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Using polynomial chaos expansion for uncertainty and sensitivity analysis of bridge structures

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Abstract: The quantification of the uncertainty effect of random system parameters, such as the loading conditions, material and geometric properties, on the system output response has gained significant attention in recent years. One of the well-known methods is the first-order second-moment (FOSM) method, which can be used to determine the mean value and variance of the system output. However, this method needs to derive the formulas for calculating the local sensitivity and it can only be used for systems with low-level uncertainties. Polynomial Chaos (PC) expansion is a new non-sampling-based method to evaluate the uncertainty evolution and quantification of a dynamical system. In this paper, PC expansion is used to represent the stochastic system output responses of civil bridge structures, which could be the natural frequencies, linear and nonlinear dynamic responses. The PC coefficients are obtained from the non-intrusive regression based method, and the statistical characteristic can be evaluated from these coefficients. The results from the proposed approach will be compared with those calculated with commonly used methods, such as Monte Carlo Simulation (MCS) and FOSM. The accuracy and efficiency of the presented PC based method for uncertainty quantification and global sensitivity analysis will be investigated. Global sensitivity analysis is performed to quantify the effect of uncertainty in each random system parameter on the variance of the stochastic system output response, which can be obtained directly from the PC coefficients. The

results demonstrate that PC expansion can be a powerful and efficient tool for uncertainty quantification and sensitivity analysis in linear and nonlinear structure analysis.

Keywords: Global Sensitivity Analysis; Polynomial Chaos Expansion; Uncertainty Quantification; Nonlinear Structural Analysis; Stochastic Response Analysis, Random System Parameters.

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1 Introduction

Uncertainties inevitably exist in the loading conditions, material and geometric properties of civil structures. Padgett and DesRoches [1] presented a comprehensive introduction on various sources of uncertainties in civil structures. With the growing needs for the optimum design of new structures, and optimum retrofit and maintenance planning of existing structures, sensitivity analysis gains a significant attention in recent years. It aims at quantifying the uncertainty in the system output response affected by different sources of uncertainties in the system parameters [2], which can be usually classified into two categories, namely local sensitivity analysis and global sensitivity analysis [3].

Local sensitivity analysis is performed to assess the ~~local~~ impact of the variations in inputs ~~factors~~ on model responses. It is usually performed to calculate the partial derivative of the output functions at these factor values [4]. Local sensitivity analysis has been used in the field of structural engineering for structural optimization analysis [5], structural identification [6, 7], finite element model updating [8, 9] and reliability analysis [10]. Local sensitivity analysis is considered as an essential component in the first-order second-moment (FOSM) method, which is a probablistic method to determine the stochastic properties of a function with random input variables. Haukaas and Kiureghian [11] conducted the detailed studies on the derivation of local response sensitivity. Using the direct differentiation method (DDM), the dynamic response sensitivities with respect to the material, load and geometric parameters can be calculated. The local response sensitivity has been widely used along with FOSM for uncertainty and reliability analysis, such as probabilistic nonlinear response analysis [12] and probabilistic push-over analysis [13].

Global sensitivity analysis, also named as variance based sensitivity analysis, aims at quantifying the effect of random input parameters (or combinations thereof) on the variance of stochastic output [3]. Global sensitivity analysis is primarily used in uncertainty analysis and quantification to obtain the effect of random inputs on the variance of stochastic output. Sobol [14] first proposed Monte Carlo (or quasi-Monte Carlo) methods to compute the global sensitivity indices. Due to his contribution to the development of the sensitivity indices, the indices are also referred as Sobol' indices. These indices gain much attention for engineering applications, and many methods, such as Fourier analysis sensitivity test method [15], support vector regression [16, 17] and Gaussian process model [18, 19], have been developed for calculating Sobol' indices.

Wiener [20] first introduced Polynomial Chaos (PC) expansion to model stochastic processes by using Hermite polynomials and Gaussian random variables. PC expansion can be used to conduct the uncertainty quantification of a dynamical system with random system parameters. Xiu and Karniadakis [21] extended the Hermite based PC to the Wiener–Askey based PC to the representation of random processes with inputs of different probability distributions. Using different types of orthogonal polynomials from the Askey scheme may provide a more efficient way to represent the random processes, compared with the original Wiener–Hermite expansions. Sudret [3] discovered the relationship between the PC expansion coefficients and the Sobol' indices. It is proved that the computation of Sobol' indices from PC coefficients can be analytical. The computational demand is reduced when the PC expansion coefficients are directly used to calculate the Sobol' indices. Sandoval [22] used PC expansion based method for global sensitivity analysis in a Mass-Spring-Damper system and a DC motor system. Other studies on PC based global sensitivity analysis can be found in [23, 24].

Uncertainty quantification and sensitivity analysis of bridge structures have gained much attention in recent years. Wan and Ren [18, 25] developed Bayesian approaches to perform global sensitivity analysis of a bridge. The Sobol' indices of natural frequencies of bridges were calculated. Zou et al. [26] presented an interval analysis method to evaluate the lower and upper bounds of bridge responses under moving loads. The uncertainties of the bridge and vehicle parameters are unknown but bounded. Jin, *et al.* [27] used pseudo-excitation method to investigate the response quantity of vehicle–bridge system. Results showed that the dynamic characteristics of vehicle is dominated by the random rail irregularities. Yu, *et al.* [28] presented a study on train–bridge system with probability density evolution method. The dynamic vibration characteristics due to rail irregularity can be calculated. An intrusive based method was proposed to evaluate the random dynamic characteristics of a bridge–vehicle system [29]. The uncertainty in the material parameters of bridge was assumed with Gaussian distributions. A stochastic finite element method was developed to evaluate the statistical characteristic of the deformation at the middle span of a bridge [30]. The bridge was simplified as a laminated composite beam. It should be noted that the intrusive based method could be difficult to be applied for large scale civil structures. As reported by Ni, *et al.* [31], to evaluate response statistics of a 20 elements beam structure, the stochastic system matrix size is up to 10000. Although numerous studies can be found on probabilistic response analysis of bridge structures, most of them considered linear dynamic problems with simplified models. It is a more challenging task to perform uncertainty analysis in complex structures, especially for eigenvalue analysis and nonlinear dynamic response analysis. Studies on performing the probabilistic nonlinear response analysis has been conducted with FOSM [12], however, the accuracy of the analysis results is low. Therefore an efficient and accurate approach for the uncertainty quantification and reliability analysis of large scale structural analysis is worth of investigation.

This paper presents a non-intrusive method for uncertainty quantification and sensitivity analysis of bridge structures by using PC expansion. Three classic problems in civil engineering, such as eigenvalue analysis, a linear bridge-vehicle sytem interaction analysis and a probabilistic nonlinear structural response analysis under seismic loading, are studied. The corresponding system outputs are structural natural frequencies, the linear dynamic response and nonlinear dynamic response, respectively. Various system random parameters are considered in these three examples. The system outputs are represented by using PC expansion, where the PC coefficients are obtained from the non-intrusive regression method. When the PC coefficients are obtained, the response statistics and global sensitivity indices can be evaluated. Results from the presented PC based method are compared with those from MCS and FOSM methods. They show that the PC based method has a very good accuracy compared with MCS, while takes much less computation time. The presented PC expansion based method can also provide more accurate results than FOSM, particularly when the system parameters have high level of uncertainties. The results demonstrate that PC expansion can be a powerful and efficient tool in civil engineering for uncertainty quantification and sensitivity analysis.

2 Theoretical Framework

2.1 PC expansion

A general second order random process θ, which is viewed as a function of $\xi = \{\xi_i\}_{i=1}^{m}$, can be represented as

$$\theta = a_0\phi_0 + \sum_{i=1}^{m} a_i\phi_1\left(\xi_i\right) + \sum_{i=1}^{m}\sum_{j=1}^{i} a_{i,j}\phi_2\left(\xi_i,\xi_j\right) + \sum_{i=1}^{m}\sum_{j=1}^{i}\sum_{k=1}^{j} a_{i,j,k}\phi_3\left(\xi_i,\xi_j,\xi_k\right) + \ldots \quad (1)$$

where m is the number of random inputs, $\phi_P\left(\bullet\right)$ defines the *P-th* order term in the PC expansion, $a_{i,j,k}$ is the deterministic coefficient. In this study, ξ is considered as a standard Gaussian process and $\phi(\bullet)$ is the Hermite polynomial function. For the random processes with different probability distributions, the corresponding polynomial functions can be found in [21].

For notational convenience, Eq. (1) can be rewritten as a more compact expression

$$\theta(\xi) = \sum_{j=0}^{\infty} \hat{a}_j \psi_j(\xi) \tag{2}$$

where $\hat{a}_j$ denotes the deterministic PC coefficients, $\psi(\bullet)$ is the polynomial function which can accommodate a random process with different probability distributions.

2.2 Representation of the output response of a dynamic system

The output response y of a physical dynamic model, e.g. structural displacement, acceleration, stain, etc., can be represented by using a deterministic mapping $y = h(\chi)$, where $\boldsymbol{\chi} = [\chi_1, \chi_2, \cdots \chi_m]$ is a $1\times m$ vector of the dynamic system parameters, i.e., geometry, material properties and loading etc. Since the input parameter vector $\boldsymbol{\chi}$ is inevitably subjected to the uncertainty effect, it is represented by using a random vector $\boldsymbol{\eta} = [\eta_1, \eta_2, \cdots, \eta_m]$. Consequently, the output response of a dynamic system is also a random variable, which is defined as $Y(\boldsymbol{\eta})$. The random variables in the vector $\boldsymbol{\eta}$ are assumed to be independent. Then the chaos representation of the output response can be written as

$$Y(\boldsymbol{\eta}) = \sum_{j=0}^{\infty} \beta_j \Phi_j(\boldsymbol{\eta}) \tag{3}$$

where $\Phi_j(\eta), j = 0, 1, 2, \ldots, \infty$ is the Hilbertian basis of the suitable Hilbert space containing the response and $\beta_j,\ j = 0, 1, 2, \ldots, \infty$ is the unknown deterministic coefficient.

When the random system parameters are following the Gaussian distributions denoted as ξ in Eq. (2), a Hilbertian basis is the family of multivariate Hermite polynomial functionals $\psi(\bullet)$, which are orthogonal with respect to the Gaussian distribution function. Using Hermite polynomials functions as described in Section 2.1, Eq. (3) can be re-written as

$$Y(\xi) = \sum_{j=0}^{\infty} \beta_j \psi_j(\xi) \tag{4}$$

To reduce the computational demand, PC expansion in Eq. (4) has to be truncated with M terms.

One retains those polynomials ψ_j with a total degree up to P as

$$Y(\xi)=\sum_{j=0}^{M-1}\beta_j\psi_j(\xi)=\boldsymbol{\psi}(\xi)\boldsymbol{\beta} \tag{5a}$$

$$\boldsymbol{\psi}(\xi)=\left[\psi_0(\xi),\psi_1(\xi),\cdots,\psi_{M-1}(\xi)\right] \tag{5b}$$

$$\boldsymbol{\beta}=\left[\beta_0,\beta_1,\cdots,\beta_{M-1}\right]^T \tag{5c}$$

A convergence analysis can be conducted to determine the PC expansion order [28]. Usually, results from the second or third order PC expansion is accurate enough. When the order is sufficiently high (i.e. the third order), the order number of PC expansion will not affect the results. In this paper, the third order PC expansion is used and the comparison studies against MCS method will be conducted to verify the accuracy of the results.

The number of unknown coefficients M is equal to

$$M=\frac{(P+m)!}{P!m!} \tag{6}$$

The PC expansion was initially formulated with standard Gaussian random system parameters and Hermite polynomials [20]. In practice, the system parameters may not necessarily always following Gaussian distributions. When non-Gaussian distributions, e.g., Lognormal distribution, Gumbel distribution, Weibull distribution, Beta distribution and Uniform distribution, are observed to better describe the random properties of several system parameters, the random parameters can be obtained by using the isoprobabilistic transformation approach [32]. The one-to-one mapping can be obtained as

$$\eta_i=F_i^{-1}\left(Z(\xi_i)\right) \tag{7a}$$

$$\xi_i=Z^{-1}\left(F_i(\eta_i)\right) \tag{7b}$$

where η_i is the system random parameters, ξ_i is standard Gaussian random parameters, F_i^{-1} is the inverse cumulative distribution function (CDF) of η_i, and $Z^{-1}(\ )$ is the inverse CDF of ξ_i.

