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Increasing the resilience level of a vulnerable rail network: The strategy of location and allocation of emergency relief trains

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Abstract:

Location and allocation of emergency vehicles plays an important role in a functional and resilience transport system. In railway, an effective strategy of location and allocation of relief train (RT) can considerably reduce the vulnerability level of the rail system in case of rail accidents and increase its resiliency. In this paper, we first introduce link importance measure by a multi-attribute decision making (MADM) technique to recognize the vital links in terms of effective attributes. Link exposure is then introduced based on the link importance and the accessibility of rail lines to the road system. We propose a bi-objective mathematical programming for locating and allocating RTs over the rail network which seeks two major aims: (i) providing maximum cooperative coverage of high exposure links, and (ii) minimizing total travel time from RT stations to the whole links of the network. An augmented e-constraint method (AUGMECON) is applied to provide Pareto optimal solutions which further combined with a fuzzy-logic approach to find the preferred solution. The proposed framework is employed to a real-world case study and the obtained results are compared with the ones of conventional maximal covering model. The comparison reveals the superiority of proposed model in providing an economical and effective layout for location and allocation of RTs in a real-world rail network.

Keywords: *Resilience; Vulnerability; Relief trains; Cooperative coverage; Railway systems; Location and allocation model.*

1. Introduction

Railway systems are part of critical infrastructures and the main backbone of economic development in every country. In Iran, railroads annually ship over 35 million tons of goods and 25 million passengers along the 10,000 km integrated network, contributing to 8.5% Gross Domestic Product (GDP) of the country(RAI 2016). During a rail accident, parts of infrastructure might be damaged, and some passengers become injured. In such case, delivery of medical services and accelerate the rate of recovery through the movement of relief equipment to disrupted lines, particularly those located in low access area, are vital. For this purpose, railways should be robust and resilient enough to mitigate the disastrous impact of incidents and restore to normal status immediately.

Resiliency is defined as the ability of a transportation network to recover from a disruption and return to normal function within a "reasonable" time frame. According to Snelder et al. (2012), resilience is one of the five essential elements to make a network more robust (i.e. less vulnerable). Rose (2004) also states that resilience is one of the several ways to reduce vulnerability. Vulnerability focus on socio-economic consequences of a network failure or degradation. Jenelius et al. (2006) first suggested the concept of link importance and exposure in the context of vulnerability analysis. They utilized the increase in generalized travel cost as a measure of link importance. In other words, the consequences of a failing link or group of links are called the importance of that link/group of links. Moreover, they defined the expected increase in travel cost in a certain place as exposure, so it is site-dependent. Nagurney and Qiang (2012) introduced the network performance efficiency measure as the average throughput on a network with a given demand vector. With such measure, they investigate the importance of network components by studying their impact on the network efficiency through their removal. Khademi et al. (2015) developed a vulnerability analysis for transportation system at a major population center in a geographic area prone to earthquakes. Bababeik et al. (2017) also proposed the increased cost of routing and scheduling in case of a multi-link blockage in a railway network. Mattsson and Jenelius (2015) introduced the concept of conditional vulnerability to show how resilience and vulnerability can be framed. This concept fits nicely into Fig. 1 which illustrate the effects of decision-making on infrastructure resilience. The conditional vulnerability can be seen as the aggregate consequences of a disruption represented by the area between the dotted line in the figure corresponding to the full system function and the relevant curve representing the reduced level of function. As Fig. 1 demonstrates, taking an appropriate action can reduce the vulnerability of the network to disruption. The latter curves depend typically on actions of ex-ante mitigation or ex post-adaptation.

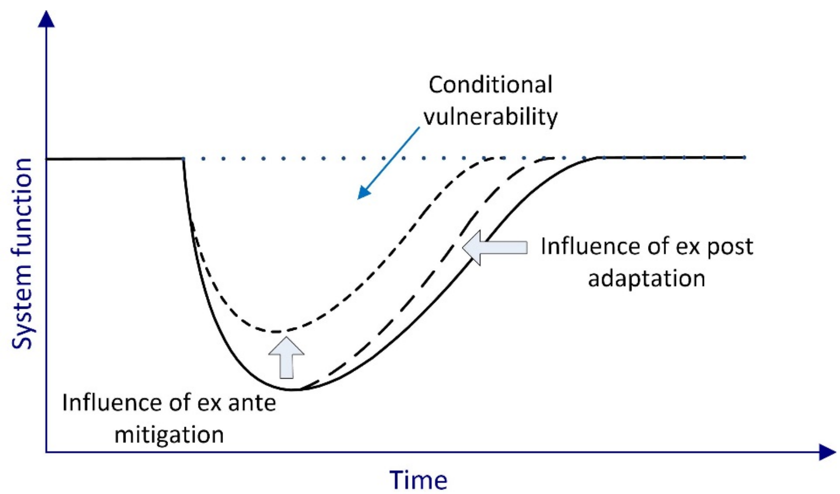

Fig. 1. Effects of decision making on resilience (adopted from Mattsson and Jenelius (2015))

In this paper, we utilize four main attributes to calculate the importance of any link in the rail network including passenger and freight traffic passing through the track, the amount of export/import transshipment through the link, and the serviceability of the link to industry. We further employ a multi-attribute decision making (MADM) technique to calculate the importance of links. A major difference of our study with previous vulnerability works is that they used to evaluate the impact of link removal by defining link importance while the current study utilizes link importance to identify the priority of network components to deal with emergency facilities after a disruption. For this purpose, we consider the dependency of the railway on road system for transport of emergency facility to the incident location. In fact, relief operations may be considerably influenced by territorial context where the railway line is located. For example, delayed relief operation in a low access tunnel of the south corridor in Iran, between Tang-haft and Andymeshk stations, caused 41-hour line blockage and many train cancellations (Kachoueyan et al. 2016). A study by Borghetti and Malavasi (2016) also proved that the topic of road accessibility of emergency services to railway line has been rarely addressed. Based on the link importance we suggest a measure called *link exposure* which represents the criticality of an important link with respect to accessibility of rail lines to a road system. Providing emergency response to high exposure elements of the network in a fast manner can be seen an effective effort to reduce the vulnerability level of the network and increase its resiliency.

Availability of emergency vehicles in railway for high exposure links is a matter of grave concern as they are the only rescue vehicles in low accessibility areas. This study aims to locate and allocate emergency vehicles in railway system called hereafter the relief train (RT). Such trains are equipped with medicine and first aid supplies, track components, a crane, re-railing and cutting equipment along with some technicians working with them. An

RT in Iran consists of open wagons containing ballast, flat wagons holding traverses, tank wagon for firefighting, heavy rail crane, and passenger wagon for technicians (see Fig. 2). In this paper, we develop a tactical plan for locating and allocating RTs to increase the resilience level of the network as demonstrated by the conceptual framework in Fig. 3.

