and FP. According to Cantarella et al. (2013, 2015, 2016), FP is the most flexible formulation among the three formulations as it can cope with a wider range of operational issues, including separable and non-separable (or asymmetric) link cost functions, additive and non-additive route cost structures, separable and non-separable demand functions, deterministic and stochastic choice models, single-user and multi-user classes, and uni-modal and multi-modal assignment problems. However, convergent solution algorithms available to FP formulation are very limited. Most algorithms rely on the method of successive averages (MSA) based on link flows or link costs (Cantarella et al., 2015, 2016), which are known to suffer from slow convergence when highly accurate solutions are required. This is partly due to the non-availability of an objective function for performing a line search step, which is known to be an important component of solution algorithms to many mathematical formulations (Chen et al., 2013). On the contrary, the development of a MP formulation for the weibit-based CSMTA model provides the following benefits:

(1) The optimality conditions directly provide the equivalency between the MP formulation and the weibit-based mode choice and route choice probabilities. This is similar to the Beckmann transformation used as the objective function for the user equilibrium MP formulation (Beckmann

et al., 1956) and its relationship to the Kuhn-Tucker conditions. These conditions are readily interpretable and easily understandable. For details, readers are directed to Boyce (2016) for the interpretation of the Kuhn-Tucker optimality conditions.

(2) Given that the MP is a convex program, many convergent algorithms are readily available for solving the weibit-based CMSTA model. A widely use algorithm for solving the combined travel demand models (e.g., including the combined distribution and assignment problem, the combined modal split and traffic assignment (CMSTA) problem, and the elastic demand traffic equilibrium problem) is the Evans' algorithm (Evans, 1976), also known as the partial linearization algorithm (Patriksson, 1994). Computational results conducted by LeBlanc and Farhangian (1981) revealed the partial linearization algorithm performed better than the complete linearization of the Frank-Wolfe algorithm suggested by Florian et al. (1975) and Florian and Nguyen (1978). Recently, Ryu et al. (2017) adapted the gradient projection (GP) algorithm for solving the CMSTA problem and demonstrated the superiority of GP algorithm over Evans' algorithm.

(3) Since the MP is a convex program, the objective function can be used not only to determine a suitable search direction and a suitable step size in a typical iterative solution algorithm, but also used as a stopping criterion to monitor the convergence of the algorithm.

In addition, many researchers have adopted the MP approach to model different applications, including advanced discrete choice models in a network equilibrium framework (e.g., cross-nested logit SUE model with fixed and elastic demand (Bekhor and Prashker, 1999; Kitthamkesorn et al., 2016); paired combinatorial logit SUE model with fixed and elastic demand (Bekhor and Prashker, 1999; Ryu et al., 2014); generalized nested logit SUE model (Bekhor and Prashker, 2001); C-logit SUE model with fixed and elastic demand (Zhou et al., 2010; Xu and Chen, 2013); path-size logit SUE model (Chen et al., 2012); weibit-based SUE model with fixed and elastic demand (Kitthamkesorn and Chen, 2013, 2014; Kitthamkesorn et al., 2015), spatially correlated logit model in the combined distribution and assignment problem (Yao et al., 2014)), and several emerging technological applications such modeling the range anxiety of electric vehicle users using a path-constrained traffic assignment model (Wang et al., 2016), public charging stations with a combined distribution and assignment model for capturing the travel demand distribution of plug-in hybrid electric vehicles (He et al., 2013), ridesharing as a new mode choice option in a network equilibrium framework (Bahat and Bekhor, 2016), just to name a few. Suffice to say, the MP formulation, despite the need to make many mathematically convenient assumptions compared to VI and FP formulations, it has its own appeal as reflected by numerous applications in integrating advanced discrete choice models and modeling emerging technologies within a network equilibrium framework.

The remainder of this paper is organized as follows. A list of notation is provided in section 2. Section 3 describes the weibit-based models for both mode choice and route choice. Specifically, a new nested weibit (NW) model is developed for mode choice, and a path-size weibit (PSW) model is adopted for route choice. Section 4 provides the MP formulation and solution properties of the NW-PSW model, and solution procedure for solving the NW-PSW model. Section 5 provides several numerical experiments to illustrate the features of the weibit-based CMSTA model and its application to evaluate green transportation policies. Finally, some concluding remarks are provided in Section 6.

2 LIST OF NOTATIONS

This section provides a list of notations used in this study unless specified otherwise. The notations are classified into three group as follows.

Indices

A: a set of links

IJ : a set of origin-destination (O-D) pairs

U_{ij} : a set of upper nests between O-D pair $ij \in IJ$

M_{iju} : a set of transportation mode alternatives under the upper nest $u \in U_{ij}$ between O-D pair $ij \in IJ$

R_{ijum} : a set of routes in mode $\mathrm{m} \in M_{iju}$ under the upper nest $u \in U_{ij}$ between O-D pair $ij \in IJ$

Variable

ξ_{ijm} : the Gumbel distributed random error of mode $\mathrm{m} \in M_{iju}$ between O-D pair $ij \in IJ$

ε_{ijm} : the Weibull distributed random error of mode $\mathrm{m} \in M_{iju}$ between O-D pair $ij \in IJ$

ε_{umr}^{ij} : the Weibull distributed random error of route $r \in R_{ijum}$ in mode $\mathrm{m} \in M_{iju}$ under the upper nest $u \in U_{ij}$ between O-D pair $ij \in IJ$

V_{ijm} : the deterministic (observed) utility of mode $\mathrm{m} \in M_{iju}$ between O-D pair $ij \in IJ$

g_{umr}^{ij} : the travel cost on route $r \in R_{ijum}$ in mode $\mathrm{m} \in M_{iju}$ under the upper nest $u \in U_{ij}$ between O-D pair $ij \in IJ$

q_{ij} : the travel demand between O-D pair $ij \in IJ$

q_{um}^{ij} : the travel demand of mode $\mathrm{m} \in M_{iju}$ in nest $u \in U_{ij}$ between O-D pair $ij \in IJ$

f_{umr}^{ij} : the traffic flow on route $r \in R_{ijum}$ in mode $\mathrm{m} \in M_{iju}$ under the upper nest $u \in U_{ij}$ between O-D pair $ij \in IJ$

τ_a : the travel cost on link $a \in A$

Parameters

φ_{iju} : the specific parameter of nest $u \in U_{ij}$ between O-D pair $ij \in IJ$

ζ_{um}^{ij} : the Weibull location parameter of mode $\mathrm{m} \in M_{iju}$ under the upper nest $u \in U_{ij}$ between O-D pair $ij \in IJ$

β_{um}^{ij} : the Weibull shape parameter of mode $\mathrm{m} \in M_{iju}$ under the upper nest $u \in U_{ij}$ between O-D pair $ij \in IJ$

ϖ_{umr}^{ij} : the path-size factor on route $r \in R_{ijum}$ in mode $\mathrm{m} \in M_{iju}$ under the upper nest $u \in U_{ij}$ between O-D pair $ij \in IJ$

3 WEIBIT-BASED MODELS

In this section, we provide some background on the nested logit (NL) model and develop the nested weibit (NW) model for mode choice. We also provide some background on the recently developed path-size weibit model for route choice.

2.1 Nested logit model

The NL model was developed to partially relax the independence assumption of the multinomial logit (MNL) model (Ben-Akiva and Lerman, 1985). It uses a two-level tree structure to account for the similarities among the alternatives. Using modes as the alternatives, Fig. 1 shows the mode alternatives $m \in M_{iju}$ sharing the same upper nest $u \in U_{ij}$ between origin-destination (O-D) pair $ij \in IJ$ are correlated (Marzano and Papola, 2008). The random utility maximization (RUM) model of the NL model can be presented as an additive form:

$$U_{ijm} = V_{ijm} + \xi_{ijm}, \forall m \in M_{iju}, u \in U_{ij}, ij \in IJ \, , \tag{1}$$

where V_{ijm} is the deterministic utility, and ξ_{ijm} is the Gumbel distributed random error with the marginal cumulative distribution function (CDF)

$$F_{\xi_{ijm}} = \exp\left(-e^{-\xi_{ijm}}\right), \tag{2}$$

and the joint CDF

$$H = \exp\left(-\sum_{u \in U_{ij}} \left(\sum_{m \in M_{iju}} e^{-\frac{\xi_{ijm}}{\varphi_{iju}}}\right)^{\varphi_{iju}}\right), \tag{3}$$

$\varphi_{iju} \in [0,1]$ is the upper-nested-specific parameter, which can be considered as a degree of correlation between alternatives (Ben-Akiva and Lerman, 1985). A smaller φ_{iju} indicates a higher correlation between modes under nest u. When φ_{iju} equal to 1, the NL model collapses to the MNL model.

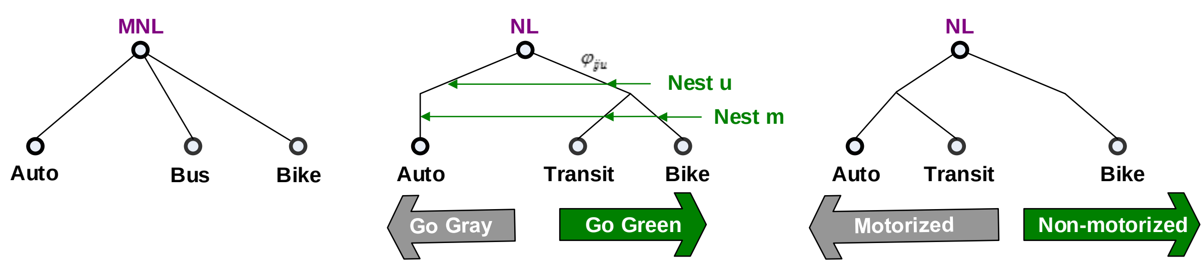


Fig. 1. Tree representations of the MNL and NL models

The NL probability can be derived from (Ben-Akiva and Lerman, 1985)

$$\begin{aligned} P_m^{ij} &= \Pr\left(V_{ijm} + \xi_{ijm} \geq V_{ijn} + \xi_{ijn}, \forall n \neq m\right), \; \forall m \in M_{iju}, u \in U_{ij}, ij \in IJ \\ &= \Pr\left(V_{ijm} - V_{ijn} + \xi_{ijm} \geq \xi_n^{ij}, \forall n \neq m\right), \; \forall m \in M_{iju}, u \in U_{ij}, ij \in IJ \end{aligned} \tag{4}$$

through

$$P_m^{ij} = \int_{-\infty}^{+\infty} H_m^{ij}\left(..,\xi_{ijm},..\right) d\xi_{ijm}, \tag{5}$$

where H_m^{ij} is the partial derivative of the joint CDF with respect to (w.r.t.) ξ_{ijm}. Note that the negative sign is added when the joint survival is used. Then, the NL probability can be expressed as

$$P_m^{ij} = \frac{\exp\left(\frac{1}{\varphi_{iju}} V_{ijm}\right)\left[\sum_{n\in M_{iju}} \exp\left(\frac{1}{\varphi_{iju}} V_{ijn}\right)\right]^{\varphi_{iju}-1}}{\sum_{t\in U_{ij}}\left[\sum_{s\in M_{ijt}} \exp\left(\frac{1}{\varphi_{ijt}} V_{ijs}\right)\right]^{\varphi_{ijt}}}, \forall m \in M_{iju}, u \in U_{ij}, ij \in IJ . \tag{6}$$

2.2 Nested weibit model

Although the NL model can relax the independent issue, it still encounters the identical perception variance issue, where each mode alternative has the same and fixed perception variance of $\pi^2/6$ (Marzano and Papola, 2008). To overcome this drawback, we develop a nested weibit (NW) model from a copula viewpoint (Nelsen, 2006) as follows. Recall that the joint Gumbel distribution for the NL model can be expressed in Eq. (3), and its marginal CDF is presented in Eq. (2). Let u_{ijm} be the marginal CDF. Thus, the random error can be presented as

$$\xi_{ijm} = -\ln\left(-\ln u_{ijm}\right). \tag{7}$$

Using the inverse method (see Nelsen, 2006), we have the copula by substituting Eq. (7) in Eq. (3) (e.g., Bhat, 2009), i.e.,

$$C = \exp\left(-\sum_{u\in U_{ij}}\left(\sum_{m\in M_{iju}}\left(-\ln u_{ijm}\right)^{\frac{1}{\varphi_{iju}}}\right)^{\varphi_{iju}}\right). \tag{8}$$

Now consider the Weibull distributed random error of mode *m* between O-D pair *ij* whose CDF is

$$F_{\varepsilon_{ijm}} = 1 - \exp\left(-\varepsilon_{ijm}\right). \tag{9}$$

Then, the marginal survival function of the weibit model can be expressed as

$$\bar{u}_{ijm} = \bar{F}_{\varepsilon_{ijm}} = \exp\left(-\varepsilon_{ijm}\right). \tag{10}$$