Generally the random system parameter vector $\boldsymbol{\eta}$ can be exactly or approximately represented by using a standard normal random vector ξ as

$$\boldsymbol{\eta}=T(\xi) \tag{8}$$

where T is the isoprobabilistic transformation [32]

The truncated Hermite polynominals based PC expansion of the output response of a dynamic system can be represented as

$$Y = Y\left(T\left(\xi\right)\right) = \sum_{j=0}^{M-1} \beta_j \psi_j\left(\xi\right) = \boldsymbol{\psi}\left(\xi\right)\boldsymbol{\beta} \tag{9}$$

2.3 Computation of the PC coefficients

The intrusive based Galerkin projection method and the non-intrusive based regression method are two widely used methods for computing the PC coefficients [33]. In the intrusive based projection method, the PC coefficients are obtained by using a Galerkin scheme, which leads to a deterministic system with a large number of equations and an intensive computational load. The non-intrusive regression method [34] is an alternative method and has been successfully used in various applications [35]. In this method, the PC coefficients are estimated by minimizing the mean square error of the response approximation. It shall be noted that the non-intrusive regression method is used in this study for the calculation of PC coefficients.

The minimization of the variance of the residual with respect to the unknown coefficients leads to

$$\boldsymbol{\beta} = \arg\min E\left[Y\left(\boldsymbol{\eta}\left(\xi\right)\right) - \boldsymbol{\psi}\left(\xi\right)\boldsymbol{\beta}\right] \tag{10}$$

To estimate the unknown coefficients in Eq. (10), a set of n regression points with standard normal random distributions are selected as $\left[\boldsymbol{\xi}^1, \boldsymbol{\xi}^2, \cdots, \boldsymbol{\xi}^n\right]^T$. Using the isoprobabilistic transform as shown in Eq. (8), the non-Gaussian distribution inputs can be obtained as $\left[\boldsymbol{\chi}^1, \boldsymbol{\chi}^2, \cdots, \boldsymbol{\chi}^n\right]^T$. The corresponding model evaluation $\overline{\boldsymbol{Y}} = \left[y^1, y^2, \cdots, y^n\right]^T$ can then be obtained, where $y^i = h\left(\boldsymbol{\chi}^i\right)$, i=1, 2, …, n. The calculation of the PC coefficients is conducted with the following mean-square minimization

$$\boldsymbol{\beta} = \left(\frac{1}{n}\sum_{i=1}^{n} \boldsymbol{\psi}\left(\xi^i\right)\boldsymbol{\psi}^T\left(\xi^i\right)\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} \boldsymbol{\psi}\left(\xi^i\right)y^i\right) \tag{11}$$

Eq. (11) can be rewritten in the matrix form as

$$\boldsymbol{\beta} = \left(\boldsymbol{\Psi}^T\boldsymbol{\Psi}\right)^{-1}\boldsymbol{\Psi}^T\overline{\boldsymbol{Y}} \tag{12}$$

where

$$\boldsymbol{\Psi}=\begin{pmatrix} \psi_0\left(\xi^1\right) & \psi_1\left(\xi^1\right) & \cdots & \psi_{M-1}\left(\xi^1\right) \\ \psi_0\left(\xi^2\right) & \ddots & \cdots & \psi_{M-1}\left(\xi^2\right) \\ \vdots & \cdots & \ddots & \vdots \\ \psi_0\left(\xi^n\right) & \psi_1\left(\xi^n\right) & \cdots & \psi_{M-1}\left(\xi^n\right) \end{pmatrix} \tag{13}$$

The number of regression points should be larger than the number of unknown PC coefficients to ensure that the numerical stability of the regression problem in Eq. (12) is achieved and the matrix $\boldsymbol{\Psi}$ is well-conditioned. According to [3], the number of sampling points are two or three times larger than the number of PC coefficients. These sampling points are generated from Latin hypercube method.

When the PC coefficients are estimated, an analytical surrogate model of the system output response can be obtained with the following PC approximation

$$\hat{Y}=\sum_{j=0}^{M-1}\hat{\beta}_j\psi_j\left(\xi\right) \tag{14}$$

The statistics of the uncertain system output response can be calculated with a much less computational demand. The mean of the system output response $E\left(Y\right)$ can be evaluated as

$$E\left(Y\right)=\hat{\beta}^0 \tag{15}$$

The variance $V\left(Y\right)$ is obtained as

$$V\left(Y\right)=\sum_{j=1}^{M-1}\left[\hat{\beta}_j\right]^2\psi_j^2 \tag{16}$$

2.4 Global sensitivity analysis

Global sensitivity analysis aims to quantify the contributions of each uncertain system parameter on the statistics of the system output response. The output response can be represented based on the variance decomposition [14] as

$$Y\left(\xi\right)=Y_0+\sum_{i=1}^{m}Y_i\left(\xi_i\right)+\sum_{i=1}^{m}\sum_{j=1}^{i}Y_{ij}\left(\xi_i,\xi_j\right)+\cdots+Y_{12\cdots m}\left(\xi_1,\xi_2\cdots,\xi_m\right) \tag{17}$$

The term Y_0 is the mean value of the output and can be obtained as

$$Y_0=\int Y\left(\xi\right)d\xi \tag{18}$$

Moreover, the summands of Eq. (16) are given by

$$\begin{aligned}Y_i(\xi_i) &= E(Y(\xi)\,|\,\xi_i) - Y_0 \\ Y_{ij}(\xi_i,\xi_j) &= E(Y(\xi)\,|\,\xi_i,\xi_j) - Y_0 - Y_i - Y_j\end{aligned} \tag{19}$$

$E(Y(\xi)\,|\,\xi_i)$ is the conditional expectation of $Y(\xi)$ when ξ_i is set. Similarly, $E(Y(\xi)\,|\,\xi_i,\xi_j)$ is the conditional expectation of $Y(\xi)$ when ξ_i and ξ_j are set.

The integral of each summand is equal to zero, which can be expressed as

$$\int Y_{i1\cdots is}(\xi_{i1},\xi_{i2},\cdots\xi_{is})d\xi_{ik} = 0 \tag{20}$$

where $ik \in \{i1, i2, \cdots, is\}$ and $1 \le i1 \le \cdots \le is \le m$. The summands are orthogonal to each other

$$\int Y_{i1\cdots is}(\xi_{i1},\xi_{i2},\cdots\xi_{is})Y_{j1\cdots jr}(\xi_{j1},\xi_{j2},\cdots\xi_{js})d\xi = 0 \tag{21}$$

where $\{i1, i2, \cdots, is\} \ne \{j1, j2, \cdots, jr\}$

The variance of the output is given by

$$V(Y) = \int\left(\left(Y(\xi)\right)^2 - \left(Y_0\right)^2\right)d\xi \tag{22}$$

Substituting Eq. (17) into Eq. (22), we have

$$V(Y) = \sum_{i=1}^{m} V_i(\xi_i) + \sum_{i=1}^{m}\sum_{j=1}^{i} V_{ij}(\xi_i,\xi_j) + + \cdots + V_{12\cdots m}(\xi_1,\xi_2\cdots,\xi_m) \tag{23}$$

The summands of Eq. (23) are given by

$$\begin{aligned}V_i &= V\left(E(Y(\xi)\,|\,\xi_i)\right) \\ V_{ij} &= V\left(E(Y(\xi)\,|\,\xi_i,\xi_j)\right) - V_i - V_j\end{aligned} \tag{24}$$

where $V\left(E(Y(\xi)\,|\,\xi_i)\right)$ (resp. $V\left(E(Y(\xi)\,|\,\xi_i,\xi_j)\right)$) is the variance of the conditional expectation of $Y(\xi)$ when ξ_i is set (resp. ξ_i and ξ_j are set)

According to [14], the global sensitivity indices are defined as

$$S_{i1\cdots is} = \frac{V_{i1\cdots is}(\xi_{i1},\xi_{i2},\cdots\xi_{is})}{V(Y)} \tag{25}$$

The sensitivity indices satisfy the following equation

$$\sum_{i=1}^{m} S_i(\xi_i) + \sum_{i=1}^{m}\sum_{j=1}^{i} S_{ij}(\xi_i,\xi_j) + + \cdots + S_{12\cdots m}(\xi_1,\xi_2\cdots,\xi_m) = 1 \tag{26}$$

The total sensitivity indices are defined to evaluate the total effect of a system parameter

$$S_{Ti} = 1 - S_{\sim i} \tag{27}$$

where $S_{\sim i}$ is the sum of all $S_{i1\cdots is}$ that does not include the input parameter ξ_i.

The variance decomposition of a function $Y(\boldsymbol{\xi})$ can be obtained from reorganising Eq. (14) based on a previous study [3],

$$\begin{aligned} Y(\boldsymbol{\xi}) = \hat{\beta}_0 &+ \sum_{i=1}^{m} \sum_{\boldsymbol{\alpha} \in \Re_i} \hat{\beta}_{\boldsymbol{\alpha}} \psi_{\boldsymbol{\alpha}}(\xi_i) \\ &+ \sum_{1 \le i1 < i2 \le n} \sum_{\boldsymbol{\alpha} \in \Re_{i1,i2}} \hat{\beta}_{\boldsymbol{\alpha}} \psi_{\boldsymbol{\alpha}}(\xi_{i1}, \xi_{i2}) + \cdots \\ &+ \sum_{1 \le i1 < i2 < \cdots < is \le n} \sum_{\boldsymbol{\alpha} \in \Re_{i1,i2,\cdots,is}} \hat{\beta}_{\boldsymbol{\alpha}} \psi_{\boldsymbol{\alpha}}(\xi_{i1}, \xi_{i2} \cdots, \xi_{is}) + \cdots \\ &+ \sum_{\boldsymbol{\alpha} \in \Re_{1,2,\cdots m}} \hat{\beta}_{\boldsymbol{\alpha}} \psi_{\boldsymbol{\alpha}}(\xi_1, \xi_2 \cdots, \xi_m) \end{aligned} \tag{28}$$

where $\Re_{(\cdot)}$ is the set of indices which make Eq. (28) equivalent to Eq. (17), that is $\Re_i$ picks up the signal variables *i;* $\Re_{i,j}$ picks the pairs of variables (*i*, *j*); $\Re_{i,j,k}$ picks the triplets (*i*, *j, k*) and so on. $\Re_{(\cdot)}$ is defined as

$$\Re_{i1,i2,\cdots is} = \left\{ \boldsymbol{\alpha} : \begin{array}{l} \alpha_k > 0 \ \forall k{=}1,2,\cdots,m,\ k \in (i1, i2, \cdots is) \\ \alpha_k = 0 \ \forall k{=}1,2,\cdots,m,\ k \notin (i1, i2, \cdots is) \end{array} \right\} \tag{29}$$

where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \cdots, \alpha_m)$, $\alpha_i \ge 0$, $\sum_{i=1}^{m} \alpha_i \le P$ is an integer sequence.

Following the derivation in a previous study [3], the first order sensitivity index can be computed directly as

$$S_i = \frac{\sum_{j \in \Re_i} \hat{\beta}_j^2 \psi_j^2(\xi_i)}{V(Y)} \tag{30}$$

where $\Re_i$ is a set of polynomial functions with the random system parameter ξ_i only.

The high order sensitivity indices can be computed as

$$S_{i1,i2,\cdots,is} = \frac{\sum_{j \in \Re_{i1,i2,\cdots,is}} \hat{\beta}_j^2 \psi_j^2(\xi_{i1}, \xi_{i2}, \cdots, \xi_{is})}{V(Y)} \tag{31}$$

where $\Re_{i1,i2,\cdots,is}$ is a set of polynomial functions with the random system parameters $\xi_{i1}, \xi_{i2}, \cdots, \xi_{is}$ only.

For a given integer sequence ($j1,\ldots,jt$), $\vartheta_{j1,\cdots,jt}$ is defined as

$$\vartheta_{j1,\cdots,jt} = \left\{ \left(i1,\cdots,is\right), \left(j1,\cdots,jt\right) \subset \left(i1,\cdots,is\right) \right\} \tag{32}$$

The total sensitivity indices can be computed as

$$S^T_{j1,\cdots,jt} = \sum_{(i1,i2,\cdots,is)\in\vartheta_{j1,\cdots,jt}} S_{i1,i2,\cdots,is} \tag{33}$$

The detailed proof can be found in [3]. Once the PC coefficients are obtained, the sensitivity indices and statistics characteristic of output responses can be evaluated. Compared with MCS method, which need several thousands of simulations or even more, the computational cost is significantly reduced. The PC based method is used for uncertainty propagation and sensitivity analysis of bridge structures. MCS and FOSM are also conducted for the comparison to verify the accuracy and efficiency of the PC based method.

