

1 Time-Variant Reliability Analysis of Widened Deteriorating Prestressed Concrete 2 Bridges considering Shrinkage and Creep

3 Bing Tu ^a; Zhi Fang ^{a,*}; You Dong ^b; and Dan M. Frangopol ^c

4 ^a College of Civil Engineering, Hunan University, Changsha, Hunan 410082, China

5 ^b Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University, Hong Kong,
6 China

7 ^c Department of Civil and Environmental Engineering, Engineering Research Center for Advanced Technology
8 for Large Structural Systems (ATLSS Center), Lehigh University, Bethlehem, PA 18015-4729, USA

9 10 **Abstract:**

11 Nowadays, bridge widening has become an economic option to tackle the increasing demand
12 of the traffic volume and to enhance the capacity of existing highway bridges. Thus, relevant
13 studies on the performance assessment of widened bridges are needed. This paper presents a
14 computational probabilistic framework for time-variant reliability analysis of widened concrete
15 highway bridges in a systematic manner considering the effects of live-load redistribution,
16 structural deterioration, and concrete shrinkage and creep. Specifically, differences and
17 inconsistencies between the new and existing structures regarding live-load distribution,
18 reinforcement corrosion, and concrete shrinkage and creep are considered. A finite element
19 grillage model is constructed to investigate live-load distribution factors and internal axial
20 forces caused by concrete shrinkage and creep. The uncertainties associated with shrinkage and
21 creep effects are accounted for within the probabilistic framework. The flexural moment
22 resistance of the bridge girder is computed considering the combined effects of the shrinkage-
23 and-creep-induced axial force and structural deterioration. Ultimately, the system reliability of
24 the widened bridge is calculated. The proposed probabilistic framework is applied to a widened
25 prestressed concrete T-girder bridge.

26 **Key words:** Widened concrete bridge; Live-load distribution factor; Shrinkage and creep;
27 Corrosion; System reliability.

28 *Corresponding author.

29 E-mail address: fangzhi@hnu.edu.cn (Z. Fang).

30 **1. Introduction**

31 Due to limited budget and conservative prediction of increase of traffic volume, a considerable
32 number of narrow bridges were built previously all over the world. With the rapid growth of
33 traffic volume, many of these existing bridges became functionally obsolete due to insufficient
34 width. Compared with complete replacement or building a new bridge, widening these bridges
35 is generally more economical and effective [1]. For most widened concrete bridges, in order to
36 improve structural integrity, the superstructures of the new and existing bridges are connected
37 by longitudinal splice joints [2]. The static and long-term behavior of the widened bridge,
38 considering the interaction between new and existing bridges, are much more complex than
39 those in the case of treating these bridges separately [3-4]. However, nearly all bridge widening
40 guidelines suggest that standards and guides used for new bridges can also be applied for the
41 widened bridges without considering the interaction between the new and existing bridges [1,
42 5-8]. Thus, it can be concluded that the specified design and assessment methodology for
43 widened bridges have not been well established and, therefore, relevant studies are needed.

44 Nowadays, the reliability-based load and resistance factor design (LRFD) method
45 dominates the design philosophy for most current design codes including the Chinese code for
46 design of highway reinforced concrete and prestressed concrete bridge and culverts (JTG D62-
47 2004) [9], the model code 2010 [10], and the AASHTO LRFD Bridge Design Specifications
48 [11], among others. The structural reliability index is also recognized as the fundamental

49 performance indicator for structural safety and performance assessment of existing structures
50 [12-17]. Thus, relevant reliability studies of widened bridges are necessary.

51 Within the performance assessment of widened bridges, the live-load redistribution should
52 be computed firstly. Nie et al. [18] modified the conventional rigid-jointed girder method
53 (RJGM) to compute the transverse distribution coefficient of concrete girders that are widened
54 with steel-concrete composite beams. Chen et al. [19] proposed a general hinge-jointed slab
55 method (HJSM) for the computation of the lateral distribution factor of widened prestressed
56 concrete hollow slab bridge. Chang et al. [20] investigated the live-load redistribution behavior
57 of widened T-girder and hollow slab bridge using finite element (FE) grillage model. Based on
58 these studies, it can be concluded that the analytical methods (e.g., RJGM, HJSM) are only
59 suitable for certain types of widened bridges, while the FE-model-based method can serve as a
60 general approach for live-load redistribution analysis of widened bridges.

61 Another well-recognized mechanical characteristic of widened concrete bridges is the
62 shrinkage and creep difference between new and existing girders. This difference could result
63 in significant time-dependent internal stresses [4, 21-23]. Fang et al. [24] investigated the
64 internal forces induced by shrinkage and creep in widened box-girder bridge using FE, and
65 compared the bending moment capacity of the box-girder before and after widening by
66 sectional nonlinear analysis. The results indicated that as the axial compressive force induced
67 by shrinkage and creep changes the failure mode of the existing box-girder (i.e., from flexural
68 failure to compressive-flexural failure), the bending moment capacity of the existing box-girder

69 increases. Thus, it is of vital importance to integrate these effects associated with concrete
70 shrinkage and creep into the structural reliability analysis process. In addition, it should be
71 noted that most studies on shrinkage and creep effects in widened bridges are deterministic.
72 Uncertainties associated with concrete shrinkage and creep will be addressed in this paper.

73 As bridges are usually directly exposed to environmental attack, their capacities will
74 decrease over time. For reinforced concrete and prestressed concrete bridges, corrosion of
75 reinforcement steel is the primary source of structural deterioration [25-27]. During the past
76 decades, the effects of reinforcement corrosion on the reliability of existing concrete structures
77 have been investigated [28-34]. Two main conclusions can be drawn from these studies. Firstly,
78 reinforcement corrosion plays an important role in time-variant reliability analysis of existing
79 concrete bridges, especially for those exposed to chloride-prone environments [35-37].
80 Secondly, the deterioration process mainly depends on corrosion initiation time and corrosion
81 rate. As there exist differences in the construction time (or service time) and design profiles
82 (e.g., concrete material properties, thickness of concrete cover) between new and existing
83 girders, the extent of reinforcement corrosion can vary between them. Therefore, it is necessary
84 to consider, for widened concrete bridges, the corrosion progress (i.e., corrosion initiation time
85 and corrosion rate) using time-variant reliability analysis.

86 Overall, bridge widening has become an economic option to enhance the capacity of
87 existing bridges. However, the reliability analysis of widened bridges is still in its infancy. The
88 interactions between new and existing girders, including live-load redistribution, concrete

89 shrinkage and creep effects, and different reinforcement corrosion, deem to have significant
90 effects on performance of the widened concrete bridges. All these effects should be carefully
91 considered within the reliability assessment process in a systematic and probabilistic manner.
92 Furthermore, the reliability analysis should be conducted at a system level to account for the
93 correlations among different girders and the redundancy of the bridge system. This paper aims
94 to propose a probabilistic approach to compute the time-variant reliability of widened concrete
95 bridges considering live-load redistribution, concrete shrinkage and creep as well as the
96 difference in reinforcement corrosion between new and existing girders. To conduct this study,
97 firstly, live-load distribution factors and internal axial forces caused by concrete shrinkage and
98 creep are computed using a FE grillage model, and an age-adjusted effective modulus
99 (AAEM)-based algorithm is proposed within the FE analysis to assess the shrinkage and creep
100 effects. The uncertainties associated with the shrinkage and creep effects are accounted for
101 within the computation process. Subsequently, considering the combined effects of
102 reinforcement corrosion and the shrinkage-and-creep-induced forces, the probabilistic flexural
103 moment resistance of girder components is assessed using Monte Carlo Simulation (MCS).
104 Then, the superstructure of widened concrete bridge is modeled as a series-parallel system, in
105 which the correlation of resistances between different girder components is considered. Finally,
106 the reliability index of girder components and the bridge system are computed. In order to
107 demonstrate the application of the proposed approach, a widened prestressed concrete T-girder
108 bridge is considered.

109 **2. Flowchart of reliability assessment of widened concrete bridge**

110 The proposed methodology is integrated using three modules: structural analysis of capacity
111 and demand, probabilistic analysis, and system reliability analysis modules, as shown in Fig.1.
112 Firstly, structural analysis of widened concrete bridge is conducted considering the interactions
113 between new and existing girders. The output of this module is the demand (i.e., live-load effect)
114 and resistance of each girder component. Then, the probabilistic analysis module is developed
115 to consider the uncertainties associated with the variables involved in the first module. The
116 simulation methods (e.g., MCS and Latin Hypercube Sampling (LHS) [38]) can be used to
117 generate these random variables. Finally, a system reliability model that describes the behavior
118 of the widened bridge system and the relationship of individual girder components to the
119 overall system is introduced. The reliability of the investigated widened bridge system can be
120 calculated using First order reliability method (FORM)/Second order reliability method
121 (SORM). In order to account for time-dependent effects (i.e., structural deterioration, concrete
122 shrinkage and creep), the proposed methodology is repeated for each time step during the
123 investigated time interval.

124 **3. Live-load redistribution of widened bridge**

125 **3.1 Lateral live-load distribution**

126 For a bridge superstructure with multi-girders, the live-load effect on an individual girder is
127 generally computed as [39]

$$128 \quad F_{refined,i} = F_{beamline} \times g_{F,i} \quad (1)$$

129 where $F_{refined,i}$ = the maximum live-load flexural moment or shear force in a certain girder for
130 all possible load combinations; $F_{beamline}$ = the maximum flexural moment or shear force
131 determined from a simple beam-line analysis under one lane of traffic; and $g_{F,i}$ = live-load
132 distribution factor (LLDF), which reflects the distribution characteristic of live load in lateral
133 direction.

134 LLDF is related with many geometric and material parameters, such as girder type, girder
135 spacing, span length, traffic lane arrangement, transverse connection stiffness and sectional
136 longitudinal stiffness, among others [40-42]. As these parameters may change during the bridge
137 widening process, LLDF as well as live-load effect of an individual girder associated with new
138 and existing bridges should be updated accordingly.

139 Though explicit formulas of LLDF are available in various bridge design codes, such as
140 AASHTO LRFD Specifications [11], these simplified formulas are generally conservative, and
141 are only applicable to certain types of bridges, such as slab-girder superstructures with uniform
142 girder spacing and longitudinal stiffness [43]. For widened bridges, the type of superstructure
143 may vary, and the spacing as well as longitudinal stiffness of girder component of new and
144 existing bridges are usually different. Therefore, a more accurate and systematic method for
145 computation of LLDF of widened structures should be established. Herein, the grillage model
146 is used to compute the LLDF. This method will be described in the following section.

147 **3.2 Grillage model for live-load effect analysis**

148 The grillage-model-based method is adopted herein to compute the live-load effects on bridges
149 before and after widening. Grillage model has been well recognized to produce a good balance
150 between accuracy and computational cost, thus it is usually recommended to analyze complex
151 bridges, such as skewed and curved bridges [44]. The grillage model can be implemented with
152 commercial finite element (FE) software, e.g., *ANSYS* [45], to aid the computational process
153 associated with complex structures. For illustrative purpose, Fig.2(a) shows a typical grillage
154 model of a widened multi-girder bridge. The model consists of eleven longitudinal girders in
155 which girders 1 to 6 are existing girders and 7 to 11 are new girders. Seven transverse members
156 are labeled as a, b, c, d, e, f, g representing the transverse diaphragms. They are uniformly
157 distributed along the span. The rest transverse members are virtual diaphragms, and represent
158 the connection contribution from the girder flange. The splice joint is also modeled as discrete
159 transverse members with corresponding cross section in the grillage model. The precise
160 determination of structural parameters (e.g., flexural stiffness) associated with the splice joint
161 should be based on experimental results and/or a refined FE model. Regarding the engineering
162 applications, some approximations can be made. For instance, in the widened T-girder bridges,
163 flange wet seams and transverse diaphragms are used to connect the adjacent new and existing
164 girders, in which the flexural stiffness of transverse diaphragms is relatively large. Accordingly,
165 the splice joint can be modeled as a rigid connection, as shown in Fig.2(b). For the widened
166 hollow-slab bridge, as the new and old slabs are connected by the weak flange wet seams, it is

167 reasonable to model the splice joint using a hinge connection, as indicated in Fig.2(c). With
168 additional information from test or refined FE modeling regarding splice joint, its model detail
169 can be refined or updated.

170 **4. Concrete shrinkage and creep effects of new and existing bridges**

171 Normally, widening is carried out after several years of service of an existing bridge. By this
172 time, concrete shrinkage and creep in the existing bridge have almost fully developed, while
173 for the newly-built bridge, these long-term deformations just start. Therefore, when subsequent
174 deformations originated from concrete shrinkage and creep of new bridge are restricted by the
175 existing bridge, long-term internal forces (i.e., axial forces and flexural moments) in horizontal
176 plane will generate. The long-term internal forces, especially axial forces, can lead to a
177 significant change of bending capacity of girders [24] and should be carefully considered
178 during the performance assessment process.

179 **4.1 Prediction of concrete shrinkage and creep**

180 The CEB-FIP-90 model [46] has been widely adopted to predict the concrete shrinkage strain
181 and creep coefficient and is used in this study. This model accounts for the effects of several
182 parameters, such as cement type, compressive strength of concrete, theoretical thickness of
183 component, ambient humidity, among others.

184 Accordingly, the shrinkage strain $\varepsilon_{cs}(t, t_s)$ at time t (days) is calculated as [46]

$$185 \quad \varepsilon_{cs}(t, t_s) = [160 + 10\beta_{sc}(9 - f_{cm}/10)]\beta_{RH} \sqrt{\frac{t - t_s}{350(h/100)^2 + (t - t_s)}} \times 10^{-6} \quad (2)$$

186 where t_s = concrete age (days) when shrinkage starts; β_{sc} = a coefficient that depends on the
 187 type of cement (e.g., for normal or rapid hardening cement, $\beta_{sc} = 5$); f_{cm} = mean compressive
 188 strength (cylinder) of concrete at the age of 28 days (MPa); h = nominal thickness of member
 189 (mm) defined as $2A_c/u$, in which A_c = cross-sectional area and u = perimeter of the member that
 190 in contact with the atmosphere; and β_{RH} = a coefficient related to relative humidity of ambient
 191 environment, and can be expressed as

$$192 \quad \beta_{RH} = \begin{cases} -1.55 \left[1 - (RH/100\%)^3 \right] & 40\% \leq RH < 99\% \\ 0.25 & RH \geq 99\% \end{cases} \quad (3)$$

193 where RH = annual relative humidity of ambient environment (%).

194 The creep coefficient $\varphi(t, t_0)$ at time t (days) can be calculated as [46]

$$195 \quad \varphi(t, t_0) = \left[1 + \frac{1 - RH / 100\%}{0.46(h / 100)^{1/3}} \right] \left(\frac{5.3}{\sqrt{f_{cm} / 10}} \right) \left(\frac{1}{0.1 + t_0^{0.2}} \right) \left[\frac{t - t_0}{\beta_H + (t - t_0)} \right]^{0.3} \quad (4)$$

196 where t_0 = initial time when load acting (days), and it corresponds to the widening time; and

$$197 \quad \beta_H = \min \left\{ 150 \left[1 + \left(1.2 \frac{RH}{100\%} \right)^{18} \right] \frac{h}{100} + 250, \quad 1500 \right\} \quad (5)$$

198 4.2 AAEM-based procedure for analysis of shrinkage and creep

199 The grillage model, shown in Fig.2(a), is combined with age-adjusted effective modulus
 200 (AAEM) method to compute the internal forces caused by the difference of concrete shrinkage
 201 and creep between new and existing bridges.

202 According to AAEM, during the time interval from widening time t_0 to the time of interest
 203 t , the relationship between incremental stress $\Delta\sigma(t, t_0) = \sigma(t) - \sigma(t_0)$ and incremental strain $\Delta\varepsilon(t, t_0)$
 204 $= \varepsilon(t) - \varepsilon(t_0)$ of concrete in new bridge can be computed as [47]

$$205 \quad \Delta\sigma(t, t_0) = E_\varphi(t, t_0) \left[\Delta\varepsilon(t, t_0) - \frac{\sigma(t_0)}{E(t_0)} \varphi(t, t_0) - \varepsilon_{cs}(t, t_0) \right] \quad (6)$$

206 where $E_\varphi(t, t_0)$ = age-adjusted effective modulus, and can be calculated as [47]

$$207 \quad E_\varphi(t, t_0) = \frac{E(t_0)}{1 + \chi(t, t_0) \varphi(t, t_0)} \quad (7)$$

208 where $\chi(t, t_0)$ = ageing coefficient that can be derived from $\varphi(t, t_0)$ and $E(t_0)$ = initial elastic
 209 modulus.