Substituting Eq. (10) into the copula in Eq. (8), we have a joint Weibull survival function

$$\bar{H} = \exp\left(-\sum_{u \in U_{ij}} \left(\sum_{m \in M_{iju}} \left(\varepsilon_{ijm} \right)^{\frac{1}{\varphi_{iju}}} \right)^{\varphi_{iju}} \right). \tag{11}$$

The NW RUM model can be presented as a multiplicative form:

$$U_{ijm} = V_{ijm} \varepsilon_{ijm}, \quad \forall m \in M_{iju}, u \in U_{ij}, ij \in IJ . \tag{12}$$

The NW probability can be derived from

$$\begin{aligned} P_{mr}^{ij} &= \Pr\left(V_{ijm}\varepsilon_{ijm} \geq V_{ijn}\varepsilon_{ijn}, \forall n \neq m \right), \ \forall m \in M_{iju}, u \in U_{ij}, ij \in IJ \\ &= \Pr\left(V_{ijm}\varepsilon_{ijm} / V_{ijn} \geq \varepsilon_{ijn}, \forall n \neq m \right), \ \forall m \in M_{iju}, u \in U_{ij}, ij \in IJ. \end{aligned} \tag{13}$$

Using Eq. (5), we have

$$P_m^{ij} = \int_0^{+\infty} \left(\varepsilon_{ijm} \right)^{\frac{1}{\varphi_{iju}}-1} \left(\sum_{\substack{l \in M_{iju} \\ u \in U_{ij}}} \left(\varepsilon_{ijl} \right)^{\frac{1}{\varphi_{iju}}} \right)^{\varphi_{iju}-1} \exp\left(-\sum_{v \in U_{ij}} \left(\sum_{n \in M_{ijv}} \left(\varepsilon_{ijn} \right)^{\frac{1}{\varphi_{ijv}}} \right)^{\varphi_{ijv}} \right) d\varepsilon_{ijm}. \tag{14}$$

Substituting Eq. (13) into Eq. (14) gives

$$P_m^{ij} = \left(V_{ijm} \right)^{1-\frac{1}{\varphi_{iju}}} \left(\sum_{\substack{l \in M_{iju} \\ u \in U_{ij}}} \left(V_{ijl} \right)^{-\frac{1}{\varphi_{iju}}} \right)^{\varphi_{iju}-1} \int_0^{+\infty} \exp\left(-V_{ijm}\varepsilon_{ijm} \sum_{v \in U_{ij}} \left(\sum_{n \in M_{ijv}} \left(V_{ijn} \right)^{-\frac{1}{\varphi_{ijv}}} \right)^{\varphi_{ijv}} \right) d\varepsilon_{ijm}. \tag{15}$$

Then, we have the NW probability expression:

$$P_m^{ij} = \frac{\left(V_{ijm} \right)^{-\frac{1}{\varphi_{iju}}} \left[\sum_{n \in M_{iju}} \left(V_{ijn} \right)^{-\frac{1}{\varphi_{iju}}} \right]^{\varphi_{iju}-1}}{\sum_{t \in U_{ij}} \left[\sum_{s \in M_{ijt}} \left(V_{ijs} \right)^{-\frac{1}{\varphi_{ijt}}} \right]^{\varphi_{ijt}}}, \forall m \in M_{iju}, u \in U_{ij}, ij \in IJ . \tag{16}$$

Following the Weibull distribution variance (see Kitthamkesorn and Chen, 2013 for more details), the NW model has a mode-specific perception variance as a function of V_{ijm} as follows:

$$\left(\sigma_m^{ij} \right)^2 = \left(V_{ijm} \right)^2, \quad \forall m \in M_{iju}, u \in U_{ij}, ij \in IJ . \tag{17}$$

Equation (17) is the result of assuming the shape parameter and location parameter of the Weibull distribution as 1 and 0, which leads to the exponential distribution with its mean equals to the standard deviation. Hence, a larger deterministic term value would lead to a higher perception variance. However, one can incorporate the shape and location parameters in the NW model similar to that in Kitthamkesorn and Chen (2013) for modelling flexibility.

Consider the three-mode example in Fig. 1. We assume that V_{ijm} for auto, transit, and bike are 4, 2.5, and 1, respectively. Without loss of generality, we compare the MNL, NL, and NW models under the go-green and go-gray scheme (i.e., middle tree structure in Fig. 1) by setting the model parameters as $\varphi_{iju} = 1$ for the auto mode, and varying φ_{iju} from 0.25, 0.5, and 1 for the transit and bike modes that share the upper nest. Table 1 presents the choice probability for all modes of each model. Some observations for the models are summarized as follows:

- The mode shares of the three modes (auto, transit, and bike) for the three models (MNL, NL and NW) satisfy conservation (i.e., the sum of the three mode choice probabilities for all values of φ_{iju} equals to 1.0).
- The MNL model gives the same mode choice probability for all values of φ_{iju}, while the NL model and NW model give different results. As φ_{iju} for the transit and bike modes increases, both NL and NW models give a higher probability to the transit and bike modes.
- When $\varphi_{iju} = 1$, the NL model gives identical results as those in the MNL model (i.e., the NL model collapses to the MNL model). However, this is not the case for the NW model since the two models use different random error distributions (Gumbel for MNL and NL and Weibull for NW).
- On the other hand, the NW model collapses to the multinomial weibit (MNW) model (Castillo et al., 2008) when $\varphi_{iju} = 1$, i.e.,

$$P_m^{ij} = \frac{\left(V_{ijm}\right)^{-1}}{\sum_{s \in M_{ij}} \left(V_{ijs}\right)^{-1}}, \forall m \in M_{ij}, ij \in IJ \ . \tag{18}$$

- The logit models seem to give a higher probability on the go-green mode choice compared to that of the NW model. This is because the MNL and NL models assume the same and fixed perception variance for all modes.

Table 1: Mode choice probability for the go-green and go-gray scheme

φ_{iju}	0.25			0.5			0.75			1		
Model	Auto	Transit	Bike	Auto	Transit	Bike	Auto	Transit	Bike	Auto	Transit	Bike
MNL	0.786	0.175	0.039	0.786	0.175	0.039	0.786	0.175	0.039	0.786	0.175	0.039
NL	0.817	0.181	0.001	0.814	0.177	0.009	0.803	0.174	0.023	0.786	0.175	0.039
NW	0.614	0.376	0.010	0.598	0.347	0.055	0.569	0.334	0.097	0.533	0.333	0.133

Next, we examine the impact of adding more utility to transit and bike using $\varphi_{iju} = 0.75$ (i.e., column 3 in Table 1). Three scenarios are created by adding an incentive of 0.25, 0.5, and 1.0 to the utility of transit and bike as shown in Table 2. Recall the base utility V_{ijm} is 2.5 for transit and 1 for bike. For scenario 1, the new utility with an incentive of 0.25 would be 2.75 (2.5+0.25) for transit and 1.25 (1+0.25) for bike. Similarly, the new utilities are 3 and 1.5 for scenario 2 with an incentive of 0.5, and 3.5 and 2 for scenario 3 with an incentive of 1, respectively. The results show that both transit and bike modes receive a higher probability as the incentive increases. The MNL and NL models seem to be more sensitive to the incentive than the NW model. This is because both logit models have the fixed perception variance of $\pi^2/6$ while the NW model has the perception variance as a function of the deterministic utility (see Eq. (17)).

Note that there exists a similar probability increasing pattern in the transit and bike modes for both MNL and NL models. This is because the lower nest of the NL model is similar to that of the MNL model. It is not sensitive to the additive utility due to the fixed and same perception variance (i.e., only concern with the utility difference in computing the probability) as shown in Fig. 2. The NL probability pattern for the auto mode does not have such a problem since the logsum propagates from the lower nest is sensitive to the additive utility. On the other hand, the NW model gives different mode choice probability patterns for each additional unit of incentive. The mode choice in the lower nest is the MNW model. The mode-specific perception variance can be calculated from Eq. (17), where a higher utility is associated with a larger perception variance. When propagating the MNW probability to the upper nest, we can represent it as an inverse of the summation of mode utility in the lower nest. For example, we can show from this go-green and go-gray scheme as

$$P_{auto}=\frac{\left(V_{auto}\right)^{-1}}{\left(V_{auto}\right)^{-1}+\left[\left(V_{transit}\right)^{-\frac{1}{\varphi_{transit,bike}}}+\left(V_{bike}\right)^{-\frac{1}{\varphi_{transit,bike}}}\right]^{\varphi_{transit,bike}}}=\frac{\left(V_{auto}\right)^{-1}}{\left(V_{auto}\right)^{-1}+\left(V_{transit,bike}\right)^{-1}}, \tag{19}$$

where

$$V_{transit,bike}=\left[\left(V_{transit}\right)^{-\frac{1}{\varphi_{transit,bike}}}+\left(V_{bike}\right)^{-\frac{1}{\varphi_{transit,bike}}}\right]^{-\frac{1}{\varphi_{transit,bike}}}. \tag{20}$$

To further explore the difference between the NL and NW models, we consider the direct elasticity and cross elasticity. Let x_{mq}^{ij} be an attribute *q* in the deterministic utility V_{ijm}, where $V_{ijm}=\rho_0^{ij}+\sum_q \rho_q^{ij} x_{mq}^{ij}$. The direct elasticity describes the effect of a change in the attribute x_{mq}^{ij} of mode *m* on the probability of choosing mode *m*, and the cross elasticity describes the effect of a change in x_{nq}^{ij} of mode *n*≠*m* on the probability of choosing mode *m*. Obviously, the direct and cross elasticities of the NW model are different from those of the NL model as presented in Table 3. There are two main differences between the elasticities of the two models. First, only the attribute x_{nq}^{ij} and the probability P_m^{ij} play a key role in the elasticity in the NL model. The elasticity of the NW model, on the other hand, includes not only the attribute x_{nq}^{ij} and the probability P_m^{ij}, but also the deterministic utility V_{ijm}. This is because the NL model is based on an exponential function while the NW model is based on a power function. Second, the sign of the elasticity is different between the NL and NW models. This indicates that the deterministic utility V_{ijm} of the NL model could be different from that of the NW model. For example, the NL model uses the negative value of the travel time to consider the disutility. The NW model, in contrast, uses the positive value of the travel time directly to compute the mode choice probability. Note that, from the cross elasticity, the NW could provide similar characteristics as the NL model for the mode choices that share the same upper nest. This is because the NW model could be insensitive to the multiplicative increasing of the utility (Xu *et al.*, 2016). We can adopt the Weibull parameters like the MNW model and PSW model to enhance the model flexibility in handling mode-specific perception variance.

Table 2: Impact of additional mode incentive under the go-green and go-gray scheme

Scenario	1			2			3		
Incentive	0	+0.25		0	+0.5		0	+1	
Model	Auto	Transit	Bike	Auto	Transit	Bike	Auto	Transit	Bike
MNL	0.741 (-5.74%)	0.212 (21.03%)	0.047 (21.03%)	0.690 (-12.21%)	0.254 (44.74%)	0.057 (44.74%)	0.574 (-26.92%)	0.348 (98.65%)	0.078 (98.65%)
NL	0.760 (-5.3%)	0.211 (21.6%)	0.029 (21.6%)	0.712 (-11.33%)	0.254 (46.18%)	0.034 (46.18%)	0.600 (-25.29%)	0.352 (103.07%)	0.048 (103.07%)
NW	0.537 (-5.49%)	0.343 (2.89%)	0.120 (22.01%)	0.509 (-10.44%)	0.351 (5.45%)	0.139 (41.99%)	0.461 (-18.98%)	0.366 (9.8%)	0.173 (76.66%)

*The number in () presents the percentage change in mode choice probability compared to those in Table 1.