Fig. 2. Relief train and its components (Source: own pictures)

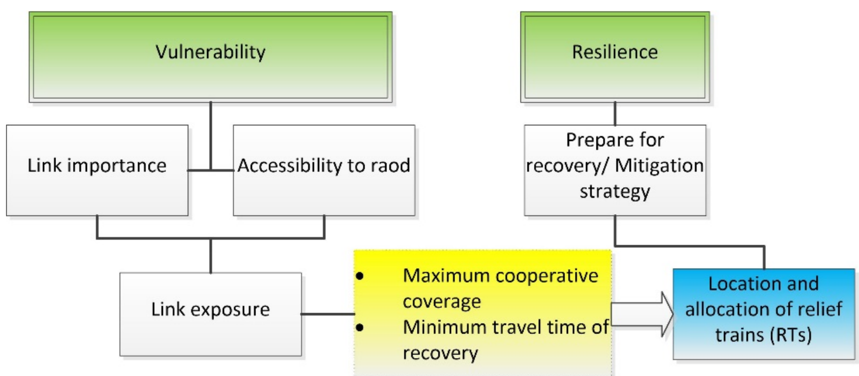

Fig. 3. Conceptual framework of the study

One of the most popular techniques to deal with the emergency facility location problem is the applications of the covering location models (Li et al. 2011). The first emergency facility location covering model was the location set covering problem (LSCP) proposed by Toregas et al. (1971). The objective of LSCP is to find the minimum number of facilities to cover all demand points, yet it is hard to achieve a full coverage due to the limited number of resources. Another covering model which has been extensively used for facility location problem is maximal covering location problem (MCLP) whose aim is to maximize the demand coverage for a limited number of facilities. The LSCP and MCLP have two drawbacks: first, when a facility is called for service, demand points under its coverage are not covered by it anymore; second, they assume that a demand point is covered only when it can be reached within a specific predefined distance by at least one facility(Li et al. 2011). Recently, some researchers have proposed gradual covering models to relax the assumption of coverage within a fixed distance. Berman et al. (2003) applied a level of partial coverage by considering the coverage decay function for the formulation of uncapacitated facility location problem. Karasakal and Karasakal (2004) developed a partial coverage version of maximal covering location problem and applied Lagrangian Relaxation to optimize it. Also, in some of the covering models, only one facility (namely the closest one) determines whether a demand point is covered or not. Berman et al. (2011) showed that this individual assumption might lead to solutions that require more facilities to cover the same amount of demands. In other words, it causes facilities are concentrated in specific parts of the network. To cope with this challenge, the cooperative behavior of facility location in covering demand has been recently considered by researchers. Cooperative coverage framework is a new trend which deems to combine (cooperate) facilities to provide coverage (Li et al. 2011). In this approach, the individual coverage assumption is replaced with a mechanism where all facilities contribute to the coverage of each demand point. In this vein, we assume that each facility location (RT station) transmits a signal dissipating with distance, and each degraded link receive signals from all RT stations. If the signal strength at the degraded link exceeds a certain threshold, the link is covered, otherwise, it is not. In this paper, we combine the idea from cooperative coverage with the problem from the resource allocation model to create a bi-objective location and allocation model for relief trains in the railway network. The objectives are (i) maximizing the cooperative coverage of link exposure by RT stations, and (ii) minimizing the total travel time from RT stations to the whole of the links.

Emergency facility location problems commonly represent demand as discrete points over the study area. In this paper, however, we consider the demand space continuously spread over the network. In other words, as every link of the network may be prone to degradation, it is treated as a demand point. For this purpose, a heuristic algorithm is applied to make modifications on the initial network so that every link is treated as a demand point.

The paper is organized as follows. Section 2 provides a review on the literature of emergency facility location problems. Section 3 presents network modification algorithm to consider a continuous demand space. Section 4 proposes the methodology of the location and allocation model of RTs. Section 5 provides the solution approach used to solve the model. Section 6 is devoted to a real-world case study of railway network to test the efficiency of the proposed model and the solution approach. Finally, Section 7 finishes the article through conclusions and directions for further works.

2. Literature review

To further explore the problem of emergency facility location problem in the literature, a survey has been conducted in terms of emergency type. A survey on the literature of emergency facility location shows that there are two major trends in this area: (1) models for disaster logistic planning, and (2) models for a routine emergency relief. The first category deals with locating facilities which come into use after a rare event like earthquake or hurricane while the next category demands high frequent facilities such as ambulance, police, or fire services.

In the first category, Caunhye et al. (2012) provided a comprehensive survey on emergency logistics operations and classified models more into two main sets: (a) facility location, and (b) relief distribution and casualty transportation. Facility location models are mainly based on mixed integer programs with binary location variables. Facilities considered for location are either shelters or warehouses (sometimes referred to as distribution centers). Location models are found to be associated with either evacuation operations, or stock pre-positioning and relief distribution (Caunhye et al. 2012). According to data type, these models use deterministic or stochastic parameters. The approach for considering the uncertainty of demand is to include stochastic parameters using probabilistic distribution or disruption scenarios. For example, Rawls and Turnquist (2010) utilize a two-stage stochastic mixed integer program (SMIP) that considers uncertainty in demand for the stocked supplies as well as uncertainty regarding transportation network availability after a natural event. Mete and Zabinsky (2010) proposed a stochastic optimization approach for the storage and distribution problem of medical supplies to be used for disaster management. Their model captures the disaster-specific information and possible effects of disasters through the use of disaster scenarios. In a different approach, Bell et al. (2014) alternatively proposed robust optimization approach in the case when no reliable probability distribution of damages exists. In robust optimization, the objective is typically to minimize the cost or the regret in the worst situation, called best-worst models. Our proposed work is distinct from above studies as the emergency occurs occasionally throughout the network and is not a "rare" event. In addition, incidents might happen in every component of the network such as remote access desert and highly populated areas. Then, despite the models for emergency logistics that assume some limited and discrete demand points, the demand in our study is scattered over the whole links of the network and is not localized in discrete points. Furthermore, robust optimization is reasonable in facing emergencies where a high performance is required from a system in the worst situation (Snyder 2006), which make it more suitable for dealing with natural disasters. Thus, it is unrealistic to regard best-worst situation for single-link failures due to human errors or attacks as they might occur in every part of the network. In addition, best-worst location problems are difficult to solve, thus exact solutions using this approach have been provided only for networks with special structures or for single supply facility cases (Bell et al. 2014).