210 Eq. (6) can be extended to a generalized vector form as [48]

$$211 \quad \{\Delta\sigma\} = [D_\varphi] \left[\{\Delta\varepsilon\} - \left\{ \frac{\sigma(t_0)}{E(t_0)} \right\} \varphi(t, t_0) - \{\varepsilon_{cs}\} \right] \quad (8)$$

212 where $\{\Delta\sigma\}$ = incremental stress vector; $[D_\varphi]$ = effective elasticity matrix, and it can be
 213 modified from conventional elasticity matrix by replacing E with $E_\varphi(t, t_0)$; $\{\Delta\varepsilon\}$ = incremental
 214 strain vector; $\{\sigma(t_0)/E(t_0)\}$ = initial strain vector caused by initial force $\sigma(t_0)$; and $\{\varepsilon_{cs}\}$ =
 215 shrinkage strain vector.

216 The three strain items in Eq. (8) can be transformed to the corresponding nodal
 217 displacement vectors as [48]

$$218 \quad \{\Delta\varepsilon\} = [B] \{\Delta\delta\}^e \quad (9)$$

219
$$\begin{Bmatrix} \sigma(t_0) \\ E(t_0) \end{Bmatrix} = [B] \{\delta(t_0)\}^e \quad (10)$$

220
$$\{\varepsilon_{cs}\} = [B] \{\delta_{cs}\}^e \quad (11)$$

221 where $[B]$ = strain matrix; $\{\Delta\delta\}^e$ = incremental nodal displacement vector; $\{\delta(t_0)\}^e$ = initial
 222 elastic nodal displacement vector caused by initial stress $\sigma(t_0)$; and $\{\delta_{cs}\}^e$ = nodal displacement
 223 vector caused by shrinkage.

224 Then, Eq. (8) can be rewritten as

225
$$\{\Delta\sigma\} = [D_\varphi][B] \left[\{\Delta\delta\}^e - \{\delta(t_0)\}^e \varphi(t, t_0) - \{\delta_{cs}\}^e \right] \quad (12)$$

226 Following the principle of conventional finite-element method and neglecting body and
 227 surface forces, equilibrium of new bridge element can be established using virtual displacement
 228 principle as [48]

229
$$\int_e [B]^T \{\Delta\sigma\} dV = \{\Delta F\}^e \quad (13)$$

230 where $\{\Delta F\}^e$ = incremental nodal force vector.

231 Then, substituting Eq. (12) into Eq. (13), the governing equation for the concrete element
 232 of new bridge is derived as

233
$$[k_\varphi]^e \{\Delta\delta\}^e - [k_\varphi]^e \{\delta(t_0)\}^e \varphi(t, t_0) - [k_\varphi]^e \{\delta_{cs}\}^e = \{\Delta F\}^e \quad (14)$$

234
$$[k_\varphi]^e = \int_e [B]^T [D_\varphi][B] dV \quad (15)$$

235 where $[k_\varphi]^e$ = effective stiffness matrix for new bridge elements.

236 For the existing bridge elements, the effects of concrete shrinkage and creep can be
 237 neglected, and the corresponding governing equation is [48]

$$238 \quad [k]^e \{\Delta\delta\}^e = \{\Delta F\}^e \quad (16)$$

239 where $[k]^e$ =element stiffness matrix for existing bridge.

240 Integrating governing equations of all elements (i.e., Eq. (14) and Eq. (16)) into global
 241 coordinate system, the governing equation of the entire widened structures is

$$242 \quad [K]\{\Delta\delta\} - \sum_{new,global} [k_\varphi]^e \{\delta(t_0)\}^e \varphi(t, t_0) - \sum_{new,global} [k_\varphi]^e \{\delta_{cs}\}^e = \{\Delta P\} \quad (17)$$

243 where $[K]$ = global stiffness matrix of the widened structure, in which $[k_\varphi]^e$ and $[k]^e$ are element
 244 stiffness matrix used for elements associated with new and existing bridges, respectively; $\{\Delta\delta\}$
 245 = global incremental nodal displacement vector; $\{\Delta P\}$ = global incremental nodal force vector;
 246 and $\sum_{new,global}$ means integration of elements within the new bridge under the global coordinate
 247 system.

248 As there is no incremental nodal force during the time interval from the widening time t_0
 249 to the time of interest t , $\{\Delta P\}$ is a zero matrix. Then the Eq. (17) can be transformed as

$$250 \quad [K]\{\Delta\delta\} = \sum_{new,global} [k_\varphi]^e \{\delta(t_0)\}^e \varphi(t, t_0) + \sum_{new,global} [k_\varphi]^e \{\delta_{cs}\}^e \quad (18)$$

251 Based on the above derivations, the authors developed a computational procedure for FE
 252 analysis of shrinkage and creep effects on widened bridge considering the interaction between
 253 the new and existing bridges, and the flowchart is shown in Fig.3. The procedure defines the
 254 sequence of the analysis process, the pre-processing of input data, and the post-processing of

255 output results. It can be translated into program code and be embedded into arbitrary FE model
256 of widened bridges.

257 **4.3 Analysis of uncertainties**

258 As stated previously, the input (e.g., shrinkage strain and creep coefficient) and the associated
259 calculation model (e.g., FE grillage model and AAEM) in the analysis procedure for shrinkage
260 and creep effects are associated with many parameters, such as concrete material properties
261 (e.g., compressive strength, elastic modulus), geometric parameters (e.g., sectional nominal
262 thickness, area), and environment conditions (e.g., relative humidity), among others. These
263 parameters are usually associated with uncertainty. Thus, it is important to incorporate the
264 uncertainty within the assessment process. Under given information, these parameters can be
265 modeled as random variables with specific distribution types.

266 Additionally, the modeling uncertainty factors associated with concrete shrinkage strain
267 and creep coefficient prediction (Ψ_1 and Ψ_2) should be incorporated within the computational
268 process. Accordingly, the prediction formula for concrete shrinkage strain $\varepsilon_{cs}(t, t_s)$ and creep
269 coefficient $\varphi(t, t_0)$ (i.e., Eq. (2) and (4)) need to be updated with a multiplier of Ψ_1 and Ψ_2 ,
270 respectively. Information on the modeling uncertainty can be obtained from the statistical study
271 conducted by Bažant and Baweja [49]. The mean value and coefficient of variation (COV) of
272 Ψ_1 , Ψ_2 within CEB-FIP model are [49]

$$273 \quad E(\psi_1) = 1; \quad V_{\psi_1} = 0.451 \quad (19a)$$

$$274 \quad E(\psi_2) = 1; \quad V_{\psi_2} = 0.339 \quad (19b)$$

275 MCS method can be utilized to generate these random variables within the analysis of
276 shrinkage and creep effects. As the finite element simulation (i.e., FE grillage model) process,
277 which aims to establish the implicit relationship between the input (e.g., shrinkage strain and
278 creep coefficient) and output parameters (e.g., internal axial forces), is usually time-consuming,
279 the Latin Hypercube Sampling (LHS) [38] can be used to generate the relevant variables
280 associated with the input and model parameters to improve the computational efficiency.

281 **5. Time-dependent corrosion model**

282 As bridges are usually directly exposed to aggressive environment, their strength and durability
283 degrade with time, and corrosion of reinforcement steel is recognized as the primary source of
284 structural deterioration for reinforced and prestressed concrete bridges [25-27].

285 Based on the geometric shape of reinforcement steel after corrosion, reinforcement
286 corrosion can be divided into two categories, one is the uniform corrosion that is mainly caused
287 by concrete carbonization and the other is chloride-induced pitting corrosion. According to
288 González et al. [50], the maximum penetration of pitting corrosion is about four to eight times
289 of that associated with uniform corrosion, thus pitting corrosion can lead to a more severe area
290 loss. Therefore, the chloride-induced pitting corrosion is considered in this study. Specifically,
291 area loss and yield (ultimate) strength decrease of reinforcement steel under pitting corrosion
292 are accounted.

293 The diffusion process of chloride ions through concrete surface is usually modeled by the
294 one-dimensional Fick's law. Accordingly, the corrosion initiation time t_i (years) can be
295 predicted as [25]

$$296 \quad t_i = \frac{d^2}{4D_c} \frac{1}{\left[\operatorname{erf}^{-1} \left(1 - \frac{C_{cr}}{C_0} \right) \right]^2} \quad (20)$$

297 where d = thickness of concrete cover (cm); D_c = diffusion coefficient of chloride ions
298 (cm^2/year); C_0 = constant concentration of chloride ions on concrete surface (% weight of
299 concrete); C_{cr} = threshold concentration of chloride ions (% weight of concrete); and erf^{-1}
300 represents inverse of error function.

301 Then the radius of the pit in reinforcement bar at time t (years), $p(t)$ (mm), can be computed
302 as [51]

$$303 \quad p(t) = 0.0116(t - t_i) i_{corr} R \quad (21)$$

304 where i_{corr} = corrosion current density ($\mu\text{A}/\text{cm}^2$) and R = the ratio between maximum and
305 average penetration.

306 The geometric model proposed by Val and Melchers [51] is used to compute the loss of
307 effective cross-sectional area under pitting corrosion. Accordingly, the remain net area of
308 reinforcement bar at time t , $A_r(t)$, is [51]

$$309 \quad A_r(t) = \begin{cases} \frac{\pi D_0^2}{4} - A_1 - A_2, & p(t) \leq \frac{\sqrt{2}}{2} D_0 \\ A_1 - A_2, & \frac{\sqrt{2}}{2} D_0 < p(t) < D_0 \\ 0, & p(t) \geq D_0 \end{cases} \quad (22)$$

$$310 \quad \text{where} \quad a = 2p(t) \sqrt{1 - \left[\frac{p(t)}{D_0} \right]^2}; \theta_1 = 2 \arcsin \left(\frac{a}{D_0} \right); \theta_2 = 2 \arcsin \left[\frac{a}{2p(t)} \right];$$

$$311 \quad A_1 = \frac{1}{2} \left[\theta_1 \left(\frac{D_0}{2} \right)^2 - a \left| \frac{D_0}{2} - \frac{p^2(t)}{D_0} \right| \right]; \quad A_2 = \frac{1}{2} \left[\theta_2 p^2(t) - a \frac{p^2(t)}{D_0} \right]; \quad \text{and } D_0 \text{ is the initial}$$

312 diameter of the reinforcement bar.

313 As the prestressed tendon usually consists of strands with several twisted wires (e.g., seven
314 5 mm-diameter wires), and is placed inside the grouted corrugated ducts, the corrosion
315 mechanism of prestressed tendons is much more complex than that of reinforcement bar. In
316 this study, the probabilistic model proposed by Darmawan and Stewart [36] is used to account
317 for the corrosion effects of prestressed tendons. This model is based on accelerated corrosion
318 tests of 54 prestressed 7-wire strands and the maximum pit depth $p(t)$ was found to follow an
319 extreme value distribution (type I). Given the corrosion density i_{corr} ($\mu A/cm^2$), wire length l
320 (mm) and initial corrosion time t_i (years), the probability distribution function (PDF) of $p(t)$
321 (cm) is [36]

$$322 \quad f_{p(t)}(t, i_{corr}, l) = \frac{\alpha}{\lambda^{0.54}} \exp \left[-\alpha \left(\frac{p(t)}{\lambda^{0.54}} - \mu \right) \right] \exp \left\{ -\exp \left[-\alpha \left(\frac{p(t)}{\lambda^{0.54}} - \mu \right) \right] \right\} \quad t > t_i \quad (23)$$

323 where

$$324 \quad \lambda = \frac{D_0^2 - [D_0 - 0.0232 i_{corr} (t - t_i)]^2}{D_0^2 - (D_0 - 0.0232 i_{corr - exp} T_{0-exp})^2} \quad (24)$$

325
$$\mu = \mu_{0\text{-exp}} + \frac{1}{\alpha_{0\text{-exp}}} \ln \left(\frac{l}{l_{0\text{-exp}}} \right), \quad \alpha = \alpha_{0\text{-exp}} \quad (25)$$

326 Herein, $T_{0\text{-exp}} = 0.03836$ years, $\mu_{0\text{-exp}} = 0.84$, $\alpha_{0\text{-exp}} = 8.10$, $i_{\text{corr-exp}} = 186 \mu\text{A}/\text{cm}^2$, $l_{0\text{-exp}} = 650$ mm
 327 and these values are obtained from statistical analysis of the accelerated corrosion tests.

328 With respect to the prestressed strand with 7 wires, it is assumed that the pitting only
 329 formed on the six outer wires [36]. Thus, the remain net area of the entire strand $A_{sr}(t)$ is $6A_{wr}(t)$
 330 $+ A_{w0}$, where $A_{wr}(t)$ is the remain net area of outer wire at time t and can be computed using Eq.
 331 (22), and A_{w0} denotes the time-invariant net area of the inner wire.

332 In addition to the loss of net area, existing laboratory results indicate that corrosion could
 333 also reduce the yield (ultimate) stress of reinforcement steel by the following equation [52,53]

334
$$f_y(t) = (1 - 100\alpha_{\text{corr}}P_{\text{corr}})f_{y0} \quad (26)$$

335 where $f_y(t)$ = deteriorated yield (ultimate) stress at time t ; f_{y0} = initial yield (ultimate) stress;
 336 P_{corr} = percentage of area loss that caused by corrosion (%); and α_{corr} = a coefficient and the
 337 value is 0.0054 and 0.0075 for reinforcement bar and prestressed strand, respectively.

338 **6. Illustrative Example**

339 **6.1 Bridge description**

340 The presented methodology is applied to a widened simply-supported concrete bridge located
 341 in Hebei Province, China. The existing part of the widened bridge was built in 1996, and
 342 consisted of 6 prestressed concrete T-shape girders to support three traffic lanes in one direction.
 343 In 2016, the existing bridge was widened with another 5 prestressed concrete T-shape girders

344 to expand it to five traffic lanes in one direction, as shown in Fig.4(a). The continuous flange
345 wet seams and seven uniformly-arranged transverse diaphragms are used to connect the girders
346 within the construction and widening process, as shown in Fig.4(b).

347 The existing and new bridges were designed based on the old and the latest version of
348 Chinese Code for Design of Reinforced Concrete and Prestressed Concrete Highway Bridges
349 and Culverts, i.e., JTJ 023-85 [54] and JTG D62-2004 [9], respectively. The nominal
350 compressive strength f_{ck1} of concrete used in existing bridge is 28.0 MPa [54] and the
351 corresponding nominal compressive strength of the concrete used in the new bridge f_{ck2} is 32.4
352 MPa [9]. The reinforcement steel (prestressed tendons and reinforcement bars) arrangement in
353 mid-span section of existing and new girders is shown in Fig.4(c) and Fig.4(d), respectively.
354 In Fig.4(c) and Fig.4(d), R_y^b and f_{pk} denote the nominal ultimate stress of prestressed tendon,
355 R_{g1} , R_{g2} and f_{sk} represent the nominal yield stress of reinforcement bar.

356 Accordingly, the FE grillage model of the illustrated bridge is constructed using *ANSYS*
357 [45], as shown in Fig.5. The longitudinal new and existing girders, the transverse diaphragms,
358 and the transverse virtual diaphragms are modeled using BEAM189 element, a 3-D quadratic
359 finite strain beam. The transverse splice joint members are modeled by rigid connections as
360 shown in Fig.2(b). The cross sections and spatial arrangements of the longitudinal and
361 transverse members are determined according to the bridge configuration as indicated in Fig.4.
362 Linear elastic behavior of all elements is assumed under the effects of live-load and concrete
363 shrinkage and creep. In the analysis for live-load effect, the elastic modulus of concrete of new

364 and existing bridge is 3.45×10^4 MPa [9] and 3.30×10^4 MPa [54], respectively. The analysis for
365 shrinkage and creep effects is conducted according to the incremental procedure indicated in
366 Fig.3. Within the analysis process, the elastic modulus of concrete used in the existing bridge
367 is fixed as 3.30×10^4 MPa. The age-adjusted effective modulus is adopted for concrete in the
368 new bridge, and the value of the age-adjusted effective modulus is updated in each time interval
369 by using Eq. (7).