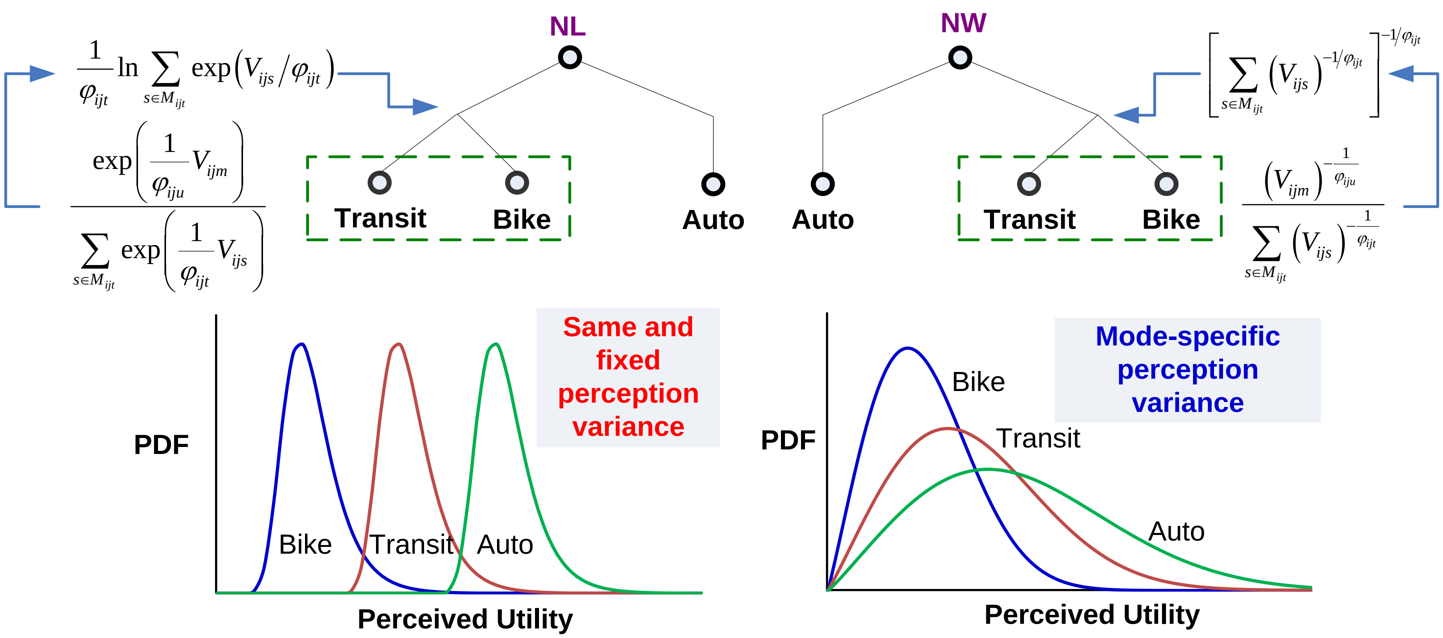


Fig. 2. Comparison between NL and NW models

Table 3: Direct elasticity and cross elasticity of the NL and NW models

Model	Direct elasticity	Cross elasticity
NL	$\frac{\rho_q^{ij} x_{mq}^{ij}}{\varphi_u^{ij}}\left(1-\varphi_u^{ij} P_m^{ij}+\left(\varphi_u^{ij}-1\right) P_{m\mid u}^{ij}\right)$	*n* shares the same upper nest: $\frac{\rho_q^{ij} x_{nq}^{ij}}{\varphi_u^{ij}}\left(-\varphi_u^{ij} P_n^{ij}+\left(\varphi_u^{ij}-1\right) P_{\mathrm{n}\mid u}^{ij}\right)$ *n* does not share the same upper nest: $-\rho_q^{ij} x_{nq}^{ij} P_n^{ij}$
NW	$\frac{\rho_q^{ij} x_{mq}^{ij} V_{ijm}^{-1}}{\varphi_u^{ij}}\left(\varphi_u^{ij} P_m^{ij}+\left(\varphi_u^{ij}-1\right) P_{m\mid u}^{ij}-1\right)$	*n* shares the same upper nest: $\frac{\rho_q^{ij} x_{nq}^{ij} V_{ijn}^{-1}}{\varphi_u^{ij}}\left(\varphi_u^{ij} P_n^{ij}-\left(\varphi_u^{ij}-1\right) P_{\mathrm{n}\mid u}^{ij}\right)$ *n* does not share the same upper nest: $\rho_q^{ij} x_{nq}^{ij} V_{ijn}^{-1} P_n^{ij}$

Remark: $P_{\mathrm{n}\mid u}^{ij}$ is the lower nest probability as shown in Fig. 2.

In addition, the NW model has the covariance as a function of V_{ijm}.

Proposition 1. *The covariance between modes in the nested weibit model is a function of the deterministic utility* V_{ijm}.

Proof. The covariance can be calculated through

$$\operatorname{cov}\left[U_{ijm}, U_{ijn}\right]=\mathrm{E}\left[U_{ijm} U_{ijn}\right]-E\left[U_{ijm}\right] E\left[U_{ijn}\right], \tag{21}$$

where E[] is the expected value. We begin with the first term of the right hand side (RHS). From Eq. (12), we have

$$\mathrm{E}\left[U_{ijm} U_{ijn}\right]=E\left[V_{ijm} \varepsilon_{ijm} V_{ijn} \varepsilon_{ijn}\right]=V_{ijm} V_{ijn} E\left[\varepsilon_{ijm} \varepsilon_{ijn}\right]. \tag{22}$$

Now, consider the second term of the RHS in Eq. (21). From the marginal of ε_{ijm} in Eq. (9), $E\left[U_{ijm}\right]=E\left[V_{ijm} \varepsilon_{ijm}\right]=V_{ijm} E\left[\varepsilon_{ijm}\right]$ and is equal to V_{ijm}. As such, the nested weibit covariance is

$$\operatorname{cov}\left[U_{ijm}, U_{ijn}\right]=V_{ijm} V_{ijn} E\left[\varepsilon_{ijm} \varepsilon_{ijn}\right]-V_{ijm} V_{ijn}. \tag{23}$$

Note that when U_{ijm} and U_{ijn} are not under the same upper nest, $E\left[\varepsilon_{ijm} \varepsilon_{ijn}\right]$ = $E\left[\varepsilon_{ijm}\right] E\left[\varepsilon_{ijn}\right]$ = 1 and the covariance is equal to zero. In contrast, if U_{ijm} and U_{ijn} are under the same upper nest, $E\left[\varepsilon_{ijm} \varepsilon_{ijn}\right]$ is not equal to zero, and hence the covariance is a function of V_{ijm}. This completes the proof. □

This is in contrast to the nested logit (NL) model. The NL model covariance is a function of the random error term alone. From Eq. (1), we have

$$\begin{aligned}\operatorname{cov}\left[U_{ijm},U_{ijn}\right] &= \mathrm{E}\left[U_{ijm}U_{ijn}\right]-E\left[U_{ijm}\right]E\left[U_{ijn}\right] \\ &= E\left[\left(V_{ijm}+\xi_{ijm}\right)\left(V_{ijn}+\xi_{ijn}\right)\right]-E\left[V_{ijm}+\xi_{ijm}\right]E\left[V_{ijn}+\xi_{ijn}\right] \\ &= E\left[V_{ijm}V_{ijn}+V_{ijm}\xi_{ijn}+V_{ijn}\xi_{ijm}+\xi_{ijm}\xi_{ijn}\right]-\left(V_{ijm}+E\left[\xi_{ijm}\right]\right)\left(V_{ijn}+E\left[\xi_{ijn}\right]\right) \\ &= E\left[\xi_{ijm}\xi_{ijn}\right]-E\left[\xi_{ijm}\right]E\left[\xi_{ijn}\right]\end{aligned}. \tag{24}$$

From the above equation, the NL model covariance is a function of the expected values of the random error term only. Hence, the NL model covariance is fixed and independent to V_{ijm}.

Note that the CDF of $\varepsilon_{ijm}\varepsilon_{ijn}$ can be determined by Manski and McFadden (1981) as follows:

$$H_{\phi_{ij1n}..\phi_{ijpm}}\left(t_{ij1n}..t_{ijpm}\right)=\int_{-\infty}^{\infty}\left.\frac{\partial H_{\varepsilon_{ij1}..\varepsilon_{ijm}}\left(t_{ij1}..t_{ijm}\right)}{\partial t_{ijm}}\right|_{t_{ijn}=\phi_{ijmn}/x_{ijm},\forall n} dx_{ijm}, \tag{25}$$

where $\phi_{ijmn}=\varepsilon_{ijm}\varepsilon_{ijn}$. From Eq. (11), we have

$$\frac{\partial H_{\varepsilon_{ij1}..\varepsilon_{ijm}}\left(t_{ij1}..t_{ijm}\right)}{\partial t_{ijm}}=\left(t_{ijm}\right)^{\frac{1}{\varphi_{iju}}-1}\left(\sum_{\substack{l\in M_{iju}\\ u\in U_{ij}}}\left(t_{ijl}\right)^{\frac{1}{\varphi_{iju}}}\right)^{\varphi_{iju}-1}\exp\left(-\sum_{v\in U_{ij}}\left(\sum_{n\in M_{ijv}}\left(t_{ijn}\right)^{\frac{1}{\varphi_{ijv}}}\right)^{\varphi_{ijv}}\right). \tag{26}$$

Substituting $\phi_{ijmn}=\varepsilon_{ijm}\varepsilon_{ijn}$ into Eq. (25) and Eq. (26) gives

$$\begin{aligned}H_{\phi_{ij1n}..\phi_{ijpm}}\left(\;\right) &= -\int_{0}^{\infty}\left(\frac{t_{ijmm}}{x_{ijm}}\right)^{\frac{1}{\varphi_{iju}}-1}\left(\sum_{\substack{l\in M_{iju}\\ u\in U_{ij}}}\left(\frac{t_{ijlm}}{x_{ijm}}\right)^{\frac{1}{\varphi_{iju}}}\right)^{\varphi_{iju}-1}\exp\left(-\sum_{v\in U_{ij}}\left(\sum_{n\in M_{ijv}}\left(\frac{t_{ijnm}}{x_{ijm}}\right)^{\frac{1}{\varphi_{ijv}}}\right)^{\varphi_{ijv}}\right)dx_{ijm} \\ &= -t_{ijmm}\left(\sum_{\substack{l\in M_{iju}\\ u\in U_{ij}}}\left(t_{ijlm}\right)^{\frac{1}{\varphi_{iju}}}\right)^{\varphi_{iju}-1}\int_{0}^{\infty}\exp\left(-\frac{1}{x_{ijm}}\sum_{v\in U_{ij}}\left(\sum_{n\in M_{ijv}}\left(t_{ijnm}\right)^{\frac{1}{\varphi_{ijv}}}\right)^{\varphi_{ijv}}\right)dx_{ijm}\end{aligned} \tag{27}$$

The second term of the RHS needs to be computed numerically. With this, one can approximate $H_{\phi_{ij1n}..\phi_{ijpm}}\left(t_{ij1n}..t_{ijpm}\right)$ and its corresponding PDF (i.e., $h_{\phi_{ij1n}..\phi_{ijpm}}\left(t_{ij1n}..t_{ijpm}\right)$. Then, we can determine $E\left[\varepsilon_{ijm}\varepsilon_{ijn}\right]=E\left[\phi_{ijmn}\right]$ to find the covariance in Eq. (23). Note that the correlation between the mode alternatives (i.e., $corr = \frac{\text{cov}\left[U_{ijm}, U_{ijn}\right]}{\sigma_m^{ij}\sigma_n^{ij}}$) is not affected by the change in V_{ijm}. From Eq. (17) and Eq. (23), the correlation between the mode alternatives is equal to

$$\text{corr}\left[U_{ijm}, U_{ijn}\right] = E\left[\varepsilon_{ijm}\varepsilon_{ijn}\right] - 1. \tag{28}$$

As such, the correlation depends on $E\left[\varepsilon_{ijm}\varepsilon_{ijn}\right]$, which is based on the joint distribution. In other words, the correlation is governed by φ_{iju}.

Similar to the previous example presented in Table 2, we examine the impact of the NW covariance feature using φ_{iju} = 0.75 (i.e., column 3 in Table 1). Three scenarios are created by adding an incentive of 1, 1.5, and 2 to the utility of transit mode only as shown in Table 4. As expected, the change in the transit probability is higher in the NL model. This is because the NL model has a fixed covariance (see e.g., Marzano and Papola, 2008). In contrast, the NW covariance is a function of the utility. The covariance is increased as the utility increases. Incorporating this feature with the perception variance, which is also a function of the utility, the change in the transit probability is thus smaller for the NW model.

Table 4: An investigation of covariance impact under the go-green and go-gray scheme

Scenario	1			2			3		
Incentive	0	+1	0	0	+1.5	0	0	+2	0
Model	Auto	Transit	Bike	Auto	Transit	Bike	Auto	Transit	Bike
NL	0.773	0.216	0.011	0.726	0.261	0.013	0.617	0.365	0.018
		[1.279]*			[1.279]			[1.279]	
NW	0.570	0.357	0.074	0.544	0.365	0.091	0.498	0.378	0.124
		[2.408]			[3.016]			[3.704]	

* [*covariance*] between transit and bike modes using a numerical technique.