3 Numerical Studies

In this section, three examples of using PC exapansion for uncertainty quantification and global sensitivity analysis in civil engineering structures are presented. These examples include the eigenvalue analysis of a bridge structure, dynamic response analysis of a linear bridge-vehicle sytem and probabilistic nonlinear structural response analysis of a bridge under seismic loading. The corresponding system outputs are structural natural frequencies, linear and nonlinear dynamic responses, respectively. Various system random parameters with uncertainties are considered in these three examples.

3.1 A simply-supported bridge

A simply-supported box-section girder bridge model is selected for uncertainty quantification and sensitivity analysis in this section. The total length of the box-section bridge deck is 30m. The plan view and cross-section of the bridge deck model are shown in Figures 1(a) and (b), respectively. The bridge deck is modeled using 60 flat shell elements with 66 nodes. Each node has six Degrees-of-Freedom (DOFs), and totally the structural system has 360 DOFs. The bridge deck is simply-supported at nodes 5, 6, 65 and 66 at two ends of the deck, and the translational restraints at the supports are represented by a large stiffness of 3×10^9 kN/m. In the deterministic analysis, Young's modulus and mass density are assumed as $2.6 \times 10^4 MPa$ and $2500 kg/m^3$, respectively. The first ten intact structural natural frequencies are from 4.44 to 21.61 Hz. The details of this bridge model can be found in [36].

To study the uncertainty effect of material and geometric properties, a total of nine selected uncertain system parameters are summarized in Table 1. The thicknesses of the top flange t_{top}, the web t_{web} and the bottom flange t_{bot} are assumed to satisfy the lognormal distributions, and the Coefficient of Variances (COVs) are assumed as 15%. Elastic moduli of the top flange (E_{top}), the web (E_{web}) and the bottom flange (E_{bot}) are assumed to follow Gaussian distributions with COVs equal to 20%. The mass densities of the top flange ρ_{top}, the web ρ_{web} and the bottom flange ρ_{bot} are assumed to have lognormal distributions with COVs of 20%. Therefore, the random system parameter vector, including the thicknesses, elastic moduli and mass densities of the girder bridge model, is defined as $\boldsymbol{\eta} = \left[t_{top}, t_{web}, t_{bot}, E_{top}, E_{web}, E_{bot}, \rho_{top}, \rho_{web}, \rho_{bot}\right]$. It is noted that these random parameters are assumed as independent.

3.1.1 Eigenvalue analysis problem

The frequencies are considered as fundamental dynamic characteristics of structures. Efforts have been made to identify the natural frequencies of structures by performing the eigenvalue analysis with the finite element model. Uncertainty quantification is conducted and considered as a forward problem by investigating the effect of uncertainty in the random system parameters on the natural frequencies of structures. The output response vector $Y(\boldsymbol{\eta})$ includes the first ten frequencies of the bridge model. For each frequency, they are represented by using the third order PC expansion, which has 219 unknown PC coefficients to be identified. Therefore, 657 regression points are generated with Latin hypercube sampling method, which is three times larger than the number of unknown PC coefficients. These parameters are scaled to the designed points $\left[\chi^1, \chi^2, \cdots, \chi^{657}\right]^T$ with the isoprobabilistic transformation approach. Then the random system output response vector $\bar{\boldsymbol{Y}} = \left[y^1, y^2, \cdots, y^n\right]^T$ and their corresponding PC coefficients can then be obtained. The response statstics, including the mean value, variance and sensitivity indices are calculated by using Eqs. (15), (16) and (30), respectively.

Figure 2 shows the statistical results, i.e. mean value and variance, of the first ten natural frequencies considering the uncertainties listed in Table 1. The results are compared with those calculated from MCS and FOSM. Good agreements in the results are observed. It is noted that when using FOSM, the local sensitivity index is obtained from the forward difference method. The difference step Δ_i is $0.001 \times \mathrm{mean}(\eta_i)$ for each random input. MCS takes 3 hours for 5×10^4 simulations. The results calculated from the presented PC expansion based method match well with MCS results, while some minor errors are observed in the results from FOSM, e.g., the fifth and the sixth natural frequencies. This is because the variance is approximated with the first-order Taylor expansion with the higher order items neglected when FOSM is used. The difference may also affect the accuracy of results.

Figure 3 shows the comparison of probability density functions (PDFs) of frequencies calculated from PC expansion based method, FOSM and MCS. It is observed that the results from PC based method match well with those from MCS, as shown in Figure 3(a). The comparison of obtained PDFs by using MCS and FOSM is shown in Figure 3(b). Minor differences are observed in the results of the lower order frequencies, i.e. the first and second ones, from FOSM and MCS. However, significant errors are observed in the higher order frequencies, indicating that FOSM is not accurate for analysing the uncertainty effect on the higher order frequencies.

Global sensitivity analysis is conducted, and PC based Sobol' indices are calculated with Eqs. (30-33). The reference Sobol' indices are obtained from MCS. For each MCS based Sobol' index calculation, it takes about 3 hours with parallel computation in Matlab and the total computation time is about $(m+2)\times 3$ hours [2], where m is equal to the number of random system parameters which is 9 in this study. The computational time to obtain the first order Sobol' indices and the total sensitivity indices is 61 hours with MCS. However, it takes only 5 minutes when using PC expansion based method. Taking the first frequency of the bridge as an example, the comparison of the first order Sobol' indices and the total sensitivity indices of the first frequency with respect to the selected parameters in Table 1 are shown in Figure 4. It is noted that the PC based sensitivity indices are very close to the results from MCS, indicating that the accuracy of the presented method is good. The first order sensitivity indices are very close to the total sensitivity indices, demonstrating that the higher order effect (combinations thereof random inputs) is minor. The sensitivity index values of geometrical parameters, such as t_{top}, t_{web}, and t_{bot}, are lower than those of material parameters, such as E_{top}, E_{web}, E_{bot}, ρ_{top}, ρ_{web}, and ρ_{bot}. This shows that uncertainties in system material properties have a more significant effect on the natural frequencies than the thicknesses of the girder elements.

3.1.2 Bridge-vehicle system response analysis

In engineering applications, moving loads induced by a vehicle is often considered as excitations to the bridge structures [36, 37]. In this example, a two-axle three-dimensional vehicle with seven DOFs is used to represent a moving vehicle travelling on the bridge model. The vehicle model is a H20-44 truck in AASHTO as shown in Figure 5. The vehicle has a length of 4.73m. Figure 1(a) shows the travelling path of the moving vehicle on the top of the girder bridge. It takes 3.47s for the vehicle passing the bridge with a velocity of 10 m/s. The time step for dynamic analysis is 0.01 second and 5 seconds vibration responses are calculated. The vertical deformation of the bridge at the middle span, namely on Node 32, is selected for uncertainty analysis. The dynamic response analysis of the bridge is conducted by solving the coupled equation of motion of the bridge-vehicle system. The road surface roughness effect is also considered in the dynamic analysis. In this study, the road surface roughness is defined as Class C, corresponding to the average road pavement condition. The details for the dynamic response analysis of the bridge-vehicle system and the corresponding equation of motion can be found in [36]. The vehicle parameters are assumed as deterministic in the analysis.

The system output response is defined as the time history of the vertical displacement at Node 32. The system output response can be represented as

$$Y(\xi,t)=\sum_{j=0}^{M-1}\beta_j\psi_j(\xi)=\boldsymbol{\psi}(\xi)\boldsymbol{\beta}(t) \tag{34}$$

Then the PC coefficients at each time instant can be obtained by minimizing the variance of the residual

$$\boldsymbol{\beta}(t)=\arg\min E\left[Y\left(\boldsymbol{\eta}(\xi),t\right)-\boldsymbol{\psi}(\xi)\boldsymbol{\beta}(t)\right] \tag{35}$$

The computational procedure is the same as described in Section 3.1.1. In this section, two cases, including a low level uncertainty case and a high level uncertainty case, are studied. In the high level uncertainty case, the selected uncertain parameters and their variances are listed in Table 1. In the low level uncertainties case, the COVs are one-tenth of the values in Table 1, while the mean values are kept the same as those in Table 1. The results calculated from MCS, FOSM and PC expansion based method for the low level and high level uncertainty cases are shown in Figures 6 and 7, respectively. The vertical displacement at the mid-span of the girder bridge reaches its maximum value of 1.8×10^{-3}m at t=1.73s when the vehicle is at the middle span. After the vehicle leaves the bridge at 3.47s, the vibration of the bridge damps out to zero quickly. In the low level uncertainty case, the mean values calculated with PC based method and FOSM match well with those from MCS, as shown in Figure 6. However, PC based method is more accurate than FOSM in predicting the variances in the output response.

Figure 7 shows the mean values and variances in the displacement response with a high level of uncertainty. Both the mean values and variances from PC based method match well with MCS results, however, significant differences are observed in the results from FOSM. These results demonstrate that FOSM may not be able to provide the accurate uncertainty quantification results with high level uncertainties. The contour of the PDF surface of the observation point is shown in Figure 8(a). The variance of the vibration is smaller when the vehicle enters and leaves the bridge. It can be observed from Figures 8(b) and (c) that the PDF and CDF of the maximum displacement in the selected response time history agree well with those calculated by using MCS. These results demonstrate that the PC based method can accurately evaluate the PDF and CDF of the dynamic response considering the effect of uncertainties in the system parameters.

The first order sensitivity indices and total sensitivity indices of the maximum displacement in the selected response time history with respect to those random parameters are calculated and shown in Figure 9. It is noted that the total sensitivity indices are larger than the first order sensitivity indices, indicating that the higher order vibrations are significant in the obtained dynamic response. It is observed that generally the sensitivity indices calculated with the PC expansion based method are close to those from MCS. However minor differences are observed since the third order PC expansion is used in this study and the truncated error may lead to minor errors in the calculation of sensitivity indices. The first three important factors are the thickness and Young's modulus of the bottom flange, that is, t_{bot} and E_{bot}, and the mass density of the top flange ρ_{top}. It is worth noting that sensitivity indices obtained by using MCS are calculated with a Microsoft virtual machine (type: D16v3, 16 cores, E5-2673 v4 2.3GHz Prozessor, 64GB RAM), taking more than 140 hours with parallel computing. However, the computational time by using the presented PC based method to calculate the sensitivity indices is less than 20 minutes. This validates that the presented approach based on PC expansion to conduct the uncertainty quantification and sensitivity analysis is much more efficient than using MCS but does not lose the accuracy. It shall be noted that PC based method is also more accurate than using the traditional method based on FOSM.

3.2 Nonlinear response analysis of a bridge considering uncertainties

Probabilistic nonlinear response analysis is a key topic in the performance-based earthquake engineering [11]. Several studies addressed this issue based on DDM-FOSM framework [11-13]. In this section, PC expansion is used to represent the nonlinear vibration responses of a bridge structure under seismic loading. The presented method will be used to evaluate the statistical characteristics of the nonlinear response analysis considering uncertainties in system parameters, and DDM-FOSM and MCS are also conducted for comparison.

The bridge as reported in [38] is taken as an example in this study. A typical 3-lane, 4-cell box girder with a width of 432 in (or 10.973 m in the standard unit system) and depth (D_s) of 84 in (or 2.134 m) is modelled. Figures 10(a) and (b) show the dimensions and cross section of the bridge deck. Figure 10(c) shows the finite element model of the bridge. Each span of the bridge superstructure has five linear beam–column elements in OpenSees [39], since the deck elements are expected to remain elastic. The lumped mass matrix is used. The cross-sectional area, moment of inertia along the z and y axes, torsional constant and the elastic modulus of the bridge deck are defined as 8960 in^2 (Aera), 164711680 in^4 (I_z), 10299150 in^4 (I_y), 31020600 in^4 (J) and 4108 ksi (E), respectively. With the international standard units, these parameters are 5.781 m^2 (Aera), 68.558 m^4 (I_z), 4.287 m^4 (I_y), 12.912 m^4 (J) and 28323 MPa (E), respectively. To model the nonlinear material behavior of the column, five nonlinear beam-column elements are used to model the pier in OpenSees. Figure 10 shows the cross section of the pier and the reinforcements. The cross section of the column is discretized into 96 and 24 fibers for the core and cover concrete, respectively. Constitutive models used to simulate the concrete and steel reinforcement are uniaxial Kent-Scott-Park concrete material model and Giuffre-Menegotto-Pinto steel material model [12], respectively.