The second group of studies focused on the location and allocation of routine emergency facilities concerning high demand emergency facilities like emergency medical service (EMS) or fire vehicles. The main characteristics which have been already considered by researchers for location and allocation of these facilities include the availability of ambulance or fire vehicle, the frequency of calls to determine the amount of demand, providing maximum coverage to ensure efficiency, and considering the same response time to ensure fairness. The earlier works were done by Hogan and ReVelle (1986) and Marianov and ReVelle (1996) were static and deterministic models and neglected stochastic considerations. The ambulance might be unavailable, causing some demand points not to be covered while there are idle ambulances in other locations. To cope with such issue, the researchers introduced stochastic parameters to consider the demands fluctuations and availability of ambulance service. Rajagopalan et al. (2008) offered a model to achieve the dynamic redeployment (relocation) of ambulances based on fluctuating demand over time. Ingolfsson et al. (2008) determine the number of ambulances needed to provide a specified service level by incorporating the randomness in the ambulance availability and in the delays and the travel times. Another approach for determining the location of routine emergency facilities is to maximize risk coverage. The accident risk levels of various areas can be calculated by the accident occurrence probability multiplied by the accident's severity. In the work by Tzeng and Chen (1999) the risk has been addressed by minimizing the distance from any fire station to the high-risk area. Yang et al. (2007) introduce various fire risk category to establish a fire station location model. In the railway application, Cheng and Liang (2014) applied a risk assessment approach by evaluating the accident occurrence frequency and the expected number of injuries to determine the location of ambulances for urban and railway emergency system. Our proposed work is distinct from above studies in some ways. First, the incident types considered in this study happen randomly, and they are not as frequent as high demand emergencies like medical or fire services. Then, we can realistically assume that emergency vehicles (RTs) are available when demand rises somewhere in the network. In addition, the probability of incident occurrence due to human errors or malicious attacks can hardly be achieved, meaning that an objective distribution function for emergency demand does not exist. Then, applying probabilistic approaches like risk assessment are not appropriate for this purpose. Risk evaluation is commonly applied where the probability of failure can be evaluated.

3. Network modification

Facility location models commonly represent demand as discrete points rather than as continuously spread over an area. Such modeling technique brings inaccuracies into the objective function and consequently into the optimal solutions. In this paper, we replace continuous demand of RTs with discrete demand through considering every link as demand point. For this purpose, we develop a heuristic algorithm to make modifications on the initial network so that every link is treated as a demand point.

Let $G = (N, L)$ represents a railway network with stations denoted by a node set $N = \{N_1, N_2, \cdots, N_n\}$, and tracks denoted by a link set $L = \{L_1, L_2, \cdots, L_l\}$. Relief trains (RTs) are required to be located only in hub stations due to operational issues. We specify them as a subset of stations called potential stations denoted by $K = \{K_1, K_2, \cdots, K_k\}$. In our problem, every link of the network may be prone to degradation due to failure, so we assume a dummy node in the middle of each link as a demand point such that an RT from any potential station may assign to it. The initial network is then modified in such a way that every link is treated as a demand point. To clarify the procedure, we explain it through a simple network depicted in Fig. 4. This network contains 4 links $\{L_1, L_2, L_3, L_4\}$, and 4 nodes $\{N_1, N_2, N_3, N_4\}$, from which two nodes are considered as potential stations $\{N_2, N_4\}$ (Fig 4.a). First, let us put a dummy node in the middle of the links to represent demand points (i.e. dots in the middle of links in Fig. 4.b). If the links end at any of potential stations, the dummy nodes are connected to the related potential station through a new link like L'_1in Fig. 4.c. The travel time of this link is half of that of original link. If two adjacent links end at a non-potential station, their dummy nodes are connected with a new link like the dashed link L''_{14} in Fig. 4.c, and the related non-potential station (N_1) is then removed. The travel time of the new link is the average of both initial links, e.g. the travel time of L''_{14} is half of the sum of travel times of links $L_1$and L_4.

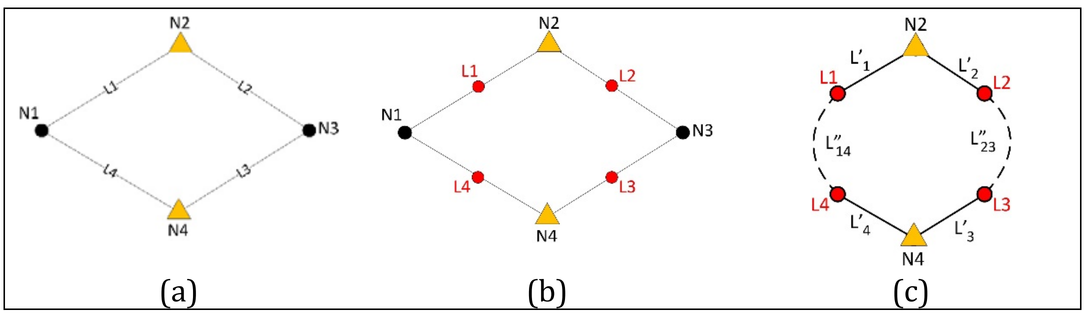

Fig. 4. Example of network modification: (a) original network, (b) dummy nodes on links, (c) modified network

With extending the above procedure, we propose Algorithm 1 to generate the modified network:

Algorithm 1. Generating the modified network

Start:
1. Take the inputs of the initial network as below:
Set of stations (N); Set of potential stations (K); Set of links (L).
2. Put a dummy node in the middle of each link and denote it with the same name of that link, e.g. dummy node $L_1$for link L_1.
3. Repeat for each dummy node L_i ($i \in L$):
Does dummy node L_i end at a potential station $k \in K$?
3.a: Yes; define a new link, L'_i, between the dummy node $L_i$and potential station k with travel time $t_{L'_i} = t_{L_i}/2$.

3.b: No; look for any adjacent dummy node (j) to station k, and define new link, L''_{ij}, between that dummy node and dummy node L_i with travel time $t_{L''_{ij}} = (t_{L_i} + t_{L_j})/2$.

4. Keep the potential stations and remove other ones.

Finish.

4. Methodology

In this section, we first introduce the notations, second, explain the procedure of determining the coverage index, and last the formulate RT location and allocation.

4.1. Notation system

The definition of sets, parameters, and variables used in the proposed model are summarized as follows:

Sets	
N	Set of nodes in the modified network indexed by n, including potential stations and dummy nodes
L	Set of network links indexed by l
K	Set of potential stations indexed by k
Parameters	
t_{mn}	Travel time from node $m \in N$ to $n \in N$
ci_l	Coverage index of link l
c_k	Supply cost of potential station k with RT
a	Cooperative coverage threshold
$\emptyset(t_{kl})$	Coverage function in terms of travel time from potential station $k \in K$ to dummy node of link $l \in L$
P	Maximum number of potential stations
p_l	Importance index of link l
ex_l	Exposure of link l
Variables	
X_{mn}^{kl}	Binary variable indicating whether link (m, n) of the modified network is located on the path from potential station $k \in K$ to link $l \in L$
Y_l	Binary variable indicating whether link $l \in L$ is covered
W_k	Binary variable indicating whether RT is located in potential station $k \in K$
U_{kl}	Binary variable indicating whether the potential station $k \in K$ is assigned to link $l \in L$