2.3 Path-size weibit model

To relax the identical perception variance issue in the MNL model, Castillo *et al*. (2008) developed the multinomial weibit (MNW) model from the Weibull distribution. Then, Kitthamkesorn and Chen (2013) introduced a path-size factor to the MNW utility function to develop the path-size weibit (PSW) model. This model can be expressed as the RUM model as

$$U_{umr}^{ij} = \frac{\left(g_{umr}^{ij} - \zeta_{um}^{ij}\right)^{\beta_{um}^{ij}}}{\varpi_{umr}^{ij}} \varepsilon_{umr}^{ij}, \quad \forall r \in R_{ijum}, \mathrm{m} \in M_{iju}, \mathrm{u} \in U_{ij}, ij \in IJ, \tag{29}$$

where ε_{umr}^{ij} is the Weibull distributed random error on route r in mode m under nest u between O-D pair ij, g_{umr}^{ij} is the travel cost on route r in mode m under nest u between O-D pair ij, ζ_{um}^{ij} and β_{um}^{ij} are the Weibull parameters related to the route-specific perception variance, and $\varpi_{umr}^{ij} \in (0,1]$ is the path-size factor which can be presented as (Ben-Akiva and Bierlaire, 1999)

$$\varpi_{umr}^{ij} = \sum_{a \in \Upsilon_{umr}} \frac{l_{uma}}{L_{umr}^{ij}} \frac{1}{\sum_{k \in R_{ijum}} \delta_{umak}^{ij}}, \quad \forall r \in R_{ijum}, \mathrm{m} \in M_{iju}, \mathrm{u} \in U_{ij}, ij \in IJ, \tag{30}$$

where l_{uma} is the length of link a under nest u in mode m, L_{umr}^{ij} is the length of route r under nest u in mode m connecting O-D pair ij, Υ_{umr} is the set of all links in route r under nest u in mode m, δ_{umar}^{ij} is equal to 1 for link a on route r under nest u in mode m between O-D pair ij and 0 otherwise. The lengths in the common part and the route ratio (i.e., l_{uma} / L_{umr}^{ij}) approximate the route correlation, and $\sum_{k \in R_{ijum}} \delta_{umak}^{ij}$ measures the contribution of link a in the route correlation (Frejinger and Bierlaire, 2007). This path-size factor accounts for different route sizes determined by the length of links within a route and the relative lengths of routes that share a link (Ben-Akiva and Bierlaire, 1999). With the PSW RUM model in Eq. (29), the PSW probability can be presented as

$$P_{umr}^{ij} = \frac{\varpi_{umr}^{ij} \left(g_{umr}^{ij} - \zeta_{um}^{ij}\right)^{-\beta_{um}^{ij}}}{\sum_{k \in R_{ijum}} \varpi_{umk}^{ij} \left(g_{k}^{ij} - \zeta_{um}^{ij}\right)^{-\beta_{um}^{ij}}}, \quad \forall r \in R_{ijum}, \mathrm{m} \in M_{iju}, \mathrm{u} \in U_{ij}, ij \in IJ. \tag{31}$$

4 COMBINED MODAL SPLIT AND TRAFFIC ASSIGNMENT PROBLEM

In this section, we provide the assumptions, the mathematical programming (MP) formulation for the combined modal split and traffic assignment (CMSTA) problem using the nested weibit (NW) mode choice model and path-size weibit (PSW) route choice model, and some properties of the MP formulation. Let q_{ij} be the travel demand between O-D pair ij, and q_{um}^{ij} be the travel demand of mode m in nest u between O-D pair ij. After applying the NW and PSW probabilities as a function of the route cost as a function of τ_a travel cost on link a and Ψ_{ijum} exogenous modal attractiveness on mode m in nest u between O-D pair ij, we have f^{ij}_{umr} traffic flow on route r in mode m and nest u between O-D pair ij. With a route/link relationship, we have v_a traffic flow on link a. To begin with, some assumptions are made.

4.1 Assumptions

Assumption 1. The travel cost τ_a which could be a function of the travel time is a strictly increasing function w.r.t. its own flow.

Since we cannot easily decompose ζ^{ij}_{um} into the link level, we make another assumption:

Assumption 2. $\zeta^{ij}_{um} = 0$.

Note that the variational inequality (VI) formulation can be adopted to incorporate ζ^{ij}_{um} (e.g., Zhou *et al*., 2009). Since the weibit model is the *multiplicative* RUM model, the deterministic part is simply a set of *multiplicative* explanatory variables (e.g., Cooper and Nakanishi, 1988). Then, the route travel cost assumption is:

Assumption 3. The route travel cost function consists of the multiplicative link travel costs, i.e.,

$$g^{ij}_{umr} = \prod_{a \in \Upsilon_{umr}} \tau_a , \quad \forall r \in R_{ijum}, m \in M_{iju}, u \in U_{ij}, ij \in IJ . \tag{32}$$

This assumption maintains the weibit relative cost criterion where the travelers are assumed to make a decision based on the relative difference of the utility (Fosgerau and Bierlaire, 2009). It makes the route travel cost decomposable to a link level and workable with the Beckmann's transformation (i.e., multiplicative Beckmann objective function). One possible non-linear travel cost functional forms could be the exponential function of travel time. With the exponential travel cost function, travelers could be assumed as a constant risk averse when selecting the route (Mirchandani and Soroush, 1987).

Following the path-size logit (PSL) SUE formulation (Chen *et al.,* 2012), the path-size factors are assumed to be flow independent:

Assumption 4. l_a and L^{ij}_{umr} are flow independent.

Note that we can also adopt the VI formulation (Zhou *et al*., 2012) to incorporate the flow dependent path-size factors.

4.2 Mathematical Programming Formulation

Based on the above assumptions, the mathematical program for the CMSTA problem can be formulated as follows:

$$\min Z = Z_1 + Z_2 + Z_3 + Z_4 + Z_5 + Z_6 \tag{33}$$

$$= \sum_{a \in A} \int_0^{v_a} \ln \tau_a(\omega) d\omega + \sum_{ij \in IJ} \sum_{u \in U_{ij}} \sum_{m \in M_{iju}} \sum_{r \in R_{ijm}} \frac{1}{\beta_{um}^{ij}} f_{umr}^{ij} \left(\ln f_{umr}^{ij} - 1 \right) - \sum_{ij \in IJ} \sum_{u \in U_{ij}} \sum_{m \in M_{iju}} \sum_{r \in R_{ijm}} \frac{1}{\beta_{um}^{ij}} f_{umr}^{ij} \ln \varpi_{umr}^{ij}$$
$$+ \sum_{ij \in IJ} \sum_{u \in U_{ij}} \sum_{m \in M_{iju}} \left(\varphi_{iju} - \frac{1}{\beta_{um}^{ij}} \right) q_{um}^{ij} \left(\ln q_{um}^{ij} - 1 \right) + \sum_{ij \in IJ} \sum_{u \in U_{ij}} \left(1 - \varphi_{iju} \right) \left(\sum_{m \in M_{iju}} q_{um}^{ij} \right) \left(\ln \left(\sum_{m \in M_{iju}} q_{um}^{ij} \right) - 1 \right)$$
$$+ \sum_{ij \in IJ} \sum_{u \in U_{ij}} \sum_{m \in M_{iju}} q_{um}^{ij} \ln \Psi_{ijum}$$

$$\text{s.t.} \sum_{r \in R_{ijum}} f_{umr}^{ij} = q_{um}^{ij}, \ \forall m \in M_{iju}, u \in U_{ij}, ij \in IJ, \tag{34}$$

$$\sum_{u \in U_{ij}} \sum_{m \in M_{iju}} q_{um}^{ij} = q_{ij}, \ \forall ij \in IJ, \tag{35}$$

$$f_{umer}^{ij} \geq 0, \ \forall r \in R_{ijum}, m \in M_{iju}, u \in U_{ij}, ij \in IJ;$$
$$q_{um}^{ij} \geq 0 \ \forall m \in M_{iju}, u \in U_{ij}, ij \in IJ, \tag{36}$$

where f_{umr}^{ij} is the traffic flow on route *r* in mode *m* and nest *u* between O-D pair *ij*; q_{um}^{ij} is the demand of mode *m* in nest *u* between O-D pair *ij*; q_{ij} is the demand between O-D pair *ij*; and Ψ_{ijum} is an exogenous modal attractiveness on mode m in nest *u* between O-D pair *ij*. Eq. (33) is the objective function of the combined NW-PSW model, which consists of six terms. Each term has its own meaning and its contribution to the Karush-Kuhn-Tucker conditions in deriving the NW and PSW probability expressions for mode choice and route choice, respectively. These six terms are: Z_1 is the multiplicative Beckmann's transformation (Kitthamkesorn and Chen, 2013) corresponding to the multiplicative route travel cost; Z_2 is the well-known entropy term (Fisk, 1980) reflecting the stochastic effect of random perception; Z_3 is the penalty term using the path-size factor (Ben-Akiva and Bierlaire, 1999) to capture the similarities among the routes; Z_4 and Z_5 are respectively related to the conditional and marginal probabilities of the NW modal split function; and Z_6 is the attractiveness term incorporated to model the exogenous modal utility. The parameters in the objective function also play a role in the derivation of the closed-form probability expressions that are consistent with the nested weibit model for mode choice and the path-size weibit model for route choice (see the proofs for Propositions 2 and 3). Eq. (34) and Eq. (35) are the conservation constraints. Eq. (36) are the non-negativity constraints on the two sets of decision variables (i.e., modal splits and route flows).

Note that the combined NW-PSW model developed in this paper can be considered as an extension of the PSW stochastic user equilibrium (SUE) with elastic demand (ED) or PSW-SUE-ED for short and the combined MNL mode choice and PSW-SUE route choice (or combined MNL-PSW) model by Kitthamkesorn et al. (2015). The main differences between the combined NW-PSW model and the PSW-SUE-ED/combined MNL-PSW models include: (a) the decision variables (modal splits and route flows) need to account for the nested structure of mode choice (i.e., q_{um}^{ij} and f_{umr}^{ij} instead of q_{ij} and f_r^{ij} for the PSW-SUE-ED model or q_m^{ij} and f_{mr}^{ij} for the combined MNL-PSW model), and (b) the conditional and marginal probabilities of the NW modal split function represented by two entropy terms in Z_4 and Z_5. When $\varphi_{iju}=1$ (i.e., no nesting structure, just a single-level tree structure), the combined NW-PSW model becomesthe combined MNW-PSW model; when $\varpi_r^{ij}=1$ (i.e., no route overlapping), the combined NW-PSW model reduces to the combined NW-MNW model; and when both $\varphi_{iju}=1$ and $\varpi_r^{ij}=1$, the combined NW-PSW model collapses to the combined MNW-MNW model.

4.3 Solution Properties

In this section, some properties of the MP formulation are provided.

Proposition 2. *The MP formulation in Eqs. (33) through (36) provides the NW mode choice and the PSW route choice.*

Proof. The Lagrangian (L) of the equivalent MP problem w.r.t. the constraints can be formulated as:

$$L = Z + \sum_{ij\in IJ}\sum_{u\in U_{ij}}\sum_{m\in M_{iju}} \phi_{um}^{ij}\left(q_{um}^{ij} - \sum_{r\in R_{ijum}} f_{umr}^{ij}\right) + \sum_{ij\in IJ}\sum_{u\in U_{ij}} \lambda_{ij}\left(q_{ij} - \sum_{u\in U_{ij}}\sum_{m\in M_{iju}} q_{um}^{ij}\right), \tag{37}$$

where ϕ_{um}^{ij} and λ_{ij} are the dual variables associated with the conservation constraints.

Given that L has to be minimized w.r.t. route flows, the following conditions have to hold:

$$\partial L/\partial f_{umr}^{ij}=0 \;\Rightarrow\; \beta_{um}^{ij}\sum_{a\in A}\ln\tau_a\delta_{ra}^{ij}+\ln f_{umr}^{ij}-\ln\varpi_{umr}^{ij}=\beta_{um}^{ij}\phi_{um}^{ij}. \tag{38}$$

From **Assumption 3**, we have

$$f_{umr}^{ij}=\exp\left(\beta_{um}^{ij}\phi_{um}^{ij}\right)\varpi_{umr}^{ij}\left(g_{umr}^{ij}\right)^{-\beta_{um}^{ij}}. \tag{39}$$

The summation of f_{umr}^{ij} gives

$$q_{um}^{ij}=\sum_{r\in R_{ijum}}f_{umr}^{ij}=\exp\left(\beta_{um}^{ij}\phi_{um}^{ij}\right)\sum_{r\in R_{ijum}}\varpi_{umr}^{ij}\left(g_{umr}^{ij}\right)^{-\beta_{um}^{ij}}. \tag{40}$$

Dividing Eq. (39) by Eq. (40) gives the PSW route choice, i.e.,

$$P_{umr}^{ij}=\frac{f_{umr}^{ij}}{q_{um}^{ij}}=\frac{\varpi_{umr}^{ij}\left(g_{umr}^{ij}\right)^{-\beta_{um}^{ij}}}{\sum_{k\in R_{ijum}}\varpi_{umk}^{ij}\left(g_{umk}^{ij}\right)^{-\beta_{um}^{ij}}}. \tag{41}$$

Then, we consider the mode choice:

$$\partial L/\partial q_{um}^{ij}=0 \;\Rightarrow\; \left(\varphi_{iju}-\frac{1}{\beta_{um}^{ij}}\right)\ln q_{um}^{ij}+\left(1-\varphi_{iju}\right)\ln\left(\sum_{m\in M_{iju}}q_{um}^{ij}\right)+\ln\Psi_{ijum}+\phi_{um}^{ij}-\lambda_{ij}=0. \tag{42}$$