This two span continuous bridge is simply supported at two ends at Nodes 1 and 11, and the pier is fixed at Node 16. El Centro earthquake with scaled PGA=0.8g is applied as the excitation along the y-direction of the bridge. The displacement at Node 6 along the y-direction is studied. Studies are conducted to investigate the effect of uncertain system parameters on the response and calculate the sensitivity indices. The equation of motion of the bridge structure under ground motion excitation is written as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)+\boldsymbol{C}\dot{\boldsymbol{x}}(t)+\boldsymbol{K}\left(\boldsymbol{x}(t),\boldsymbol{\vartheta}\right)=\boldsymbol{MI}\ddot{x}_g(t) \tag{36}$$

where $\boldsymbol{x}(t)$, $\dot{\boldsymbol{x}}(t)$ and $\ddot{\boldsymbol{x}}(t)$ are displacements, velocity and acceleration response vectors of the structure, respectively; $\boldsymbol{M}$ and $\boldsymbol{C}$ are the mass and damping matrices of the structure; $\boldsymbol{\vartheta}$ is the uncertainty material parameter vector in the material constitutive model and $\boldsymbol{K}\left(\boldsymbol{x}(t),\boldsymbol{\vartheta}\right)$ is the resisting force vector which depends on the $\boldsymbol{\vartheta}$ and $\boldsymbol{x}(t)$; and $\ddot{x}_g(t)$ is the input acceleration ground motion. The nonlinear dynamic responses are calculated in OpenSees with Newmark method and Newton-Raphson method. The time step for dynamic analysis is set as 0.005s, and 30s vibration responses are calculated. The eight uncertainty material constitutive paramete of the column are used to characterize the concrete and reinforcement, such as two parameters for the confined concrete ($f_{c,\mathrm{core}}$: peak strength and $\varepsilon_{c,\mathrm{core}}$: strain at peak strength); three parameters for the unconfined concrete ($f_{c,\mathrm{cover}}$, $\varepsilon_{c,\mathrm{cover}}$, and $\varepsilon_{cu,\mathrm{cover}}$: strain when the residual strength is reached) and three parameters for the reinforcement (f_y: yield strength, E_0: initial elastic tangent; and b=strain-hardening ratio). Table 2 provides the marginal probability distribution, mean and COV of each random material parameter. It is noted these distribution parameters are given according to [12, 13].

Figure 12 shows the statistical results of the dynamic displacement response at Node 6 along the y-direction. It is noted that 20000 samples are generated for MCS by using Latin hypercube sampling. The simulations of 41 samples, which is less than 0.3%, encounter the convergence problem in the nonlinear dynamic analysis. This may not affect the statistics of the final results. In the presented PC base method, the third order PC expansion is used to represent the nonlinear dynamic displacement at Node 6. When using FOSM, DDM method is used to calculate the local response sensitivity. MCS results are considered as the baseline for comparison. The mean values from FOSM and the presented PC based method agree well with those from MCS, as shown in Figure 12(a). However, it is observed from Figure 12(b) that PC based method gives more accurate results than FOSM. The PC based method takes about 30 mins for the computation, while MCS and FOSM take about 16 hours and 5mins, respectively. This demonstrates that the presented PC expansion based method can provide accurate statistical results, but use much less time than MCS. The PDF of different time instants from the PC based method are shown in Figure 12(a). For the responses in the first several seconds, the uncertainties are very small since the structure is still at the linear stage. After the maximum deformation, the variance in the displacement response increases. Some selected PDFs at different time instants are shown in Figure 13(b), also indicating the accuary of PC based method.

The first order sensitivity indices and total sensitivity indices from PC based method are shown in Figures 14 and 15, respectively. In the first several seconds, the column is in the linear stage, and the variance of displacement is very small, as observed from Figure 13(a). In the uniaxial Kent-Scott-Park concrete material model, the initial Young's modulus of concrete is given as $2f_{c,core}/\varepsilon_{c,core}$, $2f_{c,cover}/\varepsilon_{c,cover}$ for the core and cover concrete, respectively. Therefore, random parameters of $f_{c,\mathrm{core}}$, $\varepsilon_{c,\mathrm{core}}$, $f_{c,\mathrm{cover}}$, $\varepsilon_{c,\mathrm{cover}}$ and E_0 significantly contribute to the variance of the system output response in the first few seconds. The effect of uncertainties in the concrete strengths is more significant than the uncertainties in concrete strains at the maximum strength ($\varepsilon_{c,\mathrm{core}}/\varepsilon_{c,\mathrm{cover}}$), as concluded based on the sensitivity indices shown in Figures 14(a) and (b), and Figures 15(a) and (b). After the displacement reaches its peak value in the nonlinear stage, the uncertainty in the yield strength of the reinforcement f_y plays a more important role in the variance of the structural output response. From this study, it is realized that $f_{c,\mathrm{core}}$, $\varepsilon_{c,\mathrm{core}}$, $f_{c,\mathrm{cover}}$, $\varepsilon_{c,\mathrm{cover}}$, E_0 and f_y have relatively higher sensitivity indices compared with other system parameters on the variance of the output displacement response.

4 Conclusions

Uncertainty analysis gains more and more attention in performance based engineering. This paper mainly investigates the effect of uncertainties in the bridge structures. The uncertainties in frequencies, dynamic linear and nonlinear responses of structures due to the random material and geometric properties of the example structures are quantified. In this paper, the example structures are civil engineering structures only, but the proposed approach is generally applicable to other types of structures. The results obtained from the presented PC based method are compared with those from the widely used MCS and FOSM methods. Results show that the PC based method has a higher accuracy than FOSM and is much more efficient than MCS but still with the same accuracy. The global sensitivity indices are directly obtained from PC coefficients rather than using MCS method, which significantly reduces the computational demand. The comparison of global sensitivity indices calculated from the PC based method and MCS verifies the accuracy of the presented method in structural analysis of relatively large-scale civil engineering structures. The application of using PC expansion for uncertainty quantification and global sensitivity is also extend to a bridge-vehicle interaction response analysis and a probabilistic nonlinear dynamic response analysis. The results demonstrate that the presented PC based method is accurate and efficient compared with MCS, and is more acurrate than FOSM for the uncertainty quantification and reliability analysis. It is noted that the third order PC expansion is used in this study, and the accuracy is expected to be better than FOSM since only the second order moment is considered in using FOSM for uncertainty quantification and reliabiltiy analysis. It is worth noting that the computation time is not significantly increased by using the proposed approach with the third order PC expansion, compared with the simplified FOSM method. However, the accuracy is improved prominently, particularly for the system of high level uncertainties. The number of structural elements will not affect the uncertainty analysis results. However, a ~~higher~~ larger number of elements and consequently random inputs will lead to more unknown PC coefficients. This will increase the computational demand of the uncertainty analysis.

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Reference

[1] J.E. Padgett, R. DesRoches, Sensitivity of seismic response and fragility to parameter uncertainty, Journal of Structural Engineering, 133 (2007) 1710-1718.
[2] A. Saltelli, M. Ratto, T. Andres, F. Campolongo, J. Cariboni, D. Gatelli, M. Saisana, S. Tarantola, Global sensitivity analysis: the primer, John Wiley & Sons, 2008.
[3] B. Sudret, Global sensitivity analysis using polynomial chaos expansions, Reliability Engineering & System Safety, 93 (2008) 964-979.
[4] P. Gustafson, C. Srinivasan, L. Wasserman, Local sensitivity analysis, Bayesian statistics, 5 (1996) 197-210.
[5] K.K. Choi, N.-H. Kim, Structural sensitivity analysis and optimization 1: linear systems, Springer Science & Business Media, 2006.
[6] S.S. Law, P.H. Ni, J. Li, Parallel Decentralized Damage Detection of a Structure with Subsets of Parameters, AIAA Journal, 52 (2014) 650-656.
[7] S. Weng, H.P. Zhu, Y. Xia, L. Mao, Damage detection using the eigenparameter decomposition of substructural flexibility matrix, Mechanical Systems and Signal Processing, 34 (2013) 19-38.
[8] H. Ebrahimian, R. Astroza, J.P. Conte, R.A. de Callafon, Nonlinear finite element model updating for damage identification of civil structures using batch Bayesian estimation, Mechanical Systems and Signal Processing, (2016).
[9] R. Astroza, H. Ebrahimian, Y. Li, J.P. Conte, Bayesian nonlinear structural FE model and seismic input identification for damage assessment of civil structures, Mechanical Systems and Signal Processing, 93 (2017) 661-687.
[10] S. Law, J. Li, Updating the reliability of a concrete bridge structure based on condition assessment with uncertainties, Engineering Structures, 32 (2010) 286-296.
[11] T. Haukaas, A. Der Kiureghian, Finite element reliability and sensitivity methods for performance-based earthquake engineering, Pacific Earthquake Engineering Research Center, College of Engineering, University of California, Berkeley, 2004.
[12] M. Barbato, A. Zona, J.P. Conte, Probabilistic nonlinear response analysis of steel-concrete composite beams, Journal of Structural Engineering, 140 (2013) 04013034.
[13] M. Barbato, Q. Gu, J. Conte, Probabilistic push-over analysis of structural and soil-structure systems, Journal of Structural Engineering, 136 (2010) 1330-1341.
[14] I.M. Sobol, Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates, Mathematics and computers in simulation, 55 (2001) 271-280.
[15] J.-L. Christen, M. Ichchou, B. Troclet, O. Bareille, M. Ouisse, Global sensitivity analysis and uncertainties in SEA models of vibroacoustic systems, Mechanical Systems and Signal Processing, 90 (2017) 365-377.
[16] K. Cheng, Z. Lu, Y. Wei, Y. Shi, Y. Zhou, Mixed kernel function support vector regression for global sensitivity analysis, Mechanical Systems and Signal Processing, 96 (2017) 201-214.
[17] K. Cheng, Z. Lu, Y. Zhou, Y. Shi, Y. Wei, Global sensitivity analysis using Support Vector Regression, Applied Mathematical Modelling, (2017).
[18] H.-P. Wan, W.-X. Ren, Stochastic model updating utilizing Bayesian approach and Gaussian process model, Mechanical Systems and Signal Processing, 70 (2016) 245-268.
[19] H.-P. Wan, M.D. Todd, W.-X. Ren, Statistical Framework for Sensitivity Analysis of Structural Dynamic Characteristics, Journal of Engineering Mechanics, 143 (2017) 04017093.