370 6.2 Load effects

371 The flexural failure in the mid-span section, as the dominate failure mode for simply-supported
372 girder, is investigated in this paper. The nominal flexural moment in the mid-span section of a
373 single girder subjected to dead load, $M_{DL,n}$, can be calculated as

$$374 \quad M_{DL,n} = \frac{(W_{DC1} + W_{DC2} + W_{DW})l^2}{8} \quad (27)$$

375 where W_{DC1} = uniformly-distributed load due to self-weight of the T-type girder, and it depends
376 on the geometry size of girder section and density of concrete material; W_{DC2} = uniformly-
377 distributed load due to traffic barriers; W_{DW} = uniformly-distributed load of wearing surface;
378 and l = span length. The flexural moment associated with the dead load is assumed to follow a
379 normal distribution, and the mean value and coefficient of variation (COV) are $1.0148 M_{DL,n}$
380 and 0.0431, respectively [55].

381 The nominal live-load flexural moment $M_{LL,n}$ is computed based on the truck load specified
382 in the Chinese General Code for Design of Highway Bridges and Culverts [56]. The axle

383 spacing and weight distribution of the specified semitrailer truck are shown in Fig.6. Then
384 given the grillage model as shown in Fig.5, the largest mid-span live-load flexural moments of
385 each girder for all load cases (in both longitudinal and transverse directions) can be computed
386 with following procedure: (1) the longitudinal position of the truck load is determined using
387 influence line analysis based on the single-girder model, and the worst scenario for the flexural
388 moment in mid-span is the case that the front wheel (30 kN) is placed 1.1 m away from the left
389 support; and (2) in the transverse direction, five load combinations are considered, as shown in
390 Fig.7. In these load combinations, the minimum clearance between two adjacent trucks is 1.3
391 m, and the minimum clearance between the extreme-exterior wheel and the barrier edge is 0.5
392 m [56]. The five load combinations are moved from left to right side of the deck at a given
393 fixed step (e.g., 0.1 m for each step), respectively. Then, the load effects of the girders under
394 the loading steps can be computed using the grillage model implemented within *ANSYS*. The
395 computed load effects should be further modified by multiplying lane factor (m in Fig.7), and
396 the factor equals to 1.00, 0.78, 0.67, and 0.60 for two, three, four, and five adjacent trucks case,
397 respectively [56]. Ultimately, by searching the corresponding maximum flexural moment of
398 mid-span section under the investigated scenarios (i.e., load combinations and load action
399 positions), the worst scenario in transverse direction under the truck load associated with each
400 girder can be determined.

401 The probabilistic model proposed in Chinese Standard for Reliability Design of Highway
402 Engineering Structures [55] is utilized to account for the uncertainty associated with vehicle

403 load (i.e., live load) effect. Accordingly, the maximum flexural moment through the lifetime
 404 period that caused by vehicle load is assumed to follow an extreme value distribution (type I).
 405 For the investigated highway bridge, the design period T is specified as 100 years, then the
 406 mean value and COV of the live-load flexural moment are $0.7795 M_{LL,n}$ and 0.0862,
 407 respectively [55].

408 The uncertainty associated with the dynamic load amplification factor η_{DLA} is also
 409 considered. η_{DLA} is affected by many factors, such as road surface roughness, bridge dynamics
 410 and vehicle dynamics. The distribution of η_{DLA} used herein is based on the monitoring data of
 411 a similar bridge (a simply-supported reinforced concrete bridge with a span of 20m) [55].
 412 Accordingly, η_{DLA} is assumed to follow an extreme value distribution (type I) with a mean
 413 value of 1.1776 and COV of 0.0428 [55].

414 6.3 Time-variant resistance model

415 The time-variant resistance of the bridge is investigated in this section. According to the stress-
 416 block model used in JTG D62 2004, the time-variant flexural moment capacity $M_{u0}(t)$ of the
 417 flanged concrete section with both prestressed tendons and normal reinforcement bars (see
 418 Fig.8(a) and Fig.8(b)) is given by [9]

$$419 \quad M_{u0}(t) = \begin{cases} k_{pM} \left[0.85 f_c b'_f x_1(t) \left(h_0 - \frac{x_1(t)}{2} \right) + f'_s(t) A'_s(t) (h_0 - a'_s) \right], & x_1(t) \leq h'_f \\ k_{pM} \left\{ \begin{aligned} & 0.85 f_c \left[b x_2(t) \left(h_0 - \frac{x_2(t)}{2} \right) + (b'_f - b) h'_f \left(h_0 - \frac{h'_f}{2} \right) \right] \\ & + f'_s(t) A'_s(t) (h_0 - a'_s) \end{aligned} \right\}, & x_2(t) > h'_f \end{cases} \quad (28)$$

420 where k_{pM} = modeling uncertainty factor for flexural capacity; f_c = compressive strength of
421 concrete; b'_f = effective width of flange; b = width of web; h_0 = effective height of the section,
422 and it is the distance from the top fiber to the centroid of the tensile steel; h'_f = height of flange;
423 $f'_s(t)$, $A'_s(t)$ = yield strength and area of the compressive reinforcement at time t , and it can be
424 determined based on the corrosion model; a'_s = distance from the top fiber to the centroid of
425 the compressive reinforcement; and $x_1(t)$, $x_2(t)$ = depth of concrete compression block when
426 the concrete compression block is within and exceeds the flange height, respectively.

427 For the widened bridge, concrete shrinkage and creep effects can lead to time-variant axial
428 compressive force $N_c(t)$ or tensile force $N_t(t)$ in the girder, as seen in Fig.8(c) and Fig.8(d).
429 Considering the axial load effect, the flexural capacity of the girder is [9]

$$430 \quad M_{uN}(t) = \begin{cases} k_{pMN} \left[\begin{array}{l} 0.85 f_c b'_f x_{1N}(t) \left(h_0 - \frac{x_{1N}(t)}{2} - e_s \right) + f'_s(t) A'_s(t) e'_s + \\ (f_s(t) A_s(t) + f_p(t) A_p(t)) e_s \end{array} \right], & x_{1N}(t) \leq h'_f \\ k_{pMN} \left\{ \begin{array}{l} 0.85 f_c \left[b x_{2N}(t) \left(h_0 - \frac{x_{2N}(t)}{2} - e_s \right) + (b'_f - b) h'_f \left(h_0 - \frac{h'_f}{2} - e_s \right) \right] + \\ f'_s(t) A'_s(t) e'_s + (f_s(t) A_s(t) + f_p(t) A_p(t)) e_s \end{array} \right\}, & x_{2N}(t) > h'_f \end{cases} \quad (29)$$

431 where $M_{uN}(t)$ = flexural moment capacity of section with axial force; k_{pMN} = modeling
432 uncertainty factor; e_s and e'_s = distance from the section centroid to the centroid of the tensile
433 steel and compressive reinforcement bars, respectively; and $x_{1N}(t)$, $x_{2N}(t)$ = the depth of the
434 equivalent rectangular stress block when the block is within and exceeds the flange height,
435 respectively. $x_{1N}(t)$ and $x_{2N}(t)$ can be obtained based on the equilibrium equation of the
436 calculated section with respect to axial force. For instance, for the case with axial compressive
437 force $N_c(t)$ [9]

438
$$x_{1N}(t) = \frac{f_s(t)A_s(t) + f_p(t)A_p(t) - f'_s(t)A'_s(t) + N_c(t)}{0.85f_c b'_f} \quad (30)$$

439
$$x_{2N}(t) = \frac{f_s(t)A_s(t) + f_p(t)A_p(t) - f'_s(t)A'_s(t) + N_c(t)}{0.85f_c b} - \frac{(b'_f - b)h'_f}{b} \quad (31)$$

440 where $f_p(t)$, $A_p(t)$ = ultimate tensile strength and area of prestressed tendon, respectively.

441 Given the load effects and capacity, the performance function associated with different
442 girders at time t can be identified as

443
$$g_i(t) = M_{u,i}(t) - (M_{DL,i} + \eta_{DLA}M_{LL,i}) \quad (32)$$

444 where $M_{u,i}(t)$ = flexural resistance (capacity) of mid-span section of girder i at time t ; $M_{DL,i}$ =
445 dead load moment; η_{DLA} = dynamic load amplification factor; and $M_{LL,i}$ = live-load moment.

446 **6.4 System reliability of bridge superstructure**

447 In order to compute the reliability at the system level, a model that describes the relationship
448 between the individual components and the relationship of individual girder components to
449 overall system should be identified firstly. A series-parallel model for the investigated widened
450 bridge is shown in Fig.9. In this model, it is assumed that failure of any three adjacent girders
451 will cause the whole structure failure [13].

452 Once the system failure model is established, the reliability of structural components and
453 system are computed using RELSYS (Reliability of Systems), a FORTRAN 77 computer
454 program [57]. RELSYS firstly computes the reliability of every component in the system using
455 FORM. Then, the system is progressively reduced to a single equivalent component, and

456 equivalent alpha vectors are used to account for the correlations between the failure of different
457 equivalent components. The accuracy of the RELSYS has been ascertained by comparing its
458 results with those computed using Monte Carlo simulation [13, 57-59].

459 The random variables involved in this study are shown in Table 1, and they can be
460 classified to four categories: (1) geometric parameters and initial material properties; (2)
461 parameters that specified for calculation of concrete shrinkage and creep; (3) corrosion-related
462 parameters; and (4) parameters associated with load effects. The relevant references are also
463 indicated in this table.

464 **6.5 Results and discussion**

465 *6.5.1 Live-load distribution factor*

466 The comparison of live-load distribution factor (LLDF) of each girder before and after
467 widening is shown in Fig.10. LLDF is determined according to Eq. (1). The maximum flexural
468 moment determined from a simple beam-line analysis, $M_{beamline}$, is 2245 kN·m for the
469 investigated bridge.

470 It can be observed that the LLDF of all existing girders will decrease after widening. As
471 the existing girders that are close to the splice joint (i.e., girder 5 and 6) turn into the interior
472 girder within the widened bridge, the reduction of LLDF associated with these girders is much
473 more significant than that of other girders. For instance, the LLDF of girder 6 after the widening
474 is around 41% of that before the widening; this value associated with girder 2 is approximately
475 90% of that before widening.

476 6.5.2 Axial force induced by concrete shrinkage and creep

477 The axial force induced by concrete shrinkage and creep of the new bridge is presented in
478 Fig.11. The results are obtained from the response of 100 times of FE analysis using grillage
479 model implemented within *ANSYS* (see Fig.5). The relevant random variables within the
480 computational process, such as the variables associated with shrinkage strain and creep
481 coefficient, are generated using the LHS, which could improve the sampling efficiency
482 significantly compared with traditional sampling method.

483 Results of the Chi-square (χ^2) goodness-of-fit tests indicate that normal distribution is an
484 acceptable approximation for the probability distribution of the axial forces (Fig.11(a)). Given
485 $t = 100$ years, the expected values of the axial force of all girders are presented in Fig.11(b),
486 and it can be concluded that both tensile and compressive force can result in the girders. For
487 instance, in the existing girders, the girder 1 has tensile force, while the girder 6, which is close
488 to the splice joint, results in compressive force. In contrast, for the new girders, the compressive
489 force is associated with the girders that are far away from the splice joint (e.g., girder 11). In
490 addition, the largest expected values of the compressive and tensile forces among all girders
491 are those associated with girders 6 and 7, respectively. The absolute values of these forces are
492 3863 kN and 3464 kN, respectively. Therefore, concrete shrinkage and creep has a significant
493 effect on the girders that adjacent to the splice joint, and should not be ignored.

494 The time-variant mean (μ) and mean plus ($\mu + \sigma$) and minus ($\mu - \sigma$) one standard deviation
495 (σ) of axial force in girder 6 and girder 7 are shown in Fig.11(c) and Fig.11(d), respectively. In

496 accordance with the development law of concrete shrinkage strain and creep coefficient, the
 497 mean value as well as standard deviation of axial force increase with time, but with a decreasing
 498 rate, and the axial force converges at about 20 years after widening.

499 In addition, a sensitivity analysis is conducted to sorting the importance of the input
 500 variables on the variability of the output parameters (i.e., axial forces of girder) within the
 501 analysis for shrinkage and creep effects. The Spearman rank-order correlation coefficient
 502 (SRCC), as defined by Eq. (33), is utilized herein. The SRCC has a value that ranges from -1
 503 and 1. A larger absolute value of SRCC indicates a higher contribution of the investigated input
 504 variable to the output. Accordingly, the SRCC between the input variable x and output variable
 505 y can be computed as [66]

$$506 \quad r_{x,y} = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^n (Y_i - \bar{Y})^2}} \quad (33)$$

507 where n is the sampling size; X_i is the rank of x_i within the sampling set of x , $[x_1, x_2, \dots, x_n]$; Y_i
 508 is the rank of y_i within the result set of y , $[y_1, y_2, \dots, y_n]$; and \bar{X} and \bar{Y} is the average value of
 509 X and Y set, respectively.

510 Given $t = 30$ years and 100 years, the SRCCs between the input variables and the output
 511 axial force in girder 6 and 7 are presented in Table 2. As indicated, the most important input
 512 factor is the modeling uncertainty factor of shrinkage Ψ_1 , followed by the nominal thickness of
 513 girder h and the modeling uncertainty factor of creep Ψ_2 . Thus, the Ψ_1 should be paid special
 514 attention within the evaluation process.

515 6.5.3 *Flexural moment capacity*

516 At each time step, based on Eqs. (28) - (31), flexural moment capacity of each girder is obtained
517 using MCS with 10,000 samplings. The profiles of the flexural moment capacity of girder 6
518 and 7 are shown in Fig.12. The Chi-square (χ^2) goodness-of-fit test is utilized to search for the
519 optimum distribution of flexural moment capacity. Using this test, for both cases that whether
520 the effects of the shrinkage-and-creep-induced axial force are considered or not, it is best to
521 describe the statistical distribution of flexural moment capacity using a lognormal distribution.
522 The fitting results are shown in Fig.12(a) and (b). Table 3 lists the statistical parameters of
523 these two cases. As shrinkage and creep can induce compressive and tensile axial force to
524 girder 6 and 7 (see Fig.11(b)), the mean value of flexural moment capacity of these two girders
525 increases and decreases by 27.7% and 21.6%, respectively. This change can be intuitively
526 illustrated by the failure curve of reinforced concrete section under the combined effects of
527 flexural moment M and axial force N [67]. In addition, for both girder 6 and 7, the coefficient
528 of variation (COV) of flexural moment capacity increases significantly after including the
529 effects of the axial force. This is mainly attributed to the variability of the axial force itself (see
530 Fig.11(a)) and the increase of modeling uncertainty for flexural capacity calculation regarding
531 different failure modes. The latter is reflected with the increase of COV for the modeling
532 uncertainty factors used in Eq. (28) and Eq. (29) (i.e., 0.062 for k_{pM} versus 0.1344 for k_{pMN} , as
533 listed in Table 1).

534 Under the combined effects of the shrinkage-and-creep-induced axial force and corrosion
535 of reinforcement steel, the expected value development of time-variant flexural moment
536 capacity of girder 6 and 7 after widening can be divided into two stages, as shown in Fig.12(c)
537 and Fig.12(d). Within 20 years after widening, the effects of axial force have a larger effect on
538 the structural performance. The expected value of flexural moment capacity of girder 6 and 7
539 increases and decreases with time, respectively, but both with a decreasing rate. Then, the
540 corrosion of reinforcement steel dominates the structural capacity degradation.

541 6.5.4 Time-variant reliability of girder components

542 The reliability index of each girder is computed based on the performance function (Eq. (32))
543 at each time step. The results for girders 3, 6, 7 and 10 are shown in Fig.13. According to GB/T
544 50283-1999 [55], the reliability index threshold β_T is set as 4.2. From Fig.13(a) and Fig.13(b),
545 it can be concluded that without widening, the reliability index of existing girders begins to
546 decrease after about 25 years' service when corrosion initiations, while the reliability index
547 remains above the threshold value through the design service years (i.e., 100 years).

548 After widening, the reliability index of existing girders will increase due to the decrease
549 of live-load distribution factor. For instance, at $t = 20$ years (widening time), the reliability
550 index of girder 3 and 6 is increased by 0.2 and 3.4 after widening, respectively. However, if
551 the effects of the shrinkage-and-creep-induced axial force is included, the reliability index of
552 all girders will drop immediately with a value of 2 - 4 at the widening time under the increased
553 COV of flexural moment capacity. After that, the reliability index mainly depends on the

554 quantity of the shrinkage-and-creep-induced axial force. For girder 6 that has the largest
555 compressive axial force, the reliability index will decrease with time after widening, but with
556 a decreasing rate. The decreasing rate of reliability index in the late stage is lower than that of
557 the case without considering the effects of axial force, as shown in Fig.13(b). In contrast, the
558 reliability index of girder 7 will decrease with time at a relatively high rate, as shown in
559 Fig.13(c). The reliability index of girder 6 and 7 reaches the reliability index threshold at about
560 15 and 50 years after widening, respectively. For girder 3 and 10, as they are less effected by
561 concrete shrinkage and creep, i.e., the expected value of the shrinkage-and-creep-induced axial
562 force is relatively small (see Fig.11(b)), their reliability index will remain above the threshold
563 value through the service life, as shown in Fig.13(a) and (d).