From Eq. (40), ϕ_{um}^{ij} can be defined as

$$\phi_{um}^{ij}=\frac{1}{\beta_{um}^{ij}}\ln q_{um}^{ij}-\frac{1}{\beta_{um}^{ij}}\ln\sum_{r\in R_{ijum}}\varpi_{umr}^{ij}\left(g_{umr}^{ij}\right)^{-\beta_{um}^{ij}}, \tag{43}$$

where the second term in the right hand side is the logarithmic expected travel cost (ETC) (Kitthamkesorn et al., 2015). Let w_{um}^{ij} be the logarithmic ETC. Eq. (42) can be rearranged as

$$q_{um}^{ij}\left(\sum_{m\in M_{iju}}q_{um}^{ij}\right)^{\frac{1-\varphi_{iju}}{\varphi_{iju}}}=\exp\left(\lambda_{ij}\right)\left(\Psi_{ijum}\exp\left(-w_{um}^{ij}\right)\right)^{\frac{1}{\varphi_{iju}}}, \tag{44}$$

$$\sum_{m\in M_{iju}}q_{um}^{ij}=\left[\exp\left(\lambda_{ij}\right)\sum_{m\in M_{iju}}\left(\Psi_{ijum}\exp\left(-w_{um}^{ij}\right)\right)^{\frac{1}{\varphi_{iju}}}\right]^{\varphi_{iju}}, \text{ and} \tag{45}$$

$$q_{ij}=\sum_{u\in U_{ij}}\sum_{m\in M_{iju}}q_{um}^{ij}=\left[\exp\left(\lambda_{ij}\right)\sum_{u\in U_{ij}}\left(\sum_{m\in M_{iji}}\Psi_{ijum}\exp\left(-w_{um}^{ij}\right)\right)^{\frac{1}{\varphi_{iju}}}\right]^{\varphi_{iju}}. \tag{46}$$

From Eq. (44) through Eq. (46), we have

$$P_{um}^{ij}=\frac{q_{um}^{ij}}{q_{ij}}=\frac{\left(\Psi_{ijum}\exp\left(-w_{um}^{ij}\right)\right)^{\frac{1}{\varphi_{iju}}}\left[\sum_{m\in M_{iju}}\left(\Psi_{ijum}\exp\left(-w_{um}^{ij}\right)\right)^{\frac{1}{\varphi_{iju}}}\right]^{\varphi_{iju-1}}}{\left[\sum_{t\in U_{ij}}\left(\sum_{s\in M_{iji}}\Psi_{ijts}\exp\left(-w_{ts}^{ij}\right)\right)^{\frac{1}{\varphi_{ijt}}}\right]^{\varphi_{ijt}}}. \tag{47}$$

Since $\Psi_{ijum}\exp\left(-w_{um}^{ij}\right)$ is the utility, let $V_{ijm}=\left(\Psi_{ijum}\exp\left(-w_{um}^{ij}\right)\right)^{-1}$. Then, the above equation is the NW mode choice. This completes the proof. □

Proposition 3. *The solution of NW-PSW model is unique.*

Proof. It is sufficient to prove that the objective function in Eq. (33) is strictly convex in the vicinity of route flows and modal splits and that the feasible region is convex.

The Hessian matrix of $Z_1+Z_2+Z_3$ w.r.t. the route flow variables can be defined as

$$\frac{\partial^2\left(Z_1+Z_2+Z_3\right)}{\partial f_{umr}^{ij}\partial f_{umk}^{ij}}=\begin{cases}\dfrac{1}{\tau_a}\dfrac{d\tau_a}{dv_a}\delta_{umra}^{ij}+\dfrac{1}{\beta_{um}^{ij}f_{umr}^{ij}}>0;\ r=k\\ 0 \qquad\qquad ;otherwise\end{cases}. \tag{48}$$

This implies the positive definite matrix. The Hessian matrix of the $Z_4+Z_5+Z_6$ w.r.t. the modal demand variables can be defined as

$$\frac{\partial^2\left(Z_4+Z_5+Z_6\right)}{\partial q_{um}^{ij}\partial q_{tn}^{ij}}=\begin{cases}\left(\varphi_{iju}-\dfrac{1}{\beta_{um}^{ij}}\right)\Big/q_{um}^{ij}+\left(1-\varphi_{iju}\right)\Big/\sum_{m\in M_{iju}}q_{um}^{ij}>0;u=t,m=n\\ 0 \qquad\qquad ;otherwise\end{cases}, \tag{49}$$

This also implies the positive definite matrix. Therefore, the NW-PSW model has a unique solution for both route flows and modal splits. This completes the proof. □

Further, from the nested weibit covariance is a function of the deterministic utility, the mode covariance is also a function of the traffic condition and network topology.

Proposition 4. *The covariance between modes in the NW-PSW model is a function of the traffic condition and network topology.*

Proof. From Eq. (47), $V_{ijm}=\left(\Psi_{ijum}\exp\left(-w_{um}^{ij}\right)\right)^{-1}$, w_{um}^{ij} is the logarithmic ETC (see Eq. (43)). Then, the mode choice utility can be expressed as

$$\begin{aligned} U_{ijm} &= \left(\Psi_{ijum}\exp\left(\frac{1}{\beta_{um}^{ij}}\ln\sum_{r\in R_{ijum}}\varpi_{umr}^{ij}\left(g_{umr}^{ij}\right)^{-\beta_{um}^{ij}}\right)\right)^{-1}\varepsilon_{ijm} \\ &= \left(\Psi_{ijum}\left[\sum_{r\in R_{ijum}}\varpi_{umr}^{ij}\left(g_{umr}^{ij}\right)^{-\beta_{um}^{ij}}\right]^{\frac{1}{\beta_{um}^{ij}}}\right)^{-1}\varepsilon_{ijm}. \end{aligned} \tag{50}$$

From Eq. (22), we have the covariance between a mode pair under the same upper nest u as

$$\begin{aligned} \operatorname{cov}\left[U_{ijm},U_{ijn}\right] = &\left(\Psi_{ijum}\left[\sum_{r\in R_{ijum}}\varpi_{umr}^{ij}\left(g_{umr}^{ij}\right)^{-\beta_{um}^{ij}}\right]^{\frac{1}{\beta_{um}^{ij}}}\right)^{-1}\left(\Psi_{ijun}\left[\sum_{r\in R_{ijun}}\varpi_{unr}^{ij}\left(g_{unr}^{ij}\right)^{-\beta_{un}^{ij}}\right]^{\frac{1}{\beta_{un}^{ij}}}\right)^{-1} \\ &\times\left(E\left[\varepsilon_{ijm}\varepsilon_{ijn}\right]-1\right). \end{aligned} \tag{51}$$

It can be seen that the NW mode choice covariance resulted from the MP formulation of the NW-PSW model is a function of the traffic conditions in terms of the route travel cost g_{umr}^{ij} and the network topology in terms of the path-size factor ϖ_{umr}^{ij}. This complete the proof. □

Note that from Proposition 4, when the routes between an O-D pair are longer, the mode utility is smaller. Further, when the route overlapping ϖ_{umr}^{ij} is larger, it also lowers the mode utility. This feature suggests that the nested weibit model has the capability to adjust the mode share by lowering the utility for modes with longer routes and/or larger overlapped segments. When applying to the go-green mode promotion strategy as an example, the go-green modes should have a shorter route length compared to the other modes. In addition, the go-green modes should have a smaller overlapped segment for promoting green transportation.

4.4 Solution Procedure

This study adopts the path-based partial linearization algorithm combined with a self-regulated averaging (SRA) line search strategy to solve the proposed NW-PSW model. The path-based partial linearization method belongs to the descent direction algorithm for solving continuous optimization problems (Patriksson, 1994). In general, the partial linearization method has the search direction obtained by solving a partial linearized subproblem and an approximate stepsize obtained by the classical generalized Armijo rule (Bertsekas, 1976). Note that the stepsize approximation could be computationally expensive for a complex objective function. As such, this study adopts the SRA scheme recently proposed by Liu et al. (2009) to determine a stepsize without the need to evaluate the complex objective and/or its derivatives. This SRA scheme determine the stepsize based on the residual error and the stepsize in the current iteration to evaluate the stepsize in the next iteration. It is shown to satisfy the convergence condition (Robbins and Monro, 1951; Blum, 1954; Liu et al., 2009). A brief detail for applying the path-based partial linearization algorithm combined with a SRA line search strategy to solve the combined NW-PSW model is as follows. The search direction can be done by updating the link costs and route costs, computing the NW mode choice probabilities and PSW route choice probabilities, and assigning the auxiliary mode-specific demands and auxiliary route flows according to the NW probabilities for mode choice and the PSW probabilities for route choice, respectively. Then, the SRA scheme is used to determine the stepsize for updating the modal splits and route flows. These two steps are solved iteratively until some convergence criterion is satisfied. For small networks used in this study, we enumerate the routes and focus on the route equilibration procedure to produce the equilibrium solution that is consistent with the mode choice and route choice probabilities. However, a column generation procedure could be incorporated to generate routes as needed in the partial linearization algorithm. In other words, the partial linearization algorithm can work with both route enumeration and column generation. For the partial linearization algorithm with route enumeration, see Chen et al. (2014) for solving the paired combinatorial logit (PCL) SUE model, and Kitthamkesorn and Chen (2013, 2015) for solving the weibit-based SUE model with fixed demand and elastic demand, respectively. For the partial linearization algorithm with column generation, see Bell et al. (1997), Chen et al. (2009, 2010), and Ryu et al. (2014) for solving the path flow estimator, and Yang et al. (2013) for solving the combined travel-destination-mode-route choice model.

5 NUMERICAL EXPERIMENTS

This section provides two numerical examples to illustrate features of the NW-PSW model. The proposed model results are also compared to those provided by the MNL-MNL model and

NL-CNL model (Kitthamkesorn et al., 2016). Without loss of generality, the model parameters are assumed as $\beta_{um}^{ij} = 3.7$, $\varphi_{iju} = 0.5$ for the correlated modes, $\varphi_{iju} = 1$ for the independent mode, and $\Psi_{ijum} = 1$ unless specified otherwise. Note that $\beta_{um}^{ij} = 3.7$ is used to provide the route coefficient of variation of 0.3 (Kitthamkesorn and Chen, 2013; 2014). For the two logit models (MNL-MNL and NL-CNL), the route dispersion parameter is set equal to 0.1, and $\Psi_{ijum} = 0$.

4.1 Example 1: Two-route network

A two-route network with different trip lengths (i.e., short network and long network) shown in Table 5 is used in this example. Both networks consist of three modes, including automobiles, transit, and bicycles for each route. The free flow travel time (FFTT) of the upper route is 5 units longer than the lower route for all modes. Note that the upper route is two times longer than the lower route in the short network, while the upper route is only about 20% longer than the lower route in the long network. The O-D demand is 200 units. The nested structure is the go-green and go-gray scheme where transit and bike share the same upper nest (see Fig. 1 for the tree structure representation). The purpose of using a simple two-route network is three-fold: (a) to demonstrate the correctness of the NW-PSW model as a CMSTA problem, (b) to investigate the effect of heterogeneous perception variance on mode choice and route choice, and (c) to compare the proposed NW-PSW model with the classical MNL-MNL model, which assumes the perception error is independently and identically Gumbel distributed (i.e., not accounting for correlation among alternatives and assuming identical perception variance for all alternatives), and the NL-CNL model proposed by Kitthamkesorn et al. (2016), which adopts the Gumbel distribution as the perception error while using a two-level tree structure to only account for the correlation among alternatives (i.e., nested logit (NL) model for mode choice and cross nested logit (CNL) model for route choice). Using a simple two-route network with three modes has the benefits of clearly articulating the effects highlighted above, particularly the differences between the logit-based and weibit-based models.

Table 5. Characteristics of the two-route network

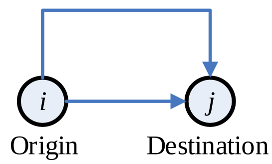
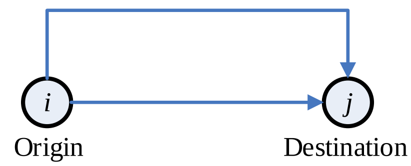
Network	 Short network		 Long network	
Route	Upper	Lower	Upper	Lower
Auto	$10+f_{umr}^{ij}/10$	$5+f_{umr}^{ij}/10$	$30+f_{umr}^{ij}/10$	$25+f_{umr}^{ij}/10$
Transit	$15+f_{umr}^{ij}/20$	$10+f_{umr}^{ij}/20$	$35+f_{umr}^{ij}/20$	$30+f_{umr}^{ij}/20$
Bike	$20+f_{umr}^{ij}/20$	$15+f_{umr}^{ij}/20$	$40+f_{umr}^{ij}/20$	$45+f_{umr}^{ij}/20$

Table 6 provides the results of three models (i.e., MNL-MNL, NL-CNL, and NW-PSW) for both short and long networks. First, we check the correctness of the results by checking conservation for both mode choice and route choice. As expected, the modal splits for both short and long networks satisfy the probability conservation (e.g., for the short network: 61.94+36.52+1.54=100.00 for the MNL-MNL model, 62.53+37.38+0.09=100.00 for the NL-CNL model, and 47.43+33.25+19.32=100.00 for the NW-PSW model).