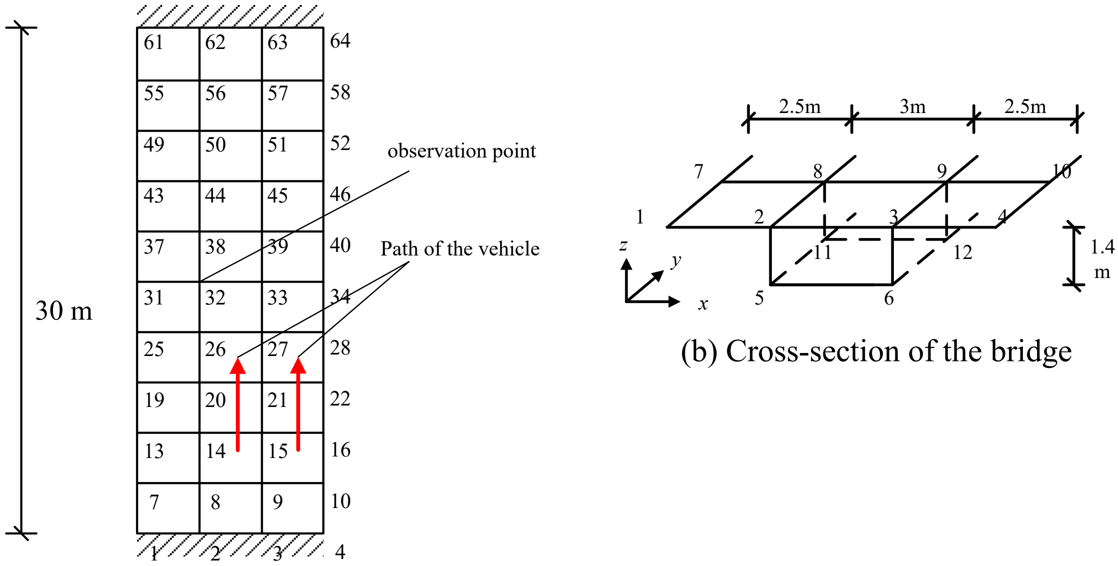
[20] N. Wiener, The Homogeneous Chaos, American Journal of Mathematics, 60 (1938) 897-936.
[21] D. Xiu, G.E. Karniadakis, Modeling uncertainty in flow simulations via generalized polynomial chaos, Journal of Computational Physics, 187 (2003) 137-167.
[22] E.H. Sandoval, F. Anstett-Collin, M. Basset, Sensitivity study of dynamic systems using polynomial chaos, Reliability Engineering & System Safety, 104 (2012) 15-26.
[23] O. Garcia-Cabrejo, A. Valocchi, Global sensitivity analysis for multivariate output using polynomial chaos expansion, Reliability Engineering & System Safety, 126 (2014) 25-36.
[24] G.T. Buzzard, Global sensitivity analysis using sparse grid interpolation and polynomial chaos, Reliability Engineering & System Safety, 107 (2012) 82-89.
[25] H.P. Wan, W.X. Ren, M.D. Todd, An efficient metamodeling approach for uncertainty quantification of complex systems with arbitrary parameter probability distributions, International Journal for Numerical Methods in Engineering, 109 (2017) 739-760.
[26] Q. Zou, L. Deng, C. Jiang, Predicting the bounds of vehicle-induced bridge responses using the interval analysis method, Journal of Bridge Engineering, 21 (2016) 04016046.
[27] Z. Jin, S. Pei, X. Li, S. Qiang, Probabilistic evaluation approach for nonlinear vehicle–bridge dynamic performances, Journal of Sound and Vibration, 339 (2015) 143-156.
[28] Z.-w. Yu, J.-f. Mao, F.-q. Guo, W. Guo, Non-stationary random vibration analysis of a 3D train–bridge system using the probability density evolution method, Journal of Sound and Vibration, 366 (2016) 173-189.
[29] S.Q. Wu, S.S. Law, Evaluating the response statistics of an uncertain bridge–vehicle system, Mechanical Systems and Signal Processing, 27 (2012) 576-589.
[30] T.-P. Chang, Stochastic dynamic finite element analysis of bridge–vehicle system subjected to random material properties and loadings, Applied Mathematics and Computation, 242 (2014) 20-35.
[31] P. Ni, J. Li, H. Hao, Y. Xia, Stochastic dynamic analysis of marine risers considering Gaussian system uncertainties, Journal of Sound and Vibration, 416 (2018) 224-243.
[32] M. Lemaire, Structural reliability, John Wiley & Sons, 2013.
[33] O. Le Maître, O.M. Knio, Spectral methods for uncertainty quantification: with applications to computational fluid dynamics, Springer Science & Business Media, 2010.
[34] S.-K. Choi, R.V. Grandhi, R.A. Canfield, C.L. Pettit, Polynomial chaos expansion with latin hypercube sampling for estimating response variability, AIAA Journal, 42 (2004) 1191-1198.
[35] M.S. Eldred, Recent advances in non-intrusive polynomial chaos and stochastic collocation methods for uncertainty analysis and design, AIAA Paper, 2274 (2009) 37.
[36] J. Li, S. Law, Damage identification of a target substructure with moving load excitation, Mechanical Systems and Signal Processing, 30 (2012) 78-90.
[37] S.Q. Wu, S.S. Law, Dynamic analysis of bridge–vehicle system with uncertainties based on the finite element model, Probabilistic Engineering Mechanics, 25 (2010) 425-432.
[38] K. Mackie, B. Stojadinović, Seismic demands for performance-based design of bridges, Pacific Earthquake Engineering Research Center Berkeley, 2003.
[39] F. McKenna, G. Fenves, M. Scott, Open system for earthquake engineering simulation, University of California, Berkeley, CA, (2000).

Table 1. Random system parameters of a girder bridge model

Parameter (unite)	Probability distribution	Mean value	COV (%)
t_{top} (mm)	lognormal	250	15
t_{web} (mm)	lognormal	500	15
t_{bot} (mm)	lognormal	250	15
E_{top}, E_{web}, E_{bot} (MPa)	normal	2.6×10^4	20
ρ_{top}, ρ_{web}, ρ_{bot} (kg/m^2)	lognormal	2600	20

Table 2. Random system parameters of a two span continuous bridge

Parameter (unite)	Probability distribution	Mean value	COV (%)
$f_{c,\mathrm{core}}$ (ksi)	lognormal	5	20
$\varepsilon_{c,\mathrm{core}}$	lognormal	0.005	20
$f_{c,\mathrm{cover}}$ (ksi)	lognormal	4	20
$\varepsilon_{c,\mathrm{cover}}$	lognormal	0.002	20
$\varepsilon_{cu,\mathrm{cover}}$	lognormal	0.006	20
f_y (ksi)	lognormal	44.6	10
E_0 (ksi)	lognormal	29152	3.3
b	lognormal	0.02	20



(a) Plan view of the box-section girder

(b) Cross-section of the bridge

Figure 1 Finite element model of the girder bridge model

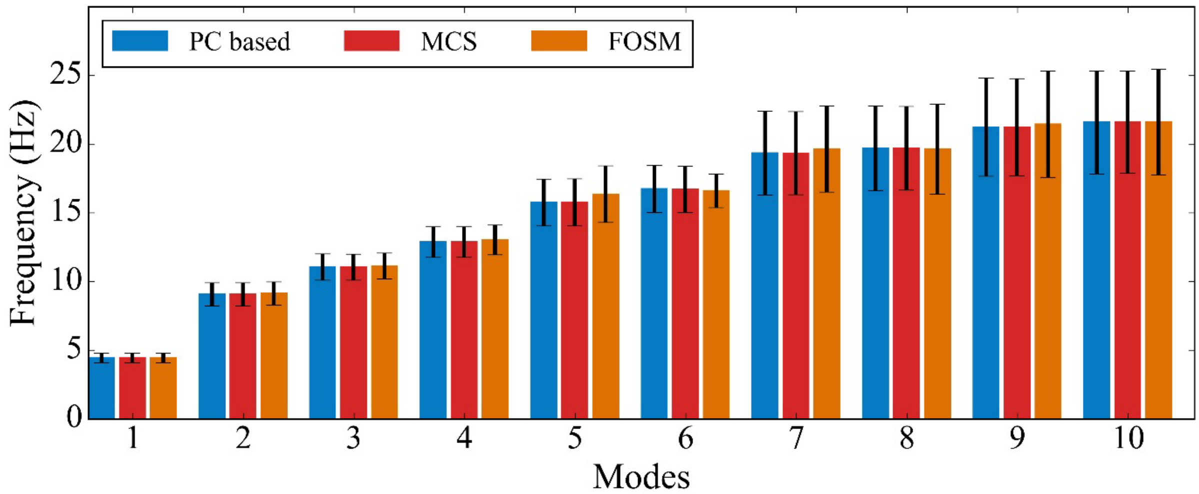
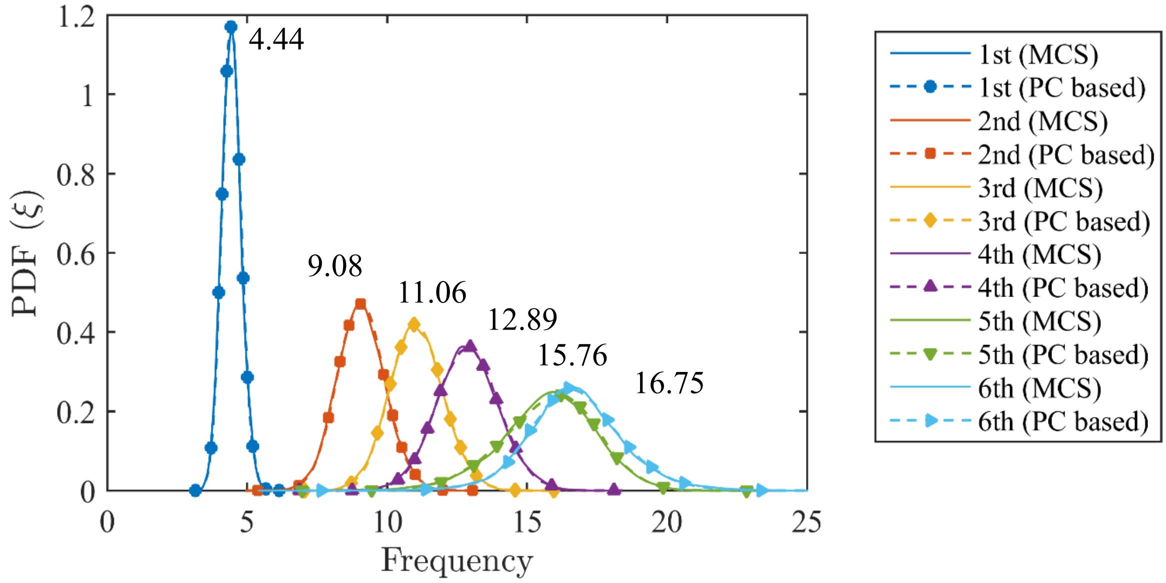
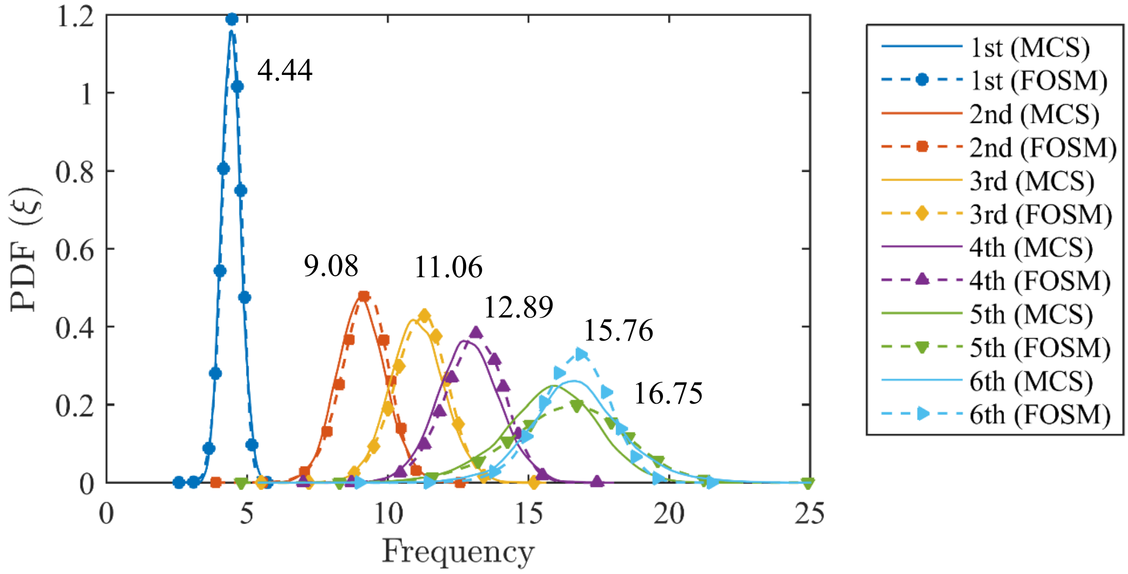


Figure 2 Statistical results in the first ten frequencies



(a) PDF results from MCS and PC based method



(b) PDF results from MCS and FOSM

Figure 3 PDF results of the first six frequencies

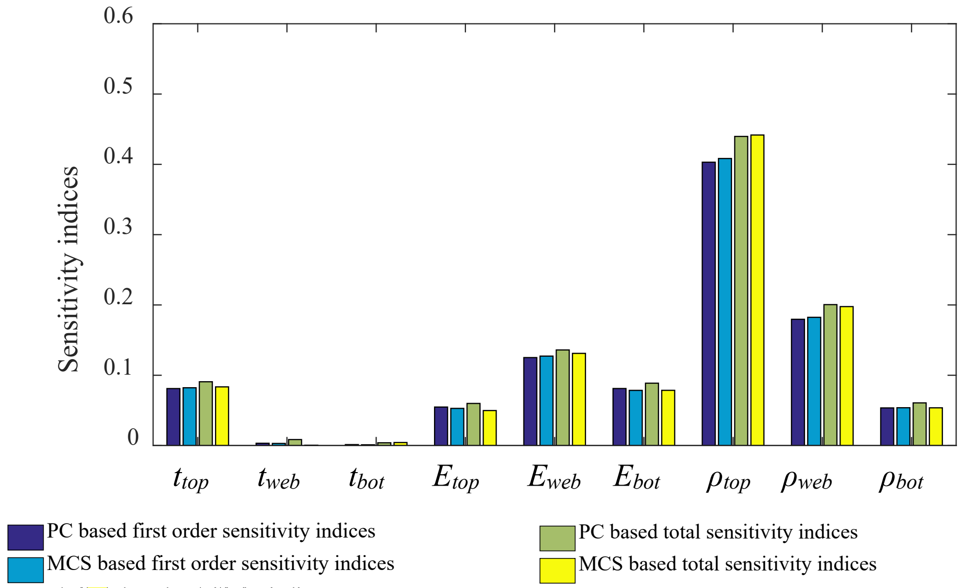


Figure 4 Sensitivity indices of the first frequency with respect to random system parameters