4.2. Calculating the link importance and link exposure

The concept of link importance in network vulnerability was first introduced by Jenelius et al. (2006) which addresses the consequences of a failing link or group of links by evaluating the network performance through their removal. Here, we utilize this concept to determine the priority of links to deal with relief trains. For this purpose, we consider the most attributes affecting the priority of rail lines in order to be covered by relief trains. For instance, a rail line with heavy passenger traffic is more important than those with light traffic in terms of emergency facilities. A multi-attribute decision making (MADM) technique is then applied to determine the *link importance*. In this paper, we consider the accessibility of rail lines to road system as an alternative way to deliver emergency facilities. Accordingly, an important link with low accessibility to road system is highly critical to be treated by RTs. Regarding both the accessibility and link importance, we define a new parameter called *link exposure*. The following procedure describes determining these values:

Step 1. *Determining attributes and their importance*:

To gain a clear understanding of factors influencing the location of relief trains and emergency management in railway, we held several brainstorming sessions with the railway authorities from October to December 2016 (Bababeik 2016). We summarized the result of the interview, and categorized the attributes affecting the emergency priority of rail lines as specified in Table 1. As the table shows, four major attributes affecting the emergency priority of links have been specified by railway specialists. The word in the first column is the abbreviation of that attribute. The number of passenger-kilometer and freight tonne-kilometer passing through a rail link per month is considered as major attributes. In addition, some rail lines are important since they provide transit paths for import or export purposes. Finally, those tracks that provide accessibility to the industry like factories and refinery plants are considered significant in terms of emergency situations.

Table 1. Attributes influencing the link importance

Attribute	Description
PT	Monthly passenger traffic of the track
FT	Monthly freight traffic of the track
T	Monthly export/import transit of the track
U	Direct serviceability of track to industry (factories, refinery plants, and customs)

Each of the above four attributes affects the priority of links with different weight. To determine the relative normalized weights of the attributes, we quantify the subjective judgment of railway specialists through the geometric mean method of the Analytic Hierarchy Process (AHP). This method can efficiently deal with objective (tangible) as well as subjective (non-tangible) attributes, where the subjective judgment of different individuals constitutes an important part of the decision process. According to Saaty (1994), we first construct a pairwise comparison matrix for these four attributes. Elements of this matrix are entered based on the judgment of the decision makers and using the fundamental scale of AHP. Then, we find the relative normalized weight of each attribute by normalizing the geometric means of rows in the comparison matrix. The consistency of judgment is also evaluated by the consistency index. Interested readers are referred to Saaty (1994) and Khademi et al. (2010) for more details.

The attributes are further divided into two categories: beneficial and non-beneficial (Saaty 1994). A beneficial attribute means its higher measures are more desirable in terms of link importance. A non-beneficial attribute is one of which the lower attribute value is desirable. In our problem, however, all attributes are considered as beneficial ones.

Step 2. *Determining the normalized decision matrix*:

We construct a decision matrix in which each row is assigned to one link and each column to one attribute. That is, any element m_{ij} of this matrix gives the value of the attribute *j* for the link *i* in terms of original real unit (i.e. non-normalized value). Then, the normalized decision matrix, R_{ij}, is calculated using the following equation:

$$R_{ij} = \frac{m_{ij}}{\sqrt{\sum_{i=1}^{M} m_{ij}^2}} \qquad (1)$$

where M is the number of attributes.

Step 3. *Constructing the weighted normalized decision matrix:*

This is done by the multiplication of each element of the column of the matrix R_{ij} with its associated weight obtained in Step 1, w_j. Therefore, the weighted normalized decision matrix V_{ij} equals to:

$$V_{ij} = W_j R_{ij} \qquad (2)$$

Step 4. *Determining the ideal and negative ideal solutions:*

We utilize the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) as an MADM technique to determine the priority of links(Tzeng and Huang 2011). TOPSIS is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution and the longest geometric distance from the negative ideal solution. The ideal (best) and negative ideal (worst) solutions can be expressed as:

$$V^+ = \{(\max_i V_{ij} \mid j \in J), (\min_i V_{ij} \mid j \in J') \mid i = 1,2,\ldots,L\} = \{V_1^+, V_2^+, \ldots, V_M^+\} \qquad (3)$$

$$V^- = \{(\min_i V_{ij} \mid j \in J), (\max_i V_{ij} \mid j \in J') \mid i = 1,2,\ldots,L\} = \{V_1^-, V_2^-, \ldots, V_M^-\} \qquad (4)$$

where L is the number of links, $J = (j = 1,2, \cdots, M)$ is associated with beneficial attributes, and $J' = (j = 1,2, \cdots, M)$ is associated with non-beneficial attributes. It should be mentioned that Eq. 3 and 4 are general expressions; however, all attributes applied in this study are beneficial ones, so Eq. 4 remains inapplicable.

Step 5. *Determining the separation measure*

The separation of each link from the most important link is given by the Euclidean distance in the following equation:

$$S_i^+ = \{\sum_{j=1}^{M}(V_{ij} - V_j^+)^2\}^{0.5}, \quad i = 1,2, \ldots, L \tag{5}$$

$$S_i^- = \{\sum_{j=1}^{M}(V_{ij} - V_j^-)^2\}^{0.5}, \quad i = 1,2, \ldots, L \tag{6}$$

where S_i^+ and S_i^- is the distance between the importance of link *i* and the most and least important link, respectively.

Step 6. *Calculating the link importance index*

The relative closeness of each link to ideal solution (most important link) explains the importance of that link. Such relative closeness can be expressed as follows:

$$p_i = \frac{S_i^-}{(S_i^+ + S_i^-)} \tag{7}$$

Step 7. Calculating the link exposure

Technical analysis of some accidents has confirmed that the inaccessibility of the railway line to the road in some cases made rescue operations difficult as far as the approach of facilities for handling the emergency is concerned. Therefore, we consider the accessibility of railway line to road system as an influencing factor to cover a link by RTs. Based on the link importance we suggest a measure called the critical coverage index which represents the criticality of an important link with respect to accessibility to the road system. Particularly, an important link is critical when it has low accessibility to the road system. Mathematically, we can say:

$$ex_i = \frac{p_i}{ac_i} \tag{8}$$

Where ex_i is the critical coverage index, p_i the priority of the link and ac_i the value of accessibility of rail line to road system. In this study, it is assumed that the accessibility can take the values in Table 2:

Table 2. Accessibility measure applied in the coverage index

Accessibility	Value	Description

Null accessibility	0.1	there is no accessibility to the road system
Low accessibility	0.4	up to 1/3 length of the link has accessibility to the road system
Medium accessibility	0.7	1/3 to 2/3 length of the link has accessibility to road system to road system
High accessibility	1	the whole length of the link has accessibility to the road system

4.3. Model formulation

In this subsection, we introduce our model of locating and allocating RT stations throughout the railway network. At first, the domain coverage of each RT station should be determined. The domain determines the extreme distance in which each RT station can provide emergency services, i.e. cover the demand points. In most location models, the domain coverage of facilities is specified in advance as the demand points are limited and localized. However, in this study, such domain is determined with according to a gradual coverage function of travel time from RT stations to the whole links of the network. For this purpose, a preprocessing stage is supposed based on the shortest path problem which determines the routes with least travel time from every potential RT station to the links of the network. The preprocessing stage is formulated as follows:

Preprocessing stage:

A) Solve:

$$Minimize \sum_{k\in K}\sum_{l\in L}\sum_{(m,n)\in N'} t_{mn}X_{mn}^{kl} \tag{9}$$

$$\sum_{(m,n)\in N} X_{mn}^{kl} - \sum_{(m,n)\in N} X_{nm}^{kl} = 0 \qquad \forall l \in L, \forall k \in K,\ m,n \neq l,k \tag{10}$$

$$\sum_{(m,n)\in N} X_{mn}^{kl} - \sum_{(m,n)\in N} X_{nm}^{kl} = 1 \qquad \forall l \in L, \forall k \in K, m\ or\ n = k \tag{11}$$

$$\sum_{(m,n)\in N} X_{mn}^{kl} - \sum_{(m,n)\in N} X_{nm}^{kl} = -1 \qquad \forall l \in L, \forall k \in K,\ m\ or\ n = l \tag{12}$$

B) Calculate Travel time:

$$t_{kl} = \sum_{(m,n)\in N} t_{mn}X_{mn}^{kl} \qquad \forall l \in L, \forall k \in K \tag{13}$$

The objective function (9) minimizes travel time from potential RT station $k \in K$ and link $l \in L$. Constraints (10)-(12) keep flow conservation at all nodes of the modified network. Eq. 13 determines total travel time from potential station $k \in K$ to link $l \in L$. It should be noted that although X_{mn}^{kl} is a binary variable, there is no need to include integrality constraint in the above formulation. Since the coefficients' matrix of model (1)-(5) holds the sufficient condition for total unimodularity, it ensures that every basic feasible solution is automatically integer (Chen et al. 2011). Therefore, it can be solved by a regular LP solver.

We define our gradual coverage function for potential RT station $k \in K$ as a monotone decreasing function, as shown below:

$$\emptyset(t_{kl}) = \begin{cases} 1 & if\, t_{kl} \leq t_a \\ \frac{t_{kl}-t_b}{t_a-t_b} & if\ t_a < t_{kl} < t_b \\ 0 & otherwise \end{cases} \tag{14}$$

where t_a is the maximum travel time for the full coverage and $t_b$is the maximum partial travel time beyond which the link is considered uncovered by that potential RT station.

In this study, we utilize the idea of cooperative coverage of links for locating RTs. The concept of cooperative coverage was first introduced in Cooperative Location Set Covering Problem (CLSCP) and Cooperative Maximum Covering Location Problem (CMCLP) by Berman et al. (2011). This concept is based on the mechanism that each facility at site k emits a “signal” that decays over distance according to a decay function. A demand point receives signals from all facility stations and is covered only if the “signal” it receives exceeds a certain threshold. In our application, the signal emitted by any potential RT station is determined by Eq. 14. If the cumulative effect of signals for any link of the network exceeds a given value, it is considered as covered.

We propose the model of location and allocation of relief trains (LART) as a bi-objective integer linear program with considering the two aims: (i) maximizing the cooperative coverage of link exposure by RT stations, and (ii) minimizing the total travel time from RT stations to the whole of the links. This model is proposed as the following:

LART model

$$\max f_1 = \sum_{l \in L} ex_l Y_l \tag{15}$$

$$\min f_2 = \sum_{k \in K} \sum_{l \in L} t_{kl} U_{kl} \tag{16}$$

$$\sum_{k \in K} \emptyset(t_{kl})\, W_k \geq a \times Y_l \qquad \forall l \in L \tag{17}$$

$$\sum_{k \in K} c_k W_k \leq P \tag{18}$$

$$U_{kl} \leq W_k \qquad \forall l \in L, \forall k \in K \tag{19}$$

$$W_k \leq \sum_{l \in L} U_{kl} \qquad \forall k \in K \tag{20}$$

$$\sum_{k \in K} U_{kl} = 1 \qquad \forall l \in L \tag{21}$$

$$Y_l, W_k, U_{kl} \in \{0,1\} \quad (22)$$

The first objective function, Eq. 15, maximizes the coverage of link exposure over the network with regarding their importance. The second objective, Eq. 16, is to assign opened RT stations to the links of the network such that the total travel time is minimized. Constraint set 17 ensures that a link is covered if its cooperative coverage of all potential RT stations is greater than the given threshold a. Each potential RT station requires a fixed cost to be equipped with RT facilities. Constraint set 18 limits the total supply cost of RT stations to an available budget. Constraint set 19 ensures that only opened RT stations are assigned to the links. Constraint set 20 makes certain that at least one link should be assigned to any opened RT station. Constraint set 21 makes sure that every link should be provided by one opened RT station. At last, constraint set 22 keeps the integrality condition for variables.

The proposed model is a bi-objective mathematical programming in which there is no single optimal solution that simultaneously optimizes both objective functions. In these cases, the concept of optimality is replaced with that of efficiency or Pareto optimality. The next section discusses the solution approach to find optimal solutions and adopt the preferred one.

5. Solution approach

In this section, we propose the solution approach of LART model. This model is categorized in multi-objective optimization problems. In a multi-objective mathematical programming, there is no single optimal solution that simultaneously optimizes all the objective functions. In these cases, the concept of optimality is replaced with that of noninferiority or Pareto optimality. A feasible solution to a multi-objective programming problem is noninferior if there exists no other feasible solution that will yield an improvement in one objective without causing a degradation in at least one other objective. A comprehensive survey of solution techniques to multi-objective mathematical programming can be found in the book of Ehrgott (2006).

We utilized augmented e-constraint (AUGMECON) technique in our problem to find efficient solutions for LART model. This method proposed by Mavrotas and Florios (2013) and eliminates some of the weak points in the conventional e-constraint method. In fact, the conventional e-constraint method has two points that need attention: the range of the objective functions over the efficient set (mainly the calculation of nadir values) and the guarantee of the efficiency of the obtained solution. To overcome the first ambiguity, they proposed the use of lexicographic optimization for every objective function to construct the payoff table (the table with the results from the individual optimization of the p objective functions) with only efficient solutions. To overcome the second ambiguity, the objective function constraints in a regular e-constraint method is transformed to equalities by explicitly incorporating the appropriate slack or surplus variables. At the same time, the sum of these slack or surplus variables is used as a second term (with lower priority) in the objective function forcing the program to produce only efficient solutions. The new problem which is called *augmented e-constraint method* becomes:

$$
\begin{aligned}
&\max(f_1(x) + \delta \times (s_2 + s_3 + \cdots + s_p)) \\
&f_2(x) - s_2 = e_2 \\
&f_3(x) - s_3 = e_3 \\
&\ldots \\
&f_p(x) - s_p = e_p \\
&x \in S \quad and \quad s_i \in R^+
\end{aligned}
$$

where $f_p(x)$ is the objective function p, $s_p$is the slack or surplus variable associated to this objective function, e_p is the RHS of objective function p in conventional e-constraint method, S is the set of constraints, and $\delta$is a small number (usually between 10^{-3} and 10^{-6}).