564 6.5.5 Time-variant reliability of superstructure system

565 RELSYS is utilized to compute the reliability index of existing and widened bridge systems
566 using a series-parallel model as shown in Fig.9. The correlations among the random variables
567 are assumed herein. Specifically, two extreme scenarios are considered regarding capacity
568 correlation among different girders: (a) independent ($\rho_{M_{ui},M_{uj}} = 0$); and (b) perfect correlation
569 among different new or existing girders ($\rho_{M_{ui},M_{uj}} = 1$). The correlation between the new girders
570 and existing girders is assumed to be zero. Given more detailed information (e.g., materials
571 used, construction methods, deterioration scenarios), the correlation effects could be easily
572 updated within the reliability analysis process. Within the computational process for system
573 reliability in RELSYS, the determined correlation coefficients among the random parameters

574 (e.g., capacity of different girders) are used to compute the equivalent alpha vectors, which are
575 then utilized to account for the correlations between the failure of different equivalent
576 components [68]. The results of system reliability are presented in Fig.14.

577 As shown in Fig.14(a), the system reliability index of existing superstructure without
578 considering correlation is higher than that of any girder, this indicates that the redundancy of
579 bridge system will improve the safety of structure. When the correlation is included, the system
580 reliability index reduces significantly, but is still much higher than reliability index threshold
581 through the service life. When the correlation and effects of shrinkage and creep are considered
582 simultaneously, the system reliability index of the widened bridge reaches the index threshold
583 in about 70 years after widening, as shown in Fig.14(b).

584 **7. Conclusions**

585 This paper presents a computational probabilistic framework for time-variant reliability of
586 individual girders as well as system of widened multi-girders concrete bridges. Additionally,
587 the effects of live-load redistribution, concrete shrinkage and creep and the difference in
588 reinforcement corrosion between new and existing girders are considered within the assessment
589 process. The presented approach is applied to a widened prestressed concrete bridge.

590 The following conclusions are drawn:

- 591 1. The live-load distribution factor (LLDF) of all existing girders is reduced after
592 widening. As the existing external girders that are close to the splice joint turn into
593 interior girders after widening, the reduction of LLDF associated with these girders is

594 more than 40%.

595 2. Normal distribution is an acceptable approximation to describe the probability
596 distribution of axial force induced by concrete shrinkage and creep. As the shrinkage-
597 and-creep-induced axial force changes the failure mode of prestressed concrete
598 sections (i.e., from flexural failure to compressive-flexural or tensile-flexural
599 combined failure), the expected value of flexural moment capacity of girder
600 component can increase and decrease when subjected to compressive and tensile axial
601 force, respectively. Simultaneously, due to the considerable variability of the
602 shrinkage-and-creep-induced axial force, as well as the significant increase of
603 modeling uncertainty for flexural capacity calculation regarding different failure
604 modes, the variation of flexural moment capacity of all girders will increase
605 significantly when considering the shrinkage and creep effects.

606 3. Concrete shrinkage and creep have more significant effects on girder reliability than
607 that of live-load redistribution and reinforcement corrosion, especially for the girders
608 adjacent to splice joint, which are subjected to maximum shrinkage-and-creep-induced
609 axial force. The reliability indices of the existing and new girders adjacent to splice
610 joint reaches the threshold value at about 15 and 50 years after widening, respectively.
611 Therefore, special emphasis, such as carefully assessment and conservative design,
612 should be placed on the girders that adjacent to the splice joint when conducting bridge
613 widening.

614 4. In addition to the shrinkage and creep effects, capacity correlation among different
615 girders has a considerable impact on the system reliability of widened bridges. When
616 correlation and effects of concrete shrinkage and creep are simultaneously considered,
617 the reliability index of the bridge system reaches the threshold value in about 70 years
618 after widening. Therefore, the correlation among different girders should be carefully
619 estimated.

620 5. The interaction effects between the new and existing parts within a widened bridge are
621 emphasized in this paper. The thermal variation and prestress loss could also have
622 some effects on the structural performance. Given more information, these effects can
623 also be incorporated within the evaluation process.

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631 **References**

632 [1] ACI Committee 345. Guide for widening highway bridges, ACI 345.2R-13, American
633 Concrete Institute, Michigan, USA; 2013.

- 634 [2] Zong Z, Xia Z, Chen Y, Zhao X. State- of- the- art and engineering example analysis of
635 longitudinal connection joints between the new bridges and existing old bridges during
636 bridge widening and constructions. Journal of Fuzhou University (Natural Science Edition)
637 2009; 37(2):248-60 (in Chinese).
- 638 [3] Huang P. Research on Static Behavior of Broaden Spliced Prestressed Concrete T Type
639 Beam Bridges. Journal of Highway and Transportation Research and Development 2010;
640 27(3):86-92 (in Chinese).
- 641 [4] Wen Q. Long-term effect analysis of prestressed concrete box-girder bridge
642 widening. Construction and Building Materials 2011; 25(4):1580-6.
- 643 [5] Wipf TJ, Fanous FS, Klaiber FW, Eapen AS. Evaluation of Appropriate Maintenance,
644 Repair and Rehabilitation Methods for Iowa Bridges. Final Report, Iowa DOT Project TR-
645 429; 2003.
- 646 [6] SCDOT. SCDOT bridge design manual (Chapter 23: Bridge Widening and Rehabilitation).
647 Columbia (SC): South Carolina Department of Transportation; 2006.
- 648 [7] Caltrans. Memo to designers (Chapter 9-3: Bridge Widening existing bridges). Sacramento
649 (CA): California Department of Transportation; 2010.
- 650 [8] WIDOT. WisDOT Bridge Manual (Chapter 40: Bridge Rehabilitation). Madison (WI):
651 Wisconsin Department of Transportation; 2016.
- 652 [9] JTG D62-2004. Code for design of highway reinforced concrete and prestressed concrete
653 bridge and culverts. Beijing: China Communications Press; 2004
- 654 [10] Fédération Internationale du Béton (fib). Model code 2010 -final draft. Fédération
655 Internationale du Béton, Bulletins 65, 66, Lausanne, Switzerland; 2012.
- 656 [11] AASHTO LRFD. Bridge design specifications. American Association of State Highway
657 and Transportation Officials; 2014.
- 658 [12] Mori Y, Ellingwood BR. Reliability-based service-life assessment of aging concrete
659 structures. Journal of Structural Engineering 1993; 119(5):1600-21.
- 660 [13] Estes AC, Frangopol DM. Repair optimization of highway bridges using system reliability
661 approach. Journal of Structural Engineering 1999; 125(7):766-75.

- 662 [14] Frangopol DM, Kong JS, Gharaibeh ES. Reliability-based life-cycle management of
663 highway bridges. *Journal of Computing in Civil Engineering* 2001; 15(1):27-34.
- 664 [15] Kong JS, Frangopol DM. Life-cycle reliability-based maintenance cost optimization of
665 deteriorating structures with emphasis on bridges. *Journal of Structural Engineering* 2003;
666 129(6):818-28.
- 667 [16] Saydam D, Frangopol DM, Dong Y. Assessment of risk using bridge element condition
668 ratings. *Journal of Infrastructure Systems* 2012; 19(3):252-65.
- 669 [17] Barone G, Frangopol DM. Reliability, risk and lifetime distributions as performance
670 indicators for life-cycle maintenance of deteriorating structures. *Reliability Engineering
671 and System Safety* 2014; 123:21-37.
- 672 [18] Nie J, Zhang X, Fan J, Li Y, Xu R. Transverse distribution coefficient of concrete bridges
673 widened with steel-concrete composite beams. *Journal of Tsinghua University (Science
674 and Technology Edition)* 2010; 50(6):805-9 (in Chinese).
- 675 [19] Chen J, Zhao S, Yao J. Method for calculating vehicle load traverse distribution in
676 widening design of existing PPCHS bridge. *Engineering Mechanics* 2012; 29(9):265-71
677 (in Chinese).
- 678 [20] Chang G, Zhang C, Hu W, Wang P. Influence of Widening and Renovation on Mechanical
679 Performance of Old Bridges. *Journal of Chongqing Jiaotong University (Natural Science)*
680 2013; 32(1):836-8 (in Chinese).
- 681 [21] Wen Q, Ye J. Analysis of shrinkage and creep effects in widening reinforced concrete
682 girder. *Journal of Southeast University (Natural Science Edition)* 2006; 36(4):596-600 (in
683 Chinese).
- 684 [22] Wen Q, Ye J. Shrinkage and creep new and effects of lateral jointing for old concrete
685 beam. *China Journal of Highway and Transport* 2007; 20(4):53-7 (in Chinese).
- 686 [23] Tu B, Fang Z. Analysis of shrinkage and creep effects on newly-built and existing concrete
687 slab bridge after lateral jointing. *China Journal of Highway and Transport* 2016;
688 29(10):66-76 (in Chinese).

- 689 [24] Fang Z, Chang H, Yang X, Yuan Y. Transverse effects resulting from concrete shrinkage
690 and creep of widened concrete box girder bridge. *China Journal of Highway and Transport*
691 2013; 26(6):65-72 (in Chinese).
- 692 [25] Enright MP, Frangopol DM. Probabilistic analysis of resistance degradation of reinforced
693 concrete bridge beams under corrosion. *Engineering structures* 1998; 20(11):960-71.
- 694 [26] Dong Y, Frangopol DM, Saydam D. Time-variant sustainability assessment of seismically
695 vulnerable bridges subjected to multiple hazards. *Earthquake Engineering and Structural*
696 *Dynamics* 2013; 42(10):1451-67.
- 697 [27] Zhou Y, Gencturk B, Willam K, Attar A. Carbonation-induced and chloride-induced
698 corrosion in reinforced concrete structures. *Journal of Materials in Civil Engineering* 2014;
699 27(9):04014245.
- 700 [28] Frangopol DM, Lin KY, Estes AC. Reliability of reinforced concrete girders under
701 corrosion attack. *Journal of Structural Engineering* 1997; 123(3):286-97.
- 702 [29] Vu KA, Stewart MG. Structural reliability of concrete bridges including improved
703 chloride-induced corrosion models. *Structural safety* 2000; 22(4):313-33.
- 704 [30] Akgül F, Frangopol DM. Time-dependent interaction between load rating and reliability
705 of deteriorating bridges. *Engineering Structures* 2004; 26(12):1751-65.
- 706 [31] Guo T, Frangopol DM, Han D, Chen Y. Probabilistic assessment of deteriorating
707 prestressed concrete box-girder bridges under increased vehicle loads and aggressive
708 environment. *Journal of Performance of Constructed Facilities* 2010; 25(6):564-76.
- 709 [32] Strauss A, Wendner R, Bergmeister K, Costa C. Numerically and experimentally based
710 reliability assessment of a concrete bridge subjected to chloride-induced
711 deterioration. *Journal of Infrastructure Systems* 2012; 19(2):166-75.
- 712 [33] Pillai RG, Trejo D, Gardoni P, Hueste MBD, Reinschmidt K. Time-Variant Flexural
713 Reliability of Posttensioned, Segmental Concrete Bridges Exposed to Corrosive
714 Environments. *Journal of Structural Engineering* 2014; 140(8): A4014018.

- 715 [34] Frangopol DM, Dong Y, Sabatino S. Bridge life-cycle performance and cost: Analysis,
716 prediction, optimization and decision making. *Structure and Infrastructure Engineering*
717 2017; doi: 10.1080/15732479.2016.1267772 (in press).
- 718 [35] Val DV, Stewart MG, Melchers RE. Effect of reinforcement corrosion on reliability of
719 highway bridges. *Engineering structures* 1998; 20(11):1010-9.
- 720 [36] Darmawan MS, Stewart MG. Spatial time-dependent reliability analysis of corroding
721 pretensioned prestressed concrete bridge girders. *Structural Safety* 2007; 29(1):16-31.
- 722 [37] Akiyama M, Frangopol DM, Yoshida I. Time-dependent reliability analysis of existing
723 RC structures in a marine environment using hazard associated with airborne chlorides.
724 *Engineering Structures* 2010; 32(11):3768-79.
- 725 [38] McKay MD, Beckman RJ, Conover WJ. Comparison of three methods for selecting values
726 of input variables in the analysis of output from a computer code. *Technometrics* 1979;
727 21(2):239-45.
- 728 [39] Harris DK. Assessment of flexural lateral load distribution methodologies for stringer
729 bridges. *Engineering Structures* 2010; 32(11):3443-51.
- 730 [40] Barr PJ, Eberhard MO, Stanton JF. Live-load distribution factors in prestressed concrete
731 girder bridges. *Journal of Bridge Engineering* 2001; 6(5):298-306.
- 732 [41] Huo XS, Wasserman EP, Zhu P. Simplified method of lateral distribution of live load
733 moment. *Journal of Bridge Engineering* 2004; 9(4):382-90.
- 734 [42] Hughs E, Idriss R. Live-load distribution factors for prestressed concrete, spread box-
735 girder bridge. *Journal of Bridge Engineering* 2006; 11(5):573-81.
- 736 [43] Yousif Z, Hindi R. AASHTO-LRFD live load distribution for beam-and-slab bridges:
737 Limitations and applicability. *Journal of Bridge Engineering* 2007; 12(6): 765-73.
- 738 [44] Hambly EC. *Bridge deck behavior*. 2nd ed. E & FN Spon; 1991.
- 739 [45] *Ansys 17.2 Documentation*. Ansys, Inc., Canonsburg, PA, USA; 2017.
- 740 [46] CEB-FIP MC 90. *Design of concrete structures*. CEB-FIP Model Code 1990. Thomas
741 Telford; 1993.

- 742 [47] Bažant ZP. Prediction of Concrete Creep Effects Using Age-Adjusted Effective. ACI
743 Journal 1972; 69:212-7.
- 744 [48] Zienkiewicz OC, Taylor RL. The finite element method. New York: McGraw-Hill; 1977.
- 745 [49] Bažant ZP, Baweja S. Justification and refinements of model B3 for concrete creep and
746 shrinkage 1. Statistics and sensitivity. Materials and structures 1995; 28(7):415-30.
- 747 [50] González JA, Andrade C, Alonso C, Feliu S. Comparison of rates of general corrosion and
748 maximum pitting penetration on concrete embedded steel reinforcement. Cement and
749 concrete research 1995; 25(2):257-64.
- 750 [51] Val DV, Melchers RE. Reliability of deteriorating RC slab bridges. Journal of Structural
751 Engineering 1997; 123(12):1638-44.
- 752 [52] Du YG, Clark LA, Chan AHC. Residual capacity of corroded reinforcing bars. Magazine
753 of Concrete Research 2005; 57(3):135-48.
- 754 [53] Vu NA, Castel A, François R. Effect of stress corrosion cracking on stress–strain response
755 of steel wires used in prestressed concrete beams. Corrosion Science 2009; 51(6):1453-9.
- 756 [54] JTG D62-85. Code for design of highway reinforced concrete and prestressed concrete
757 bridge and culverts. Beijing: China Communications Press; 1985 (in Chinese).
- 758 [55] GB/T 50283-1999. Unified standard for the reliability design of highway engineering
759 structures. Beijing: China Plan Press; 1999 (in Chinese).
- 760 [56] JTG D60-2004. General specifications for design of highway bridges and culverts. Beijing:
761 China Communications Press; 2004 (in Chinese).
- 762 [57] Estes AC, Frangopol DM. RELSYS: A computer program for structural system reliability.
763 Structural Engineering and Mechanics 1998; 6(8):901-19.
- 764 [58] Estes AC, Frangopol DM. Bridge lifetime system reliability under multiple limit states.
765 Journal of Bridge Engineering 2001; 6(6):523-8.
- 766 [59] Imai K, Frangopol DM. Reliability-based assessment of suspension bridges: Application
767 to the Innoshima Bridge. Journal of Bridge Engineering 2001; 6(6):398-411.
- 768 [60] Attard MM, Stewart MG. A two-parameter stress block for high-strength concrete. ACI
769 Structural Journal 1998; 95(3):305-17.