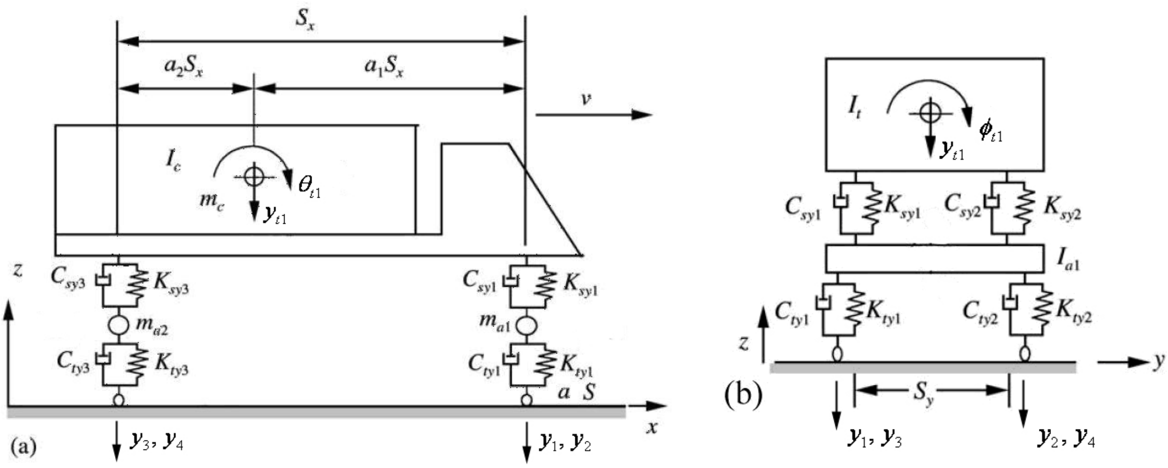
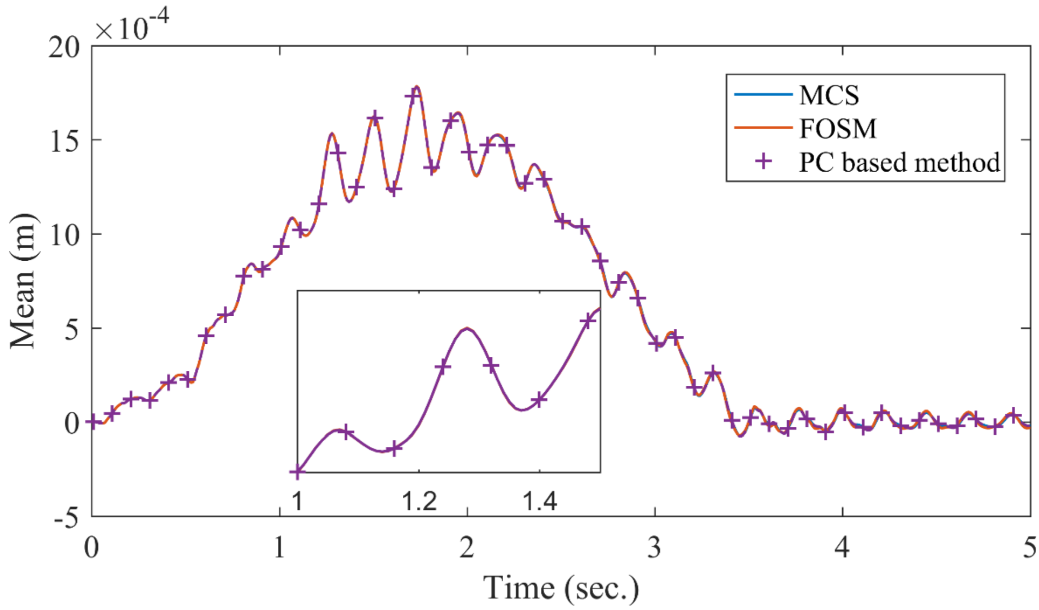
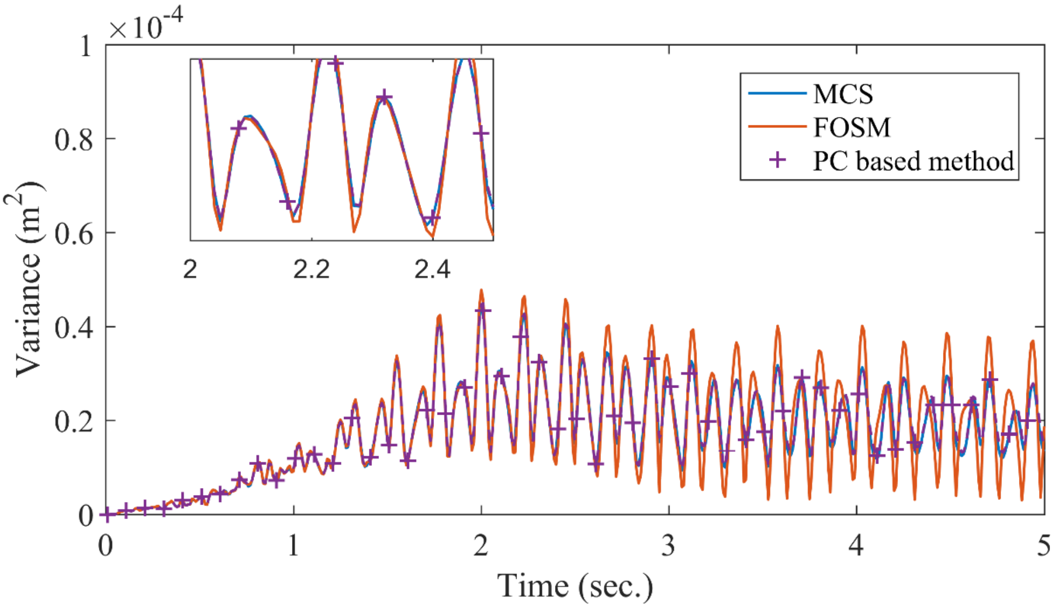


Figure 5 A two-axle three-dimensional vehicle with seven DOFs

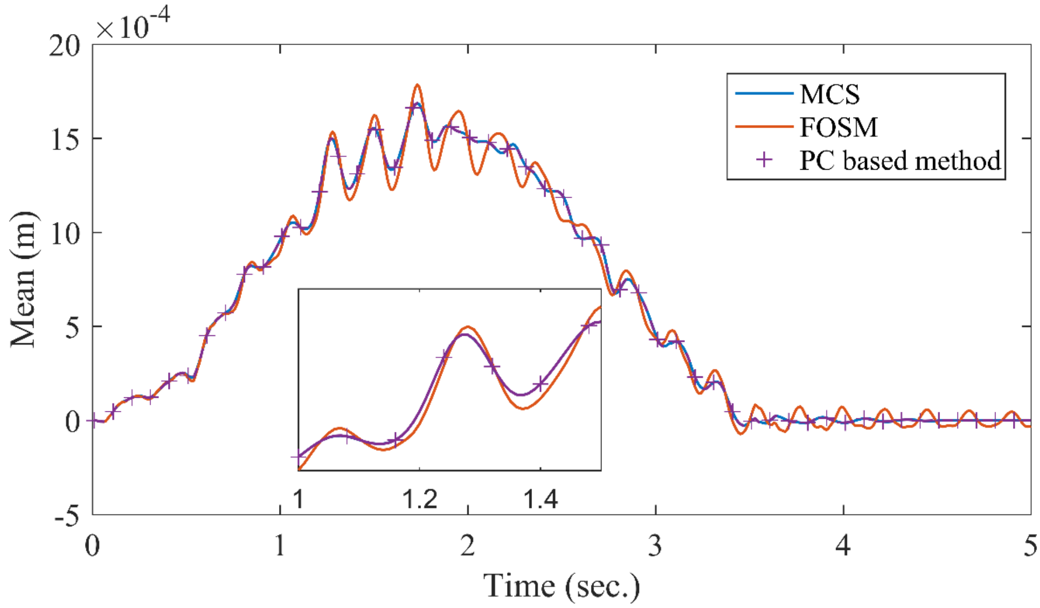


(a) Mean value

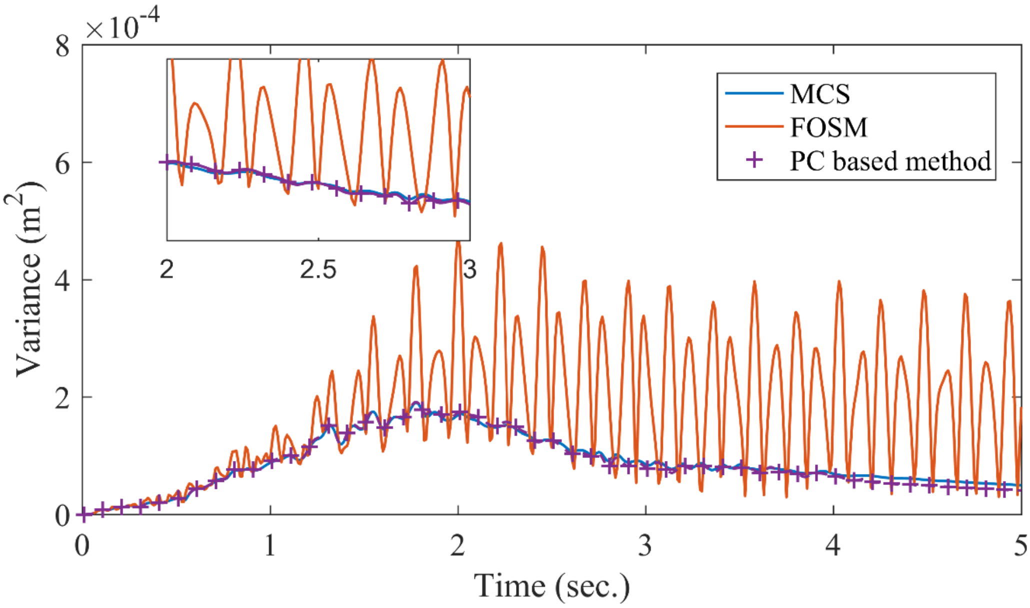


(b) Variance

Figure 6 Statistical response results from MCS, FOSM and PC based method for the low level uncertainty case