Next, we test the impact of heterogeneous perception variance from the short and long networks. The results show that both MNL-MNL and NL-CNL models cannot handle the heterogeneous perception variance. These two models give the same route choice probability for both short and long networks. This is because both MNL-MNL and NL-CNL models have the identically distributed assumption (i.e., all perception variances are the same and fixed) despite that the NL-CNL model uses a two-level tree structure to handle the independence assumption for both mode choice and route choice. On the other hand, the NW-PSW model gives different route choice results for the short and long networks. This is due to the perception variance of the Weibull distribution is a function of route cost. Hence, the lower route in the short network receives a larger share than that of the long network.

Table 6. Route share and mode share of three models for both short and long networks

	Route share				Mode share	
	Short network		Long network		Short network	Long network
Model	Upper Route	Lower Route	Upper Route	Lower Route		
Auto						
MNL-MNL	42.32%	57.68%	42.32%	57.68%	61.94%	61.94%
NL-CNL	42.35%	57.65%	42.35%	57.65%	62.53%	62.53%
NW-PSW	34.67%	65.33%	40.61%	59.39%	47.43%	44.57%
Transit						
MNL-MNL	39.56%	60.44%	39.56%	60.44%	36.52%	36.52%
NL-CNL	39.60%	60.40%	39.60%	60.40%	37.38%	37.38%
NW-PSW	28.23%	71.77%	38.71%	61.29%	33.25%	34.01%
Bike						
MNL-MNL	37.84%	62.16%	37.84%	62.16%	1.54%	1.54%
NL-CNL	37.75%	62.25%	37.75%	62.25%	0.09%	0.09%
NW-PSW	29.94%	70.06%	40.34%	59.66%	19.32%	21.42%

When considering the mode share, both MNL-MNL model and NL-CNL model give the same results for both short and long networks. This is because the exponential proportion of the expected perceived travel time (i.e., the log sum) is the same for both networks. In contrast, the NW-PSW model has different mode shares for each network. Its logarithmic expected perceived travel time of all modes becomes more similar for the longer network (i.e., both alternatives becomes more similar) in Table 7 and Fig. 3. As such, the go-green modes receive a higher share in the long network.

Table 7. Expected perceived travel time of three models for both short and long networks

Model	Short network	Long network
Auto		
MNL-MNL	6.64	26.71
NL-CNL	6.64	26.71
NW-PSW*	2.30	3.27
Transit		
MNL-MNL	7.17	27.22
NL-CNL	7.17	27.22

NW-PSW*	2.43	3.34
Bike		
MNL-MNL	10.34	35.26
NL-CNL	10.34	35.26
NW-PSW*	2.70	3.58

*The NW-PSW model uses the logarithmic expected travel time

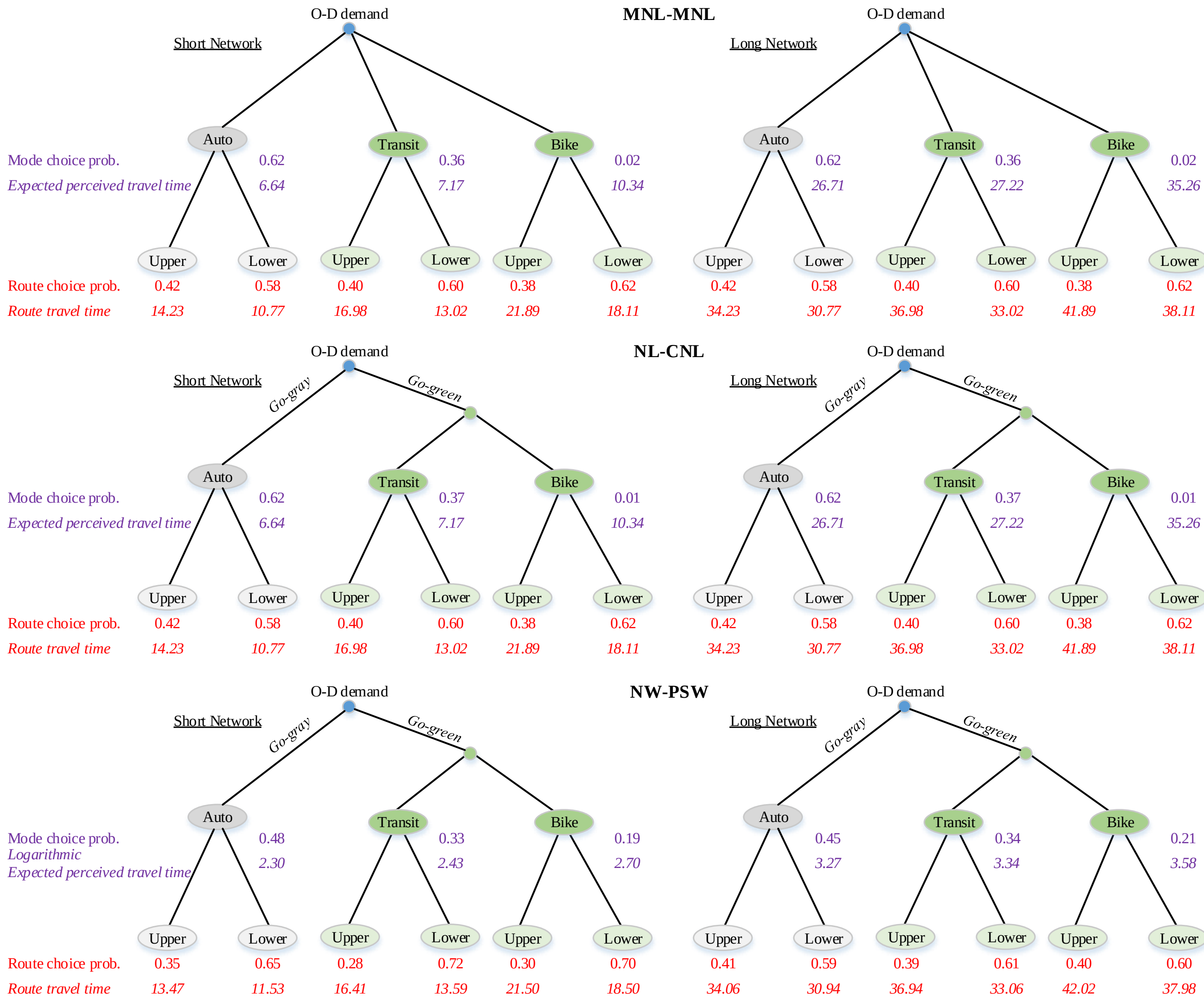


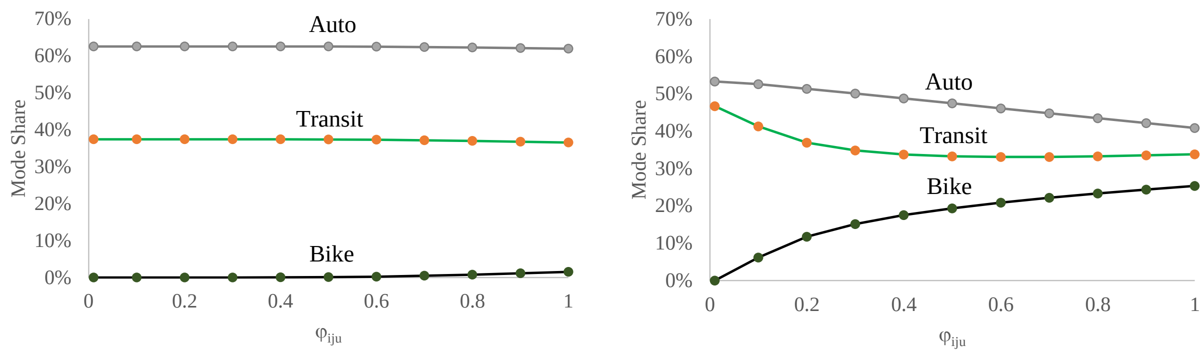
Fig. 3. Multi-dimensional equilibrium demand and choice probability patterns

Then, we investigate the impact of the mode similarity in the short network. The parameter φ_{iju} for the go-green modes is varied from 0.01 to 1. We can observe that the NW-PSW model is more sensitive to the change in φ_{iju} as shown in Fig. 4. This is because the covariance and perception variance of the NW-PSW model is a function of the deterministic utility. The change in φ_{iju} impacts the choice correlation, covariance, and perception variance. By using a numerical method to compute the correlation, the covariance and correlation between transit and bike can be presented in Fig. 5 (see Eq. (21) and Eq. (27)).

Both NL-CNL and NW-PSW models have the correlation approaches to one as $\varphi_{iju} \rightarrow 0$ and the correlation equals to zero when $\varphi_{iju} = 1$. The NL-CNL model seems to have a stronger correlation than the NW-PSW model. However, the NW-PSW model presents a larger covariance between transit and bike modes. This results in a more dispersed mode share. At a lower φ_{iju} or a higher correlation, the covariance is high, and the expected perceived travel cost of each mode is smaller according to the dispersed assignment results. At a higher φ_{iju} or a smaller correlation, the covariance is low, but the expected perceived travel cost is high according to maintain the attractiveness of each mode at equilibrium. Hence, the mode perception variance is high. In other words, for the NW model, the covariance dominates the mode share at a lower φ_{iju} while the mode perception variance dominate the result at a higher φ_{iju}.

To visualize the change in the perception variance, we use the perception variance ratio of transit over bicycle. The perception variance ratio resulted from the NL-CNL model equals to one since the NL model has the predetermined perception variance of $\pi^2/6$ for both modes under the same nest (e.g., Ben-Akiva and Lerman, 1985). In contrast, the perception variance ratio resulted from the NW-PSW model changes according to the value of φ_{iju}. This is because the NW-PSW model has the perception variance as a function of the route travel time and logarithmic expected perception variance (see Eq. (17)). The change in φ_{iju} would change the choice pattern, route travel time, logarithmic expected perception variance, and hence the perception variance. As φ_{iju}

increases, the perception variances of transit and bike become more similar. With these, the choice patterns between the NL-CNL and NW-PSW models could be different. In sum, the change in φ_{iju} impacts correlation, covariance, and perception variance in the choice patterns of the NW-PSW model.



a) NL-CNL model b) NW-PSW model

Fig. 4. Mode share by varying φ_{iju} of the go-green modes in the short network

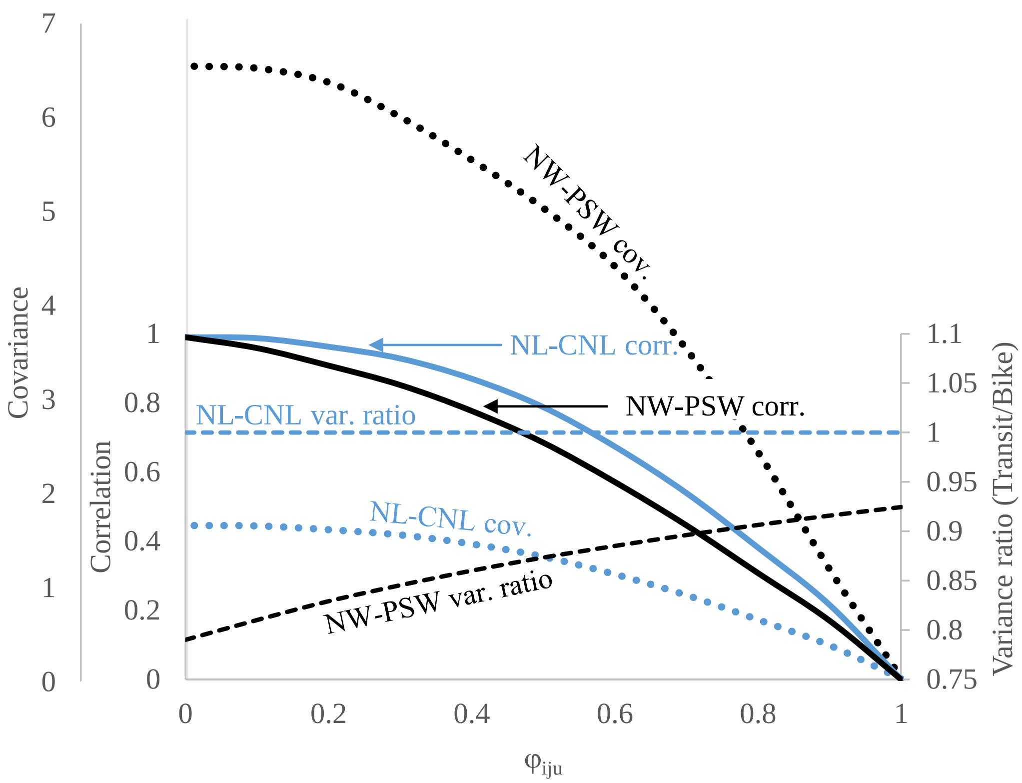


Fig. 5. Correlation, covariance, and variance ratio by varying φ_{iju} of the go-green modes in the

short network

4.2 Example 2: Nguyen-Dupius network

Similar to the simple two-route network used in example 1, example 2 adopts the modified Nguyen-Dupuis network shown in Fig. 6 to obtain important insights of using different nested structure configurations (see Fig. 1 for the tree structure representations) and different models (i.e., logit-based NL-CNL model and weibit-based NW-PSW model) to model the go-green and go-gray scheme and the motorized and non-motorized scheme. Compared to the two-route network, the Nguyen-Dupuis network has four O-D pairs with routes consisting of multiple links (i.e., link equals to route as in the two-route network) and multiple modes. The four O-D pairs are (1,2), (1,3), (4,2), and (4,3) with the O-D demands of 500, 500, 600, and 300 travelers per hour, respectively. The link travel time function is the Bureau of Public Road (BPR) type function. The link travel time for the auto is

$$t_a = t_a^0\left[1+0.15\left(v_a/c_a\right)^4\right], \tag{52}$$

and the link travel time for the transit is

$$t_a = t_a^0\left[1+0.5\left(v_a/c_a\right)^2\right], \tag{53}$$

where t_a is the link travel time function on link *a*, t_a^0 is the free flow travel time on link *a*, and c_a is the capacity on link *a*. For the bike mode, the travel time is fixed at 30 minutes for each route. Detailed network characteristics can be found in Table 8. The value of Ψ_{ijum} for the go-green modes (i.e., transit and bike) is equal to 1. Since the logit model is an additive RUM model and the weibit model is a multiplicative RUM model, we set the travel cost for comparison reason as follows:

$$h_a = 0.25t_a \text{ and } \tau_a = e^{0.075t_a}, \quad \forall a \in A, \tag{54}$$

where h_a is the link travel cost for the logit model.