After the generation of the Pareto optimal solutions, the next step is to find the most appropriate solution, i.e. preferred solution. In the literature, there are several approaches related to the selection of preferred solutions, such as the k-mean clustering procedure, the weighted-sum approach and the fuzzy-logic-based approach. Cluster analysis can classify data into groups in which individuals are similar to each other. This method chooses a set of solutions rather than a single solution; moreover, it is usually used together with evolution algorithms such as genetic algorithms. The decision maker can choose a preferred solution by the weighted-sum approach when a preferred weight vector of objectives is provided. However, the method only gives the absolute weighted sum of objective values of a solution but fails to indicate the degree of optimality of a solution. The degree of optimality is a measure which indicates the degree to which each Pareto optimal solution satisfies the decision maker viewpoint. The fuzzy-logic-based approach not only provides a most preferred solution but also indicate its degree of optimality. Hence, we adopt this method to choose a preferred solution among Pareto optimal solutions of problem P1. In this respect, we develop the following algorithm based on a fuzzy logic approach to find the preferred solution:

Algorithm 2: Finding the preferred solution in LART Model
1. Convert the maximum objective function f_1 to a minimum one by a negative coefficient. 2. Find the optimum values of each objective function by solving it individually through problems P1 and P2, and find the ideal objective vector for them: Problem P1: Problem P2:

	$\min f_1 = -\sum_{l \in L} ex_l Y_l$ st. Constraints (17)-(22)	$\min f_2 = \sum_{k \in K} \sum_{l \in L} t_{kl} U_{kl}$ st. Constraints (17)-(22)
Optimal values	$(Y_l^{*1}, W_l^{*1}, U_{kl}^{*1})$	$(Y_l^{*2}, W_l^{*2}, U_{kl}^{*2})$
ideal objective	f_1^I	f_2^I

3. Find the Nadir objective vector. It represents the upper limit of each objective in the entire Pareto set but not in the entire objective space. To calculate the upper limit of each objective, solve each objective with new constraints according to problems P3 and P4:

	Problem P3: $\min f_1 = -\sum_{l \in L} ex_l Y_l$ st. $U_{kl} = U_{kl}^{*2}$ Constraints (17)-(22)	Problem P4: $\min f_2 = \sum_{k \in K} \sum_{l \in L} t_{kl} U_{kl}$ st. $Y_L = Y_l^{*1}$ Constraints (17)-(22)
Nadir objective	f_1^N	f_2^N

4. Consider there are J non-inferior points after solving the LART model by AUGMECON method. We then define the membership function to indicate the degree of optimality for every objective function of each non-inferior point through the equation 23. Let $MF(f_i^j)$ is the membership function of objective function *p* in the non-inferior solution *j*, so we have:

$$MF(f_i^j) = \begin{cases} 1 & f_i^j \leq f_i^I \\ \frac{f_i^N - f_i^j}{f_i^N - f_i^I} & f_i^I < f_i^j < f_i^N \\ 0 & f_i^j \geq f_i^N \end{cases} \quad (23)$$

5. Calculate the membership degree for each noninferior solution j based on its individual membership functions as follows:

$$\alpha^j = \frac{\sum_{i=1}^{2} MF(f_i^j)}{\sum_{j=1}^{J} \sum_{i=1}^{2} MF(f_i^j)} \quad (24)$$

6. If decision maker can offer a preferred weight for each objective, another way of calculating membership degree is provided, i.e.,

$$\alpha^j = \frac{\sum_{i=1}^{2} \omega_i MF(f_i^j)}{\sum_{i=1}^{2} \omega_i} \quad (25)$$

where ω_i is the weight of the objective function *p*.

7. Select the solution with the maximum value of α^j as the most preferred solution.

6. Case study

In this section, we present a numerical example based on the railway network of Iran and its operation data to examine the practicability of the developed methodology and solution approach.

6.1. Characteristics of the case network

The railway network of Iran is a national state-owned railway system which manages 35 million tons of goods and 25 million passengers annually, accounting for 12.5 and 10 percent of the whole transport system in Iran, respectively (RAI 2016). The network also consists of 446 links and 440 stations covering a length of 10,000 km across the country. In terms of strategic importance, this network has a western extension to Turkey, a North-South corridor from Azerbaijan and Russia to the Persian Gulf, and an eastern connection to the bogie-changing station at Sarakhs (see Fig. 5). Therefore, such widespread network demands a lot of attention to an efficient emergency management plan.

Fig. 5. Case study network

6.2. Determining link exposure

According to the structure of multi-criteria decision making described in Section 4.2, the exposure of each link should be determined. After interviews with the railway experts (Bababeik 2016), the comparison matrix is obtained based on the pairwise comparison of the attributes as follows:

$$
CM = \begin{array}{c} \\ PT \\ FT \\ T \\ U \end{array}
\begin{array}{c}
\begin{array}{cccc} PT & FT & T & U \end{array} \\
\begin{bmatrix} 1 & 5 & 5 & 7 \\ 0.2 & 1 & 1 & 5 \\ 0.5 & 1 & 1 & 5 \\ 0.143 & 0.2 & 0.2 & 1 \end{bmatrix}
\end{array}
$$

The relative normalized weights of the attributes are obtained using the geometric mean method of AHP method:

	PT	**FT**	**T**	**U**
Weights	0.61	0.17	0.17	0.05

It should be noted that the consistency ratio of the normalized comparison matrix is obtained 0.077, and thus there exists a good consistency in the judgment of analyst regarding the problem under study (Saaty 1994).

The TOPSIS approach is applied to calculate the link importance in terms of four attributes, as described in Section 4.2. The exposure of each link is then determined by dividing the importance of each link to its road accessibility measure. Fig. 6 illustrates the result of calculating link exposure for the whole set of links. Also, this figure shows the position of 24 potential RT stations in the network.