- 770 [61] Mirza SA, Hatzinikolas M, MacGregor JG. Statistical descriptions of strength of concrete.
771 Journal of the Structural Division 1979;105(6):1021-37.
- 772 [62] Zhang G, Jin X, Guan Y, Yan X. Sultry weather in Shijiazhuang, China and the influencing
773 factors in 1957-2010. PROGRESSUS INQUISITIONES DE MUTATIONE CLIMATIS
774 2013; 9(3):199-203 (in Chinese).
- 775 [63] Val DV, Trapper PA. Probabilistic evaluation of initiation time of chloride-induced
776 corrosion. Reliability Engineering and System Safety 2008; 93(3):364-72.
- 777 [64] Stewart MG, Rosowsky DV. Time-dependent reliability of deteriorating reinforced
778 concrete bridge decks. Structural Safety 1998; 20(1):91-109.
- 779 [65] Li Y, Bao W, Guo X, Cheng X. Structural reliability and probabilistic limit state design.
780 Beijing: China Communications Press; 1997 (in Chinese).
- 781 [66] Conover WJ, Iman RL. Rank transformations as a bridge between parametric and
782 nonparametric statistics. The American Statistician 1981; 35(3):124-9.
- 783 [67] Park R, Paulay T. Reinforced concrete structures. New York: John Wiley & Sons; 1975.
- 784 [68] Gollwitzer S, Rackwitz R. Equivalent components in first-order system reliability.
785 Reliability Engineering 1983; 5(2):99-115.
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Variable	Properties	Mean	COV	Distribution type	Reference
b'_f	Effective width of flange	1.0013*	0.0081	Normal	[55]
b	Width of web	1.032*	0.1019	Normal	[55]
h'_f	Height of flange	1.032*	0.1019	Normal	[55]
h_0	Effective height of section	1.0124*	0.0229	Normal	[55]
a'_s	Distance from the top fiber to the centroid of the compressive reinforcement	1.0178*	0.0496	Normal	[55]
h	Nominal thickness of girder	1.032*	0.1019	Normal	[55]
d	Thickness of concrete cover	1.0178*	0.0496	Normal	[55]
A_s, A'_s, A_p	Initial area of tensile reinforcement, compressive reinforcement and prestressed tendon	1.000*	0.0350	Normal	[55]
$f_{c,e}$ (MPa)	Compressive strength of concrete in existing bridge	34.1	0.16	Normal	[55]
$f_{c,n}$ (MPa)	Compressive strength of concrete in new bridge	39.9	0.15	Lognormal	[60]
$f_{s1,e}$ (MPa)	Initial yield strength of reinforcement bar (grade I) in existing bridge	259	0.1211	Normal	[55]
$f_{s2,e}$ (MPa)	Initial yield strength of reinforcement bar (grade II) in existing bridge	369	0.0719	Normal	[55]
$f_{s2,n}$ (MPa)	Initial yield strength of reinforcement bar in new bridge	369	0.0719	Normal	[55], [9]
$f_{p,e}$ (MPa)	Initial ultimate strength of prestressed tendon in existing bridge	1662	0.0142	Normal	[61]
$f_{p,n}$ (MPa)	Ultimate strength of prestressed tendon in new bridge	1932	0.0142	Normal	[61]
RH (%)	Annual relative humidity of ambient environment	54.4	0.075	Normal	[62]
ψ_1	Modeling uncertainty factor of concrete shrinkage	1.000	0.451	Normal	[49]
ψ_2	Modeling uncertainty factor of concrete creep	1.000	0.339	Normal	[49]
D_c (cm ² /year)	Diffusion coefficient	0.631	0.2	Lognormal	[63]
C_0 (kg/m ³)	Surface chloride content	15	0.2	Normal	[63]
C_{cr} (kg/m ³)	Threshold chloride concentration	2.0	0.2	Normal	[63]
i_{corr} (μ A/cm ²)	Corrosion current density	1.0	0.2	Normal	[50]
R	Penetration ratio	3.00	0.33	Normal	[64]

Variable	Properties	Mean	COV	Distribution type	Reference
$M_{DL,i}$	Dead load moment	1.0148*	0.0431	Normal	[55]
$M_{LL,i}$	Live load moment	0.7995*	0.0862	Extreme value (type I)	[55]
η_{DLA}	Dynamic load amplification factor	1.1776	0.0428	Extreme value (type I)	[55]
k_{pM}	Modeling uncertainty factor for flexural moment capacity	1.110	0.062	Normal	[65]
k_{pMN}	Modeling uncertainty factor for flexural moment capacity with axial force	1.1389	0.1344	Normal	[65]

791 * indicates the normalized value divided by nominal value.

792

793 **Table 2.** SRCCs between the input and out variables within the analysis for shrinkage and

794 creep effects

Service time t (years)	Output variable	Input variable				
		$f_{c,n}$ (Compressive strength of concrete in new bridge)	RH (Annual relative humidity of ambient environment)	h (Nominal thickness of girder)	Ψ_1 (Modeling uncertainty factor of shrinkage)	Ψ_2 (Modeling uncertainty factor of creep)
30	Axial force in girder 6	0.078	0.092	0.280	-0.923	-0.196
	Axial force in girder 7	0.003	-0.085	-0.276	0.967	0.060
100	Axial force in girder 6	0.053	0.101	0.207	-0.947	-0.103
	Axial force in girder 7	0.013	-0.074	0.199	0.977	-0.035

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798 **Table 3.** Statistical parameters of flexural moment capacity at $t = 100$ years

Girder number	Without axial force effects			With axial force effects		
	Mean (kN·m)	Standard deviation (kN·m)	COV	Mean (kN·m)	Standard deviation (kN·m)	COV
6#	6014	544	0.090	7682	2126	0.277
7#	7686	662	0.086	6025	1230	0.204

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802

803 **FIGURES**

804 **Fig.1.** Flowchart of reliability assessment of widened concrete bridges

805 **Fig.2.** Schematic diagram of grillage model for a widened multi-girders bridge (a) basic
806 grillage model; (b) model of splice joint for widened T-girder bridges; and (c) model of splice
807 joint for widened hollow-slab bridges

808 **Fig.3.** Flowchart of the proposed AAEM-based procedure

809 **Fig.4.** Bridge configuration (a) mid-span section after widening; (b) bottom view (two new
810 girders and one existing girder are included only); (c) arrangement of reinforcement steel of
811 existing girder (mid-span section); and (d) arrangement of reinforcement steel of new girder
812 (mid-span section) (Units: cm)

813 **Fig.5.** FE grillage model of the investigated bridge implemented with *ANSYS*

814 **Fig.6.** Axle spacing and weight distribution of the semitrailer truck specified in JTG D60 2004
815 (a) lateral view; and (b) plan view (Units: m) [56]

816 **Fig.7.** Transverse truck load cases and positions (Units: m)

817 **Fig.8.** Simplified stress-block model for flexural capacity calculation (a) case without axial
818 force; (b) geometric parameters of simplified section; (c) case with compressive force; and (d)
819 case with tensile force [9]

820 **Fig.9.** System reliability model of bridge superstructure

821 **Fig.10.** Comparison of live-load distribution factor before and after widening

822 **Fig.11.** Axial force of mid-span section caused by concrete shrinkage and creep (a) probability
823 density function associated with girder 7 at $t = 100$ years; (b) expect value in different girder

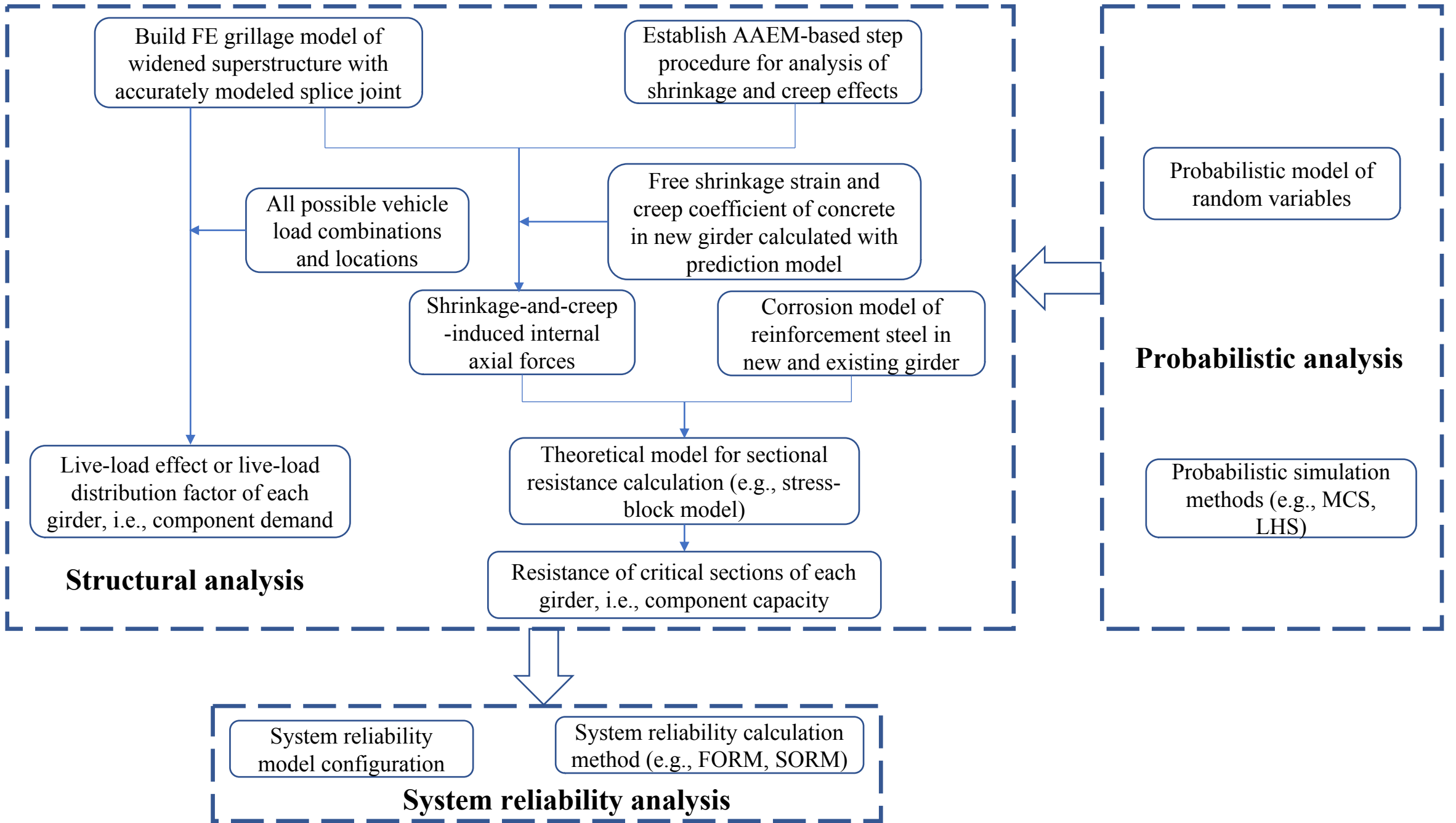
824 at $t = 100$ years; (c) time-variant value of statistic parameters of girder 6; and (d) time-variant
825 value of statistic parameters of girder 7

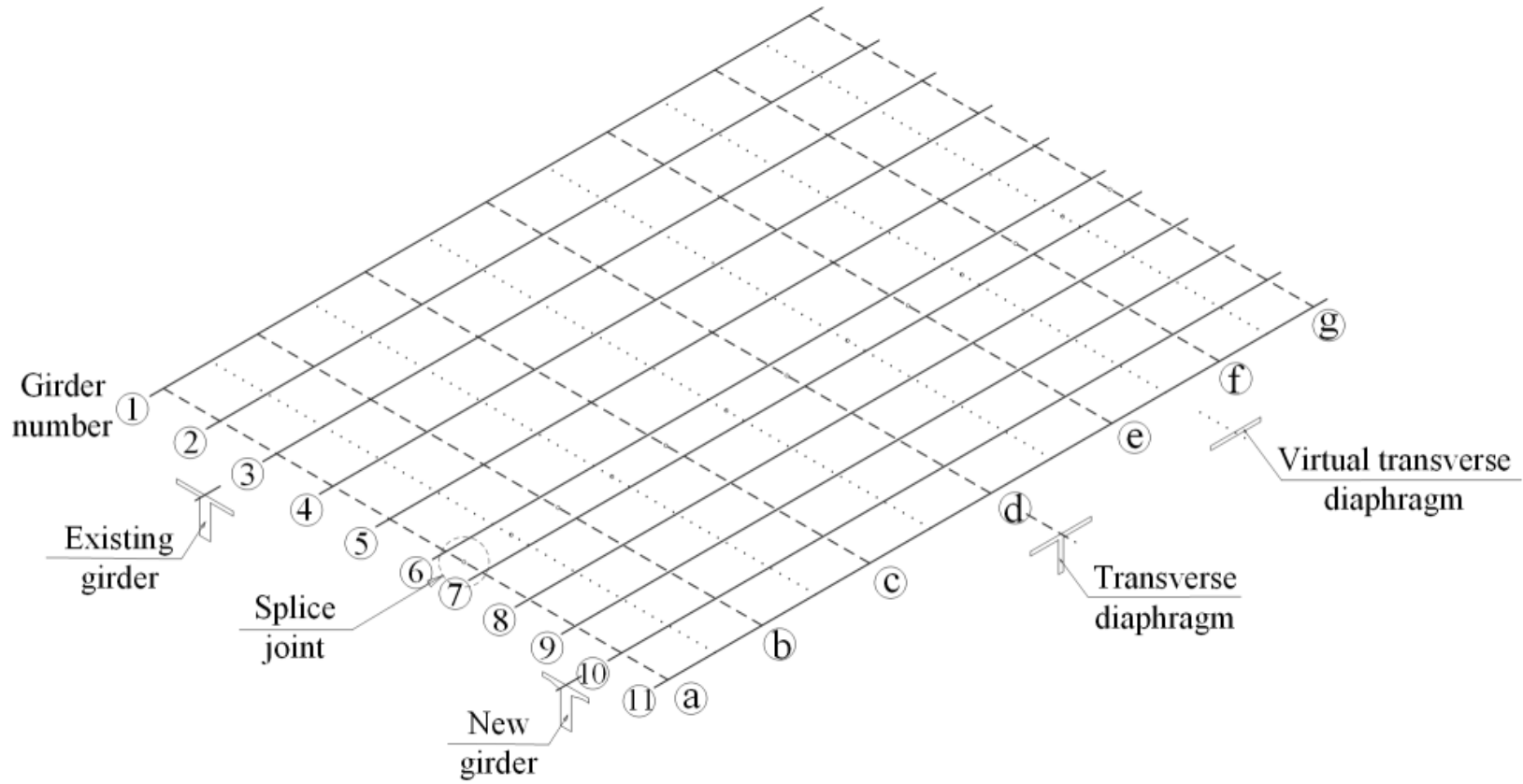
826 **Fig.12.** Flexural moment capacity profiles (a) probability density function associated with
827 girder 6 at $t = 100$ years; (b) probability density function associated with girder 7 at $t = 100$
828 years; (c) time-variant development of expected value of girder 6; and (d) time-variant
829 development of expected value of girder 7

830 **Fig.13.** Time-variant reliability index of single girder (a) girder 3; (b) girder 6; (c) girder 7;
831 and (d) girder 10

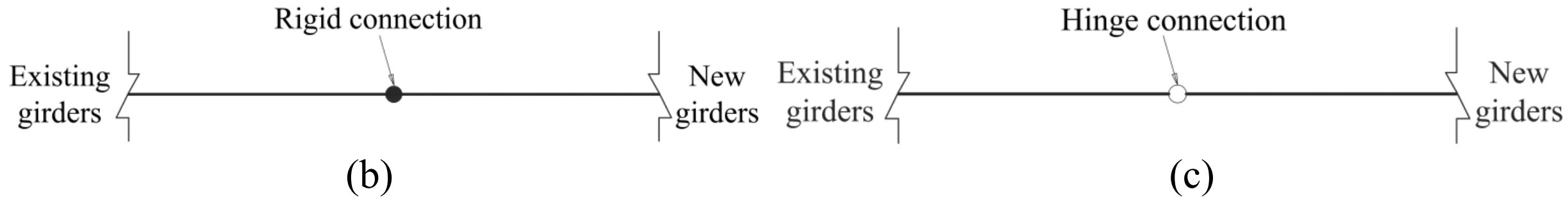
832 **Fig.14.** Time-variant reliability index of superstructure system (a) existing bridge; and (b)
833 widened whole bridge