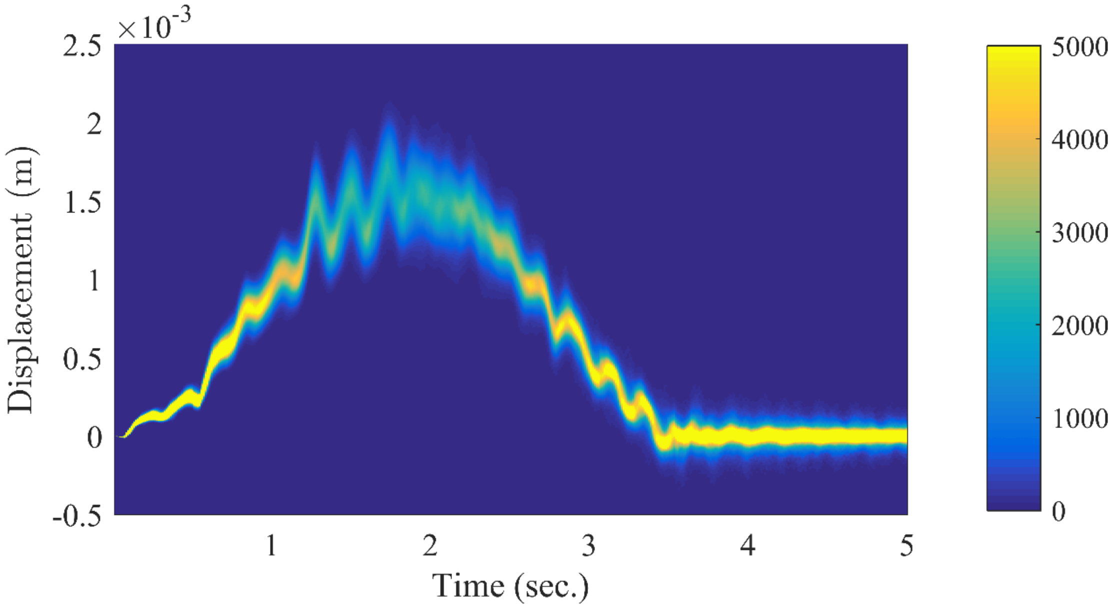


(a) Mean value

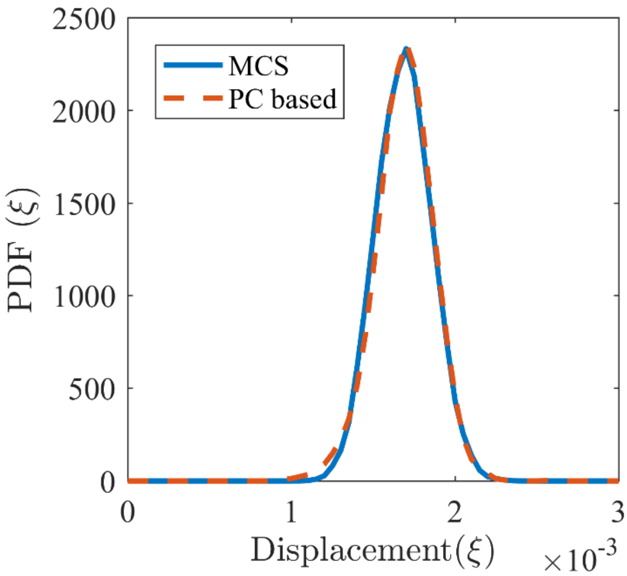


(b) Variance

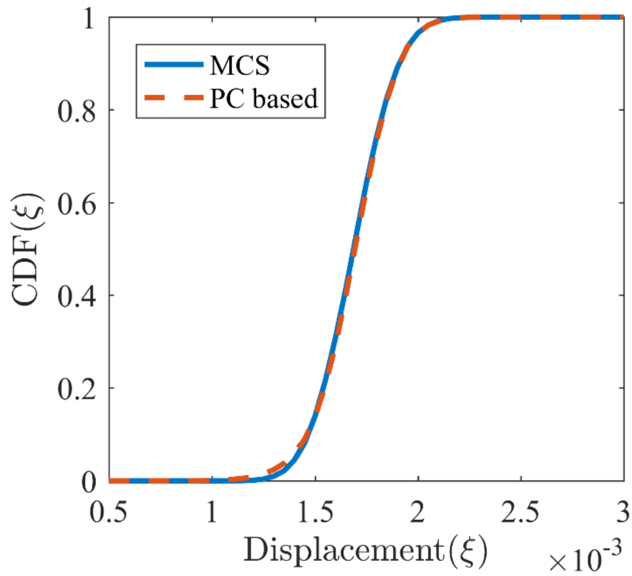
Figure 7 Statistical response results from MCS, FOSM and PC based method for the high level uncertainty case



(a) PDF contour



(b) PDF of the maximun deformation



(c) CDF of the maximun deformation

Figure 8 Probability density evolution of the observation point

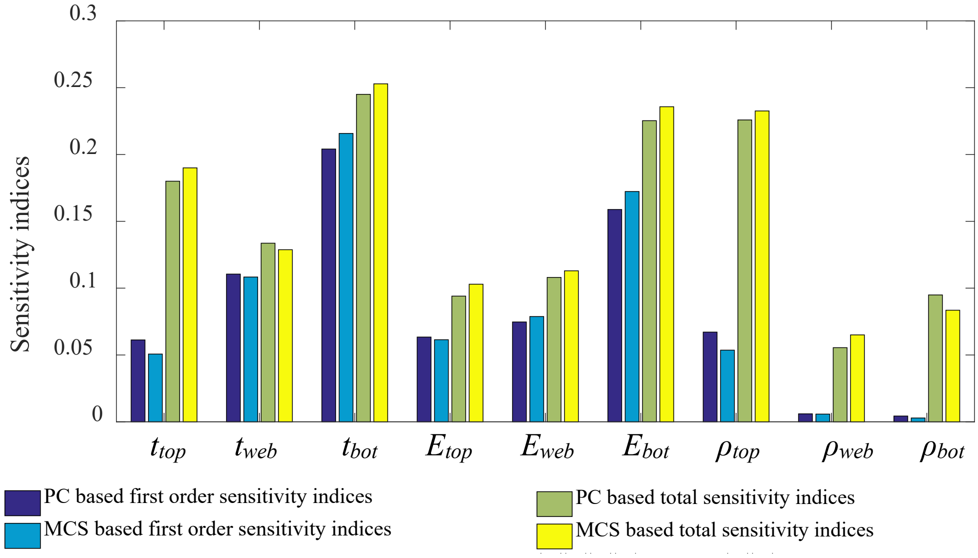
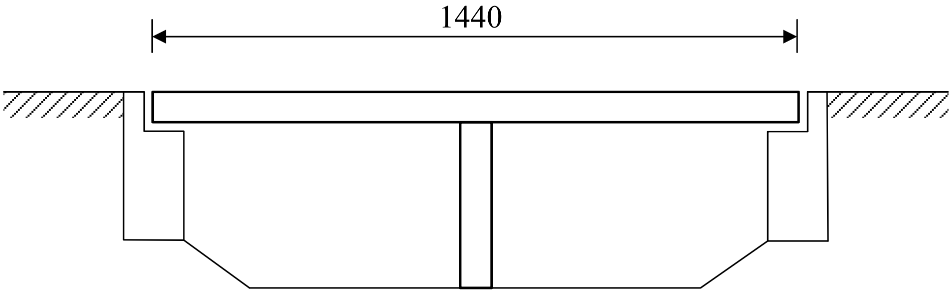
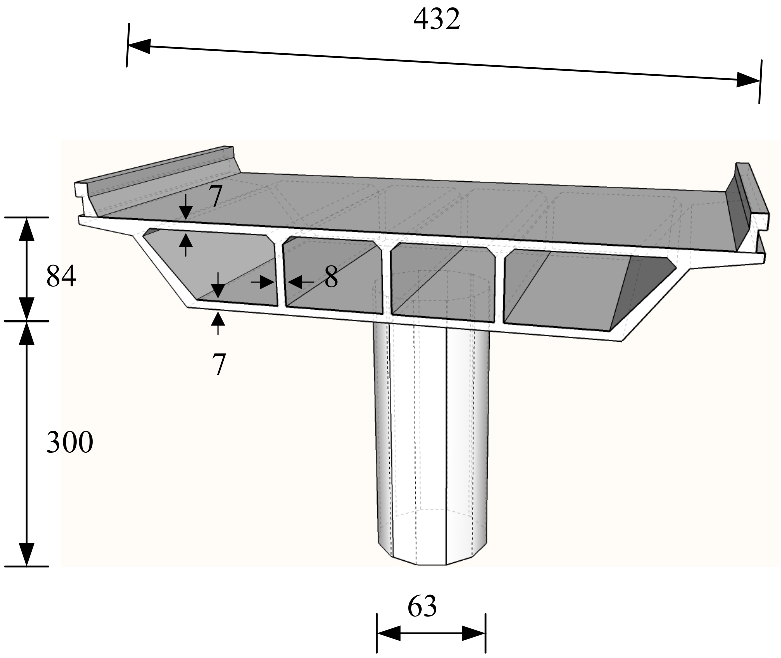


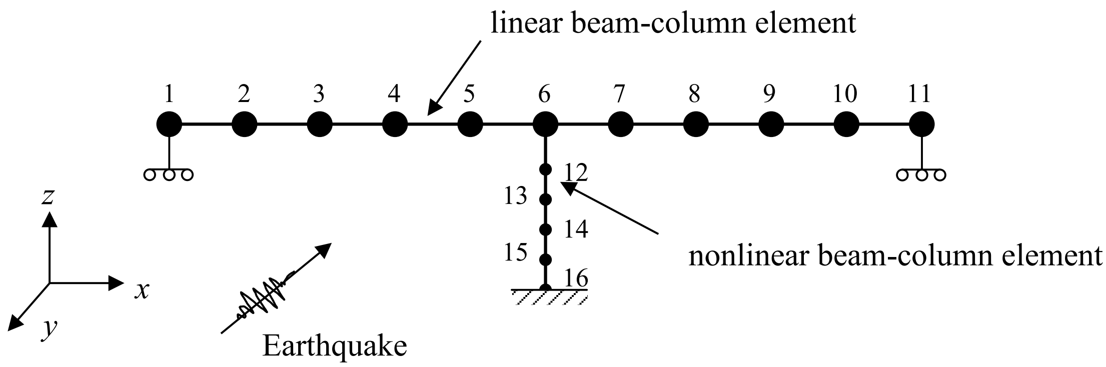
Figure 9 Sensitivity indices of the maximun deformation



(a) The elevation length of the bridge model (unit: in)



(b) Cross section of the bridge deck (unit: in)



(c) Finite element model

Figure 10 The bridge model used in the nonlinear response analysis

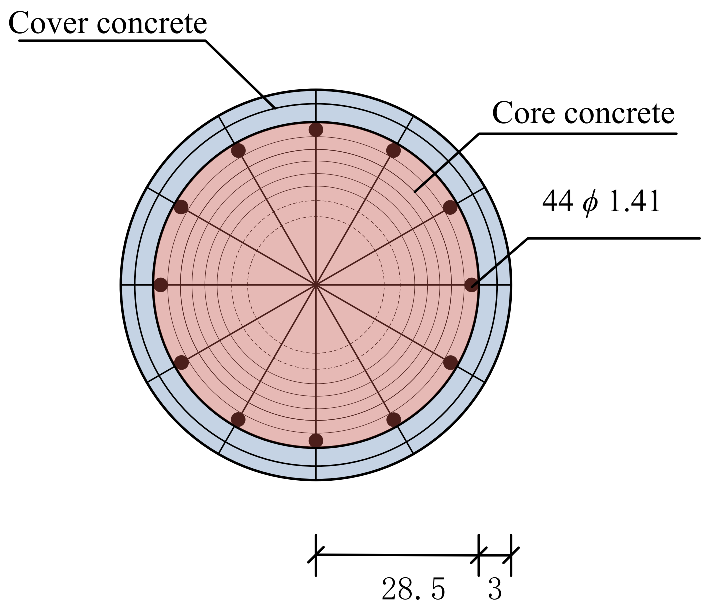
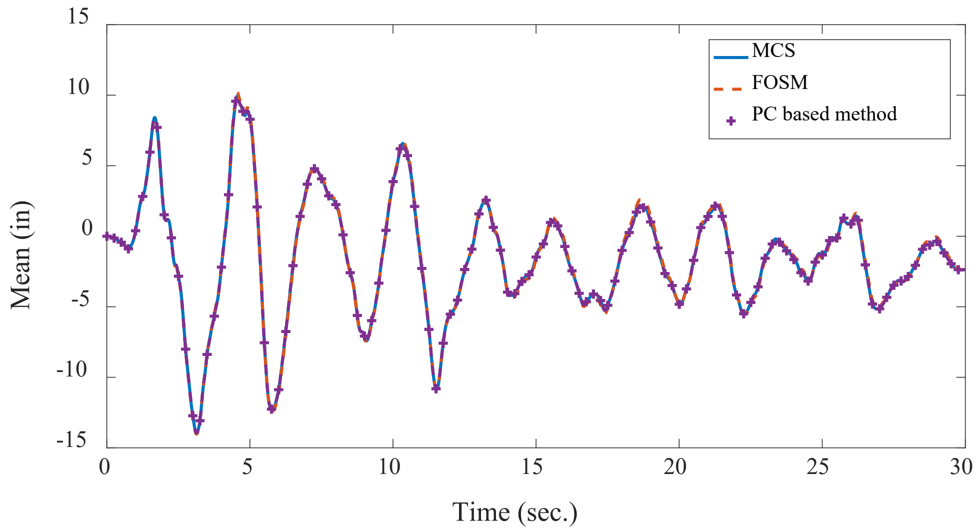
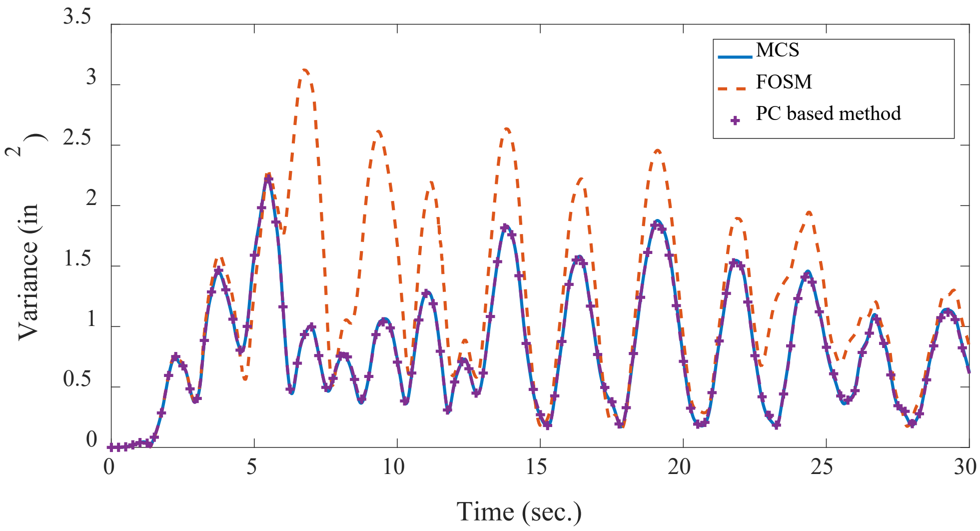