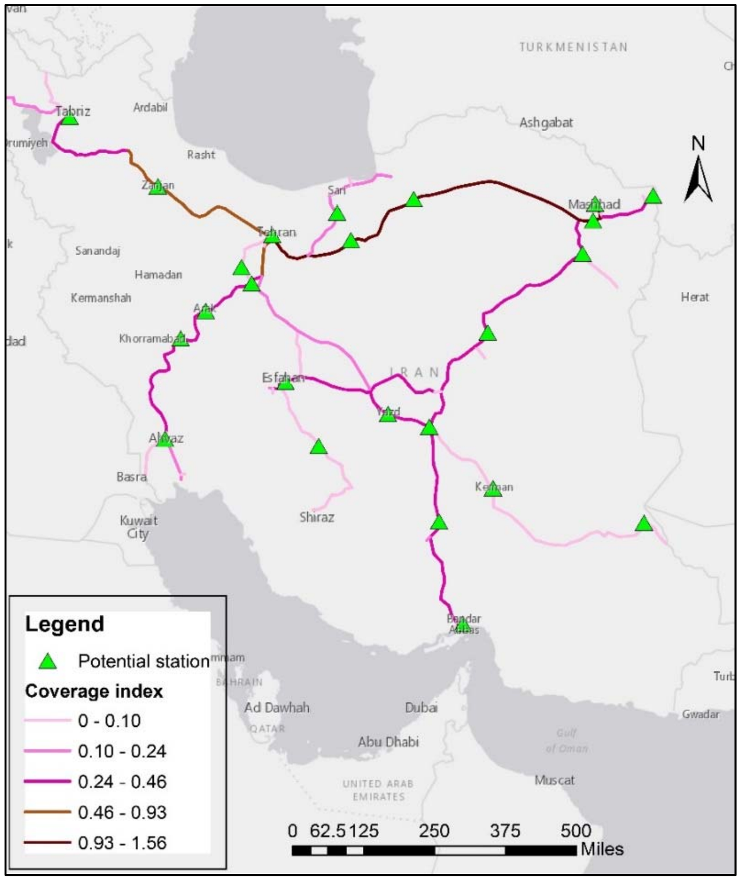

Fig. 6. Coverage index of links and potential ERT stations

6.3. Solution results

We employ the LART model for the case network using the data obtained in previous stages and solve it with AUGMECON approach to find the Pareto optimal solutions. The results are recorded in Table 3. There are 8 non-dominated solutions obtained by this approach. The second and third columns show the value of objective functions $f_1$and $f_2$in LART model, respectively. The results denote that it is impossible to make any reduction on the total travel time (second objective function) without decreasing the total coverage of link exposure (first objective function) or vice versa. To offer a suitable solution to a real application, we perform the fuzzy-logic approach using Algorithm 2. The results are provided in the last three columns of Table 3 for each non-dominated solution. The obtained values of membership degree of different solutions show that the optimal solution No.5 is among the most preferred ones.

Table 3. Pareto optimal solutions of case study

Pareto optimal solution No.	1st obj. function	2nd obj. function	MF of 1st obj.	MF of 2nd obj.	Membership degree

1	104.99	88564.86	1	0	0.5
2	103.67	86225.25	0.94	0.1	0.52
3	102.78	80416.84	0.89	0.36	0.63
4	102.71	72746.63	0.89	0.7	0.79
5	101.39	70407.02	0.83	0.8	0.81
6	99.91	69869.88	0.76	0.82	0.79
7	97.45	67689.87	0.64	0.92	0.78
8	84.06	65830.07	0	1	0.5

6.4. Analysis of the results

In this analysis, the same weight is considered for both objectives, but the decision-maker can consider the priority of objectives by assigning different weight to them and calculate membership degree through Eq. 25. Accordingly, a further experiment is carried out to investigate the effect of choosing different weights of objectives on the preferred solution in the fuzzy logic selection procedure. For this purpose, we made a comparison between the normalized value of objective functions in preferred solution when different weights are selected for them. The result has been depicted in Fig. 7. As we can see, when the weight of objective function f_1(maximizing cooperative coverage of link exposure) increases, its value increases continuously while the negative value of objective function f_2 (minimizing total travel time of recovery) decrease continuously. As the lower value of total travel time is preferable, its negative value has been used. This figure reveals that the weights in which the most cooperative coverage is reached whereas the least total travel time is provided occurs in the weights of $\omega_1 = 0.57$ and $\omega_2 = 0.43$. The result of solving the case network with such weights has been depicted in Fig. 8. According to budget limit of 16000, there will be 16 opened stations and then they are allocated to links. The green circles indicate opened RT stations, and the links assigned to each one are depicted with the same color.

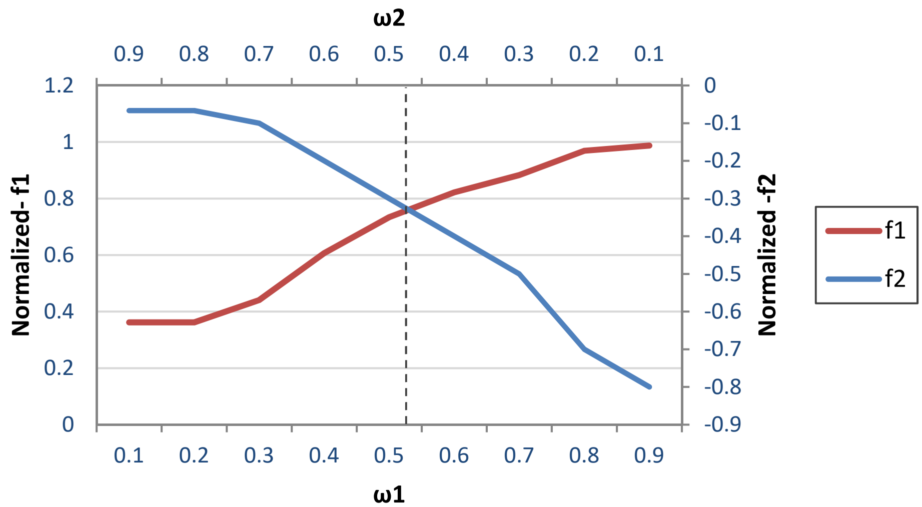

Fig. 7. The effect of different weight factor on the objective functions in the preferred solution

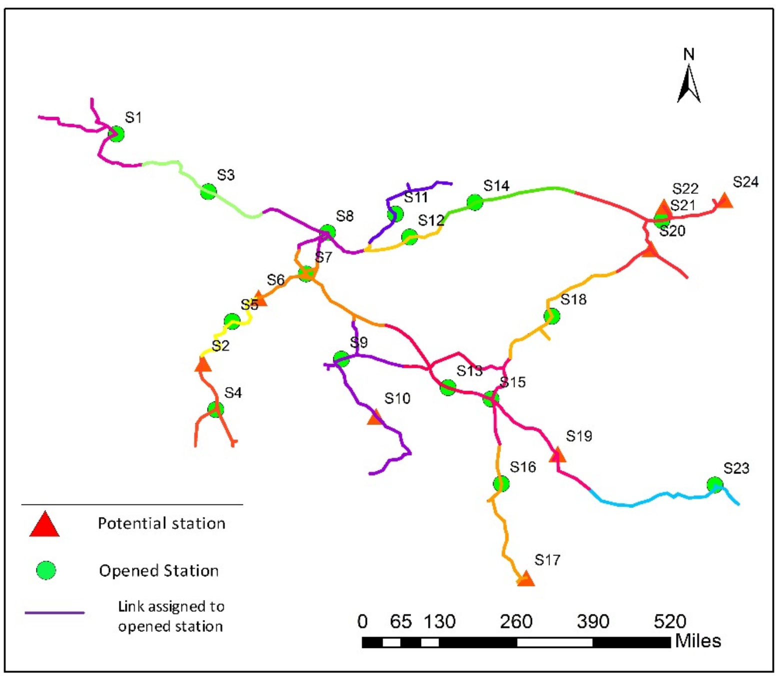

Fig. 8. Opened RT stations and allocation layout of the preferred solution

A sensitive analysis is also used to determine how different values of budget level impact the preferred solution. As Fig. 9 shows, the ratio of total cooperative coverage of exposure increases monotonically by enhancing the level of budget. However, when the budget level exceeds 18000 unit, the increase in this ratio is negligible. Hence, the decision maker might choose this level of the budget as an economical option while he concerns providing a satisfactory coverage of high exposure elements. It can also be seen that even when the most budget level is available, some links remain uncovered in the preferred solution (total cooperative coverage ratio is 69%). It implies the fact that the available potential RT stations are not properly spread throughout the network to cover all links. Therefore, the railway authority requires to scatter potential RT stations throughout the network or consider additional ones.