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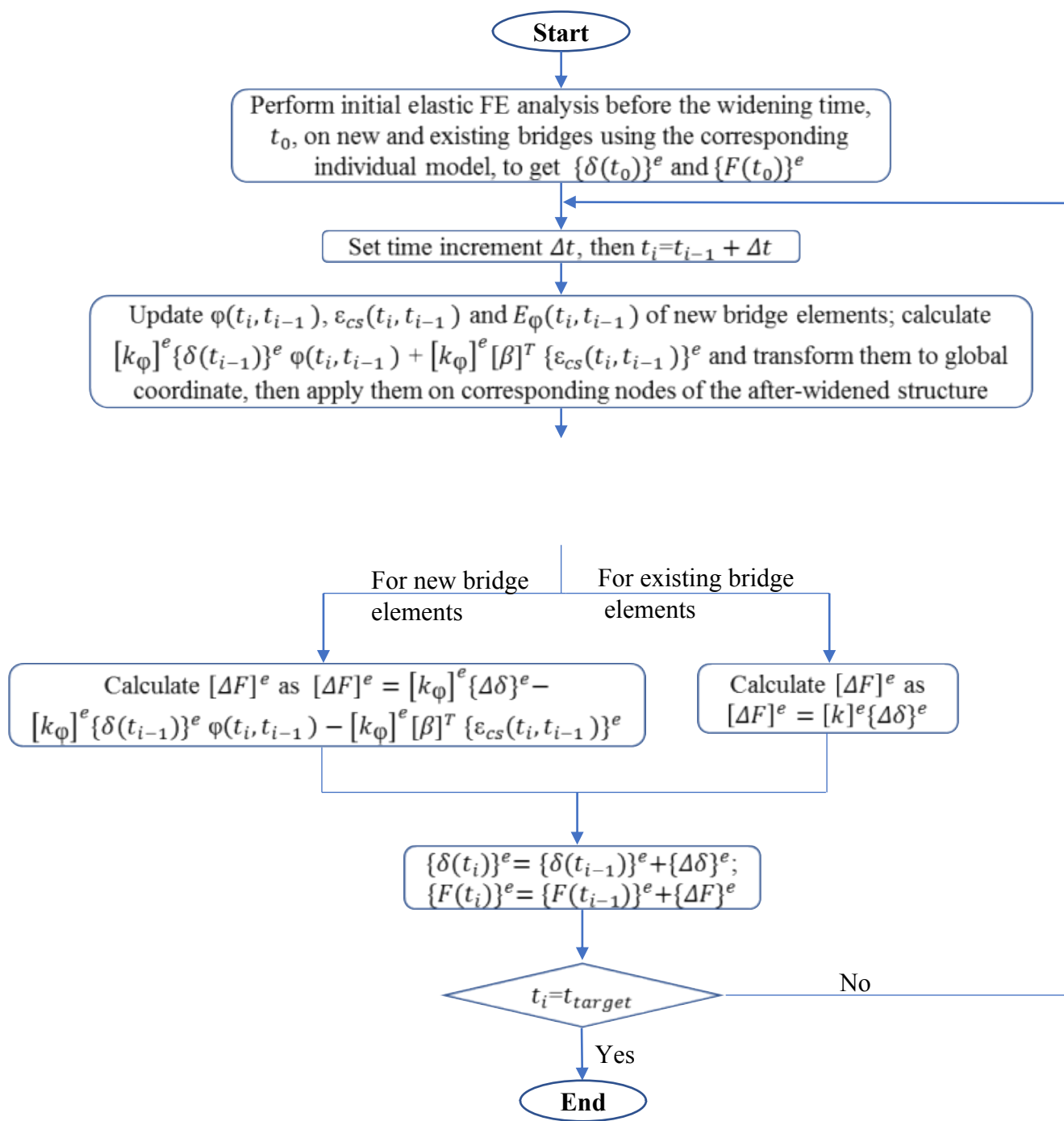


(a)



(b)

(c)

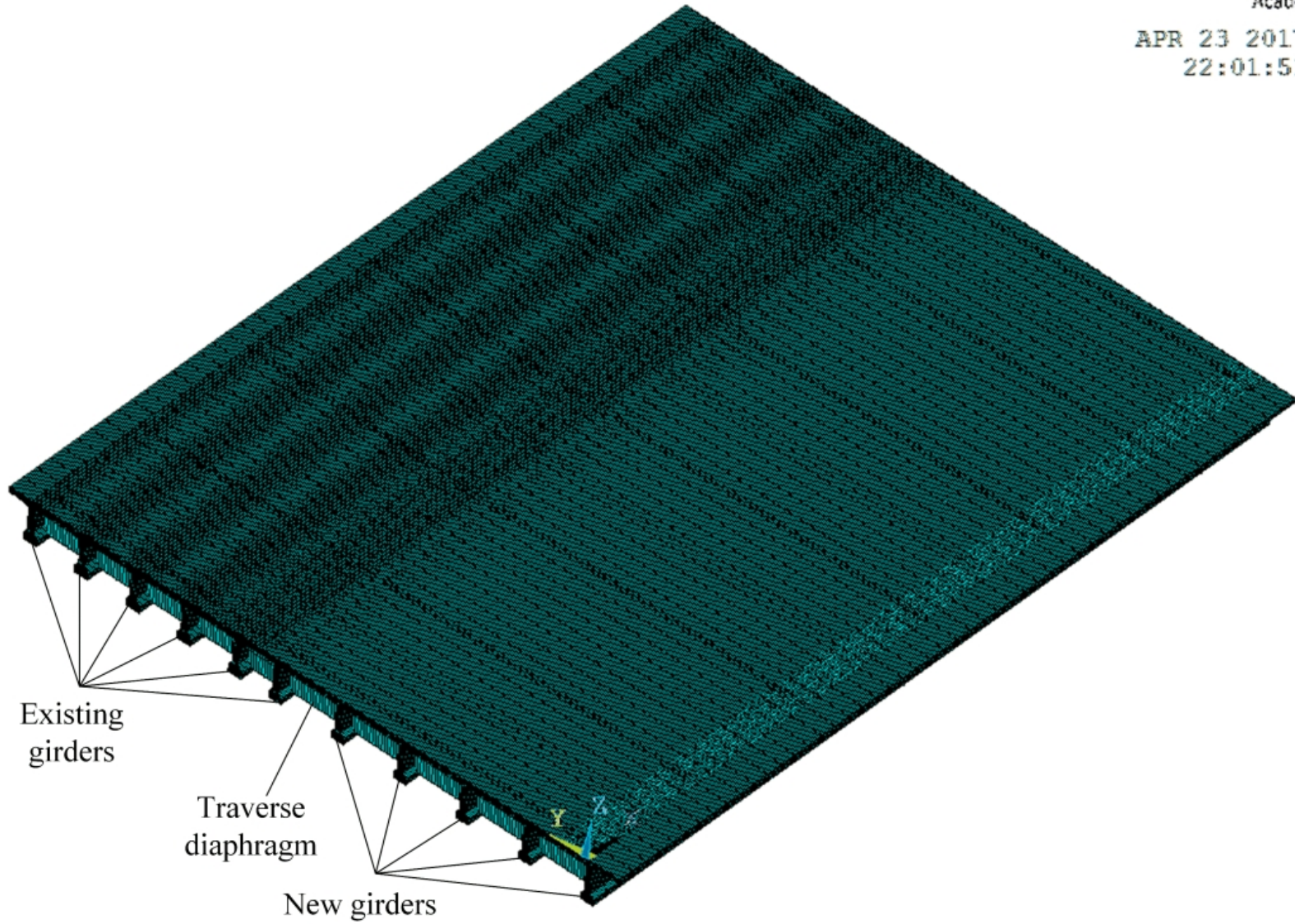


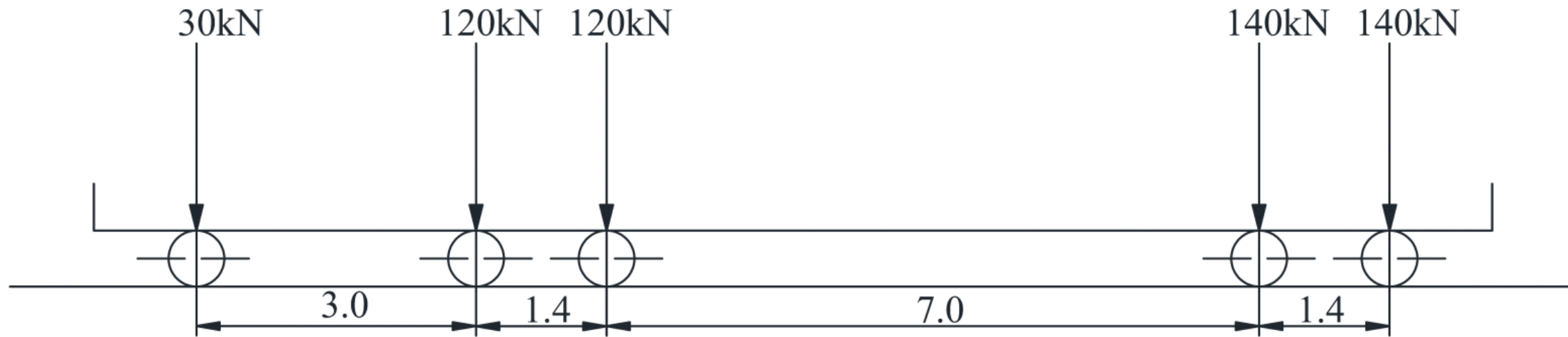
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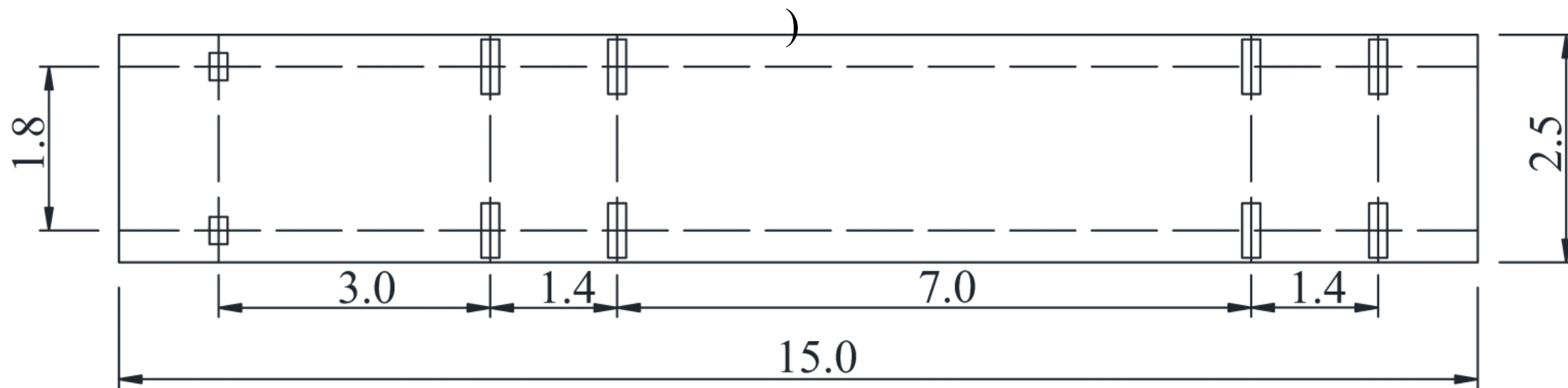
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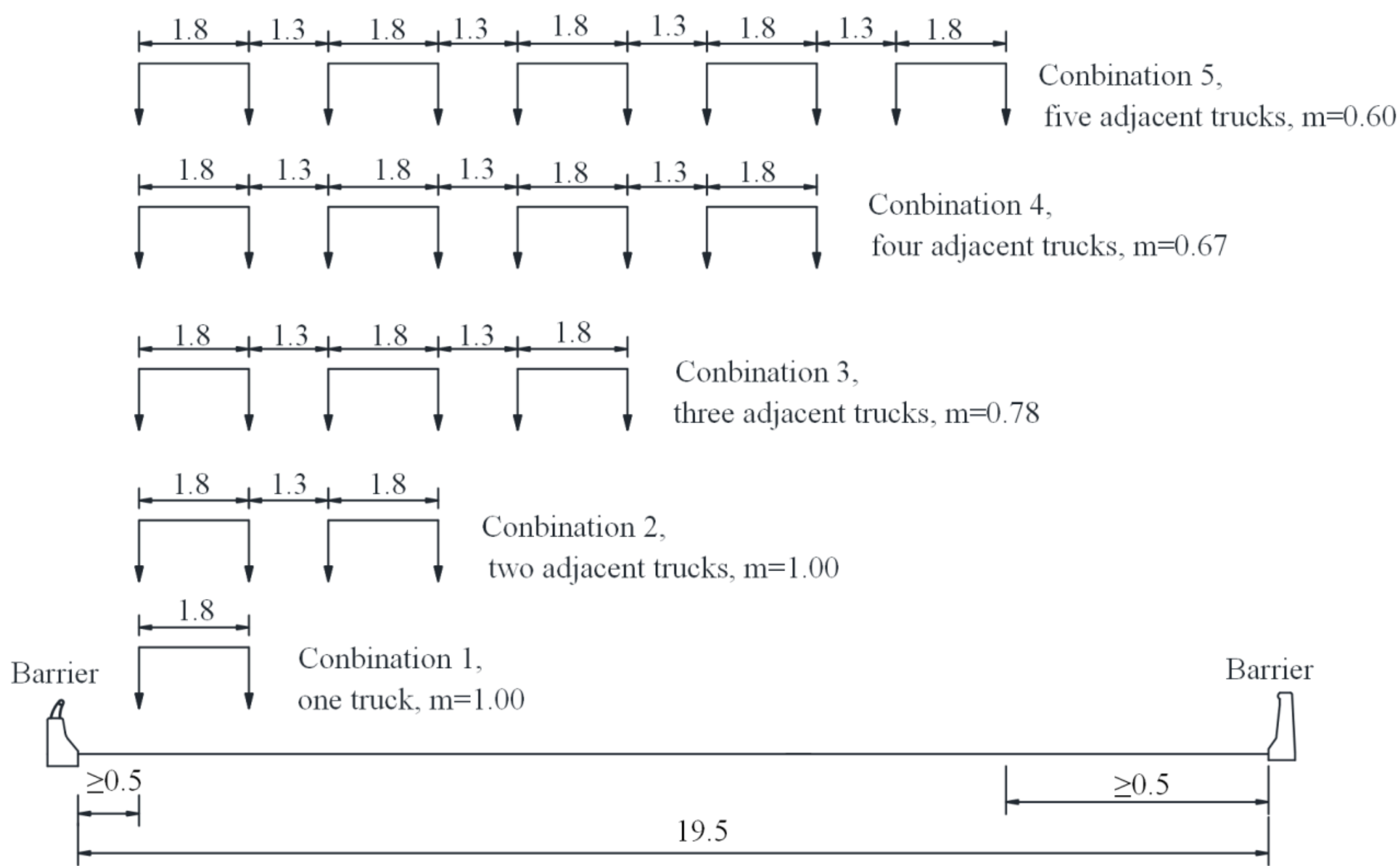