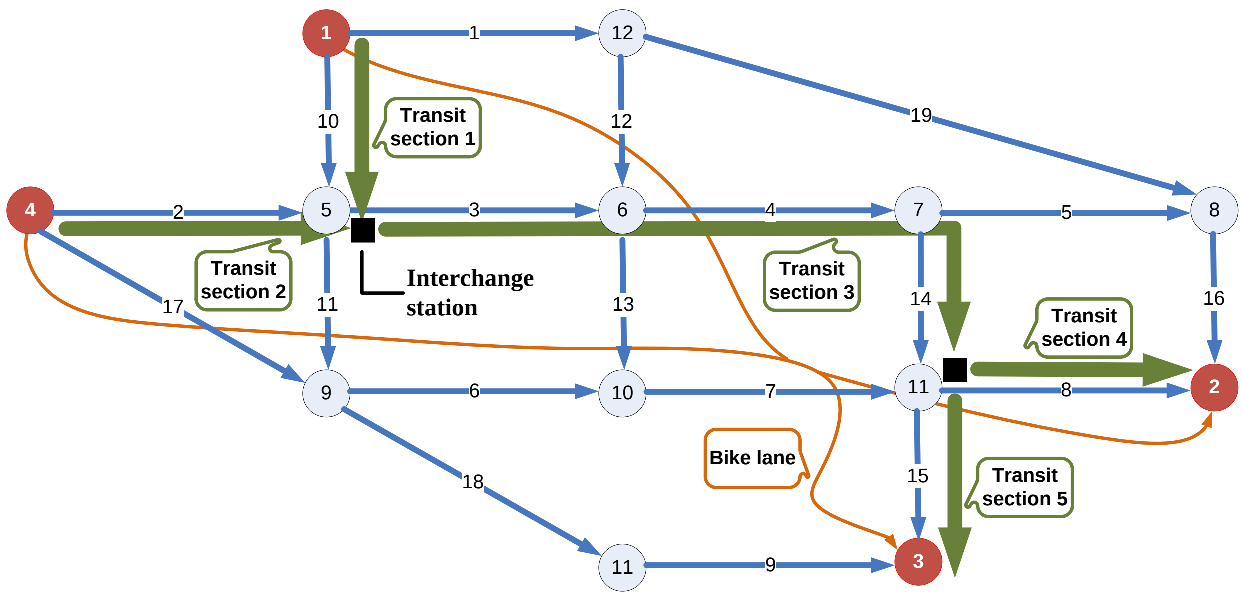


Fig. 6. Modified Nguyen-Dupius network

Table 8: Characteristics of the modified Nguyen-Dupius network

Auto	Distance (km)	FFTT (minute)	Capacity (vph)
Link 1	2.5	2.5	500
Link 2	2.5	2.5	300
Link 3	2.0	2.0	400
Link 4	4.0	4.0	500
Link 5	3.0	3.0	400
Link 6	2.5	2.5	300
Link 7	3.0	3.0	600
Link 8	2.5	2.5	200
Link 9	2.5	2.5	200
Link 10	2.5	2.5	250
Link 11	2.5	2.5	400
Link 12	2.5	2.5	400
Link 13	3.75	3.75	400
Link 14	2.5	2.5	400

Auto	Distance (km)	FFTT (minute)	Capacity (vph)
Link 15	2.0	2.0	400
Link 16	2.5	2.5	500
Link 17	2.5	2.5	300
Link 18	7.0	7.0	400
Link 19	3.0	3.0	400

Transit	Distance (km)	FFTT (minute)	Capacity (pph)
section 1	2.5	2.5	300
section 2	2.5	2.5	300
section 3	10	10	600
section 4	2.5	2.5	300
section 5	2.5	2.5	300
Bike	Distance (km)	FFTT (minute)	Capacity
Bike lane	10	30	--

The mode share results for the two nested structure configurations are presented in Fig. 7. From the results, it is apparent that the nested structure configurations do have an impact on the mode shares. The independent mode (i.e., the auto mode in the go-green and go-gray scheme and the bike mode in the motorized and non-motorized scheme) receives a higher share. This result is consistent with Kitthamkesorn et al. (2016). Note that the NW-PSW model give a higher go-green mode share since it considers both mode-specific perception variance and route-specific perception variance.

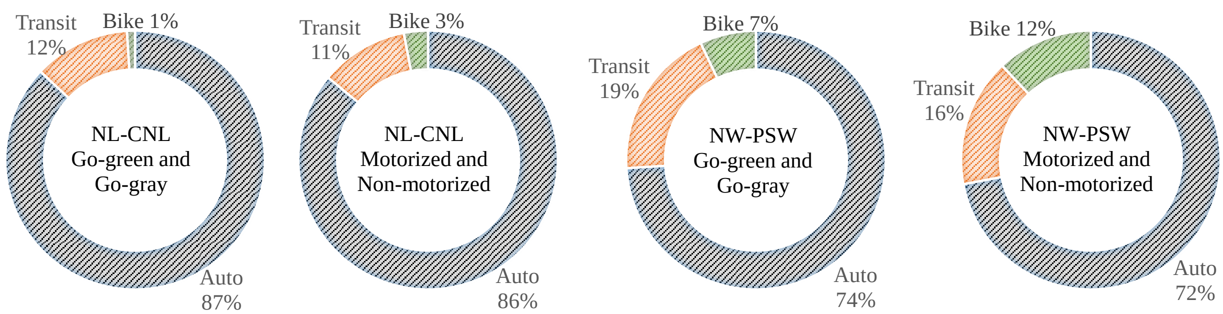
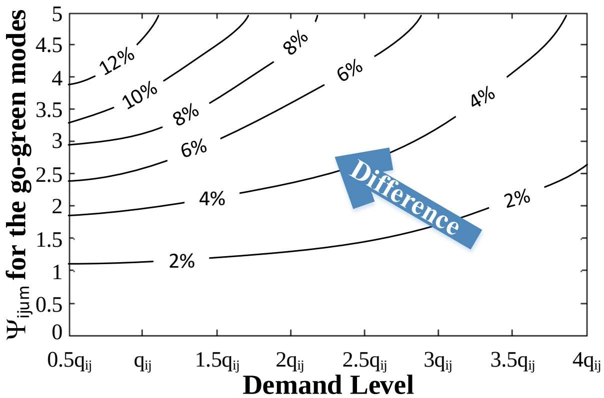
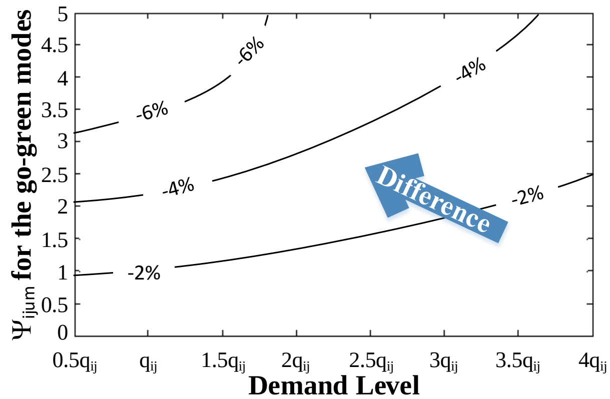


Fig. 7. Mode share for the modified Nguyen-Dupius network

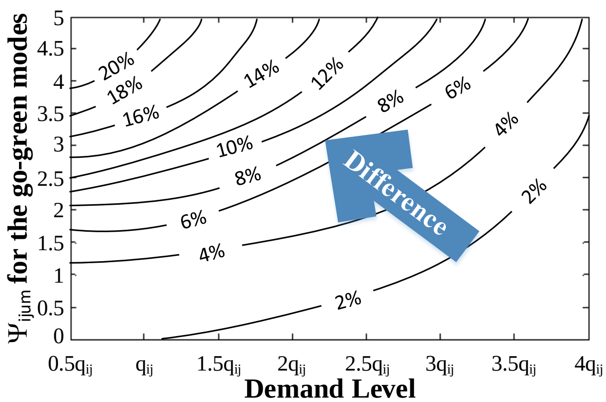
Then, we vary the demand level and the exogenous modal utility Ψ_{ijum}. The demand level is varied from 0.5 to 4 times of the base O-D demands, and Ψ_{ijum} of the go-green modes (i.e., transit and bike) is varied from 0 to 5. As the demand level increases, the difference between the mode share estimated under each scheme (i.e., mode share from the go-green go-gray scheme minus mode share from the motorized and non-motorized scheme) is decreased. This is because the congestion effect dominates the results. On the other hand, when Ψ_{ijum} increases, the mode share difference also increases. The NW-PSW model seems to provide a larger mode share difference than the NL-CNL model. This is because the NW-PSW exogenous utility is a multiplicative type as the weibit model, which is a member of the multiplicative RUM model. This is unlike the NL-CNL model which has Ψ_{ijum} as an additive form. As such, the mode share resulted from the NW-PSW model will be more sensitive to the change in Ψ_{ijum}.



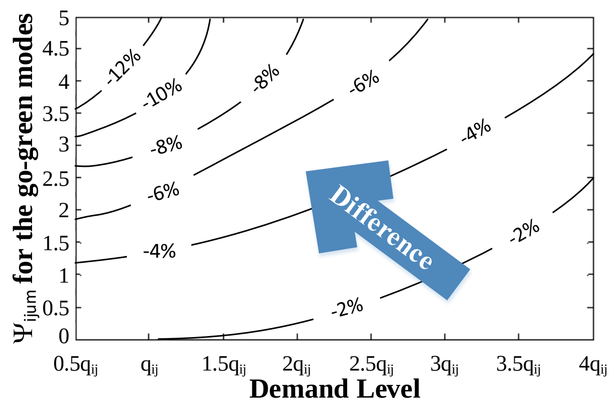
a) NL-CNL transit mode choice probability difference



b) NL-CNL bike mode choice probability difference



c) NW-PSW transit mode choice probability difference



d) NW-PSW bike mode choice probability difference

Fig. 8. Mode share difference when varying Ψ_{ijum} and demand level

6 CONCLUDING REMARKS

This paper presented an alternate weibit-based combined modal split and traffic assignment (CSMTA) model based on random utility theory derived from the Weibull distribution. The main contributions are twofold: (1) the development of a nested weibit model, and (2) the development of a weibit-based CSMTA model as a mathematical programming (MP) formulation. The nested weibit (NW) model was developed by adapting the nested structure of the well-established nested logit (NL) model with the Weibull distributed random error for modeling mode choice, while the recently developed path-size weibit (PSW) model was adopted for modeling route choice. The development of a MP formulation for the combined NW-PSW model was provided to simultaneously consider both similarities and heterogeneous perception variance in the joint mode-route travel choice decisions under congestion. The benefits of a MP formulation are: (1) Optimality conditions are readily interpretable and easily understandable (i.e., the Kuhn-Tucker conditions provide the equivalency between the MP formulation and the weibit-based mode choice and route choice probabilities), and (2) convergent algorithms (e.g., partial linearization algorithm) are readily available for solving the weibit-based CMSTA model. Numerical examples were performed to illustrate features of the proposed combined NW-PSW model. Through the examples, the mode share resulting from the combined NW-PSW model is more sensitive to changes in model parameters and network characteristics than those produced by the two logit-based models (MNL-MNL and NL-CNL). This is because the proposed model has the mode-specific and route-specific perception variance. The perception variance is a function of the (dis)utility. Moreover, its exogenous utility is a multiplicative type as the weibit model, which is a member of the multiplicative RUM model.