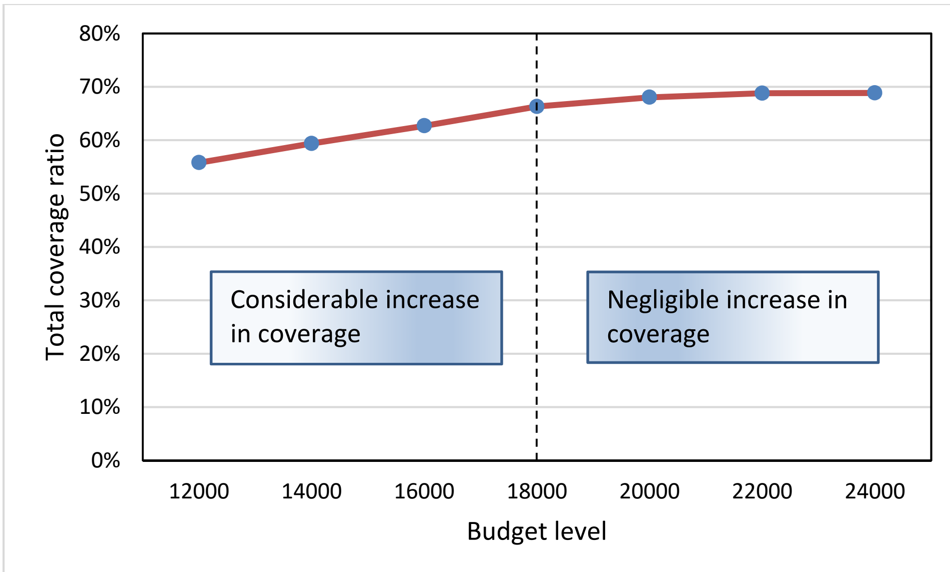

Fig. 9. The effect of weight factor on preferred solutions

To highlight the role of cooperative coverage, we made a comparison of the preferred optimal solution in the proposed LART model with the optimal solutions of a maximal covering location problem (MCLP) for various budgetary level, as illustrated in Fig. 10. We observe that the LART model provides a better total coverage index in comparison with MCLP with the same level of budget. Hence, the developed cooperative model can achieve a better performance than the conventional MCLP in providing coverage in the network. Also, as the budget level for supplying RT increases, the solution by LART model provides a better total coverage index than MCLP.

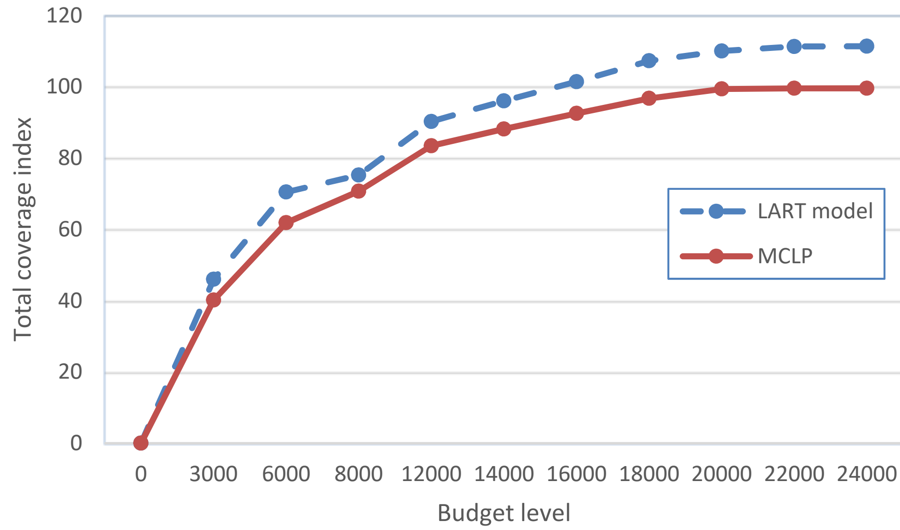

Fig. 10. Comparison of optimal solutions in LART and MCLP models

7. Conclusion

Location and allocation of emergency vehicles can be an effective mechanism for improving the capability of a system to recover from an incident immediately after it occurs. In this paper, we addressed optimizing the location and allocation of Relief Trains (RTs) in the rail network to enhance the resilience of the system. In this respect, we propose a mathematical programming model which consists of two conflicting objectives: (i) maximizing the cooperative coverage of the link exposure by RT stations, and (ii) minimizing the total travel time of relief trains from RT stations to the whole links of the network. The first objective incorporates the *link exposure* which numerically indicates the criticality of the link to be covered by RTs. Such index is determined by a multi-attribute decision making (MADM) technique and the accessibility of links to road system as a backup service in case of emergency. The second objective assigns opened RT stations to tracks throughout the network such that the total travel time of RTs is minimized. The proposed problem is solved with a modified version of an e-constraint method called AUGMECON to produce Pareto optimal solutions. A fuzzy logic approach is then applied to select the preferred solution.

A case study addressing the railway network of Iran illustrates the application of proposed model and solution approach. Here, we used the same weight for both objectives, but deciding about proper weights is a tradeoff between maximizing coverage and minimizing total travel time of RTs to the links. Especially, we made an experiment to investigate the effect of adopting various weight factors on the preferred solution. The result of experiment reveals the weights in which the highest total coverage is reached whereas the least total travel time is provided. Based on the obtained result of this experiment, we made a location and allocation layout for RTs in the case network. A sensitive analysis is also used to determine how different values of a budget level might impact the total coverage index. The result of this analysis shows that when the budget level exceeds a certain level, the increase in coverage of total exposure throughout the network is negligible. Hence, this analysis might help the decision maker to choose an economical decision while he concerns providing maximum coverage of critical links. We also observe that about one-third of total cooperative coverage still does not satisfy in case of opening all potential ERT stations, so the railway authority should scatter the potential stations over the network to provide an efficient coverage. A comparison between the bi-objective LART model and conventional maximal covering location problem (MCLP) also reveals that the proposed model provides a better performance in terms of covering the important links with the same level of budget.

Several avenues present themselves as directions for further work. Different categories of emergency vehicles are required to provide cooperative emergency services in a rail incident. Especially, rail-road rescue vehicles provide more flexibility in case of emergency. A generalization that treats location and allocation of such categories is certainly worth developing. As another avenue, adding new RT stations to existing railway rescue centers is expected to be developed.

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