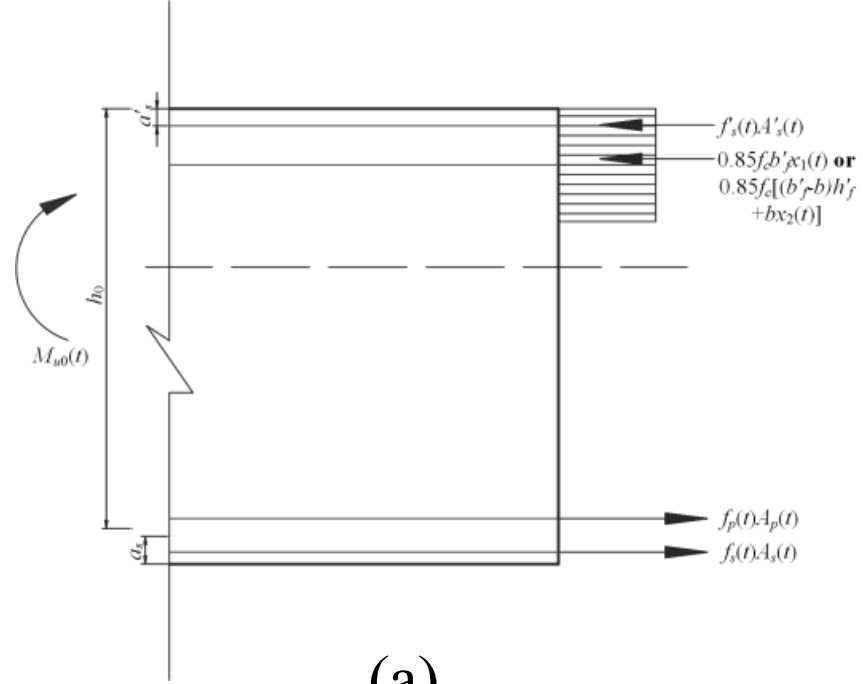


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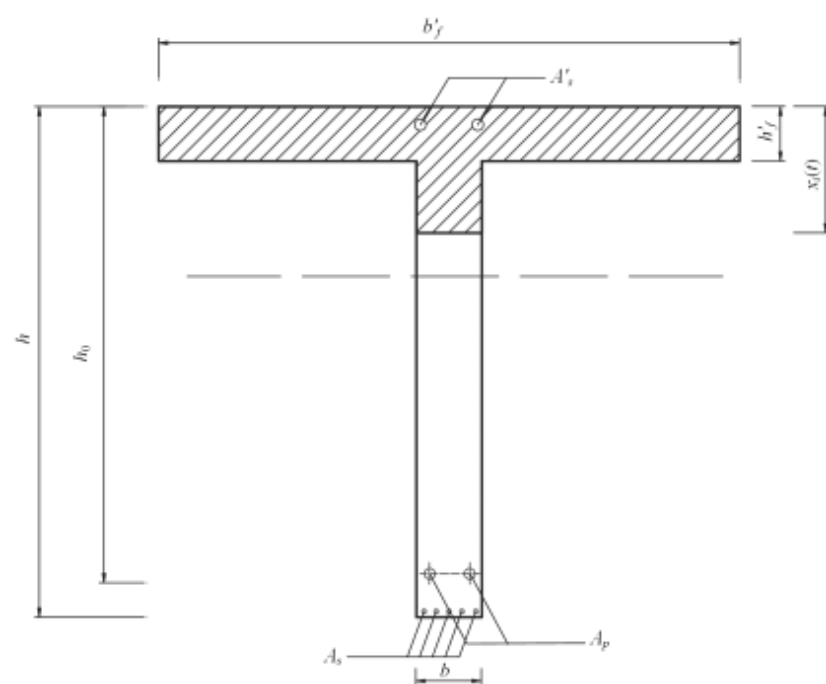


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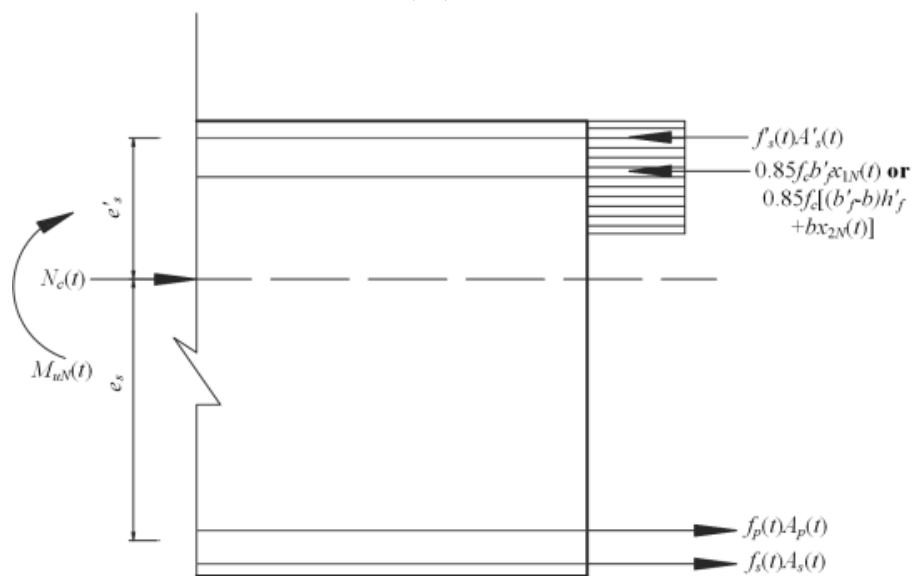




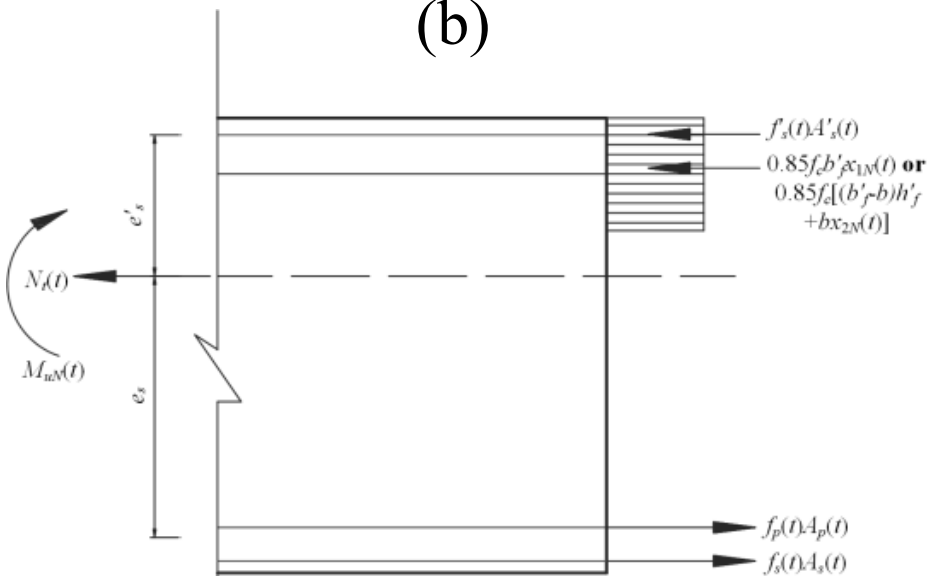
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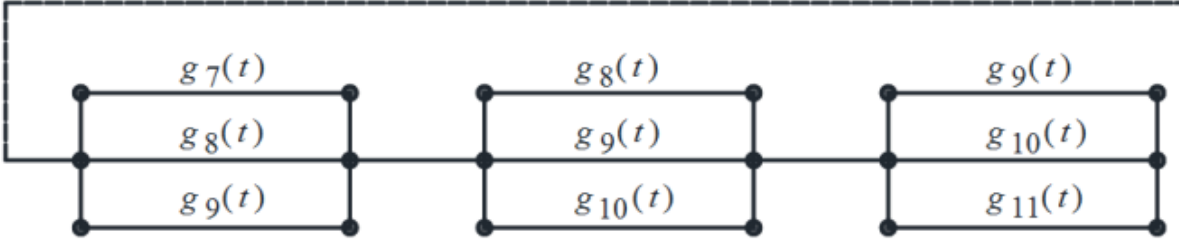
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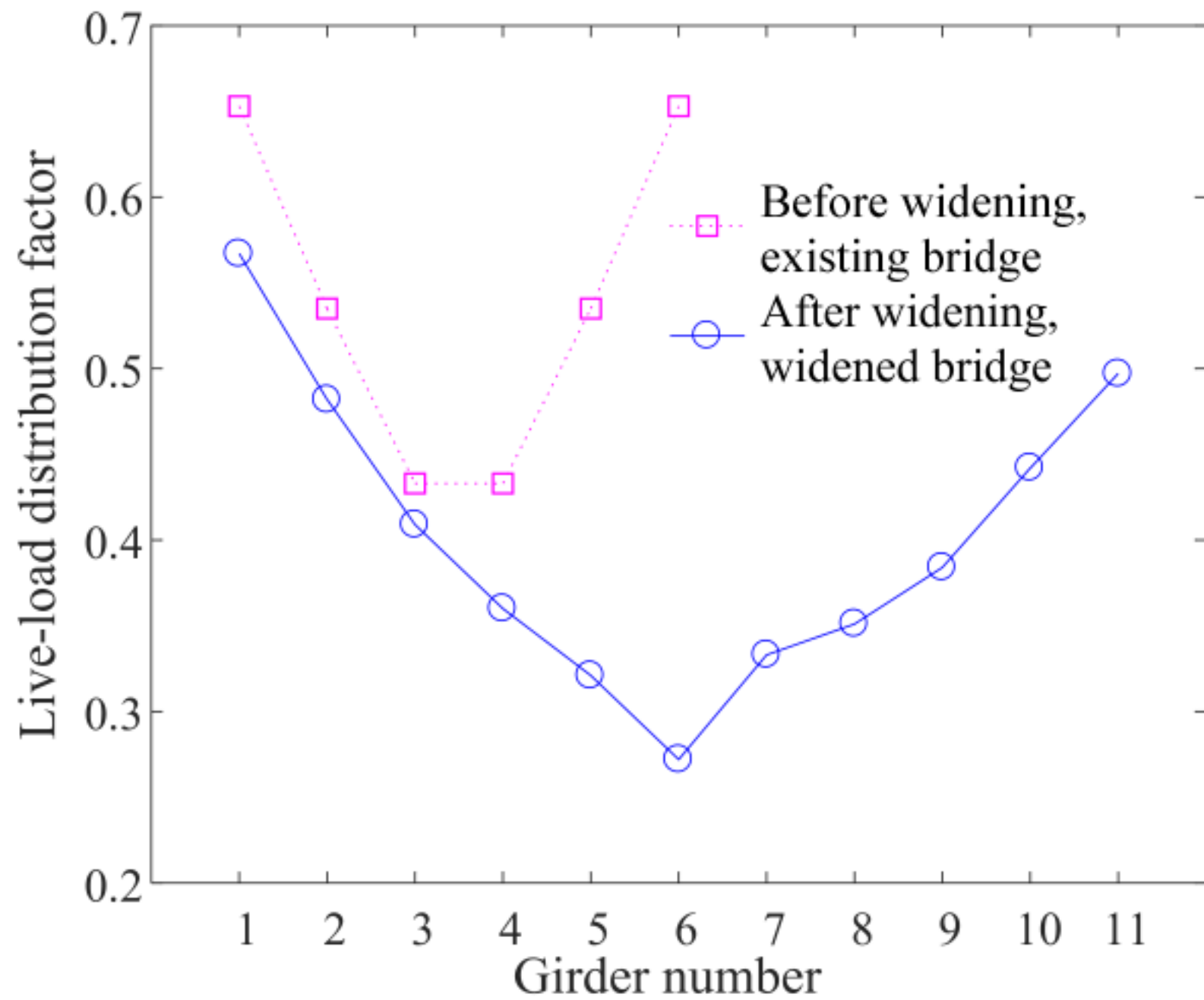
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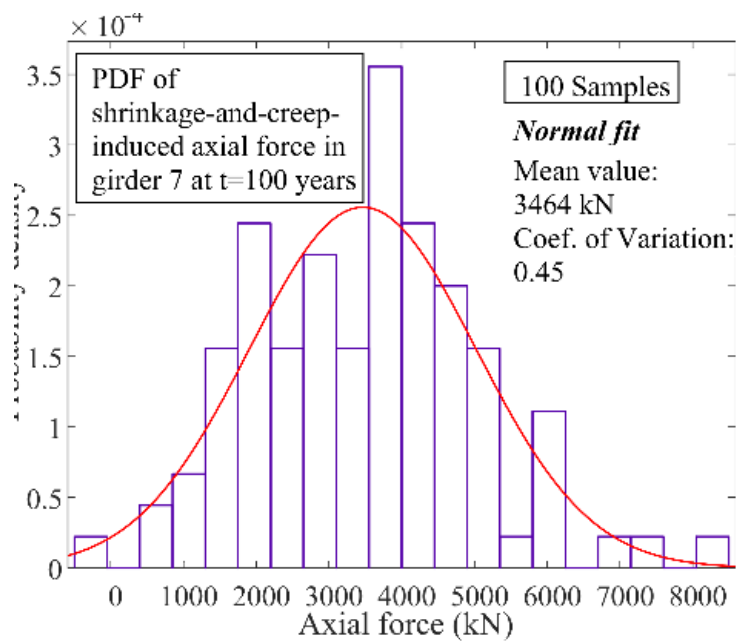
Existing-new girders



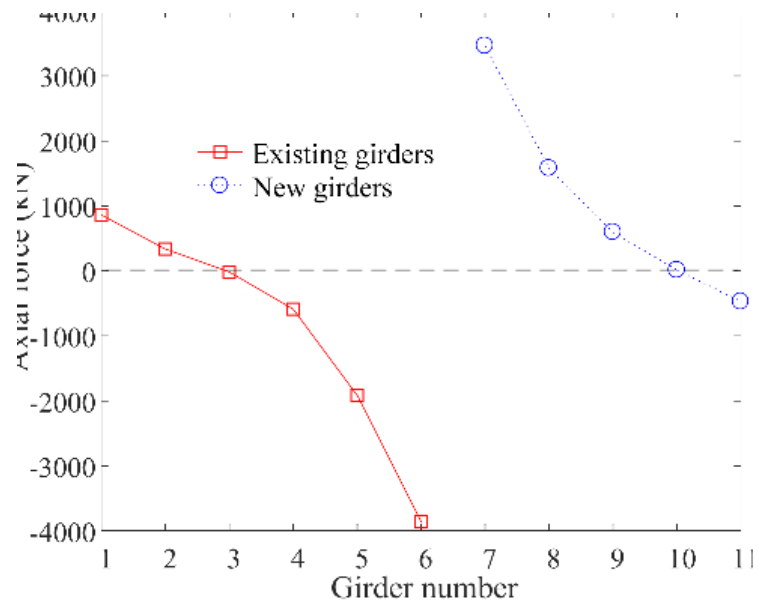
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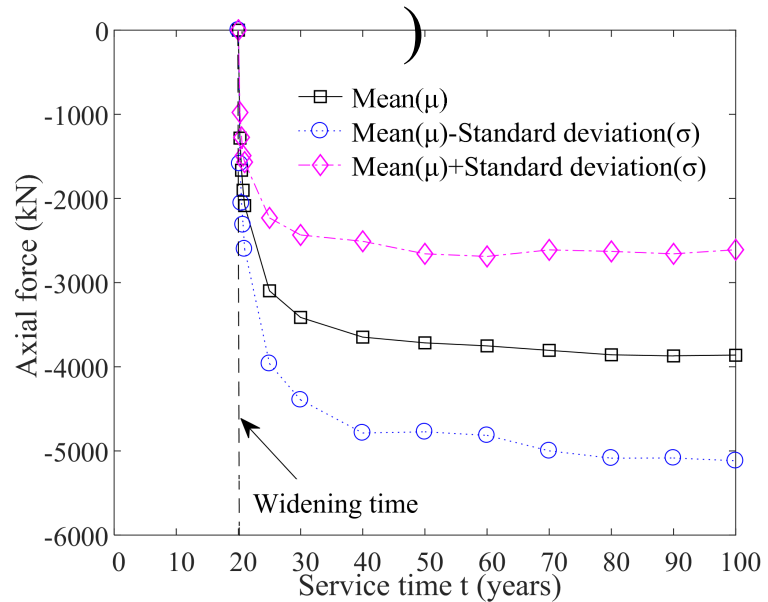