For future research, parameter calibration (e.g., Oppenheim, 1995; de Grange *et al*., 2010) should be performed for the combined NW-PSW problem. An investigation on how to shift between the go-green and go-gray nested structure and the motorized and non-motorized nested structure is also interesting since we could encourage a mode choice more effective with one nested structure than the other (e.g., Kitthamkesorn et al., 2016). Applications of the combined NW-PSW model should be tested in real networks to demonstrate proof of concept. In addition, several assumptions have been made for simplifying the MP formulation so that the contribution of this paper is clear and focused. As suggested by Cantarella et al. (2016), fixed point formulation could be considered as a general framework to relax the assumptions for modeling more complex issues (e.g., asymmetric interactions, non-additive route cost structures, multi-user classes, etc.) and specific operational details in public transport modes (e.g., bus, tram, and mass rapid transit).

ACKNOWLEDGEMENTS

The authors are grateful to the three anonymous referees for their constructive comments and suggestions to improve the quality and clarity of the paper. This research was supported by the Thailand Research Fund, the Research Committee of the Hong Kong Polytechnic University (Project No. 1-ZE5T and 1-ZVJV), the Research Grants Council of the Hong Kong Special

Administrative Region, and the Chang Jiang Chair Professorship Program sponsored by the Ministry of Education in China. These supports are gratefully acknowledged.

REFERENCES

Abdulaal, M., LeBlanc, L.J., 1979. Methods for combining modal split and equilibrium assignment models. *Transportation Science* 13(4), 292–314.

Bahat, O., Bekhor, S., 2016. Incorporating ridesharing in the static traffic assignment model. *Networks and Spatial Economics 16(4)*, 1125-1149.

Beckmann, M.J., McGuire, C.B., Winsten, C.B., 1956. Studies in Economics of Transportation. New Haven: Yale University Press, Connecticut.

Bekhor, S., Prashker, J., 1999. Formulations of extended logit stochastic user equilibrium assignments. In: Proceedings of the 14th International Symposium on Transportation and Traffic Theory, Jerusalem, Israel, 351-372.

Bekhor, S., Prashker, J., 2001. A stochastic user equilibrium formulation for the generalized nested logit model. *Transportation Research Record* 1752, 84-90.

Bell, M.G.H., Shield, C.M., Busch, F., Kruse, G., 1997. A stochastic user equilibrium path flow estimator. *Transportation Research Part C* 5 (3-4), 197-210.

Ben-Akiva, M., Bierlaire M., 1999. Discrete Choice Methods and Their Applications to Short Term Travel Decisions. Handbook of Transportation Science, R.W. Hall (Editor), Kluwer Publishers.

Ben-Akiva, M.E., Lerman, S.R., 1985. Discrete Choice Analysis: Theory and Application to Travel Demand (Vol. 9), MIT press, Cambridge.

Bertsekas, D.R., 1976. On the Goldstein-Levitin-Polyak gradient projection method. IEEE Transactions on automatic control, 21(2), 174-184.

Bhat, C. R., 2009. A new generalized Gumbel copula for multivariate distributions. Technical paper, Department of Civil, Architectural and Environmental Engineering, The University of Texas at Austin.

Blum, J.R., 1954. Multidimensional stochastic approximation methods. The Annals of Mathematical Statistics 25(4), 737-744.

Boyce, D., 2007. Forecasting travel on congested urban transportation networks: Review and prospects for network equilibrium models. Networks and Spatial Economics 7(2), 99-128.

Boyce, D., 2013. Beckmann's transportation network equilibrium model: Its history and relationship to the Kuhn-Tucker conditions. *Economics of Transportation* 2, 47-52.

Briceño, L., Cominetti, R., Cortés, C., Martínez, F. J., 2008. An integrated behavioral model of land-use and transport system: an extended network equilibrium approach. Networks and Spatial Economics 8(2–3), 201–224.

Cantarella, G.E., 1997. A general fixed-point approach to multimode multi-user equilibrium assignment with elastic demand. Transportation Science,31(2), pp.107-128.

Cantarella, G.E., de Luca, S., Di Gangi, M., Di Pace, R., 2013. Stochastic equilibrium assignment with variable demand: Literature review, comparisons and research needs. WIT Transactions on the Built Environment 130, 349-364.

Cantarella, G.E., de Luca, S., Di Gangi, M., Di Pace, R., 2015. Approaches for solving the stochastic equilibrium assignment with variable demand: Internal vs. external solution algorithms. Optimization Methods and Software 30(2), 338-364.
Cantarella, G.E., Carteni, A., de Luca, S., 2015. Stochastic equilibrium assignment with variable demand: Theoretical and implementation issues. *European Journal of Operational Research* 241, 330-347.
Castillo, E., Menéndez, J.M., Jiménez, P., Rivas, A., 2008. Closed form expression for choice probabilities in the Weibull case. Transportation Research Part B 42(4), 373-380.
Chen, A., Chootinan, P., Recker, W., 2009. Norm approximation method for handling traffic count inconsistencies in path flow estimator. *Transportation Research Part B* 43(8), 852-872.
Chen, A., Pravinvongvuth, S., Xu, X., Ryu, S., Chootinan, P., 2012. Examining the scaling effect and overlapping problem in logit-based stochastic user equilibrium models. *Transportation Research Part A* 46(8), 1343-1358.
Chen, A., Ryu, S., Chootinan, P., 2010. L_∞-norm path flow estimator for handling traffic count inconsistencies: Formulation and solution algorithm. *Journal of Transportation Engineering* 136(6), 565-575.
Chen, A., Ryu, S., Xu, X., Choi, K., 2014. Computation and application of the paired combinatorial logit stochastic traffic equilibrium problem. *Computers and Operations Research* 43(1), 68-77.
Chen, A., Xu, X., Ryu, S., Zhou, Z., 2013. A self-adaptive Armijo stepsize strategy with application to traffic assignment models and algorithms. *Transportmetrica A: Transport Science* 9(8), 695-712.
Chen, A., Zhou, Z., Xu, X., 2012. A self-adaptive gradient projection algorithm for the nonadditive traffic equilibrium problem. Computer & Operation Research 39(2), 127-138.
Cooper L.G., Nakanishi, M., 1988. Market-share Analysis: Evaluating Competitive Marketing Effectiveness. Kluwer Academic Publishers, London.
de Grange, L., Fernández, E., de Cea, J., Irrazábal, M., 2010. Combined model calibration and spatial aggregation. Networks and Spatial Economics 10(4), 551-578.
Evans, S.P., 1976. Derivation and analysis of some models for combining trip distribution and assignment. *Transportation Research* 10(1), 37–57.
Fernandez, E., de Cea, J.D., Florian, M., Cabrera, E., 1994. Network equilibrium models with combined modes. Transportation Science 28(3), 182-192.
Fisk, C., 1980. Some developments in equilibrium traffic assignment. Transportation Research Part B 14(3), 243-255.
Florian, M., 1977. A traffic equilibrium model of travel by car and public transit modes. Transportation Science 11(2), 166-179.
Florian, M., Nguyen, S., 1978. A combined trip distribution, modal split and trip assignment model. Transportation Research 12(4), 241-246.
Fosgerau, M., Bielaire, M., 2009. Discrete choice models with multiplicative error terms. Transportation Research Part B 43, 494-505.
García, R, Marín, A., 2005. Network equilibrium models with combined modes: Models and solution algorithms. Transportation Research Part B 39（3），223-254.
He, F., Wu, D., Yin, Y., Guan, Y., 2013. Optimal deployment of public charging stations for plug-in hybrid electric vehicles. Transportation Research Part B 47, 87-101.

Kitthamkesorn, S., Chen, A., 2013. Path-size weibit stochastic user equilibrium model, Transportation Research Part B 57, 378-397.
Kitthamkesorn, S., Chen, A., 2014. Unconstrained weibit stochastic user equilibrium with extensions. Transportation Research Part B 59, 1-21.
Kitthamkesorn, S., Chen, A., Xu, X., 2015. Elastic demand with weibit stochastic user equilibrium flows and application in a motorised and non-motorised network. Transportmetrica A: Transport Science 11(2), 158-185.
Kitthamkesorn, S., Chen, A., Xu, X., Ryu, S., 2016. Modeling mode and route similarities in network equilibrium problem with go-green modes. *Networks and Spatial Economics* 16(1), 33-60.
LeBlanc, L.J., Farhangian, K., 1981. Efficient algorithm for solving elastic demand traffic assignment problems and model split-assignment problems. *Transportation Science* 15, 306–317.
Liu, H., He, X., He, B.S., 2009. Method of successive weighted averages (MSWA) and self-regulated averaging schemes for solving stochastic user equilibrium problem. Network and Spatial Economics 9(4), 485-503.
Maher, M., 1992. SAM – A stochastic assignment model. In: Mathematics in Transport Planning and Control, ed. J.D. Griffiths. Oxford University Press, Oxford.
Manski, C.F., McFadden, D.L., 1981. Structural Analysis of Discrete Data and Economietric Applications. MIT Press, Cambridge.
Marzano, V., Papola, A., 2008. On the covariance structure of the cross-nested logit model. Transportation Research Part B 42(2), 83–98
Meng Q, Liu Z, 2012. Impact analysis of cordon-based congestion pricing on mode-split for a bimodal transportation network. Transportation Research Part C 21 (1): 134-147.
Mirchandani, P., Soroush, H., 1987. Generalized traffic equilibrium with probabilistic travel times and perceptions. Transportation Science 21(3), 133-152.
Nelsen, R.B., 2006. An Introduction to Copulas. Springer Series in Statistics, Springer: Berlin.
Oppenheim, N., 1995. Urban Travel Demand Modeling. John Wiley and Sons Inc., New York
Patriksson, M., 1994. The traffic assignment problem: Models and Methods. VSP, Utrecht, The Netherlands.
Robbins, H., Monro, S., 1951. A stochastic approximation method. The Annals of Mathematical Statistics 22(3), 400-407.
Rosa, A., Maher, M.J., 2002. Algorithms for solving the probit path-based stochastic user equilibrium traffic assignment problem with one or more user classes. Proceedings of the 15th International Symposium on Transportation and Traffic Theory, Australia, 371-392.
Ryu, S., Chen, A., Xu, X., Choi, K., 2014. Modeling demand elasticity and route overlapping in stochastic user equilibrium through paired combinatorial logit model. *Transportation Research Record* 2429, 8-19.
Ryu, S., Chen, A., Zhang, H.M., Recker, W., 2014. Path flow estimator for planning applications of small communities. *Transportation Research Part A* 69, 212-242.
Sheffi, Y., 1985. Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods. Prentice-Hall, Englewood Cliffs, N.J.
Sheffi, Y., Powell, W.B., 1982. An algorithm for the equilibrium assignment problem with random link times. *Networks* 12(2)**,** 191–207.
Szeto, W.Y., Jaber, X., Wong, S.C., 2012. Road network equilibrium approaches to environmental sustainability. Transport Reviews 32(4), 491-518.

Wang, T.G., Xie, C., Xie, J., Waller, S.T., 2016. Path-constrained traffic assignment: A trip chain analysis under range anxiety. *Transportation Research Part C* 68, 447-461.

Wu, Z.X., Lam, W.H.K., 2003. A combined modal split and stochastic assignment model for congested networks with motorized and non-motorized modes. *Transportation Research Record* 1831, 57-64.

Xu, H., Ordonez, F., Dessouky, M., 2015. A traffic assignment models for a ridesharing transportation market. *Journal of Advanced Transportation 49*, 793-816.

Xu, X., Chen, A., 2013. C-logit stochastic user equilibrium problem with elastic demand. *Transportation Planning and Technology* 36(5), 463-478.

Xu, X., Chen, A., Kitthamkesorn, S., Yang, H., Lo, H. K., 2015. Modeling absolute and relative cost differences in stochastic user equilibrium problem. Transportation Research Part B 81, 686-703.

Yang, C., Chen, A., Xu, X., 2013. Improved partial linearization algorithm for solving the combined travel-destination-mode-route choice problem. *Journal of Urban Planning and Development* 139(1), 22-32.

Yang, H., Bell, M.G.H., 1998. Models and algorithms for road network design: a review and some new developments. Transport Reviews 18(3), 257-278.

Yao, J., Chen, A., Ryu, S., Shi, F., 2014. A general unconstrained optimization formulation for the combined distribution and assignment problem. *Transportation Research Part B* 59, 137-160.

Zhou, Z., Chen A., Bekhor, S., 2012. C-logit stochastic user equilibrium model: Formulations and solution algorithm. Transportmetrica A: Transport Science 8(1), 17-41.

Zhou, Z., Chen, A., Wong, S.C., 2009. Alternative formulations of a combined trip generation, trip distribution, modal split, and trip assignment model. European Journal of Operational Research 198(1), 129-138.