Figure 11 The cross-section of the bridge pier (unit: in)

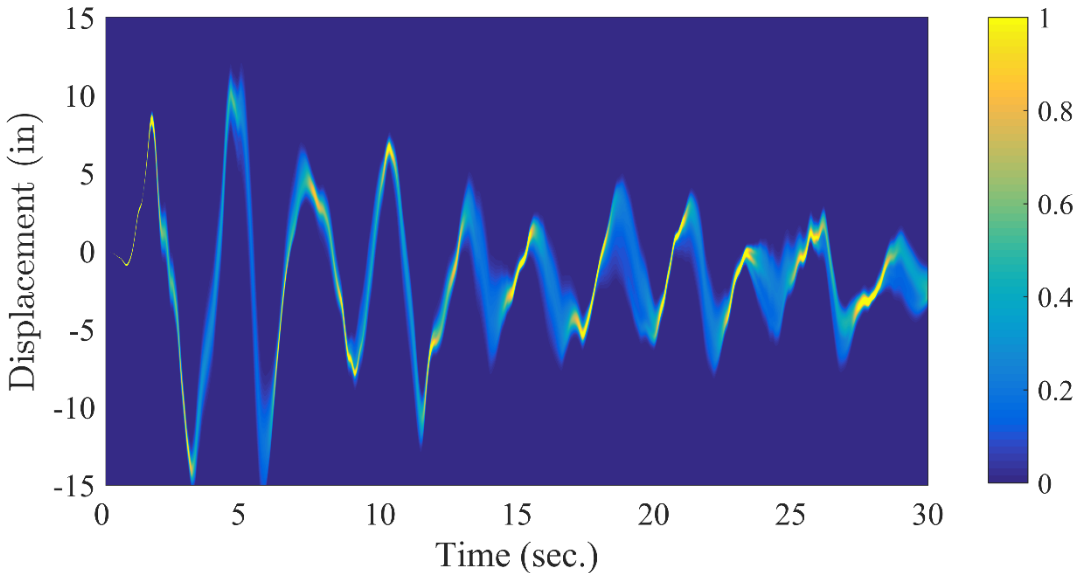


(a) Mean value

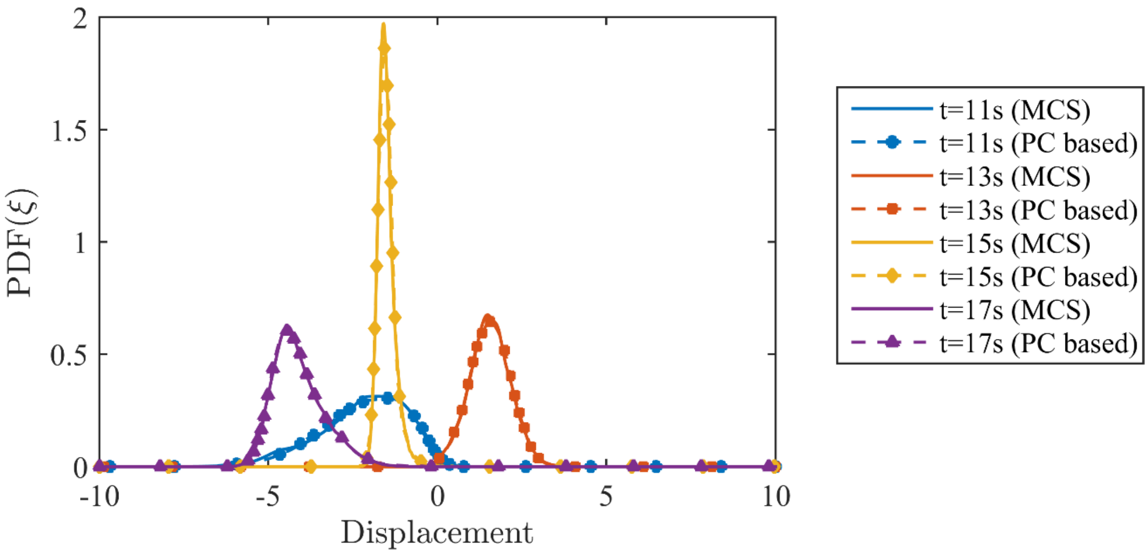


(b) Variance

Figure 12 Statistical displacement response results at Node 6 along *y*-direction

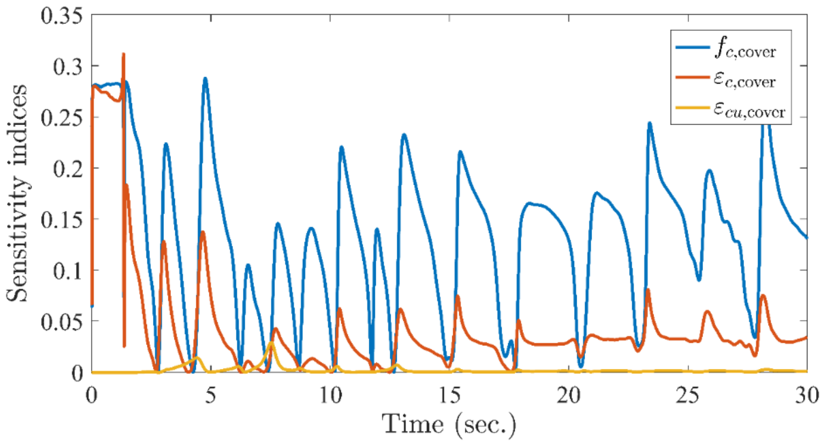


(a) Time history of PDF

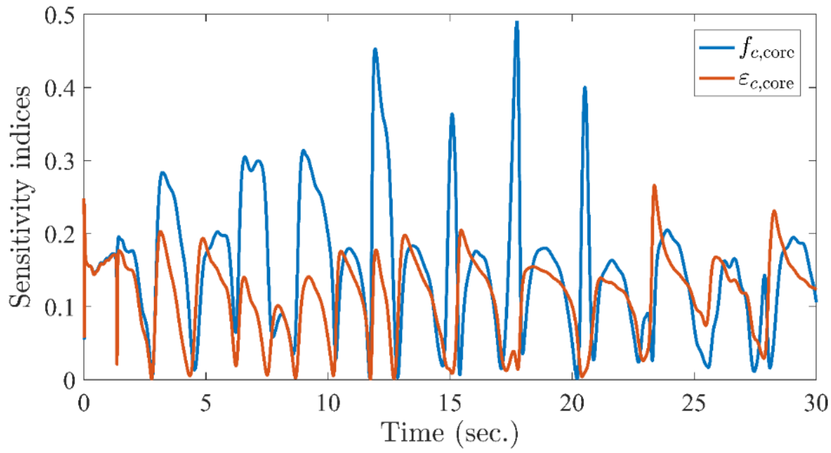


(b) PDF at different time instants

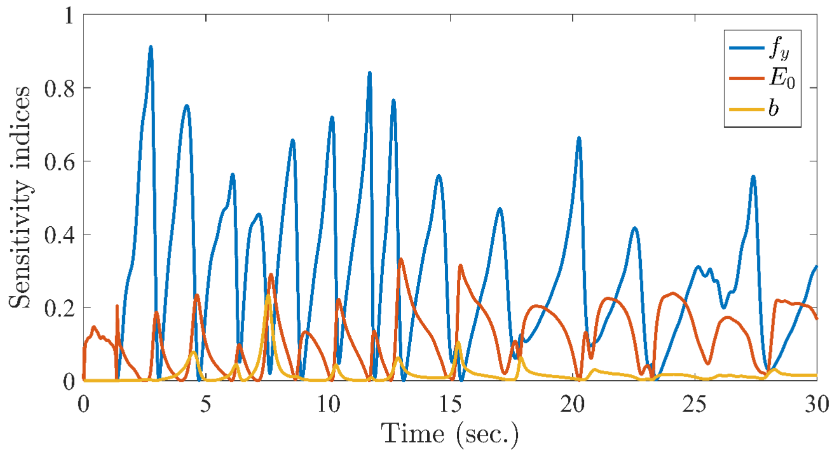
Figure 13 Probability density evolution of the displacement response at Node 6 along *y*-direction



(a) Cover concrete

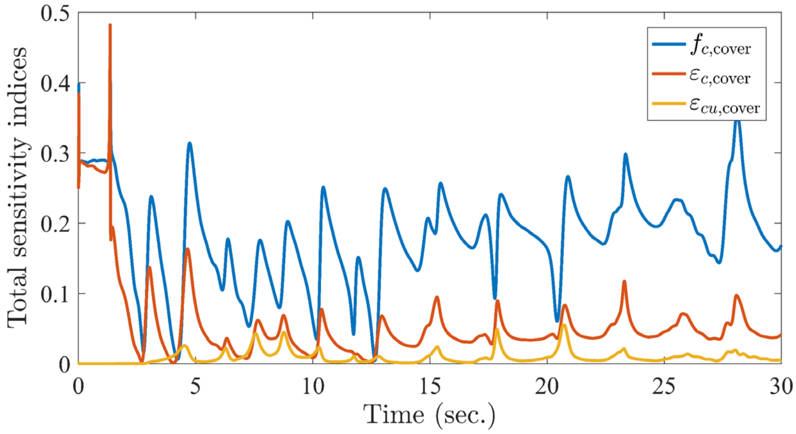


(b) Core concrete

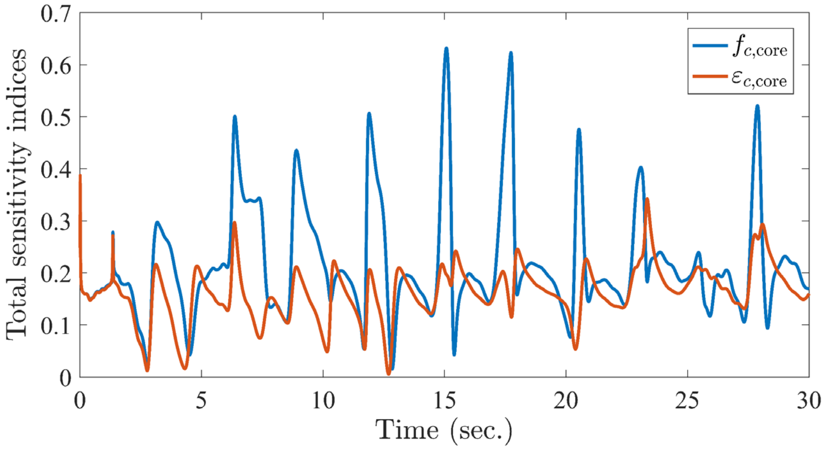


(c) Reinforcement

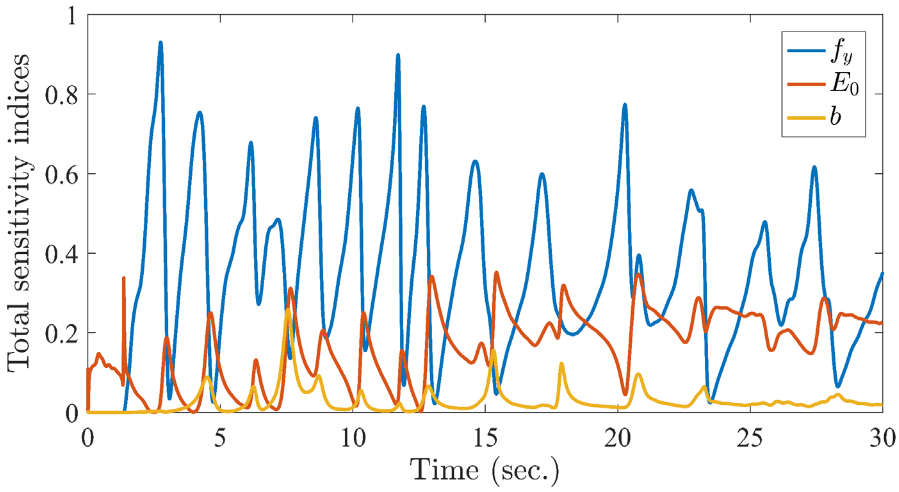
Figure 14 The first order sensitivity indices from PC based method



(a) Cover concrete



(b) Core concrete



(c) Reinforcement

Figure 15 The total sensitivity indices from PC based method