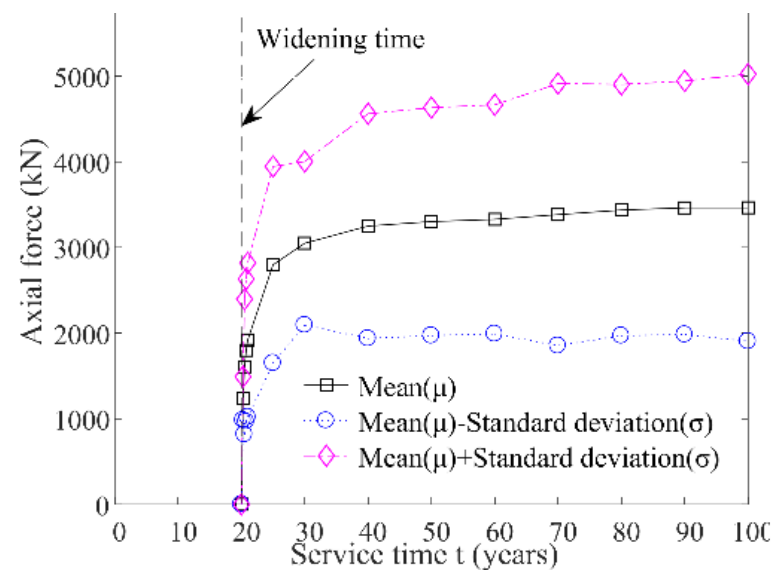
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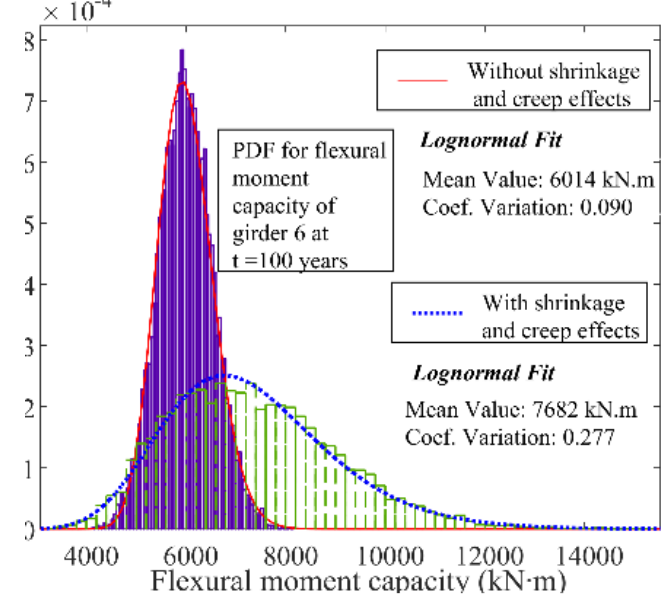
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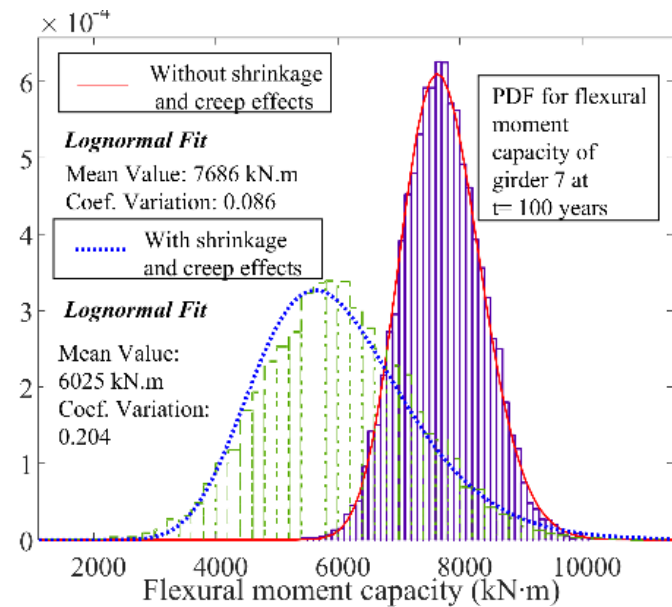
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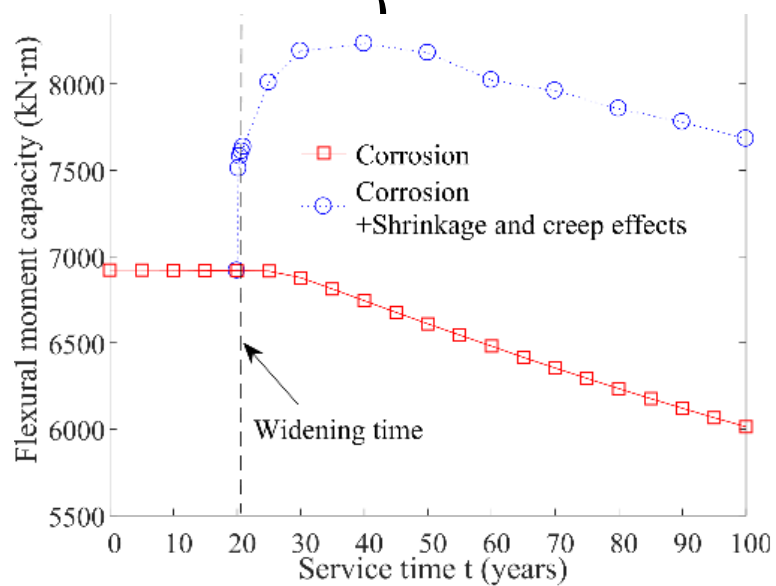
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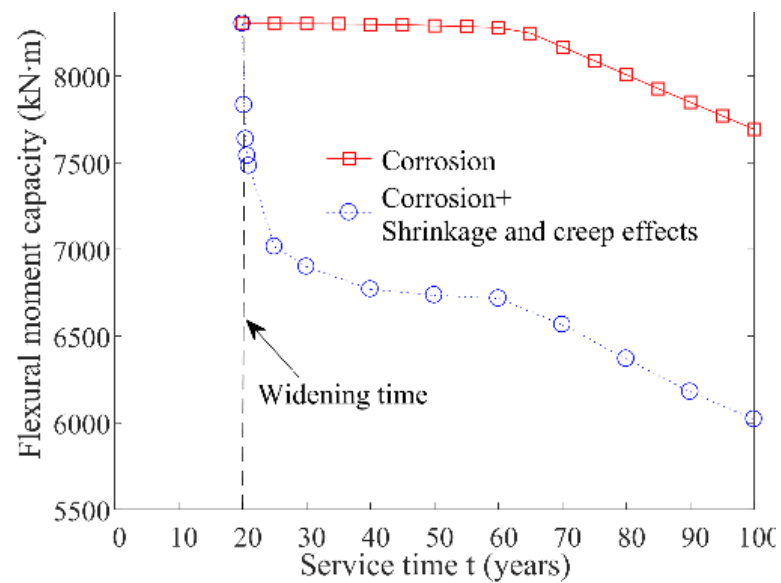
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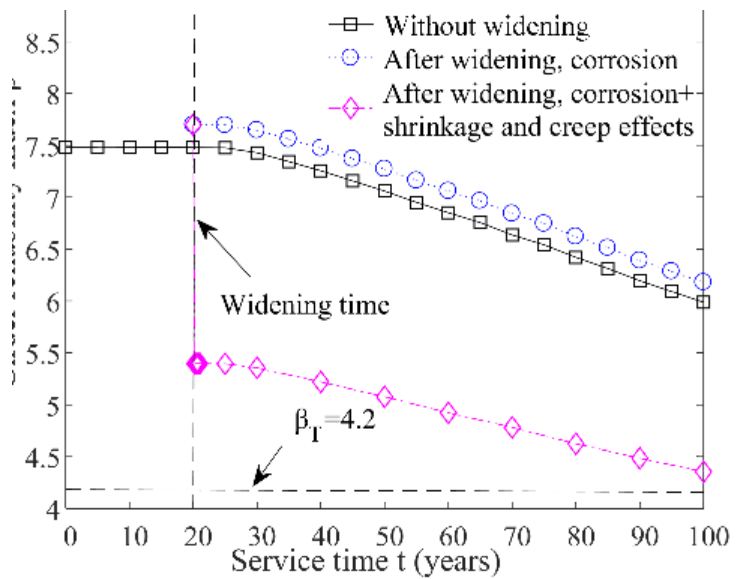
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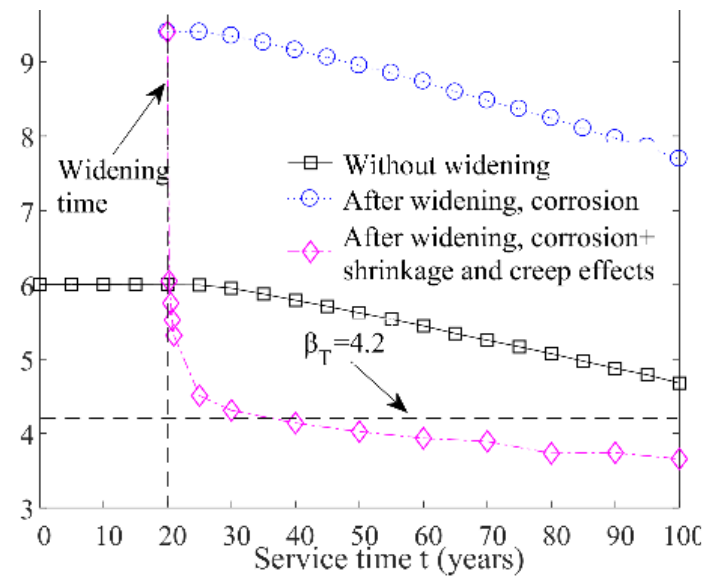
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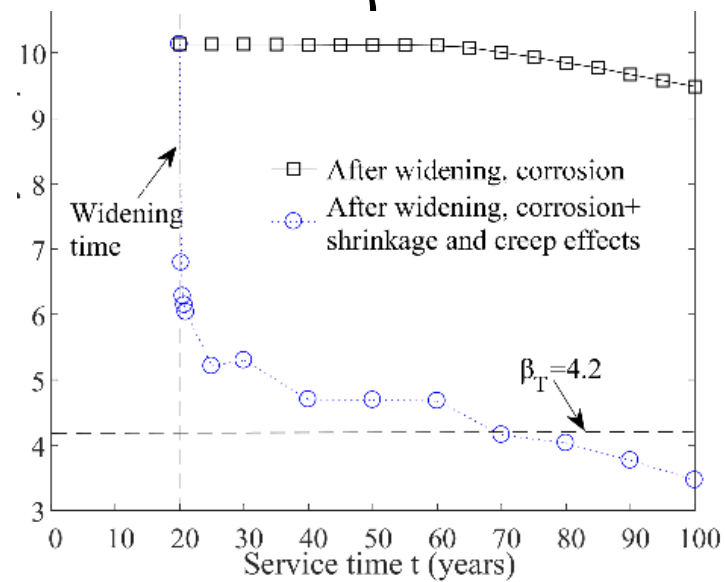
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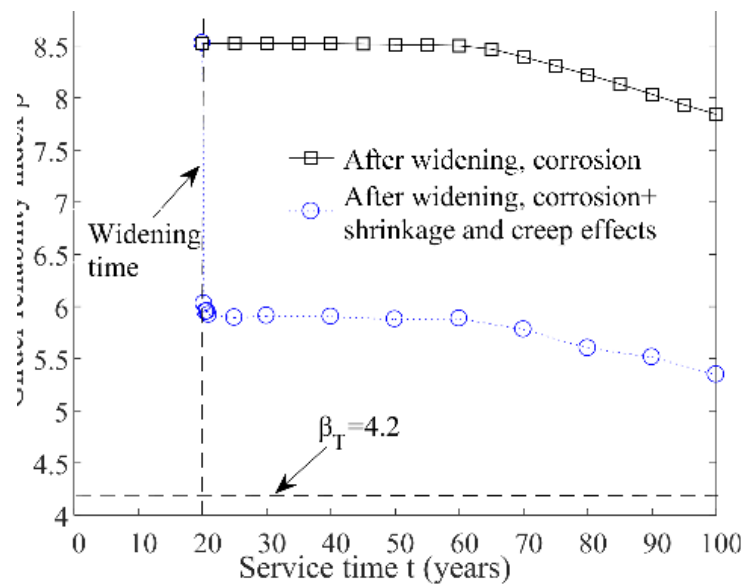
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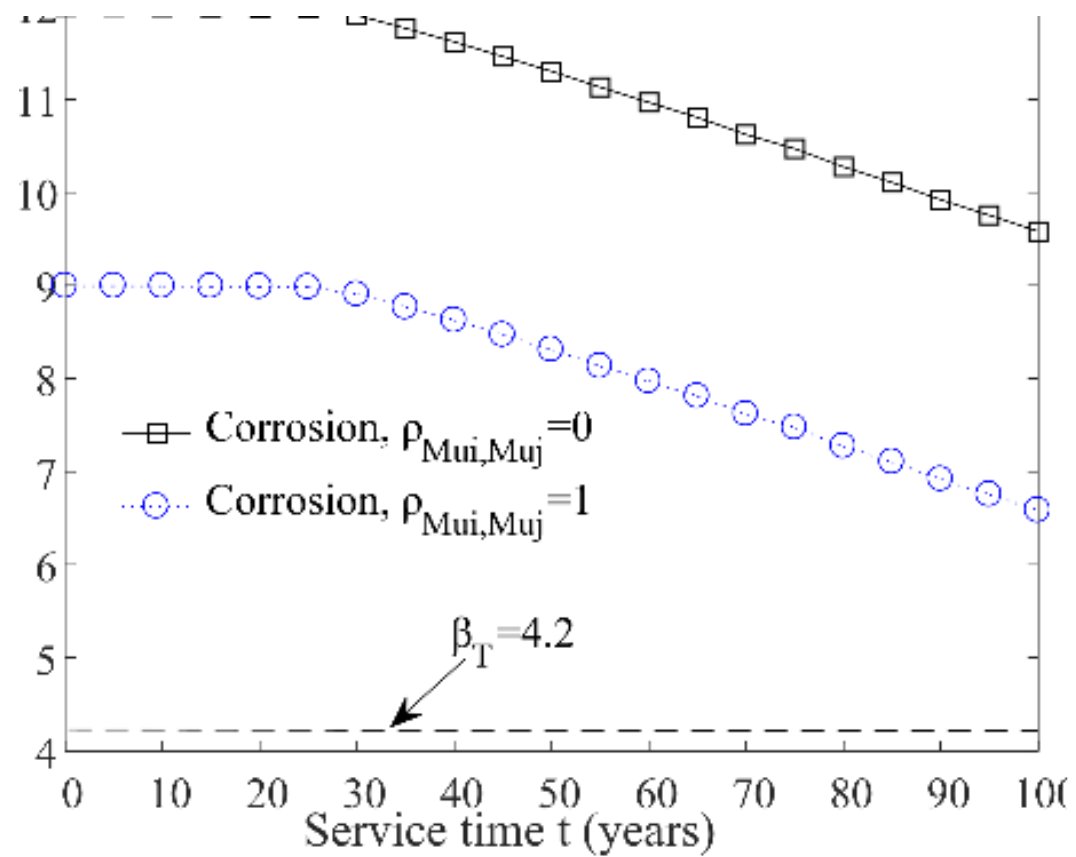
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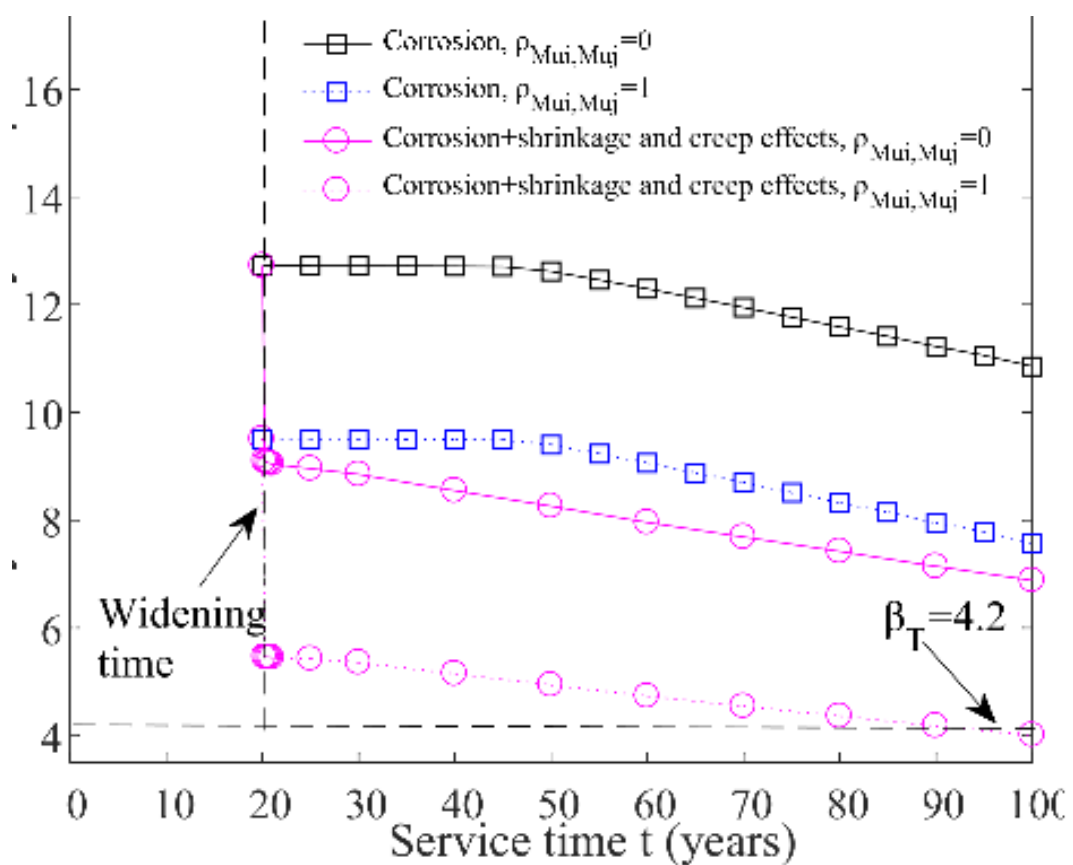
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